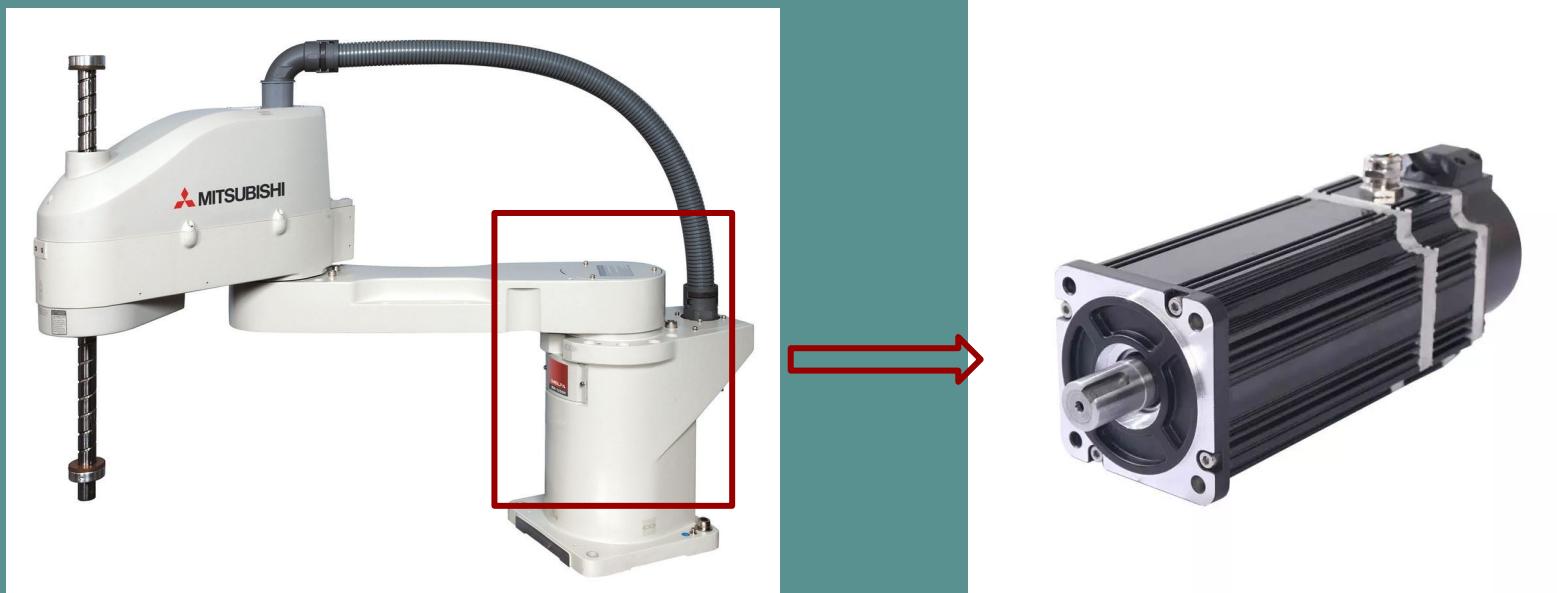
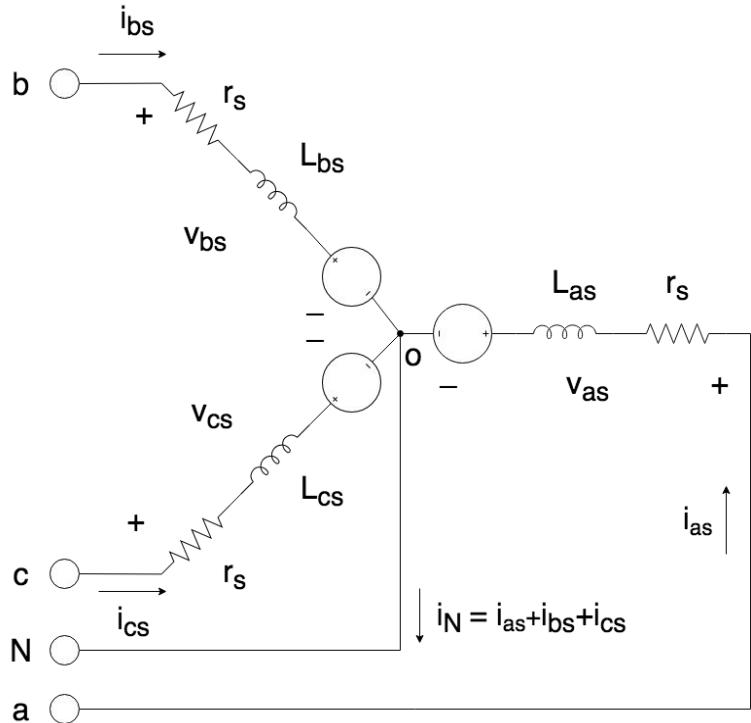


Control de Brazo Robot (1 GDL) accionado por **Máquina Síncrona de Imanes Permanentes (PMSM)**



Máquina Eléctrica y Transf. de Park



Círcuito Equivalente

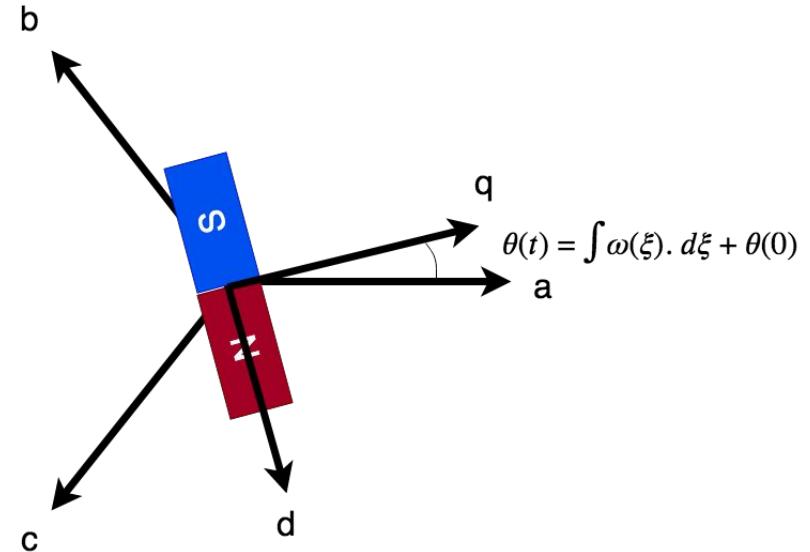


Diagrama Fasorial
abc → qd0

$$\begin{cases} v_{as} = r_s \cdot i_{as} + \frac{d\lambda_{as}}{dt} \\ v_{bs} = r_s \cdot i_{bs} + \frac{d\lambda_{bs}}{dt} \\ v_{cs} = r_s \cdot i_{cs} + \frac{d\lambda_{as}}{dt} \end{cases}$$

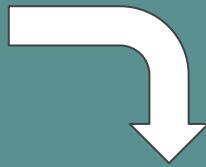
Aplicando Transf. Park

$$\begin{cases} v_{qs}^\theta(t) = r_s \cdot i_{qs}^\theta(t) + \frac{d\lambda_{qs}^\theta}{dt} + \omega(t) \cdot \lambda_{ds}^\theta(t) \\ v_{ds}^\theta(t) = r_s \cdot i_{ds}^\theta(t) + \frac{d\lambda_{ds}^\theta}{dt} - \omega(t) \cdot \lambda_{qs}^\theta(t) \\ v_{0s}(t) = r_s \cdot i_{0s}(t) + \frac{d\lambda_{0s}}{dt} \end{cases}$$

Máquinas General

Si tenemos una PMSM, sin saturación magn.:

$$\begin{cases} \theta = P_p \cdot \theta_m \\ \omega = P_p \cdot \omega_m \end{cases} \Rightarrow \begin{cases} \lambda_{qs}^r = L_q \cdot i_{qs}^r \\ \lambda_{ds}^r = L_d \cdot i_{ds}^r + \lambda_m \\ \lambda_{0s} = L_{ls} \cdot i_{0s} \end{cases}$$



$$v_{qs}^r(t) = R_s(t) \cdot i_{qs}^r(t) + L_q \cdot \frac{di_{qs}^r(t)}{dt} + [\lambda_m' + L_d \cdot i_{ds}^r(t)] \cdot \omega_r(t)$$

$$v_{ds}^r(t) = R_s(t) \cdot i_{ds}^r(t) + L_d \cdot \frac{di_{ds}^r(t)}{dt} - L_q \cdot i_{qs}^r(t) \cdot \omega_r(t)$$

$$v_{0s}(t) = R_s(t) \cdot i_{0s}(t) + L_{ls} \cdot \frac{di_{0s}(t)}{dt}$$

Modelo Mecánico

1) **Modelo Mecánico Brazo (referido al “hombro”):**

$$J_l \cdot \dot{\omega}_l(t) = T_q(t) - b_l \cdot \omega_l(t) - T_l(t)$$

2) **Transmisión “Rígida”**

$$\omega_l(t) = \frac{1}{r} \cdot w_m(t) \quad T_q(t) = r \cdot T_d(t)$$

3) **Modelo Mecánico Rotor (referido al estator = sistema inercial)**

$$J_m \cdot \dot{\omega}_m(t) = T_m(t) - b_m \cdot \omega_m(t) - T_d(t)$$

Modelo Mecánico Simplificado

Consideraciones:

- NO hay backlash, transmisión rígida
- Transmisión de potencia SIN pérdidas (solo por amort “b”)
- Es IDEAL la transmisión

Trabajando algebraicamente:

$$\underbrace{\left(J_m + \frac{J_l}{r^2} \right)}_{J_{eq}} \cdot \dot{\omega}_m(t) = T_m(t) - \underbrace{\left(b_m + \frac{b_l}{r^2} \right)}_{b_{eq}} \cdot \omega_m(t) - \underbrace{\frac{T_l}{r}}_{T_{eq}}$$

Modelo Global NL

$$\left\{
 \begin{array}{l}
 \dot{\theta}_m(t) = \omega_m(t) \\
 \boxed{\dot{\omega}_m(t) = \frac{3}{2} \cdot \frac{P_p}{J_{eq}} \cdot [\lambda_m^{r'} + (L_d - L_q) \cdot i_{ds}^r(t) \cdot i_{qs}^r(t)] - \frac{b_{eq}}{J_{eq}} \cdot \omega_m(t) - \frac{T_{eq}(t)}{J_{eq}}} \\
 \frac{di_{qs}^r(t)}{dt} = \frac{v_{qs}^r(t)}{L_q} - \frac{R_s(t)}{L_q} \cdot i_{qs}^r(t) - \frac{P_p}{L_q} \cdot [\lambda_m^{r'} + L_d \cdot i_{ds}^r(t)] \cdot \omega_m(t) \\
 \frac{di_{ds}^r(t)}{dt} = \frac{v_{ds}^r(t)}{L_d} - \frac{R_s(t)}{L_d} \cdot i_{ds}^r(t) + \frac{P_p \cdot L_q}{L_d} \cdot i_{qs}^r(t) \cdot \omega_m(t) \\
 \frac{di_{0s}^r(t)}{dt} = \frac{v_{0s}^r(t)}{L_{ls}} - \frac{R_s(t)}{L_{ls}} \cdot i_{0s}^r(t) \\
 \boxed{\frac{dT_s^\circ(t)}{dt} = \frac{\frac{3}{2} \cdot R_s(t) \cdot (i_{qs}^{r^2}(t) + i_{ds}^{r^2}(t) + 2 \cdot i_{0s}^{r^2}(t))}{C_{ts}} - \frac{T_s^\circ(t) - T_{amb}^\circ(t)}{R_{ts-amb}}} \\
 R_s(t) = R_{sREF} \cdot (1 + \alpha_{Cu} \cdot (T_s^\circ(t) - T_{sREF}^\circ))
 \end{array}
 \right\}$$

Subsistema Electromagnético

Subsistema Mecánico

Subsistema Eléctrico

Subsistema Térmico

Modelo Global NL

$$\left\{ \begin{array}{l} \dot{\theta_m}(t) = \omega_m(t) \\ \\ \dot{\omega_m}(t) = \frac{3}{2} \cdot \frac{P_p}{J_{eq}} \cdot [\lambda_m^{r'} + (L_d - L_q) \cdot i_{ds}^r(t) \cdot i_{qs}^r(t)] - \frac{b_{eq}}{J_{eq}} \cdot \omega_m(t) - \frac{T_{eq}(t)}{J_{eq}} \\ \\ \frac{di_{qs}^r(t)}{dt} = \frac{v_{qs}^r(t)}{L_q} - \frac{R_s(t)}{L_q} \cdot i_{qs}^r(t) - \frac{P_p}{L_q} \cdot [\lambda_m^{r'} + L_d \cdot i_{ds}^r(t)] \cdot \omega_m(t) \\ \\ \frac{di_{ds}^r(t)}{dt} = \frac{v_{ds}^r(t)}{L_d} - \frac{R_s(t)}{L_d} \cdot i_{ds}^r(t) + \frac{P_p \cdot L_q}{L_d} \cdot i_{qs}^r(t) \cdot \omega_m(t) \\ \\ \frac{di_{0s}^r(t)}{dt} = \frac{v_{0s}^r(t)}{L_{ls}} - \frac{R_s(t)}{L_{ls}} \cdot i_{0s}^r(t) \\ \\ \frac{dT_s^\circ(t)}{dt} = \frac{\frac{3}{2} [R_s(t) \cdot (i_{qs}^{r^2}(t) + i_{ds}^{r^2}(t) + 2 \cdot i_{0s}^{r^2}(t)) - \frac{T_s^\circ(t) - T_{amb}^\circ(t)}{R_{ts-amb}}]}{C_{ts}} \\ \\ R_s(t) = R_{sREF} \cdot (1 + \alpha_{Cu} \cdot (T_s^\circ(t) - T_{sREF}^\circ)) \end{array} \right.$$

NL producto de variables de estado

Parámetro en función de variable de estado

Producto de variable de estado con parámetro variable

Linealización Jacobiana

Modelo NL

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) , \quad x(t_0) = x_0 \\ y(t) = C \cdot x(t) \end{cases}$$

Asumimos

$$z(t) \equiv Z_0 + \Delta z(t)$$

Obtenemos

$$\begin{cases} \frac{dX_o(t)}{dt} + \frac{d\Delta x(t)}{dt} = f(X_o(t) + \Delta x(t), U_o(t) + \Delta u(t)) \rightarrow X_o(0) = x_0 ; \Delta x(0) = 0 \\ Y_o(t) + \Delta y(t) = C \cdot (X_o(t) + \Delta x(t)) \rightarrow Y_o(t) = C \cdot X_o(t) ; \Delta y(t) = C \cdot \Delta x(t) \end{cases}$$

Expandiendo mediante la serie de taylor truncada a 1º orden:

$$f(X_o(t) + \Delta x(t), U_o(t) + \Delta u(t)) \approx \dots$$

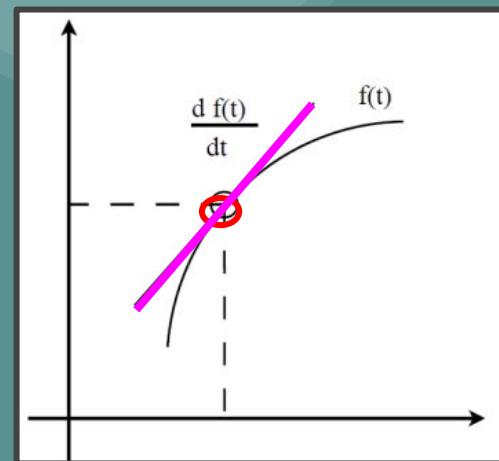
$$f(X_o(t), U_o(t)) + \left[\frac{\partial f}{\partial x_1} \quad \dots \quad \frac{\partial f}{\partial x_n} \right] \Big|_0 (t) \cdot \Delta x(t) + \left[\frac{\partial f}{\partial u_1} \quad \dots \quad \frac{\partial f}{\partial u_n} \right] \Big|_0 (t) \cdot \Delta u(t)$$

Lo que nos permite separar el modelo en dos:

Espacio de operación global NL
(cuasi-estacionario)

+

Modelo dinámico LPV (pequeñas
variaciones locales)



Espacio de operación global NL (cuasi-estacionario)

$$\left\{ \begin{array}{l} \text{cte.} \simeq \Omega_{mo}(t) \\ \\ 0 \simeq \frac{3}{2} \cdot \frac{P_p}{J_{eq}} \cdot \left[\lambda_m^{r'} + (L_d - L_q) \cdot I_{dso}^r(t) \right] \cdot I_{qso}^r(t) - \frac{b_{eq}}{J_{eq}} \cdot \Omega_{mo}(t) - \frac{T_{eqo}(t)}{J_{eq}} \\ \\ 0 \simeq \frac{V_{qso}^r(t)}{L_q} - \frac{R_{so}(t)}{L_q} \cdot I_{qso}^r(t) - \frac{P_p}{L_q} \cdot [\lambda_m^{r'} + L_d \cdot I_{dso}^r(t)] \cdot \Omega_{mo}(t) \\ \\ 0 \simeq \frac{V_{dso}^r(t)}{L_d} - \frac{R_{so}(t)}{L_d} \cdot I_{dso}^r(t) + \frac{P_p \cdot L_q}{L_d} \cdot I_{qso}^r(t) \cdot \Omega_{mo}(t) \\ \\ 0 \simeq \frac{V_{0so}^r(t)}{L_{ls}} - \frac{R_{so}(t)}{L_{ls}} \cdot I_{0so}^r(t) \\ \\ 0 \simeq \frac{\frac{3}{2} \cdot R_{so}(t) \cdot (I_{qso}^{r^2}(t) + I_{dso}^{r^2}(t) + 2 \cdot I_{0so}^{r^2}(t)) - \frac{T_{so}^\circ(t) - T_{amb}^\circ(t)}{R_{ts-amb}}}{C_{ts}} \\ \\ R_{so}(t) = R_{sREF} \cdot (1 + \alpha_{Cu} \cdot (T_{so}^\circ(t) - T_{sREF}^\circ)) \end{array} \right.$$

Modelo dinámico LPV (pequeñas variaciones locales)

$$\left\{ \begin{array}{l} \dot{\Delta\theta_m}(t) = \Delta\omega_m(t) \\ \\ \dot{\Delta\omega_m}(t) = \frac{3P_p}{2J_{eq}} \cdot \left[[\lambda_m^{r'} + (L_d - L_q) \cdot I_{dso}^r(t)] \cdot \Delta i_{qs}^r(t) + [(L_d - L_q) \cdot I_{qso}^r(t)] \cdot \Delta i_{ds}^r(t) \right] \dots \\ \quad - \frac{b_{eq}}{J_{eq}} \cdot \Delta\omega_m(t) - \frac{\Delta T_{eq}(t)}{J_{eq}} \\ \\ \frac{d\Delta i_{qs}^r(t)}{dt} = \frac{\Delta v_{qs}^r(t)}{L_q} - \left[\frac{R_{so}(t)}{L_q} \cdot \Delta i_{qs}^r(t) + \frac{\Delta R_s(t)}{L_q} \cdot I_{qso}^r(t) \right] \dots \\ \quad - \frac{P_p}{L_q} \cdot \left[[\lambda_m^{r'} + L_d \cdot I_{dso}^r(t)] \cdot \Delta\omega_m(t) + [L_d \cdot \Omega_{mo}(t)] \cdot \Delta i_{ds}^r(t) \right] \\ \\ \frac{d\Delta i_{ds}^r(t)}{dt} = \frac{\Delta v_{ds}^r(t)}{L_d} - \left[\frac{R_{so}(t)}{L_d} \cdot \Delta i_{ds}^r(t) + \frac{\Delta R_s(t)}{L_d} \cdot I_{dso}^r(t) \right] \dots \\ \quad + \frac{P_p \cdot L_q}{L_d} \cdot [I_{qso}^r(t) \cdot \Delta\omega_m(t) + \Delta i_{qs}^r(t) \cdot \Omega_{mo}(t)] \\ \\ \frac{d\Delta i_{0s}^r(t)}{dt} = \frac{\Delta v_{0s}^r(t)}{L_{ls}} - \left[\frac{R_{so}(t)}{L_{ls}} \cdot \Delta i_{0s}^r(t) + \frac{\Delta R_s(t)}{L_{ls}} \cdot I_{0so}^r(t) \right] \\ \\ \frac{d\Delta T_s^\circ(t)}{dt} = \frac{3R_{so}(t)}{C_{ts}} \cdot [I_{qso}^r \cdot \Delta i_{qs}^r + I_{dso}^r \cdot \Delta i_{ds}^r + 2I_{0so}^r \cdot \Delta i_{0s}^r] + \frac{3\Delta R_s(t)}{2C_{ts}} \cdot [I_{qso}^r{}^2 + I_{dso}^r{}^2 + 2 \cdot I_{0so}^r{}^2] \dots \\ \quad - \frac{[\Delta T_s^\circ(t) - \Delta T_{amb}^\circ(t)]}{R_{ts-amb} \cdot C_{ts}} \\ \\ \Delta R_s(t) = R_{sREF} \cdot \alpha_{Cu} \cdot \Delta T_s^\circ(t) \end{array} \right.$$

Linealización por Realimentación NL.

