Ex.1 若将y ← uniform(0, 1) 改为 y ← x, 则上 述的算法估计的值是什么?

当y=x时,随机点分散在线段 $y=x, (-1 \le x \le 1)$ 上,当满足 $x^2+y^2 \le 1$ 时,显然当且仅当 $x \in [-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$, 此时若随机选取满足条件的点,那么该算法估算的结果趋近于 $\frac{2}{\sqrt{2}}$

```
import random
import math
tot =1000000
cnt = 0;
for i in range (0,tot):
    x = random.random()
    y = x
    if(x*x+y*y<1) : cnt += 1;
print("估计值:"+str(1.0*cnt/tot))
print("1/sqrt(2):"+str(2/2.0/math.sqrt(2)))
```

```
估计值:0.707156
1/sqrt(2):0.7071067811865475
```

Ex2. 在机器上用 $4\int_0^1 \sqrt{1-x^2}\, dx$ 估计π值，给出不 同的n值及精度。

```
tot = 100.0
for i in range(0,5):
    tot *= 10
    cnt = 0;
    for j in range(0,int(tot)):
        x = random.random()
        y = random.random()
        if(1-x*x>=y*y ) : cnt+=1;
    print("重复"+str(tot)+"次时估计的π为:"+str(4*cnt/tot)+"   相对误差
为:"+str(abs(4*cnt/tot-math.pi)/math.pi))
```

```
重复1000.0次时估计的π为:3.22   相对误差为:0.024957833511806065
重复10000.0次时估计的π为:3.1248   相对误差为:0.005345267652890864
重复100000.0次时估计的π为:3.14208   相对误差为:0.00015512718036502713
重复1000000.0次时估计的π为:3.141604   相对误差为:3.61167454156795e-06
重复10000000.0次时估计的π为:3.1407948   相对误差为:0.00025396468535834096
```

Ex3. 设a, b, c和d是实数，且a ≤ b, c ≤ d, f:[a, b] → [c, d]是一个连续函数，写一概率算法计算积分：

我们不妨设函数为$y = sin(x), a = 0, b = 1, c = 0, d = 1$ 因此其表达式可以表示为:

$$\int_0^1 sin(x)dx$$

```
def func(x,y):
    if(math.sin(x)>=y) : return 1
    return 0

tot = 100000
cnt = 0
for i in range(0,tot):
    x = random.random()
    y = random.random()
    cnt += func(x,y)
print("估算的积分值为:"+str(cnt/tot))
```

```
估算的积分值为:0.45925
```

EX. 用上述算法(setCount)，估计整数子集1~n的大小，并分析n对估计值的影响

```
import numpy as np
def SetCount(X):
    k = 0
    S = {}
    a = np.random.choice(X,size = 1)[0]
    while True :
        k += 1;
        S[a] = 1;
        a = np.random.choice(X,size = 1)[0]
        if(a in S.keys()) : break
    #print(k)
    return k
n = 10
tot=1000
for i in range (0,6):
    n *= 2
    X = []
    for j in range(0,n) : X.append(j)
#     X = random.shuffle(X)
    ans = 0;
    for k in range(0,1):
        cnt = 0;
        for j in range(0,tot):
            cnt += SetCount(X)
        print(cnt/tot)
        cnt /= tot
        cnt = 2*cnt*cnt/math.pi
        ans += cnt/1

    print("当n="+str(n)+"时估计的n为:"+str(ans)+"  相对误差为:"+str(abs(ans-
n)/n))
```

```
5.291
当n=20时估计的n为:17.821967445723057  相对误差为:0.10890162771384712
7.542
当n=40时估计的n为:36.212055649546485  相对误差为:0.09469860876133787
10.885
当n=80时估计的n为:75.42876372887692  相对误差为:0.057140453389038545
15.526
当n=160时估计的n为:153.4614462028058  相对误差为:0.040865961232463735
22.166
当n=320时估计的n为:312.7913833377296  相对误差为:0.022526927069595006
31.599
当n=640时估计的n为:635.6628061623782  相对误差为:0.006776865371284124
```

上面代码实现了SetCount的功能,对于相同的n,进行10组实验,每组实验重复计数1000次,k取1000次的平均值,最后结果为这10组实验平均值,我们不难看出随着n的增大器相对误差也逐渐减小

Ex. 分析dlogRH的工作原理，指出该算法相应的u和v

使用一个随机生成的随机数与a进行运算,保证生成的数据的均匀性,计算这个新的结果通常情况下能保证其平均性能

之后对生成的数据进行拟操作使得还原原先a的结果.

