通信專題　　Lab_1　B08901019 吳冠緯

1. (20 points) Manipulating a single qubit state

(a)

```
import qiskit
import numpy as np
from qiskit import *
from qiskit import QuantumCircuit, execute, Aer
from qiskit.visualization import plot_histogram, plot_bloch_multivector
from math import sqrt, pi

from qiskit.tools.visualization import plot_histogram
```

(b)

```
q = QuantumRegister(2)
c = ClassicalRegister(2)
```

(c)

```
circuit = QuantumCircuit(q, c)
circuit.draw()
```

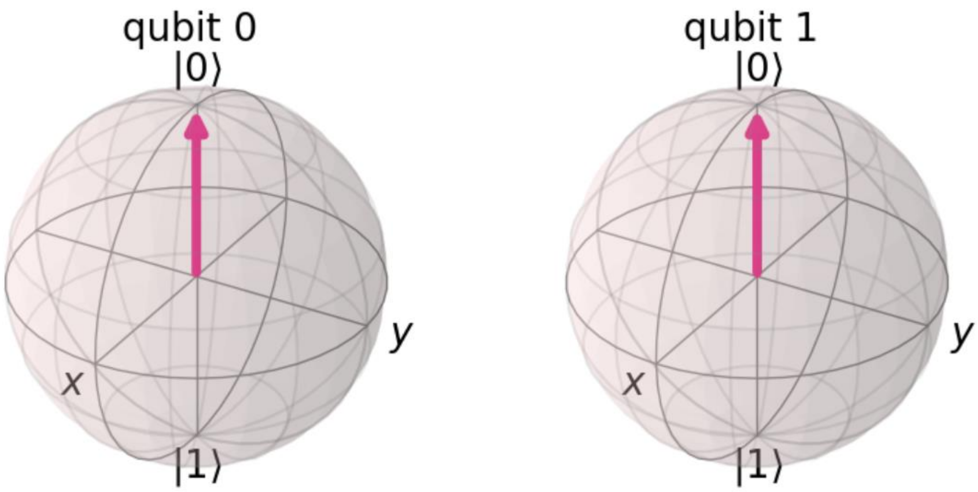
```
q0_0:

q0_1:

c0: 2/
```

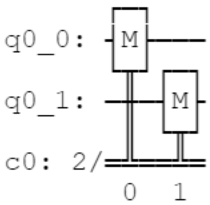
```
simulator = Aer.get_backend('statevector_simulator')
job = execute(circuit, simulator)
result = job.result()
statevector = result.get_statevector()
print(statevector)
plot_bloch_multivector(statevector)
```

```
Statevector([1.+0.j, 0.+0.j, 0.+0.j, 0.+0.j],
            dims=(2, 2))
```



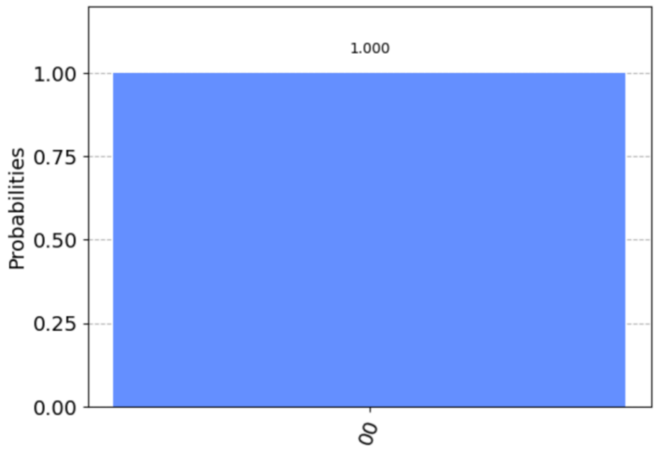
(d)

```
circuit.measure(q, c)
circuit.draw()
```



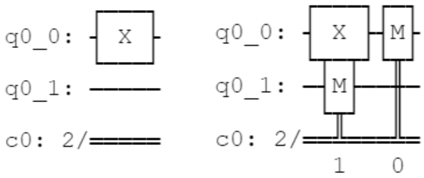

```
simulator = Aer.get_backend('qasm_simulator')
job = execute(circuit, simulator, shots=1024)
result = job.result()
counts = result.get_counts()
print(counts)
plot_histogram(counts)
```

```
{'00': 1024}
```

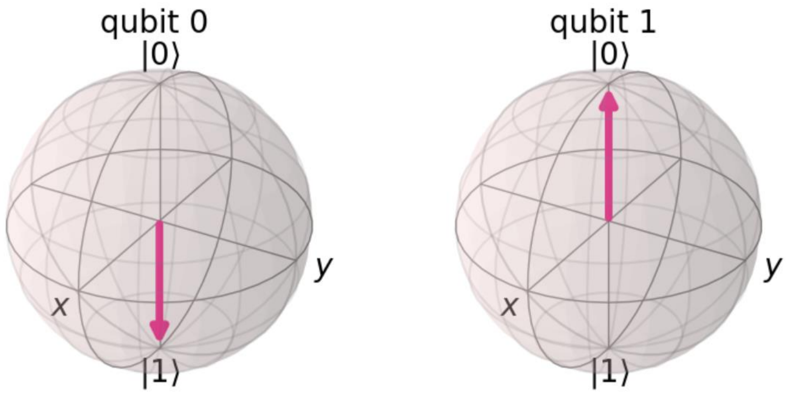


(e)

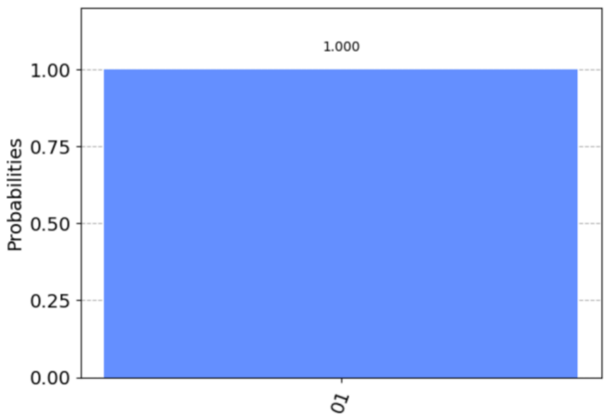
X-gate



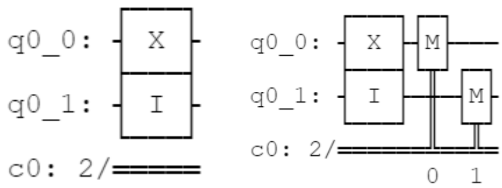

```
Statevector([0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j],
            dims=(2, 2))
```



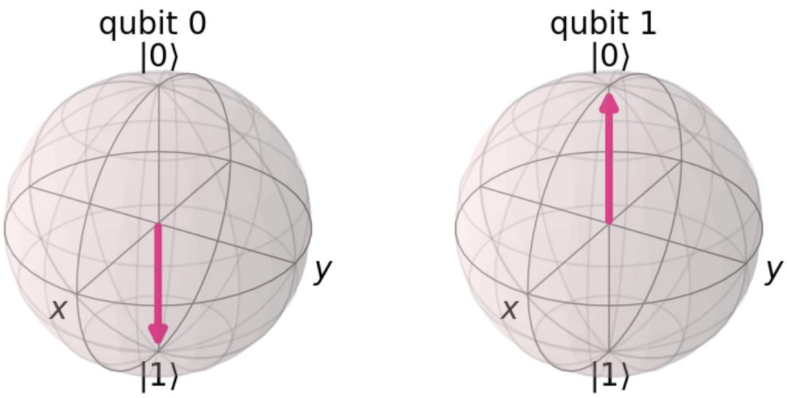

```
{'01': 1024}
```

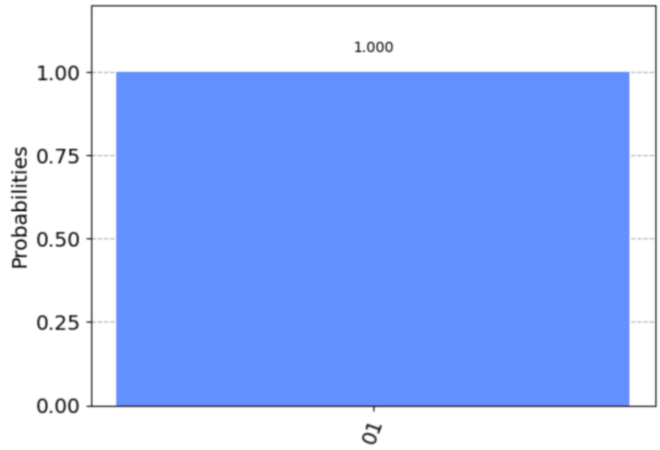


With identity gate

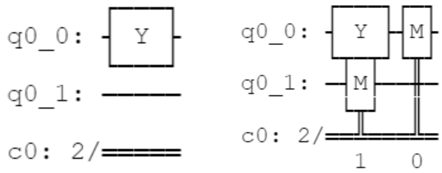



```
Statevector([0.+0.j, 1.+0.j, 0.+0.j, 0.+0.j],
            dims=(2, 2))
```