Si imponemos $i_d=0$, sin considerar acoplamiento térmico:

$$\left\{ \begin{array}{l} \dot{\theta}_m(t) = \omega_m(t) \\ \\ \dot{\omega}_m(t) = \frac{3}{2} \cdot \frac{P_p}{J_{eq}} \cdot [\lambda_m^{r'} + ((L_d - L_q) \cdot i_{ds}^r(t))] \cdot i_{qs}^r(t) - \frac{b_{eq}}{J_{eq}} \cdot \omega_m(t) - \frac{T_{eq}(t)}{J_{eq}} \\ \\ \frac{di_{qs}^r(t)}{dt} = \frac{v_{qs}^r(t)}{L_q} - \frac{R_s(t)}{L_q} \cdot i_{qs}^r(t) - \frac{P_p}{L_q} \cdot [\lambda_m^{r'} + L_d \cdot i_{ds}^r(t)] \cdot \omega_m(t) \\ \\ \boxed{\frac{di_{ds}^r(t)}{dt}} = \frac{v_{ds}^r(t)}{L_d} - \frac{R_s(t)}{L_d} \cdot i_{ds}^r(t) + \frac{P_p \cdot L_q}{L_d} \cdot i_{qs}^r(t) \cdot \omega_m(t) \\ \\ \frac{di_{0s}^r(t)}{dt} = \frac{v_{0s}^r(t)}{L_{ls}} - \frac{R_s(t)}{L_{ls}} \cdot i_{0s}^r(t) \\ \\ \frac{dT_s^\circ(t)}{dt} = \frac{\frac{3}{2} \cdot R_s(t) \cdot (i_{qs}^{r^2}(t) + i_{ds}^{r^2}(t) + 2 \cdot i_{0s}^{r^2}(t)) - \frac{T_s^\circ(t) - T_{amb}^\circ(t)}{R_{ts-amb}}}{C_{ts}} \\ \\ R_s(t) = R_{sREF} \cdot (1 + \alpha_{Cu} \cdot (T_s^\circ(t) - T_{sREF}^\circ)) \end{array} \right.$$

Términos que se cancelan

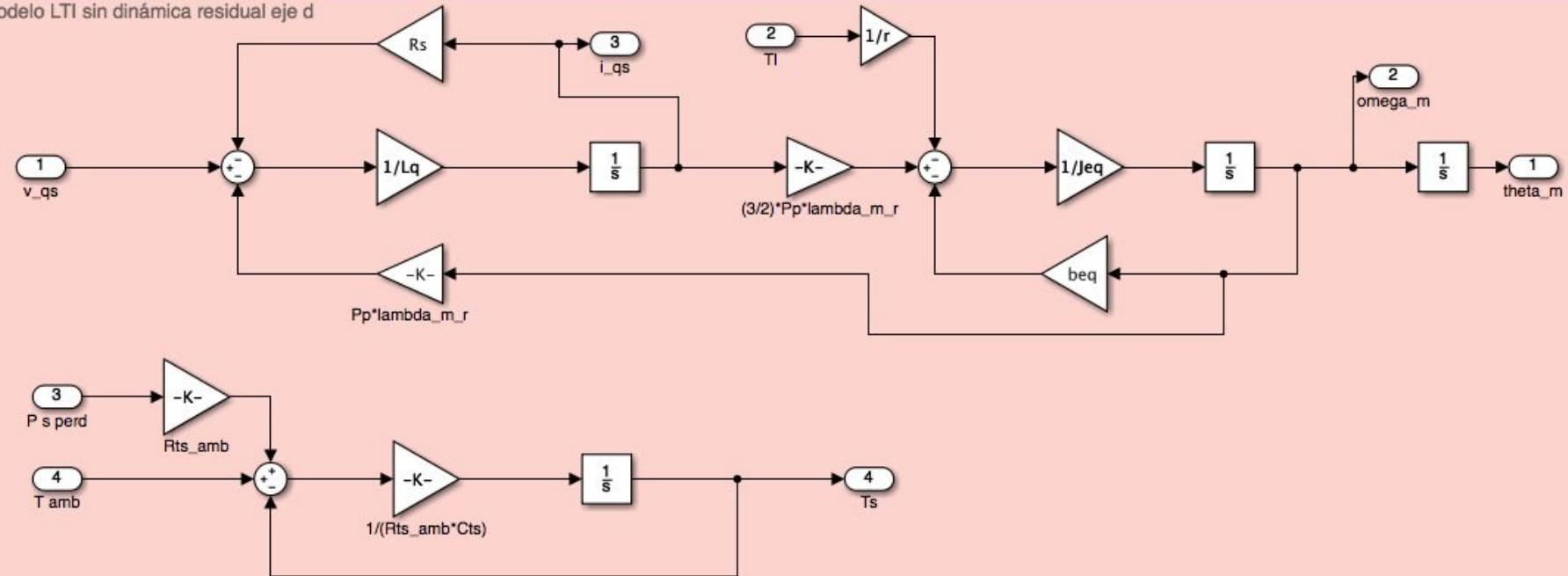
Restricción

Modelo simplificado lineal invariante (LTI) equivalente.

Quedando el sistema:

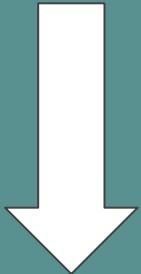
$$\begin{cases} \dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p\lambda_m^{r'}}{2J_{eq}} & 0 \\ 0 & \frac{-P_p\lambda_m^{r'}}{L_q} & \frac{-R_s}{L_q} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb}C_{ts}} \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{J_{eq}} & 0 & 0 \\ \frac{1}{L_q} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_{ts}} & \frac{1}{R_{ts-amb}C_{ts}} \end{bmatrix} \cdot u(t); \\ X(t_0) = x_0 \\ y(t) = [1 \ 0 \ 0 \ 1] \cdot X(t) \end{cases}$$

Modelo LTI sin dinámica residual eje d

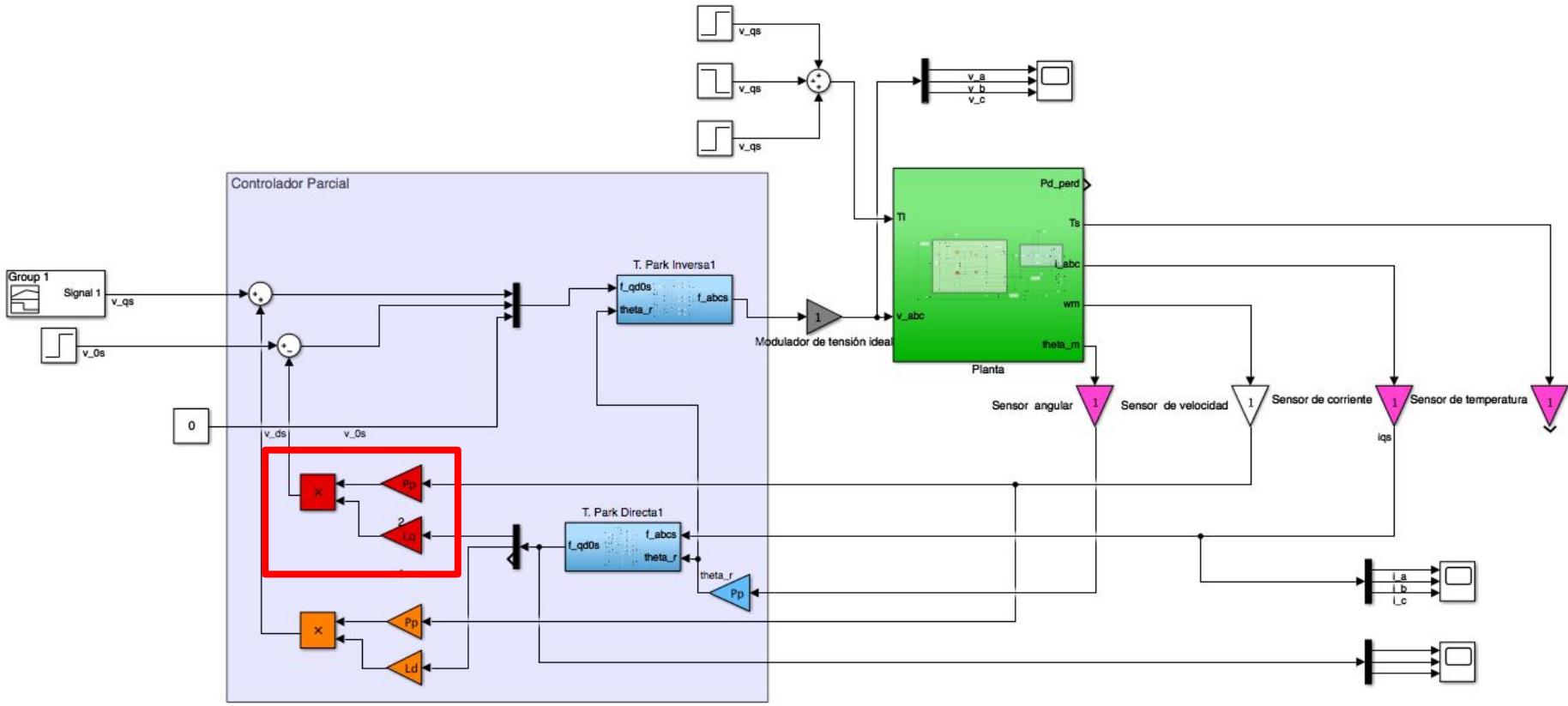


Restricción o Ley de Control mínima.

$$v_{ds}^r(t) = R_s(t) \cdot i_{ds}^r(t) + L_d \cdot \frac{di_{ds}^r(t)}{dt} - L_q \cdot i_{qs}^r(t) \cdot \omega_r(t)$$



$$v_{ds}^r(t) = -L_q \cdot i_{qs}^r(t) \cdot P_p \cdot \omega_m(t); i_{ds}^r(t_0) = 0$$



Dinámica residual en el eje d

$$v_{ds}^r(t) = R_s(t) \cdot i_{ds}^r(t) + L_d \cdot \frac{di_{ds}^r(t)}{dt} - L_q \cdot i_{qs}^r(t) \cdot \omega_r(t)$$



$$\frac{di_{ds}^r}{dt} = -\frac{R_s}{L_d} i_{ds}^r(t)$$

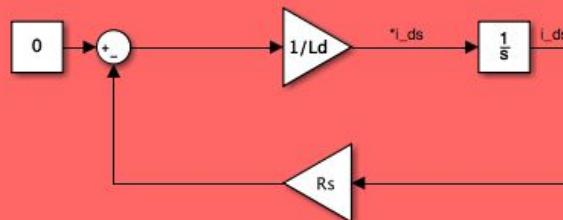
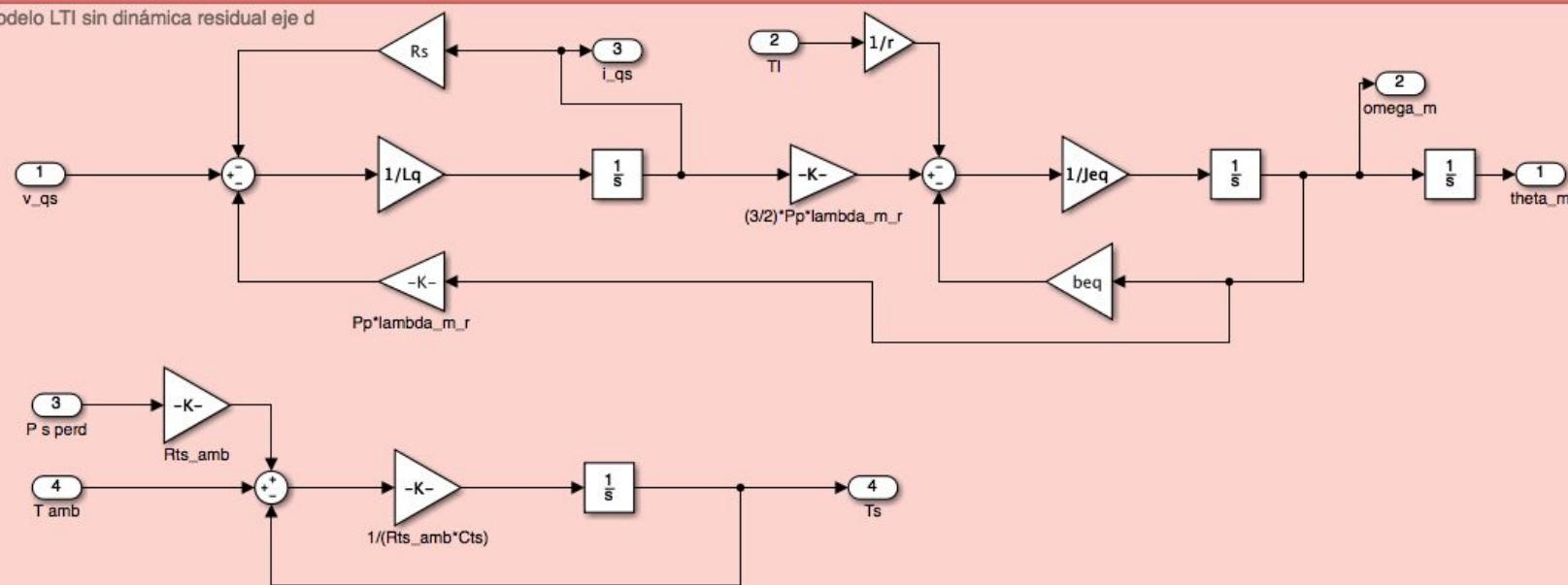
$$\frac{I_{ds}^r(s)}{I_{ds0}^r} = \frac{1}{s + \frac{R_s}{L_d}}$$

$$s = -154.55$$

$$\tau = 6.47ms$$

Modelo LTI con dinámica residual eje d

Modelo LTI sin dinámica residual eje d



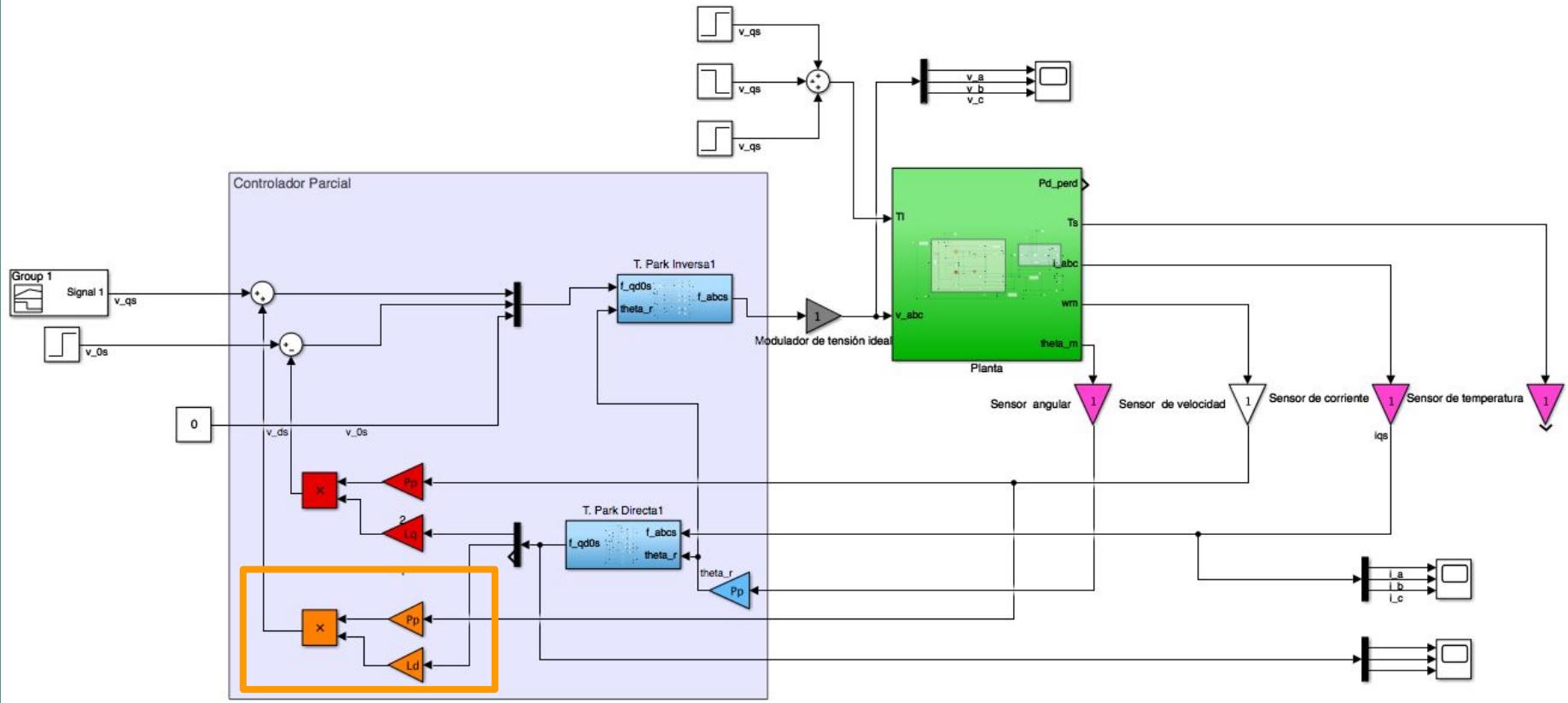
Restricción o Ley de Control complementaria mínima en el eje q

$$v_{qs}^r(t) = R_s(t) \cdot i_{qs}^r(t) + L_q \cdot \frac{di_{qs}^r(t)}{dt} + [\lambda_m' r + L_d \cdot i_{ds}^r(t)] \cdot \omega_r(t)$$

$$v_{qs}^{r*}(t) = L_d \cdot i_{ds}^r(t) \cdot P_p \cdot \omega_m(t)$$

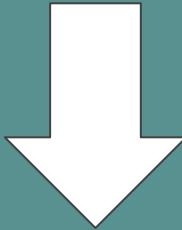


$$\begin{cases} \dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p\lambda_m^{r'}}{2J_{eq}} & 0 & 0 \\ 0 & \frac{-P_p\lambda_m^{r'}}{L_q} & \frac{-R_s}{L_q} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb}C_{ts}} & 0 \\ 0 & 0 & 0 & 0 & \frac{R_s}{L_d} \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{J_{eq}} & 0 & 0 & 0 \\ \frac{1}{L_q} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_{ts}} & \frac{1}{R_{ts-amb}C_{ts}} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot u(t); \\ X(t_0) = x_0 \\ y(t) = [1 \ 0 \ 0 \ 0 \ 0] \cdot X(t) \end{cases}$$



Comparación del modelo dinámico LTI equivalente aumentado vs. el modelo LPV forzando $I_{d0} \equiv 0$.

$$\Delta v_{ds}^r(t) = -\frac{P_p \cdot L_q}{L_d} \cdot [I_{qso}^r(t) \cdot \Delta \omega_m(t) + \Delta i_{qs}^r(t) \cdot \Omega_{mo}(t)]$$



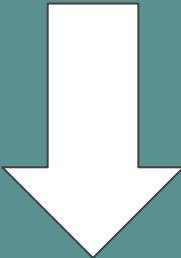
$$\begin{bmatrix} \Delta \dot{\theta}_m(t) \\ \Delta \dot{\omega}_m(t) \\ \frac{d\Delta i_{qs}^r(t)}{dt} \\ \Delta T_s(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p}{2J_{eq}}[\lambda_m^{r'} + (L_d - L_q) \cdot I_{ds0}^r(t)] & 0 \\ 0 & \frac{-P_p}{L_q}[\lambda_m^{r'} + L_d \cdot I_{ds0}^r(t)] & \frac{-R_a}{L_q} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb} C_{ts}} \end{bmatrix} \begin{bmatrix} \Delta \theta_m(t) \\ \Delta \omega_m(t) \\ \Delta i_{qs}^r(t) \\ \Delta T_s(t) \end{bmatrix}$$

Comparación del modelo dinámico LTI equivalente aumentado vs. el modelo LPV forzando $\mathbf{Id}_0 \equiv 0$.

