Strong Relevant Logic
as the Universal Basis of Various Applied Logics for Knowledge Representation and Reasoning

Jingde Cheng

Saitama University

Strong Relevant Logic as the Universal Basis of Various Applied Logics

- Background, Motivation, and Goal
- Essential Requirements for the Universal Basis of Various Applied Logics
- Strong Relevant (Relevance) Logics
- Temporal Relevant Logics
- Spatial Relevant Logics
- Spatio-temporal Relevant Logics
- Deontic Relevant Logics
- Epistemic Relevant Logics
- Concluding Remarks
- Bibliography

Background, Motivation, and Goal

- Various applied logic systems
  - In various applications, in order to represent, specify, verify, and reason about various objects and relationships among them, we often need a right fundamental logic to provide us with a criterion of logical validity for reasoning and/or proving as well as a formal representation and specification language.
  - Different applications may require different logics.
- A right fundamental logic system as the core logic?
  - Can we have a set of essential requirements as the universal core requirements for various applied logics?
  - Can we have a logic system satisfying the essential requirements as the universal core logic such that we can construct various applied logics by extending the logic?

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1st Essential Requirement for the Fundamental Logic

- Underlie relevant reasoning and truth-preserving reasoning in the sense of conditional
  - As a general logical criterion for the validity of reasoning as well as proving, the logic must be able to underlie relevant reasoning in the sense of conditional as well as truth-preserving reasoning in the sense of conditional.
- Notes
  - Require both relevant reasoning and truth-preserving reasoning.
  - "in the sense of conditional" means that we consider the notion of conditional as a whole (i.e., intensional primitive), but not a combination of its antecedent and consequent.

1st Essential Requirement for the Fundamental Logic

- Relevant reasoning
  - For a reasoning to be valid (in the sense of weak relevance), its premises must somehow be "*relevant*" to its conclusion, and vice versa.
- Truth-preserving reasoning
  - For a reasoning to be valid (in the sense of truth-preservation), if its premises are "*true*", then its conclusion must also be "*true*".
- Relevant and truth-preserving reasoning in the sense of conditional
  - For a reasoning to be valid (in the sense of conditional), if its premises are "*true*" in the sense of conditional, then its conclusion must be "*relevant*" in the sense of conditional to the premises and must be "*true*" in the sense of conditional.

2nd Essential Requirement for the Fundamental Logic

- **Underlie ampliative reasoning**
  - The logic must be able to underlie ampliative reasoning.
  - From the viewpoint to regard reasoning as the process of drawing new conclusion from given premises, any meaningful reasoning must be ampliative but not circular and/or tautological.
- **Ampliative reasoning**
  - For a reasoning to be valid (in the sense of ampliation), the "*truth*" of conclusion of the reasoning should be recognized "*after the end*" of the reasoning process but not be invoked in deciding the "*truth*" of premises of the reasoning.

3rd Essential Requirement for the Fundamental Logic

- **Underlie paracomplete and paraconsistent reasoning**
  - The logic must be able to underlie paracomplete reasoning and paraconsistent reasoning.
  - Reasoning with incomplete and/or inconsistent information and/or knowledge is the rule rather than the exception in our everyday lives and almost all scientific disciplines.
- **Notes**
  - Prefix "para (para-)" means "beyond".
  - In general, "para-A" means "NOT A, but also NOT completely deny A".

3rd Essential Requirement for the Fundamental Logic

- **Paracomplete reasoning**
  - For a reasoning to be valid (in the sense of paracompleteness), its conclusion "*may not be*" the negation of a sentence when (even if) the sentence is not a conclusion of the premises of that reasoning.
- **Paraconsistent reasoning**
  - For a reasoning to be valid (in the sense of paraconsistency), its conclusion "*may not be*" an arbitrary sentence when (even if) its premises are inconsistent.
  - In particular, the so-called principle of Explosion (ex contradictione quodlibet, ECQ) that everything follows from a contradiction cannot be accepted as a valid principle.

Classical Mathematical Logic and Its Extensions

- **Classical mathematical logic (CML)**
  - The classical account of validity (CAV): an argument (reasoning) is valid if and only if it is impossible for all its premises to be true while its conclusion is false.
  - A reasoning based on CML is not necessarily relevant.
  - The classical truth-preserving property of a reasoning based on CML is meaningless in the sense of conditional.
  - A reasoning based on CML must be circular and/or tautological but not ampliative.
  - Paraconsistent reasoning (reasoning under inconsistency) is impossible within the framework of CML.
- **Classical conservative extensions of CML**
  - The same as the above facts of CML.

The Notion of Conditional (Entailment) in CML

- **The notion of material implication [Philo of Megara, 400 B.C.]**
  - The notion of conditional is represented in CML by the extensional notion of *material implication* (denoted by '→') which is defined as an extensional truth-functional connective as follows:

$$A\rightarrow B =_{df} \neg(A \land \neg B)\,,\quad A\rightarrow B =_{df} \neg A \lor B$$

- **Notes**
  - The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication (i.e., NOT in the sense of conditional).
  - '$\neg(A\land\neg B)$' and/or '$\neg A\lor B$' is necessary but not sufficient to a conditional A⇒B, because the relevance between A and B is not accounted.

The Notion of Conditional (Entailment) in CML

- **Proof-theoretical deduction theorems in CML**
  - $\Gamma \cup \{A\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} A\rightarrow B$
  - $\{A\} \vdash_{CML} B$ *iff* $\vdash_{CML} A\rightarrow B$
  - $\Gamma \cup \{A_1, \ldots, A_n\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} A_1\rightarrow(\ldots(A_n\rightarrow B)\ldots)$
  - $\Gamma \cup \{A_1, \ldots, A_n\} \vdash_{CML} B$ *iff* $\Gamma \vdash_{CML} (A_1\land \ldots \land A_n)\rightarrow B$
- **Model-theoretical deduction theorems in CML**
  - $\Gamma \cup \{A\} \models_{CML} B$ *iff* $\Gamma \models_{CML} A\rightarrow B$
  - $\{A\} \models_{CML} B$ *iff* $\models_{CML} A\rightarrow B$
  - $\Gamma \cup \{A_1, \ldots, A_n\} \models_{CML} B$ *iff* $\Gamma \models_{CML} A_1\rightarrow(\ldots(A_n\rightarrow B)\ldots)$
  - $\Gamma \cup \{A_1, \ldots, A_n\} \models_{CML} B$ *iff* $\Gamma \models_{CML} (A_1\land \ldots \land A_n)\rightarrow B$
- **Note**
  - The notion of material implication is "equivalent" to the logical consequence relation '$\vdash_{CML}$' and/or '$\models_{CML}$'.

Comparison of Conditional and Material Implication

- **The notion of conditional**
  - **The notion of conditional is intrinsically intensional but not truth-functional (the von Wright-Geach-Smiley criterion).**
  - **The notion of conditional requires that there is a necessarily relevant and conditional relation between its antecedent and consequent.**
  - **The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and conditional relation between them.**
  - **The truth of the consequent (or the falsity of the antecedent) of a conditional is by itself insufficient for the truth of that conditional.**

Comparison of Conditional and Material Implication

- **The notion of material implication**
  - **The notion of material implication is no more than an extensional truth-function of its antecedent and consequent, and therefore, the truth of a material implication depends totally on the truth of its antecedent and consequent.**
  - **The notion of material implication does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent.**
  - **The truth of a material implication depends only on the truth of its antecedent and consequent, without regard to any relevance between them.**
  - **The truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication.**

Comparison of Conditional and Material Implication

antecedent A	consequent B	necessarily relevant relation between A and B	conditional "if A then B" $A \Rightarrow B$	material implication "A implies B" $A \rightarrow B$
T	*T*	Existence	*T*	*T*
T	*T*	Not existence	*F*	*T*
T	*F*	Existence	*F*	*F*
T	*F*	Not existence	*F*	*F*
F	*T*	Existence	*T*	*T*
F	*T*	Not existence	*F*	*T*
F	*F*	Existence	*T*	*T*
F	*F*	Not existence	*F*	*T*

