Relevant Reasoning:

Its Key Role in Discovery and Prediction

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Relevant Reasoning

An Example [E. D. Mares, 2004]

- A short proof of Fermat's Last Theorem
  - Since 1993, when Andrew Wiles completed his difficult proof of Fermat's Last Theorem, mathematicians have wanted a shorter, easier proof.
  - Suppose when someone addressing a conference of number theorists, suggests the following proof of the theorem:

    The sky is blue

    ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers $x, y, z, x^n = y^n + z^n$.
  - This proof would not be well received. It is a non-sequitur - its conclusion does not, in any intuitive sense, follow from its premise. It is a bad proof.
- The classical validity and soundness of the proof
  - Validity: It is impossible for the premises all to be true and the conclusion false.
  - Soundness: The premise is true.

A Modification of Mares's Example [J. Cheng, 2005]

- Classical valid proofs of Fermat's Last Theorem
  - Taniyama-Shimura conjecture & The sky is blue & …

    ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers $x, y, z, x^n = y^n + z^n$.
  - Taniyama-Shimura conjecture

    ∴ There is no integer n greater than or equal to 3 such that for any non-zero integers $x, y, z, x^n = y^n + z^n$ or The sky is blue or … .
- The classical and weak-relevant validity of the proofs
  - If a proof is valid, then a new proof that is obtained by adding any conjunct into the premises of the proof is still valid.
  - If a proof is valid, then a new proof that is obtained by adding any disjunct into the conclusion of the proof is still valid.

The Propositions This Tutorial Will Show You

- Logic in general
  - Reasoning is intrinsically different from proving.
  - The major role of logic is to underlie valid reasoning rather than to provide a formal language for representation.
  - The notion of conditional is the heart of logic (and hence, mathematics).
  - Any valid reasoning in scientific discoveries as well as our everyday lives should be both truth-preserving and relevant in the sense of conditional.

The Propositions This Tutorial Will Show You

- Relevant reasoning and relevant (relevance) logic
  - Relevant (relevance) logic is the logic for reasoning (and hence, for discovery and prediction) rather than proving; while classical mathematical logic (and hence, its various classical conservative extensions) is the logic for (describing) proving rather than reasoning.
  - Strong relevance in the sense of conditional is indispensable to any valid reasoning for discovery and prediction.
  - You should invoke relevant reasoning, if you really want to discover some new things or predict some future events by reasoning.
  - You indeed did relevant reasoning when you discovered some new things or predicted some future events.

Relevant Reasoning

- Introduction
- Basic Notions: Reasoning, Arguments, Truth, Validity, Proving, Discovery, Prediction, Logic Systems, Formal Theories
- The Notion of Conditional: The Heart of Logic and Mathematics
- Relevant (Relevance) Logic: What Is It and Why Study It?
- Relevant Reasoning Based on Strong Relevant (Relevance) Logic
- Relevant Reasoning in Discovery and Prediction
- Research Directions and Challenging Problems
- Bibliography

Definitions of 'Reasoning' in Dictionaries

- "The process by which one judgment is deduced from another or others which are given." [The Oxford English Dictionary, 2nd Edition]
- "The drawing of inferences or conclusions through the use of reason." [Longman Dictionary of the English Language]
- "The drawing of inferences or conclusions through the use of reason." [Webster's Third New International Dictionary of the English Language]
- "The process of forming conclusions, judgments, or inferences from facts or premises." [The Random House Dictionary of the English Language, 2nd Edition]
- "Use of reason, especially to form conclusions, inferences, or judgments." [The American Heritage Dictionary of the English Language, 3rd Edition]

The Importance of Reasoning Ability

- Reasoning ability as a part of human intelligence
  - The great ability to reason, in particular, to reason conceptually, is the most intrinsic difference between human being and animals.
- The importance of reasoning ability
  - The ability to reason is extremely important in our daily lives because it is the way of getting most of our knowledge.
  - Most of our knowledge is inferential; it is gained not through direct observation, but by inferring one thing from another.
  - The ability to reason is our only way to predict various dangers for avoiding natural or man-made disasters.

Reasoning: What Is It?

- Reasoning as a process of drawing new conclusions
  - *Reasoning* is the *process* of drawing *new conclusions* from given *premises*, which are already known facts or previously assumed hypotheses to provide some *evidence* for the conclusions.
- Notes
  - "process", "new conclusions", "premises", "evidence".
- Reasoning as an ordered process
  - In general, a reasoning consists of a number of *arguments* (or *inferences*) in some *order*.
- Notes
  - "arguments", "inferences", "order".

Reasoning: What Is It?

- Reasoning as a way to acquire new knowledge
  - Reasoning is the process of going from what we do know (the premises) to what we previously did not know (the new conclusions).
- Reasoning as a way to expand known knowledge
  - Reasoning is intrinsically *ampliative*, i.e., it has the function of enlarging or extending some things, or adding to what is already known or assumed.

Reasoning: What Is It?

- An example

(1) All rational number are expressible as a ratio of integers.

(2) π is not expressible as a ratio of integers.

Therefore,

(3) π is not a rational number.

(4) π is a number.

Therefore,

(5) There exists at least one nonrational number.

The Characteristics of Reasoning

- Evidential relation between premises and conclusions
  - The premises of a reasoning are supposed to present evidence for the conclusions of that reasoning.
  - Though the premises of a reasoning are intended to provide some evidence for the conclusions of that reasoning, they need not actually do so.
- New conclusions
  - The conclusions of a reasoning are supposed to be new to the premises of that reasoning.
  - How to define the notion of 'new' formally and satisfactorily is still a difficult philosophical problem until now.

The Characteristics of Reasoning

- The correctness and/or validity of reasoning
  - Good and bad reasonings
  - Correct and incorrect reasonings
  - Valid and invalid reasonings
- Fundamental problems about reasoning
  - What is a good, correct, valid reasoning and how we do it?
  - What is the criterion by which one can decide whether or not the conclusion of a reasoning is really new to the premises of that reasoning?
  - What is the criterion by which one can decide whether or not the premises of a reasoning really provide some evidence for the conclusions of that reasoning?
  - What is an argument (or inference)?

Reasoning and Logic

- Where can we find the solutions to the fundamental problems about reasoning?
  - It is logic that deals with the correctness and/or validity of reasoning in a general theory.
- Reasoning and logic
  - Logic is primarily about inferring, about reasoning; in particular, it is the study of what constitutes correct reasoning.
  - Logic is the study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.

Definitions of 'Inference' in Dictionaries

- "The action or process of inferring; the drawing of a conclusion from known or assumed facts or statements; *esp.* in *Logic*, the forming of a conclusion from data or premisses, either by inductive or deductive methods; reasoning from something known or assumed to something else which follows from it. Also (with *pl.*), a particular act of inferring; the logical form in which this is expressed." [The Oxford English Dictionary, 2nd Edition]
- "The act or an instance of passing from one proposition accepted as true to another whose truth is believed to follow from that of the former." [Longman Dictionary of the English Language]

Definitions of 'Inference' in Dictionaries

- "The act of passing from one or more propositions, statements, or judgments considered as true to another the truth of which is believed to follow from that of the former." [Webster's Third New International Dictionary of the English Language]
- "The process of deriving the strict logical consequences of assumed premises." [The Random House Dictionary of the English Language, 2nd Edition]
- "The act or process of deriving logical conclusions from premises known or assumed to be true." [The American Heritage Dictionary of the English Language, 3rd Edition]

Definitions of 'Argument' in Dictionaries

- "A statement or fact advanced for the purpose of influencing the mind; a reason urged in support of a proposition; *spec.* in *Logic*, the middle term in a syllogism." "A connected series of statements or reasons intended to establish a position (and, hence, to refute the opposite); a process of reasoning; argumentation." "Statement of the reasons for and against a proposition; discussion of a question; debate." [The Oxford English Dictionary, 2nd Edition]
- "A reason given in proof or rebuttal." "The act or process of arguing; argumentation, debate." "A coherent series of reasons offered." [Longman Dictionary of the English Language]

Definitions of 'Argument' in Dictionaries

- "A reason or matter for dispute or contention." [The American Heritage Dictionary of the English Language, 3rd Edition]
- "A course of reasoning aimed at demonstrating truth or falsehood." [The American Heritage Dictionary of the English Language, 3rd Edition]
- "A fact or statement put forth as proof or evidence." [The American Heritage Dictionary of the English Language, 3rd Edition]

Argument: What Is It?

- **Argument as a set of statements**
  - An *argument* is a set of *statements* (or *declarative sentences*) of which one statement is intended as the *conclusion*, and one or more statements, called '*premises*,' are intended to provide some *evidence* for the conclusion.
- **Argument as an evidential relation**
  - An argument is a conclusion standing in relation to its supporting evidence.
  - In an argument, a claim is being made that there is some sort of *evidential relation* between its premises and its conclusion: the conclusion is supposed to *follow from* the premises, or equivalently, the premises are supposed to *entail* the conclusion.

Argument: What Is It?

- **An example (an argument by Sherlock Holmes):**
  - premises
    - This is a large hat.
    - Someone is the owner of this hat.
    - The owners of large hats are people with large heads.
    - People with large heads have large brains.
    - People with large brains are highly intellectual.
  - conclusion
    - The owner of this hat is highly intellectual.