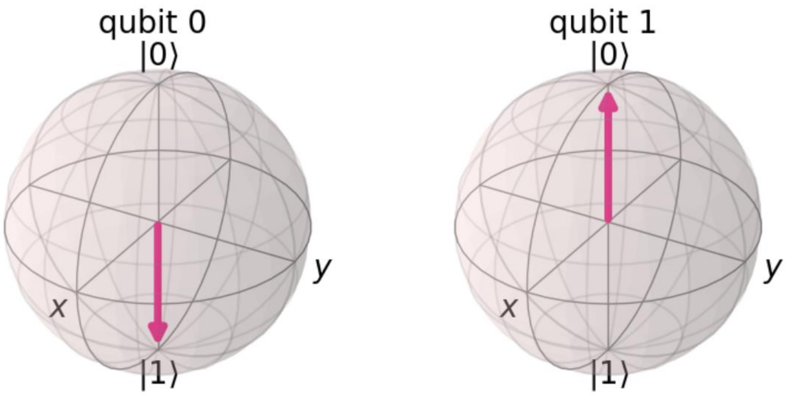




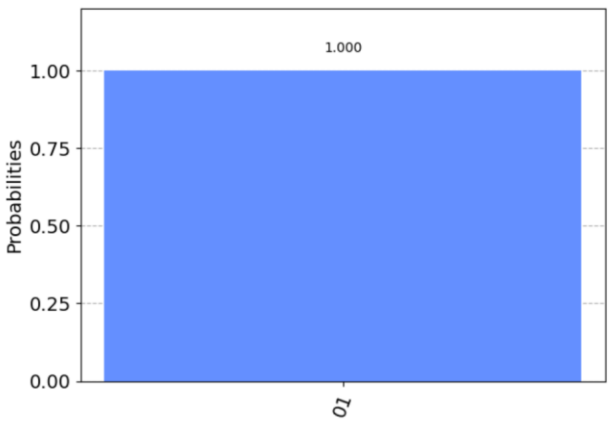
Y-gate



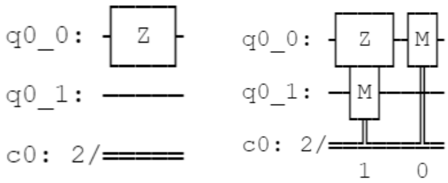

```
Statevector([0.-0.j, 0.+1.j, 0.-0.j, 0.+0.j],
            dims=(2, 2))
```



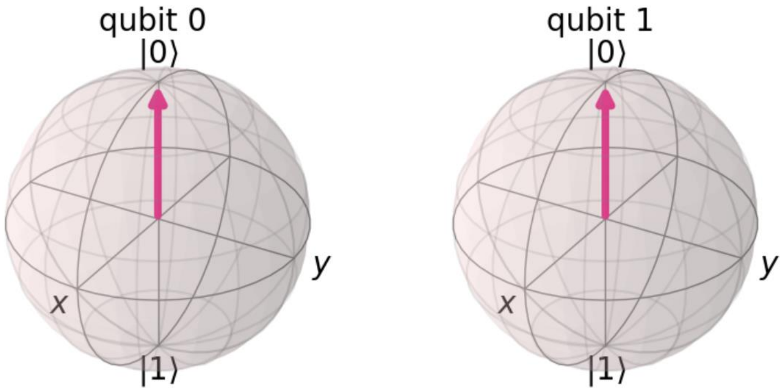

```
{'01': 1024}
```



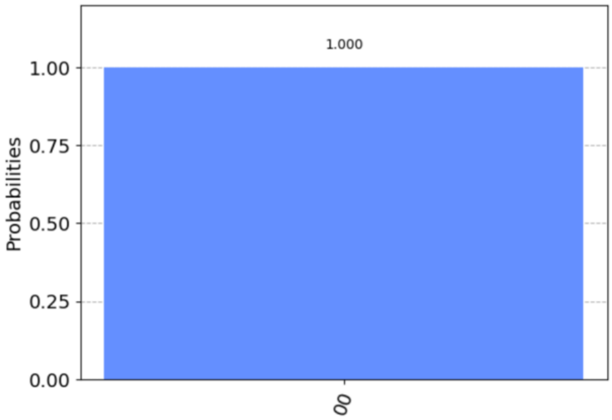
Z-gate




```
Statevector([ 1.+0.j, -0.+0.j,  0.+0.j, -0.+0.j],
            dims=(2, 2))
```

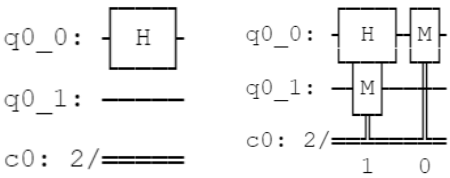



```
{'00': 1024}
```

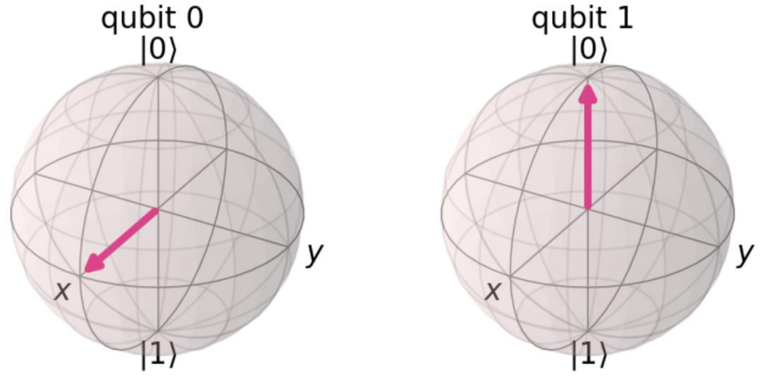


(f)

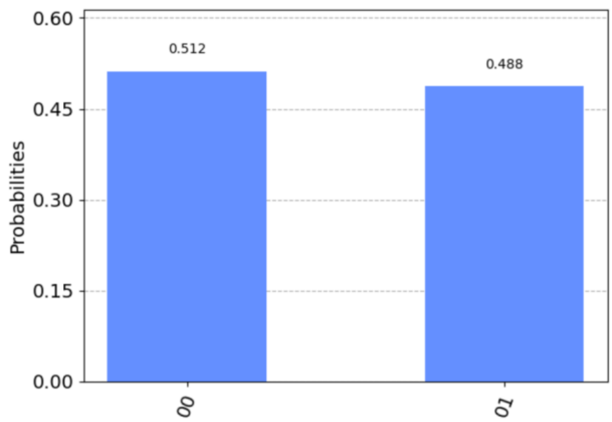
H-gate



```
Statevector([0.70710678+0.j, 0.70710678+0.j, 0.        +0.j,
             0.        +0.j],
            dims=(2, 2))
```