Paradoxes of Material Implication in CML

- **The problem of implicational paradox**
  - **If one thinks of the material implication as the notion of conditional and thinks of every logical theorem of CML as a valid reasoning form or entailment, then a great number of logical axioms and logical theorems of CML present some paradoxical properties and therefore they have been referred to in the literature as "*implicational paradoxes*."**
- **Notes**
  - **"If one thinks of …… and thinks of ……, then …… ."**
  - **It is to think of the notion of material implication as the notion of conditional, or in other words, it is to use material implication in the sense of conditional, that leads to the problem of implicational paradoxes.**
  - **If we use a material implication as an extensional truth-value function (and hence not conditional) in the sense of its original definition in CML, then no problem occurs.**

Paradoxes of Material Implication in CML

- **Paradoxes of material implication as empirical conditionals**

  snow is white → 1 + 1 = 2
  snow is black → 1 + 1 = 2
  snow is black → 1 + 1 = 3

- **Paradoxes of material implication as entailments**
  - $A\rightarrow(B\rightarrow A)$, $B\rightarrow(\neg A\lor A)$,
    $\neg A\rightarrow(A\rightarrow B)$, $(\neg A\land A)\rightarrow B$ **(ECQ!)**,
    $(A\rightarrow B)\lor(\neg A\rightarrow B)$, $(A\rightarrow B)\lor(A\rightarrow\neg B)$,
    $(A\rightarrow B)\lor(B\rightarrow A)$,
    $((A\land B)\rightarrow C)\rightarrow((A\rightarrow C)\lor(B\rightarrow C))$
  - **Do you think that from 'if *A* and *B* then *C*' you can say 'if *A* then *C* or if *B* then *C*'?**

Implicational Paradoxes: Problems and Results

- **Necessary but not sufficient to the notion of conditional**
  - **'$\neg(A\land\neg B)$' or '$\neg A\lor B$' (definitions of material implication '$A\rightarrow B$', CAV) is necessary but not sufficient to the notion of conditional '$A\Rightarrow B$' because the relevance between A and B, another necessary condition required by conditional, is not accounted.**
- **Necessary but not sufficient to the notion of conditional**
  - **The notion of material implication cannot be used for distinguishing conditionals from implicational statements.**

CML Cannot Satisfy the 1st Essential Requirement

- **CML cannot underlie relevant and truth-preserving in the sense of conditional**
  - In the framework of CML (or any of its classical conservative extensions), even if a reasoning is classically valid, both the relevance relationship between its premises and conclusion and the truth of its conclusion in the sense of conditional cannot be guaranteed necessarily.
- **Reason: material implication and implicational paradoxes**
  - We cannot directly accept a conclusion of a reasoning with implicational paradoxes of entailment as a relevant and true conclusion in the sense of conditional, even if all premises of the reasoning are true and the conclusion is true in the sense of material implication.

CML Cannot Satisfy the 1st Essential Requirement

- **Examples**
  - From A, we can infer $B \rightarrow A$, $C \rightarrow A$, ... where B, C, ... are arbitrary formulas, by using logical axiom $A \rightarrow (B \rightarrow A)$ of CML and Modus Ponens for material implication (from A and $A \rightarrow B$ to infer B).
  - However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be considered to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B and A, C and A, …, and therefore we cannot say 'if B then A', 'if C then A', and so on.

CML Cannot Satisfy the 2nd Essential Requirement

- **CML cannot underlie ampliative reasoning**
  - Any reasoning based on CML (or any of its classical conservative extensions) is circular and/or tautological but not ampliative.
- **Reason: material implication is an extensional truth-function**
  - Since any material implication is an extensional truth-function of its antecedent and consequent, the truth of a material implication depends totally on the truth of its antecedent and consequent, i.e., one cannot determine the truth of a material implication without knowing truths of its antecedent and consequent.
  - On the other hand, the truth of the consequent (or the falsity of the antecedent) of a material implication is by itself sufficient for the truth of that material implication. However, when we reason, we do not know the truth of the consequent and do not use false antecedent.

CML Cannot Satisfy the 2nd Essential Requirement

- **Examples (reasoning by Modus Ponens for material implication)**
  - Modus Ponens: If A holds then B holds, now A holds, therefore B holds.
  - Before the reasoning is performed, we do not know whether B holds or not. (If we do, we do not need reasoning at all.)
  - Modus Ponens in CML: From A and $A \rightarrow B$ to infer B.
  - According to the extensional truth-functional semantics of the material implication, if we know 'A is true' but do not know the truth-value of B, then we cannot decide the truth-value of '$A \rightarrow B$'.
  - In order to know the truth-value of B using Modus Ponens for material implication, we have to know the truth-value of B before the reasoning is performed!

CML Cannot Satisfy the 3rd Essential Requirement

- **Paraconsistent logic (NOT allow ECQ)**
  - For a paraconsistent logic with Modus Ponens as an inference rule, the paraconsistence requires that the logic does not have $(\neg A \wedge A) \Rightarrow B$ as a logical theorem where A and B are any two different formulas and '$\Rightarrow$' is the notion of conditional used in Modus Ponens.
  - If a logic is not paraconsistent, then infinite propositions (even negations of those logical theorems of the logic) may be reasoned out based on the logic from a set of premises that directly or indirectly include a contradiction.
- **Paraconsistent reasoning based on CML is impossible**
  - CML (or any of its classical conservative extensions) is explosive but not paraconsistent.
  - CML uses Modus Ponens for material implication as its inference rule, and has "$(\neg A \wedge A) \rightarrow B$" as a logical theorem.

Traditional Relevant (Relevance) Logics

- **Traditional relevant (or relevance) logics**
  - Relevant logics ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called 'paradoxes' of material and strict implication.
- **Paradoxes of material and strict implication**
  - $A \rightarrow (B \rightarrow A)$, $B \rightarrow (\neg A \vee A)$, $\neg A \rightarrow (A \rightarrow B)$, $(\neg A \wedge A) \rightarrow B$, $(A \rightarrow B) \vee (\neg A \rightarrow B)$, $(A \rightarrow B) \vee (A \rightarrow \neg B)$, $(A \rightarrow B) \vee (B \rightarrow A)$, $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
  - $(\neg A \wedge A) > B$, $B > (\neg A \vee A)$

Traditional Relevant Logics

- **Well-known major relevant logics**
  - system Π' of *rigorous implication* [Ackermann, 1956]
  - system E of *entailment* [Anderson and Belnap, 1958]
  - system R of *relevant implication* [Belnap, 1967]
  - system T of *ticket entailment* (*entailment shorn of modality*) [Anderson, 1960]
- **Characteristic features of the relevant logics**
  - A primitive intensional connective to represent the notion of conditional (entailment)
  - Variable-sharing and the relevance principle
  - Free from the paradoxes of material and strict implication
  - Relevant (in the sense of weak relevance!) reasoning
  - Ampliative reasoning

Paradoxes of Relevant Implication

- **Paradoxes of relevant implication (with conjunction and disjunction)**
  - In traditional (weak) relevant logics, there are still some paradoxes in the sense of conditional.
  - 'The relevance principle' is necessary but still not so sufficient to the notion of conditional A⇒B.
  - Traditional relevant logics are certainly 'relevant' but not so strongly relevant.
- **The problem of equivalence between the notion of entailment and the logical consequence relations**
  - In traditional (weak) relevant logics, $\{A, B\} \vdash_{RL} A$ does not hold, but $(A \wedge B) \Rightarrow A$ is an entailment.

Paradoxes of Relevant Implication

- **Conjunction-implicational paradoxes** [Cheng, 1991]
  - The antecedent includes some conjuncts that are not relevant to the consequent.
  - Ex. $(A \wedge B) \Rightarrow A$, $(A \wedge B) \Rightarrow B$, $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow B)$
  - Ex. If snow is white and 1+1=2 (even 3!), then snow is white.
- **Disjunction-implicational paradoxes** [Cheng, 1991]
  - The consequent includes some disjuncts that are not relevant to the antecedent.
  - Ex. $A \Rightarrow (A \vee B)$, $B \Rightarrow (A \vee B)$, $(A \Rightarrow B) \Rightarrow (A \Rightarrow (B \vee C))$
  - Ex. If snow is white, then snow is white or 1+1=2 (even 3!).