Deduction

- **Definitions of 'deduction' in dictionary [The Oxford English Dictionary, 2nd Edition]**
  - "The process of deducing or drawing a conclusion from a principle already known or assumed; spec. in Logic, inference by reasoning from generals to particulars; opposed to induction."
- **Deductive argument (reasoning)**
  - A *deductive argument* is an argument in which the premises are intended to provide absolute support (evidence) for the conclusion.
  - Ex. : If A then B, A, therefore B.

Induction

- **Definitions of 'induction' in dictionary [The Oxford English Dictionary, 2nd Edition]**
  - "The process of inferring a general law or principle from the observation of particular instances (opposed to deduction, q.v.)."
- **Inductive argument (reasoning)**
  - An *inductive argument* is an argument in which the premises are intended to provide some degree of support (evidence) for the conclusion.
  - Ex. : If A_1 is a B, A_2 is a B, …, A_n is a B, then all A is B.

Abduction

- **Definitions of 'abduction' in dictionary [The Oxford English Dictionary, 2nd Edition]**
  - "A syllogism, of which the major premiss is certain, and the minor only probable, so that the conclusion has only the probability of the minor."
- **Abductive argument (reasoning)**
  - An *abductive argument* is an argument that produces a hypothesis as its end result.
  - "The surprising fact, C, is observed. But if A were true, C would be a matter of course. Hence, there is reason to suspect that A is true." [C. S. Peirce]
  - Ex. : If A then C, C, therefore A.

Deduction, Induction, and Abduction

- "if … then …" in deduction, induction, and abduction
  - If A then B, A, therefore B.
  - If A_1 is a B, A_2 is a B, …, A_n is a B, then all A are B.
  - If A then C, C, therefore A.
- The logic of scientific discovery : three major approaches
  - The hypothetico-deductive account [Popper, Hempel]
  - The inductive-probability account [Reichenbach, Salmon]
  - The abductive inference account [Peirce, Hanson]

The Characteristics of Argument

- The evidential relation between premises and conclusion
  - The premises of an argument are supposed to present evidence for the conclusion of that argument.
  - Though the premises of an argument are intended to provide some evidence for the conclusion of that argument, they need not actually do so.
- The correctness of arguments
  - The correctness of an argument is a matter of the connection between its premises and its conclusion, and concerns the strength of the evidential relation between them.

The Characteristics of Argument

- The correctness and/or validity of arguments
  - Good and bad arguments
  - Correct and incorrect arguments
  - Valid and invalid arguments
- Fundamental problems about arguments
  - What is a correct argument and how we do it?
  - What is the criterion by which one can decide whether or not the conclusion of an argument really does follow from its premises?
  - Is there the only one criterion, or are there many criteria?
  - If there are many criteria, what are the intrinsic differences between them?
  - What is a statement (declarative sentence)?

Arguments and Logic

- Where can we find the solutions to the fundamental problems about argument?
  - It is logic that deals with the correctness and/or validity of argument in a general theory.
- Argument and logic
  - Logic may be defined as the organized body of knowledge, or science, that evaluates arguments.
  - The purpose of logic, as the science that evaluates arguments, is thus to develop methods and techniques that allow us to distinguish good arguments from bad.

Statement (Declarative Sentence)

- Statement
  - A ***statement*** is an assertion that is either ***true*** or ***false*** and is typically expressed by a declarative sentence.
- Declarative sentence
  - A ***declarative sentence*** is a grammatically correct sentence that can be put in place of '…' in the sentence 'Is it true that …?' with the effect that the resulting sentence is a grammatically correct question.
  - A declarative sentence typically expresses an assertion that is either ***true*** or ***false***, as opposed to questions, commands, or exclamations.
- Examples
  - "Mary is a student" is a declarative sentence, but "How pretty Mary is!" is not a declarative sentence.

Truth and Validity

- Truth of a statement
  - ***Truth*** is the attribute of a statement that asserts what really is the case.
  - Truth and falsity are attributes of individual statements.
- Validity of a deductive argument
  - When a deductive argument makes the claim that its premises (if correct) provide irrefutable grounds for the correctness of its conclusion, that claim will be either correct or not correct. If it is correct, that argument is said to be '***valid***.'
  - ***Validity*** and ***invalidity*** are attributes of deductive arguments.

Validity of Deductive Arguments

- Validity of deductive arguments
  - A deductive argument is *valid* if and only if it is necessary that its conclusion *follows from* its premises, or, equivalently, its premises *entail* its conclusion.
- Notes
  - To define the notion of validity of deductive arguments formally is nothing more than to define 'follows from' and/ or 'entail' formally.
  - According to the definition of 'follows from' and/or 'entail,' there may be different definitions of validity which may lead to different logic systems.

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The Classical Account of Validity

- The *classical account of validity* (CAV)
  - A deductive argument is valid IF AND ONLY IF it is impossible for all its premises to be true while its conclusion is false.
- Notes
  - What 'true' and 'false' mean have to be defined.
  - "if and only if": The CAV is defined as a sufficient and necessary condition.
  - The truth of the conclusion of an argument is by itself sufficient for the classical validity of that argument.
  - The CAV says nothing about how the premises are relevant to the conclusion.

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The Classical Account of Validity

- Fundamental questions about the CAV
  - What is 'true' and what is 'false'?
  - How can we define 'true' and 'false' formally?
  - Is the CAV satisfactory to the validity of deductive arguments in scientific reasoning as well as our everyday reasoning?
  - Is the CAV a primitive, absolute first-principle or assumption of logic?

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Classically Valid and Invalid Arguments

- Classical validity of deductive arguments
  - A deductive argument is *classically valid* if and only if it is necessary that if all its premises are true, then its conclusion is true
  - A deductive argument is *classically valid* if and only if it is impossible for all the premises to be true while the conclusion is false.
- Classical invalidity of deductive arguments
  - A deductive argument is *classically invalid* if and only if its conclusion may be false when all its premises are true.

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Classically Valid and Invalid Arguments: Examples

- Example 1
  - { (1) PRICAI 2004 is held in Auckland,
(2) Auckland is in New Zealand }
|- (3) PRICAI 2004 is held in New Zealand
  - Note: (1), (2), (3) are true (OK by CAV)
- Example 2
  - { (1) PRICAI 2004 is held in Auckland,
(2) Auckland is in New Zealand }
|- (3) PRICAI 2004 is held in China
  - Note: (1), (2) are true, but (3) is false (not OK by CAV)

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Classically Valid and Invalid Arguments: Examples

- Example 3
  - { (1) PRICAI 2004 is held in Auckland,
(2) Auckland is in China }
|- (3) PRICAI 2004 is held in New Zealand
  - Note: (1), (3) are true, but (2) is false (OK by CAV)
- Example 4
  - { (1) PRICAI 2004 is held in Auckland,
(2) Auckland is in China }
|- (3) PRICAI 2004 is held in Japan
  - Note: (1) is true, but (2), (3) are false (OK by CAV)

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Classically Valid and Invalid Arguments: Examples

- Example 5
  - { (1) PRICAI 2004 is held in Auckland, (2) Auckland is in New Zealand, (3) Auckland is NOT in New Zealand } |- (4) PRICAI 2004 is held in New Zealand
  - Note: (1), (2), (4) are true, but (3) is false; (2) and (3) is a contradiction (OK by CAV)
- Example 6
  - { (1) PRICAI 2004 is held in Auckland, (2) Auckland is in New Zealand, (3) Auckland is NOT in New Zealand } |- (4) PRICAI 2004 is held in China
  - Note: (1), (2) are true, but (3), (4) are false; (2) and (3) is a contradiction (OK by CAV)

Classically Valid and Invalid Arguments: Examples

- Example 7
  - { (1) PRICAI 2004 is held in Auckland, (2) Auckland is in New Zealand, (3) 1 + 1 = 2 } |- (4) PRICAI 2004 is held in New Zealand
  - Note: (1), (2), (3), (4) are true; (3) is irrelevant to (4) (OK by CAV)
- Example 8
  - { (1) PRICAI 2004 is held in Auckland, (2) Auckland is in China, (3) 1 + 1 = 3 } |- (4) PRICAI 2004 is held in New Zealand
  - Note: (1), (4) are true, but (2), (3) are false; (3) is irrelevant to (4) (OK by CAV)

Consistent and Inconsistent Arguments

- *Consistency* of arguments
  - An argument is *consistent* if and only if its conclusion may be true when all its premises are true.
- *Inconsistency* of arguments
  - An argument is *inconsistent* if and only if its conclusion must be false when all its premises are true.
- Notes
  - Classically invalid arguments may be either consistent or inconsistent.
  - Consistent arguments may be either classically valid or classically invalid.

Sound and Unsound Arguments

- *Soundness* of arguments
  - An argument is *sound* if and only if it is valid and all its premises are true.
- *Unsoundness* of arguments
  - An argument is *unsound* if and only if it is not sound, i.e., either it is invalid or one of its premises is false.
- Notes
  - Sound arguments must be valid; invalid arguments are unsound.
  - Valid arguments may be either sound or unsound.

