

# 1. (20 points) The Deutsch-Jozsa Algorithm

(a) (10 points)

$$\begin{array}{c} |x\rangle \text{ --- } \boxed{I} \text{ --- } |x\rangle \\ |y\rangle \text{ --- } \text{---} |f(x) \oplus y\rangle \end{array}$$

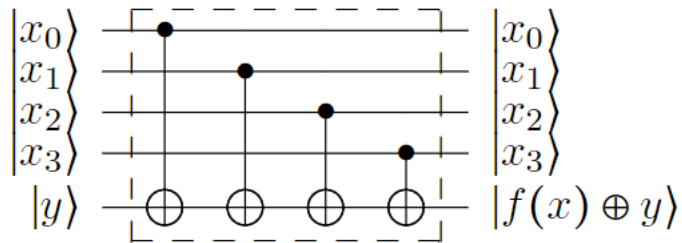
x0x1x2x3	f(x)⊕y	x0x1x2x3	f(x)⊕y
0000	0	1000	0
0001	0	1001	0
0010	0	1010	0
0011	0	1011	0
0100	0	1100	0
0101	0	1101	0
0110	0	1110	0
0111	0	1111	0

Constant function

$$\begin{array}{c} |x\rangle \text{ --- } \boxed{I} \text{ --- } |x\rangle \\ |y\rangle \text{ --- } \boxed{X} \text{ --- } |f(x) \oplus y\rangle \end{array}$$

x0x1x2x3	f(x)⊕y	x0x1x2x3	f(x)⊕y
0000	1	1000	1
0001	1	1001	1
0010	1	1010	1
0011	1	1011	1
0100	1	1100	1
0101	1	1101	1
0110	1	1110	1
0111	1	1111	1

Constant function

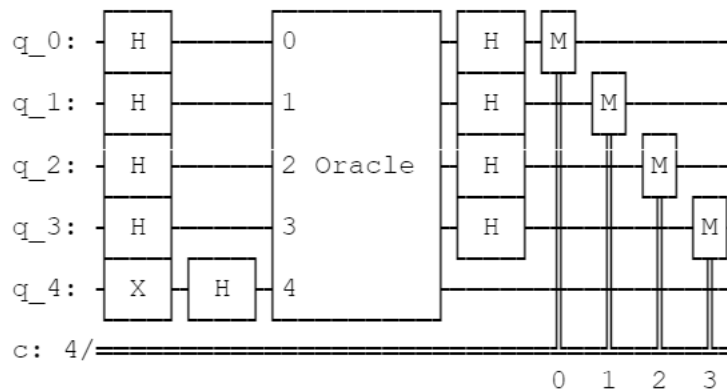


<b>x<sub>0</sub>x<sub>1</sub>x<sub>2</sub>x<sub>3</sub></b>	<b>f(x)⊕y</b>	<b>x<sub>0</sub>x<sub>1</sub>x<sub>2</sub>x<sub>3</sub></b>	<b>f(x)⊕y</b>
<b>0000</b>	<b>0</b>	<b>1000</b>	<b>1</b>
<b>0001</b>	<b>1</b>	<b>1001</b>	<b>0</b>
<b>0010</b>	<b>1</b>	<b>1010</b>	<b>0</b>
<b>0011</b>	<b>0</b>	<b>1011</b>	<b>1</b>
<b>0100</b>	<b>1</b>	<b>1100</b>	<b>0</b>
<b>0101</b>	<b>0</b>	<b>1101</b>	<b>1</b>
<b>0110</b>	<b>0</b>	<b>1110</b>	<b>1</b>
<b>0111</b>	<b>1</b>	<b>1111</b>	<b>0</b>