Paradoxes of Relevant Implication

- **Necessary but not sufficient to the notion of conditional**
  - The relevance principle is necessary but not sufficient to the notion of conditional $A \Rightarrow B$ because the relevance between A and B is not fully accounted.
  - The notion of relevant implication cannot be used for distinguishing conditionals from implicational statements.
- **The cause of the implicational paradox problem**
  - In general, a necessary condition for something is not necessarily a sufficient condition, and vice versa.
  - It is to consider one of necessary conditions for the notion of conditional as the sufficient condition that leads to the problem of implicational paradoxes.

Paradoxes of Relevant Implication

- **Reasoning based on RL are not truth-preserving in the sense of conditional**
  - One cannot directly accept a conclusion of a reasoning with (conjunction or disjunction) implicational paradoxes of entailment as a correct and true conclusion in the sense of conditional, even if all premises of the reasoning are true or valid and the conclusion is true in the sense of relevant implication.

Paradoxes of Relevant Implication

- **Examples**
  - From any given premise $A \Rightarrow B$, we can infer $(A \wedge C) \Rightarrow B$, $(A \wedge C \wedge D) \Rightarrow B$, …, and so on by using logical theorem $(A \Rightarrow B) \Rightarrow ((A \wedge C) \Rightarrow B)$ of traditional (weak) relevant logics and Modus Ponens for entailment (from A and $A \Rightarrow B$ to infer B).
  - However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded as valid in the sense of conditional because there may be no necessarily relevant and conditional relation between C and B, D and B, …, and therefore we cannot say 'if A and C then B', 'if A and C and D then B', …, and so on.

Relevant Logic vs Relevant Reasoning

- **Relevant logic**
  - Relevant logic is intended to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, but did not pay attentions so much to relevant reasoning.
- **Relevant reasoning (in the sense of strong relevance)** [Cheng, 1998]
  - For a reasoning to be valid (in the sense of strong relevance), its premises must not contain conjuncts irrelevant to its conclusion, and its conclusion must not contain disjuncts irrelevant to its premises.
  - Relevant (and ampliative) reasoning is the heart of discovery and prediction.
  - Traditional (weak) relevant logics cannot underlie relevant reasoning in the sense of strong relevance.

Strong Relevant (Relevance) Logics

- **Strong relevant (relevance) logics** [Cheng, 1992]
  - As a modification of R, E, and T, strong relevant logics Rc, Ec, and Tc rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in R, E, and T, respectively.
  - What underlies the strong relevant logics Rc, Ec, and Tc is the strong relevance principle.
- **The strong relevance principle**
  - If *A* is a theorem of Rc, Ec, and Tc, then every sentential variable in *A* occurs at least once as an antecedent part and at least once as a consequent part in *A*.

Strong Relevant Logics as the Core Logic

- **Strong relevant logics as the core logic**
  - Strong relevant logics can underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional.
  - Relevant logics can underlie ampliative reasoning.
  - Relevant logics can underlie paracomplete reasoning and paraconsistent reasoning.
  - Strong relevant logics satisfy all three essential requirements for the fundamental logic.
- **More details about strong relevant logics**
  - Please refer to my papers.

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Formal Language for Propositional Relevant Logics

- **Alphabet (Symbols)**

  $\{ \neg, \rightarrow, \wedge, \vee, \leftrightarrow, \Rightarrow, \otimes, \oplus, \Leftrightarrow, L, T, F, (,), p_1, p_2, ..., p_n, ... \}$
- **Propositional constants**

  $T,\ F$
- **Propositional variables (proposition symbols)**

  $p_1, p_2, ..., p_n, ...,$

Formal Language for Predicate Relevant Logics

- **Alphabet (Symbols)**

  $\{ \neg, \rightarrow, \wedge, \vee, \leftrightarrow, \Rightarrow, \otimes, \oplus, \Leftrightarrow, L, \forall, \exists, T, F, (,),$
  $x_1, x_2, ..., x_n, ..., c_1, c_2, ..., c_n, ...,$
  $f_1^1, ..., f_1^n, ..., f_2^1, ..., f_2^n, ..., f_k^1, ..., f_k^n, ...,$
  $p_0^1, ..., p_0^n, ..., p_1^1, ..., p_1^n, ..., p_2^1, ..., p_2^n, ..., p_k^1, ..., p_k^n, ... \}$
- **Propositional constants**

  $T,\ F$
- **Individual variables (variable symbols)**

  $x_1, x_2, ..., x_n, ...,$
- **(Individual) Constants (Names) (constant symbols, name symbols)**

  $c_1, c_2, ..., c_n, ...,$
- **(Individual) Functions (function symbols)**

  $f_1^1, ..., f_1^n, ..., f_2^1, ..., f_2^n, ..., f_k^1, ..., f_k^n, ...,$
- **(Individual) Predicates (Relations) (predicate symbols, relation symbols)**

  $p_0^1, ..., p_0^n, ..., p_1^1, ..., p_1^n, ..., p_2^1, ..., p_2^n, ..., p_k^1, ..., p_k^n, ...$

Formal Language for Relevant Logics

- **Primitive logical connectives**
  - $\Rightarrow$: entailment (primitive! And therefore, intensional!)
  - $\neg$: negation
  - $\wedge$: extensional conjunction
- **Defined logical connectives**
  - $\otimes$: intensional conjunction (fusion), $A\otimes B =_{df} \neg(A\Rightarrow\neg B)$
  - $\oplus$: intensional disjunction, $A\oplus B =_{df} \neg A\Rightarrow B$
  - $\Leftrightarrow$: intensional equivalence, $A\Leftrightarrow B =_{df} (A\Rightarrow B)\otimes(B\Rightarrow A)$
  - $\vee$: extensional disjunction, $A\vee B =_{df} \neg(\neg A\wedge\neg B)$
  - $\rightarrow$: material implication, $A\rightarrow B =_{df} \neg(A\wedge\neg B)$ or $\neg A\vee B$
  - $\leftrightarrow$: extensional equivalence, $A\leftrightarrow B =_{df} (A\rightarrow B)\wedge(B\rightarrow A)$
  - L : necessity operator, $LA =_{df} (A\Rightarrow A)\Rightarrow A$
- **Terms and formulas**
  - Similar to that of classical propositional/predicate calculus.

Hilbert Style Axiomatic Systems for Relevant Logics

- **Axiom schemata on entailment**

E1: $A\Rightarrow A$ (Self-Implication)
E2: $(A\Rightarrow B)\Rightarrow((C\Rightarrow A)\Rightarrow(C\Rightarrow B))$ (Prefixing)
E2′: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow C))$ (Suffixing)
E3: $(A\Rightarrow(A\Rightarrow B))\Rightarrow(A\Rightarrow B)$ (Contraction)
E3′: $(A\Rightarrow(B\Rightarrow C))\Rightarrow((A\Rightarrow B)\Rightarrow(A\Rightarrow C))$ (Self-Distribution)
E3″: $(A\Rightarrow B)\Rightarrow((A\Rightarrow(B\Rightarrow C))\Rightarrow(A\Rightarrow C))$ (Permuted Self-Distribution)
E4: $(A\Rightarrow((B\Rightarrow C)\Rightarrow D))\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow D))$ (Restricted Permutation)
E4′: $(A\Rightarrow B)\Rightarrow(((A\Rightarrow B)\Rightarrow C)\Rightarrow C)$ (Restricted Assertion)
E4″: $((A\Rightarrow A)\Rightarrow B)\Rightarrow B$ (Specialized Assertion)
E4‴: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(((A\Rightarrow C)\Rightarrow D)\Rightarrow D))$
E5: $(A\Rightarrow(B\Rightarrow C))\Rightarrow(B\Rightarrow(A\Rightarrow C))$ (Permutation)
E5′: $A\Rightarrow((A\Rightarrow B)\Rightarrow B)$ (Assertion)
E5″: $A\Rightarrow((A\Rightarrow A)\Rightarrow A)$ (Demodalizer)

Hilbert Style Axiomatic Systems for Relevant Logics

- **Axiom schemata on conjunction**

C1: $(A\wedge B)\Rightarrow A$ (Conjunction Elimination)
C2: $(A\wedge B)\Rightarrow B$ (Conjunction Elimination)
C3: $((A\Rightarrow B)\wedge(A\Rightarrow C))\Rightarrow(A\Rightarrow(B\wedge C))$ (Conjunction Introduction)
C4: $(LA\wedge LB)\Rightarrow L(A\wedge B)$, where $LA =_{df} (A\Rightarrow A)\Rightarrow A$ (Distribution of Necessity over Conjunction)

