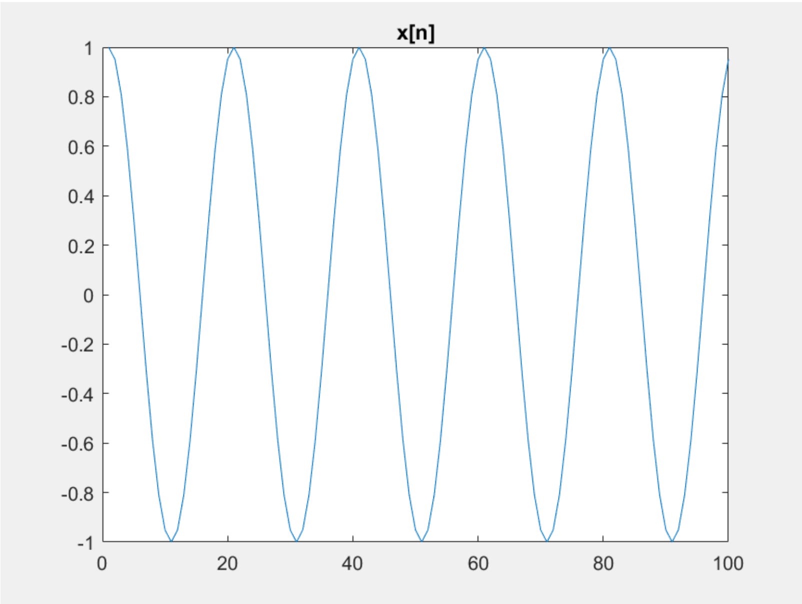
信號與系統 MATLAB 作業三 b08901019 吳冠緯

1. A discrete-time signal is written as

$$x[n] = cos(2\pi(n-1)T_s),\ n = 1,2,\cdots,100$$

where T_s denotes the sampling interval and the sampling frequency is $f_s = 1/T_s$=20Hz. Program a MATLAB script (save as **mybutter1.m** file) to do the following:

(a) (5%) Use the MATLAB function **plot** to plot $x[n]$ v.s. n.



(b) (15%) Obtain a Butterworth lowpass digital filter $H(e^{jw})$ by the MATLAB function **butter** with the following specifications :

Filter order: L=3

Cutoff frequency: $f_c = 0.02$

Please write down the transfer function $H(e^{jw})$ of the filter in your report and use the MATLAB function **plot** to plot the magnitude response (in dB) v.s. w interval $[0,\pi]$ and the phase response (in degree) v.s. w interval $[0,\pi]$ of this filter $H(e^{jw})$. Moreover, use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n of inputting $x[n]$ into the filter $H(e^{jw})$. So, there are total 3 figures in this problem.

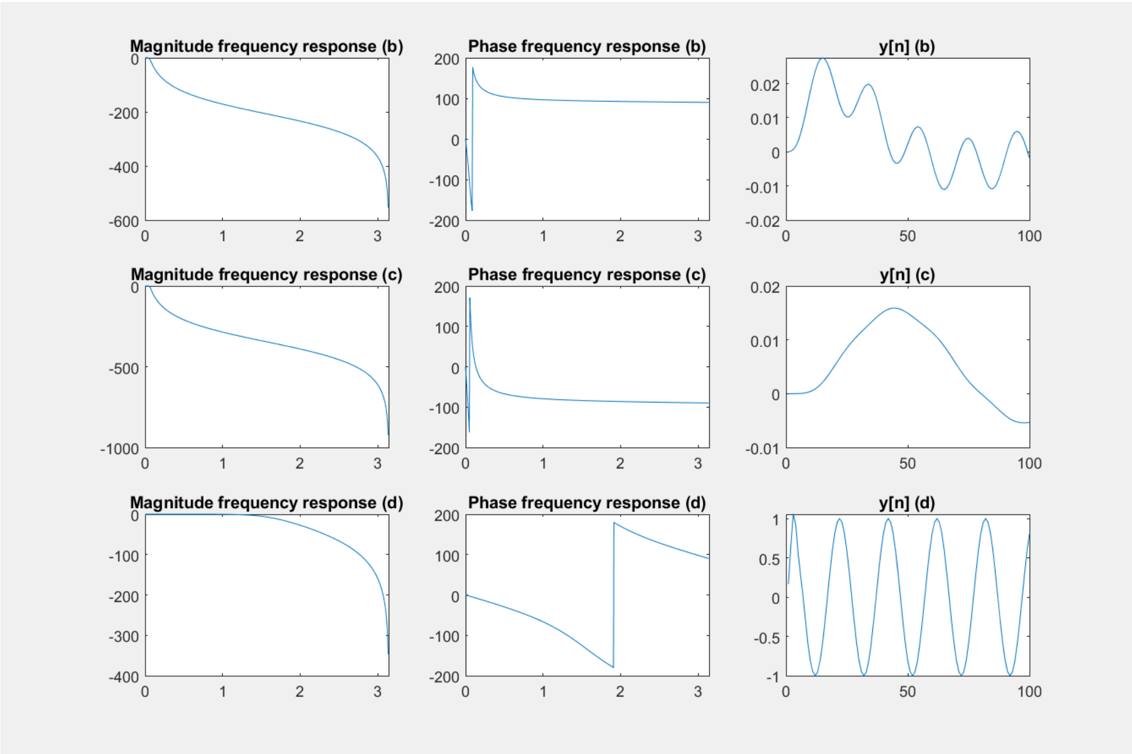
$$H(e^{j\omega}) = 10^{-4} \times \frac{0.2915 + 0.8744e^{-j\omega} + 0.8744e^{-j2\omega} + 0.2915e^{-j3\omega}}{1 - 2.8744e^{-j\omega} + 2.7565e^{-j2\omega} - 0.8819e^{-j3\omega}}$$