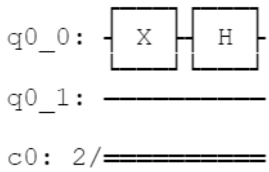


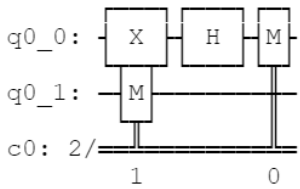

```
{'01': 500, '00': 524}
```



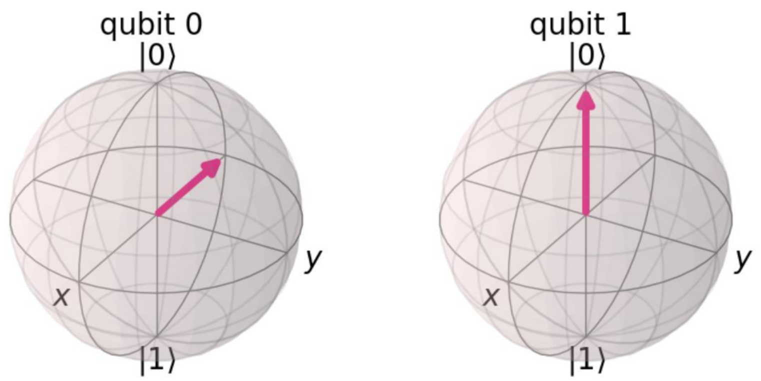
$$HH = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix} = I$$

State $|1\rangle$

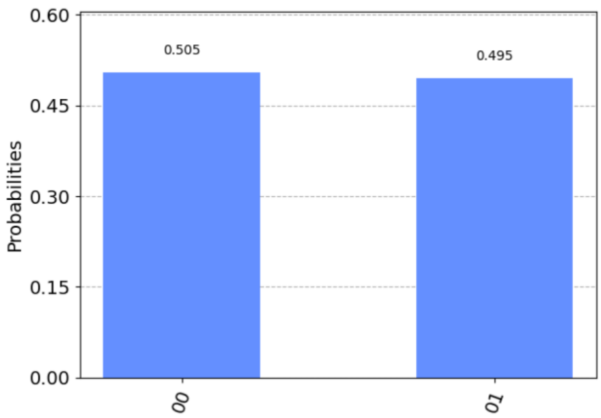





```
Statevector([ 0.70710678+0.00000000e+00j, -0.70710678-8.65956056e-17j,
              0.        +0.00000000e+00j,  0.        +0.00000000e+00j],
            dims=(2, 2))
```

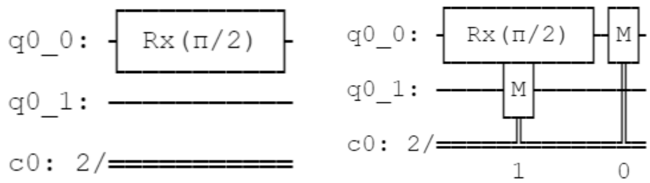


```
{'00': 517, '01': 507}
```

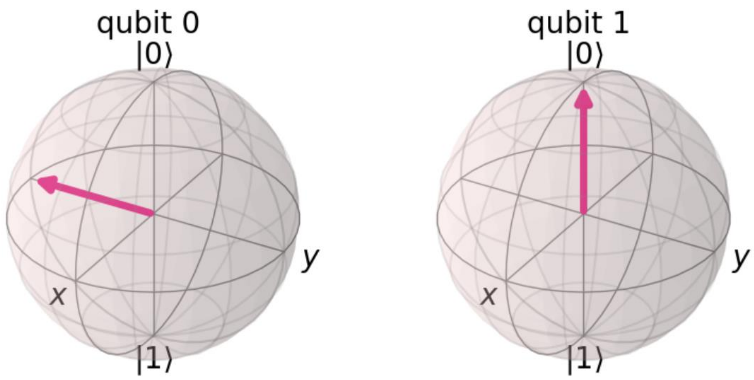


(g)

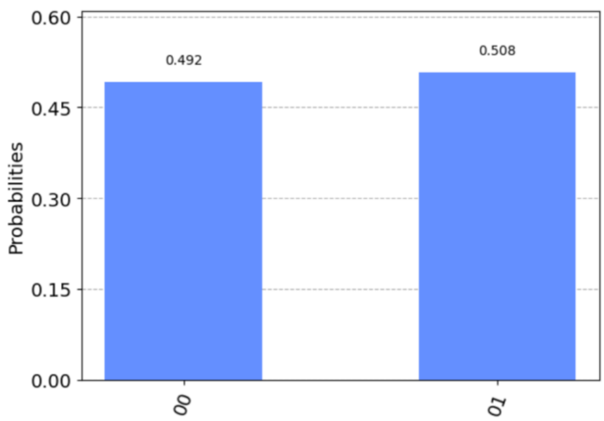
Rx-gate



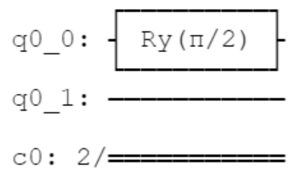

```
Statevector([0.70710678+0.j        , 0.        -0.70710678j,
             0.        +0.j        , 0.        +0.j        ],
            dims=(2, 2))
```

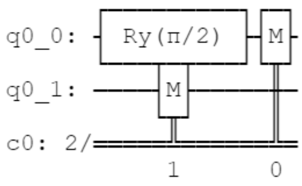



```
{'01': 520, '00': 504}
```

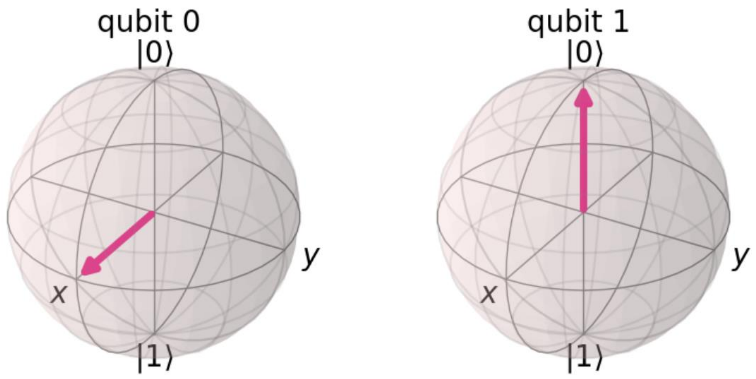


Ry-gate

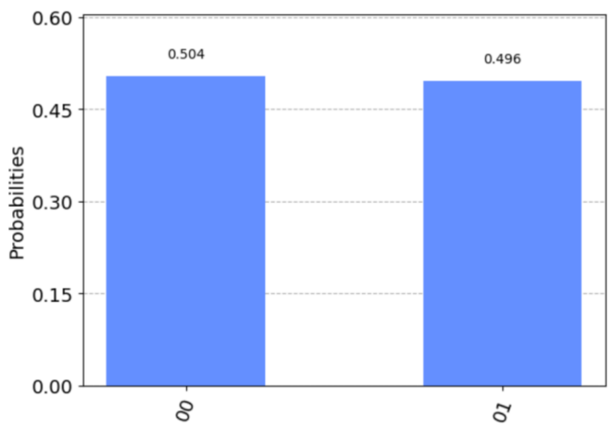




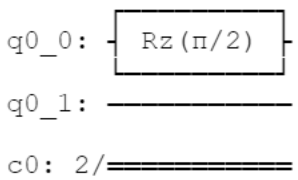

```
Statevector([0.70710678+0.j, 0.70710678+0.j, 0.        +0.j,
             0.        +0.j],
            dims=(2, 2))
```