The Relevant Account of Validity

- Relevance in arguments
  - For any correct argument in scientific reasoning as well as our everyday reasoning, its premises must somehow be *relevant* to its conclusion, and vice versa.
  - In general, we do not accept irrelevant statements as premises and/or conclusion in a correct argument.
- The *relevant account of validity* (RAV)
  - For an argument to be valid THERE MUST BE some *connection* of meaning, i.e., some *relevance*, between its premises and its conclusion.

The Relevant Account of Validity

- Notes
  - What the terms 'relevant' and 'relevance' mean have to be defined.
  - "must be": The RAV is defined as a necessary condition.
  - The relevance between the premises and conclusion of an argument is by itself insufficient for the 'validity' of that argument.
  - The RAV says nothing about truth of the premises and truth of the conclusion.

Fundamental Problems about the RAV

- **Fundamental problems about the RAV**
  - What is ‘relevance’?
  - How can we define ‘relevance’ formally?
  - Is the RAV satisfactory to the validity of deductive arguments in scientific reasoning as well as our everyday reasoning?
  - Is the RAV is a primitive, absolute first-principle or assumption of logic?

Relevant and Irrelevant Arguments

- ***Relevance* of arguments**
  - An argument is ***relevant*** if and only if its conclusion is relevant in some way to its premises.
- ***Irrelevance* of arguments**
  - An argument is ***irrelevant*** if and only if its conclusion is not relevant at all in any way to its premises.
- **Notes**
  - Classically invalid arguments may be either relevant or irrelevant.
  - Relevant arguments may be either classically valid or classically invalid.

Logically Valid (Deductive) Reasoning

- **Logically valid (deductive) reasoning**
  - A ***logically valid reasoning*** is a reasoning such that its arguments are justified based on some ***logical validity criterion*** provided by a logic system in order to obtain correct conclusions.
- **Notes**
  - A reasoning may be valid on a logical validity criterion but invalid on another.
  - The term ‘correct’ does not mean ‘true’; a logically valid reasoning with some false premise may draw a false conclusion.

Logically Valid (Deductive) Reasoning

- **Fundamental problems about logical validity criteria**
  - What should we consider in establishing a logical validity?
  - How can we establish various logical validity criteria to underlie various reasoning?
  - How can we formalize a logical validity criterion?

Truth-preserving Reasoning

- ***Truth-preserving reasoning***
  - For any correct argument in a deductive reasoning, the conclusion of the argument must be ***true*** if all premises of that argument are ***true***.
- **Notes**
  - What the term ‘true’ means has to be defined.
  - The above ‘truth-preserving reasoning’ says only about truth of premises and conclusion but does not take other things into account.

Fundamental Problems about Truth-preserving Reasoning

- **Fundamental problems about truth-preserving reasoning**
  - How can we define truth formally?
  - How can we formalize the classical account of validity?
  - Is the classical account of validity both sufficient and necessary to scientific reasoning as well as our everyday reasoning?
  - Is the classical account of validity satisfactory to scientific reasoning as well as our everyday reasoning?
- **Where can we find the solutions?**
  - Note: The classical account of validity is NOT a primitive, absolute first-principle or assumption of logic.
  - It is logic that deals with the validity of argument and reasoning in a general theory.

Relevant Reasoning

- *Relevant reasoning*
  - For any correct argument in scientific reasoning as well as our everyday reasoning, its premises must be *relevant* to its conclusion, and vice versa.
- Notes
  - What the term 'relevant' means has to be defined.
  - The above 'relevant reasoning' says only about relevance between premises and conclusion but does not take other things into account.

Fundamental Problems about Relevant Reasoning

- Fundamental problems about relevant reasoning
  - How can we define relevance formally?
  - How can we formalize the relevant account of validity?
  - Is the relevant account of validity both sufficient and necessary to scientific reasoning as well as our everyday reasoning?
  - Is the relevant account of validity satisfactory to scientific reasoning as well as our everyday reasoning?
- Where can we find the solutions?
  - Note: The relevant account of validity is NOT a primitive, absolute first-principle or assumption of logic.
  - It is logic that deals with the validity of argument and reasoning in a general theory.

Paraconsistent Reasoning

- *Paraconsistent reasoning* (*Reasoning under inconsistency, Reasoning with inconsistent knowledge*)
  - For any correct argument in scientific reasoning as well as our everyday reasoning, the conclusion of the argument MAY NOT BE an arbitrary sentence when (even if) the premises of that argument is inconsistent.
- Note
  - Reasoning with inconsistent knowledge is the rule rather than the exception in our everyday lives and all scientific disciplines.
- Fundamental problems about paraconsistent reasoning
  - What may be the conclusion of inconsistent premises?
  - How can we establish a formal logical validity criterion to underlie paraconsistent reasoning?

Paracomplete Reasoning

- *Paracomplete reasoning* (*Reasoning under uncertainty, Reasoning with incomplete knowledge*)
  - For any correct argument in scientific reasoning as well as our everyday reasoning, the conclusion of the argument MAY NOT BE the negation of a sentence when (even if) the sentence is not a conclusion of the premises of that argument.
- Note
  - Reasoning with incomplete knowledge is the rule rather than the exception in our everyday lives and all scientific disciplines.
- Fundamental problems about paracomplete reasoning
  - What may be the conclusion of incomplete premises?
  - How can we establish a formal logical validity criterion to underlie paracomplete reasoning?

Proving: What Is It?

- Definitions of 'proving' in dictionary [The Oxford English Dictionary, 2nd Edition]
  - "The action of showing to be true, genuine, or valid; demonstration."
  - "The action of establishing a claim."
- Proving
  - *Proving* is the process of finding a *justification* for an explicitly *specified statement* from given premises, which are already known facts or previously assumed hypotheses to provide some *evidence* for the specified statement.
  - A *proof* is a *description* of a found justification.

Proving and Logic

- Proving and logic
  - *Classical mathematical logic* was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them; its principal aim is a precise and adequate understanding of the notion of mathematical proof.
- Logically valid proving
  - A *logically valid proving* is a proving such that it is justified based on some logical validity criterion in order to obtain a correct proof.

Reasoning and Proving: Intrinsic Difference?

- **Nature**
  - The most intrinsic difference between reasoning and proving is that the former is intrinsically prescriptive and predictive while the latter is intrinsically descriptive and non-predictive.
- **Aim**
  - The purpose of reasoning is to find some new facts previously unknown or unrecognized, while the purpose of proving is to find a justification for some fact previously known or assumed.
- **Goal**
  - Proving has an explicitly defined target as its goal while reasoning does not.

Reasoning and Proving: Intrinsic Difference?

- **Typical pattern of reasoning**
  - From A, B, C, …, what we can say?
  - Before reasoning, we do not know what conclusion we can draw from the premises.
- **Typical pattern of proving**
  - From A, B, C, …, can we say D?
  - Before proving, we do know what statement we have to justify from the premises.

Discovery: What Is It?

- **Definitions of 'discovery' in dictionary [The Oxford English Dictionary, 2nd Edition]**
  - "The action of uncovering or fact of becoming uncovered."
  - "The finding out or bringing to light of that which was previously unknown; making known: also with a and pl., an instance of this."
- **Discovery**
  - *Discovery* is the process to find out or bring to light of that which was previously *unknown*.
  - For any discovery, both the discovered thing and its truth must be unknown before the completion of the discovery process.

Discovery and Reasoning

- **Discovery must invoke reasoning**
  - Reasoning is the only way to draw new conclusions from given premises.
  - There is no discovery process that does not invoke reasoning, because the discovered thing and its truth in a discovery process is unknown before the completion of the discovery process.
- **Discovery must be based on correct reasoning**
  - Since any discovery process has no completely explicitly defined target, the only criterion the discovery process must act according to is to reason correct conclusions from the premises.

Prediction: What Is It?

- **Definitions of 'prediction' in dictionary [The Oxford English Dictionary, 2nd Edition]**
  - "The action of predicting or foretelling future events; also, an instance of this, a prophecy."
  - "A statement made beforehand."
- **Prediction**
  - *Prediction* is the process to make some future event *known in advance*, especially on the basis of special knowledge.
  - For any prediction, both the predicted event and its occurrence must be unknown before the completion of the prediction process.

Prediction and Reasoning

- **Prediction must invoke reasoning**
  - Reasoning is the only way to draw new conclusions from given premises.
  - There is no prediction process that does not invoke reasoning, because the predicted event and its occurrence in a prediction process is unknown before the completion of the prediction process.
- **Prediction must be based on correct reasoning**
  - Since any prediction process has no completely explicitly defined target, the only criterion the prediction process must act according to is to reason correct conclusions from the premises.

Reasoning and Logic

- Logical validity as the only general criterion any reasoning must act according to
  - Since a reasoning has no previously explicitly defined target, the only general criterion it must act according to is to reason correct conclusions from its premises, i.e., the logical validity criterion.
- What is the right logic(s) ?
  - Right for what?
  - Is there 'the only one logic (The One True Logic)' right for any reasoning, or 'various logics' right for various types of reasoning?
  - From the viewpoint of application, practices are the only criterion for testing any theory and truth.

Logic: What Is It?