对于dlogRH其对应的u,v如下:

$$u: a \rightarrow [(g^r, mod, p) * a], mod, p, (r \in [0, p-2]) \qquad v: (r+x), mod(p-1) \rightarrow x$$

Ex. 写一Sherwood算法C，与算法A, B, D比较， 给出实验结果。

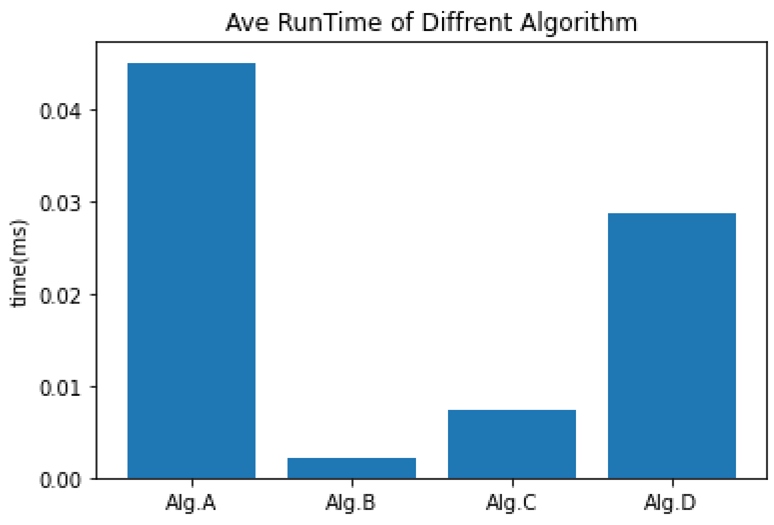
```
## 初始数据生成
import math
import random as rd
def RandData(n,val,ptr):
    pre = []
    val = rd.sample(range(0,n*10),n) # 生成无重复的序列
    val.append(-1)
    val.sort()                      # 排序
    for i in range (0,n):          # 生成指针数组ptr为后继元素 pre为前驱元素
        ptr.append(i+1)
        pre.append(i-1)
    ptr.append(0)
    pre.append(n-1)

    for i in range (0,n) :          # 随机打乱
        while(True) :             # 随机挑选a,b位置元素进行交换
            a = rd.randint(1,n)
            b = rd.randint(1,n)
            if val[a] > val[b] : a,b = b,a
            if(val[ptr[a]]<val[pre[b]]): break  #要求交换元素间距大于3(否则会出
现指针错误)

        pre[ptr[a]],pre[ptr[b]]= pre[ptr[b]],pre[ptr[a]] # 前驱/后继节点指针交
换
        ptr[pre[a]],ptr[pre[b]]=ptr[pre[b]],ptr[pre[a]]

        pre[a],pre[b]=pre[b],pre[a]                      # 节点前驱后继指针交
换
        ptr[a],ptr[b]=ptr[b],ptr[a]

        val[a],val[b]=val[b],val[a]                      # 值交换
    return (val,ptr)
```

```
## 算法实现
def search(x,i,val,ptr):  # 基础算法
    cnt = 0;
    while(x > val[i] and i!=0):
        i=ptr[i]
        cnt += 1
    return (i,cnt);

def A(x,n,val,ptr): #朴素算法A,时间复杂度O(n)
    return search(x,ptr[0],val,ptr)

def B(x,n,val,ptr): #算法B,时间复杂度O(sqrt(n))
    i = ptr[0]
    maxi = val[i]
    sqrt = int(math.sqrt(n))
    for j in range(0,sqrt):
        y = val[j]
        if maxi < y and y<= x:
            i = j
            maxi = y
    return search(x,i,val,ptr)

def C(x,n,val,ptr): #Sherwood算法C,时间复杂度O(sqrt(n))
    i = ptr[0]
    maxi = val[i]
    sqrt = int(math.sqrt(n))
    sample = rd.sample(range(1,n),sqrt)
    for j in sample:
        y = val[j]
```

