Contents

About this document

This document gives a comprehensive discussion of my current academic interests. They are categorized into broad fields as sections, and each subsection, describing my interest for a subfield of it, will generally give my motivation for studying that field and usually end with a current open problem that I aim to solve during my research in that subfield.

If you are a professor, I might have sent the whole, a part, or a modified version of this document to you, as my statement of interest. Generally, you only need to read the subfields that I want to be working in with you; see the above table of contents. If you have time, however, I still recommend a linear read through the document to see the whole picture of my ambition and passion. Finally, to get the latest version of the document, go to `https://github.com/guanyuming-he/guanyuming-he/blob/main/interests/main.pdf`.

Part I
My academic interests

1 Security: Introduction into my interests

I believe security is a good entrance into my current interests. It awards both attention to detail and high-level systematic view; its subfields connects directly with both theoretical and practical areas that fascinates me, and it offers effective tools to solve the imminent problems I face in real life. This section will take such a flow that the subfields of my interest will be first led to from real world problems and motivation, and then be explored with my more academical and philosophical pursuits.

The term *security* in computer science generally carries the sense of achieving *security goals* through *mechanisms or properties* of a system, despite the presence of adversaries in a *threat model*. The threat model has to be carefully chosen and reviewed; there would be no security against an omnipotent adversary.

Unlike in math, one cannot arbitrarily wiggle the threat model to one's desire. On the contrary, one has little control to how one's digital rights are treated, which are increasingly threatened (e.g. [28, 71, 62]). A more prominent example is the continuous push for exceptional access to people's data and communication in various forms by governments[1], ignoring expert opinions against such measures [2, 1, 3]. Prof. Anderson has coined the fight for the control of cryptography (thus a large portion of peoples' digital rights) between governments and all who try to fight back using the term "(thirty years of) crypto war" [4]. In this section, you will see a similar tone to Prof. Anderson's honesty which I admire — I aim to approach my work with the same honesty — to confront problems as they are, to fight against injustice even when inconvenient, and to maintain my integrity throughout my academic journey. In fact, Prof. Anderson's section *Privacy and freedom issues* [4] on his homepage was one major inspiration that set me on the path of cybersecurity, but which I will not elaborate in the document.

Moreover, I unfortunately have lived in a place where peoples' digital rights are much more aggressively invaded, particularly in the form of systematic, comprehensive, and far-reaching censorship and surveillance [5] [79, Sect. 5]. These threats present one of the major real world problems I aim to address by my research in security and my main realistic motivation. Through the document, you will see my philosophical motivations as well.

[1] For a recent instance, EU revived the Chat Control proposal in 2025. See `https://euteachloop.com/time-is-running-chat-control/`.

1.1 Secure Systems: Understanding the mechanisms

1.1.1 Low-level system details

Beside the threat model, when we talk about security, we also implicitly assume another model, environment, or host, that encapsulates the problem. For instance, in the context of isolation,

- The host that encapsulates process isolation is the operating system kernel.
- The host that encapsulates virtual machine isolation is the hypervisor, often including hardware support.
- The host that encapsulates air-gapped machine isolation is the physical world, or more abstractly, the physical laws.

These hosts form a hierarchy. For instance, processes inside an OS may run inside a virtual machine, on hardware, within the physical world.

As a result, a security researcher has to understand these hosts, each to a different degree, depending on how secure she wants her systems to be. Even if a system is formally verified to be secure within a host, it can still be attacked from interactions with an outer host. That is well examplified by the Meltdown attack, which breaks both kernel and hypervisor's isolation of memory, because an outer host, the hardware, has a vulnerability [44]. In fact, Meltdown touches more than logical flaws in hardware; it relies on the timing difference of cache accesses to extract information from it [80], one that directly connects with the physical world. At this point, I feel that I should also mention the Spectre attacks family [41], which, unlike Meltdown, does not exploit a clear logic bug or fault in the hardware. Instead, it work by observing the side effects of various implementations of speculative execution, which is arguably not a bug but simply a feature implemented without caution to how information may be leaked via the outer hosts.