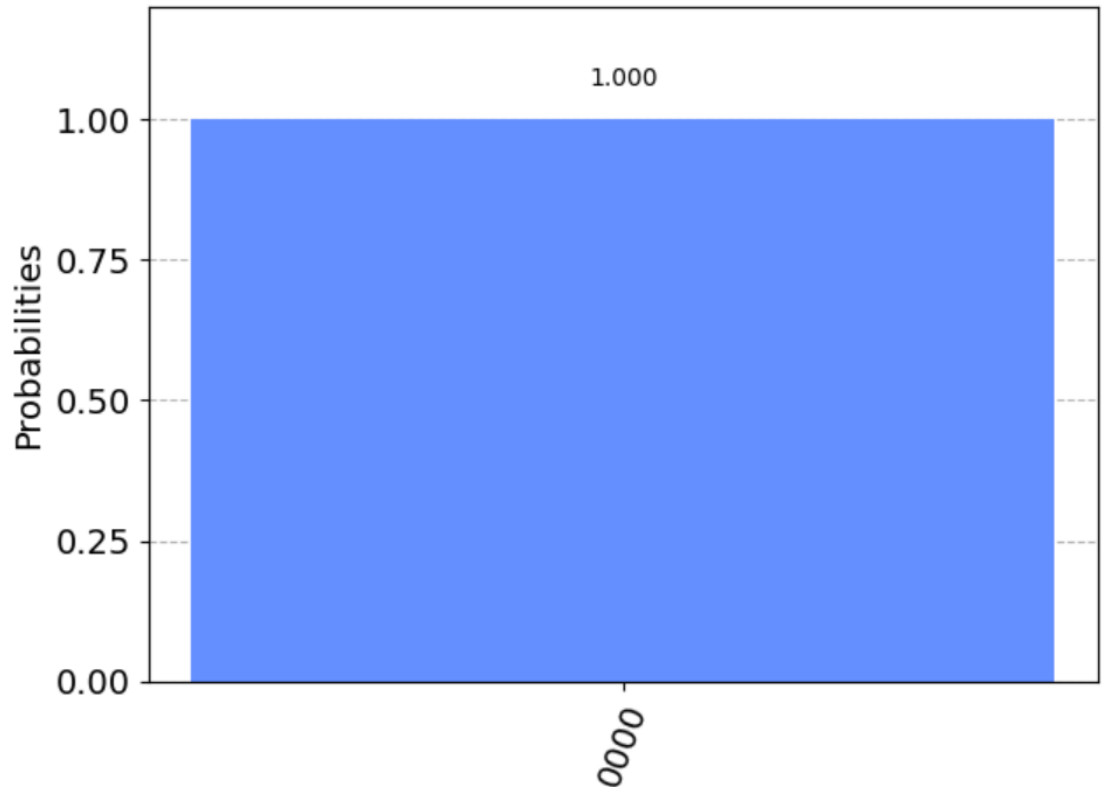
**Balanced function**

**(b)(15 points)**

**(use '1a\_1' for 'dj\_oracle')**

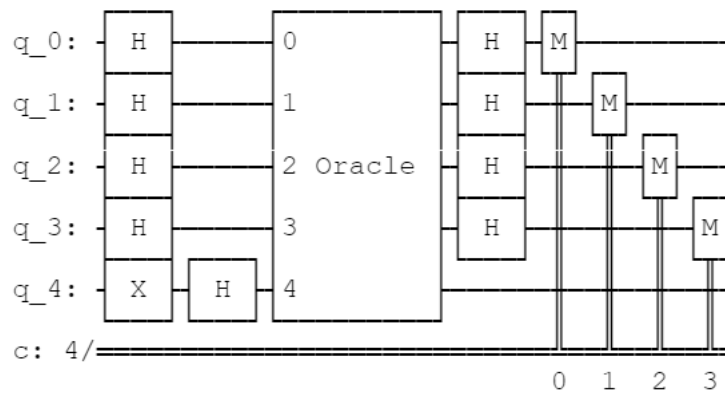


**Oracle content: I-gate on q<sub>0</sub> ~ q<sub>3</sub>**

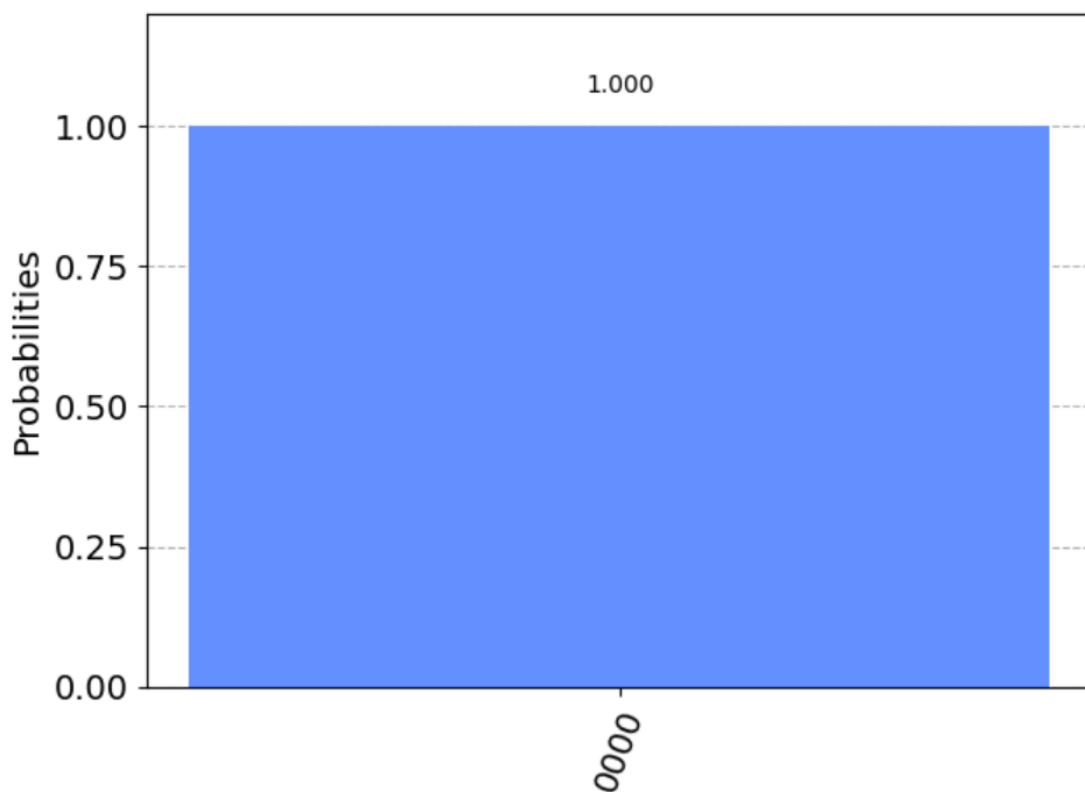


**Constant function all 0**

**(use '1a\_2' for 'dj\_oracle')**

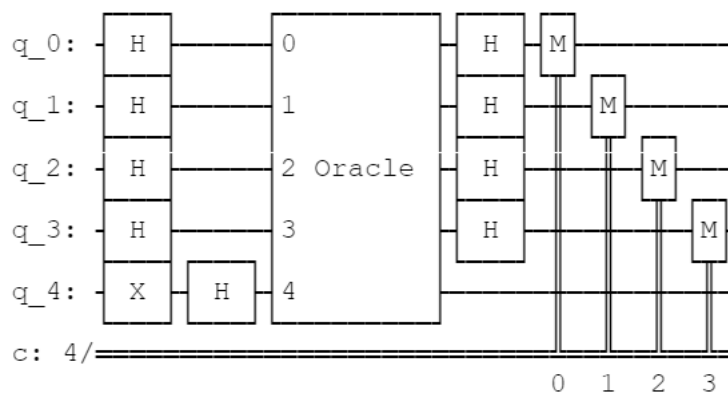


**Oracle content: I-gate on q\_0 ~ q\_3 ; X-gate on q\_4**

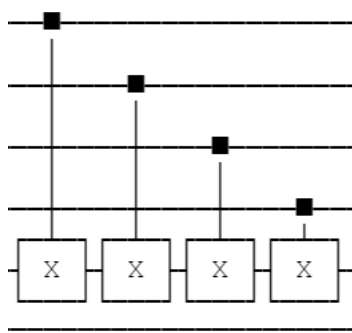


**Constant function all 0**

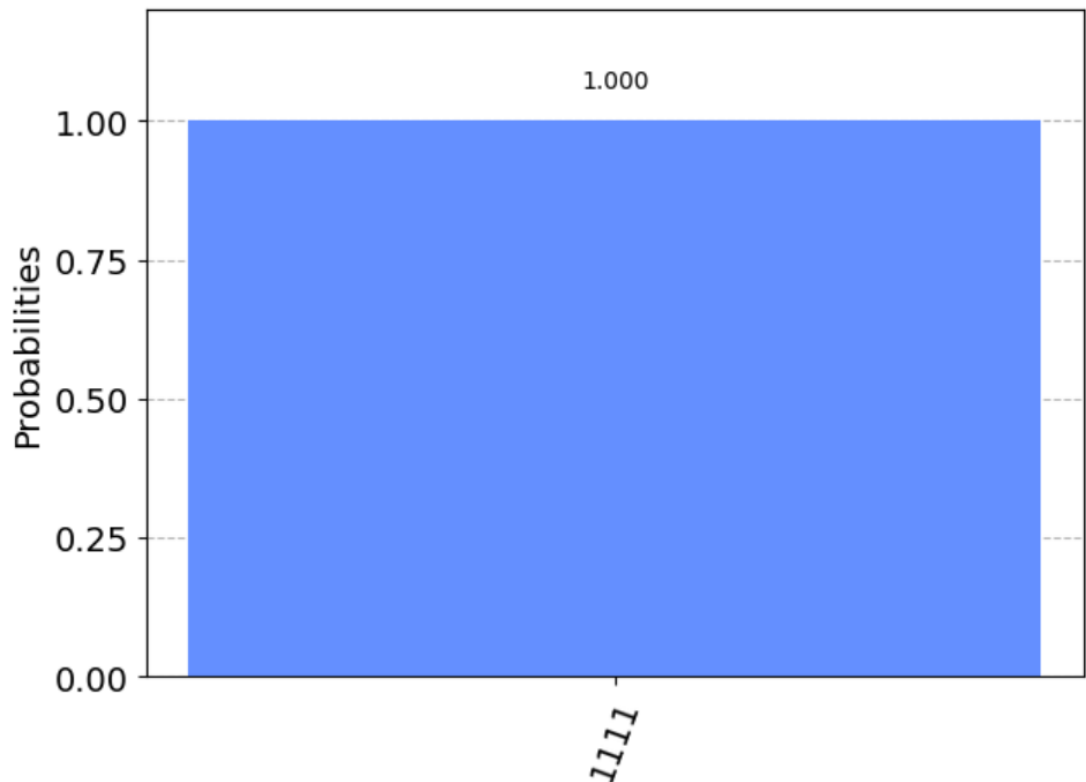
(use '1a\_3' for 'dj\_oracle')