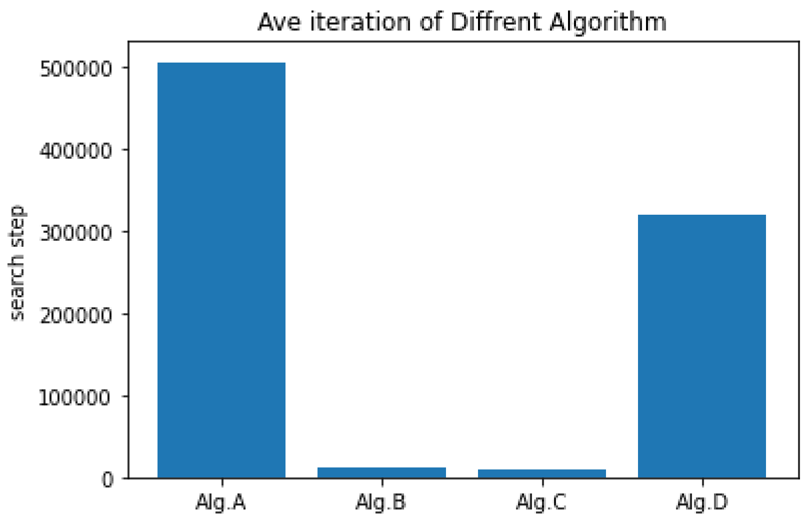
```
            if maxi < y and y<= x:
                i = j
                maxi = y
        return search(x,i,val,ptr)

    def D(x,n,val,ptr): # 朴素算法A优化,时间复杂度O(n/2)
        i = rd.randint(1,n)
        y = val[i]
        if x < y : return search(x,ptr[0],val,ptr)
        elif x > y : return search(x,ptr[i],val,ptr)
        return (i,0)
```

```
## 实验对比
import time
import matplotlib.pyplot as plt

it = [0,0,0,0]  #迭代时间
ave = [0,0,0,0] #搜索次数
init_time = 0
for i in range (0,100): # 测试程序
    val = []
    ptr = []
    n = 10000
    begin = time.perf_counter()
    (val,ptr)=RandData(n,val,ptr)
    end = time.perf_counter()
    init_time += end-begin
    tar = val[rd.randint(1,n)]

    begin = time.perf_counter()
    ave[0] += A(tar,n,val,ptr)[1]
    end = time.perf_counter()
    it[0] += end-begin

    begin = time.perf_counter()
    ave[1] += B(tar,n,val,ptr)[1]
    end = time.perf_counter()
    it[1] += end-begin

    begin = time.perf_counter()
    ave[2] += C(tar,n,val,ptr)[1]
    end = time.perf_counter()
    it[2] += end-begin

    begin = time.perf_counter()
    ave[3] += D(tar,n,val,ptr)[1]
    end = time.perf_counter()
    it[3] += end-begin
```

```
labels = ["Alg.A","Alg.B","Alg.C","Alg.D"]
plt.bar(range(4),it,tick_label=labels)
plt.ylabel('time(ms)')
plt.title("Ave RunTime of Diffrent Algorithm")
plt.show()
print("Ave RunTime of Diffrent Algorithm:"+str(it))

plt.bar(range(4),ave,tick_label=labels)
plt.ylabel('search step')
plt.title("Ave iteration of Diffrent Algorithm")
plt.show()
print("Ave iteration of Diffrent Algorithm:"+str(ave))
```



```
Ave RunTime of Diffrent Algorithm:[0.045119399999999637,
0.002061100000005034, 0.007289000000000101, 0.028673099999994456]
```



```
Ave iteration of Diffrent Algorithm:[505127, 12299, 10478, 319520]
```