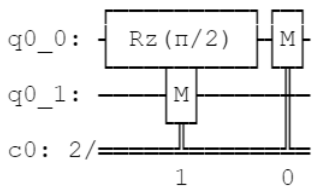



```
{'00': 516, '01': 508}
```

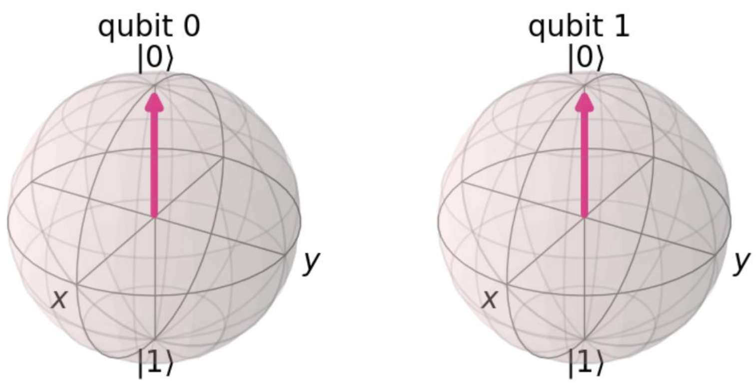


Rz-gate

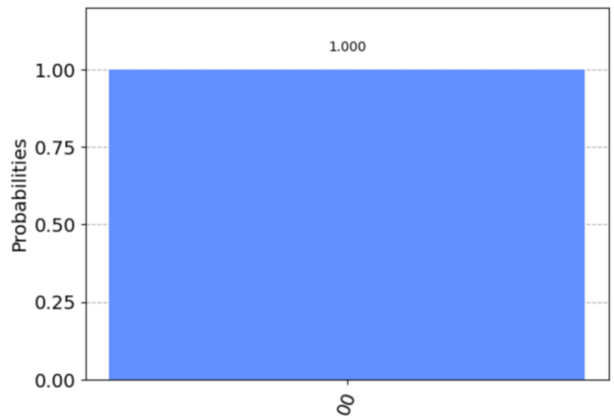




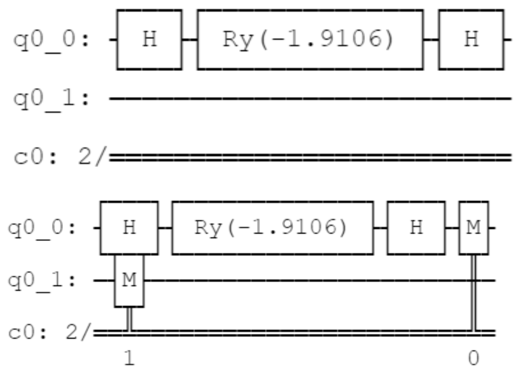

```
Statevector([0.70710678-0.70710678j, 0.        +0.j        ,
             0.        +0.j        , 0.        +0.j        ],
            dims=(2, 2))
```



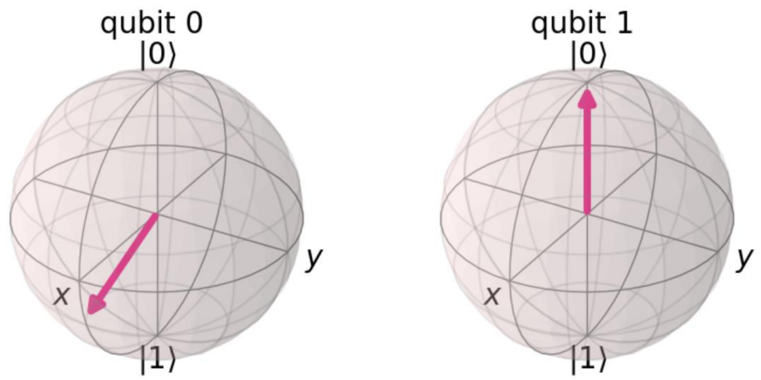
{'00': 1024}



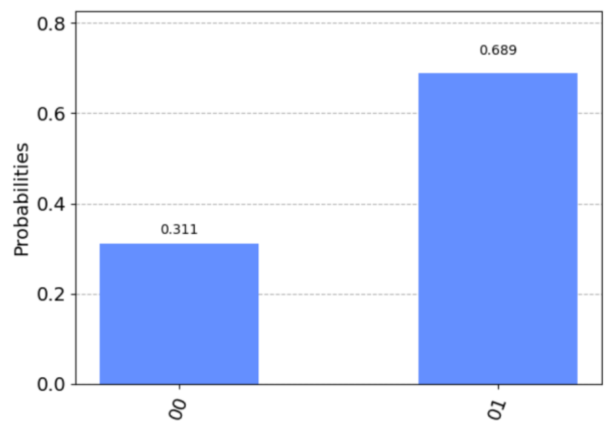
(h)



Statevector([0.57735027+0.j, 0.81649658+0.j, 0.   +0.j,
             0.   +0.j],
            dims=(2, 2))



{'01': 706, '00': 318}



$$\left(\frac{1}{3}\right)^{1/2}|+\rangle+\left(\frac{2}{3}\right)^{1/2}|-\rangle=\left(\frac{1}{\sqrt{3}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}+\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{\sqrt{3}}&\frac{\sqrt{2}}{\sqrt{3}}\\\frac{-\sqrt{2}}{\sqrt{3}}&\frac{1}{\sqrt{3}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}\quad\rightarrow\quad|0\rangle\rightarrow H\_gate\rightarrow|+\rangle\rightarrow Ry\_gate$$

$$change\ basis\rightarrow H\_gate$$

$$\cos\frac{\varphi}{2}=\frac{1}{\sqrt{3}}\quad and\quad\sin\frac{\varphi}{2}=\frac{-\sqrt{2}}{\sqrt{3}}\quad\rightarrow\quad\varphi=-2\cos^{-1}\frac{1}{\sqrt{3}}$$

Bonus

Ref: [Summary of Quantum Operations — Qiskit 0.38.0 documentation]

A single qubit quantum state can be written as

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$$

where α and β are complex numbers. In a measurement the probability of the bit being in $|0\rangle$ is $|\alpha|^2$ and $|1\rangle$ is $|\beta|^2$. As a vector this is

$$|\psi\rangle=\begin{pmatrix}\alpha\\\beta\end{pmatrix}.$$

Note, due to the conservation of probability $|\alpha|^2+|\beta|^2=1$ and since global phase is undetectable $|\psi\rangle:=e^{i\delta}|\psi\rangle$ we only require two real numbers to describe a single qubit quantum state.

A convenient representation is

$$|\psi\rangle=\cos(\theta/2)|0\rangle+\sin(\theta/2)e^{i\phi}|1\rangle$$

where $0\leq\phi<2\pi$, and $0\leq\theta\leq\pi$. From this, it is clear that there is a one-to-one correspondence between qubit states ($\mathbb{C}^2$) and the points on the surface of a unit sphere ($\mathbb{R}^3$). This is called the Bloch sphere representation of a qubit state.

Quantum gates/operations are usually represented as matrices. A gate which acts on a qubit is represented by a 2×2 unitary matrix U. The action of the quantum gate is found by multiplying the matrix representing the gate with the vector which represents the quantum state.

$$\psi'\rangle=U|\psi\rangle$$

A general unitary must be able to take the $|0\rangle$ to the above state. That is

$$U=\begin{pmatrix}\cos(\theta/2)&a\\e^{i\phi}\sin(\theta/2)&b\end{pmatrix}$$

where a and b are complex numbers constrained such that $U^\dagger U=I$ for all $0\leq\theta\leq\pi$ and $0\leq\phi<2\pi$.

$$\begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2}e^{-i\phi} \\ a^* & b^* \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2} & a \\ \sin\frac{\theta}{2}e^{i\phi} & b \end{pmatrix}$$

$$= \begin{pmatrix} 1 & a\cos\frac{\theta}{2} + b\sin\frac{\theta}{2}e^{-i\phi} \\ a^*\cos\frac{\theta}{2} + b^*\sin\frac{\theta}{2}e^{i\phi} & 1 \end{pmatrix}$$

$$let \quad a = p + qi \quad b = r + si$$

$$\begin{cases} p\cos\frac{\theta}{2} + \sin\frac{\theta}{2}[r\cos\phi + s\sin\phi] = 0 \\ q\cos\frac{\theta}{2} + \sin\frac{\theta}{2}[-r\sin\phi + s\cos\phi] = 0 \end{cases}$$

$$let \quad p = -k\sin\frac{\theta}{2} \quad q = -h\sin\frac{\theta}{2}$$

$$r = k\cos\frac{\theta}{2}\cos\phi - h\cos\frac{\theta}{2}\sin\phi$$

$$s = k\cos\frac{\theta}{2}\cos\phi + h\cos\frac{\theta}{2}\sin\phi$$

$$now\ take \quad k = \cos\lambda \quad h = \sin\lambda$$

$$we\ obtain\ that \quad a = -e^{i\lambda}\sin\frac{\theta}{2} \quad b = e^{i(\phi+\lambda)}\cos\frac{\theta}{2}$$