Indeed, there is little hope to understand and model the universe thoroughly and accurately in one's life-time, and one can expect endless discoveries of how an otherwise secure system can leak information in unexpected ways, such as power and radiation via physical effects, which opens to a large range of attacks [42, 9, 26, 80], collectively defined as side-channel attacks. Then, one must ask, is it futile trying to cope with every such possible attack? My opinion is that it depends on the current understanding of the whole cybersecurity community. One novel attack just discovered may be increasingly understood and gain popularity by the whole community and thus gradually become a necessary part every secure system designer should take into account. I view this as an everlasting dynamics where people continuously push the boundary of our understanding, and consequently expanding the requirements for all in the community.

Therefore, a good understanding of the most common and well-understood hosts, the operating systems and the hardware (including the architecture), will be essential for me as a security researcher. Additionally, figuring out how to

perform clever hacks (as in the hacking culture of MIT, not the mainstream meaning of cracking) based on knowledge of details of a system gives a great sense of achievement to me. Moreover, this continuous expansion of knowledge frontier carries a special implication in security. Whereas a mathematician is perfectly fine to specialize in one direction without knowing every details in other fields, a computer system can never be secure if only designed with one perspective. For now, a good system is usually designed by collaboration; and it will be a very interesting challenge to figure out how one single researcher can be empowered to do this task better; could the LLMs be a potential assistant who fills the knowledge gaps? Finally, this is one major place where cybersecurity connects directly with a broad range of other fields, including software engineering, systems architecture, networking, and physics. These together contribute to my interest in the low-level details of secure systems.

1.1.2 A high-level view of the Internet

A high-level understanding of how the digital infrastructures work is essential to understand what enables the threats to digital rights on top of them. The current networking infrastructure, in my opinion, has conflicting properties. On one hand, the fundamental problem of the impossibility to link every two computers requires sharing of links, a solution that welcomes centralization. On the other hand, the poor scalability of simple sharing schemes of a link (usually via a switch) necessitates a better scheme to extend a small network globally. To achieve this, the Internet relies heavily on delegation and distributed structures, such as topological divisions of its address space and delegation of most routing responsibilities to each individual networks (e.g. ISPs). Its BGP, opearing in between them, is mainly concerned exchanging reachability information, whereas its IGPs handle routing paths and allow different networks to implement different routing policies.

Therefore, the Internet has become a mixture of centralization and decentralization, where each end node is managed by an ISP network, yet no single ISP runs the whole Internet. Perhaps surprisingly, I found this hybrid structure more optimal for localized sabotage at the state level, creating "sub-Internet"s, each of which is crippled at a different level.

Unfortunately, the lower layers of the Internet have proven to have a great inertia for change. Handley gave a nice discussion on it in 2006 [34], and the trend he described has mostly been the same since then: "*the core Internet protocols have not changed significantly in more than a decade, in spite of exponential growth in the number of Internet users and the speed of the fastest links.*" [34]. On the other hand, the protocols in the higher layers evolved to a much greater degree. As the transport layer welcomes QUIC[36], the application layer has accumulated an enormous amount of innovation and progress, in particular, onion routing and Tor[31, 17], Bitcoin[52], VPN protocols[55, 20] and decentralized instant chat[48, 69], that fight for digital rights and/or promote decentralization. Yet one must not overlook the crypto-constructs that lay the foundation for all of them, which I discuss in detail in Section 1.4.

Unfortunately again, because all of these are built on the Internet, a state-level censor could easily abuse its local authority to target them. A few regimes are notorious to have blocked a vast amount of them to various extents, employing complicated passive analysis and active probing techniques [22, 40, 73, 77, 24, 23]. Although a state would need to consider the collatoral damage, a totalitarian regime would not hesitate to block an entire protocol before it can figure out how to block it selectively [81, 77]. Apart from state-level actors, because commerical local ISPs additionally have an incentive in income and profit, not only do they perform surveillance and censorship like state-actors [8, 7], they also implement unjust policies easily with their local control of the infrastructure, like unfairly limiting the use of certain P2P protocols [19, 56, 50, 8]. Although ISPs argue that these P2P protocols can consume too much bandwidth, the other side of the story is that ISPs often oversubscribe and fraudulently advertise the bandwidth of Internet service they provide [59, 11]. This essentially is a probabilistic exploit on its customers — when almost all of the users happen to use the maximal bandwidth the ISP sells to them, congestion and throttling occur, and the users, not the ISP, ultimately pay the price — this is also the same kind of injustice imposed by airlines who oversell tickets.

The question of how to build protocols and systems that preserve one's rights on top of the Internet that powerful adversaries control, fighting against surveillance, censorship, and overall other unfair practices which the infrastructures of the current networks happen to enable, is central to my interest in the high-level design of secure distributed systems.