- Logic is the basis for all other sciences and prior to all others
  - "*Logic is the science of sciences, and the art of arts*." [Duns Scotus, 13th century]
  - "There is a special discipline, called logic, which is considered to be *the basis for all other sciences*." [Tarski, 1941]
  - "It (Mathematical Logic) is *a science prior to all others, which contains the ideas and principles underlying all sciences*." [Godel, 1944]
- Logic is a normative and/or prescriptive science
  - Logic deals with *what entails what* or *what follows from what*, and aims at determining which are the correct conclusions of a given set of premises, i.e. to determine which arguments are valid.
  - It is a normative science to evaluate various arguments.

Logic: What is It all About?

- The notion of logical consequence relation
  - The most essential and central concept in logic is the *logical consequence relation* that relates a given set of premises to those conclusions, which validly follow from the premises.
  - To define a logical consequence relation is nothing else but to provide a logical validity criterion by which one can decide whether the conclusion of an argument or a reasoning really does follow from its premises or not.
- Various logic systems
  - Any science is established based on some fundamental principles and assumptions.
  - Different philosophical motivations on fundamental principles and assumptions lead to different logical validity criteria and different logic systems.

Formal Logic Systems

- Formal logic system
  - *Formal logic system* $\mathrm{L} =_{df} (F(\mathrm{L}), \vdash_{\mathrm{L}})$
  - $F(\mathrm{L})$: A *formal language* (call the '*object language*'), which is the set of all *well-formed formulas* of L
  - $\vdash_{\mathrm{L}}$: A *logical consequence relation* among the formulas of $F(\mathrm{L})$, defined as $2^{F(\mathrm{L})} \rightarrow F(\mathrm{L})$, such that for premises $P \subseteq F(\mathrm{L})$ and conclusion $C \in F(\mathrm{L})$, $P \vdash_{\mathrm{L}} C$ means that within the framework of L, C *validly follows from* P, or equivalently, P *validly entails* C.
- Notes
  - '$\vdash_{\mathrm{L}}$' is read as 'the turnstile with subscript L'.
  - Both $F(\mathrm{L})$ and $\vdash_{\mathrm{L}}$ have to be defined in detail.

Formal Logic Systems

- Representations of formal logic systems
  - A formal logic system can be represented by different forms, such as Hilbert style axiomatic system, Gentzen natural deduction system, Gentzen sequent calculus system, semantic tableau system, and so on.
- Note
  - The equivalence between different forms of the same logic system?

Formal Logic Systems

- Logic theorems
  - For a formal logic system $(F(\mathrm{L}), \vdash_{\mathrm{L}})$, a *logical theorem* T of L is a formula such that $\phi \vdash_{\mathrm{L}} T$ where ϕ is the empty set.
  - $Th(\mathrm{L})$: the set of all logical theorems of L.
- Notes
  - $Th(\mathrm{L})$ is completely determined by $\vdash_{\mathrm{L}}$.
  - It is $Th(\mathrm{L})$ that characterizes the logic L, i.e., if $Th(\mathrm{L}) = Th(\mathrm{L}')$, then we consider the L and L′ to be the same logic system.

Explosive and Paraconsistent Logic Systems

♣ Explosive logic systems

- L is said to be *explosive* if and only if $\{A, \neg A\} \vdash_L B$ for any two different formulas A and B.
- In an explosive logic system, anything is validly follows from premises with a contradiction, or equivalently, premises with a contradiction validly entails anything.

♣ Paraconsistent logic systems

- L is said to be *paraconsistent* if and only if it is not explosive.

Formal Theory

♣ L-theory with premises *P*

- Let $(F(\mathrm{L}), \vdash_{\mathrm{L}})$ be a formal logic system and $P \subseteq F(\mathrm{L})$ be a non-empty set of *sentences* (i.e., *closed well-formed formulas*). A *formal theory* with premises P based on L, called a *L-theory with premises P* and denoted by $T_{\mathrm{L}}(P)$, is defined as

$$T_L(P) =_{df} Th(\mathrm{L}) \cup Th_{\mathrm{L}}^{e}(P)$$
$$Th_{\mathrm{L}}^{e}(P) =_{df} \{ et \mid P \vdash_{\mathrm{L}} et \text{ and } et \notin Th(\mathrm{L})\}$$

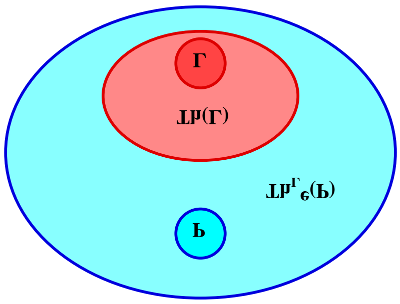
- $Th(\mathrm{L})$: The *logical part*.
- $Th_{\mathrm{L}}^{e}(P)$: The *empirical part*, any element of $Th_{\mathrm{L}}^{e}(P)$ is called an *empirical theorem*.
- P: The *empirical axioms*.

Formal Theory

♣ Formal theory: What is it ?

- A formal theory is a representation of an area of the real world characterized by premises P in a symbolic world characterized by logic L.
- An area of the real world characterized by premises P may be represented by different formal theories characterized by different logics.

Formal Theory



Consistent and Inconsistent Formal Theories

♣ Inconsistent formal theories

- $T_{\mathrm{L}}(P)$ is said to be *directly inconsistent* if and only if there exists a formula A of L such that both $A \in P$ and $\neg A \in P$ hold.
- $T_{\mathrm{L}}(P)$ is said to be *indirectly inconsistent* if and only if it is not directly inconsistent but there exists a formula A of L such that both $A \in T_{\mathrm{L}}(P)$ and $\neg A \in T_{\mathrm{L}}(P)$.

♣ Consistent formal theories

- $T_{\mathrm{L}}(P)$ is said to be *consistent* if and only if it is neither directly inconsistent nor indirectly inconsistent.

Explosive and Paraconsistent Formal Theories

♣ Explosive formal theories

- $T_{\mathrm{L}}(P)$ is said to be *explosive* if and only if $A \in T_{\mathrm{L}}(P)$ for arbitrary formula A of L.
- If L is explosive, then any directly or indirectly inconsistent L-theory $T_{\mathrm{L}}(P)$ must be explosive.

♣ Paraconsistent formal theories

- $T_{\mathrm{L}}(P)$ is said to be *paraconsistent* if and only if it is not explosive.

Object Logic and Meta-logic

- Object logic
  - The logic we are studying.
- Object language
  - The formal language of the object logic.
- Meta-logic (Observer's logic)
  - The logic we are using to study the object logic.
- Meta-language (Observer's language)
  - The language we are using to study the object logic.

Relevant Reasoning

- Introduction
- Basic Notions: Reasoning, Arguments, Truth, Validity, Proving, Discovery, Prediction, Logic Systems, Formal Theories
- The Notion of Conditional: The Heart of Logic and Mathematics
- Relevant (Relevance) Logic: What Is It and Why Study It?
- Relevant Reasoning Based on Strong Relevant (Relevance) Logic
- Relevant Reasoning in Discovery and Prediction
- Research Directions and Challenging Problems
- Bibliography

The Notion of Conditional

- Definitions of 'conditional' in dictionary [OED, 2nd Edition]
  - "A word or clause expressing a condition."
  - "A conditional conjunction; the conditional mood of the verb."
  - "A conditional proposition or syllogism."
- The Notion of Conditional
  - A sentence of the form 'if ... then ...' is usually called a *conditional proposition* or simply *conditional* which states that there exists a relationship of sufficient condition between the 'if' part (called *antecedent*) and the 'then' part (called *consequent*) of the sentence.
  - '… if …' , '… only if …'

The Notion of Conditional

- The roles of conditional in various scientific disciplines
  - Mathematical, natural, and social scientists always use conditionals in their descriptions of various definitions, propositions, theorems, and laws to connect a concept, fact, situation or conclusion and its sufficient conditions.
  - Conditionals are objects of scientific discovery.
- The roles of conditional in our daily live
  - The notion of conditional plays the most essential role in human logical thinking because any reasoning must invoke it.

The Notion of Conditional as the Heart of Logic

- Conditional as the most important subject in logic
  - There is no reasoning that does not invoke the notion of conditional.
  - The notion of conditional was discussed by the Greek ancients, and it is historically always the most important subject studied in logic.
  - Any logical consequence relation is nothing other than a conditional.
- Conditional as the heart of logic
  - "We take the heart of logic to lie in the notion 'if … then …' " [Anderson and Belnap, 1975]

The Notion of Conditional as the Heart of Mathematics

- Conditional as the heart of mathematics
  - The notion of conditional is also the heart of mathematics, because there is no branch of mathematics which does not use logic as the most general correct criterion for reasoning and proving.
- Conditional and pure mathematics [B. Russell, "The Principles of Mathematics," 1903]
  - "Pure Mathematics is the class of all propositions of the form 'p implies q,' where p and q are propositions *containing one or more variables, the same in the two propositions*, and neither p nor q contains any constants except logical constants. And logical constants are all notions definable in terms of the following: Implication, the relation of a term to a class of which it is a member, the notion of such that, the notion of relation, and such further notions as may be involved in the general notion of propositions of the above form. In addition to these, mathematics uses a notion which is not a constituent of the propositions which it considers, namely the notion of truth."