(c) (15%) Please repeat part (b) with L=5, f_c=0.02 and f_s=20Hz.

$$H(e^{j\omega}) = 10^{-6} \times \frac{0.0277 + 0.1384e^{-j\omega} + 0.2769e^{-j2\omega} + 0.2769e^{-j3\omega} + 0.1384e^{-j4\omega} + 0.0277e^{-j5\omega}}{1 - 4.7967e^{-j\omega} + 9.2072e^{-j2\omega} - 8.8404e^{-j3\omega} + 4.2485e^{-j4\omega} - 0.816e^{-j5\omega}}$$

(d) (15%) Please repeat part (b) with L=3, f_c=0.5 and f_s=20Hz.

$$H(e^{j\omega}) = \frac{0.1667 + 0.5e^{-j\omega} + 0.5e^{-j2\omega} + 01667e^{-j3\omega}}{1 + 0.3333e^{-j2\omega}}$$



(e) (10%) What is the effect of increasing L? What about increasing f_c? Please explain it in your report.

Increasing L :

濾波器的效果相較之下好上許多，包含了從 magnitude frequency response 及 phase frequency response 上可以發現，僅保留較低頻的訊號，相對高頻的訊號之 magnitude 下降速度更為急遽，phase 變化的頻率點也前移。在輸出時可以發現較原本的波形而言更為緩和。

Increasing fc:

濾波器保留了一些相對高頻的訊號，因此在 magnitude frequency response 上可以看到相對高頻的訊號 magnitude 是較大的；在 phase frequency response 上可以看到 phase 變化的頻率點也後移。這樣的結果導致在輸出時會觀察到相較於原本上下變化較多較快的波形。

2. An input signal is written as

$$x[n] = cos(2\pi(n-1)T_s) + 2\,cos(2\pi f_1(n-1)T_s)\,, n = 1,2,\cdots, M.$$

where T_s=0.001, f_l=100 and M=1000.

Program a MATLAB script (save as **mybutter2.m** file) to do the following:

(a) (10%) Use the MATLAB function **plot** to plot $x[n]$ v.s. n.

(b) (15%) Obtain a 8-order Butterworth lowpass digital filter by the MATLAB function **butter** such that the output

$$y[n] \approx cos(2\pi(n-1)T_s)\,, n = 1,2,\cdots, M$$

when inputting $x[n]$ into the filter.

Please write down transfer function $H(e^{jw})$ of this filter and what is your cutoff frequency in your report and use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n.

$$H(e^{j\omega}) = 10^{-7} \times \frac{0.0018 + 0.0142e^{-j\omega} + 0.0498e^{-j2\omega} + 0.0996e^{-j3\omega} + 0.1245e^{-j4\omega} + 0.0996e^{-j5\omega} + 0.0498e^{-j6\omega} + 0.0142e^{-j7\omega} + 0.0018e^{-j8\omega}}{1 - 7.3559e^{-j\omega} + 23.6970e^{-j2\omega} - 43.6658e^{-j3\omega} + 50.3366e^{-j4\omega} - 37.1713e^{-j5\omega} + 17.1713e^{-j6\omega} - 4.5367e^{-j7\omega} + 0.5248e^{-j8\omega}}$$

$$cutoff\ frequency : \frac{20}{\frac{fs}{2}} = \frac{20}{\frac{1000}{2}} = \frac{20}{500} = 0.04$$

(c) (15%) Obtain a 8-order Butterworth <u>bandpass</u> digital filter by the MATLAB function **butter** such that the output

$$y[n] \approx 2\cos(2\pi f_1 (n-1)T_s), n = 1,2,\cdots,M$$

when inputting $x[n]$ into the filter.