1.2 Data-driven analysis: Lights through artifical clouds

Despite my intention to understand systems, many of them are not open for analysis, for various reasons. In particular, the aforementioned systems for digital surveillance and censorship are not only proprietary but often state secrets. However, just as no material can perfectly insulate against heat in physics, no system can perfectly isolate information; some of it inevitably leaks through side channels (see Section 1.1.1).

By carefully observing their behaviors and probing their responses under controlled conditions, we can gain insight. Combined with data science techniques, these observations reveal hidden structures and enable us to draw meaningful conclusions.

The process usually involves 1) raising questions and claims about some properties of a target system 2) conducting experiments (often involving purchased servers within such as Aliyun) 3) analyzing data 4) answering questions and verifying claims. As a simple example, Sheffey et al. very recently studied the IP addresses injected by the GFW censorship system. By first finding injected IPs and then probing them within the GFW, they found three categories of such IPs [65].

Thanks to continuous work from both academic scholars and dedicated organizations, (e.g. [78, 53, 23]), I and a small set of others who suffer from such systems and who are fortunate enough to know these works, could know what

we are facing everyday better. Conversely, I strongly hope to contribute to the free-side of the arms race, making the free Internet reachable to everyone.

1.3 LLMs and security: A timely matter

The contemporary popularity of LLMs in both the academia and the general public is so obvious that I don't need to cite to support it. One particular public usage of them is for information retreival, despite the fact that they are essentially probalilistic guessing machines.

1.4 Cryptography: A mathematical savior

Security is often described as an arms race — that who controls more resources tends to discover more vulnerabilities, devise more attacks, and harden their systems more. Where is hope, then, when most self-censor and give up fighting the regime [12, 51, 72] and even fewer of the fighters have sufficient skills to join in the security war [58, 38]?

I believe one solution to what might sound like a power struggle lies within (modern) cryptography. For thousands of years since the use of the earliest symmetric encryption methods like the Ceasar cipher, the ability to securely communicate through an insecure channel had still been mostly limited to the privileged who could afford persistent access to physical secure channels to exchange the keys and reliable safeguarding of the keys. A stunning turning point was found in what was widely regarded as the beginning of modern cryptography, Deffie and Hellman's *New Directions in Cryptography* [16], where they gave a practical mathematical procedure to Merkle's original idea for establishing a key known only to both parties over an insecure channel.

At first glance, the idea sounded so impossible that Merkel himself faced rejections when presenting the idea to his then professor Hoffman and to the CACM [49]:

> "I am sorry to have to inform you that the paper is not in the main stream of present cryptography thinking and I would not recommend that it be published in the Communications of the ACM."
>
> "Experience shows that it is extremely dangerous to transmit key information in the clear."[49]

The rejections represented a concensus of the old cryptography community that even Shannon concurred with: "*The key must be transmitted by non-interceptible means from transmitting to receiving points*" [63, p. 670], which demonstrates how "paradoxical" the idea was.

Such "paradoxical" ideas of modern cryptography are what attract me most to it, because the more paradoxical such an idea is, the greater extent we know one can preserve one's rights, even when threatened by extremely powerful adversaries[2]. Thus, such ideas become the enabler that underlies those systems

[2]Albeit unfortunately they all rely on some assumptions, particularly that some problems are hard.

mentioned in section 1.1.2: TOR, Bitcoin, Matrix, etc., and the catalyst that leads to the massive adoption of society advancements such as e-commerce. Remarkably, in less than 50 years since 1976, we already have a number of such discoveries in addition, and here's a list of some of them.

Pseudorandom functions The kind of deterministic algorithms whose outputs look like that of a random oracle, by block ciphers like AES[14] or other ways [29], has a strong link with various other essential constructs like one-way functions.

Zero-knowledge proofs By interactively leveraging challenges that only a true prover could easily solve, zero-knowledge proof [32] enables one to verify that the prover knows a witness of a problem in NP [30] without revealing the witness.

Homomorphic encryptions Doing computation on encrypted data has been a very desirable property for many years. Although many earliest public-key encryption schemes, such as RSA [61] and ElGamal [21], already natively supported limited homomorphism like modular multiplication by their design, the first full scheme that allowed arbitrary computation on encrypted data was only proposed in 2009 [27].

