2023 级 线性代数 IIB 参考答案

1. -11

2. $2x^2+7xy+5y^2$

3. $\frac{1}{25}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$

4. 27A

5. $\frac{9}{2}$

6. $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

7. A+3E

8. B　9. A　10. B　11. A　12. D

13. 解:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} \lambda+2 & \lambda+2 & \lambda+2 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix}$$

$$= (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = (\lambda+2)(\lambda-1)^2$$

…… 4′

① 当 $\lambda \neq -2$ 且 $\lambda \neq 1$ 时，方程组有唯一解

2′

② 当 $\lambda = 1$ 时:

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right) \to \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore r(A) = r(A|b) = 1 < 3$

$\therefore$ 当 $\lambda = 1$ 时，方程组有无穷多解，通解为:

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + k_1 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + k_2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

4′

③ 当$\lambda=-2$时

$$(A|b)=\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -2 \\ 1 & 1 & -2 & 4 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 3 & -3 & 9 \\ 0 & -3 & 3 & -6 \end{array}\right) \rightarrow \begin{pmatrix} 1 & 1 & -2 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore r(A)=2 < r(A|b)=3$ 2'

$\therefore$ 当$\lambda=-2$时，方程组无解.

14. 解

$$\begin{pmatrix} 1 & -2 & -4 & -1 & 5 \\ 0 & 1 & 3 & 1 & -2 \\ 1 & 1 & 5 & 3 & -2 \\ 2 & -1 & 1 & 4 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -4 & -1 & 5 \\ 0 & 1 & 3 & 1 & -2 \\ 0 & 3 & 9 & 4 & -7 \\ 0 & 3 & 9 & 6 & -9 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -4 & -1 & 5 \\ 0 & 1 & 3 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 & -3 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -2 & -4 & -1 & 5 \\ 0 & 1 & 3 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -4 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4'

$\therefore$向量组的秩为3, 2' 极大无关组取为 $\alpha_1, \alpha_2, \alpha_4$ 2'

$\alpha_3=2\alpha_1+3\alpha_2$, 2' $\alpha_5=2\alpha_1-\alpha_2-\alpha_4$ 2'

15. $\begin{vmatrix} 2024-\lambda & 2024 & 2024 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = \lambda^2(2024-\lambda)$

$\therefore \lambda_1 = \lambda_2 = 0, \quad \lambda_3 = 2024$ 4'

$A - \lambda_1 E = \begin{pmatrix} 2024 & 2024 & 2024 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\therefore \quad p_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \qquad p_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 4'

$A - \lambda_3 E = \begin{pmatrix} 0 & 2024 & 2024 \\ 0 & -2024 & 0 \\ 0 & 0 & -2024 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$\therefore \quad p_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 2'

令 $P = (p_1\ p_2\ p_3)$，则

$P^{-1}AP = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 2024 \end{pmatrix} = \Lambda$ 2'

16. 证：设 $k_1\alpha+k_2\beta+k_3\gamma=0$ 2'

$\because \alpha,\beta,\gamma$ 相互正交

$\therefore \alpha,\beta,\gamma\neq 0$，且 $(\alpha,\beta)=(\beta,\gamma)=(\gamma,\alpha)=0$ 4'

$\therefore (\alpha, k_1\alpha+k_2\beta+k_3\gamma)=k_1|\alpha|^2=0$

$\therefore k_1=0$

同理 $k_2=0$, $k_3=0$ 2'

17. $Ax=0 \Rightarrow A^TAx=0$ 2'

$A^TAx=0 \Rightarrow x^TA^TAx=0 \Rightarrow (Ax)^T(Ax)=0$

$\Rightarrow \|Ax\|^2=0 \Rightarrow Ax=0$

4'

$\therefore$ 方程组 $Ax=0$ 和 $A^TAx=0$ 同解

$\therefore$ 它们解空间的维数相同 2'

$\therefore r(A^TA)=r(A)$.