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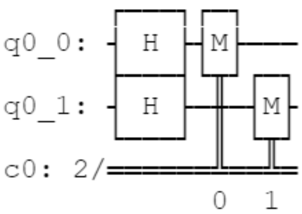
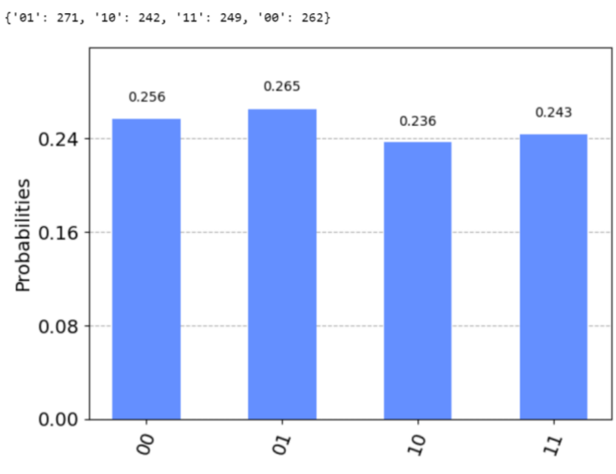
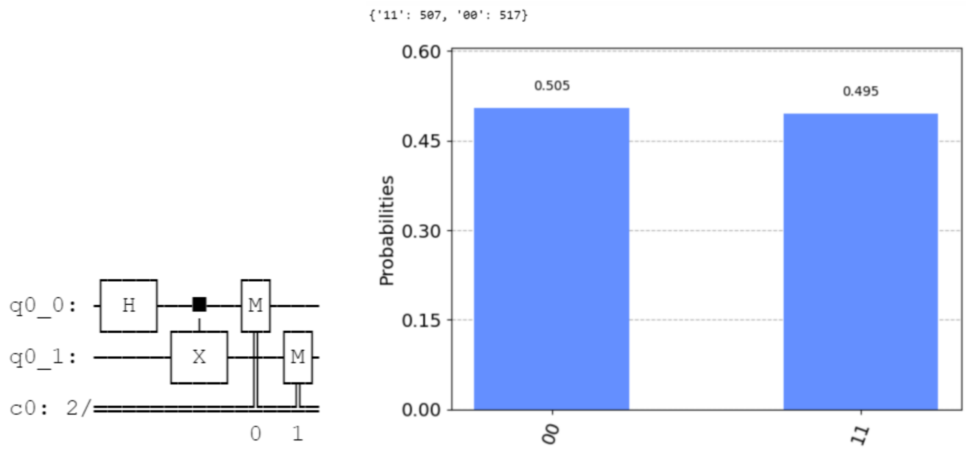
2. (20 points) Manipulating Multi-Qubit Gates

(a)





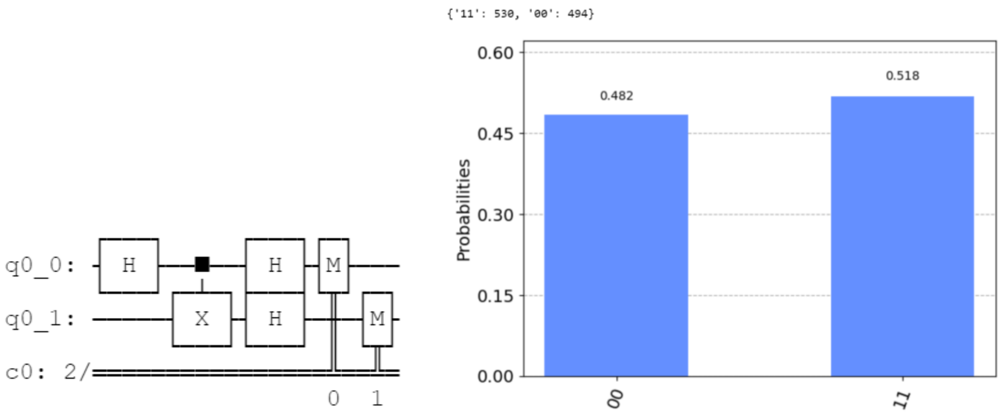
(b)



If the first classical register (say, possessed by Alice) is ‘1’, what is the outcome of the second classical register (say, possessed by Bob)?
Ans. ‘1’

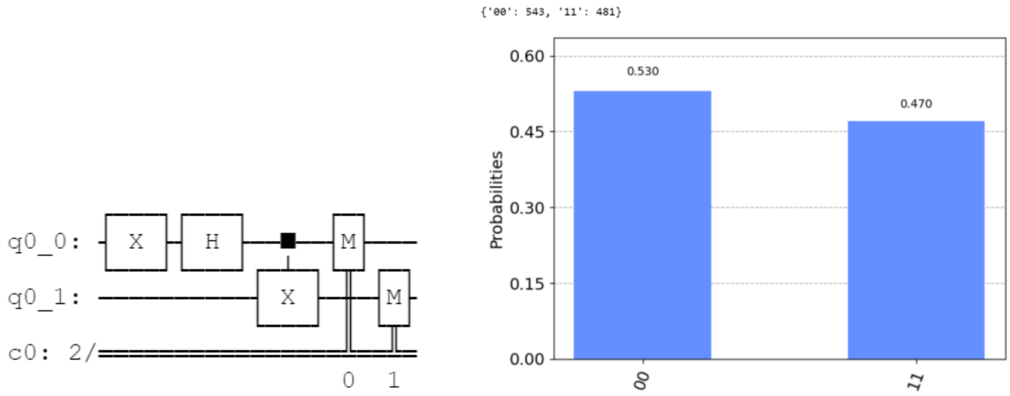
What is the probability of getting outcome ‘1’ from the first register?
Ans. 50%

(c)

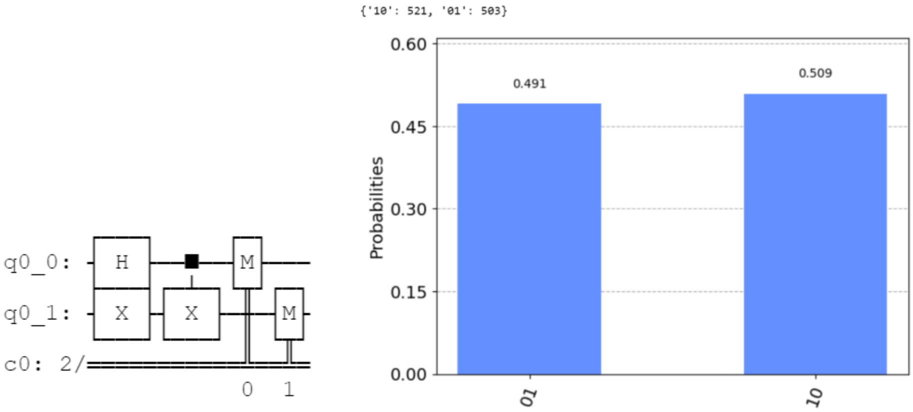


(d)

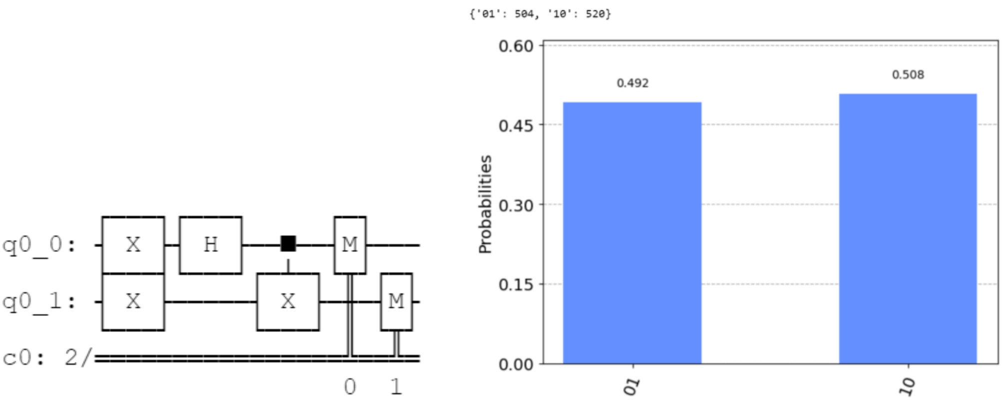
State $|\Phi -\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$