Ex.证明：当放置（k+1)th皇后时，若有多个位置是开放的,则算法 QueensLV选中其中任一位置的概率相等

原命题:对于放置（k+1)th皇后第i轮时($1 \le i \le k+1$),算法选择[1,i]中间任意位置概率相等,为$\frac{1}{i}$

归纳基础:对于i=1时显然选择[1,1]概率相等,均为1

归纳假设:假设当i=n时原命题成立

归纳递推:当i=n+1时,首先显然对于第n+1个元素其被选中的概率为$\frac{1}{n+1}$,对于其他元素被选中的概率之和为$\frac{n}{n+1}$,由于当i=n时各个元素被选取的可能性均相等为$\frac{1}{n}$因此对于单个元素,在第n+1轮时其被选中的概率为: $\frac{n}{n+1} * \frac{1}{n} = \frac{1}{n+1}$,因此对于i=n+1时也成立,因此对于所有i<=k+1均有上述结论

Ex. 写一算法，求n=12~20时最优的StepVegas值。

```python
import random as rd
import matplotlib.pyplot as plt
import time
def dfs(depth,col,dia45,dia135,n,ans):
    if(depth == 0) :
        return True,col,dia45,dia135,ans

    for j in range(1,n+1):
        if(j in col or depth+j in dia45 or depth-j in dia135): continue
        col.add(j)
        dia45.add(depth+j)
        dia135.add(depth-j)
        ans.append((depth,j))

        a = dfs(depth-1,col,dia45,dia135,n,ans)
        if a[0] == True : return a
        ans.pop(-1)
        col.remove(j)
        dia45.remove(depth+j)
        dia135.remove(depth-j)
    return False,col,dia45,dia135,ans

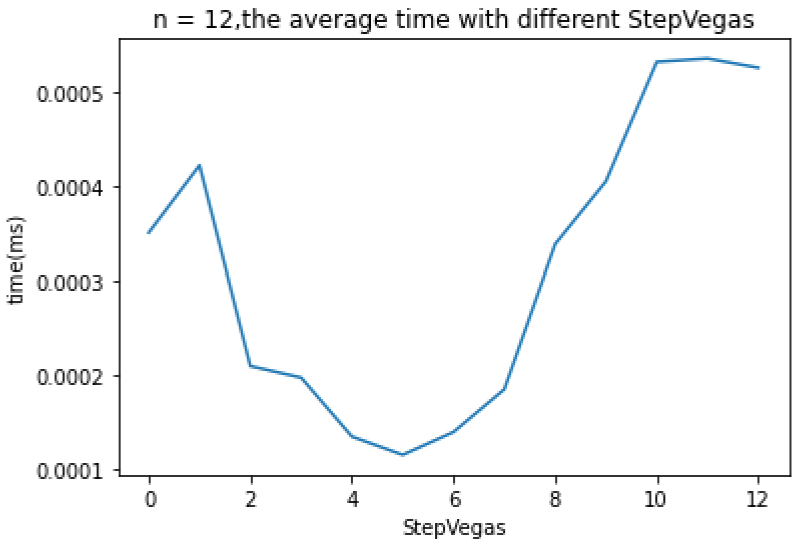
def QueensLv(col,dia45,dia135,n,StepVegas,ans):
    col = set()
    dia45 = set()
    dia135 = set()
    ans = []
    for i in range(n,n-StepVegas,-1):
        nb = []
        for j in range(1,n+1):
            if(j in col or (i+j) in dia45 or (i-j) in dia135) : continue;
            nb.append(j)
        if(len(nb)==0) : return False,col,dia45,dia135,ans;
        j = rd.sample(nb,1)[0]
        col.add(j)
        dia45.add(i+j)
        dia135.add(i-j)
```

```
            ans.append((i,j))

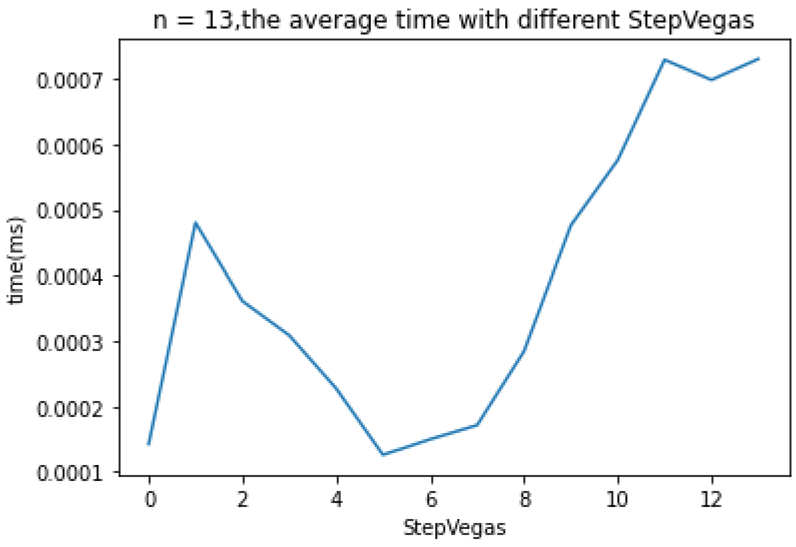
    return (True,col,dia45,dia135,ans)

cntF = cntT = 0
record = []
for n in range(12,20):
    record.append([])
    for setp in range (0,n+1):
        round = 100
        begin = time.perf_counter()
        for i in range(0,round):
            col = set()
            dia45 = set()
            dia135 = set()
            ans = []
            while True:
                while True :
                    a = QueensLv(col,dia45,dia135,n,setp,[])
                    flag,col,dia45,dia135,ans = a
                    if flag == True : break;
                flag,col,dia45,dia135,ans=dfs(n-
setp,col,dia45,dia135,n,ans)
                if flag == False : continue
                break
        end = time.perf_counter()
        record[-1].append((end-begin)/round)
```