**Oracle content:**



```
{'1111': 1024}
```

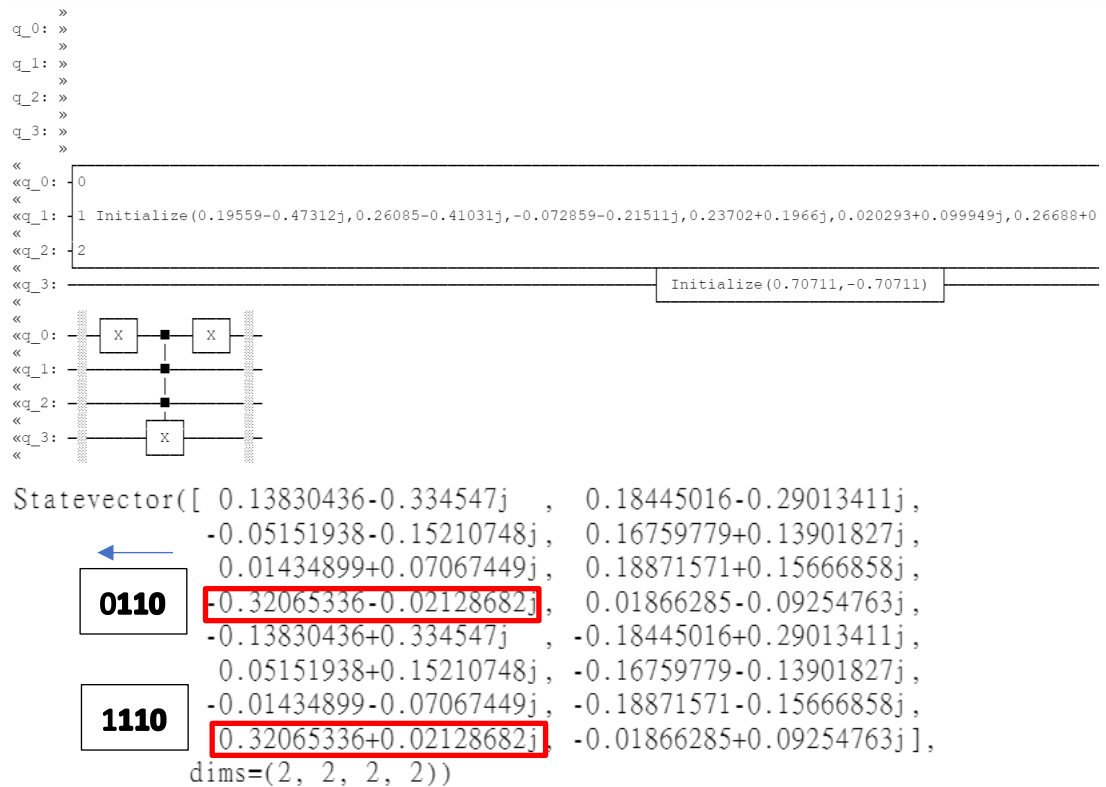


**Balanced function not all 0**

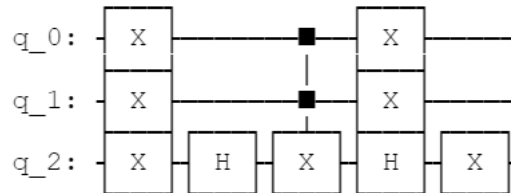
## 2. (40 points) Grover's Search

(a)

```
Statevector([ 0.13830436-0.334547j , 0.18445016-0.29013411j ,  
             -0.05151938-0.15210748j, 0.16759779+0.13901827j ,  
             0.01434899+0.07067449j, 0.18871571+0.15666858j ,  
             0.32065336+0.02128682j, 0.01866285-0.09254763j ,  
             -0.13830436+0.334547j , -0.18445016+0.29013411j ,  
             0.05151938+0.15210748j, -0.16759779-0.13901827j ,  
             -0.01434899-0.07067449j, -0.18871571-0.15666858j ,  
             -0.32065336-0.02128682j, -0.01866285+0.09254763j ],  
            dims=(2, 2, 2, 2))
```



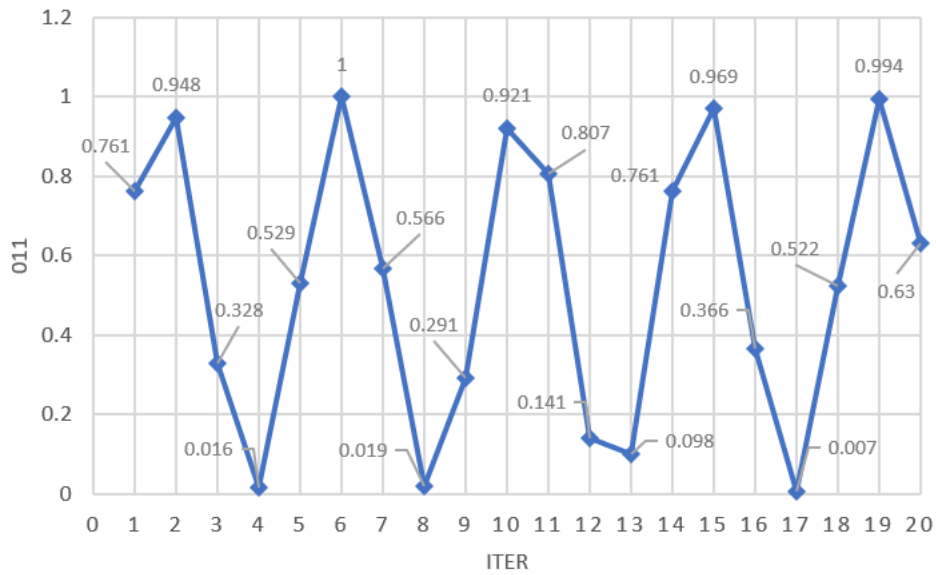
(b)



**For 3-qubit version**

**In the front and the end, remember H-gate**

(c)

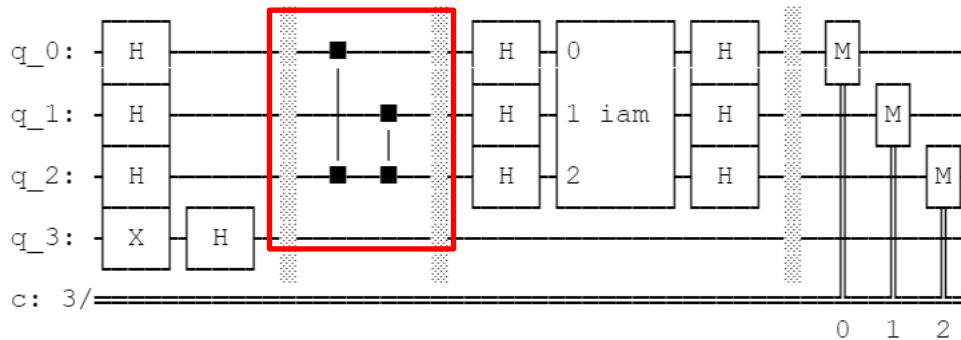


$$8^{(1/2)} = 2....$$

After 2 iter, get the acc of 0.948, not always right

(d)

```
Statevector([ 0.45673475-0.23247405j, -0.17471694-0.03892088j,
              0.1417575 +0.24842453j,  0.37406777+0.03859122j,
              0.33060483-0.34510371j,  0.20371487-0.11982149j,
              -0.2925818 +0.30906573j, -0.12864081-0.01245685j],
            dims=(2, 2, 2))
```