Conditional in Object Logic and Meta-logic

- **Conditional in the object logic**
  - In the object language, there usually is a connective to represent the notion of conditional.
  - Both the logical consequence relation in the proof theory (usually denoted by '|-') and the logical consequence relation in the model theory (usually denoted by '|=') can be considered as a representation of the notion of conditional, respectively.
- **Conditional in the meta-logic**
  - The notion of conditional, usually in the form of natural language, is used for defining various meta-notions and describe various meta-theorems about the object logic.

Truth of a Conditional

- **Truth of a conditional**
  - The truth of a conditional depends not only on the truth of its antecedent and consequent but also more essentially on a *necessarily relevant and conditional relation* between its antecedent and consequent.
- **Notes**
  - The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.
  - Discussion on the notion of conditional is continuous in philosophy and logic.
  - How to define the truth of a conditional (and hence entailment) is a fundamental issue in logic.

Truth of a Conditional

- **Examples (relevant conditionals)**
  - If Auckland is in New Zealand, then Auckland is in Oceania.
  - If Auckland is in Australia, then Auckland is in Oceania.
  - If Auckland is in New Zealand, then Auckland is in Asia.
  - If Auckland is in Australia (China), then Auckland is in Asia.
- **Examples (irrelevant conditionals)**
  - If today is Tuesday, then Auckland is in Oceania.
  - If today is Sunday, then Auckland is in Oceania.
  - If today is Tuesday, then Auckland is in Asia.
  - If today is Sunday, then Auckland is in Asia.

Empirical and Logical Conditionals

- **Empirical conditionals**
  - For a logic, a conditional is called an *empirical conditional* of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent and therefore cannot be determined only by its abstract form.
  - From the viewpoint of that logic, the relevant relation between the antecedent and the consequent of an empirical conditional is considered to be empirical but not logical.
- **Examples**
  - $A \Rightarrow B$
  - $(A \wedge B) \Rightarrow C$

Empirical and Logical Conditionals

- **Logical conditionals**
  - For a logic, a conditional is called a *logical conditional* of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false.
  - From the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is considered to be logical but not empirical.
- **Examples**
  - $(A \wedge (A \Rightarrow B)) \Rightarrow B$
  - $(A \wedge \neg A) \Rightarrow B$
  - $A \Rightarrow (A \wedge B)$

Entailments: Logically Valid Conditionals

- **Valid conditionals**
  - For a logic, a logical conditional is called a *valid conditional* (or *entailment*) of the logic if its truth-value, in the sense of that logic, is considered to be universally true.
- **Invalid conditionals**
  - For a logic, a logical conditional is called an *invalid conditional* of the logic if its truth-value, in the sense of that logic, is considered to be universally false.
- **Intrinsic difference between different logic systems**
  - The most intrinsic difference between different logics is to consider what class of conditionals as entailments.
- **The most important problem in modern logic**
  - "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [Diaz, 1981]

The Notion of Degree

- The degree of a formula
  - The *degree* of a formula is the largest number of nesting of connective, e.g. ⇒, that represents the notion of conditional within it.
- Zero degree formulas
  - A formula is called a *zero degree formula (zdf)* if and only if there is no occurrence of ⇒ in it.
- First degree conditionals
  - A formula of the form $A \Rightarrow B$ is called a *first degree conditional* if and only if both A and B are zero degree formulas.

The Notion of Degree

- First degree formulas
  - A formula A is called a *first degree formula (fdf)* if and only if it satisfies the one of the following conditions:
    (1) A is a first degree conditional,
    (2) A is in the form $+B$ (+ is a one-place connective such as negation and so on) where B is a first degree formula, and
    (3) A is in the form $B * C$ (* is a non-implicational two-place connective such as conjunction or disjunction and so on) where both of B and C is first degree formulas, or one of B and C is a first degree formula and the another is a zero degree formula.

The Notion of Degree

- k^{th} degree conditionals
  - Let k be a natural number. A formula of the form $A \Rightarrow B$ is called a *k^{th} degree conditional* if and only if both A and B are $(k\text{-}1)^{th}$ degree formulas, or one of A and B is a $(k\text{-}1)^{th}$ degree formula and another is a j^{th} ($j<k$-1) degree formula.
- k^{th} degree formulas
  - Let k be a natural number. A formula A is called a *k^{th} degree formula* if and only if it satisfies the one of the following conditions:
    (1) A is a k^{th} degree conditional,
    (2) A is in the form $+B$ where B is a k^{th} degree formula, and
    (3) A is in the form $B*C$ where both of B and C is k^{th} degree formulas, or one of B and C is a k^{th} degree formula and another is a j^{th} ($j<k$) degree formula.

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Fundamental Principles/Assumptions Underlying CML

- The classical abstraction
  - The only properties of a proposition that matter to logic are its form and its truth-value.
- The Fregean assumption
  - The truth-value of a proposition is determined by its form and the truth-values of its constituents.
- The principle of bivalence
  - There are exactly two truth-values, TRUE and FALSE. Every declarative sentence has one or other, but not both, of these truth-values.

Fundamental Principles/Assumptions Underlying CML

- The classical account of validity (CAV)
  - An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.
- Notes:
  - Usually, classical mathematical logic (CML) is often called to be ‘classical’ in the sense of the principle of bivalence.
  - From the viewpoint of conditional (entailment), it is the CAV that makes CML ‘classical’.

The Notion of Conditional (Entailment) in CML

- The notion of material implication [Philo of Megara, 400 B.C.]
  - The notion of conditional is represented in CML by the extensional notion of *material implication* (denoted by '→') which is defined as an extensional truth-functional connective as follows:

$$A \rightarrow B =_{df} \neg(A \wedge \neg B)\ ,\ A \rightarrow B =_{df} \neg A \vee B$$

- Notes
  - The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.
  - '$\neg(A \wedge \neg B)$' and/or '$\neg A \vee B$' is necessary but not sufficient to a conditional A⇒B, because the relevance between A and B is not accounted.

The Notion of Conditional (Entailment) in CML

- Proof-theoretical deduction theorems in CML
  - $\Gamma \cup \{A\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} A \rightarrow B$
  - $\{A\} \vdash_{CML} B$ *iff* $\vdash_{CML} A \rightarrow B$
  - $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} A_1 \rightarrow (\ldots(A_n \rightarrow B)\ldots)$
  - $\Gamma \cup \{A_1, ..., A_n\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$
- Note
  - The notion of material implication is "equivalent" to the logical consequence relation '$\vdash_{CML}$'.

The Notion of Conditional (Entailment) in CML

- Model-theoretical deduction theorems in CML
  - $\Gamma \cup \{A\} \models_{CML} B$ *iff* $\Gamma \models_{CML} A \rightarrow B$
  - $\{A\} \models_{CML} B$ *iff* $\models_{CML} A \rightarrow B$
  - $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ *iff* $\Gamma \models_{CML} A_1 \rightarrow (\ldots(A_n \rightarrow B)\ldots)$
  - $\Gamma \cup \{A_1, ..., A_n\} \models_{CML} B$ *iff* $\Gamma \models_{CML} (A_1 \wedge \ldots \wedge A_n) \rightarrow B$
- Note
  - The notion of material implication is "equivalent" to the logical consequence relation '$\models_{CML}$'.

Comparison of Conditional and Material Implication

- The notion of conditional
  - The notion of conditional is intrinsically intensional but not truth-functional.
  - The notion of conditional requires that there is a necessarily relevant and conditional relation between its antecedent and consequent.
  - The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and conditional relation between them.
  - The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.

Comparison of Conditional and Material Implication

- The notion of material implication
  - The notion of material implication is no more than an extensional truth-function of its antecedent and consequent.
  - The notion of material implication does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent.
  - The truth-value of a material implication depends only on the truth-values of its antecedent and consequent, without regard to any relevance between them.
  - The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.

Comparison of Conditional and Material Implication

antecedent *A*	consequent *B*	necessarily relevant relation between *A* and *B*	conditional 'if *A* then *B*' *A*⇒*B*	material implication *A*→*B*
T	*T*	Existence	*T*	*T*
T	*T*	Not existence	*F*	*T*
T	*F*	Existence	*F*	*F*
T	*F*	Not existence	*F*	*F*
F	*T*	Existence	*T*	*T*
F	*T*	Not existence	*F*	*T*
F	*F*	Existence	*T*	*T*
F	*F*	Not existence	*F*	*T*

Paradoxes of Material Implication in CML

- **The problem of implicational paradox**
  - If one considers the material implication as the notion of conditional and considers every logical theorem of CML as a valid reasoning form or entailment, then a great number of logical axioms and logical theorems of CML present some paradoxical properties and therefore they have been referred to in the literature as "*implicational paradoxes*."
- **Note**
  - "If one considers …… and considers ……, then …… ."