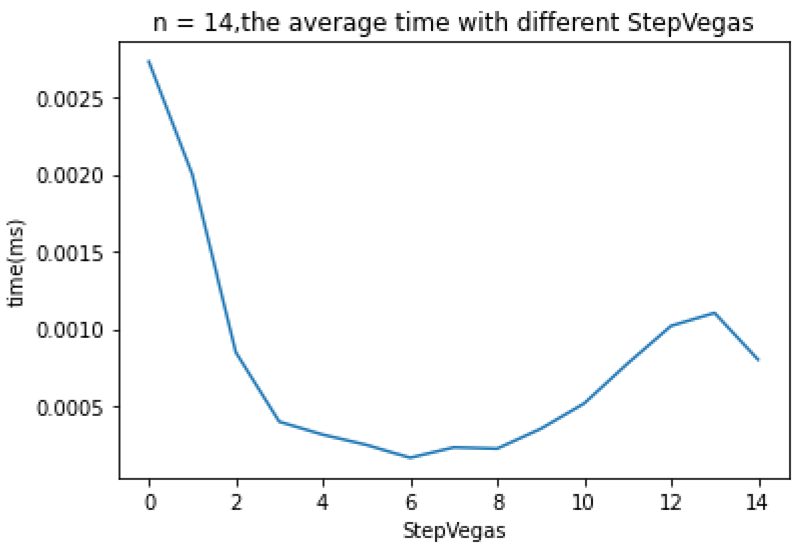
```
import matplotlib.pyplot as plt
for i in record:
    xx = [i for i in range(len(i))]
    plt.title("n = "+str(len(i)-1)+",the average time with different
StepVegas")
    plt.xlabel('StepVegas')
    plt.ylabel('time(ms)')
    plt.plot(xx,i)
    plt.show()
    mini = 0
    for j in range(len(i)):
        if(i[j]<i[mini]): mini = j
    print("When n="+str(len(i))+",the Best SetpVegas:"+str(mini))
```