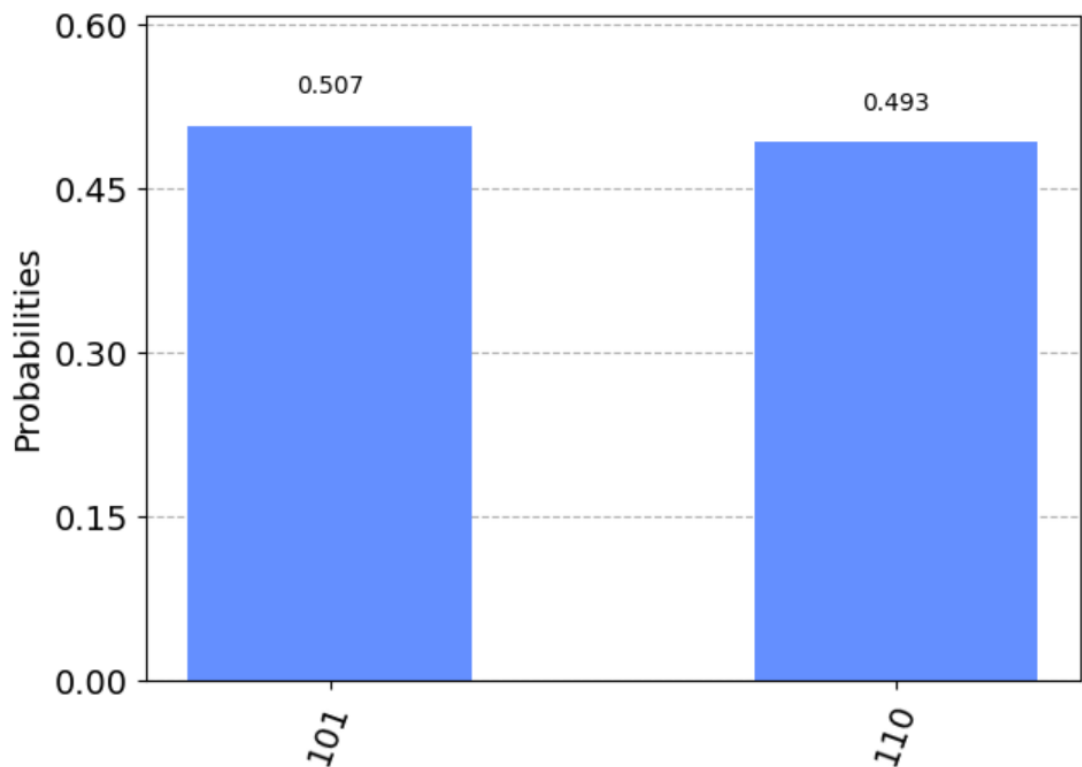
```
Statevector([ 0.45673475-0.23247405j, -0.17471694-0.03892088j,
              0.1417575 +0.24842453j,  0.37406777+0.03859122j,
              0.33060483-0.34510371j,  0.20371487+0.11982149j,
              0.2925818 -0.30906573j, -0.12864081-0.01245685j],
            dims=(2, 2, 2))
```

110

101

(e)

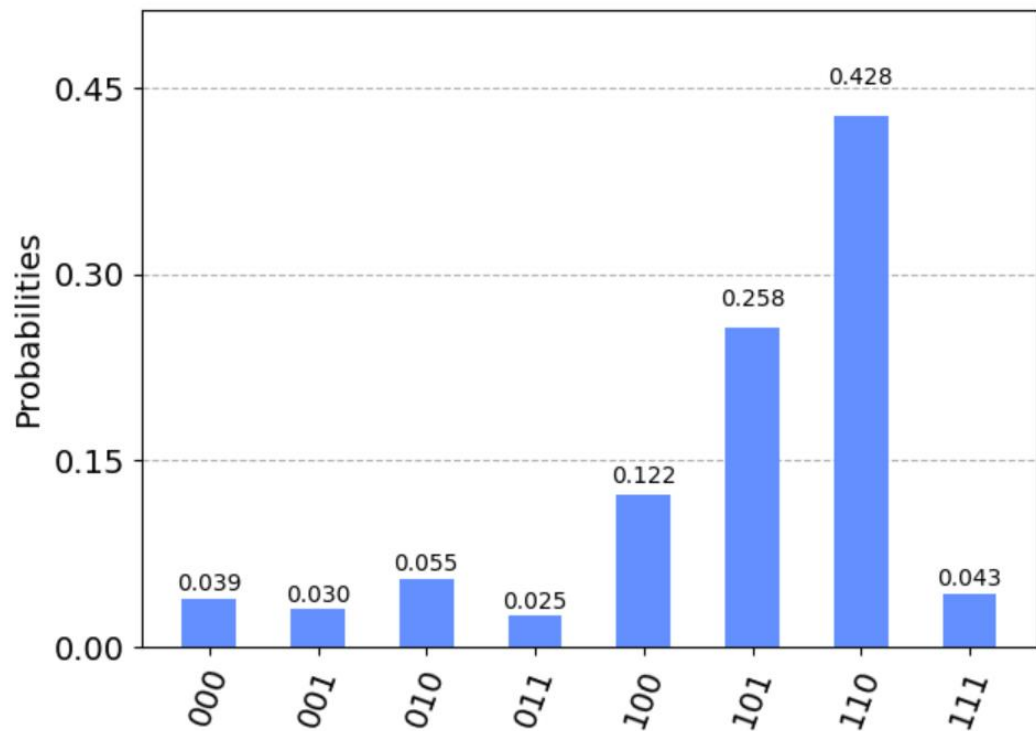
```
{'110': 505, '101': 519}
```



### One query is enough

Job Status: job has successfully run

```
{'000': 40, '001': 31, '010': 56, '011': 26, '100': 125, '101': 264, '110': 438, '111': 44}
```