$$\begin{bmatrix} \Delta\dot{\theta}_m(t) \\ \Delta\dot{\omega}_m(t) \\ \frac{d\Delta i_{qs}^r(t)}{dt} \\ \Delta T_s(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{J_{eq}} & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p}{2J_{eq}}[\lambda_m^{r'} + (L_d - L_q) \cdot I_{ds0}^r(t)] & 0 \\ 0 & \frac{-P_p}{L_q}[\lambda_m^{r'} + L_d \cdot I_{ds0}^r(t)] & \frac{-R_s}{L_q} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb}C_{ts}} \end{bmatrix} \begin{bmatrix} \Delta\theta_m(t) \\ \Delta\omega_m(t) \\ \Delta i_{qs}^r(t) \\ \Delta T_s(t) \end{bmatrix}$$

$$\left\{ \begin{array}{l} \dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p\lambda_m^{r'}}{2J_{eq}} & 0 & 0 \\ 0 & \frac{-P_p\lambda_m^{r'}}{L_q} & \frac{-R_s}{L_q} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb}C_{ts}} & 0 \\ 0 & 0 & 0 & 0 & \frac{R_s}{L_d} \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{J_{eq}} & 0 & 0 \\ \frac{1}{L_q} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_{ts}} & \frac{1}{R_{ts-amb}C_{ts}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot u(t); \\ X(t_0) = x_0 \\ y(t) = [1 \ 0 \ 0 \ 0 \ 0] \cdot X(t) \end{array} \right.$$

$$T_m(t) = \frac{3}{2} \cdot P_p \cdot \left[\lambda_m^{r'} + (L_d - L_q) \cdot i_{ds}^r(t) \right] \cdot i_{qs}^r(t)$$



- a) $I_{ds0}^r(t) = 0$
- b) $I_{ds0}^r(t) > 0$
- c) $I_{ds0}^r(t) < 0$

$$\left\{
\begin{array}{l}
\text{cte.} \simeq \Omega_{mo}(t) \\
\\
0 \simeq \frac{3}{2} \cdot \frac{P_p}{J_{eq}} \cdot \left[\lambda_m^{r'} + (L_d - L_q) \cdot I_{dso}^r(t) \right] \cdot I_{qso}^r(t) - \frac{b_{eq}}{J_{eq}} \cdot \Omega_{mo}(t) - \frac{T_{eqo}(t)}{J_{eq}} \\
\\
0 \simeq \frac{V_{qso}^r(t)}{L_q} - \frac{R_{so}(t)}{L_q} \cdot I_{qso}^r(t) - \frac{P_p}{L_q} \cdot [\lambda_m^{r'} + L_d \cdot I_{dso}^r(t)] \cdot \Omega_{mo}(t) \\
\\
0 \simeq \frac{V_{dso}^r(t)}{L_d} - \frac{R_{so}(t)}{L_d} \cdot I_{dso}^r(t) + \frac{P_p \cdot L_q}{L_d} \cdot I_{qso}^r(t) \cdot \Omega_{mo}(t) \\
\\
0 \simeq \frac{V_{0so}^r(t)}{L_{ls}} - \frac{R_{so}(t)}{L_{ls}} \cdot I_{0so}^r(t) \\
\\
0 \simeq \frac{\frac{3}{2} \cdot R_{so}(t) \cdot (I_{qso}^{r^2}(t) + I_{dso}^{r^2}(t) + 2 \cdot I_{0so}^{r^2}(t)) - \frac{T_{so}^\circ(t) - T_{amb}^\circ(t)}{R_{ts-amb}}}{C_{ts}} \\
\\
R_{so}(t) = R_{sREF} \cdot (1 + \alpha_{Cu} \cdot (T_{so}^\circ(t) - T_{sREF}^\circ))
\end{array}
\right.$$

Función de Transferencia

$$G_{v_{qs}^r}(s) = \frac{\theta_m(s)}{V_{qs}(s)} = \frac{\frac{3}{2}P_p\lambda_m^{r'}}{(J_{eq}L_qs^2 + (R_sJ_{eq} + L_qb_{eq})s + \frac{3}{2}P_p^2\lambda_m^{r'2} + R_sb_{eq}).s}$$

$$G_{T_{leq}}^\theta(s) = \frac{\theta_m(s)}{T_{leq}(s)} = \frac{L_q.s + R_s}{(J_{eq}L_qs^2 + (R_sJ_{eq} + L_qb_{eq})s + \frac{3}{2}P_p^2\lambda_m^{r'2} + R_sb_{eq}).s}$$

$$G_{v_{qs}^r}(s) = \frac{i_{qs}^r(s)}{V_{qs}(s)} = \frac{J_{eq}.s + b_{eq}}{J_{eq}L_qs^2 + (R_sJ_{eq} + L_qb_{eq})s + \frac{3}{2}P_p^2\lambda_m^{r'2} + R_sb_{eq}}$$

$$G_{T_{leq}}^{i_{qs}^r}(s) = \frac{i_{qs}^r}{T_{leq}(s)} = \frac{P_p\lambda_m^{r'}}{J_{eq}L_qs^2 + (R_sJ_{eq} + L_qb_{eq})s + \frac{3}{2}P_p^2\lambda_m^{r'2} + R_sb_{eq}}$$

Análisis de Estabilidad a lazo abierto

- Subsistema Mecánico
- Subsistema Eléctrico

Ceros :

$$z_1 = -175,86 \text{ rad/s}$$

Polos :

$$s_1 = 0 \text{ rad/s}$$

$$s_2 = -89,26 + 301,57i \text{ rad/s}$$

$$s_3 = -89,26 - 301,57i \text{ rad/s}$$

$$\omega_n = 314,5 \text{ rad/s}$$

$$\xi = 0.2838$$

- Subsistema Térmico

i_{qs}^r :

Ceros :

$$z_1 = -2,654 \text{ rad/s}$$

Polos :

$$s_1 = -89,26 + 301,57i \text{ rad/s}$$

$$s_2 = -89,26 - 301,57i \text{ rad/s}$$

i_{ds}^r :

Polo :

$$s_1 = -154,55 \text{ rad/s}$$

$$\tau = 6,47 \text{ ms}$$

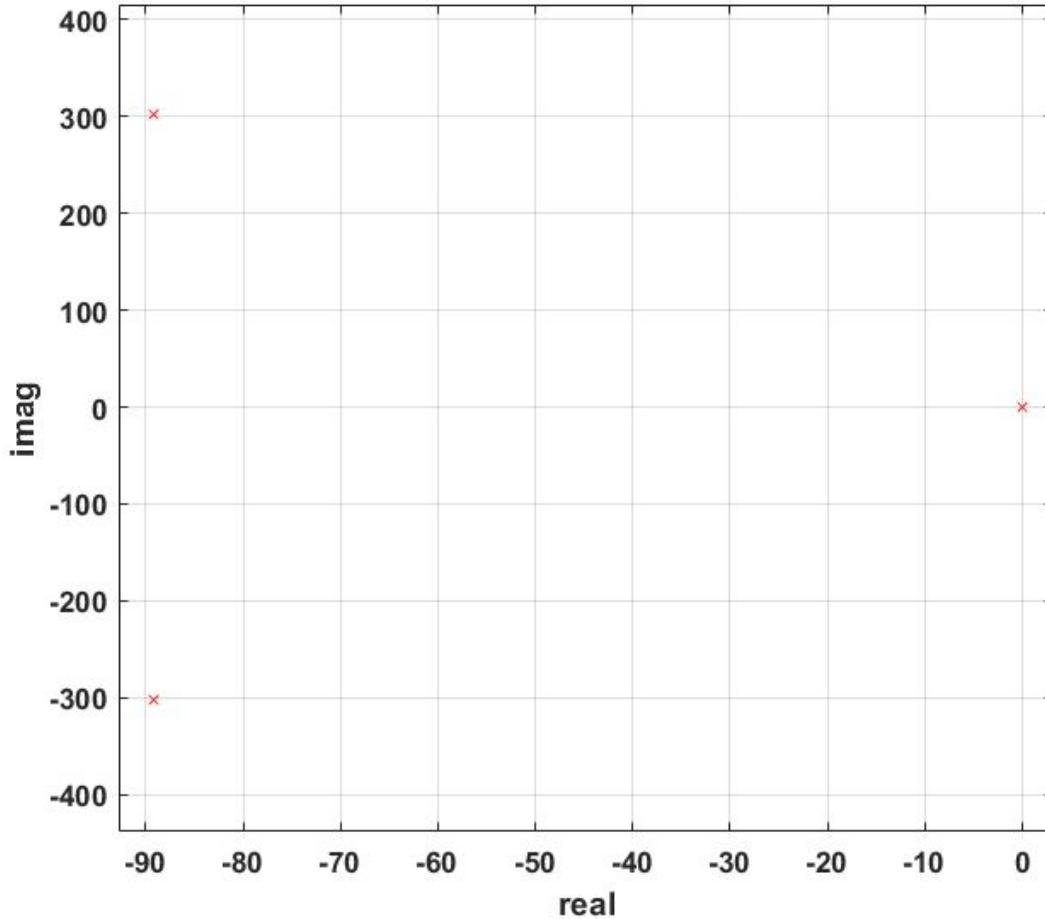
- Subsistema Térmico

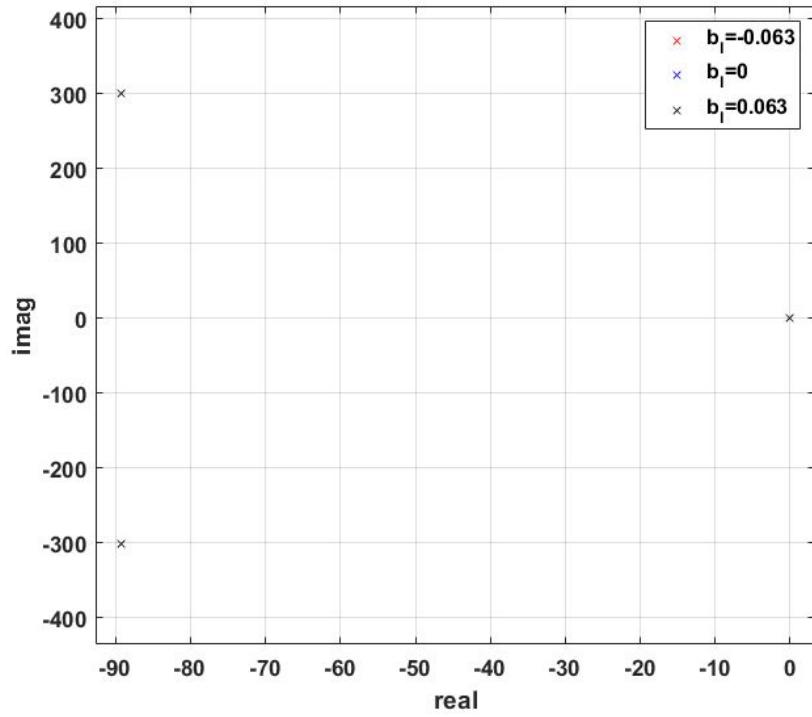
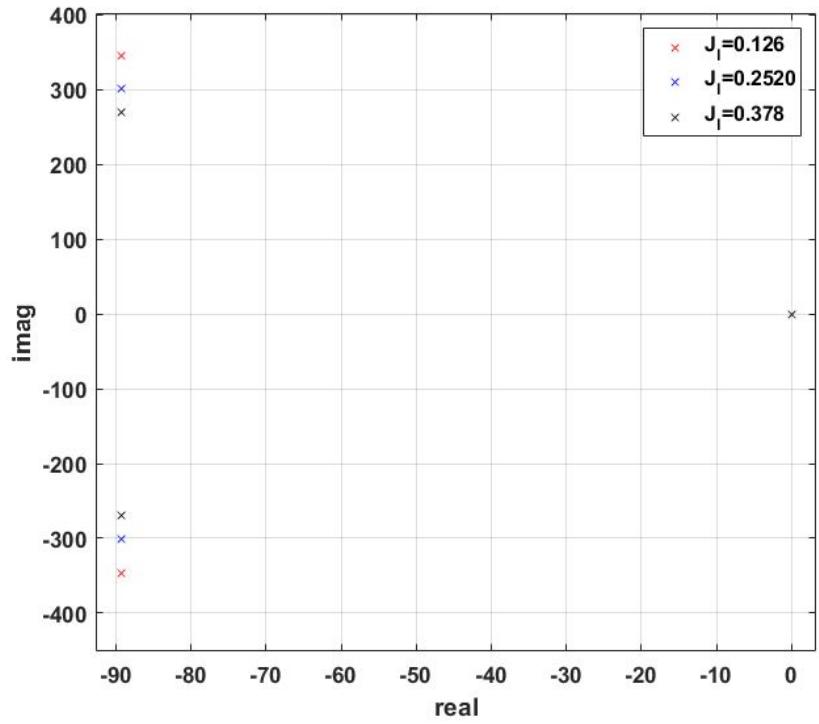
T_s :

Polo :

$$s_1 = -0,0083 \text{ rad/s}$$

$$\tau = 120 \text{ s}$$





Observabilidad

$$\text{Rango de } O = \begin{bmatrix} C \\ C.A \\ \dots \\ C.A^{n-1} \end{bmatrix} = n$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{b_{eq}}{J_{eq}} & \frac{3P_p\lambda_m^{r'}}{2J_{eq}} & 0 & 0 \\ 0 & \frac{-P_p\lambda_m^{r'}}{L_q} & \frac{-R_s}{L_q} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{ts-amb}C_{ts}} & 0 \\ 0 & 0 & 0 & 0 & \frac{R_s}{L_d} \end{bmatrix}$$

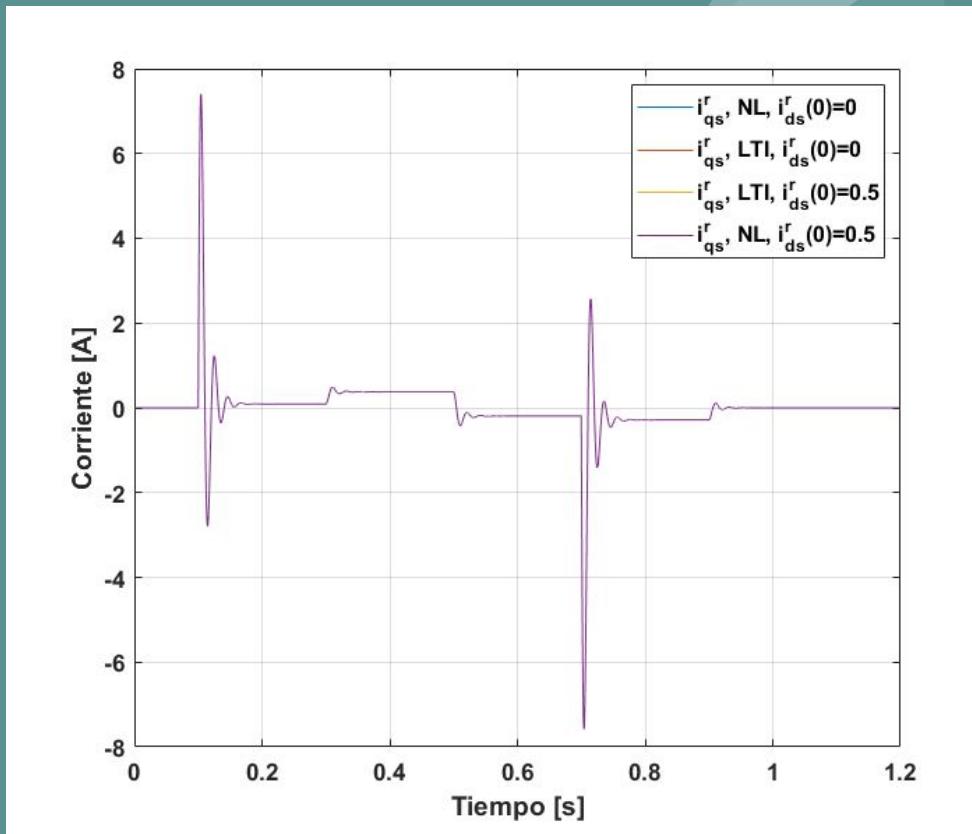
$$C = [1 \ 0 \ 0 \ 0 \ 0]$$

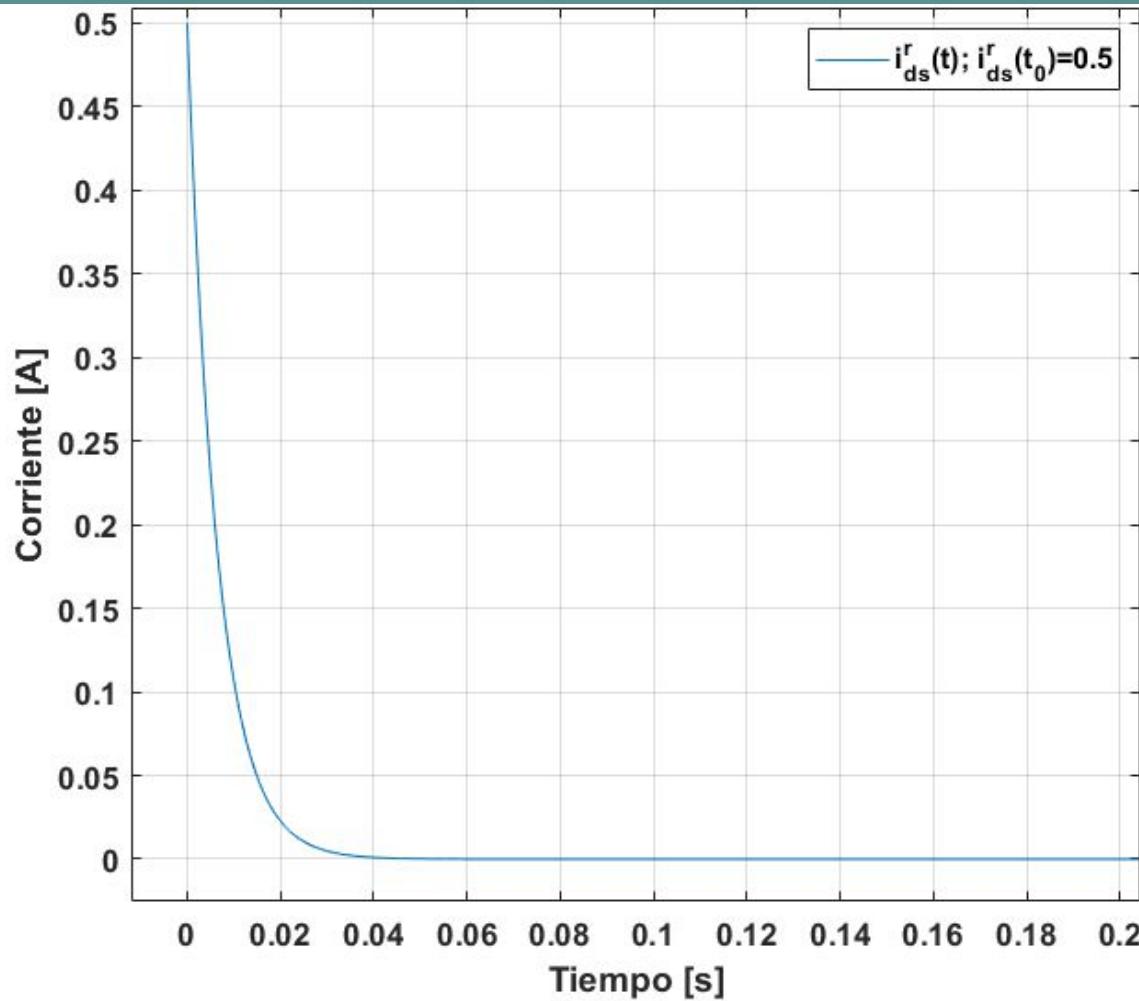
Controlabilidad

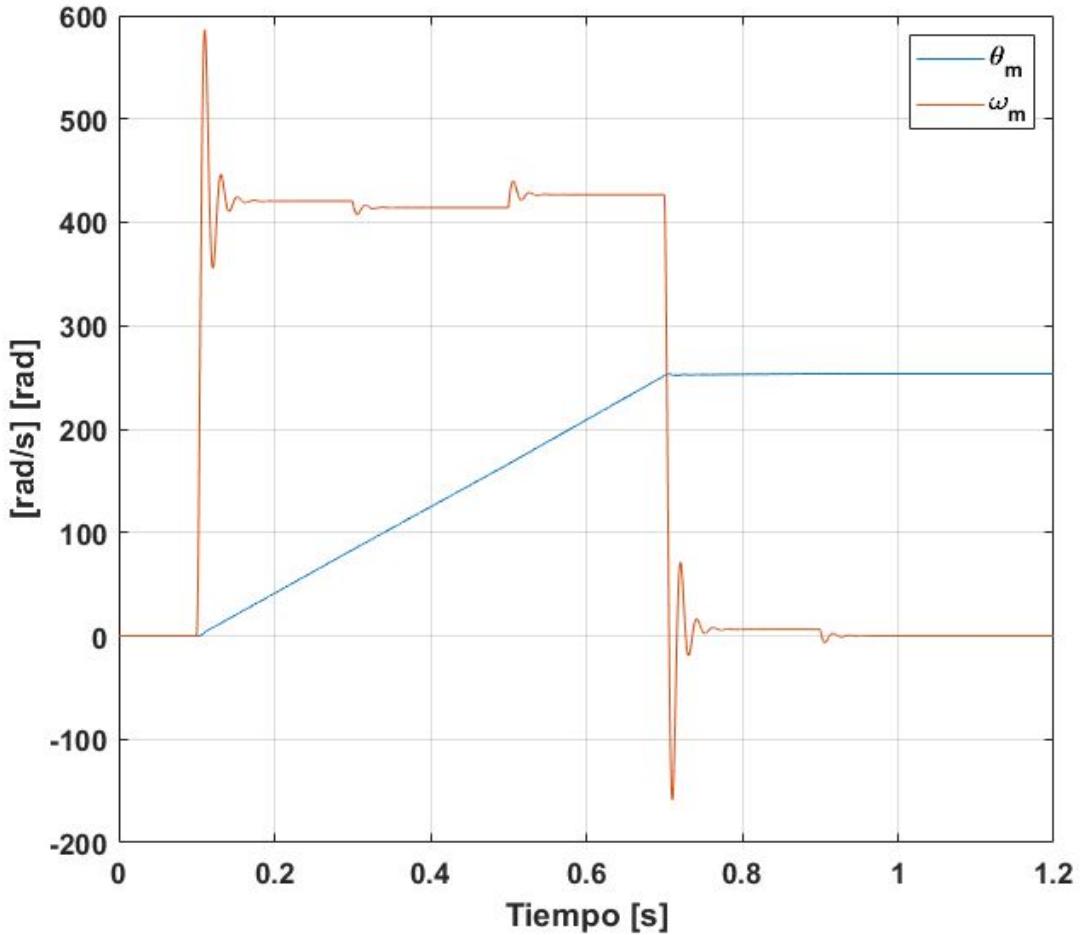
$$\text{Rango de } C = [B \ A.B \ \dots \ A^{n-1}.B] = n$$

