Homework 2

Due Date

2016/06/01 (FRIDAY), 23:59 。LATE submission is not accepted.

　　You are allowed to use any taught algorithms that fit to solve the problems unless otherwise specified. Please be advised that it will not be graded if any of MATLAB's embedded functions for integration or differentiation is found.

Problem 1: Use numerical differentiation and integration to solve the following problems.

[Principal m-file name: F<your student id >_hw2_prob1.m]

1.　A 5-kg brick undergoes a damped oscillation. The velocity of the brick can be expressed by

$$\vec{v}(t) = -20e^{-0.01t^2} \sin\left(20\pi t + \frac{2\pi}{5}\right) \frac{m}{s} \; \hat{x} \quad \text{for} \quad t \geq 0$$

(a) Draw the v-t plot of the object. (put on appropriate axis label on the figure)

(b) Plot the temporal changes of the net force on the object, an F-t plot.
　　(put on appropriate axis label on the figure)

(c) Find $\vec{F}(8)$. The precision should be up to 6 significant digits.

(d) If $\vec{x}(t) = 0$, find $\vec{x}(10)$. The precision should be up to 8 significant digits.

(e) Find the total travel distance of the object from t = 0 to t = 10. The precision should be up to 8 significant digits.

　　Print out the result in the following format after executing the principal m file

　　(c)F(8) = xxxxxxx N

　　(d) x(10) = xxxxxx m

　　(e)The total travel length from t = 0 to 10 is xxxxxxxx　m

Problem 2:

　　Use "Finite Riemman Sum", "Trapezoid Method", or "Simpson's Method" to find out following integral. The precision should be up to 8 significant digits.

　　[Principal m-file name: F<your student id >_hw2_prob2.m]

(i)　　$\int_0^4 \sqrt{1 - x^2} \; dx$

(ii)　　$\int_0^\pi \frac{1}{\sqrt{1+2\sin(x)}} \; dx$

(iii)　　$\int_0^1 \frac{\sin(x)}{1+x^2} \; dx$

　　Print out the result in the following format after executing the principal m file
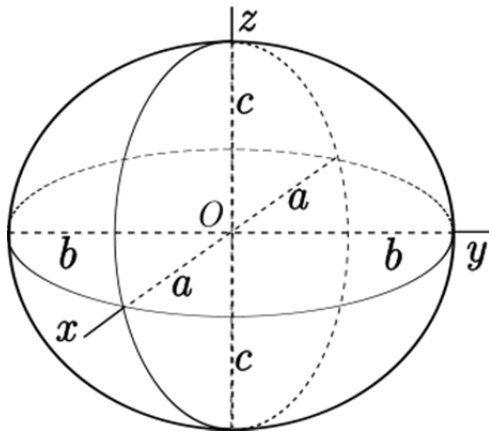
　　(i) xxxxxx

　　(ii) xxxxxx

　　(iii)......

Problem3: Monte Carlo Integration

[Principal m-file name: F<your student id >_hw2_prob3.m]

Use Monte Carlo Integration to estimate the volume of an ellipsoid enclosed by $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$

Print out the result in the following format after executing the principal m file

A total of xxxxx random points are used and the volume of the ellipsoid is xxxxxxx.

Problem4: Numerical Proof of Gauss's Law of Electrostatics

[F<your student id >_hw2_prob4.m]

The Gauss's Law of Electrostatics states that the electrical flux through a close surface should be proportional to the total amount of charge enclosed by the surface:

$$\oint_S \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0 \quad \text{or} \quad \oiint_S \vec{E} \cdot (d\vec{x} \cdot d\vec{y}) = Q_{enc}/\epsilon_0 \ .$$

Assume that there is a charged particle of 1 Coulomb placed at the origin (0,0,0) of a space . From homework1, you must have been familiar with calculating the electrical field of the charge in the space.

(1) Suppose there is a closed surface of square cube of which the center is right on the (0,0,0) and the length of the edge is 1m enclosing the charge. Calculate the total electric flux through the square cube.

(2) Find the total flux if each edge of the square cubic grows to 2m while its center remains at (0,0,0)

(3) Following (2), find the total flux if the charge moves to (0.4m, 0.4m,0.4m).

(4) Following (3), find the total flux if the charge moves to (2m, 2m, 2m).

Print out the result in the following format after executing the principal m file [at least 6 significant digits]

(1) The total flux is xxxxxxx $(Volt \cdot m)$.

(2) The total flux is xxxxxxx $(Volt \cdot m)$.

(3) The total flux is xxxxxxx $(Volt \cdot m)$.

(4) The total flux is xxxxxxx $(Volt \cdot m)$.

Contents to submit:

1. All the m-files you compose for the assignment. PLEASE DO NOT COMPRESS.
2. All the m-files should include proper COMMENTS.

   (No comment, no score)

3. A PDF includes Your Name, Your Student ID Number,

   In the document, you will need to include all the contents described above including: (1) Which method do you use to solve numerical differentiation and integration. (2) How should TA use the

programs to solve any specific differentiation or integration when needed (3) The answers including the plots, the step length you choose for each problem and the explanation of how you justify the precision of the answers should be included.

Notice:

1. DO NOT PLAGIARIZE. You are encouraged to ask and to discuss the homework content with your fellow classmates, the TAs and the instructor. But identical core program wording is NEVER ACCEPTABLE.

2. Upload all the files without archiving. Do not upload files that don't work well. Any missing file or function that leads to fail of the execution will be regarded as a program that never works.