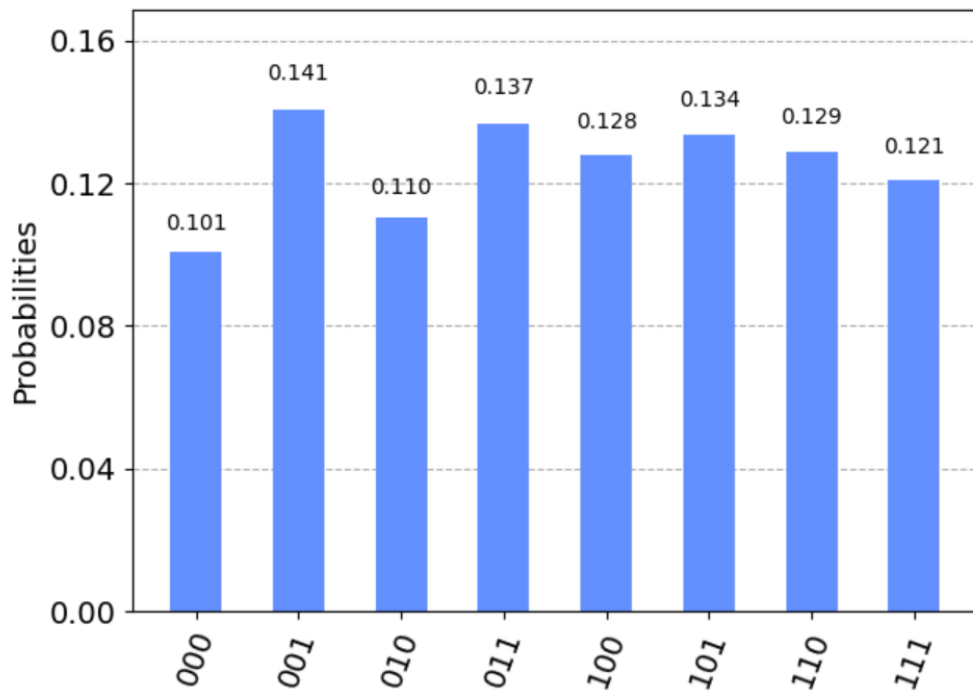


Drop to 0.686 using real device



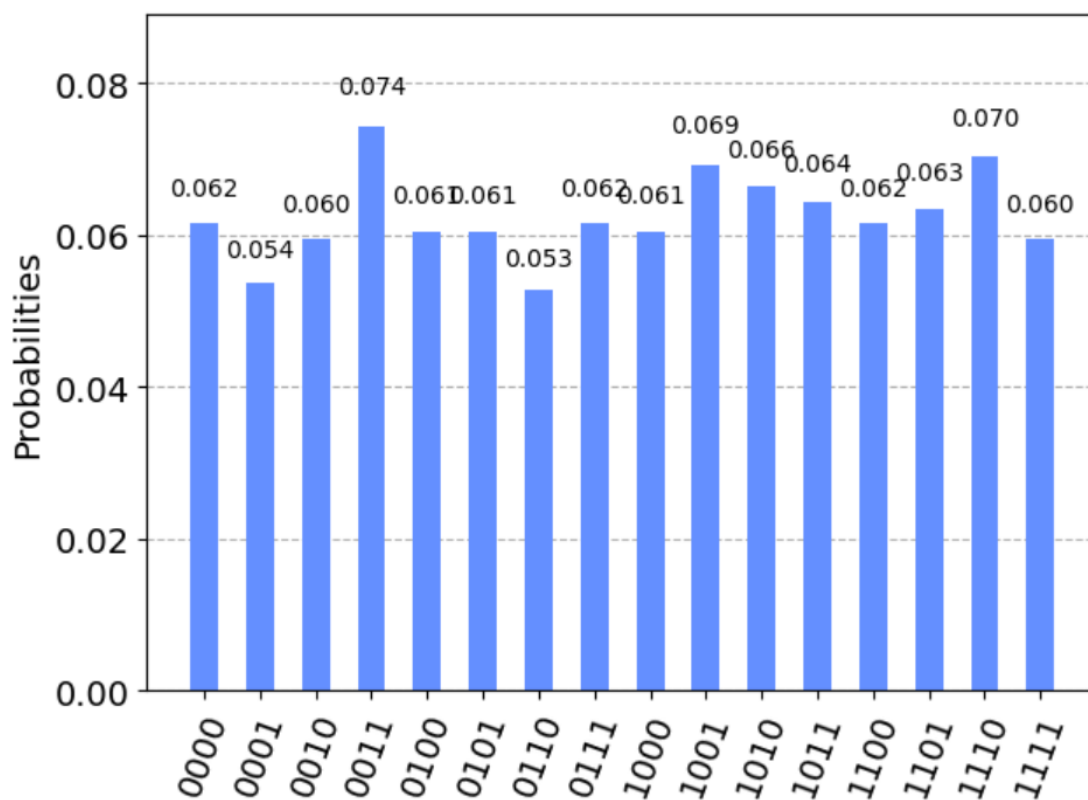
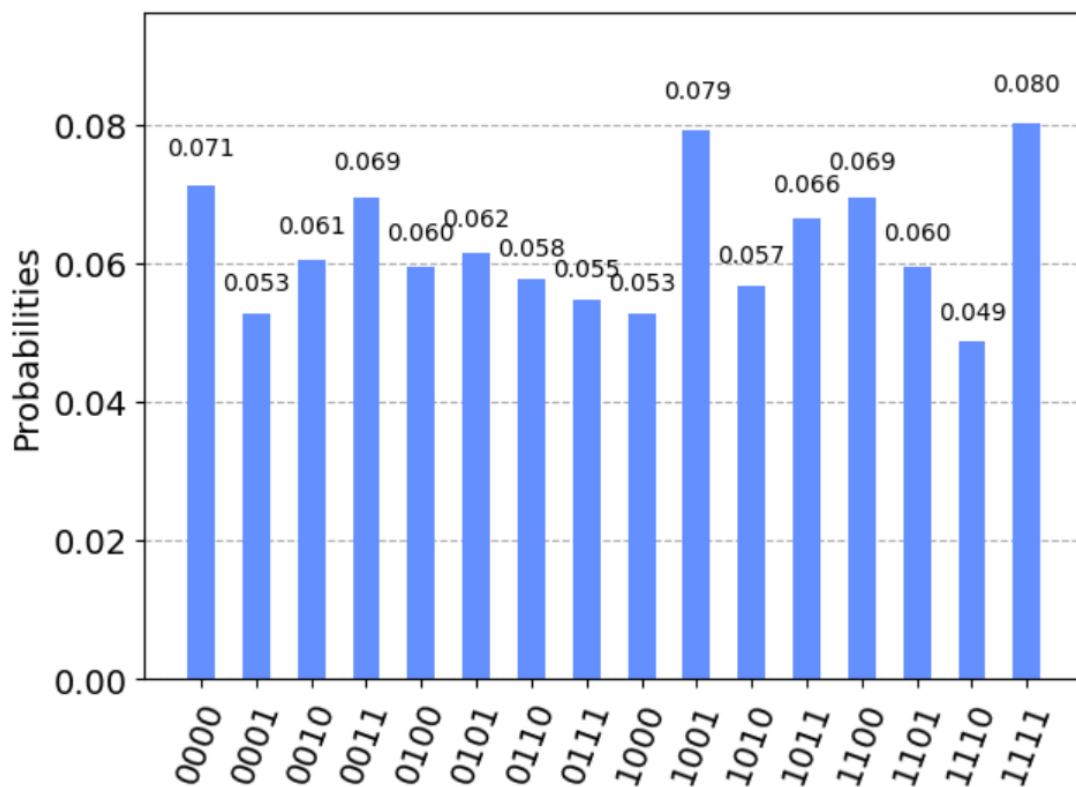
(f)