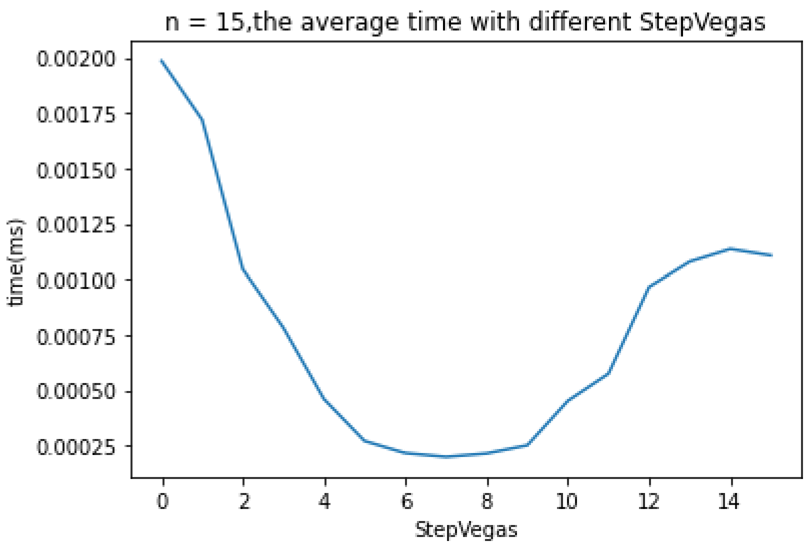
```
When n=13,the Best SetpVegas:5
```



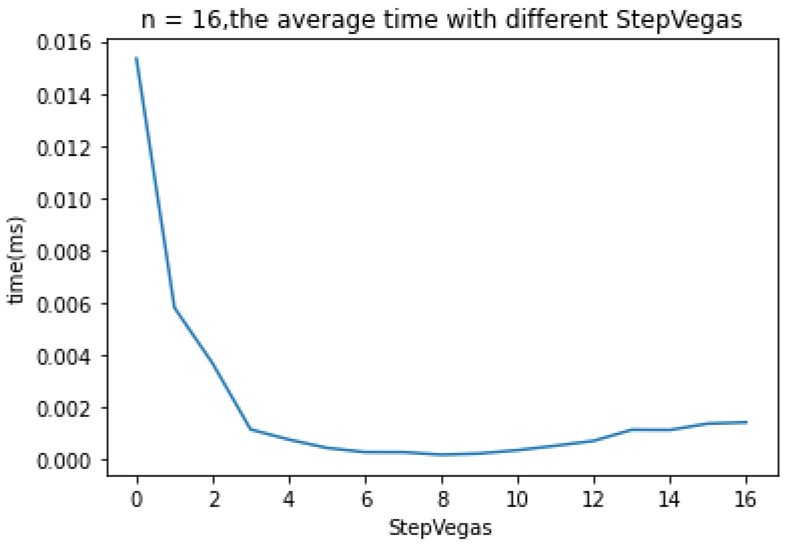
```
When n=14,the Best SetpVegas:5
```



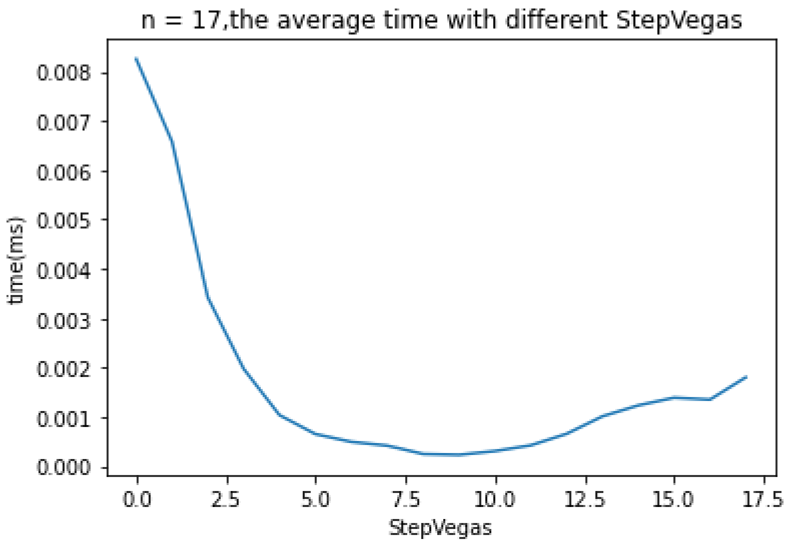
```
When n=15,the Best SetpVegas:6
```



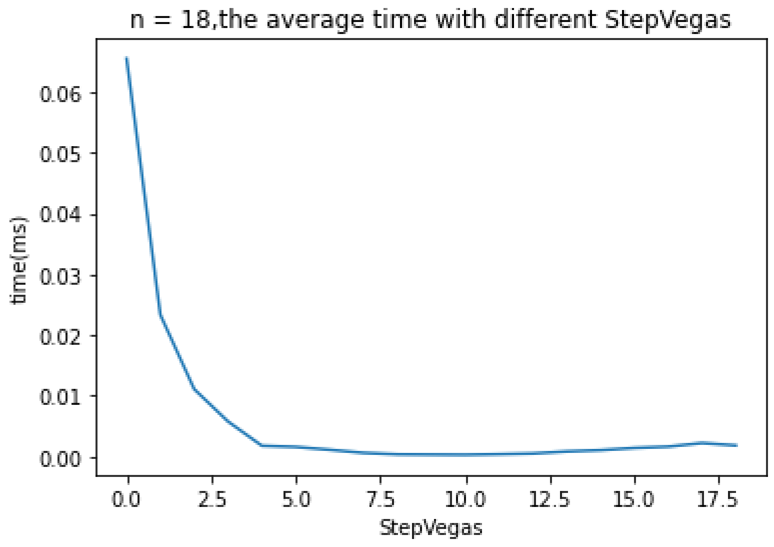