- **Axiom schemata on disjunction**

D1: $A\Rightarrow(A\vee B)$ (Disjunction Introduction)
D2: $B\Rightarrow(A\vee B)$ (Disjunction Introduction)
D3: $((A\Rightarrow C)\wedge(B\Rightarrow C))\Rightarrow((A\vee B)\Rightarrow C)$ (Disjunction Elimination)

- **Distribution axiom schema**

DCD: $(A\wedge(B\vee C))\Rightarrow((A\wedge B)\vee C)$ (Distribution of Conjunction over Disjunction)

Hilbert Style Axiomatic Systems for Relevant Logics

- **Axiom schemata on negation**

N1: $(A\Rightarrow(\neg A))\Rightarrow(\neg A)$ (Reduction)
N2: $(A\Rightarrow(\neg B))\Rightarrow(B\Rightarrow(\neg A))$ (Contraposition)
N3: $(\neg(\neg A))\Rightarrow A$ (Double Negation)

- **Mingle axiom schemata**

EM0: $(A\Rightarrow B)\Rightarrow((A\Rightarrow B)\Rightarrow(A\Rightarrow B))$
RM0: $A\Rightarrow(A\Rightarrow A)$

- **Axiom schemata on necessity**

$L1$: $LA\Rightarrow A$
$L2$: $L(A\Rightarrow B)\Rightarrow(LA\Rightarrow LB)$
$L3$: $(LA\wedge LB)\Rightarrow L(A\wedge B)$
$L4$: $LA\Rightarrow LLA$
$L5$: $LA\Rightarrow((LA\Rightarrow LA)\Rightarrow LA)$

Hilbert Style Axiomatic Systems for Relevant Logics

- **Axiom schemata on individual quantification**

IQ1: $\forall x(A\Rightarrow B)\Rightarrow(\forall xA\Rightarrow\forall xB)$
IQ2: $(\forall xA\wedge\forall xB)\Rightarrow\forall x(A\wedge B)$
IQ3: $\forall xAx\Rightarrow Ay$
IQ4: $\forall x(A\Rightarrow B)\Rightarrow(A\Rightarrow\forall xB)$ (x not free in A)
IQ5: $\forall x(A\vee B)\Rightarrow(A\vee\forall xB)$ (x not free in A)
IQ6: $\forall x_1 \ldots \forall x_n(((A\Rightarrow A)\Rightarrow B)\Rightarrow B)$ $(n\geq 0)$ (for E and EM)
IQ7: $Ay\Rightarrow\exists xAx$
IQ8: $\forall x(A\Rightarrow B)\Rightarrow(\exists xA\Rightarrow B)$ (x not free in B)
IQ9: $(\exists xA\wedge B)\Rightarrow\exists x(A\wedge B)$ (x not free in B)

Axiom clause: if A is an axiom, so is $\forall xA$.

Hilbert Style Axiomatic Systems for Relevant Logics

- **Inference rules**

$\Rightarrow$E: From A and $A\Rightarrow B$ to infer B (Modus Ponens)

$\wedge$I: From A and B to infer $A\wedge B$ (Adjunction)

LI: If A is a theorem, so is LA

$\otimes$I: If $A\Rightarrow(B\Rightarrow C)$ is a theorem, so is $(A\otimes B)\Rightarrow C$

$\otimes$E: If $(A\otimes B)\Rightarrow C$ is a theorem, so is $A\Rightarrow(B\Rightarrow C)$

TI: If A is a theorem, so is $T\Rightarrow A$

TE: If $T\Rightarrow A$ is a theorem, so is A

Hilbert Style Axiomatic Systems for Relevant Logics

♣ **The pure entailment (implication) fragments of relevant logics**

$T_{\Rightarrow} = \{E1, E2, E2', E3 \mid E3''\} + \Rightarrow E$

$E_{\Rightarrow} = \{E1, E2 \mid E2', E3 \mid E3', E4 \mid E4'\} + \Rightarrow E$
$E_{\Rightarrow} = \{E2', E3, E4''\} + \Rightarrow E$
$E_{\Rightarrow} = \{E1, E3, E4'''\} + \Rightarrow E$
$E_{\Rightarrow} = T_{\Rightarrow} + E4$ [E4: $(A\Rightarrow((B\Rightarrow C)\Rightarrow D))\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow D))$]
$E_{\Rightarrow} = T_{\Rightarrow} + E4'$ [E4′: $(A\Rightarrow B)\Rightarrow(((A\Rightarrow B)\Rightarrow C)\Rightarrow C)$]
$E_{\Rightarrow} = T_{\Rightarrow} + E4''$ [E4″: $((A\Rightarrow A)\Rightarrow B)\Rightarrow B$]

$R_{\Rightarrow} = \{E1, E2 \mid E2', E3 \mid E3', E5 \mid E5'\} + \Rightarrow E$
$R_{\Rightarrow} = E_{\Rightarrow} + E5''$ [E5″: $A\Rightarrow((A\Rightarrow A)\Rightarrow A) = A\Rightarrow LA$]

Hilbert Style Axiomatic Systems for Relevant Logics

♣ **The entailment (implication) with negation fragments of relevant logics**

$T_{\Rightarrow,\neg} = T_{\Rightarrow} + \{N1, N2, N3\}$

$E_{\Rightarrow,\neg} = E_{\Rightarrow} + \{N1, N2, N3\}$

$R_{\Rightarrow,\neg} = R_{\Rightarrow} + \{N2, N3\}$

♣ **The positive (negation-free) fragments of relevant logics**

$T_{+} = T_{\Rightarrow} + \{C1\sim C3, D1\sim D3, DCD\} + \wedge I$

$E_{+} = E_{\Rightarrow} + \{C1\sim C4, D1\sim D3, DCD\} + \wedge I$
$E_{+} = T_{+} + \{E4 \mid E4' \mid E4'', C4\}$

$R_{+} = R_{\Rightarrow} + \{C1\sim C3, D1\sim D3, DCD\} + \wedge I$
$R_{+} = E_{+} + E5''$

Hilbert Style Axiomatic Systems for Relevant Logics

♣ **Propositional relevant logics**

$T = T_{\Rightarrow,\neg} + \{C1\sim C3, D1\sim D3, DCD\} + \wedge I$

$E = E_{\Rightarrow,\neg} + \{C1\sim C4, D1\sim D3, DCD\} + \wedge I$
$E = T + \{E4 \mid E4' \mid E4'', C4\}$

$R = R_{\Rightarrow,\neg} + \{C1\sim C3, D1\sim D3, DCD\} + \wedge I$
$R = E + A\Rightarrow LA$, $LR = R - DCD$

EM = E + EM0 (semi-relevant logic)

RM = R + RM0 (semi-relevant logic)
RM = EM + $A\Rightarrow LA$ (semi-relevant logic)

$E^{L} = E + \{L1\sim L5\} + LI$

$R^{L} = R + \{L1\sim L4\} + LI$

Hilbert Style Axiomatic Systems for Relevant Logics

♣ **Predicate relevant logics**

$S^{\forall\exists x} = S + \{IQ1, IQ3, IQ4, IQ7, IQ8\}$
where $S = T_{\neg}, T_{\Rightarrow,\neg}, R_{\neg}, R_{\Rightarrow,\neg}$

$S^{\forall\exists x} = S + \{IQ1\sim IQ5, IQ7\sim IQ9\}$
where $S = T_{+}, T, R_{+}, R, RM$

$S^{\forall\exists x} = S + \{IQ1, IQ3, IQ4, IQ6\sim IQ8\}$
where $S = E_{\neg}, E_{\Rightarrow,\neg}$

$S^{\forall\exists x} = S + \{IQ1\sim IQ9\}$
where $S = E_{+}, E, EM$

Hilbert Style Axiomatic Systems for Relevant Logics

♣ **Axiom schemata on conjunction**

C5: $(A\wedge A)\Rightarrow A)$
C6: $(A\wedge B)\Rightarrow(B\wedge A)$
C7: $((A\Rightarrow B)\wedge(B\Rightarrow C))\Rightarrow(A\Rightarrow C)$
C8: $(A\wedge(A\Rightarrow B))\Rightarrow B)$
C9: $\neg(A\wedge\neg A)$
C10: $A\Rightarrow(B\Rightarrow(A\wedge B))$

♣ **Strong relevant logics**

$Tc =_{df} T_{\Rightarrow,\neg} + \{C3, C5\sim C10\}$ $TcQ =_{df} Tc + \{IQ1\sim IQ5\} + \forall I$

$Ec =_{df} E_{\Rightarrow,\neg} + \{C3\sim C10\}$ $EcQ =_{df} Ec + \{IQ1\sim IQ5\} + \forall I$

$Rc =_{df} R_{\Rightarrow,\neg} + \{C3, C5\sim C10\}$ $RcQ =_{df} Rc + \{IQ1\sim IQ5\} + \forall I$

Strong Relevant Logic as the Universal Basis of Various Applied Logics

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- Spatial Relevant Logics
- Spatio-temporal Relevant Logics
- Deontic Relevant Logics
- Epistemic Relevant Logics
- Concluding Remarks
- Bibliography

Temporal Logic: What Is It and Why Study It?