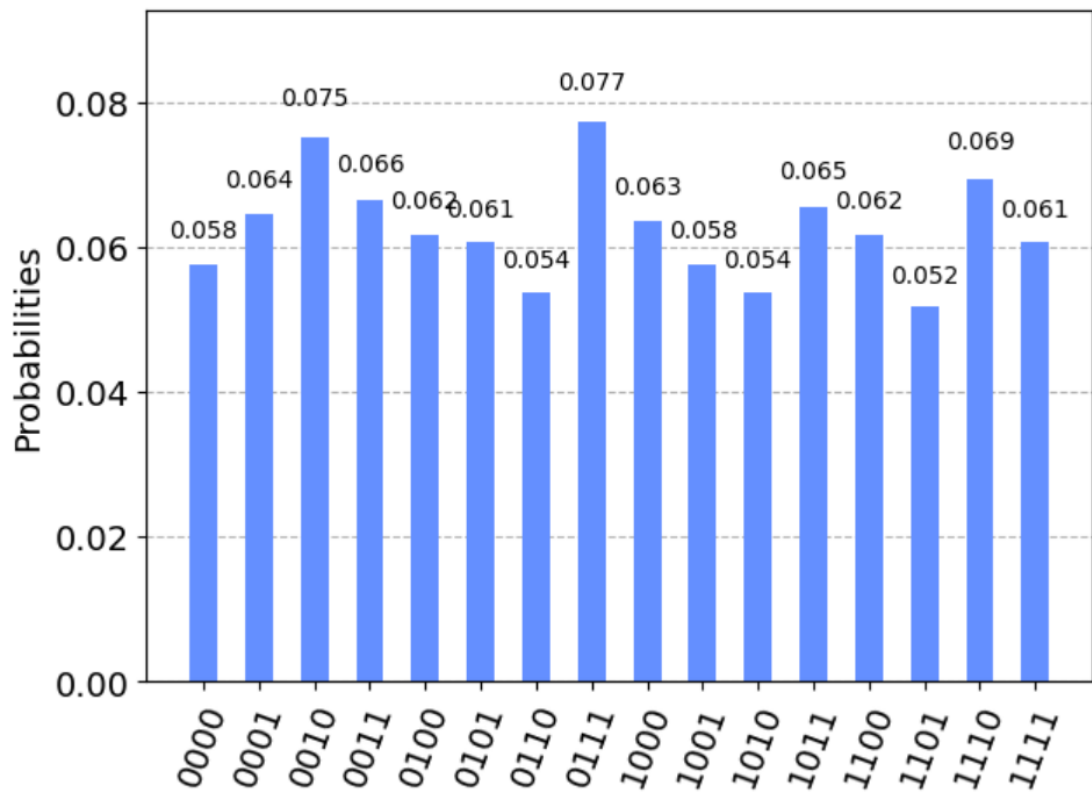
{'011': 140, '100': 131, '101': 137, '010': 113, '111': 124, '110': 132, '000': 103, '001': 144}



Every iteration shows the same results, can't tell the different from two groups' amplitude.

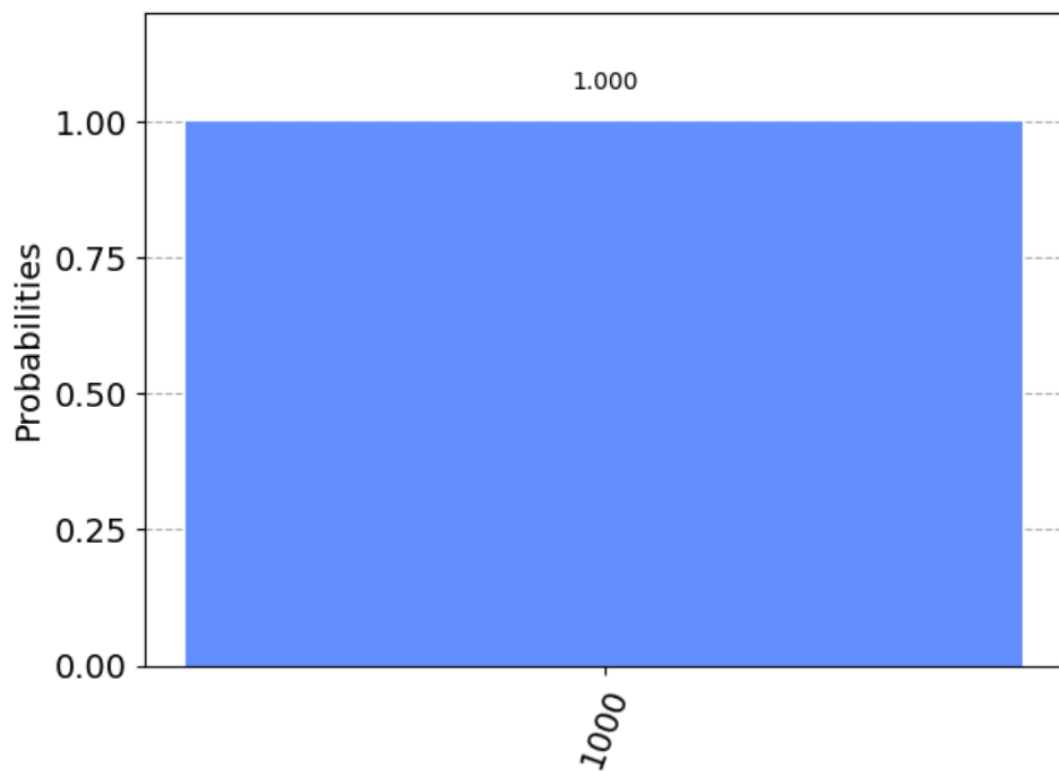
### 3. (15 points) Quantum Fourier Transform