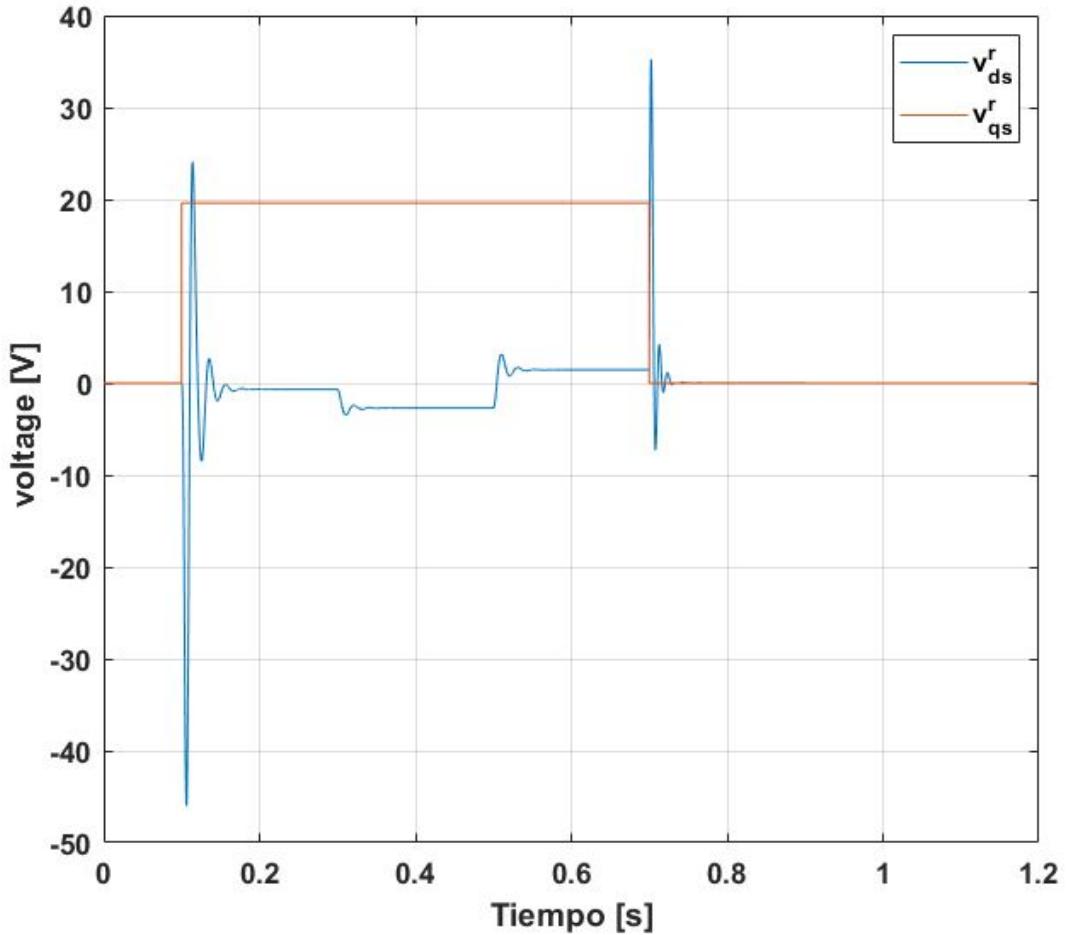
$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_q} \\ 0 \\ 0 \end{bmatrix}$$

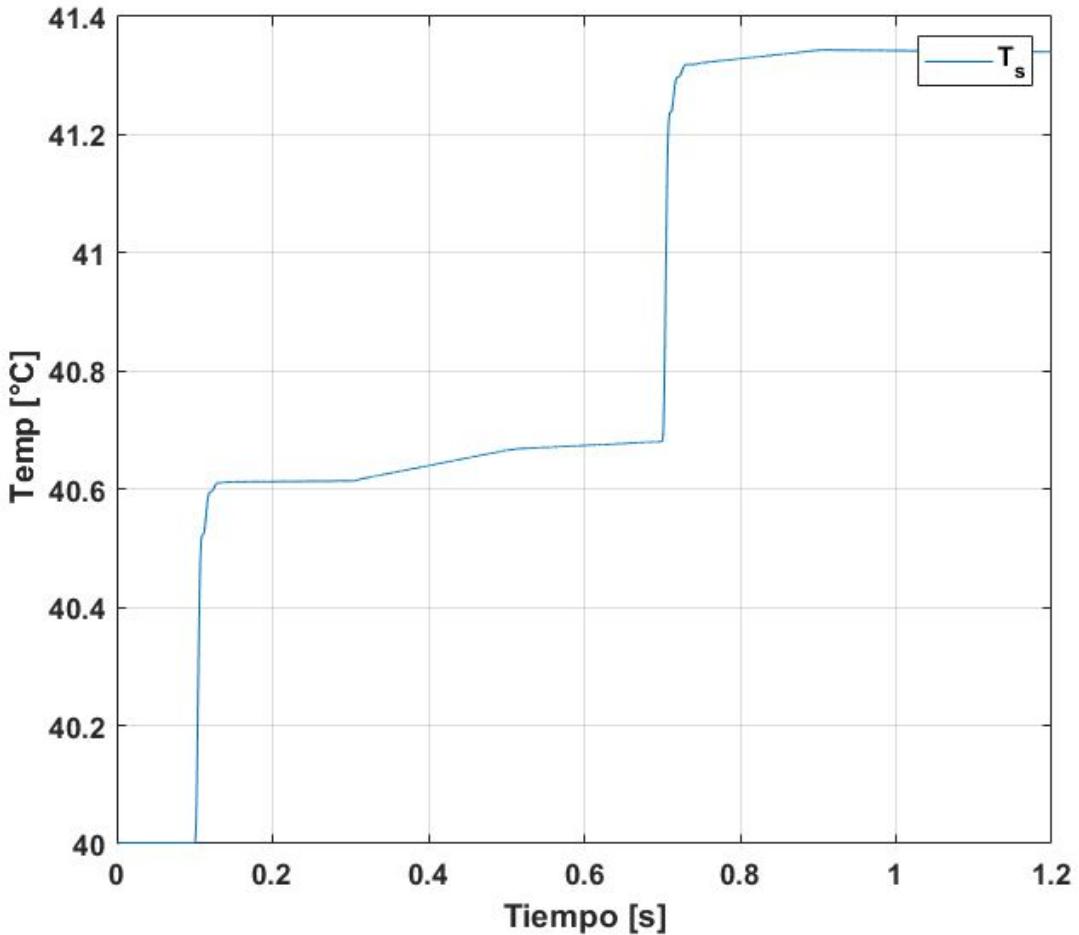
Simulación dinámica en DT

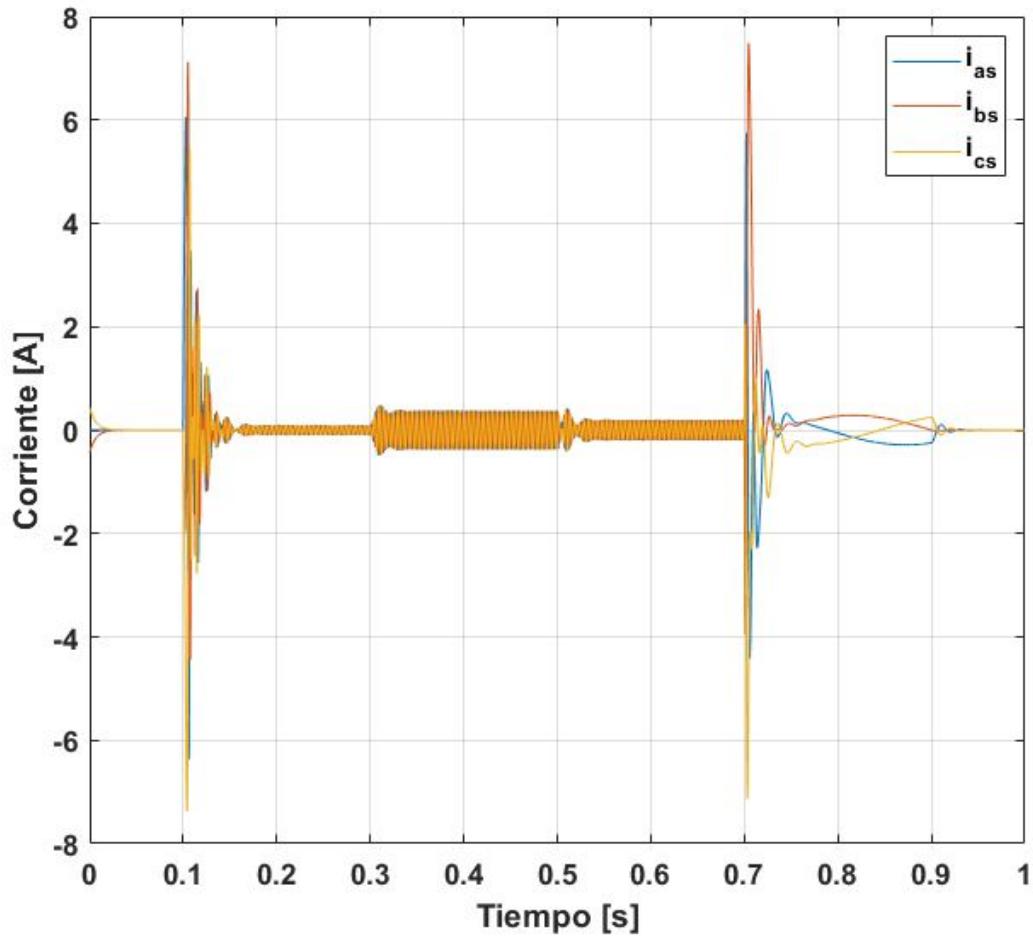


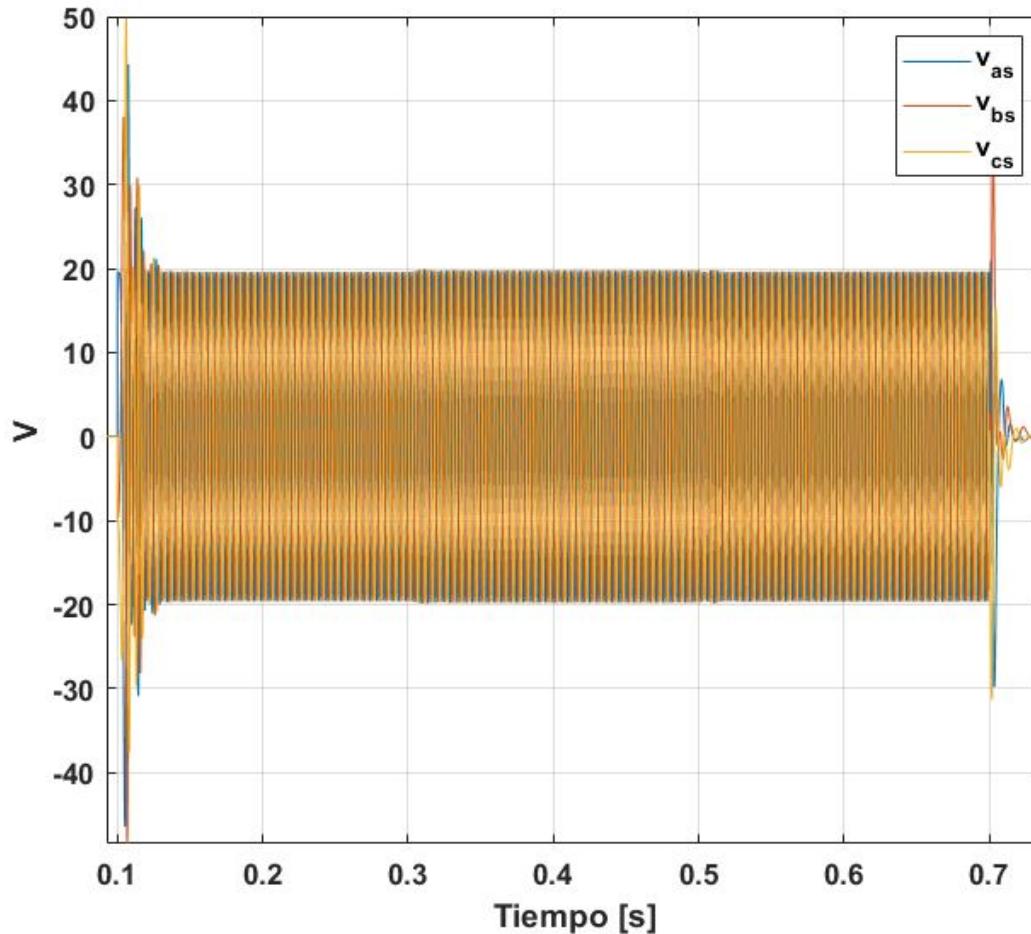












Tensiones y corrientes en ambas coordenadas (qd0<- ->abcs)

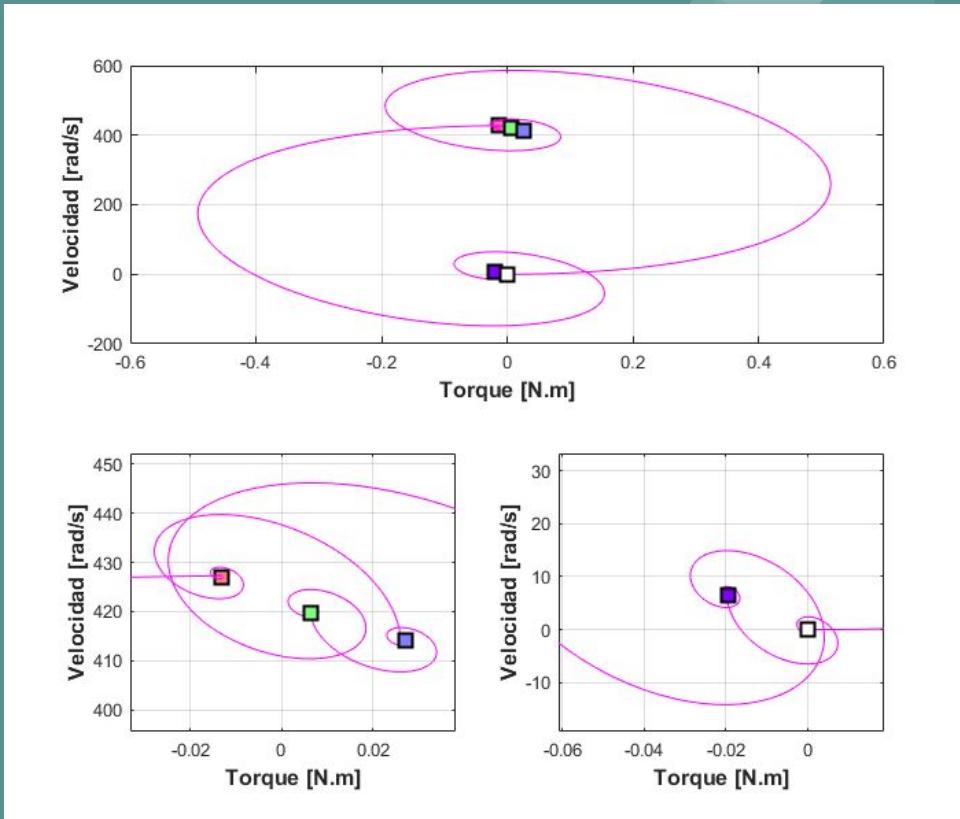
$$\begin{aligned}i_{as}(t) &= \cos(\theta_r(t)).i_{qs}^r(t) \\i_{bs}(t) &= \cos(\theta_r(t) - \frac{2}{3}\pi).i_{qs}^r(t) \\i_{cs}(t) &= \cos(\theta_r(t) + \frac{2}{3}\pi).i_{qs}^r(t) \\v_{as}(t) &= \cos(\theta_r(t)).v_{qs}^r(t) - \sin(\theta_r(t)).v_{ds}^r(t) \\v_{bs}(t) &= \cos(\theta_r(t) - \frac{2}{3}\pi).v_{qs}^r(t) - \sin(\theta_r(t) - \frac{2}{3}\pi).v_{ds}^r(t) \\v_{cs}(t) &= \cos(\theta_r(t) + \frac{2}{3}\pi).v_{qs}^r(t) - \sin(\theta_r(t) + \frac{2}{3}\pi).v_{ds}^r(t)\end{aligned}$$



Reemplazando Ley
de Control

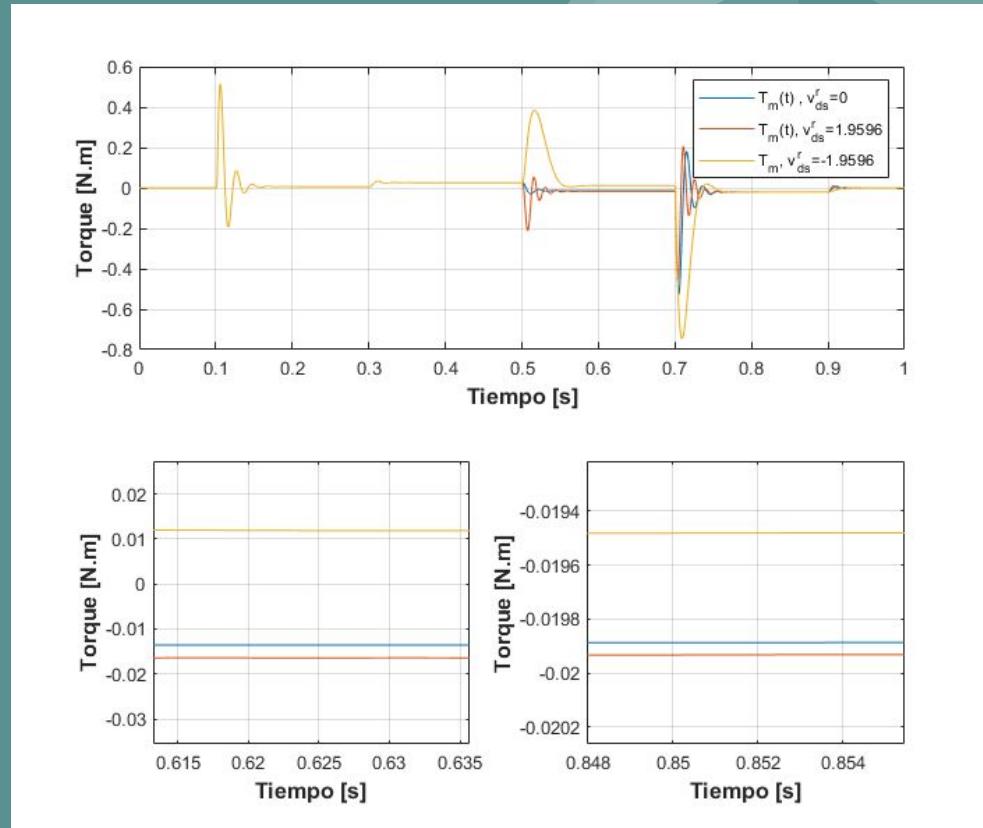
$$\begin{aligned}i_{as}(t) &= i_{qs}^r(t) \cdot \cos(\theta_r(t)) \\i_{bs}(t) &= i_{qs}^r(t) \cdot \cos(\theta_r(t) - \frac{2}{3}\pi) \\i_{cs}(t) &= i_{qs}^r(t) \cdot \cos(\theta_r(t) + \frac{2}{3}\pi) \\v_{as}(t) &= v_{qs}^r(t) \cdot \cos(\theta_r(t)) + (L_q \cdot i_{ds}^r(t) \cdot P_p \cdot \omega_m(t)) \cdot \sin(\theta_r(t)) \\v_{bs}(t) &= v_{qs}^r(t) \cdot \cos(\theta_r(t) - \frac{2}{3}\pi) + (L_q \cdot i_{ds}^r(t) \cdot P_p \cdot \omega_m(t)) \cdot \sin(\theta_r(t) - \frac{2}{3}\pi) \\v_{cs}(t) &= v_{qs}^r(t) \cdot \cos(\theta_r(t) + \frac{2}{3}\pi) + (L_q \cdot i_{ds}^r(t) \cdot P_p \cdot \omega_m(t)) \cdot \sin(\theta_r(t) + \frac{2}{3}\pi)\end{aligned}$$