- **Temporal logic: What is it?**
  - Temporal logic, or tense logic, is a branch of modal logic.
  - Temporal logic deal with the validity of reasoning that is relevant to time (tense) attributes of objects.
- **Temporal logic: Why study it?**
  - Time is one of the most fundamental notions in our cognition about the real world.
  - The ability of representing and reasoning about temporal knowledge conceptually is one of the most intrinsic characteristics of human intelligence.
  - No account of reasoning can properly be considered to be complete if it does not say something about how we reason about change.

Why Temporal Relevant Logics ?

- **Classical temporal logics**
  - Any of classical temporal logics is a classical conservative extension of CML, and therefore, has those problems of CML.
- **Temporal relevant logics**
  - Introducing the temporal operators and related axiom schemata and inference rules into strong relevant logics.

Temporal Relevant Logics

- **Temporal operators**
  - ***G*** (future-tense always or henceforth operator, GA means 'it will always be the case in the future from now that A')
  - ***H*** (past-tense always operator, HA means 'it has always been the case in the past up to now that A')
  - ***F*** (future-tense sometime or eventually operator, FA means 'it will be the case at least once in the future from now that A')
  - ***Pa*** (past-tense sometime operator, PaA means 'it has been the case at least once in the past up to now that A')
- **Relationships among temporal operators**
  - $GA =_{df} \neg F \neg A$, $FA =_{df} \neg G \neg A$,
    $HA =_{df} \neg Pa \neg A$, $PaA =_{df} \neg H \neg A$

Temporal Relevant Logics

- **Axiom schemata**

T1: $G(A \Rightarrow B) \Rightarrow (GA \Rightarrow GB)$
T2: $H(A \Rightarrow B) \Rightarrow (HA \Rightarrow HB)$
T3: $A \Rightarrow G(PA)$
T4: $A \Rightarrow H(FA)$
T5: $GA \Rightarrow G(GA)$
T6: $(FA \wedge FB) \Rightarrow F(A \wedge FB) \vee F(A \wedge B) \vee F(FA \wedge B)$
T7: $(PaA \wedge PaB) \Rightarrow Pa(A \wedge PaB) \vee Pa(A \wedge B) \vee Pa(PaA \wedge B)$
T8: $GA \Rightarrow FA$
T9: $HA \Rightarrow PA$
T10: $FA \Rightarrow F(FA)$
T11: $(A \wedge HA) \Rightarrow F(HA)$
T12: $(A \wedge GA) \Rightarrow Pa(GA)$

- **Inference rules**

TG: from A to infer GA and HA (Temporal Generalization)

Temporal Relevant Logics

- **The Minimal (or weakest) propositional temporal relevant logics**
  - T_0Tc = Tc + {T1~T4} + TG
  - T_0Ec = Ec + {T1~T4} + TG
  - T_0Rc = Rc + {T1~T4} + TG
- **Various propositional temporal relevant logics**
  - Add characteristic axioms such as T5~T12 that correspond to various assumptions about time into T_0Tc, T_0Ec, and T_0Rc respectively.

Temporal Relevant Logics

- **The Minimal (or weakest) predicate temporal relevant logics**
  - T_0TcQ = T_0Tc + {IQ1~IQ5} + ∀I
  - T_0EcQ = T_0Ec + {IQ1~IQ5} + ∀I
  - T_0RcQ = T_0Rc + {IQ1~IQ5} + ∀I
- **Various predicate temporal relevant logics**
  - Add characteristic axioms such as T5~T12 that correspond to various assumptions about time into T_0TcQ, T_0EcQ, and T_0RcQ respectively.

Applications of Temporal Relevant Logics

- **Characteristics of temporal relevant logics**
  - They can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and temporal reasoning (in particular, anticipatory reasoning).
- **Applications of temporal relevant logics**
  - They can be used as the fundamental logic to underlie reasoning about dynamics in a knowledge-based system where not only truth-values of propositions and/or formulas but also relevant relationships between them may depend on time.
  - Probably, the most important application of temporal relevant logics is to underlie anticipatory reasoning in prediction.

Strong Relevant Logic as the Universal Basis of Various Applied Logics

Spatial Logic: What Is It and Why Study It?

- **Spatial logic: What is it?**
  - Until now, any spatial logic is a conservative extension of some base logic.
  - Spatial logic deal with the validity of reasoning that is relevant to space (region) attributes of objects.
- **Spatial logic: Why study it?**
  - Space is another one of the most fundamental notions in our cognition about the real world.
  - The ability of representing and reasoning about spatial knowledge conceptually is one of the most intrinsic characteristics of human intelligence.
  - No account of reasoning can properly be considered to be complete if it does not say something about how we reason about space (move).

Why Spatial Relevant Logics ?

- **Classical spatial logics**
  - Any of classical spatial logics is a classical conservative extension of CML, and therefore, has those problems of CML.
- **Spatial relevant logics**
  - The logics are obtained by introducing region connection predicates and axiom schemata, point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into predicate strong relevant logics.
  - There is no propositional spatial relevant logics.

Spatial Relevant Logics

- **Many-sorted language**
  - Solid-region variables : $\{r_1, r_2, r_3, \ldots\}$
  - Point variables : $\{p_1, p_2, p_3, \ldots\}$
  - Object variables : $\{o_1, o_2, o_3, \ldots\}$
  - Individual constant of point *TCP* : the central point
  - Primitive and defined binary predicates about solid-region connection, point position, and motion of mobile objects

Spatial Relevant Logics

- **Primitive binary predicates about spatiality**
  - *C* : connection, $C(r_1, r_2)$ means 'r_1 connects with r_2'
  - *I* : inclusion, $I(p_1, r_1)$ means 'p_1 is included in r_1'
  - *Id* : the same point, $Id(p_1, p_2)$ means 'p_1 is identical with p_2'
  - *Arc* : arc, $Arc(p_1, p_2)$ means 'points p_1, p_2 are adjacent such that there is an arc from point p_1 to point p_2, or more simply, point p_1 is adjacent to point p_2'
  - *Reachable* : Reachable, $Reachable(p_1, p_2)$ means 'there is at least one directed path (i.e., a sequence of arcs such that one connects to the next one) from point p_1 to point p_2'

Spatial Relevant Logics

♣ **Primitive binary predicates about spatiality**

- ***NH*** : not higher than, $NH(p_1, p_2)$ means 'taking *TCP* as the reference point, the position of point p_1 is not higher than the position of point p_2, i.e., the length of the vertical line from p_1 to *TCP* is not longer than the length of the vertical line from p_2 to *TCP*'
- ***A*** : arrives at, $A(o_1, p_1)$ means 'object o_1 arrives at point p_1'
- ***NS*** : not speedier than, $NS(o_1, o_2)$ means 'the speed of object o_1 is not faster than the speed of object o_2'