These discoveries have many applications and implications, two of which I pay most attention to. One is that they update our understanding of certain theoretical lower bounds of how much security one can achieve in the face of strong adversaries, no matter the power difference between them (provide the adversary is still bounded within feasible computational limits, of course). The other is how they help minimize the trust required to perform global-level of collaboration that involves people from vastly different backgrounds and beliefs; such a collaboration that would typically require a mutually trusted central authority to organize, now only requires the participants to trust a few foundamental assumptions of cryptography.

Personally, I would go a step further. I believe modern cryptography offers not only technical tools, but a possible mitigation for one of the big challenges our democracies face today: fragmentation, polarization, and the erosion of shared trust[3]. By design, cryptography redistributes power — it makes mass surveillance prohibitively costly, even for state-level actors, by forcing them to target individuals rather than populations. For example, suppose breaking one person's encryption, on average, required a single day of dedicated effort (through side-channels, vulnerabilities, or social engineering rather than mathematics), that constraint alone would have the potential to prevent constant massive surveillance. A real challenge is the constant discoveries of vulnerabilities that affect systems used by a large proportion of people, but it does not

[3]It's only a mitigation because that problem is fundamental. When people lose trust in each other, many will instead turn to support a strong hand, hoping that it can save them [25, 43, 54]. But a strong hand almost always turns into a true tyrant that only benefits a few and enslaves the rest. This is something that cryptography alone cannot solve.

invalidate the power of cryptography, because massive exploitation of a zero-day vulnerability will result in a speedy patch, and this dynamics makes the surveillance based on even universal vulnerabilities discrete, instead of a continuous event. Indeed, if the governemnts could easily ignore all the crypto constructs, then they wouldn't, as mentioned at the start of the section, insist on overwriting the laws to legally plant universal backdoors to snoop on people.

In this sense, cryptography does something profound: it automatically and passively unites individuals, protecting each not by active coordination that is increaingly difficult, but by the collective shield of widespread adoption. Fortunately, as many modern infrastructures already deploy encryption and other crypto schemes by default, this "passive solidarity" is not a dream but a reachable reality, one that shows how mathematics can serve as a partial safeguard for democracy in an increasingly divided world.

To end this section, cryptography not only flows towards my passion for purer philosophical understanding, because of its deep connection with theoretical computer science and mathematics, to which I turn in Section 3, but also carries my hope to break free from the strong grasp of the tyrannies today, one that represents my free will which refuses to lose my agency and independence, in a world that too often seeks to reduce us into interchangable screws for the system.

2 Software engineering: My earliest and continuing passion

2.1 Software construction

The seed for my passion in questioning how things should be built and be amazed at how complex things interact with each other when assembled together, was there in me long before I had an idea of what research or the academia were. In high school, I was obsessed with creating my own video games, what I hoped would be my sanctuary from my then miserable life. My unsuspectingly ambitious goal of starting from building a game engine unsurprisingly failed, but it did introduce me to the book *Game Engine Architecture* [33], which isn't academically significant, but its contents made me realize that I was more into how things work in the engine such as how threads coordinate with each other than into actually making a game.

This interest of figuring out how a complex system work and how to build such one, leads me into learning various software engineering principles, in particular, those of Liskov [45, 46], learning them deeper when offered by my undergraduate courses, and applying some of them in all of my big projects, since I got to know them.

With all these principles, engineering of large software systems still remains a big challenge, particularly in interface design, separation of components, and all that we still have no fixed rules to follow. In other words, large systems engineering largely remains an art instead of a science [10, 64]. Could it become a

science one day, governed by well-established rules that guide us toward optimal software systems in all or at least most situations? That is an intriguing question I aspire to explore while continuing to construct software in the following years.

2.2 Formal methods/verification

Many software engineering principles are concerned with mistakes made by humans. Not only do we try to minimize bugs from programs, we also need to reduce them from the specification or requirements of the software. Therefore, formalizing the specification and verifying that the software built indeed satisfies it could be seen as an ultimate way to eliminate bugs [35].

2.2.1 The speed problem of formalization

Perhaps unsurprisingly, the industry has a low usage of fully verified software [76], since the business dynamics changes rapidly, and approaches that allow quick iteration of software, such as agile development, are much more preferred [18]. However, even in the academia, formal verification is not frequent, outside of critical areas. One central reason could be that formalization of software is simply too time-consuming [76]; as a price of having reduced software logic into primitive steps, much more of such steps are required.