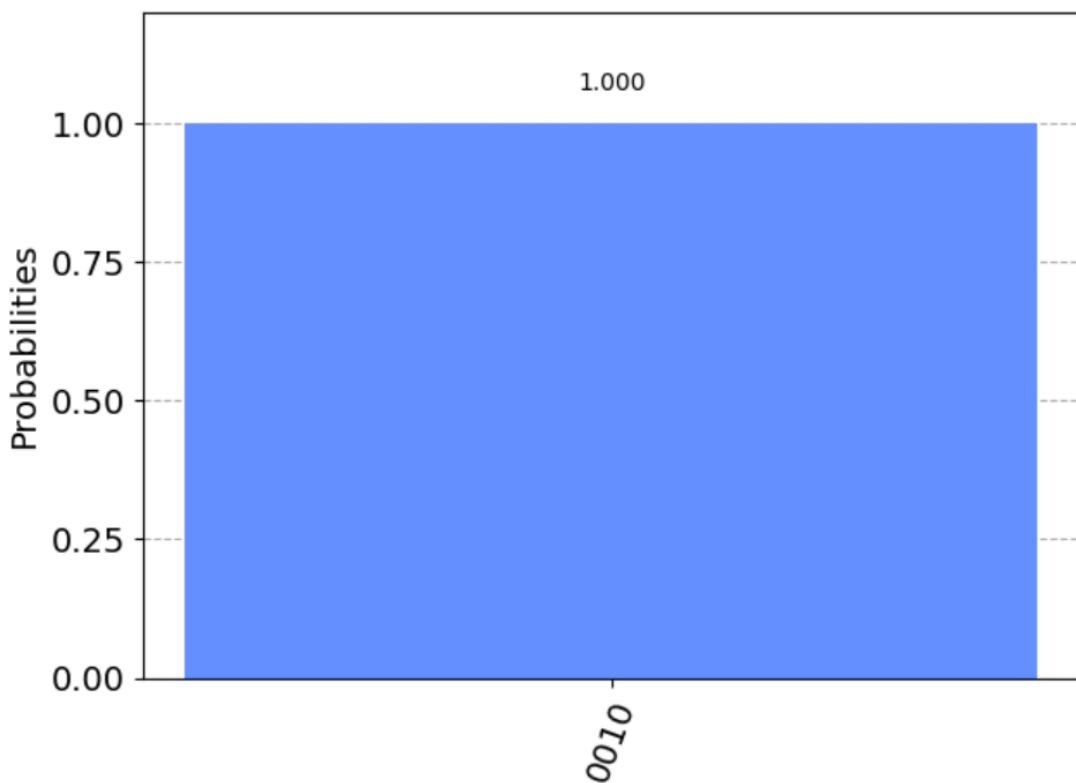
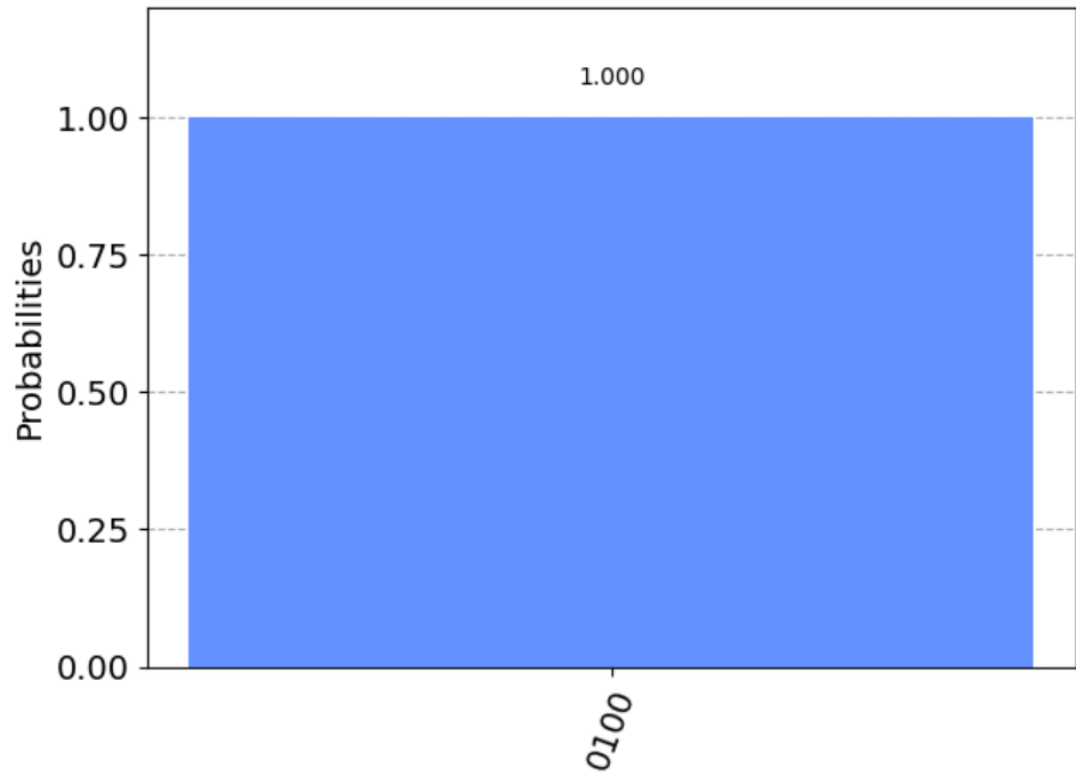




**Three graphs look the same**

**After QFT**





Here, numbers using different rotations around the Z-axis are stored

1000 = 8 and the four qubits rotate to  $2\pi/16 * 8 = \text{pi}$  、  $2\pi/8 * 8 =$

$$2\pi = 0 \text{ , } 2\pi/4 * 8 = 4\pi = 0 \text{ , } 2\pi/2 * 8 = 8\pi = 0 \rightarrow \{\text{Rz}(\pi), x, x, x\}$$

$$0100 = 4 \text{ and the four qubits rotate to } 2\pi/16 * 4 = \pi/2 \text{ , } 2\pi/8 * 4 =$$

$$\pi \text{ , } 2\pi/4 * 4 = 2\pi = 0 \text{ , } 2\pi/2 * 4 = 4\pi = 0 \rightarrow \{\text{Rz}(\pi/2), \text{Ry}(\pi), x, x\}$$

(Doing  $\text{Ry}(\pi)$  is same as doing  $\text{Rz}(\pi)$  on '+')

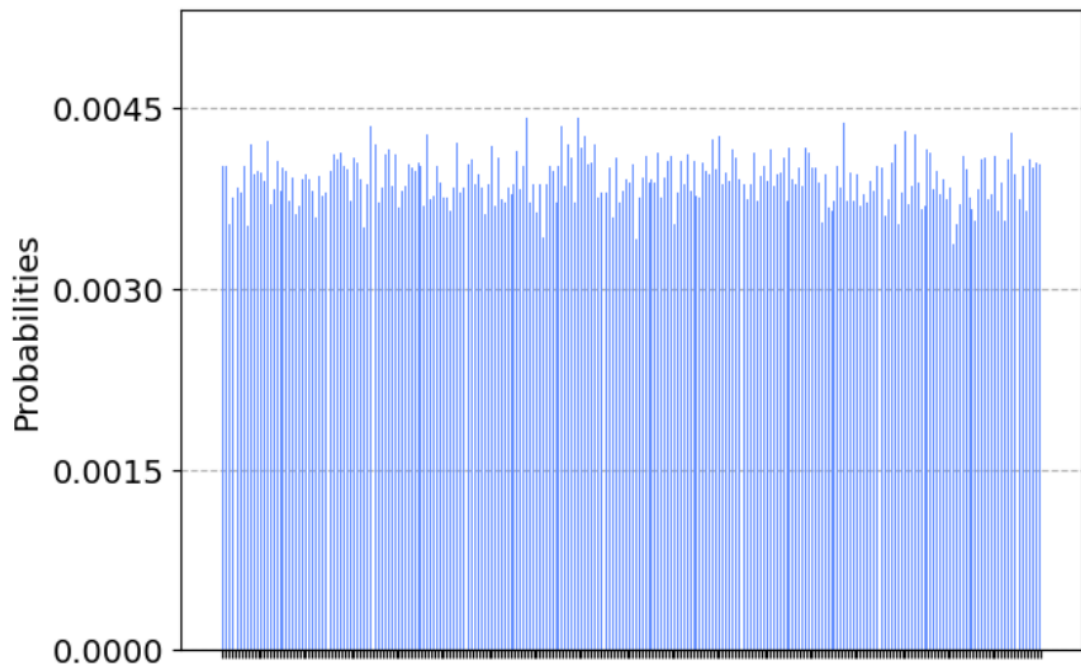
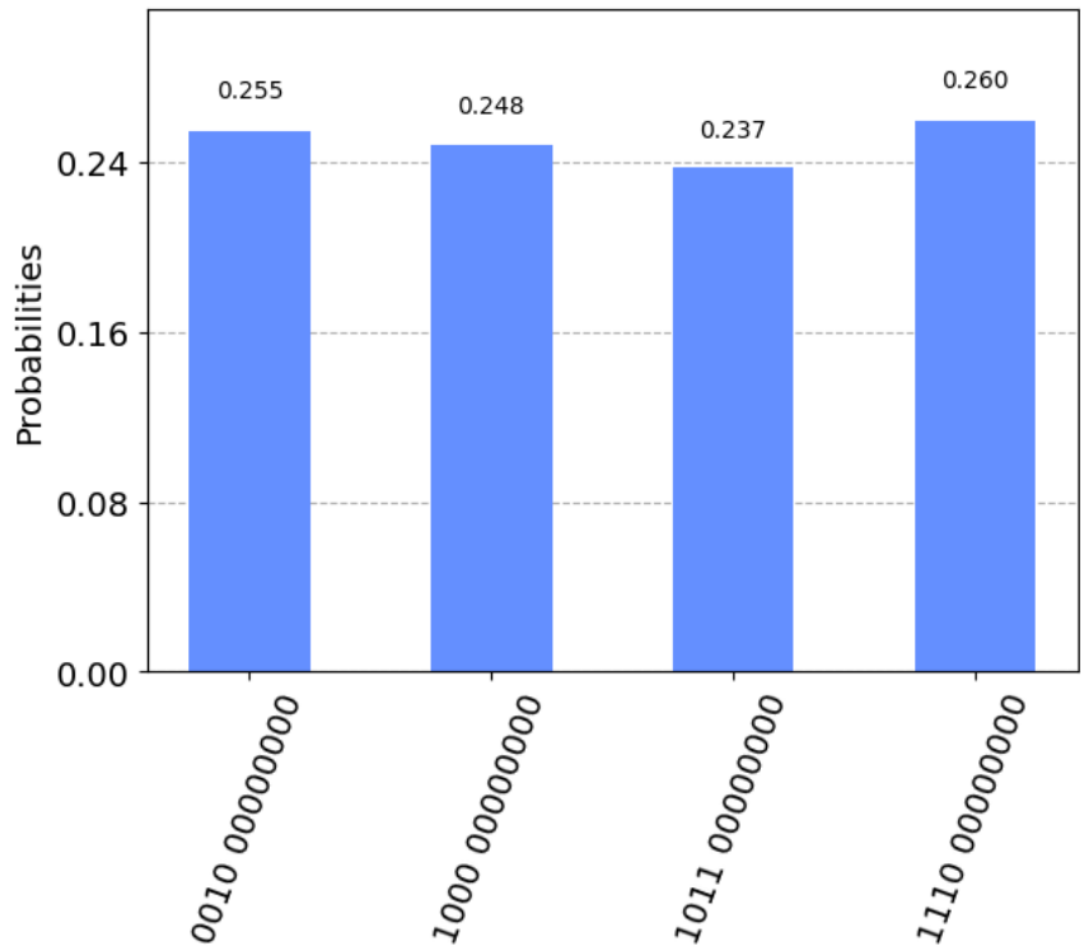
$$0010 = 2 \text{ and the four qubits rotate to } 2\pi/16 * 2 = \pi/4 \text{ , } 2\pi/8 * 2 =$$