```
When n=16,the Best SetpVegas:7
```



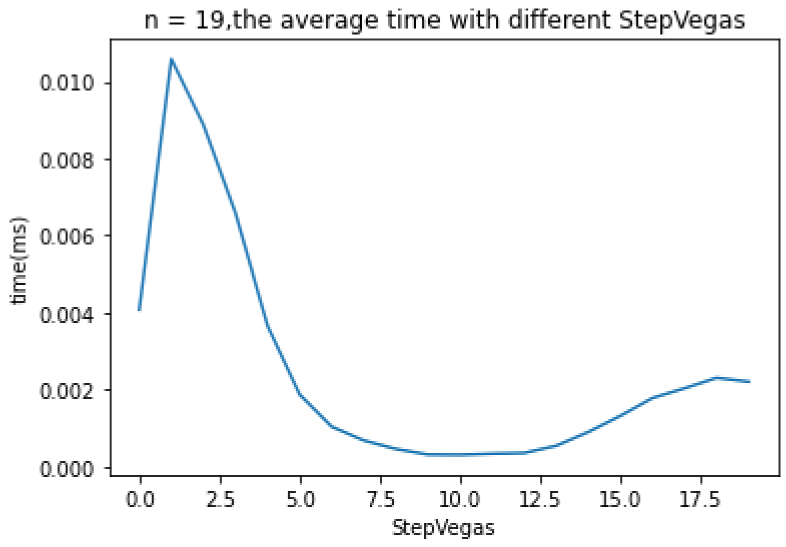
```
When n=17,the Best SetpVegas:8
```



```
When n=18,the Best SetpVegas:9
```



```
When n=19,the Best SetpVegas:10
```



```
When n=20,the Best SetpVegas:10
```