That might seem to be an essential difficulty preventing the wide-adoption of formal methods in software engineering, but another way of dealing with the problem is increasing the formalization speed. One approach is to formally build more advanced and verified steps out of primitive steps and define them in *proof assistants*. For example, Coq (Rocq) allows one to build custom tactics and use a meta-language such as Ltac to manipulate tactics at a higher level [6]. Furthermore, LLMs, the current hot topic, have the capacity to output seemingly correct formalization at high speed [57]. Combined with proof assistants, which can automatically verify the correctness of the output, we could have the potential to greatly speed up formalization of not only software, but also of informal mathematical proofs found everywhere in mathematical texts, if the product of the correct rate and the output speed of LLMs could surpass that of ours.

2.2.2 How formalization may change collaboration

One interesting application of proof assistants is Massot's `Blueprint` [47]. It is mostly a dependent graph that links each piece (lemma) of a big proof as its nodes, which one can click on to go to its rendered LaTeX, LaTeX source, and Lean (a proof assistant) source. Depending on a node's formalization status, it's displayed differently in the graph [67]. Beside being used in various math projects (see its README), it also has the potential to support building interactive math textbooks for students.

In another direction, the deep trust that was previously required for each collaborator's math skills, can now be reduced by automatic verification with

proof assistants. Thus, whereas a research-level math project used to be done by a few professional mathematicians only, such one now can be instead conducted this way: 1) the few expert lead mathematicians design a whole picture of the theorem to prove and divide it into pieces 2) people, even those who are not professional mathematicians, can claim pieces that they think they can solve. 3) when they give solutions, it's the proof assistants' job to verify them. As a proof of concept, Tao has led a pilot project conducted such way [68].

While most software engineering projects require far less trust than a math project does, projects of engineering critical secure systems could still demand a high-level of trust and thus benefit from proof assistants in the same way as math projects above. In particular, could the development of a fully-verified and quite complete (in the sense of containing drivers, file systems, and other components, unlike a microkernel) OS kernel be possible, if collaborated this way? That would be an interesting question to answer in my future research.

2.3 Free software, free society[4]

As our lives are more digitalized, digital computers are playing an increasingly important role in them. Consequently, she who controls the computers will have great power over our lives. But do we control the computers (in the general sense, including mobile phones, smart devices, etc.) that we purchased and own? By the very definition, software controls our computers. Then, the question becomes, do we control the software that runs on our computers?

Unfortunately, the answer is no for proprietary software. Stallman has a number of nice essays on this topic [66], and I would just use a simple analogy here: using proprietary software is like entrusting one's money to another person, whose operations are opaque, whose decisions one has little or no influence on, whose interests are mainly profit, and who otherwise has no connection with one. Clearly, no sane person would choose to do that. The core problem of proprietary software is that via them the developers impose such an unjust power imbalance over its users, which tends to corrupt and lead to mistreatment of users, as history has repeatedly shown us.

By launching the free software movement, Stallman sought to restore power to users by securing their essential freedoms [66, Essay 1]. The movement has achieved remarkable successes, yet today it still faces serious obstacles. An prominent example is the current situation of open source, perhaps surprisingly. Often associated with Raymond's *The Cathedral and the Bazaar* [60], open source in fact emerged from within the free software community as a deviation from its core philosophy. As Stallman emphasizes, "The term 'open source' quickly became associated with ideas and arguments based only on practical values, such as making or having powerful, reliable software" [66, Essay 14]. By shifting focus from freedom to utility, open source has obscured the very principles the free software movement was founded to protect.

There are two significant problems of the current situation of open source.

[4]The title here is the book title of the essays collection of Dr. Richard Stallman [66].

The first is about the licenses. Some licenses only open the source code but prevents unauthorized modification and sharing; some others grant total access but fail to prevent the work from being stolen (i.e. creating a proprietary fork from it).

