Proposition 1 *Let $\boldsymbol{b} \in \mathbb{R}^n$ be the vector to be normalized, sort the elements in $\boldsymbol{b}$ in descending order, $\boldsymbol{c} = Sparsemax\left(\boldsymbol{b}\right)$, k is the number of non-zero elements in $\boldsymbol{c}$, $\forall j : 1 \leq j \leq n$, if $\exists j : b_j - b_{j+1} \geq 1$, then $j > k$.*

Proof of Prop.1

From the algorithm in the paper, there are the following conclusions:

$$k = \max\left\{k \in [n] | 1 + kb_k > \sum\nolimits_{j \leq k} b_j\right\} \tag{1}$$

Thus,

$$1 + kb_k > \sum\nolimits_{j \leq k} b_j \tag{2}$$

Then there is the following formula:

$$\sum\nolimits_{j \leq k} \left(b_j - b_k\right) < 1 \tag{3}$$

$\forall 1 \leq l \leq k-1$, because of $b_l - b_{l+1} \geq 0$, Therefore, the following formula holds

$$b_l - b_k \geq b_l - b_{l+1} \tag{4}$$

Then, proof by contradiction. if $\exists 1 \leq j \leq k, b_j - b_{j+1} \geq 1$

from Eq.(4):

$$b_j - b_k \geq 1 \tag{5}$$

Then,

$$\sum\nolimits_{j \leq k} \left(b_j - b_k\right) \geq 1 \tag{6}$$

Eq.(6) contradicts Eq.(3).

Therefore, For any j, $1 \leq j \leq n$, if $\exists b_j - b_{j+1} \geq 1$, then $j > k$.