Spatial Relevant Logics

♣ **Defined binary predicates about spatiality**

- $DC(r_1, r_2)$ means 'r_1 is disconnected from r_2'
- $Part(r_1, r_2)$ means 'r_1 is a part of r_2'
- $PrPart(r_1, r_2)$ means 'r_1 is a proper part of r_2'
- $EQ(r_1, r_2)$ means 'r_1 is identical with r_2'
- $Overlap(r_1, r_2)$ means 'r_1 overlaps r_2'
- $DR(r_1, r_2)$ means 'r_1 is discrete from r_2'
- $PaOverlap(r_1, r_2)$ means 'r_1 partially overlaps r_2'
- $EC(r_1, r_2)$ means 'r_1 is externally connected to r_2'
- $TPrPart(r_1, r_2)$ means 'r_1 is a tangential proper part of r_2'
- $NTPrPart(r_1, r_2)$ means 'r_1 is a nontangential proper part of r_2'

Spatial Relevant Logics

♣ **Defined binary predicates about spatiality**

- $SA(p_1, p_2)$ means 'the position of point p_1 and the position of point p_2 are in the same altitude'
- $Hi(p_1, p_2)$ means 'the position of point p_1 is higher than the position of point p_2'
- $SS(o_1, o_2)$ means 'the motion of object o_1 and the motion of object o_2 are in the same speed'
- $Sp(o_1, o_2)$ means 'the motion of object o_1 is faster than the motion of object o_2'
- $ND(p_1, p_2, p_3)$ means 'the distance of between point p_2 and point p_3 is not more distant than the distance of between point p_1 and point p_3'

Spatial Relevant Logics

♣ **Defined binary predicates about spatiality**

- $SD(p_1, p_2, p_3)$ means 'the distance of between point p_1 and point p_3 is equal to the distance of between point p_2 and point p_3'
- $Ne(p_1, p_2, p_3)$ means 'the distance of between point p_1 and point p_3 is nearer than the distance of between point p_2 and point p_3'

Spatial Relevant Logics

♣ **Axiom schemata about solid-region connection and point position**

- RCC1: $\forall r_1(C(r_1, r_1))$
- RCC2: $\forall r_1\forall r_2(C(r_1, r_2)\Rightarrow C(r_2, r_1))$
- PRCC1: $\forall p_1\forall r_1\forall r_2((I(p_1, r_1)\wedge DC(r_1, r_2))\Rightarrow\neg I(p_1, r_2))$
- PRCC2: $\forall p_1\forall r_1\forall r_2((I(p_1, r_1)\wedge Part(r_1, r_2))\Rightarrow I(p1, r_2))$
- PRCC3: $\forall r_1\forall r_2(O(r_1, r_2)\Rightarrow\exists p_1(I(p_1, r_1)\wedge I(p_1, r_2)))$
- PRCC4: $\forall r_1\forall r_2(PaOverlap(r_1, r_2)\Rightarrow(\exists p_1(I(p_1, r_1)\wedge I(p_1, r_2))\wedge \exists p_2(I(p_2, r_1)\wedge\neg I(p_2, r_2))\wedge\exists p_3(\neg I(p_3, r_1)\wedge I(p_3, r_2))))$
- PRCC5: $\forall r_1\forall r_2(EC(r_1, r_2)\Rightarrow\exists p_1(I(p_1, r_1)\wedge I(p_1, r_2)\wedge \forall p_2(\neg Id(p_2, p_1)\Rightarrow(\neg I(p_2, r_1)\wedge\neg I(p_2, r_2)))))$
- PRCC6: $\forall p_1\forall r_1\forall r_2((I(p_1, r_1)\wedge TPrPart(r_1, r_2))\Rightarrow I(p_1, r_2))$
- PRCC7: $\forall p_1\forall r_1\forall r_2((I(p_1, r_1)\wedge NTPrPart(r_1, r_2))\Rightarrow I(p_1, r_2))$

Spatial Relevant Logics

♣ **Axiom schemata about solid-region connection, point position, and motion of mobile objects**

- RC1: $\forall p_1\forall p_2(Arc(p_1, p_2)\Rightarrow Reachable(p_1, p_2))$
- RC2: $\forall p_1\forall p_2\forall p_3((Reachable(p_1, p_2)\wedge Reachable(p_2, p_3))\Rightarrow Reachable(p_1, p_3))$
- HC1: $\forall p_1(NH(p_1, p_1))$
- HC2: $\forall p_1\forall p_2\forall p_3((NH(p_1, p_2)\wedge NH(p_2, p_3))\Rightarrow NH(p_1, p_3))$
- MC1: $\forall o_1(NS(p_1, p_1))$
- MC2: $\forall o_1\forall o_2\forall o_3((NS(o_1, o_2)\wedge NS(o_2, o_3))\Rightarrow NS(o_1, o_3))$
- DC1: $\forall p_1\forall p_3(ND(p_1, p_1, p_3))$
- DC2: $\forall p_1\forall p_2\forall p_3\forall p_4((ND(p_1, p_2, p_4)\wedge ND(p_2, p_3, p_4))\Rightarrow ND(p_1, p_3, p_4))$

Spatial Relevant Logics

Spatial relevant logics

- RTcQ = TcQ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2}
- REcQ = EcQ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2}
- RRcQ = RcQ + {RCC1, RCC2, PRCC1~PRCC7, RC1, RC2, HC1, HC2, MC1, MC2, DC1, DC2}

Spatial relevant logics as conservative extensions

- Spatial relevant logics are conservative extensions of strong relevant logics.

Applications of Spatial Relevant Logics

Characteristics of spatial relevant logics

- They can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and spatial reasoning.

Applications of spatial relevant logics

- They can be used as the fundamental logic to underlie reasoning about geometric and/or topological entities, notions, relations, and properties.
- Automated theorem finding in geometry or topology.
- Geographic information systems.

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Why Spatio-temporal Relevant Logics ?

Space and time as the most fundamental notions

- Space and time are the most fundamental notions in our cognition about the real world.
- Spatio-temporal logic deal with the validity of reasoning that is relevant to both time (tense) and space (region) attributes of objects.
- To represent, specify, verify, and reason about spatial objects and relationships among them that may change over time, a right fundamental logic to underlie both spatial reasoning and temporal reasoning is indispensable.

Classical spatio-temporal logics

- Any of classical spatio-temporal logics is a classical conservative extension of CML, and therefore, has those problems of CML.

Spatio-temporal Relevant Logics

Spatio-temporal relevant logics

- The logics are obtained by introducing region connection predicates and axiom schemata of RCC, point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into predicate temporal relevant logics.
- There is no propositional spatio-temporal relevant logics.
- $ST_0TcQ = T_0TcQ$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}
- $ST_0EcQ = T_0EcQ$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}
- $ST_0RcQ = T_0RcQ$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}

Spatio-temporal relevant logics as conservative extensions

- Spatio-temporal relevant logics are conservative extensions of strong relevant logics, temporal relevant logics, and spatial relevant logics.

Applications of Spatio-temporal Relevant Logics

Characteristics of spatio-temporal relevant logics

- They can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, spatial reasoning, and temporal reasoning.

Applications of spatio-temporal relevant logics

- Representing and reasoning about mobile agents with incomplete or even inconsistent knowledge acting concurrently in spatial regions changing over time.
- We can add epistemic operators and related axiom schemata into the logics in order to reason about epistemic interaction among mobile agents.

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Deontic Logic: What Is It and Why Study It?

- Deontic logic: What is it?
  - Deontic logic is a branch of modal logic (the normative notions obligation (ought), permission (may), and prohibition (may not) are related to each other the alethic modalities necessity, possibility, and impossibility).
  - Deontic logic investigates normative notions and deal with the validity of normative reasoning that is relevant to normative attributes of objects.
  - The word "deontic" is derived from the Greek expression "deon", which means "what is binding" or "proper".
- Deontic logic: Why study it?
  - Deontic logics often directly involve topics of considerable practical significance such as morality, law, social and business organizations (their norms, as well as their normative constitution), and security systems.

Why Deontic Relevant Logics ?

- Classical deontic logics
  - Any of classical deontic logics is a classical conservative extension of CML, and therefore, has those problems of CML.
  - Deontic paradoxes.
- Deontic relevant logics
  - Introducing the deontic operators and related axiom schemata and inference rules into strong relevant logics.