The second problem concerns the nature of freedom itself. Freedom is not passive; it requires active exercise. Many users of free or open source software almost never inspect, modify, or share the code [37], thereby remaining as dependent on developers as users of proprietary systems. Their situation differs only in that the developers of free software may act more ethically and the part of the users who do exercise their freedoms can somewhat prevent the developers from going rogue. Yet the open source movement's deliberate avoidance of freedom as a guiding value undermines awareness of it within the broader community. As much as convenience is important, one should realize that the underlying free software principles which encourages modification and sharing are the enabler of the Bazaar model. The danger of discarding those principles is that it creates an opportunity for corporate corruption and take over, undermining not only freedom, but eventually the convenience the users seek at the beginning. Today, corporate-led open-source projects, such as Microsoft's VS Code, illustrate this dynamic. Although they are sustained by community contributions, their governance, licensing, and branding remain firmly under corporate control [15, 13]. The result is software that strengthens corporate power while exploiting users (e.g. VSCode has built-in telemetry that is difficult to disable). They may be powerful to use at first, but should there's a conflict with one's interest, inconvenience then arises.

That situation of open source, in my observation, is dangerously similar to the rise of authoritarian populism discussed by Norris and Inglehard, who understand populism as "a style of rhetoric reflecting first-order principles about who should rule, claiming that legitimate power rests with 'the people' not the elites. It remains silent about second-order principles, concerning what should be done, what policies should be followed, what decisions should be made" [54, p. 4]. Similarly, I view most popular open source projects as reflecting shallow principles of allowing everyone to contribute to add more features, but ignoring deeper principles as to what direction the software should take, which decisions to make when convenience conflicts with freedom, and what are not allowed, even if they add more resources to the development of the software. Although a difference is that many populists reject the previous "ruling elites" whereas many open source members may not strongly oppose free software, they all involve people who commit to good principles such as democracy and open contribution only in the form, but ignoring the ideas and actions behind. Consequently, just as open source projects are corrupted and taken over by corporations, Norris and Inglehard observed that such shallow commitment to democracy in turn damages democracy itself and welcomes authoritarian [54] — those populist parties that once claimed to be the cure for "corrupt elites" and "ruinous rule", actually make decisions that damage the collatoral good more, such as Brexit and questionable tarrifs; and some of them became the "corrupt elites" they swore to fight against once they took power (perhaps they didn't

mean it anyway and were just opportunistic).

I see this as a profound yet subtle undermining of the free software movement, and addressing this challenge is essential as computing freedom is increasingly important in achieving a free society.

3 Mathematics: My purer philosophical pursuits

As much as I am driven by real-life problems to understand systems better, the force behind my sometimes unusally deep dive into even more primitive principles is my strong desire to understand the foundational reasonings. In this section, I will describe how that has led me to explore the purer mathematical areas and my current interests in them.

3.1 Type theory and metamathematics

In the manuals of various proof assistant languages, technical terms from type theory such as universe, sort, and introduction & elimination rules, are frequently mentioned. For me who is driven to understand what's going on behind, contact with type theory is my destiny. Furthermore, I ventured deeper into its history and the philosophical doctrines it was born from.

I have thought of a few ways to present this section, and in the end I decided that I will present it as a narrative of the basic history. If you share my curiosity for foundational reasonings, then after seeing the history I presented in my style, I believe you will also see my passion. A good starting point is probably Cantor's work on his theory of abstract sets, particularly his ingenious abstraction of sets to cardinal numbers for their comparsion. In 1638 Galileo noted that there is a bijection between the natural numbers and their squares: $f : n \mapsto n^2$, raising questions about the principle that the whole is greater than any of its parts. Cantor took this further to systematically compare sets, using bijections as the primary tool.

An essential mental construct that Cantor used was his diagnoal argument, which is often presented when proving the real numbers is not countable. Yet the central philosophical reasoning behind this argument applies to general sets which we don't know are countable or not, and this leads to this important theorem, whose proof I present because it's short and raises questions about many things.

Cantor's Theorem. *There is no surjection from any set S to the set of all of its subsets $\mathcal{P}(S)$. In Cantor's words, the cardinal number of $\mathcal{P}(S)$ is always larger than that of S.*

Proof. When S is empty, it is trival that there's no surjection between $\emptyset$ and $\{\emptyset\}$. Then we consider when S is not empty.

Suppose for now there exists a bijection $f : S \to \mathcal{P}(S)$. f's being a bijection enables us to apply Cantor's diagonal argument, because now we can make

sure the number of rows and columns are the same: arrange such a matrix that each column corresponds to an element $s \in S$, and each row is arranged to correspond to the sequence $f(s)$ for all s, in the same order as the columns. Along the diagonal, flip each element to define this subset $T \subset S$ such that $s \in T$ iff $s \notin f(s)$. Although we used the bijection fact to get to this definition, the definition itself does not require a f's being a bijection; it can be any function $f : S \to \mathcal{P}(S)$. But we will need it to be a surjection for the argument that follows. So we now demote f to be any surjection.