Please write down transfer function $H(e^{jw})$ of this filter and what is your bandpass frequency in your report and use the MATLAB function **plot** to plot the output signal $y[n]$ v.s. n.

$$H(e^{j\omega}) = 10^{-8} \times \frac{b_1 + b_2 e^{-j\omega} + b_3 e^{-j2\omega} + b_4 e^{-j3\omega} + \cdots + b_{17} e^{-j16\omega}}{a_1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega} + a_4 e^{-j3\omega} + \cdots + a_{17} e^{-j16\omega}}$$

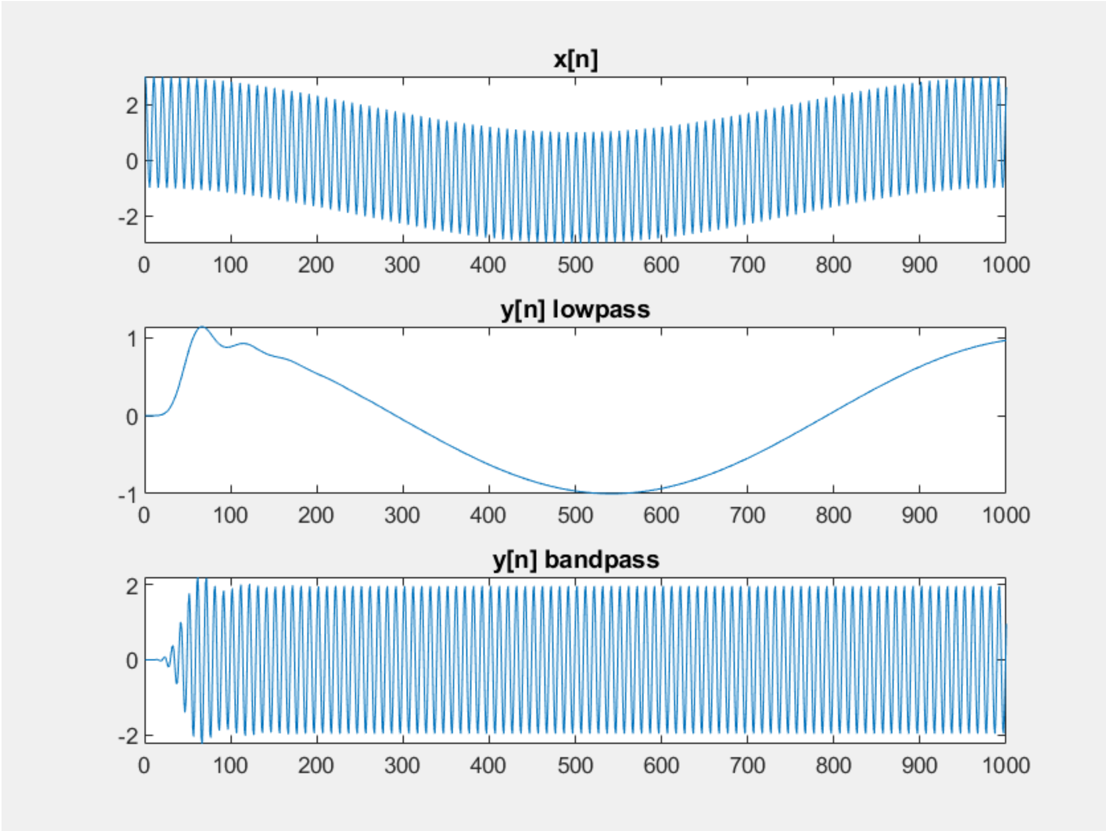
where

$$b_1 = 0.0034, b_2 = 0, b_3 = -0.0272, b_4 = 0, b_5 = 0.0952, b_6 = 0,$$
$$b_7 = -0.1903, b_8 = 0, b_9 = 0.2379, b_{10} = 0, b_{11} = -0.1903, b_{12} = 0,$$
$$b_{13} = 0.0952, b_{14} = 0, b_{15} = -0.0272, b_{16} = 0, b_{17} = 0.0034$$

and

$$a_1 = 0.001, a_2 = -0.012, a_3 = 0.0697, a_4 = -0.26, a_5 = 0.6944,$$
$$a_6 = -1.4068, a_7 = 2.2344, a_8 = -2.8362, a_9 = 2.9066, a_{10} = -2.4128,$$
$$a_{11} = 1.6171, a_{12} = -0.8662, a_{13} = 0.3637, a_{14} = -0.1159, a_{15} = 0.0264,$$
$$a_{16} = -0.0039, a_{17} = 0.0003$$

$$cutoff\ frequency: \left[\frac{80}{\frac{fs}{2}}, \frac{120}{\frac{fs}{2}}\right] = \left[\frac{80}{\frac{1000}{2}}, \frac{120}{\frac{1000}{2}}\right] = \left[\frac{80}{500}, \frac{120}{500}\right] = [0.16, 0.24]$$



x[n]
2
0
-2
0 100 200 300 400 500 600 700 800 900 1000
y[n] lowpass
1
0
-1
0 100 200 300 400 500 600 700 800 900 1000
y[n] bandpass
2
0
-2
0 100 200 300 400 500 600 700 800 900 1000