Deontic Relevant Logics

- Deontic operators
  - O (obligation operator, OA means 'it is obligatory that A')
  - Pe (permission operator, PeA means 'it is permitted that A')
  - $PeA =_{df} \neg O(\neg A)$
- Axiom schemata

  DR1: $O(A \Rightarrow B) \Rightarrow (OA \Rightarrow OB)$
  DR2: $OA \Rightarrow PA$
  DR3: $\neg(OA \wedge O\neg A)$
  DR4: $O(A \wedge B) \Rightarrow (OA \wedge OB)$
  DR5: $Pe(A \wedge B) \Rightarrow (PeA \wedge PeB)$
- Inference rules

  O-necessitation: "if A is a logical theorem, then so is OA" (Deontic Generalization)

Deontic Relevant Logics

- Propositional deontic relevant logics
  - DTc = Tc + {DR1~DR5} + O-necessitation
  - DEc = Ec + {DR1~DR5} + O-necessitation
  - DRc = Rc + {DR1~DR5} + O-necessitation
- Predicate deontic relevant logics
  - DTcQ = DTc + {IQ1~IQ5} + ∀I
  - DEcQ = DEc + {IQ1~IQ5} + ∀I
  - DRcQ = DRc + {IQ1~IQ5} + ∀I

Applications of Deontic Relevant Logics

- Characteristics of deontic relevant logics
  - They can underlie truth-preserving and relevant reasoning in the sense of conditional, ampliative reasoning, paracomplete and paraconsistent reasoning, and normative reasoning.
- Actual behavior vs ideal (or normative) behavior
  - In general, the actual behavior (as it is) of a computing system in its running is somewhat different from the ideal (or normative) behavior (as it should be) of the system which is specified by requirements of the system.
  - Therefore, to distinguish between ideal behavior and actual behavior of a computing system is important to defining what behavior is illegal and specifying what should be done if such illegal but possible behavior occurs.

Applications of Deontic Relevant Logics

- **Applications of deontic relevant logics in SE**
  - Deontic relevant logics can be used as the fundamental logic to underlie specifying, verifying, and reasoning about ideal and actual behavior and dynamics of computing systems.
- **Applications of deontic relevant logics in KE**
  - Deontic relevant logics can be used as the fundamental logic to represent, specify, verify, reason about, and ensure various laws, legal rules, and precedents in legal information systems, to make legal decisions based on legal information systems, and to discover new legal knowledge from legal information systems.

Applications of Deontic Relevant Logics

- **Applications of deontic relevant logics in ISE**
  - Information security engineering is intrinsically an engineering discipline to deal with normative requirements and their implementation and verification (validation) techniques for secure information systems.
  - Deontic relevant logics can be used as the fundamental logic to underlie specifying, verifying, and reasoning about information security and information assurance, ideal and actual behavior and dynamics of computing systems including behavior of attackers.

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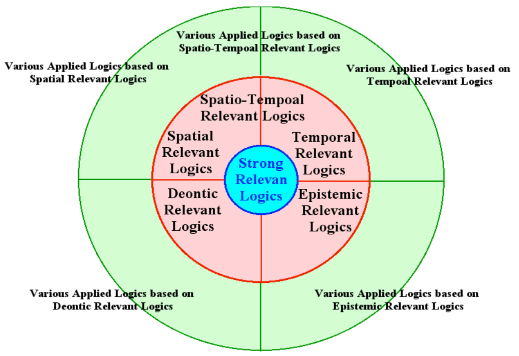
- Bibliography

Concluding Remarks

- **Spatial relevant logics as a model example**
  - Various applied logics can be constructed in the same way.
- **Reciprocal logics**
  - Logics for specifying, verifying, and reasoning about various reciprocal relationships.
- **Temporal deontic relevant Logics**
  - Logics for specifying, verifying, and reasoning about ideal and actual behavior and dynamics of computing systems.

Strong Relevant Logics as the Core Logic



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- Spatial Relevant Logics
- Spatio-temporal Relevant Logics
- Concluding Remarks
- **Bibliography**

Bibliography: Pioneers' Seminal / Primitive Works

- W. Ackermann, "Begrundung Einer Strengen Implikation," The Journal of Symbolic Logic, Vol. 21, pp. 113-128, 1956 (in German).
- S-K. Moh, "The Deduction Theorems and Two New Logical Systems," Methodos, Vol. 2, pp. 56-75, 1950.
- A. Church, "The Weak Theory of Implication," in A. Menne, A. Wilhelmy, and H. Angsil (Eds.), "Kontrolliertes Denken, Untersuchungen zum Logikkalkul und zur Logik der Einzelwissenschaften," pp. 22-37, 1951.
- I. E. Orlov, "The Calculus of Compatibility of Propositions," Matematicheskii Sbornik, Vol. 35, pp. 263-286, 1928 (in Russian). (Known by the community of relevant logic from a report in 1990 by K. Dosen)

Bibliography: Books

- A. R. Anderson and N. D. Belnap Jr., "Entailment: The Logic of Relevance and Necessity," Vol. I, Princeton University Press, Princeton, 1975.
- M. R. Diaz, "Topics in the Logic of Relevance," Philosophia Verlag, München, 1981.
- R. Routley, V. Plumwood, R. K. Meyer, and R. T. Brady, "Relevant Logics and their Rivals, Part I, The Basic Philosophical and Semantical Theory," Ridgeview, Atascadero, California, 1982.
- P. B. Thistlewaite, M. A. McRobbie, and R. K. Meyer, "Automated Theorem Proving in Non-Classical Logics," Pitman, London, 1988.
- S. Read, "Relevant Logic: A Philosophical Examination of Inference," Basil Blackwell, Oxford, 1988, 2012 (Corrected Edition).
- J. Norman and R. Sylvan (Eds.), "Directions in Relevant Logic," Kluwer Academic, Dordrecht, 1989.
- A. R. Anderson, N. D. Belnap Jr., and J. M. Dunn, "Entailment: The Logic of Relevance and Necessity," Vol. II, Princeton University Press, Princeton, 1992.

Bibliography: Books

- G. Restall, "An Introduction to Substructural Logics," Routledge, London, 2000.
- G. Priest, "An Introduction to Non-Classical Logic: From If to Is," Cambridge University Press, Cambridge, 2001, 2008 (2nd Edtion).
- D. M. Gabbay and J. Woods, "Agenda Relevance: A Study in Formal Pragmatics," Elsevier, Amsterdam, 2003.
- R. Brady (Ed.), "Relevant Logics and their Rivals, Volume II, A Continuation of the Work of Richard Sylvan, Robert Meyer, Val Plumwood, and Ross Brady," Ashgate Publishing, Farnham, 2003.
- E. D. Mares, "Relevant Logic: A Philosophical Interpretation," Cambridge University Press, Cambridge, 2004.

Bibliography: Survey Articles

- J. M. Dunn, "Relevance Logic and Entailment," in D. Gabbay and F. Guenthner (eds.), "Handbook of Philosophical Logic," Vol. III, pp. 117-224, D. Reidel, Dordrecht, 1986.
- E. D. Mares and R. K. Meyer, "Relevant Logics," in L. Goble (Ed.), "The Blackwell Guide to Philosophical Logic," pp. 280-308, Blackwell, Oxford, 2001.
- E. D. Mares, "Relevance Logic," in D. Jacquette (Ed.), "A Companion to Philosophical Logic," pp. 609-627, Blackwell, Oxford, 2002.
- J. M. Dunn and G. Restall, "Relevance Logic," in: D. Gabbay and F. Guenthner (Eds.), "Handbook of Philosophical Logic, 2nd Edition," Vol. 6, pp. 1-128, Kluwer Academic, Dordrecht, 2002.
- K. Bimbo, "Relevance Logics," in D. Jacquette (Ed.), "Philosophy of Logic," pp. 723-789, Elsevier, Amsterdam, 2007.
- E. D. Mares, "Relevance Logic," in Stanford Encyclopedia of Philosophy, Center for the Study of Language and Information (CSLI), Stanford University, 2012.