Because f is a surjection, there is such s_0 that $f(s_0) = T$. Now we ask if $s_0 \in T$. Note that here we implicitly assume either $s_0 \in T$ or $s_0 \notin T$. In either case, by definition of T, we will arrive at a contradiction. Thus, by *reductio ad absurdum*, it is not the case that there exists such a surjection f. □

Cantor and Russell arrived at two similar paradoxes, both involving the set of all sets.

Cantor's paradox Consider the set of all sets Ω and its power set $\mathcal{P}(\Omega)$. By Cantor's theorem, the latter should be larger than the former. On the other hand, $\mathcal{P}(\Omega)$, being a set of sets, should be a subset of Ω, a contradiction.

Russell's paradox When Russell was later analyzing Cantor's theorem, he wondered what if S itself was Ω. Under the assumption that all objects are sets, we thus have $S = \mathcal{P}(S)$, and the simple $f : s \mapsto s$ serves as a bijection between them. In that case, Cantor's diagnoal subset T becomes the famous form that we know of: $T := \{s \mid s \notin s\}$.

For Russell, his paradox was discovered at a bad time, when he was on his way of finishing his *Principles of Mathematics*, and when he assumed that there was a universal set. He had hoped for a single, unified solution to the paradoxes of logic. But to his discouragement, after a few years of various attempts to fix them in the way he hoped, he was eventually forced to come to his ramified theory of types, using a hierachy of types to classify propositions [70].

Perhaps one reason why the paradoxes were so difficult to fix was that Cantor's diagonal method had revealed something deeply wrong about the previous mathematical reasoning. To avoid the paradoxes, one prominent work was Zermelo's axiomatic set theory (later refined by others) which restricted what sets can be. But there are still solved problems. Previously, mathematicians thought that they were working with real and complete math objects such as sets and numbers and that they were speaking truths of them as theorems. Since that had lead to contradictions, there must have been something wrong in the foundation of what we believed these objects to be or our logic. Even if we discard the previous foundation entirely and adopt the axiomatic theory as the new foundation, there must still be the matter of truth and meaning — we cannot adopt the axiomatic set theory entirely formally as the new foundation, without a meaning about what we think is true or not [39, Ch. 12].

Unfinished.

Part II
Additional information

4 My interest in teaching and assistantship

Although I have not had any experience as a true teaching assistant, I have always been attracted to the prospect of teaching others with my knowledge. In fact, when I am learning something exciting, I will often imagine presenting it to others and be excited about it if I believe I found a nice way to do that. (You know, I believe Section 1.1.1 and 1.4 are such examples. Why not take a look if you have not already?) Similarly, when I am writing something that I want others to see, like this document, I often take the position of the reader and wonder if she will be touched by my writing and if she can get my ideas clearly.

To talk more specifically, I am particularly fond of Prof. Winston's teaching style, which I discovered when I started self-learning his 6.034 at MIT [74]. One important thing I learned from him is that he demonstrated passion consistently during his teaching, whereas some other professors I experienced have chosen to present things more objectively. That comparison made a huge difference in my perspective — I feel more engaged and inspired by his teaching style. I also went on to listen to his *How to Speak* lecture [75] after finishing learning his course.

5 My career plan

I plan to enter the academia after my PhD and become a professor somewhere there is enough freedom of academic expression. The condition is important because I come from a place where politics and ideology alignment is the priority, and I see firsthand how damaging that is.

My plan to become a professor is a result of these three things:

1. My strong academic intersets and my passion in teaching as described early.

2. My wish to spread knowledge, advancing my own as well as the others' understanding of the subjects I study.

3. Finally, my personal commitment to live and work ethically in a way that does not require freedom to come at someone else's expense. It may be easier to gain freedom by aligning with exploitative structures, but I cannot accept that path. I hope to take a different approach: to find and strengthen spaces within academia where knowledge can be shared openly, collaboration can be fair, and students can pursue ideas without fear or compromise. Though I recognize that academia is not perfect and can be exploitative (to a less degree), I believe it still offers the best opportunity

to practice and promote a more just environment in which I work and devote my passion.

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