习题: 使用Miller-Rabin并与确定性算法相比较，并给出100~10000以内错误的比例。

```
def Btest(a,n):
    s = 0
    t = n-1
    while True:
        if t %2 == 1 : break
        s += 1
        t /= 2
    x = 1
    for i in range(int(t)):
        x = x*a%n
    if x == 1 or x == n-1 : return True
    for i in range(1,s):
        x = x*x%n
        if x == n-1 : return True
    return False

def MillRab(n):
    a = random.randint(2,n-2)
    return Btest(a,n)

def RepeatMillRab(n,k):
    for i in range(k):
        if MillRab(n) == False: return False
    return True

def printPrimes():
    ans = [2,3]
    for n in range(5,10000,2):
        k = math.floor(math.log(n))
        if RepeatMillRab(n,k) : ans.append(n)
    print(ans)
    return ans

def shai():
    ans = [2,3]
    for n in range(5,10000,2):
        sqrt = math.ceil(math.sqrt(n))
        flag = 0
        for i in ans:
            if i>sqrt : break;
            if n%i == 0 :
                flag = 1
                break
        if flag == 0 : ans.append(n)
    return ans


print("由Miller-Rabin算法求出的指数为:")
ans1 = len(printPrimes());
ans2 = len(shai());
print("Miller-Rabin算法求出的10000内质数共:%d个,实际存在:%d个,错误率为:%f"%
(ans1,ans2,(ans1-ans2)/10000))
```

```
由Miller-Rabin算法求出的指数为:
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,
71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149,
151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229,
233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313,
317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409,
419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499,
503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601,
607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691,
701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809,
811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907,
911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997, 1009,
1013, 1019, 1021, 1031, 1033, 1039, 1049, 1051, 1061, 1063, 1069, 1087,
1091, 1093, 1097, 1103, 1109, 1117, 1123, 1129, 1151, 1153, 1163, 1171,
1181, 1187, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259,
1277, 1279, 1283, 1289, 1291, 1297, 1301, 1303, 1307, 1319, 1321, 1327,
1361, 1367, 1373, 1381, 1399, 1409, 1423, 1427, 1429, 1433, 1439, 1447,
1451, 1453, 1459, 1471, 1481, 1483, 1487, 1489, 1493, 1499, 1511, 1523,
1531, 1543, 1549, 1553, 1559, 1567, 1571, 1579, 1583, 1597, 1601, 1607,
1609, 1613, 1619, 1621, 1627, 1637, 1657, 1663, 1667, 1669, 1693, 1697,
1699, 1709, 1721, 1723, 1733, 1741, 1747, 1753, 1759, 1777, 1783, 1787,
1789, 1801, 1811, 1823, 1831, 1847, 1861, 1867, 1871, 1873, 1877, 1879,
1889, 1901, 1907, 1913, 1931, 1933, 1949, 1951, 1973, 1979, 1987, 1993,
1997, 1999, 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081,
2083, 2087, 2089, 2099, 2111, 2113, 2129, 2131, 2137, 2141, 2143, 2153,
2161, 2179, 2203, 2207, 2213, 2221, 2237, 2239, 2243, 2251, 2267, 2269,
2273, 2281, 2287, 2293, 2297, 2309, 2311, 2333, 2339, 2341, 2347, 2351,
2357, 2371, 2377, 2381, 2383, 2389, 2393, 2399, 2411, 2417, 2423, 2437,
2441, 2447, 2459, 2467, 2473, 2477, 2503, 2521, 2531, 2539, 2543, 2549,
2551, 2557, 2579, 2591, 2593, 2609, 2617, 2621, 2633, 2647, 2657, 2659,
2663, 2671, 2677, 2683, 2687, 2689, 2693, 2699, 2707, 2711, 2713, 2719,
2729, 2731, 2741, 2749, 2753, 2767, 2777, 2789, 2791, 2797, 2801, 2803,
2819, 2833, 2837, 2843, 2851, 2857, 2861, 2879, 2887, 2897, 2903, 2909,
2917, 2927, 2939, 2953, 2957, 2963, 2969, 2971, 2999, 3001, 3011, 3019,
3023, 3037, 3041, 3049, 3061, 3067, 3079, 3083, 3089, 3109, 3119, 3121,
3137, 3163, 3167, 3169, 3181, 3187, 3191, 3203, 3209, 3217, 3221, 3229,
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9941, 9949, 9967, 9973]
Miller-Rabin算法求出的10000内质数共:1229个,实际存在:1229个,错误率为:0.000000
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