Paradoxes of Material Implication in CML

- **Paradoxes of material implication as empirical conditionals**

  snow is white → 1 + 1 = 2
  snow is black → 1 + 1 = 2
  snow is black → 1 + 1 = 3
- **Paradoxes of material implication as entailments**
  - $A\rightarrow(B\rightarrow A)$, $B\rightarrow(\neg A\vee A)$,
    $\neg A\rightarrow(A\rightarrow B)$, $(\neg A\wedge A)\rightarrow B$,
    $(A\rightarrow B)\vee(\neg A\rightarrow B)$, $(A\rightarrow B)\vee(A\rightarrow\neg B)$,
    $(A\rightarrow B)\vee(B\rightarrow A)$,
    $((A\wedge B)\rightarrow C)\rightarrow((A\rightarrow C)\vee(B\rightarrow C))$
  - Do you think that from 'if A and B then C' you can say 'if A then C or if B then C'?

Paradoxes of Material Implication in CML

- **Notes**
  - It is to consider the notion of material implication as the notion of conditional, or in other words, it is to use material implication in the sense of conditional, that leads to the problem of implicational paradoxes.
  - If we use a material implication as an extensional truth-value function (and hence not conditional) in the same sense as its original definition in CML, then no problem occurs.

Implicational Paradoxes: Problems and Results

- **Necessary but not sufficient to the notion of conditional**
  - '$\neg(A\wedge\neg B)$' or '$\neg A\vee B$' (definitions of material implication '$A\rightarrow B$', CAV) is necessary but not sufficient to the notion of conditional '$A\Rightarrow B$' because the relevance between A and B, another necessary condition required by conditional, is not accounted.
  - The notion of material implication cannot be used for distinguishing conditionals from implicational statements.

Implicational Paradoxes: Problems and Results

- **Reasoning based on CML are not truth-preserving in the sense of conditional**
  - We cannot directly accept a conclusion of a reasoning with implicational paradoxes of entailment as a correct and true conclusion in the sense of conditional, even if all premises of the reasoning are true or valid and the conclusion is true in the sense of material implication.
  - In the framework of CML and its various classical conservative extensions, even if a reasoning is classically valid, the truth of its conclusion in the sense of conditional cannot be guaranteed necessarily.

Implicational Paradoxes: Problems and Results

- **Examples**
  - From A, we can infer $B\rightarrow A$, $C\rightarrow A$, ... where B, C, ... are arbitrary formulas, by using logical axiom $A\rightarrow(B\rightarrow A)$ of CML and Modus Ponens for material implication (from A and $A\rightarrow B$ to infer B).
  - However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be considered to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B and A, C and A, …, and therefore we cannot say 'if B then A', 'if C then A', and so on.

Implicational Paradoxes: Problems and Results

- **Reasoning based on CML are circular and/or tautological but not ampliative**
  - From the viewpoint to regard reasoning as the process of drawing new conclusions from given premises, any meaningful reasoning must be ampliative but not circular and/or tautological, i.e., may not be a vicious circle.
  - The truth of conclusion of the reasoning should be recognized after the completion of the reasoning process but not be invoked in deciding the truth of premises of the reasoning.
  - Since any material implication is an extensional truth-function of its antecedent and consequent, any reasoning based on CML are circular and/or tautological but not ampliative.

Implicational Paradoxes: Problems and Results

- **Examples (reasoning by Modus Ponens)**
  - Modus Ponens: If A holds then B holds, now A holds, therefore B holds.
  - Before the reasoning is performed, we do not know whether B holds or not. (If we do, we do not need reasoning at all.)
  - Modus Ponens in CML: From A and $A \rightarrow B$ to infer B.
  - According to the extensional truth-functional semantics of the material implication, if we know 'A is true' but do not know the truth-value of B, then we cannot decide the truth-value of '$A \rightarrow B$'.
  - In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is performed!

Implicational Paradoxes: Problems and Results

- **Paraconsistent reasoning based on CML is impossible**
  - For a paraconsistent logic with Modus Ponens as an inference rule, the paraconsistence requires that the logic does not have $(\neg A \wedge A) \Rightarrow B$ as a logical theorem where A and B are any two different formulas and '$\Rightarrow$' is the notion of conditional used in Modus Ponens.
  - If a logic is not paraconsistent, then infinite propositions (even negations of those logical theorems of the logic) may be reasoned out based on the logic from a set of premises that directly or indirectly include a contradiction.
  - It is well known that CML is explosive but not paraconsistent, and therefore, any directly or indirectly inconsistent CML-theory $T_{CML}(P)$ must be explosive.
  - This is because CML uses Modus Ponens for material implication as its inference rule, and has "$(\neg A \wedge A) \rightarrow B$" as a logical theorem, which, in fact, is the most typical implicational paradox.

The Notion of Conditional in Lewis's Modal Logics

- **Lewis's work on modal logic**
  - The main aim of Lewis's work beginning in 1912 on the establishment of modern modal logic was to find a satisfactory theory of implication which is better than CML in that it can avoid those implicational paradoxes.

The Notion of Conditional in Lewis's Modal Logics

- **Lewis's strict implication**
  - Lewis's idea is to define the notion of conditional by using necessity operator.
  - $A > B =_{df} L(A \rightarrow B)$ where L is necessity operator
- **Paradoxes of strict implication**
  - Lewis's plan was not successful in the sense that some implicational paradoxes in terms of strict implication remained in his modal logics (S1 ~ S5).
  - Ex.

$LA \rightarrow (B > A)$	$[A \rightarrow (B \rightarrow A)]$
$L\neg A \rightarrow (A > B)$	$[\neg A \rightarrow (A \rightarrow B)]$
$(\neg A \wedge A) > B$	$[(\neg A \wedge A) \rightarrow B]$
$B > (\neg A \vee A)$	$[B \rightarrow (\neg A \vee A)]$

Sugihara's Characterization of Implication Paradoxes

- **Strongest and weakest formulas [Sugihara, 1955]**
  - Relative to a given connective, $\rightarrow$, intended as implication, a formula A is said to be *strongest* if one can prove $A \rightarrow B$ for every formula B, and a formula A is said to be *weakest* if $B \rightarrow A$ is provable for all B.
- **Paradoxical logic systems**
  - A logic system is *paradoxical in the sense of Sugihara* just in case it has either a weakest or a strongest formula.
  - The existence of a proposition that is implied by all or implies all others does not fit the concept of implication as a logical connection between two propositions.
- **CML is paradoxical in the sense of Sugihara**
  - CML (and Lewis's S1 ~ S5) has $\neg A \wedge A$ as strongest formula and $\neg A \vee A$ as weakest formula, and therefore, it is paradoxical.

Ackermann's Rigorous Implication

- Rigorous implication [Ackermann, 1956]
  - "*Rigorous implication*, which we write as $A \to B$, should express the fact that a logical connection holds between A and B, that the content of B is part of that of A. … That has nothing to do with the truth of falsity of A or B. Thus one would *reject* the validity of the formula $A \to (B \to A)$, since it permits the inference from A of $B \to A$, and since the truth of A has nothing to do with whether a logical connection holds between B and A."
- Logical connection and relevance
  - For an entailment (argument) to be valid there should be some connection of meaning, i.e. some relevance, between its premises and its conclusion, among other things.

Wright-Geach-Smiley Criterion for Entailment

- "A entails B, if and only if, by means of logic, it is possible to come to know the truth of $A \to B$ without coming to know the falsehood of A or the truth of B" [Von Wright, 1957]
- "A entails B if and only if there is an a priori way of getting to know that $A \to B$ which is not a way of getting to know whether A or whether B" [Geach, 1958]
- "$A_1 \& \ldots \& A_n \to B$ should not only be itself a tautology, but should also be a substitution instance of some more general implication $A_1' \& \ldots \& A_n' \to B'$, where neither B' nor $\neg(A_1' \& \ldots \& A_n')$ are themselves tautologies" [Smiley, 1959]
- However, it is hard until now to know exactly how to formally interpret such epistemological phrases as 'coming to know' and 'getting to know' in the context of logic.

Relevant (Relevance) Logic

- Motivation
  - *Relevant logics* ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called 'paradoxes' of material and strict implication.
- Paradoxes of material and strict implication
  - $A \to (B \to A)$, $B \to (\neg A \lor A)$, $\neg A \to (A \to B)$, $(\neg A \land A) \to B$, $(A \to B) \lor (\neg A \to B)$, $(A \to B) \lor (A \to \neg B)$, $(A \to B) \lor (B \to A)$, $((A \land B) \to C) \to ((A \to C) \lor (B \to C))$
  - $(\neg A \land A) > B$, $B > (\neg A \lor A)$

Relevant (Relevance) Logic

- Well-known main relevant logics
  - system Π' of *rigorous implication* [Ackermann, 1956]
  - system E of *entailment* [Anderson and Belnap, 1958]
  - system R of *relevant implication* [Belnap, 1967]
  - system T of *ticket entailment* (*entailment shorn of modality*) [Anderson, 1960]
- Characteristic features of the relevant logics
  - A primitive intensional connective to represent the notion of conditional (entailment)
  - Variable-sharing and the relevance principle
  - Free from the paradoxes of material and strict implication
  - Relevant (but not so strongly relevant!) reasoning

The Notion of Conditional (Entailment) in RL

- Conditional as a primitive intensional connective
  - RL has a primitive intensional connective (relevant implication, entailment, ……) to represent the notion of conditional (entailment).
  - The notion of material implication can be defined in RL as an extensional truth-function of its antecedent and consequent in the same way as in CML.
  - In RL, the principle connective to represent the notion of conditional is the primitive intensional connective but not material implication.
  - The language of RL is a conservative extension of that of CML.