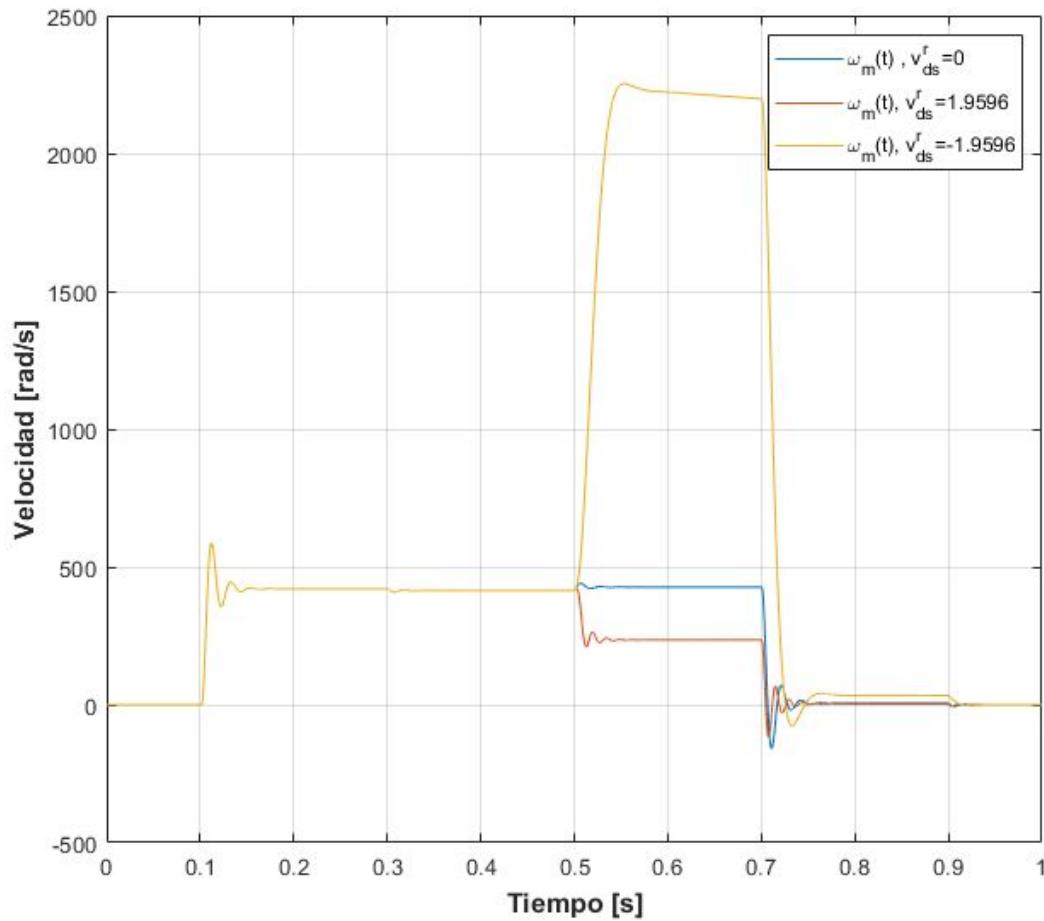
Curva paramétrica torque vs velocidad



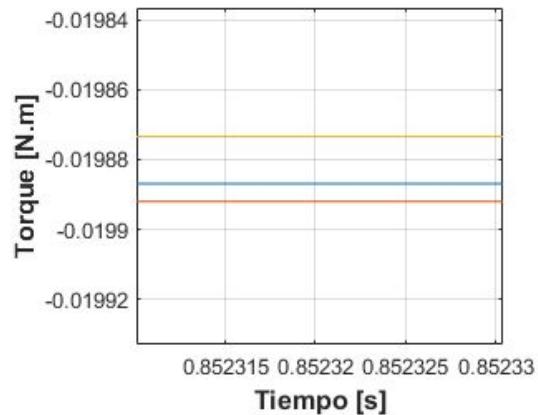
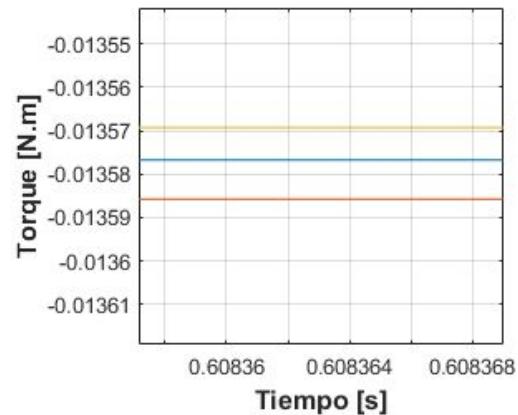
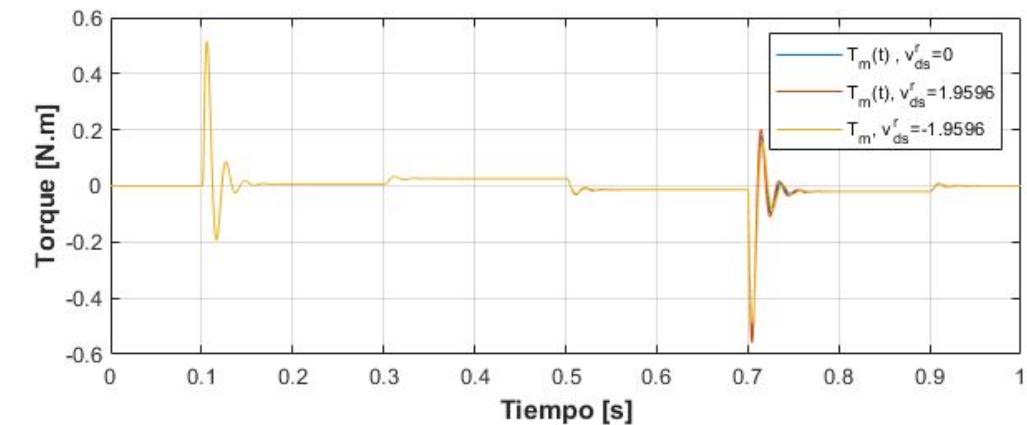
Field forcing and weakening a lazo abierto

Modelo NL con
desacoplamiento NL



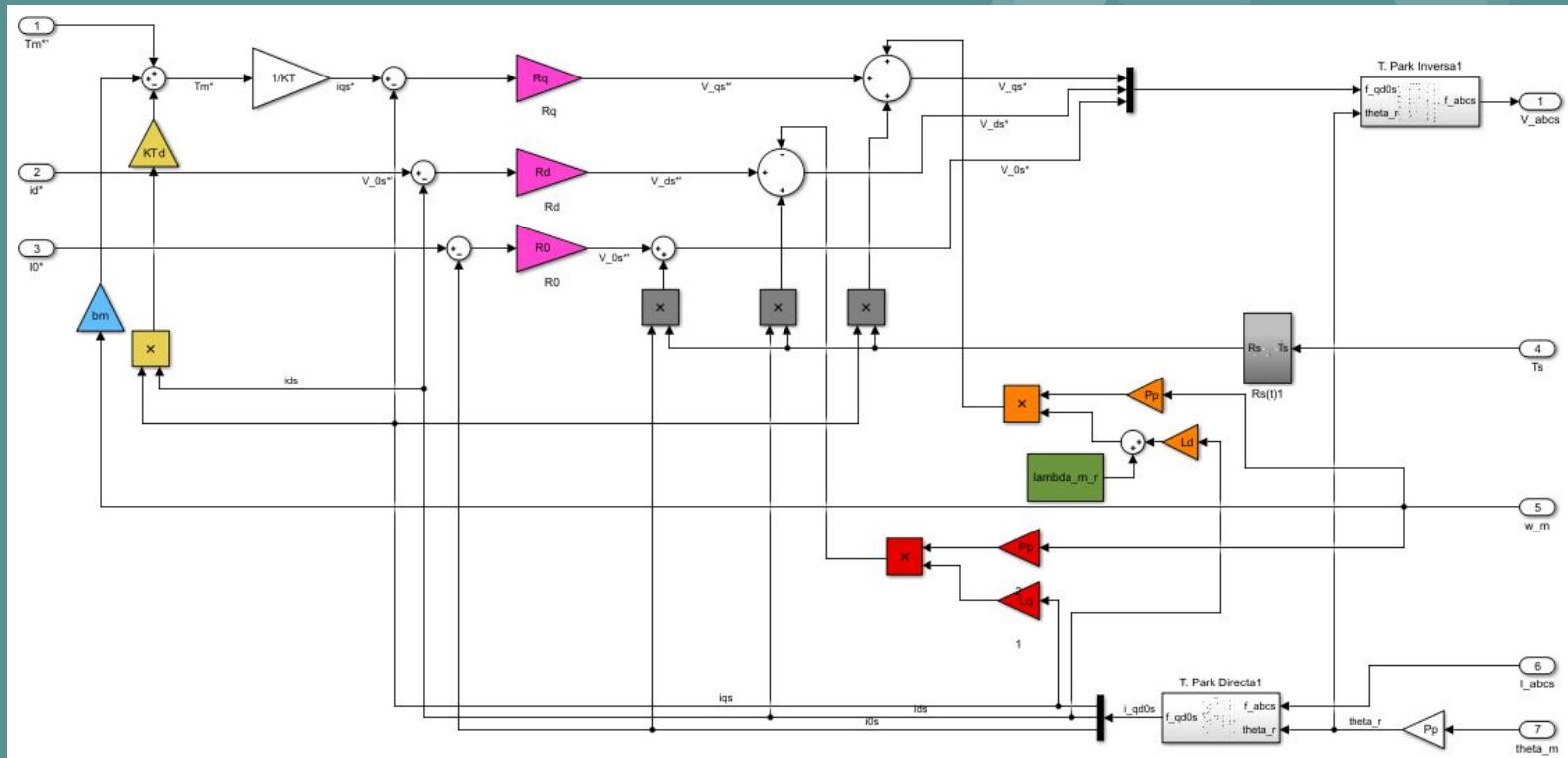


Modelo LTI equivalente
aumentado



Modulador de torque

Desacoplamiento de todas las realimentaciones físicas naturales de estado hacia la entrada



Diseño de lazos de control de corrientes

Si el modulador de tensión es ideal:

$$v_{qs}^*(t) \simeq v_{qs}(t)$$

Y teniendo en cuenta el desacoplamiento realizado anteriormente:

$$v_{qs}^*(t) = v_{qs}^{*\prime}(t) + R_s \cdot i_{qs}(t) + [\lambda_m^{r'} + L_d \cdot i_{ds}^r(t)] \cdot \omega_m(t)$$

Obtenemos:

$$\frac{I_{qs}^{r'}(s)}{I_{qs}^{r*}(s)} = \frac{1}{s \frac{L_q}{R_q} + 1}$$

	Escalón i_{qs}^{r*}
i_{qs}^r Establecimiento (A)	0.09067
Rise Time (ms)	0.439
Settling Time (ms)	0.78
Overshoot	0 %

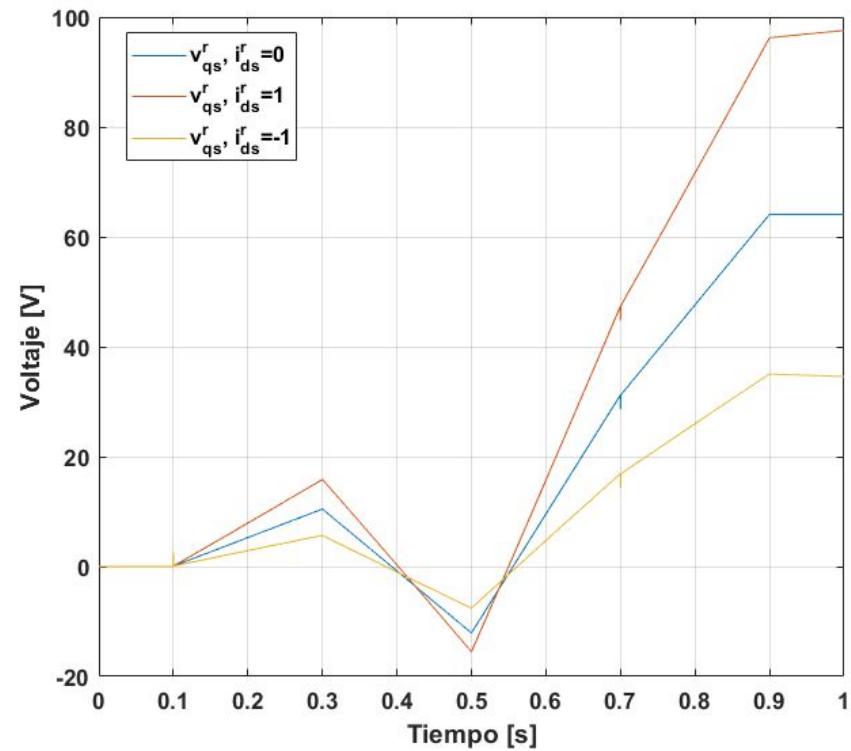
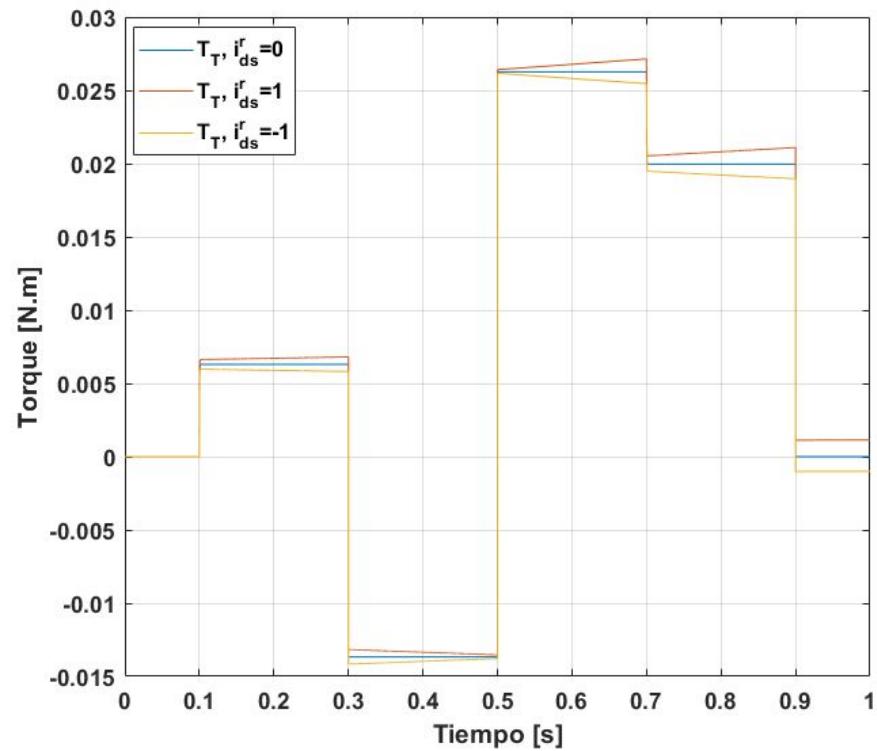
Incorporación adecuada de consigna de torque

Constante de escala 1/KT:

$$K_T = \frac{3}{2} P_p \lambda_m^r$$

Considerando el desacoplamiento del torque generado por la fricción viscosa del motor y la caja:

$$i_{qs}^{r*}(t) = \frac{1}{K_T} [T_m^{*''} + b_m \omega_m(t)] = \frac{1}{K_T} T_m^*$$

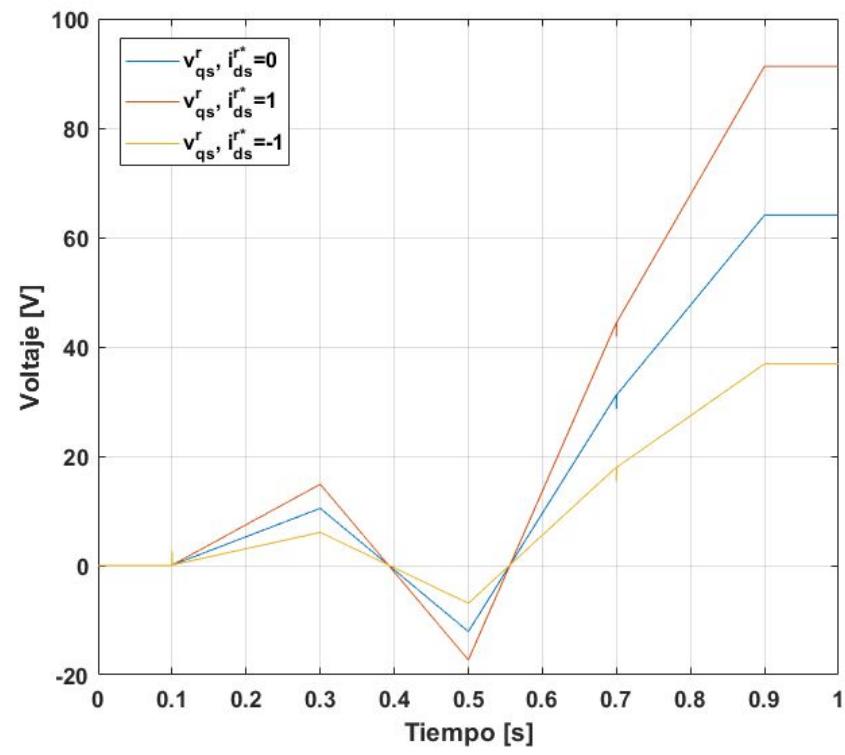
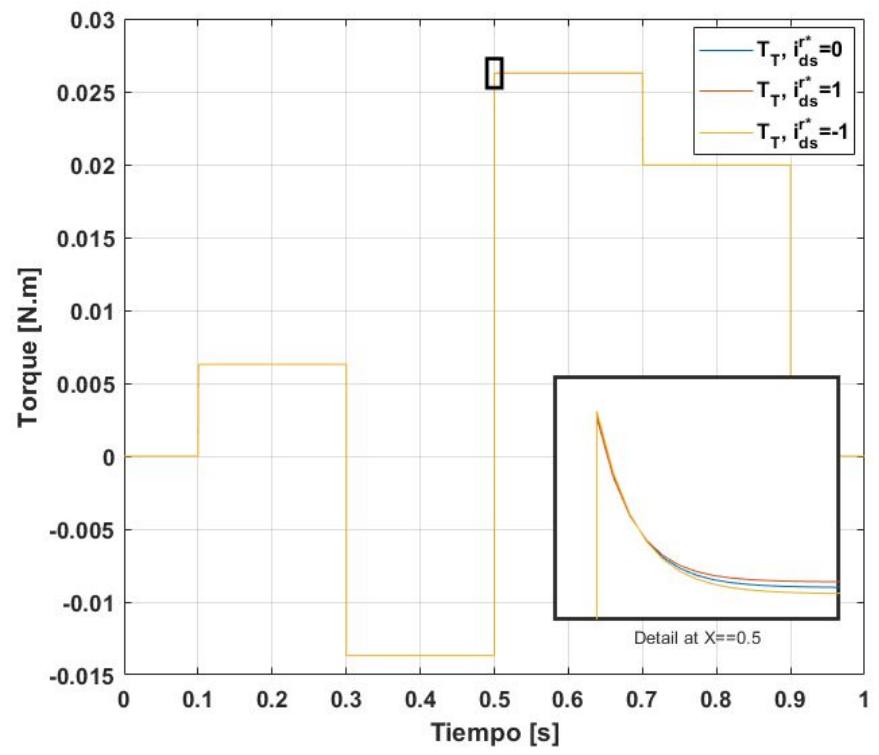