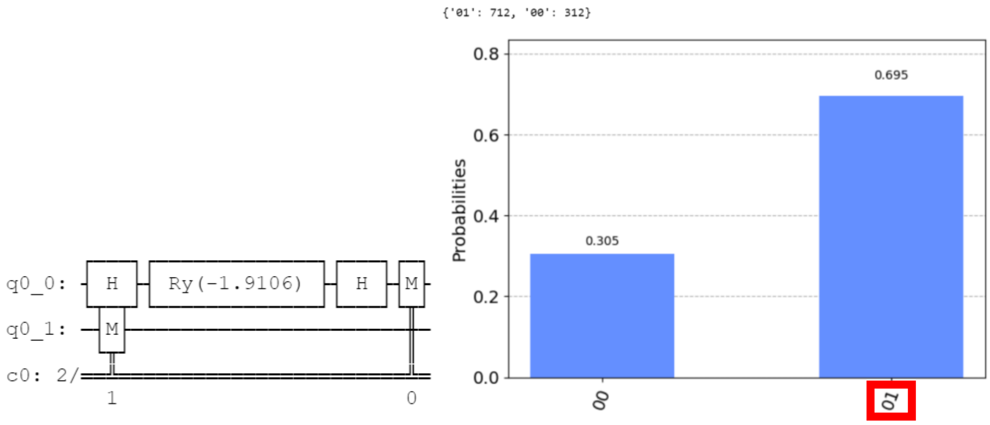
State $|\Psi +\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$

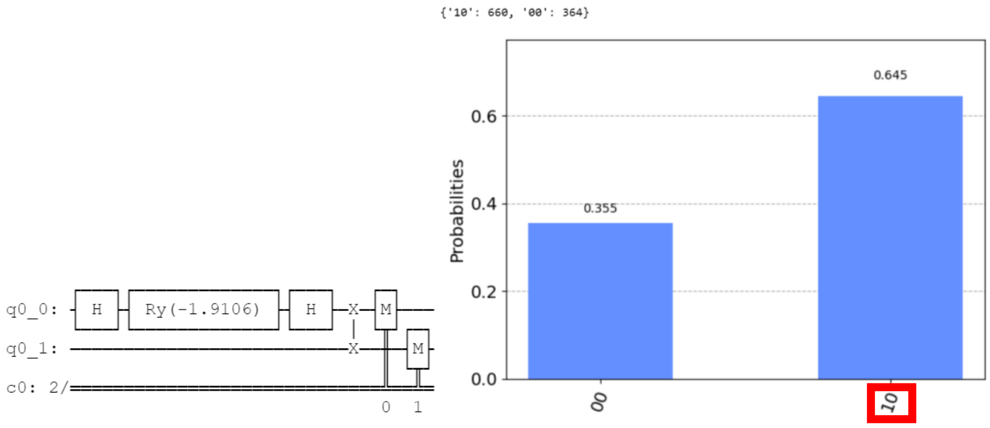


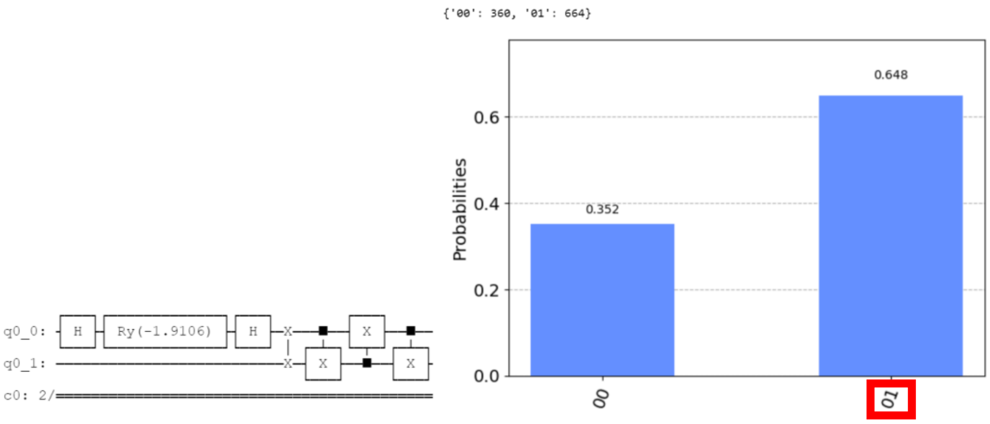
State $|\Psi -\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$



(e)







$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$$

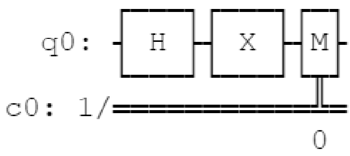
---

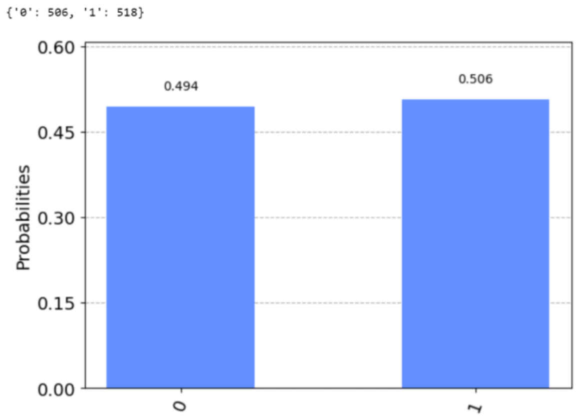
3. (20 points) Global phase does not matter

(a)

$|\Psi 1\rangle$

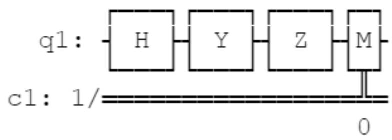
```
Statevector([0.70710678+0.j, 0.70710678+0.j],
            dims=(2,))
```

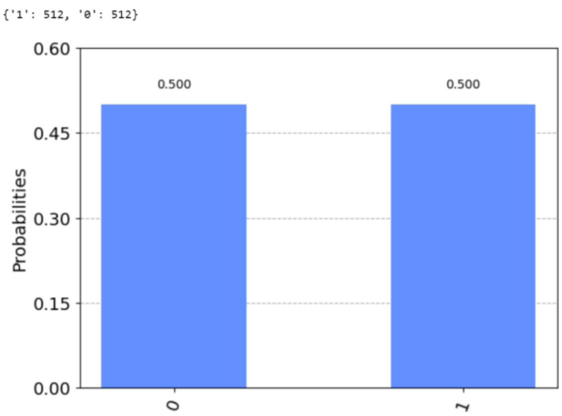




$|\Psi 2\rangle = (-i)|\Psi 1\rangle$

```
Statevector([-1.29893408e-16-0.70710678j, -1.29893408e-16-0.70710678j],
            dims=(2,))
```





Can you distinguish them (i.e. tell it is up or down circuit) from the measurement outcomes?