Variable-Sharing

- Variable-sharing as a necessary condition for relevance
  - The antecedent and consequent of an entailment should share some propositional variable(s).
  - Variable-sharing is a formal notion designed to reflect the idea that there be a meaning connection between the antecedent and consequent of an entailment.
  - Variable-sharing is a necessary, but by no means sufficient, formal condition for the relevance of the antecedent to the consequent of an entailment.

The Relevance Principle

- **The relevance principle [Anderson and Belnap, 1975]**
  - The relevance principle: If $A \Rightarrow B$, where $\Rightarrow$ denotes the notion of entailment, is a logical theorem of a relevant logic, for any two propositional formulas A and B, then A and B share at least one propositional variable.
- **The relevance principle and implicational paradoxes**
  - As a result of the relevance principle, all relevant logics are free from the paradoxes of material and strict implication.

Relationship between RL and CML

- **ZDF theorem [Anderson and Belnap, 1959]**
  - The zero degree formulas provable in E (R, T) are precisely the theorems of CML.
- **CML is exactly the extensional fragment of E (R, T)**
  - CML is exactly the extensional fragment of E, R, or T in the sense that if one define the material implication as $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$ then all tautologies of CML are theorems of E, R, or T.
  - RL is a conservative extension of CML.

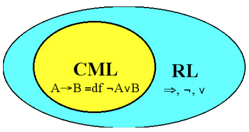
Relationship between RL and CML

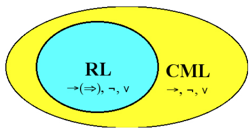
- **Entailments of E (R, T) is a "proper subsystem" of that of CML**
  - The theoremhood of E (R, T) can be regarded as a "proper subset" of the theoremhood of CML, in the sense that if all entailment connectives in a theorem of E (R, T) are replaced by material implication connectives then the resultant formula must be a theorem of CML, and E (R, T) rejects some theorems (like implicational paradoxes) of CML.

Relationship between RL and CML

RL is a conservative extension of CML

Entailments of E (R, T) is a "proper subsystem" of CML





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Paradoxes of Relevant Implication

- **Paradoxes of relevant implication (with conjunction and disjunction)**
  - In traditional (weak) relevant logics, there are still some paradoxes in the sense of conditional.
  - 'The relevance principle' is necessary but still not so sufficient to the notion of conditional $A \Rightarrow B$.
  - Traditional relevant logics are certainly 'relevant' but not so strongly.
- **The problem of equivalence between the notion of entailment and the logical consequence relations**
  - In traditional (weak) relevant logics, $\{A, B\} \vdash_{RL} A$ does not hold, but $(A \wedge B) \Rightarrow A$ is an entailment.

Paradoxes of Relevant Implication

- **Conjunction-implicational paradoxes [Cheng, 1991]**
  - The antecedent includes some conjuncts that are not relevant to the consequent.
  - Ex. $(A\wedge B)\Rightarrow A$, $(A\wedge B)\Rightarrow B$, $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$
  - Ex. If snow is white and 1 + 1 = 2(even 3!), then snow is white.
- **Disjunction-implicational paradoxes [Cheng, 1991]**
  - The consequent includes some disjuncts that are not relevant to the antecedent.
  - Ex. $A\Rightarrow(A\vee B)$, $B\Rightarrow(A\vee B)$, $(A\Rightarrow B)\Rightarrow(A\Rightarrow(B\vee C))$
  - Ex. If snow is white, then snow is white or 1 + 1 = 2(even 3!).

Paradoxes of Relevant Implication

- **Necessary but not sufficient to the notion of conditional**
  - The relevance principle is necessary but not sufficient to the notion of conditional $A\Rightarrow B$ because the relevance between A and B is not fully accounted.
  - The notion of relevant implication cannot be used for distinguishing conditionals from implicational statements.
- **The cause of the implicational paradox problem**
  - In general, a necessary condition for something is not necessarily a sufficient condition, and vice versa.
  - It is to consider one of necessary conditions for the notion of conditional as the sufficient condition that leads to the problem of implicational paradoxes.

Paradoxes of Relevant Implication

- **Reasoning based on RL are not truth-preserving in the sense of conditional**
  - One cannot directly accept a conclusion of a reasoning with (conjunction or disjunction) implicational paradoxes of entailment as a correct and true conclusion in the sense of conditional, even if all premises of the reasoning are true or valid and the conclusion is true in the sense of relevant implication.

Paradoxes of Relevant Implication

- **Examples**
  - From any given premise $A\Rightarrow B$, we can infer $(A\wedge C)\Rightarrow B$, $(A\wedge C\wedge D)\Rightarrow B$, …, and so on by using logical theorem $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$ of traditional (weak) relevant logics and Modus Ponens for entailment (from A and $A\Rightarrow B$ to infer B).
  - However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded as valid in the sense of conditional because there may be no necessarily relevant and conditional relation between C and B, D and B, …, and therefore we cannot say 'if A and C then B', 'if A and C and D then B', …, and so on.

Relevant Logic vs Relevant Reasoning

- **Relevant logic**
  - Relevant logic is intended to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, but did not pay attentions so much to relevant reasoning.
- **Relevant reasoning**
  - Relevant (and ampliative) reasoning is the heart of discovery.
  - Traditional (weak) relevant logics cannot underlie relevant reasoning in the sense of strong relevance.

Strong Relevant (Relevance) Logic

- **Strong relevant (relevance) logics [Cheng, 1992]**
  - As a modification of R, E, and T, strong relevant logics Rc, Ec, and Tc rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in R, E, and T, respectively.
  - What underlies the strong relevant logics Rc, Ec, and Tc is the strong relevance principle.
- **The strong relevance principle**
  - If A is a theorem of Rc, Ec, and Tc, then every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part in A.

Relevant Reasoning Based on Strong Relevance Logic

- Reasoning based on SRL are truth-preserving in the sense of conditional
  - A reasoning based on a strong relevant logic is truth-preserving in the sense of conditional.
  - The truth in the sense of conditional is guaranteed necessarily.
- Are there other paradoxes?
  - In strong relevant logic, are there some other unknown type of paradoxes that is not natural in the sense of conditional?

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Applications of Relevant Logic (Relevant Reasoning)

- Conditional as the heart of knowledge science
  - The notion of conditional (entailment) plays the most essential role in knowledge representation and reasoning.
  - The notion of conditional (entailment) is the heart of logic and mathematics, and is the heart of knowledge science.
- The logical basis for discovery and prediction
  - Any discovery and prediction must ask relevant and ampliative reasoning.
  - It is the primitive intensional notion of conditional (entailment) that plays the most fundamental role in any reasoning for discovery and prediction.

Ampliative Reasoning

- Ampliative reasoning
  - From the viewpoint to consider reasoning as the process of drawing new conclusions from given premises, any meaningful reasoning should be ampliative, i.e., the conclusion of a meaningful reasoning must be NEW, in some certain meaning, to the premises of that reasoning.
- A fundamental problem (open problem)
  - How can we define what is an ampliative reasoning formally?
  - An ampliative reasoning must be non-circular or non-tautological, i.e., any circular or tautological reasoning, any vicious circle is not ampliative.

Ampliative Reasoning

- A necessary condition for ampliative reasoning
  - The truth of the conclusion of an ampliative reasoning should be recognized after the completion of the reasoning process but not be invoked in deciding the truth of premises of the reasoning.
  - The truth-value of the conclusion of an ampliative reasoning must be decided by the reasoning itself but not be previously used in deciding the truth-values of premises of the reasoning.

Conditional in Ampliative Modus Ponens

- If *A* holds then *B* holds, now *A* holds, therefore *B* holds.
- When we reason using Modus Ponens, what we know are 'if *A* holds then *B* holds' and '*A* holds'. Before the reasoning, we do not know whether or not '*B* holds'. If we know, then we would not need to reason at all. Therefore, Modus Ponens is ampliative and non-circular.
- How can we know '*B* holds' by using Modus Ponens? Indeed, by using Modus Ponens, we can know '*B* holds', which is unknown until the reasoning is done, based on the following reasons:
  (i) '*A* holds'
  (ii) 'There is no case such that *A* holds but *B* does not hold' (iii) we know (ii) without investigating either 'whether *A* holds or not' or 'whether *B* holds or not'.
- Note: the Wright-Geach-Smiley criterion for entailment is corresponding to the above (ii) and (iii).

Conditional in Tautological Modus Ponens

- Modus Ponens for material implication: from A and $A \rightarrow B$ to infer B
- Note: $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$.
- According to the extensional truth-functional definition of the material implication, if we know 'A is true' but do not know the truth-value of B, then we cannot decide the truth-value of $A \rightarrow B$.
- In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is done!
- Modus Ponens for material implication is non-ampliative, circular, or tautological.