Desacoplamiento de la influencia de I_d sobre el torque motor

$$i_{qs}^{r*}(t) = \frac{1}{K_T} [T_m^{*''} + b_m \omega_m(t) - K_{T_d} \cdot i_{ds}^r \cdot i_{qs}^r] = \frac{1}{K_T} T_m^*$$

Donde:

$$K_{T_d} = \frac{3}{2} P_p \cdot (L_d - L_q)$$



Controlador externo de movimientos

Dada la ec. del sistema teniendo en cuenta los desacoplamientos realizados:

$$J_{eq}\theta(s)s^2 + b_{eq}\theta(s)s = T_m'(s) + K_{T_d} \cdot i_{ds}^r \cdot i_{qs}^r - T_{leq}(s); \quad T_{leq} = \frac{T_l}{r}$$

Siendo la ecuación del controlador PID:

$$T_m^*(s) = [ba + \frac{1}{s}Ksa + \frac{1}{s^2}Ksia] \cdot e_w(s)$$

Y teniendo en cuenta:

$$i_{qs}^{r*}(t) = \frac{1}{K_T}[T_m^{*'} + b_m\omega_m(t) - K_{T_d} \cdot i_{ds}^r \cdot i_{qs}^r] = \frac{1}{K_T}T_m^*$$

$$\frac{I_{qs}^{r*}(s)}{I_{qs}^{r*}(s)} = \frac{1}{s\frac{L_q}{R_q} + 1}$$

Obtenemos:

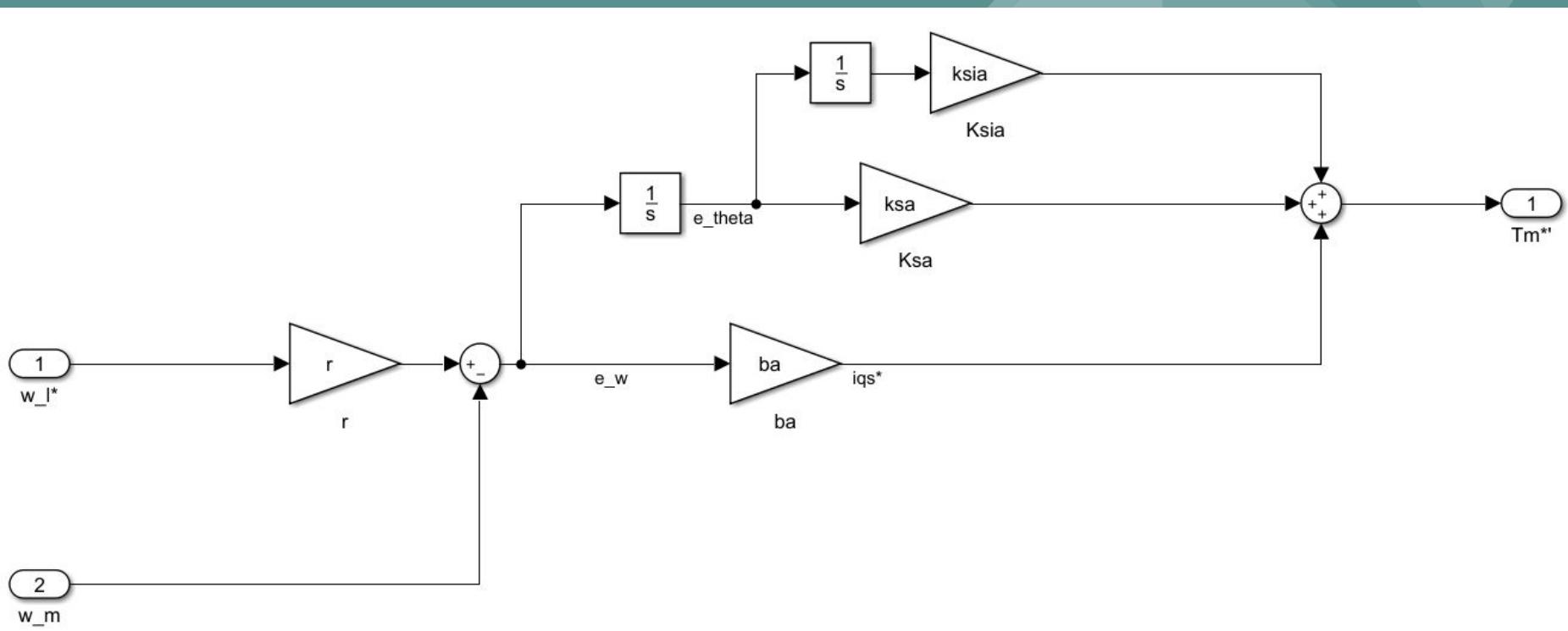
$$G_{\theta_m^*}(s) = \frac{G_I(s)[s^2ba + sKsa + Ksia]}{J_{eq}s^3 + [b_{eq} - G_I(s).b_m]s^2 + G_I(s)[s^2ba + sKsa + Ksia]}$$
$$G_{T_{leq}}(s) = \frac{s}{J_{eq}s^3 + [b_{eq} - G_I(s).b_m]s^2 + G_I(s)[s^2ba + sKsa + Ksia]}$$

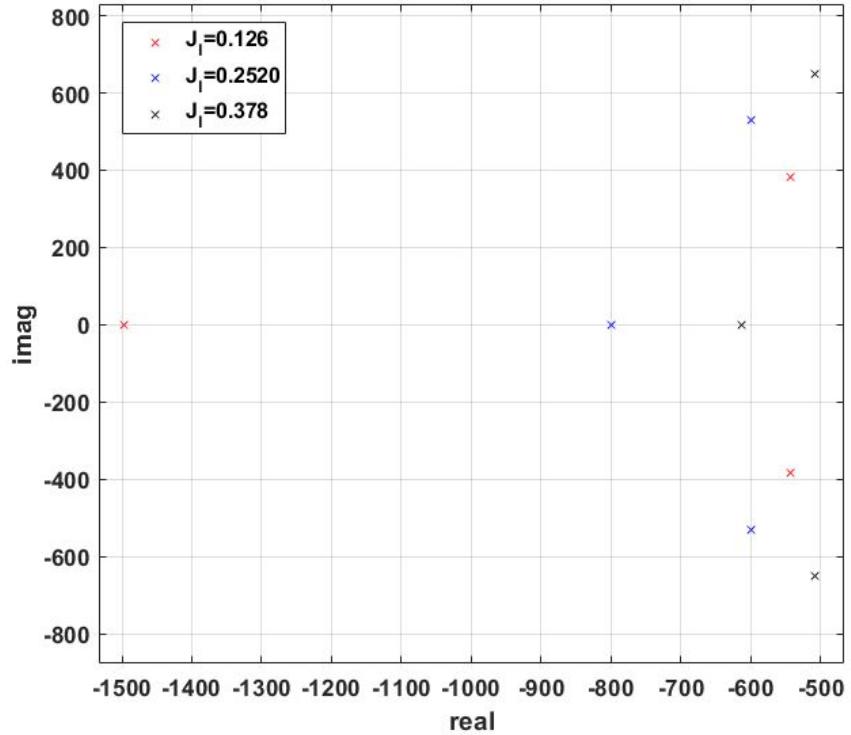
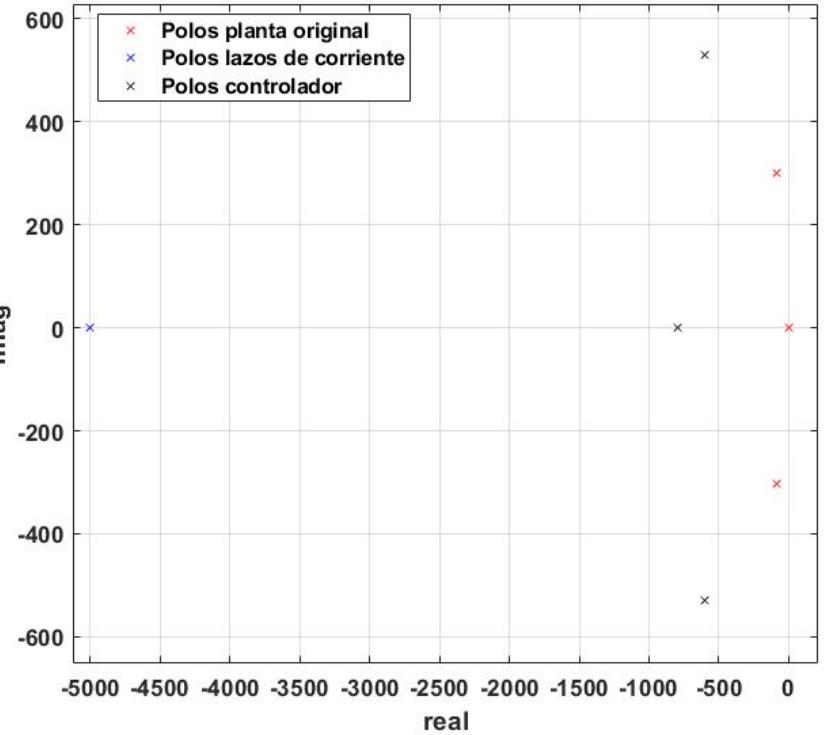
Aplicando el método de sintonía serie:

$$n\omega_{pos} = \frac{ba + b_{leq}}{J_{eq}} ; ba = n\omega_{pos}J_{eq} - b_{leq}$$

$$n\omega_{pos}^2 = \frac{Ksa}{J_{eq}} ; Ksa = n\omega_{pos}^2 J_{eq}$$

$$\omega_{pos}^3 = \frac{Ksia}{J_{eq}} ; Ksia = \omega_{pos}^3 J_{eq}$$





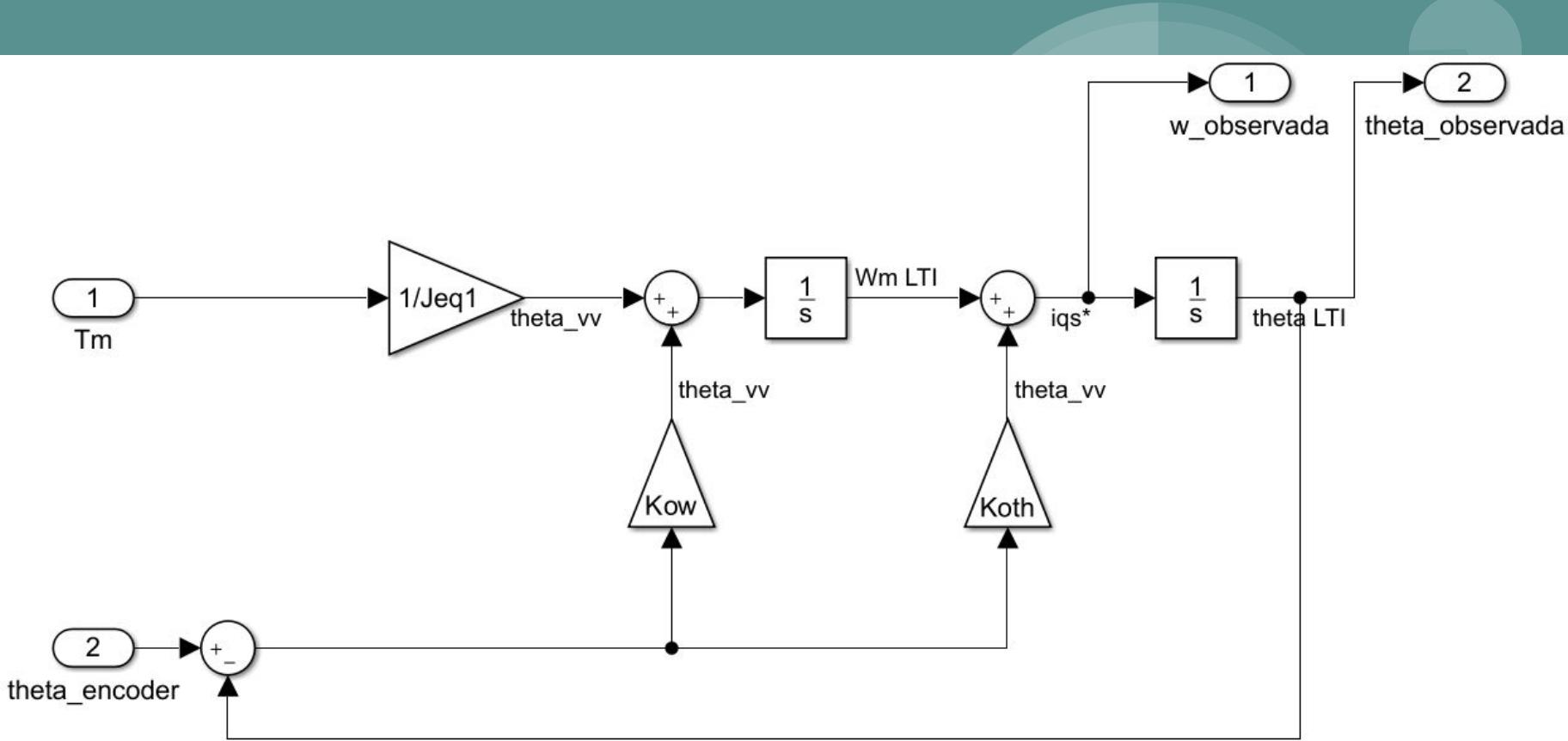
Observador de Estado

Planta:

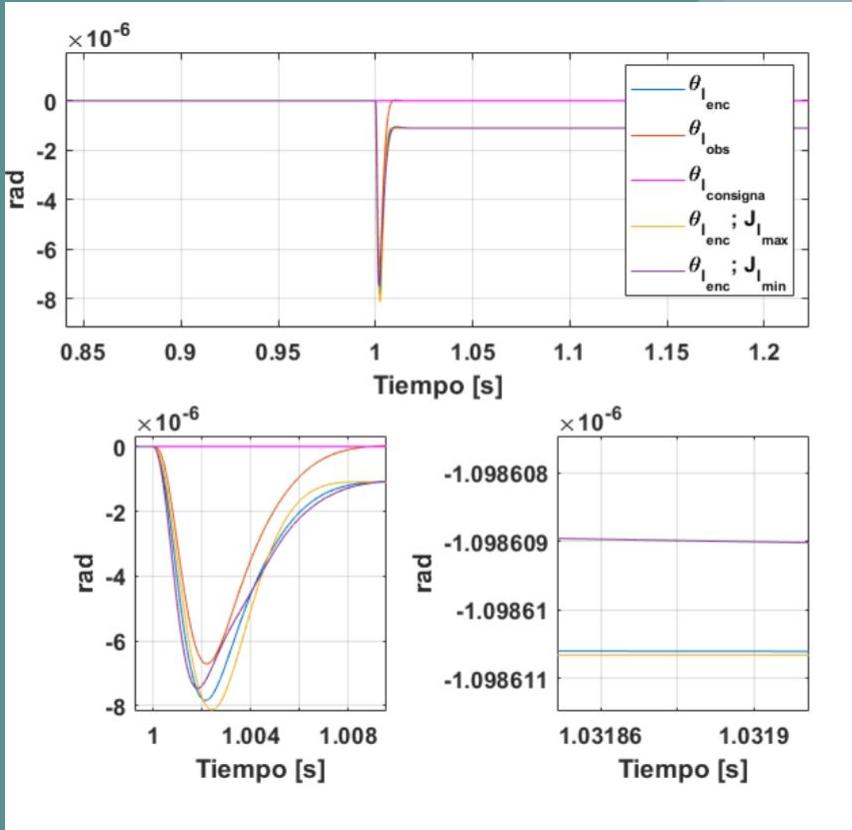
$$S_{LTI} : \begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{J_{eq}} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\frac{1}{J_{eq}} \end{bmatrix} d(t) \\ y(t) = \mathbf{C} \cdot \mathbf{x}(t) \end{cases}$$
$$\mathbf{x}(t) = \begin{bmatrix} \theta_m(t) \\ \omega_m(t) \end{bmatrix} ; \quad u(t) = T'_m(t) ; \quad d(t) = T_{eq}(t) ; \quad \mathbf{C} = [1 \quad 0]$$

Observador:

$$Observador : \begin{cases} \dot{\tilde{\mathbf{x}}}(t) = \tilde{A} \cdot \tilde{\mathbf{x}}(t) + \tilde{B} \cdot u(t) + \mathbf{K}_e (y(t) - \tilde{y}(t)) ; \quad \dot{\tilde{\mathbf{x}}}(t_0) = 0 \\ \tilde{y}(t) = \mathbf{C} \cdot \tilde{\mathbf{x}}(t) \end{cases}$$
$$\tilde{\mathbf{x}}(t) = \begin{bmatrix} \tilde{\theta}_m(t) \\ \tilde{\omega}_m(t) \end{bmatrix} ; \quad u(t) = T'_m(t) ; \quad \mathbf{K}_e = \begin{bmatrix} K_{oth} \\ K_{ow} \end{bmatrix}$$



Rechazo a perturbaciones



Observador aumentado

Dinámica del error

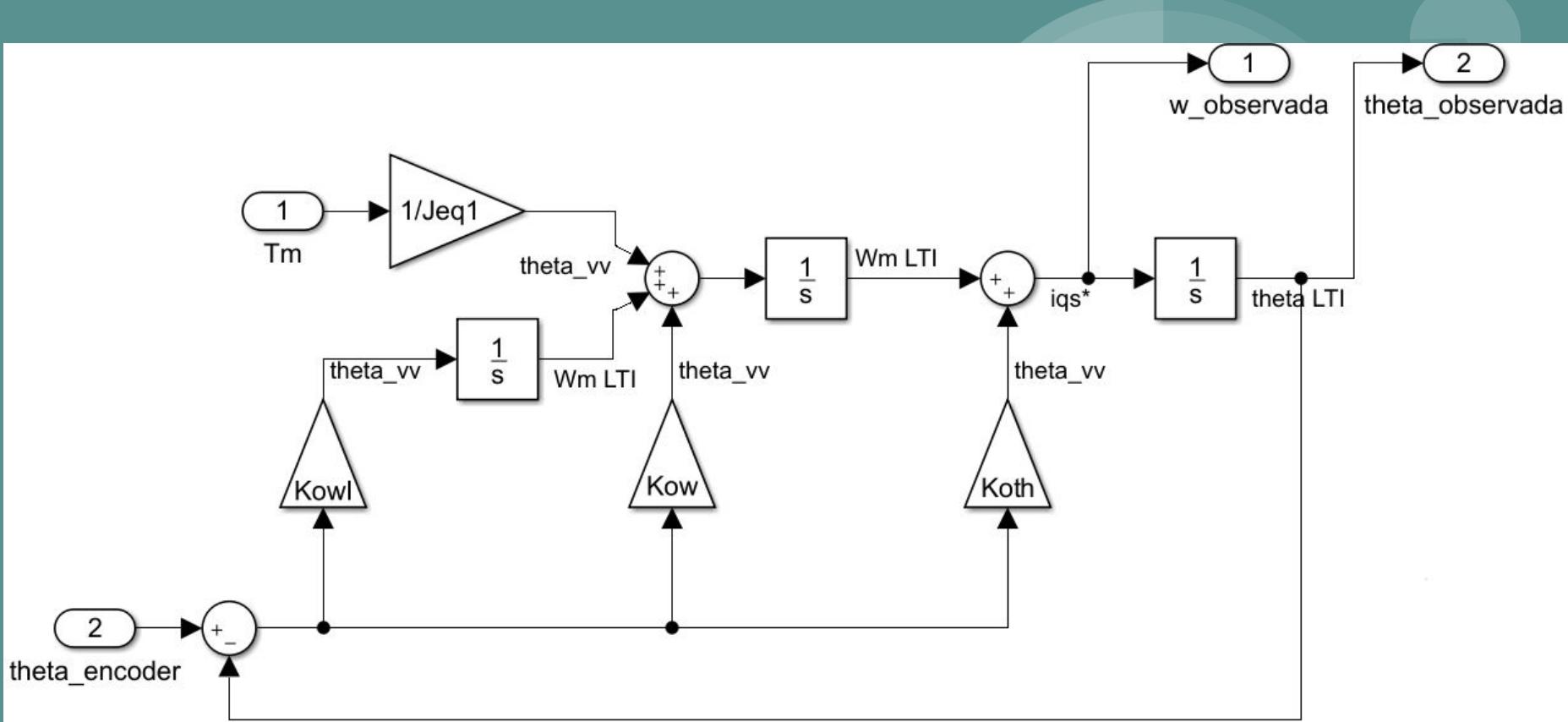
$$\begin{aligned}\dot{e}(t) &= [A - K_e C]e(t) + B_d d(t); \quad e(t_0) = x_0 \\ e(t) &= x(t) - \tilde{x}(t)\end{aligned}$$