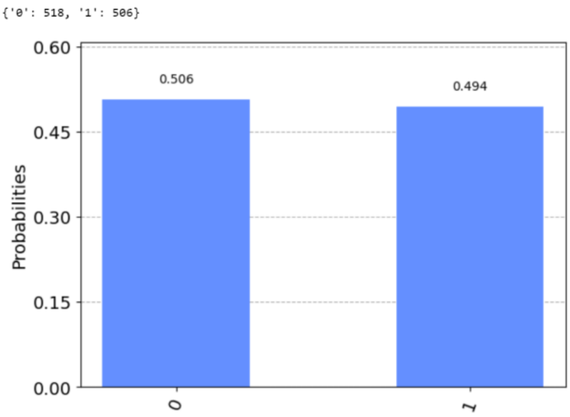
Ans. No

(b)

$|\Psi 1\rangle$

```
Statevector([-1.29893408e-16-0.70710678j, -1.29893408e-16-0.70710678j],
            dims=(2,))
```

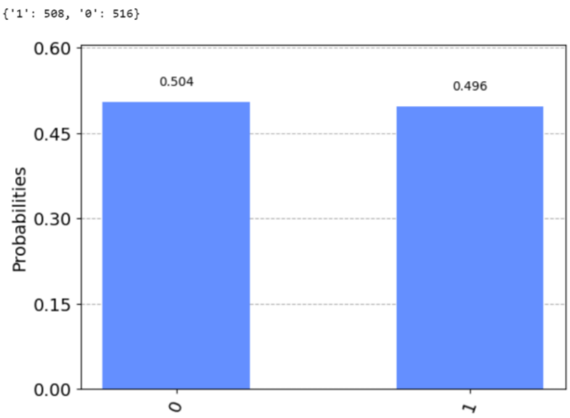
```
q0: ─┤ H ├─┤ Y ├─┤ Z ├─┤M├─
c0: 1/══════════════════╩═
                        0
```



$|\Psi 2\rangle = (-1)|\Psi 1\rangle$

```
Statevector([4.32978028e-17+0.70710678j, 4.32978028e-17+0.70710678j],
            dims=(2,))
```

```
q1: ─┤ H ├─┤ Y ├─┤ Z ├─┤ Y ├─┤ Z ├─┤ Y ├─┤ Z ├─┤M├─
c1: 1/══════════════════════════════════════════╩═
                                                0
```

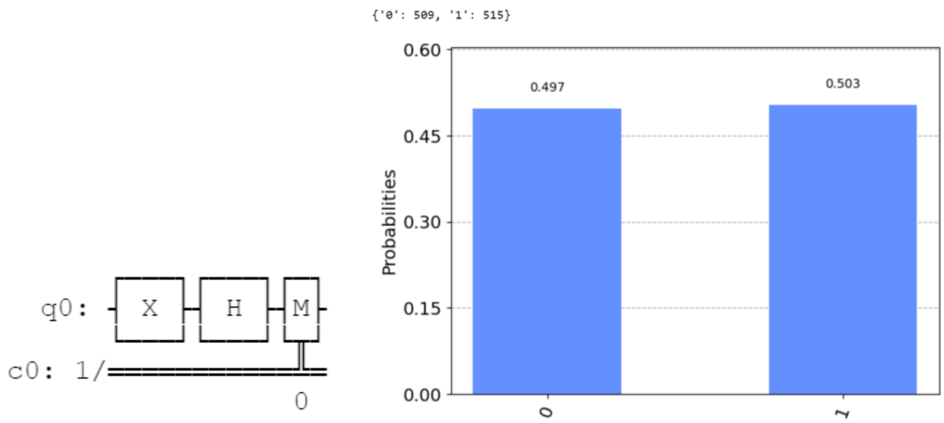


Again, I can't distinguish them from the measurement outcomes

(c)

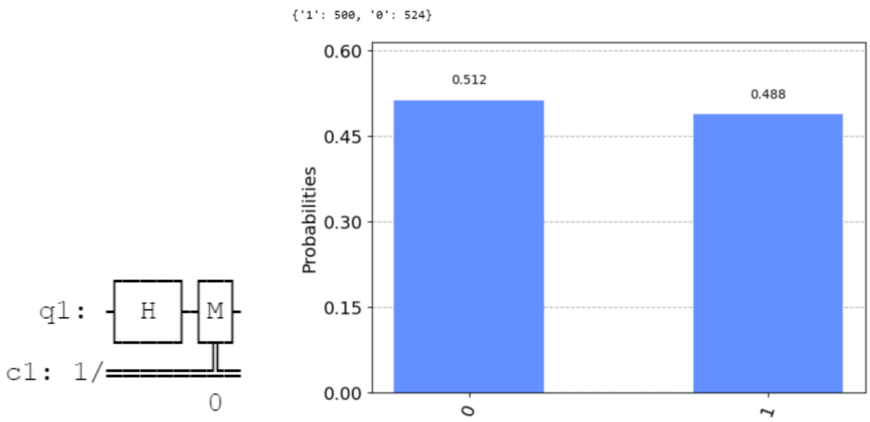
$|\Psi 1\rangle$

```
Statevector([ 0.70710678+0.00000000e+00j, -0.70710678-8.65956056e-17j],
            dims=(2,))
```



$|\Psi 2\rangle$ *only the second term is multipled by* (-1)

```
Statevector([0.70710678+0.j, 0.70710678+0.j],
            dims=(2,))
```



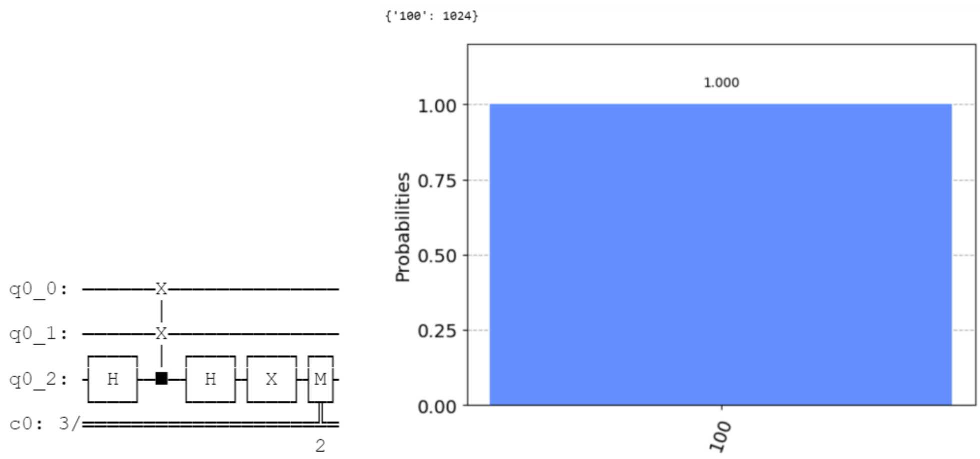
Since the outcomes seemly are measured the same, the relative phase does matter sometimes. To prove that they are not equivalent, we may change it with another input set or directly use the swap test that mentioned in 4. to check that they are the same (both actually case and indeed case) or different.

---

4. (20 points) The Swap test

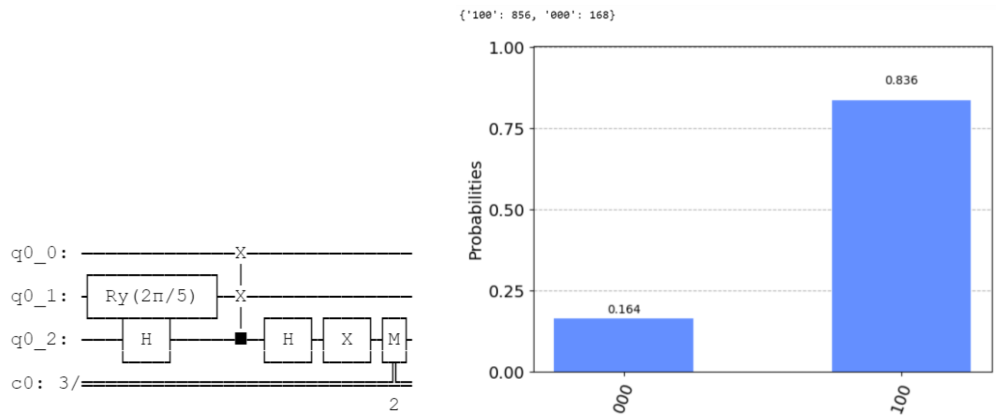
(a)