Ampliative Reasoning Based on RL

- **Modus Ponens for entailment**
  - From A and $A \Rightarrow B$ to infer B.
  - Note: The notion of entailment ($\Rightarrow$) is primitive intensional.
- **Ampliative reasoning based on RL**
  - Because relevant logics have a primitive intensional connective to represent the notion of conditional (entailment) which satisfies the Wright-Geach-Smiley criterion, Modus Ponens for entailment is ampliative, non-circular or non-tautological in the sense of relevance.

Paraconsistent Reasoning

- **Fundamental problem**
  - Our knowledge about a domain or a problem may be incomplete and even inconsistent in many ways.
  - Reasoning with inconsistent knowledge is the rule rather than the exception in our everyday lives and all scientific disciplines.
- **Logical basis for paraconsistent reasoning**
  - Any logic to underlie paraconsistent reasoning (reasoning under inconsistency) must reject the principle of explosion.
  - The principle of explosion: Everything follows from a contradiction ($(A \wedge \neg A) \rightarrow B$).

Paraconsistent Reasoning Based on RL

- **CML is explosive**
  - '$(A \wedge \neg A) \rightarrow B$,' a typical paradox of material implication, is a logical theorem of CML, and therefore, CML is explosive.
  - Any logic based on the principle of explosion, e.g., CML and its various classical conservative extensions, cannot underlie paraconsistent reasoning.
- **Paraconsistent reasoning based on relevant logic**
  - Relevant logics are paraconsistent because they use Modus Ponens for entailment as their inference rule but do not accept '$(\neg A \wedge A) \Rightarrow B$' as a logical theorem.

Abductive Reasoning Based on RL

- **Abductive reasoning [Peirce]**
  - The surprising fact, C, is observed.
    But if A were true, C would be a matter of course.
    Hence, there is reason to suspect that A is true.
  - From 'C' and 'if A then C' to infer 'A'
- **The notion of conditional in abduction**
  - Since 'C' is an observed fact and 'A' is the result of the inference, the logical validity of such an inference is totally determined by the validity of conditional 'if A then C.'
  - The key point in abduction is how to get and use genuine conditionals that are certainly relevant to the observed fact.
  - Those implicational paradoxes used as conditionals in abduction are not only useless but also harmful.

Conditional and Nonmonotonic Reasoning

- **Two fundamental facts in CML**
  - (1) in CML, if B is right then so is $A \rightarrow B$
    if $P \vdash_{CML} B$ then $P \vdash_{CML} A \rightarrow B$
    if $P \models_{CML} B$ then $P \models_{CML} A \rightarrow B$
  - (2) deduction theorem of CML
    $P \cup \{A\} \vdash_{CML} B$ *iff* $P \vdash_{CML} A \rightarrow B$
    $P \cup \{A\} \models_{CML} B$ *iff* $P \models_{CML} A \rightarrow B$
- **Why CML is monotonic?**
  - As a direct result of the two facts, CML has the following proof-theoretical and model-theoretical monotonicity:
    if $P \vdash_{CML} B$ then $P \cup \{A\} \vdash_{CML} B$
    if $P \models_{CML} B$ then $P \cup \{A\} \models_{CML} B$

Nonmonotonic Reasoning Based on RL

- Natural nonmonotonicity of relevant logics
  - The relevant logic R is naturally nonmonotonic. Its deducibility relation is already nonmonotonic and no new logical primitives need be introduced to get the desired effect.
- Nonmonotonic reasoning based on RL
  - To get a nonmonotonic deducibility relation by formalizing the notion of entailment satisfactorily is an important research direction for establishing a satisfactory logic system underlying nonmonotonic reasoning.

Automated Theorem Finding (ATF)

- The open problem of ATF [Wos, 1988]
  - "What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?"
- The difficulty
  - The most important and difficult requirement of the problem is that, in contrast to proving conjectured theorems supplied by the user, it asks for criteria that an automated reasoning program can use to find some theorems in a field that must be evaluated by theorists of the field as new and interesting theorems.

ATF by Entailment Calculus Based on SRL

- Entailment calculus as the logic basis for ATF [Cheng, 1995]
  - Paradoxical logics cannot underlie ATF.
  - The strong relevant logics are hopeful candidates for the fundamental logic to underlie ATF.
- EnCal [Cheng et al., 1995 ~]
  - An automated forward deduction system for general-purpose Entailment Calculus.
- FreeEnCal [Cheng et al., 2005 ~]
  - A forward reasoning engine with general-purpose.

Inference Rule Generation Based on SRL

- Inadequacy of the current knowledge-based systems
  - They cannot autonomously generate new and valid inference rules from those existing rules that are programmed or inputted in the systems by their developers or users.
  - Autonomous evolution of a system is impossible if it has no ability of autonomous generation of new and valid inference rules.
- Notes
  - Conceptually speaking, any inference rule is a conditional.
  - In fact, inference rules are usually represented in the form of conditional.

Inference Rule Generation Based on SRL

- Paradoxical logics are hopeless
  - Any paradoxical logic cannot underlie inference rule generation because a reasoning based on the logic may deduce false conditionals as correct conclusions.
- Inference rule generation by entailment calculus based on SRL
  - The conclusion of a valid reasoning based on a strong relevant logic must be true in the sense of conditional, if all premises of the reasoning are sound.

Epistemic Process Modeling

- A fundamental observation on scientific discovery
  - New conditionals are epistemic goals of any scientific discovery: Any scientific discovery process must include an epistemic process to gain knowledge of or to ascertain the existence of some empirical or logical conditionals previously unknown or unrecognized.

Epistemic Process Modeling based on CML

- **Problem on relevant reasoning**
  - CML cannot guarantee the truth of a belief in the sense of conditional, in an epistemic state even if all premises in the primary epistemic state are true or valid because CML cannot underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional.
- **Problem on paraconsistent reasoning**
  - To simply assume that every belief set of an agent is consistent must result in the neglect of investigating how to reason under inconsistency which is an ordinary work of many scientists in their epistemic processes in scientific discovery.

Epistemic Process Modeling Based on SRL

- **Strong relevant logic model of epistemic processes [Cheng, 1998]**
  - All problems in classical logic based approaches do not exist in our strong relevant logic model of epistemic processes.

Epistemic Programming [Cheng, 1996 ~]

- **Motivation**
  - Provide scientists with a computational methodology and its computational tools to program their epistemic processes in scientific discovery and prediction, and therefore, make scientific discovery and prediction become a 'science' and/or an 'engineering.'
- **Goal**
  - Construct a realistic computational model of epistemic processes in scientific discovery.
  - Establish a new programming paradigm, named 'Epistemic Programming.'

Epistemic Programming [Cheng, 1996 ~]

- **Epistemic programming**
  - Regards conditionals as the subject of computing, takes primary epistemic operations as basic operations of computing, and regards epistemic processes as the subject of programming.
  - Computing 'conditionals' rather than 'values.'
  - Programming 'epistemic processes' rather than 'assignment control processes.'

Anticipatory Reasoning based on TRL [Cheng, 2003 ~]

- **Anticipatory reasoning**
  - An anticipatory reasoning is a reasoning to draw new, previously unknown and/or unrecognized conclusions about some future event or events whose occurrence and truth are uncertain at the point of time when the reasoning is being performed.
- **Anticipatory reasoning based on temporal relevant logic**
  - Temporal relevant logic can underlie relevant, truth-preserving ampliative, paraconsistent, and temporal reasoning.

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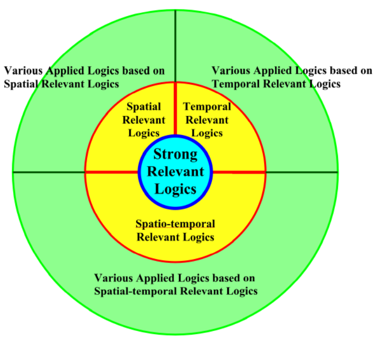
Research Directions and Challenging Problems

- **On relevant reasoning**
  - The most satisfactory way of grasping the notion of entailment
  - The most satisfactory logic to underlie truth-preserving and relevant reasoning in the sense of conditional
  - Model theory and decision problems for strong relevant logic
  - Normal form to represent equivalence classes of formulas in a logic where the notion of conditional is represented by a primitive connective

Research Directions and Challenging Problems

- **On extensions of relevant (relevance) logic**
  - Temporal relevant logic
  - Many-valued and/or fuzzy relevant logic
  - Deontic relevant logic
  - Spatial relevant logic
  - Spatial-temporal relevant logic
  - …… relevant logic

Strong Relevant Logics as the Core Logic



Research Directions and Challenging Problems

- **On applications of relevant (relevance) logic**
  - Mathematical knowledge representation and reasoning based on strong relevant logic, Automated theorem finding
  - Autonomous generation of inference rules
  - Epistemic programming: Programming epistemic processes for computing conditionals
- **On applications of extensions of relevant (relevance) logic**
  - Anticipatory reasoning based on temporal relevant logic
  - Specifying and reasoning about information assurance and security based on deontic relevant logic
  - Representing and reasoning about spatial knowledge based on spatial relevant logic
  - Specifying, verifying, and reasoning about mobile malti-agent systems based on spatio-temporal relevant logic

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Thanks

**Thank you
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Relevant Reasoning Based on Strong Relevant Logic

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