Ecuaciones observador:

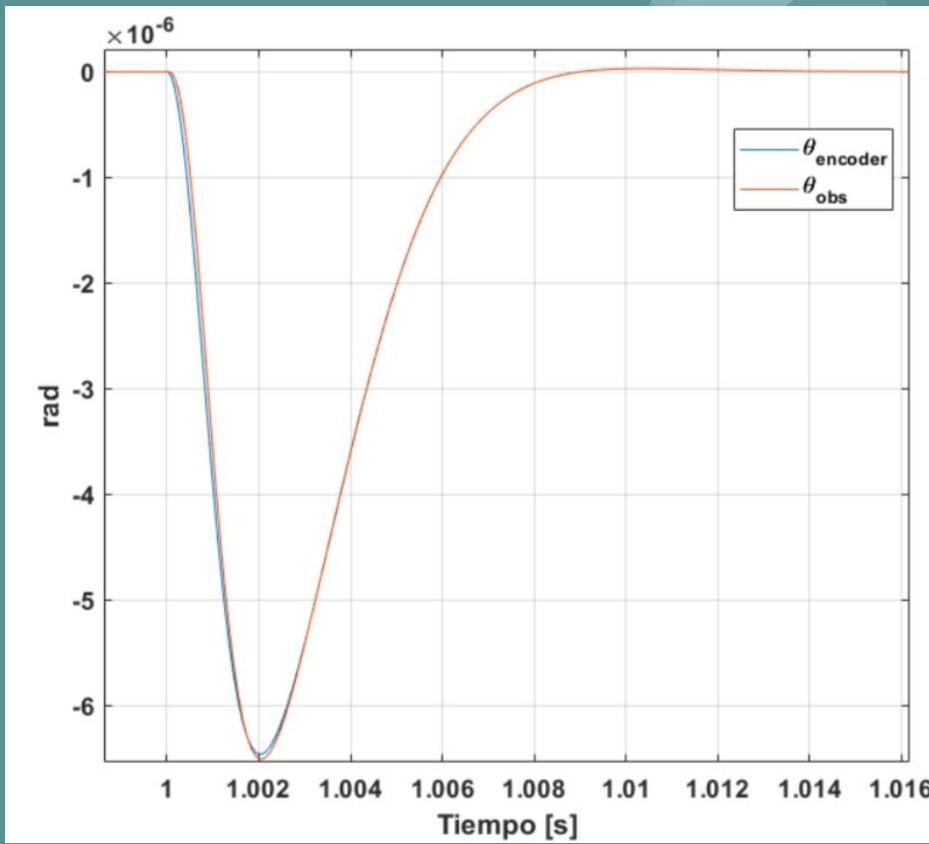
$$\begin{cases} \frac{d\tilde{\theta}_m(t)}{dt} = \tilde{\omega}_m(t) + K_\theta(\theta_m(t) - \tilde{\theta}_m(t)) \\ \frac{d\tilde{\omega}_m(t)}{dt} = \frac{T_m^{*'}(t)}{J_{eq}} + K_\omega(\theta_m(t) - \tilde{\theta}_m(t)) + K_{\omega I} \int (\theta_m(t) - \tilde{\theta}_m(t)) dt \end{cases}$$

Polinomio característico:

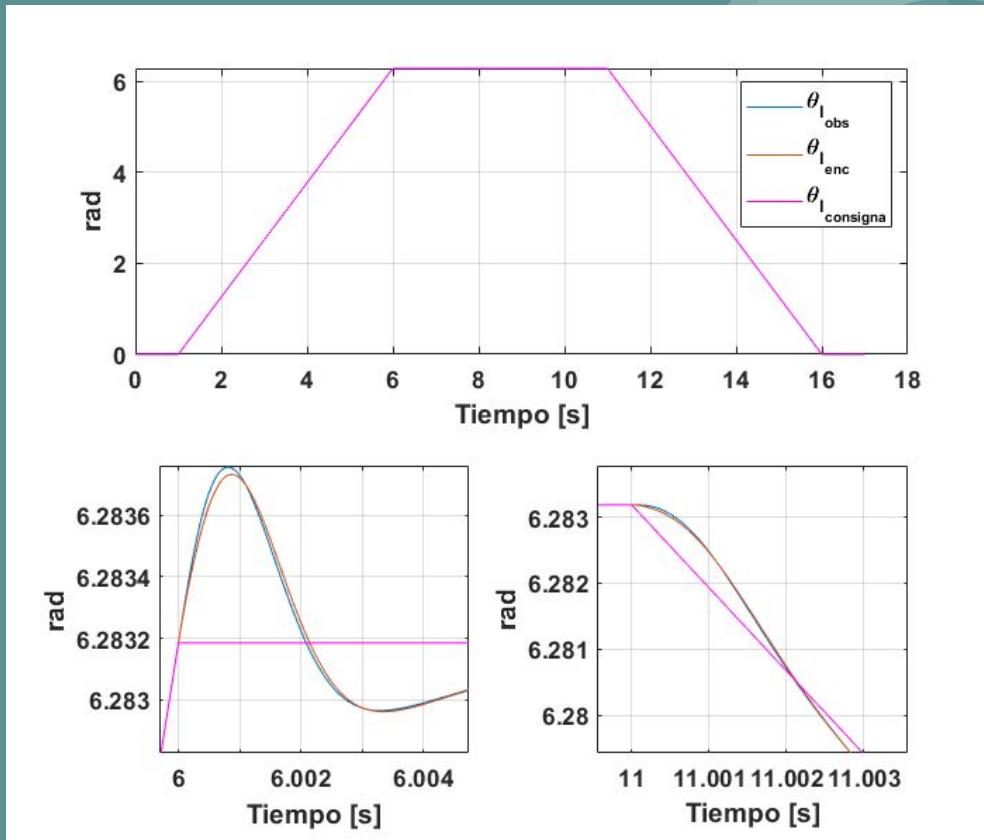
$$P_{obs} = s^3 + K_\theta s^2 + K_\omega s + K_{\omega I}$$

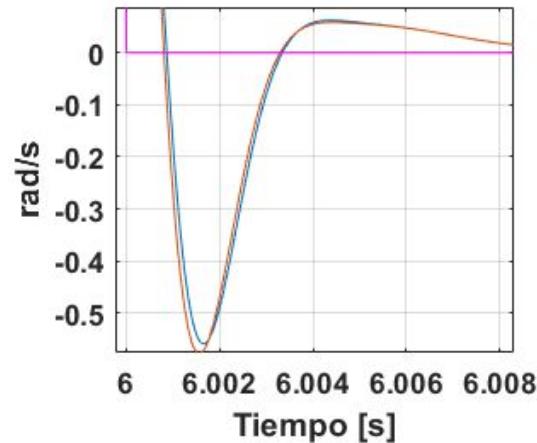
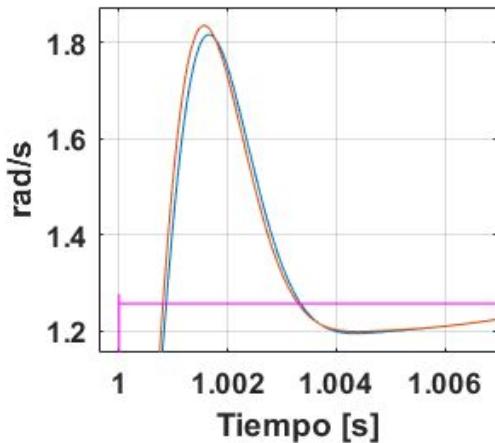
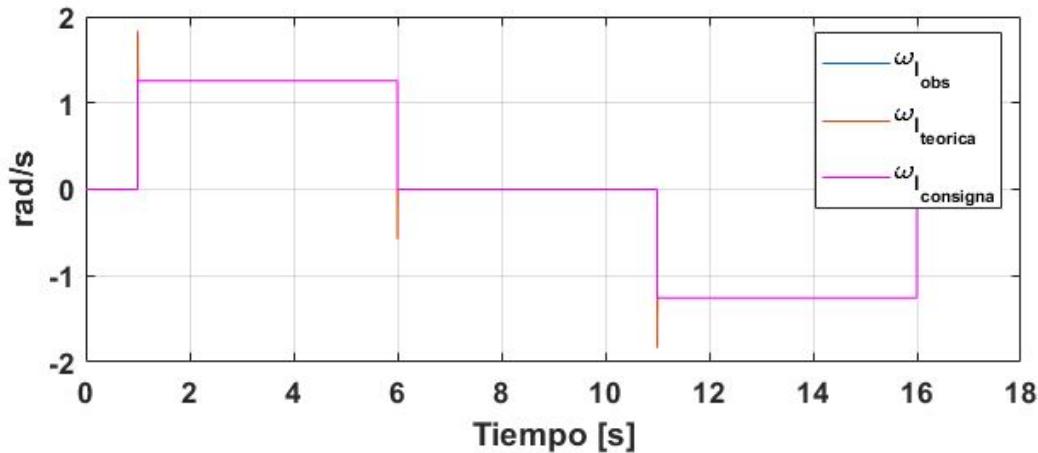


Observador agregado: Rechazo a perturb.



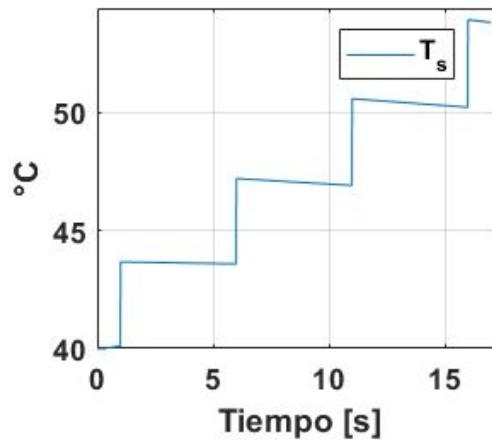
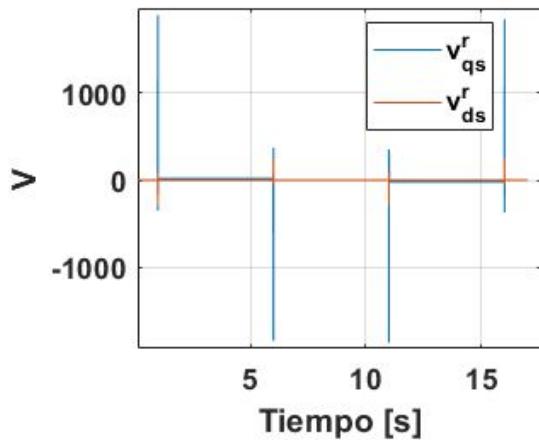
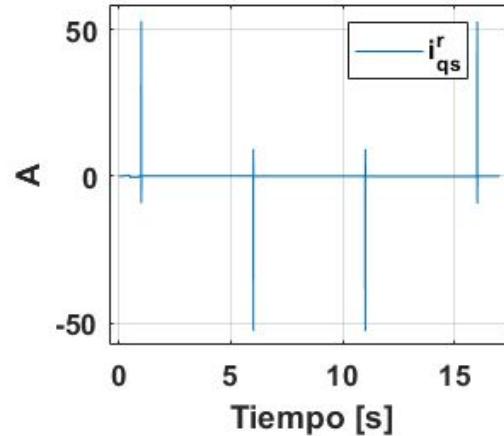
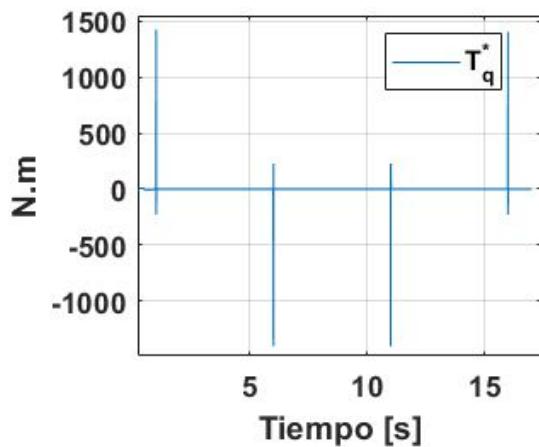
Seguimiento de consignas



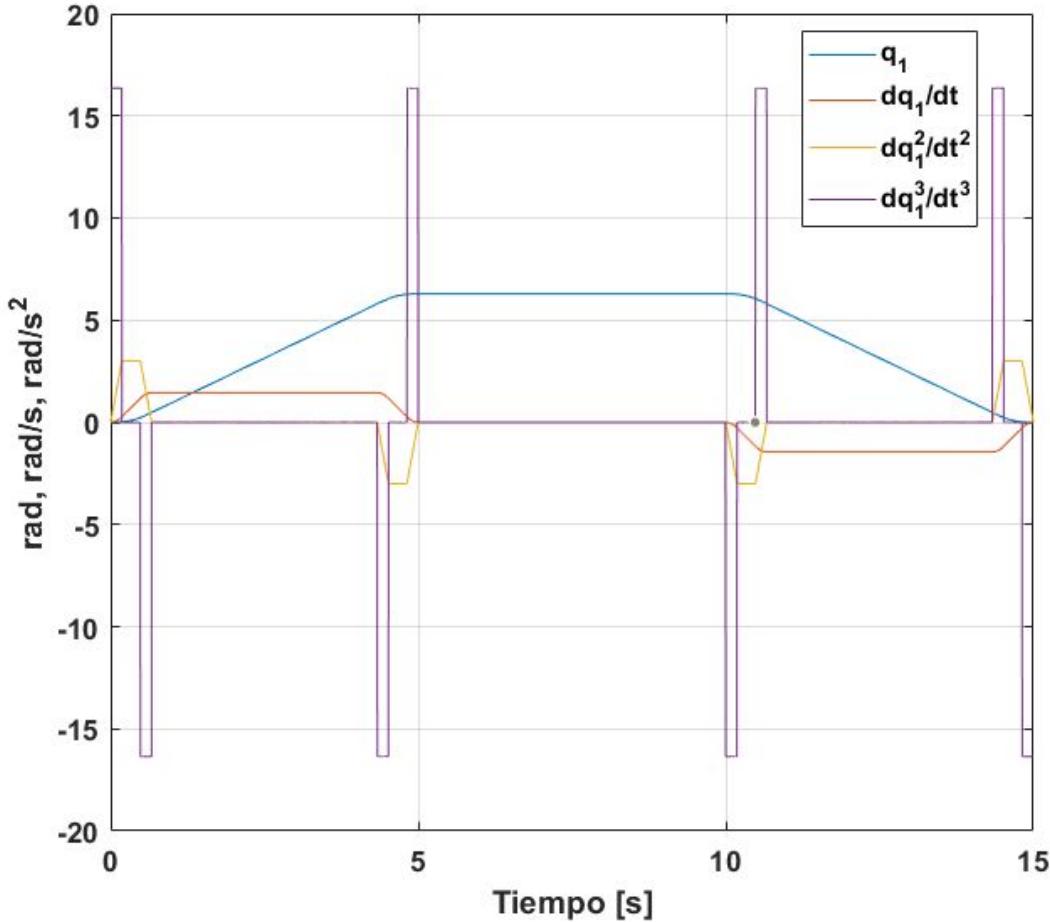


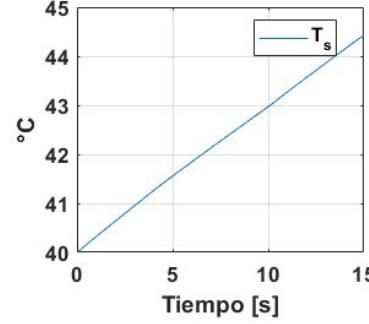
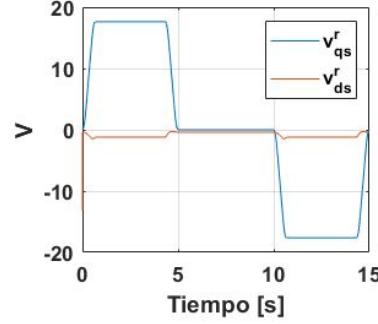
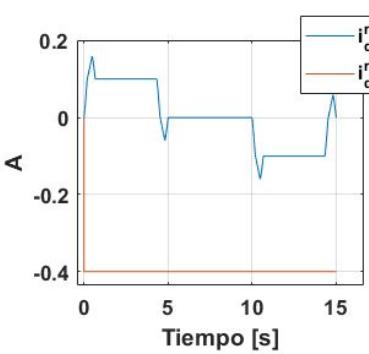
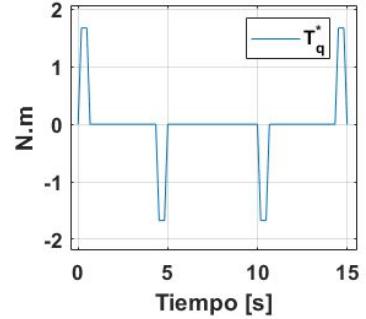
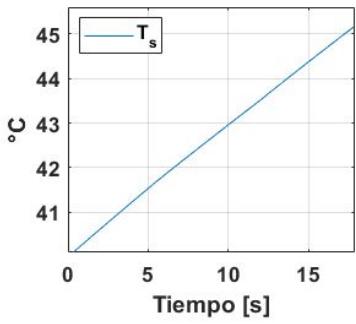
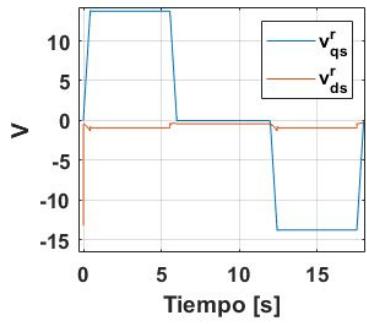
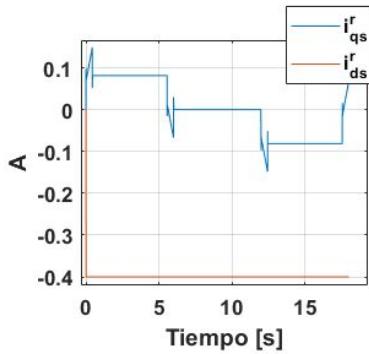
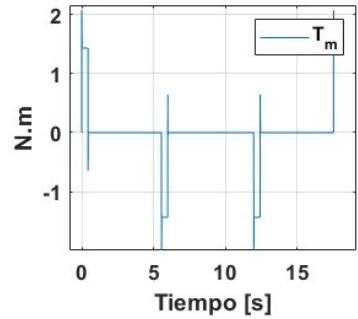
Especificaciones de operación