$$\pi/2 \text{ , } 2\pi/4 * 2 = \pi \text{ , } 2\pi/2 * 2 = 2\pi = 0 \rightarrow \{\text{Rz}(\pi/4), \text{Rz}(\pi/2), \text{Rz}(\pi), x\}$$

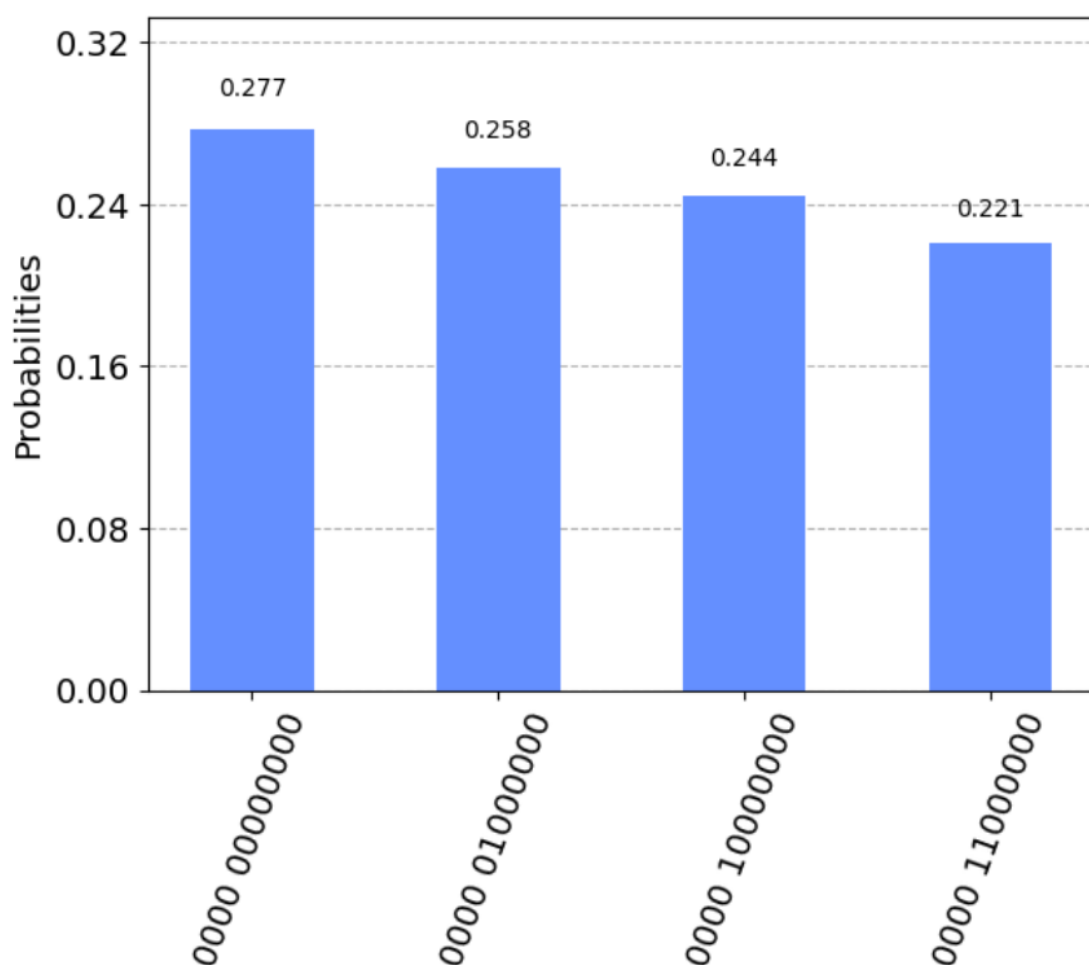
#### 4. (25 points) Period-Finding Algorithm

7, 4, 13, 1, 7, 4, 13, 1, 7, ... ( $7 \bmod 15$ ,  $49 \bmod 15$ ,  $343 \bmod 15$ , ...)

Qr2, probability for 1, 4, 7, 13 is the same because it's periodic



**Periodic with  $r$ , random selection and become uniform in the probability.**



**0, 64, 128, 192 (64 \* {0, 1, 2, 3})**

**a=7 r=4**

	Phase	Fraction	Guess for r
0	0.00	0/1	1
1	0.75	3/4	4
2	0.25	1/4	4
3	0.50	1/2	2

**a=2 r=4 (2, 4, 8, 1)**

	Phase	Fraction	Guess for r
0	0.00	0/1	1
1	0.50	1/2	2
2	0.25	1/4	4
3	0.75	3/4	4

**a=8 r=4 (8, 4, 2, 12)**

	Phase	Fraction	Guess for r
0	0.00	0/1	1
1	0.75	3/4	4
2	0.50	1/2	2
3	0.25	1/4	4

**a=11 r=2 (11, 1)**

	Phase	Fraction	Guess for r	
0	0.0	0/1		1
1	0.5	1/2		2

**a=13 r=4 (13, 4, 7, 1)**

	Phase	Fraction	Guess for r	
0	0.00	0/1		1
1	0.75	3/4		4
2	0.25	1/4		4
3	0.50	1/2		2

**All the above results have a 0.5 probability**