Actually the same



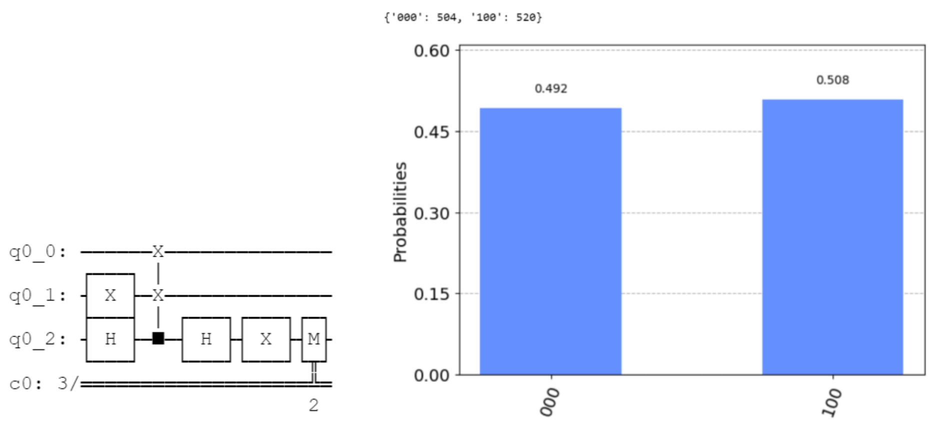
All of the outcome is measured as ‘1’

Not the same



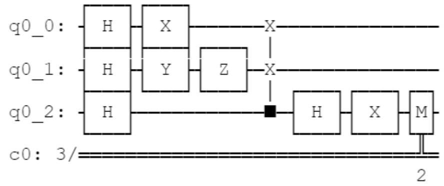
Both ‘0’ and ‘1’ may be measured as outcomes

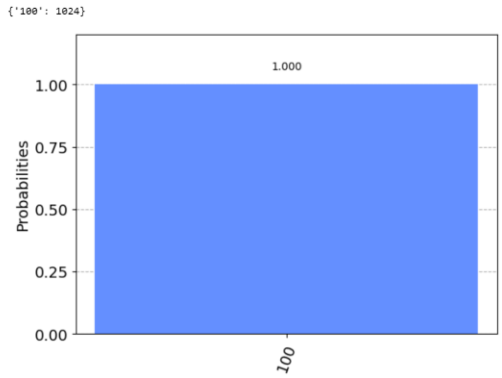
Orthogonal



Outcome is measured as ‘0’ with a 50% probability
Outcome is measured as ‘1’ with a 50% probability

Indeed the same



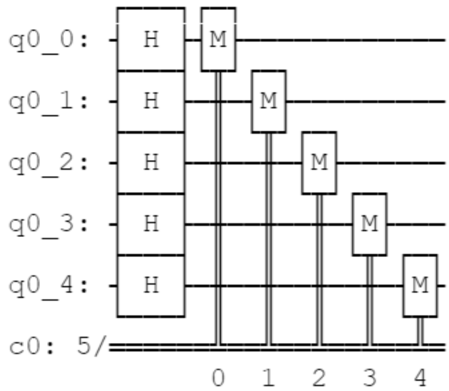


All of the outcome is measured as ‘1’

---

5. (20 Points) Quantum Random Number Generator (QRNG)

(a)



Simulator


```
['00111', '00000', '01000', '00110', '00000', '11100', '11011', '11110', '11000', '10011', '01010', '11010', '00100', '00000',
'00100', '00111', '01000', '10101', '00110', '01111', '01011', '00000', '11010', '01001', '11100', '10000', '11110', '00011',
'00011', '11101', '00011', '11010', '11011', '10001', '00010', '10110', '01010', '10001', '01010', '01100', '10100', '00100',
'00000', '11000', '10001', '01101', '11011', '10110', '00101', '01000', '11100', '01101', '10001', '10100', '01011', '10001',
'10111', '00110', '11100', '10001', '00111', '10010', '11100', '11011', '11111', '10100', '11010', '11110', '00010', '10100',
'10100', '11000', '10011', '01111', '01001', '00100', '01010', '11011', '10010', '11000', '10011', '00110', '10111', '10000',
'01101', '10110', '00110', '01111', '11011', '10111', '01001', '10100', '01010', '10111', '10011', '11000', '00011', '10100',
'11100', '01100', '00011', '01010', '11011', '11110', '01101', '01001', '00000', '01001', '00100', '01001', '10100', '11010',
'00011', '10110', '11001', '00011', '01001', '00100', '11110', '10101', '11110', '11011', '11100', '10010', '10001', '10110',
'01110', '11111', '00100', '00111', '00110', '00010', '11100', '11001', '01011', '01010', '01110', '10110', '11000', '01100',
```


…… Random

(b)

Real device


```
['00111', '01011', '11001', '11110', '10000', '01010', '01100', '10110', '10101', '10110', '11000', '01111', '00000', '10011',
'11011', '00100', '00010', '10000', '10011', '11010', '00101', '00000', '01010', '01100', '10111', '01110', '00010', '10111',
'10110', '11001', '01101', '11101', '00011', '11011', '00100', '11000', '11110', '00011', '10101', '01101', '10110', '01000',
'11000', '10110', '00101', '00011', '01001', '01011', '01111', '00111', '01001', '11011', '01111', '10010', '10101', '10110',
'00001', '00011', '10001', '00011', '11101', '01101', '11000', '10011', '01101', '11110', '00010', '10110', '11100', '11011',
'10000', '10010', '01000', '01100', '00110', '11000', '11011', '10010', '01001', '10011', '00101', '00111', '01101', '00001',
'10010', '01100', '01011', '11011', '11011', '01000', '01100', '00000', '11100', '11110', '00010', '00110', '00111', '00101',
'01100', '01001', '00100', '01000', '00110', '11000', '00000', '10111', '01001', '11001', '11011', '01111', '10111', '10010',
'10010', '01110', '01001', '01100', '10111', '10100', '01111', '00000', '00010', '11100', '11110', '01110', '00000', '10101',
'11110', '10101', '11010', '01000', '10010', '11110', '01110', '11010', '01010', '00010', '00010', '10111', '00001', '10010',
```


…… Random

Bonus

Simulator

Test name	Result value (P-value)	Status
1. Frequency (Monobit) Test	0.8013829984038171	Passed
2. Frequency Test within a Block	0.18435468419790033	Passed
3. Runs Test	0.9547150420645617	Passed
4. Test for the Longest Run of Ones in a Block	0.4601625714573061	Passed
5. Binary Matrix Rank Test		Error
6. Non-overlapping Template Matching Test		Error
7. Overlapping Template Matching Test		Error
8. Maurer's "Universal Statistical" Test		Error
9. Linear Complexity Test		Error
10. Serial Test		Error
11. Approximate Entropy Test	0.9140063841325397	Passed
12. Cumulative Sums (Cusum) Test	P-value Forward: 0.6621558639383602 P-value Reverse: 0.8638107552650531	Passed
13. Random Excursions Test		Error
14. Random Excursions Variant Test		Error

Real device

Test name	Result value (P-value)	Status
1. Frequency (Monobit) Test	0.5716076449533316	Passed
2. Frequency Test within a Block	0.14718185616880508	Passed
3. Runs Test	0.8839559805220345	Passed
4. Test for the Longest Run of Ones in a Block	0.996608294024617	Passed
5. Binary Matrix Rank Test		Error
6. Non-overlapping Template Matching Test		Error
7. Overlapping Template Matching Test		Error
8. Maurer's "Universal Statistical" Test		Error
9. Linear Complexity Test		Error
10. Serial Test		Error
11. Approximate Entropy Test	0.4858745927799353	Passed
12. Cumulative Sums (Cusum) Test	P-value Forward: 0.30634247073578225 P-value Reverse: 0.7573633290584671	Passed
13. Random Excursions Test		Error
14. Random Excursions Variant Test		Error

It seems that the simulator generates more randomly.

Now we can use 5 qubits to generate completely 5-bit random numbers (from 0 to 2^5-1). If it is possible to measure and map more than two states (ex. $|0\rangle$ 、$|1\rangle$) on only one qubit, we can use n qubits to generate more than n-bit (completely) random numbers (ex. three states on one qubit) → can represent numbers from 0 to 3^5-1, which is a wider range compared to completely 7-bit random numbers (from 0 to 2^7-1).

這次的實驗可以說是我第一次接觸量子模擬的領域，在實作上遇到花比較多時間的地方是在熟悉整個系統的表示方法以及概念轉換。在課堂上學習到的知識在透過視覺化的模擬配合實際去操作的矩陣運算會變得比較直觀，也透過這次作業逐漸加深一些基礎的概念，讓我很有收穫。