Bibliography: Strong Relevant (Relevance) Logic

- J. Cheng, "Logical Tool of Knowledge Engineering: Using Entailment Logic rather than Mathematical Logic," Proceedings of the 19th ACM Annual Computer Science Conference, pp. 228-238, San Antonio, USA, March 1991.
- J. Chng, "Rc -- A Relevant Logic for Conditional Relation Representation and Reasoning," Proceedings of the 1st Singapore International Conference on Intelligent Systems, pp. 171-176, Singapore, September 1992.
- J. Cheng, "Entailment Calculus as a Logical Tool for Reasoning Rule Generation and Verification," in J. Liebowitz (Ed.), "Moving Towards Expert Systems Globally in the 21st Century - Refereed papers accepted and presented at the 2nd World Congress on Expert Systems, Lisbon, Portugal, January 1994," pp. 386-392, Cognizant Communication Co., March 1994.
- J. Cheng, "The Fundamental Role of Entailment in Knowledge Representation and Reasoning,"Journal of Computing and Information, Vol. 2, No. 1, Special Issue: Proceedings of the 8th International Conference of Computing and Information, pp. 853-873, June 1996.
- J. Chng, "Relevant Reasoning as the Logical Basis of Knowledge Engineering," in F. J. Cantu, R. Soto, J. Liebowitz, and E. Sucar (Eds.), "Application of Advanced Information Technologies, -- 4th World Congress on Expert Systems, Mexico City, Mexico, March 1998," Vol.1, pp. 449-457, Cognizant Communication Co., March 1998.

Bibliography: Strong Relevant (Relevance) Logic

- J. Chng, "A Strong Relevant Logic Model of Epistemic Processes in Scientific Discovery," in E. Kawaguchi, H. Kangassalo, H. Jaakkola, and I. A. Hamid (Eds.), "Information Modelling and Knowledge Bases XI," Frontiers in Artificial Intelligence and Applications, Vol. 61, pp. 136-159, IOS Press, February 2000.
- J. Cheng, "Automated Knowledge Acquisition by Relevant Reasoning Based on Strong Relevant Logic," in V. Palade, R. J. Howlett, and L. C. Jain (Eds.), "Knowledge-Based Intelligent Information & Engineering Systems, 7th International Conference, KES 2003, Oxford, UK, September 3-5, 2003, Proceedings, Part I," Lectures Notes in Artificial Intelligence, Vol. 2773, pp. 68-80, Springer-Verlag, September 2003.
- J. Cheng, "Strong Relevant Logic as the Universal Basis of Various Applied Logics for Knowledge Representation and Reasoning," J. Henno, H. Jaakkola, H. Kangassalo, and Y. Kiyoki (Eds.), "Information Modelling and Knowledge Bases XVII," Frontiers in Artificial Intelligence and Applications, Vol. 136, pp. 310-320, IOS Press, February 2006.
- J. Cheng, "Representing, Managing, and Reasoning about Mathematical Knowledge based on Strong Relevant Logic," Proceedings of the 7th International Conference on Machine Learning and Cybernetics, pp. 299-306, Kunming, China, IEEE Systems, Man, and Cybernetics Society, July 2008.

Bibliography: Temporal Relevant Logic

- J. Cheng, "Temporal Relevant Logic as the Logic Basis for Reasoning about Dynamics of Concurrent Systems," Proceedings of the 1998 IEEE-SMC Annual International Conference on Systems, Man, and Cybernetics, Vol. 1, pp. 794-799, San Diego, USA, October 1998.
- J. Cheng, "Temporal Relevant Logic: What Is It and Why Study It?" Abstracts of the 11th IUHPS/DLMPS International Congress of Logic, Methodology and Philosophy of Science, p. 253, Cracow, Poland, August 1999.
- J. Cheng, "Temporal Relevant Logic as the Logical Basis of Anticipatory Reasoning-Reacting Systems (Invited Paper)," in D. M. Dubois (Ed.), "Computing Anticipatory Systems: CASYS 2003 - Sixth International Conference, Liege, Belgium, 11-16 August 2003," AIP Conference Proceedings, Vol. 718, pp. 362-375, American Institute of Physics, August 2004.

Bibliography: Spatial Relevant Logic

- J. Cheng and Y. Goto, "Representing and Reasoning about Spatial Knowledge Based on Spatial Relevant Logic," in S. Wang, K. Tanaka, et al (Eds.), "Conceptual Modeling for Advanced Application Domains, ER 2004 Workshops CoMoGIS, CoMWIM, ECDM, CoMoA, DGOV, and eCOMO, Shanghai, China, November 2004, Proceedings," Lecture Notes in Computer Science, Vol. 3289, pp. 114-126, Springer-Verlag, November, 2004.

Bibliography: Spatio-temporal Relevant Logic

- J. Cheng, "Spatio-temporal Relevant Logic as the Logical Basis for Specifying, Verifying, and Reasoning about Mobile Malti-agent Systems," in S. Wang, K. Tanaka, et al (Eds.), "Conceptual Modeling for Advanced Application Domains, ER 2004 Workshops CoMoGIS, CoMWIM, ECDM, CoMoA, DGOV, and eCOMO, Shanghai, China, November 2004, Proceedings," Lecture Notes in Computer Science, Vol. 3289, pp. 470-483, Springer-Verlag, November, 2004.
- J. Cheng, "Spatio-temporal Relevant Logic as the Logical Basis for Spatio-temporal Information Systems," Proceedings of the 17th International Conference on Tools with Artificial Intelligence, pp. 270-274, Hong Kong, China, IEEE Computer Society Press, November 2005.
- J. Cheng, "Qualitative Spatio-temporal Reasoning about Moving Objects in Three-dimensional Space," in L. Kang, et al. (Eds.), "Advances in Computation and Intelligence: Third International Symposium, ISICA 2008, Wuhan, China, December 2008, Proceedings," Lecture Notes in Computer Science, Vol. 5370, pp. 637–648, Springer-Verlag, December 2008.

Bibliography: Deontic Relevant Logic

- T. Tagawa and J. Cheng, "Deontic Relevant Logic: A Strong Relevant Logic Approach to Removing Paradoxes from Deontic Logic," in M. Ishizuka and A. Sattar (Eds.), "PRICAI 2002: Trends in Artificial Intelligence, 7th Pacific Rim International Conference on Artificial Intelligence, Tokyo, Japan, August 18-22, 2002. Proceedings," Lectures Notes in Artificial Intelligence, Vol. 2417, pp. 39-48, Springer-Verlag, August 2002.
- J. Cheng, "Deontic Relevant Logic as the Logical Basis for Specifying and Reasoning about Information Security and Assurance" Abstracts of the 12th IUHPS/DLMPS International Congress of Logic, Methodology and Philosophy of Science, Oviedo, Spain, August 2003.
- J. Cheng and J. Miura, "Deontic Relevant Logic as the Logical Basis for Specifying, Verifying, and Reasoning about Information Security and Information Assurance," Proceedings of the 1st International Conference on Availability, Reliability and Security, pp. 601-608, Vienna, Austria, IEEE Computer Society Press, April 2006.
- J. Cheng, "Deontic Relevant Logic as the Logical Basis for Representing and Reasoning about Legal Knowledge in Legal Information Systems," in I. Lovrek, R. J. Howlett, and L. C. Jain (Eds.), "Knowledge-Based Intelligent Information and Engineering Systems, 12th International Conference, KES 2008, Zagreb, Croatia, September 3-5, 2007, Proceedings," Lecture Notes in Artificial Intelligence, Vol. 5178, pp. 517-525, Springer-Verlag, September 2008.

Bibliography: Temporal Deontic Relevant Logic

- J. Cheng, "Temporal Deontic Relevant Logic as the Logical Basis for Decision Making Based on Anticipatory Reasoning," Proceedings of the 2006 IEEE Annual International Conference on Systems, Man, and Cybernetics, pp. 1036-1041, Taipei, Taiwan, IEEE Systems, Man, and Cybernetics Society, October 2006.

Bibliography: Fuzzy Relevant Logic

- J. Cheng, "Fuzzy Relevant Logic: What Is It and Why Study It?" Proceedings of the 1999 IEEE-SMC Annual International Conference on Systems, Man, and Cybernetics, Vol. 5, pp. 250-255, Tokyo, Japan, October 1999.
- J. Cheng, "Fuzziness and Relevance: Can We Establish a Unified Logical Basis for Approximate and Relevant Reasoning?" Proceedings of the 4th Asian Fuzzy Systems Symposium, pp. 875-880, Tsukuba, Japan, June 2000.
- J. Cheng and Y. Goto, "A Strong Relevant Logic Approach to the Calculus of Fuzzy Conditionals," in D. Ruan, P. D'hondt, and E. E. Kerre (Ed.), "Computational Intelligent Systems for Applied Research," pp. 66-74, World Scientific, Singapore, September 2002.