Signals and Systems MATLAB HW3
Deadline: 2021/05/20 before 23:59

The objective of this homework is to learn some MATLAB commands about the digital filter design and one of its applications.

Digital Filters

In this section, you will learn how to use the following MATLAB commands:

1.1 Background

A causal, nonideal lowpass filter is designed with frequency response $H(e^{jw})$ and its difference equation is specified as

$$a_1 y[n] = \sum_{k=1}^{N+1} b_k x[n-k+1] - \sum_{k=2}^{M+1} a_k y[n-k+1]$$

where $x[n]$ and $y[n]$ denote the input signal and the output signal, respectively.

- To obtain the frequency response of the filter, the MATLAB command **freqz**, ex:

  $$[H, w] = \text{freqz}(b, a, K);$$

  returns the K-point complex frequency response vector H and the K-point frequency vector w in radians/sample of the filter:

  $$H(e^{jw}) = \frac{b_1 + b_2 e^{-jw} + \cdots + b_{N+1} e^{-jNw}}{a_1 + a_2 e^{-jw} + \cdots + a_{M+1} e^{-jMw}}$$

  given numerator and denominator coefficients in vectors b and a where

  $$a = [a_1, a_2, \cdots, a_{M+1}]$$

  and

  $$b = [b_1, b_2, \cdots, b_{N+1}].$$

- The MATLAB command **butter**, ex:

  $$[b, a] = \text{butter}(L, f_c)$$

  designs an IIR filter with a Butterworth response of order L and returns the transfer function of numerator b and denominator a coefficient vectors (length L+1) of an Lth-order lowpass digital Butterworth filter with normalized cutoff frequency f_c. The cutoff frequency f_c of the filter is normalized so that it lies in the interval [0,1], with 1 corresponding to $w = \pi$.

- An input signal vector x if filtered by a lowpass filter with numerator b and denominator a. To obtain the output signal y, we may use the MATLAB command **filter**, ex:

$$y = \text{filter}(b, a, x).$$

1.2 Questions

1. A discrete-time signal is written as

$$x[n] = cos(2\pi(n-1)T_s)\,, n = 1,2,\cdots,100$$

where T_s denotes the sampling interval and the sampling frequency is $f_s = 1/T_s$=20Hz. Program a MATLAB script (save as **mybutter1.m** file) to do the following:

(a) (5%) Use the MATLAB function **plot** to plot $x[n]$ v.s. n.

(b) (15%) Obtain a Butterworth lowpass digital filter $H(e^{jw})$ by the MATLAB function **butter** with the following specifications :

Filter order: L=3

Cutoff frequency: f_c = 0.02

Please write down the transfer function $H(e^{jw})$ of the filter in your report and use the MATLAB function **plot** to plot the magnitude response (in dB) v.s. w interval [0,π] and the phase response (in degree) v.s. w interval [0,π] of this filter $H(e^{jw})$. Moreover, use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n of inputting $x[n]$ into the filter $H(e^{jw})$. So, there are total 3 figures in this problem.

(c) (15%) Please repeat part (b) with L=5, f_c=0.02 and f_s=20Hz.

(d) (15%) Please repeat part (b) with L=3, f_c=0.5 and f_s=20Hz.

(e) (10%) What is the effect of increasing L? What about increasing f_c? Please explain it in your report.

Note: It is better that total 9 figures in 1. (b)(c)(d) can become 9 sub-figures, which are integrated into single figure, ex: using MATLAB function **subplot**, while executing your **mybutter1.m** file.

2. An input signal is written as

$$x[n] = cos(2\pi(n-1)T_s) + 2\,cos(2\pi f_1(n-1)T_s)\,, n = 1,2,\cdots, M.$$

where T_s=0.001, f_1=100 and M=1000.

Program a MATLAB script (save as **mybutter2.m** file) to do the following:

(a) (10%) Use the MATLAB function **plot** to plot $x[n]$ v.s. n.

(b) (15%) Obtain a 8-order Butterworth lowpass digital filter by the MATLAB function **butter** such that the output

$$y[n] \approx cos(2\pi(n-1)T_s)\,, n = 1,2,\cdots, M$$

when inputting $x[n]$ into the filter.

Please write down transfer function $H(e^{jw})$ of this filter and what is your cutoff frequency in your report and use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n.

(c) (15%) Obtain a 8-order Butterworth bandpass digital filter by the MATLAB function **butter** such that the output

$$y[n] \approx 2\,cos(2\pi f_1(n-1)T_s)\,, n = 1,2,\cdots, M$$

when inputting $x[n]$ into the filter.

Please write down transfer function $H(e^{jw})$ of this filter and what is your bandpass frequency in your report and use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n.

Note: It is better that total 3 figures in 2. (a)(b)(c) can become 3 sub-figures, which are integrated into single figure, ex: using MATLAB function **subplot**, while executing your **mybutter2.m** file.

1.3 CEIBA Submission

1. Please upload a compressed file (.zip, .rar or .tar), which includes your **m-files (**save as **mybutter1.m** & **mybutter2.m** file**)** and a **word file (**save as **report.doc** file**)**. Please show the relevant plots mentioned above in the word file (report.doc) and answer the questions.
2. The compressed file name should be **ID_MATLAB3**.
   (ex: B09901xxx_MATLAB3)