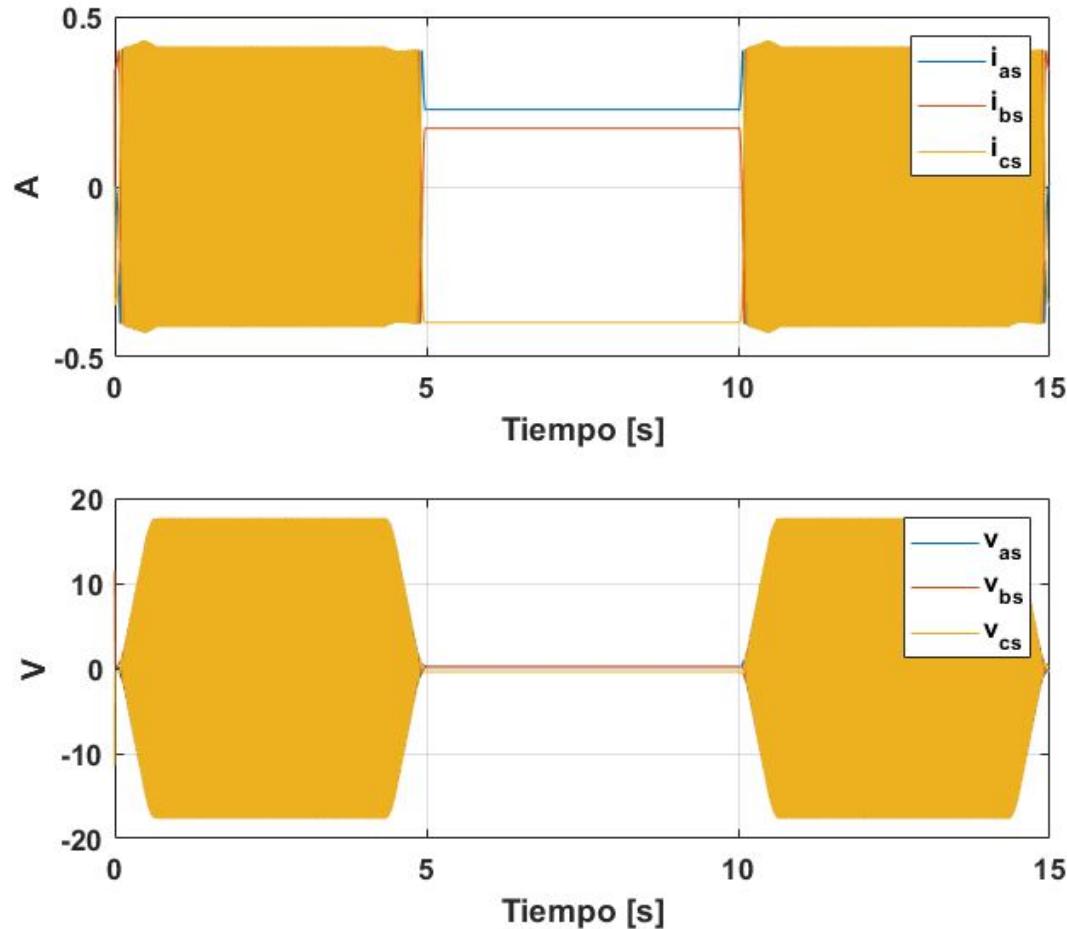
	Valores medidos	Valor nominal
Tensión de fase (V)	1889 (Pico), 18.4 (R. continuo)	19.5959
Vel. rotor [rad/s]	556.2 (Pico) 395 (R. continuo)	691.15
Corriente (R. continuo) (A)	0.085	0.5656
Corriente (corta duración) (A)	52.82	2.8284
Temperatura (°C)	54.1	115
Torque corta duración (N.m)	1425	29.42
Torque R. continuo (N.m)	0	7.26



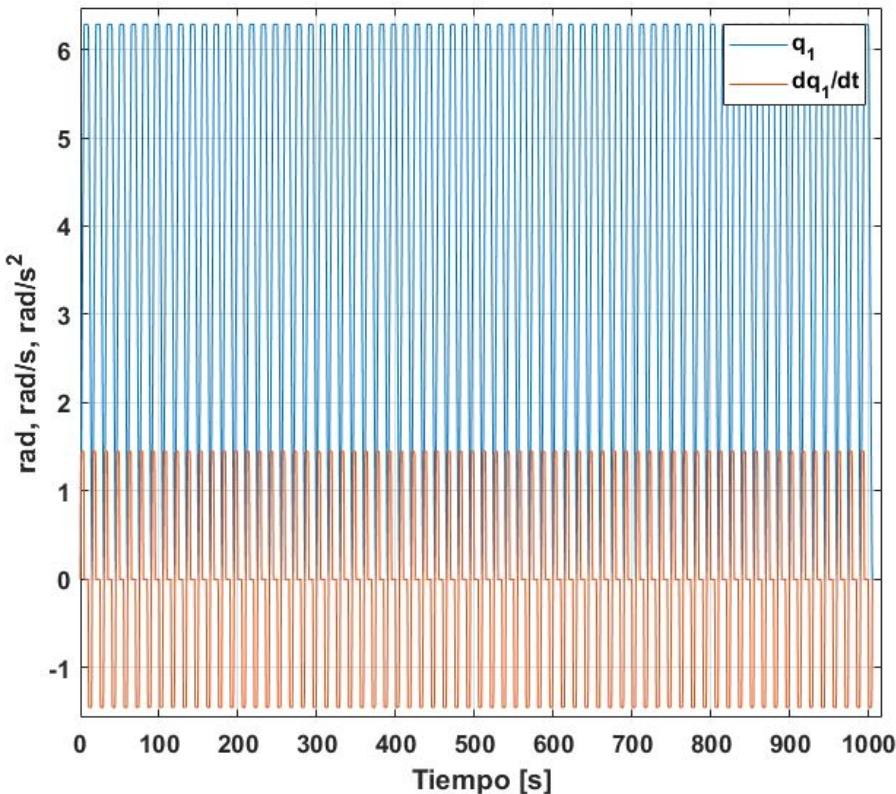
Nueva consigna



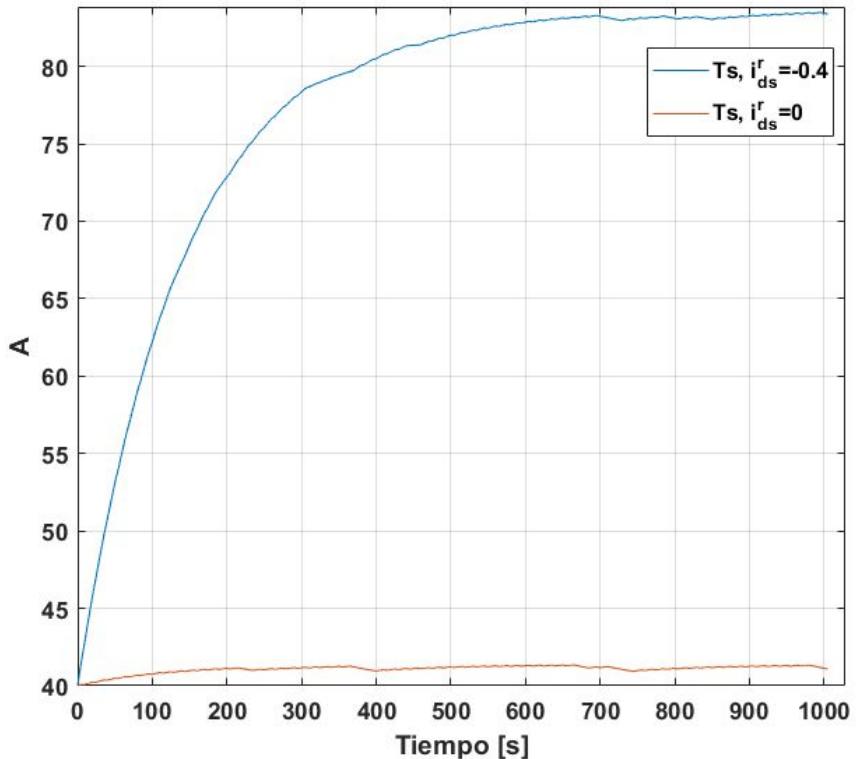




Perfil de Movimiento



Comportamiento térmico



Respuesta no ideal de los sensores y acondicionadores de señal

Filtro Pasa Bajos 2do Orden: en SS

$$G_{sensores}(s) = \frac{\omega_o^2}{s^2 + 2 \cdot \xi \cdot \omega_o \cdot s + \omega_0^2}$$

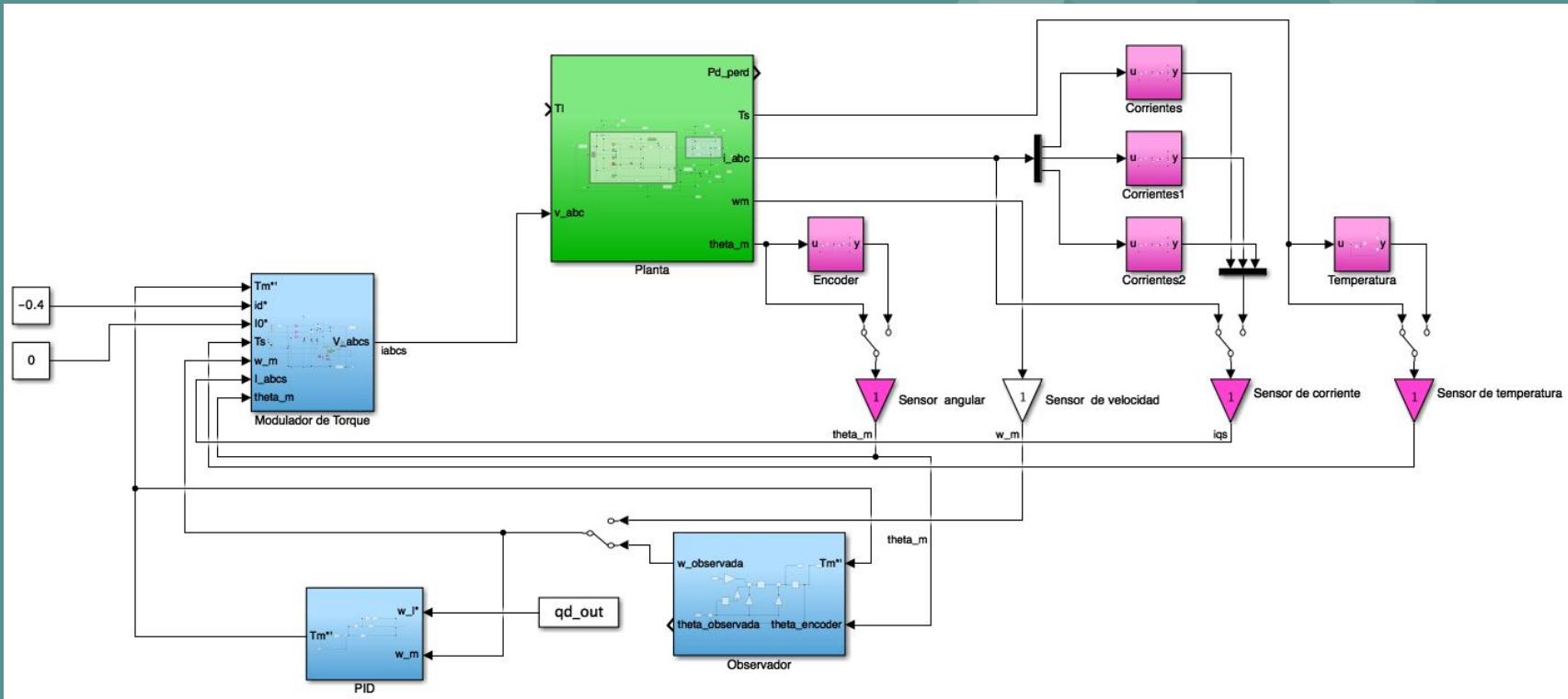
$$\begin{cases} \dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2 \cdot \xi \cdot \omega_0 \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u(t) \\ Y(t) = [\omega_0^2 \ 0] \cdot X(t) \end{cases}$$

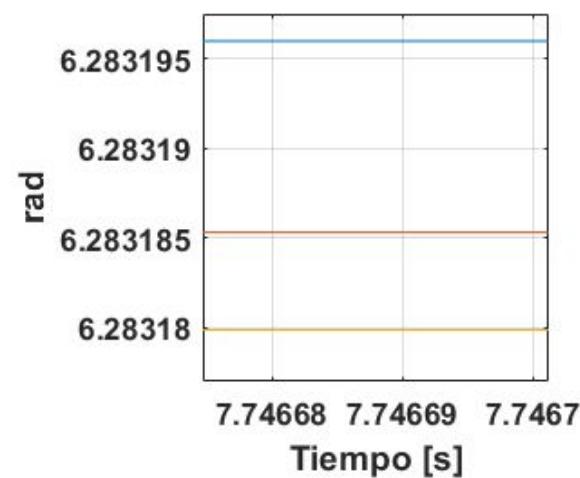
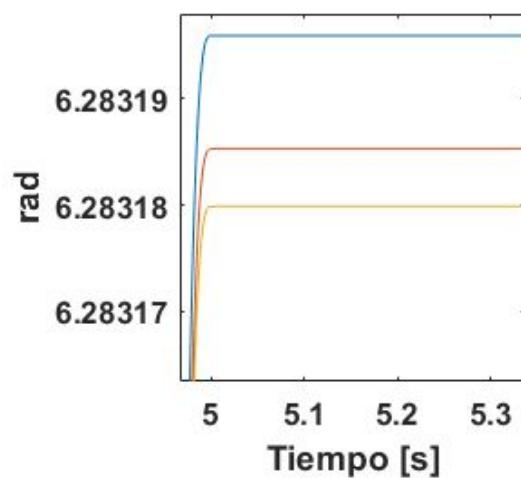
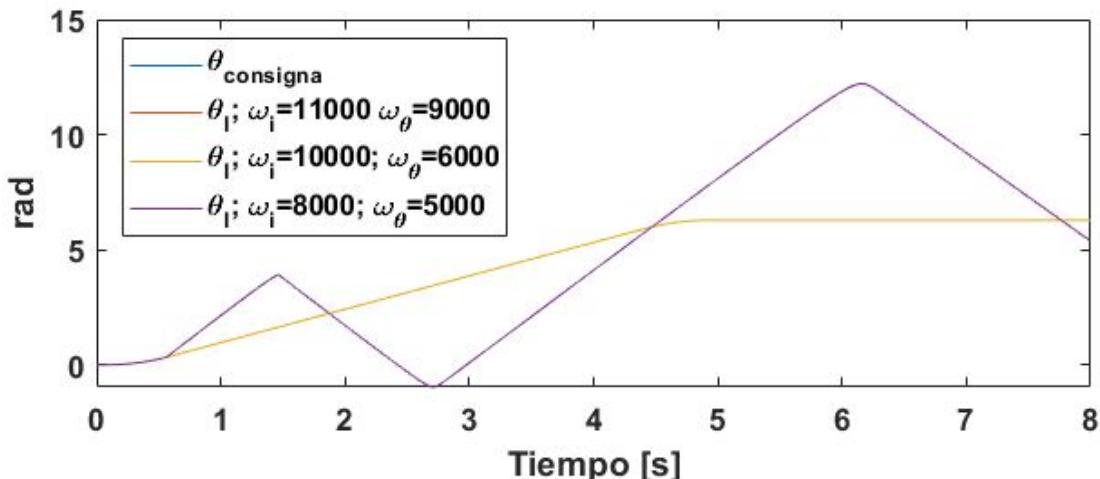
Filtro Pasa Bajos 1er Orden: en SS

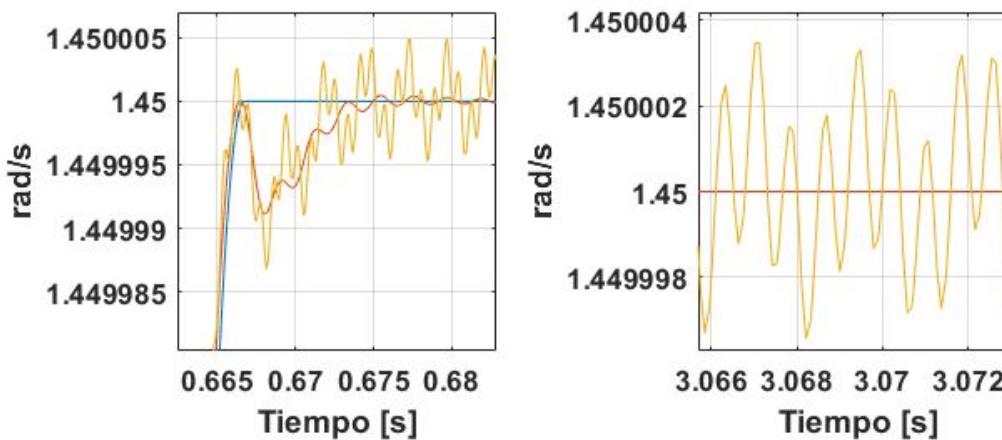
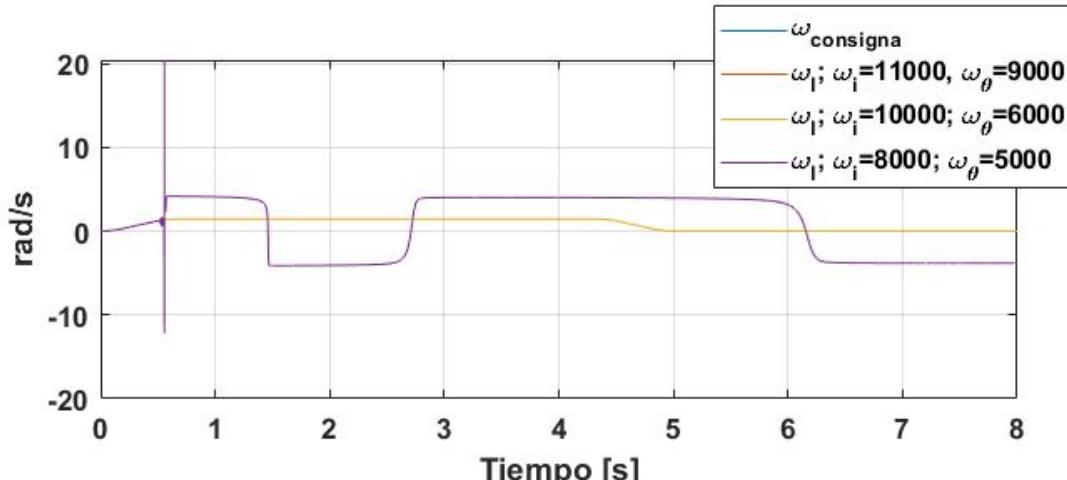
$$G_{sensor\ temp.}(s) = \frac{\frac{1}{\tau}}{s + \frac{1}{\tau}}$$

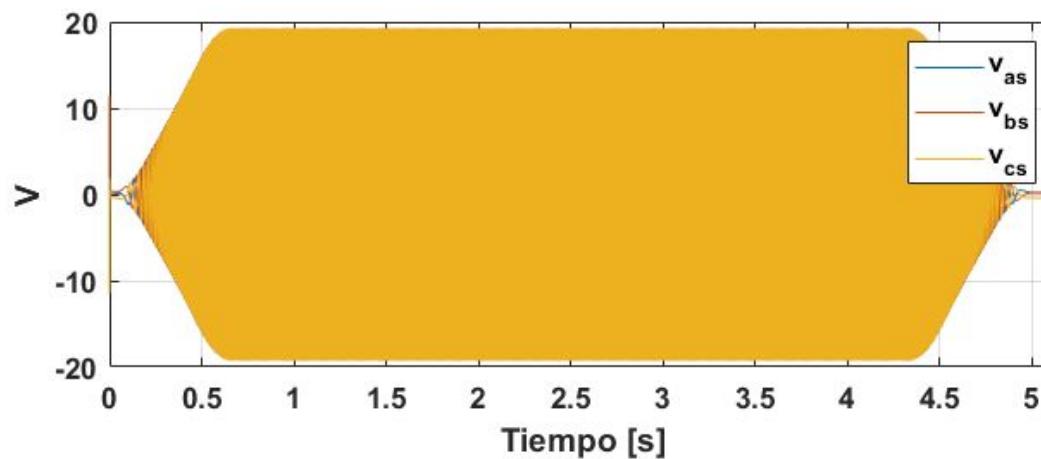
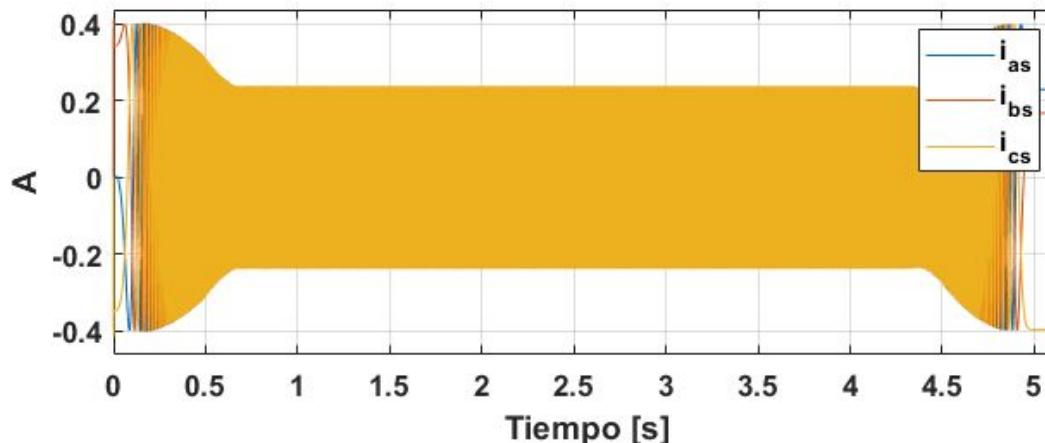
$$\begin{cases} \dot{X}(t) = \begin{bmatrix} -\frac{1}{\tau} \end{bmatrix} \cdot X(t) + \begin{bmatrix} 1 \end{bmatrix} \cdot u(t) \\ Y(t) = \begin{bmatrix} \frac{1}{\tau} \end{bmatrix} \cdot X(t) \end{cases}$$

Respuesta no ideal de los sensores y acondicionadores de señal









Muchas Gracias!!