江南大学《高等数学下》2019-2020学年第二学期期末试卷

考试形式: 闭卷

特别提醒:请将答案填写在答题纸上,若填写在试卷纸上无效.

- 一. 选择题: (每小题 3 分, 共 15 分)
- 1. 曲面 $\frac{x^2}{16} \frac{y^2}{9} + \frac{z^2}{16} = 1$ 是由()绕 y 轴旋转得到的. B



- 2. 如果在点 (x_0, y_0) 的某邻域内 $\lim_{\substack{x \to x_0 \\ y \to y_0}} f(x, y)$ 存在,则 f(x, y)在 (x_0, y_0) 处(D).
 - A. 连续 B. 可微 C. 间断 D. 不一定连续
- 3. 已知 $(axy^3 y^2\cos x)dx + (1+by\sin x + 3x^2y^2)dy$ 为某函数 f(x,y) 的全微分,则

(). A
A.
$$a = 2, b = -2$$
 B. $a = -2, b = 2$

C.
$$a = 2, b = 0$$
 D. $a = 0, b = -2$

- 4. 设有下列命题
- (1) 若 $\sum_{n=1}^{\infty} (u_{2n-1} + u_{2n})$ 收敛,则 $\sum_{n=1}^{\infty} u_n$ 收敛.

(2) 若
$$\sum_{n=1}^{\infty} u_n$$
 收敛,则 $\sum_{n=1}^{\infty} u_{n+100}$ 收敛.

(3) 若
$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$$
, 则 $\sum_{n=1}^{\infty} u_n$ 发散.

(4) 若
$$\sum_{n=1}^{\infty} (u_n + v_n)$$
收敛,则 $\sum_{n=1}^{\infty} u_n, \sum_{n=1}^{\infty} v_n$ 都收敛.

更多考试真题请扫码获取



以上命题中正确的是().

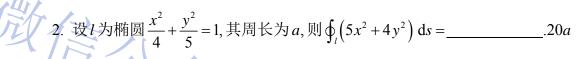
- B. (2) (3) C. (3) (4)
- D. (1) (4)

5. 已知 $D: x^2 + y^2 \le 1$, $D_1: x^2 + y^2 \le 1$, $x \ge 0$, $y \ge 0$. 则下列四个等式中**不成立**的是 ().

- A. $\iint_D x \ln(1+x^2+y^2) dxdy = 0$ B. $\iint_D \sqrt{1-x^2-y^2} dxdy = 4 \iint_{D_1} \sqrt{1-x^2-y^2} dxdy$
- C. $\iint_{D} xy dx dy = 4 \iint_{D_{1}} xy dx dy$ D. $\iint_{D} |xy| dx dy = 4 \iint_{D_{1}} xy dx dy$

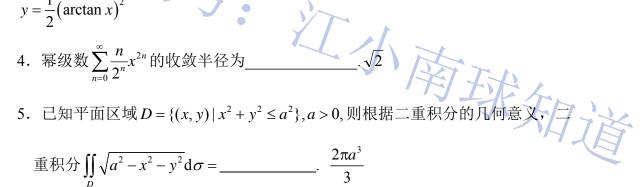
填空题: (每小题 3 分, 共 15 分)

1. 设函数 z = z(x, y) 由方程 $z = e^{2x-3z} + 2y$ 确定,则 $3\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \underline{\qquad}$



微分方程 $(1+x^2)y' = \arctan x$ 满足条件 $y|_{x=0} = 0$ 的特解是______

$$y = \frac{1}{2} \left(\arctan x\right)^2$$



三.解下列各题:(每小题8分,共40分)

1. 把 $\sin x$ 展开成 $(x + \frac{\pi}{4})$ 的幂级数. 解:

$$sin x = sin\left[-\frac{\pi}{4} + (x + \frac{\pi}{4})\right]$$

$$= \frac{1}{\sqrt{2}} \left[sin(x + \frac{\pi}{4}) - cos(x + \frac{\pi}{4})\right].....(2\%)$$

$$sin(x + \frac{\pi}{4}) = (x + \frac{\pi}{4}) - \frac{(x + \frac{\pi}{4})^3}{3!} + \frac{(x + \frac{\pi}{4})^5}{5!} -(2\%)$$

$$cos(x + \frac{\pi}{4}) = 1 - \frac{(x + \frac{\pi}{4})^2}{2!} + \frac{(x + \frac{\pi}{4})^4}{4!} -(2\%)$$

$$sin x = \frac{\sqrt{2}}{2} \left(-1 + (x + \frac{\pi}{4}) + \frac{(x + \frac{\pi}{4})^2}{2!} - (-\infty < x < +\infty)......(2\%)$$

2. 求微分方程 $x^2y' + xy - y^2 = 0$ 的通解

 $(0 \le t \le 2\pi), a > 0, b > 0.$

解:
$$ds = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} dt = \sqrt{a^2 + b^2} dt$$
, ...(2 分)

原式= $\int_0^{2\pi} (a^2 \cos^2 t + a^2 \sin^2 t + b^2 t^2) \sqrt{a^2 + b^2} dt$...(3 分)
$$= \sqrt{a^2 + b^2} \int_0^{2\pi} (a^2 + b^2 t^2) dt = \sqrt{a^2 + b^2} (2\pi a^2 + \frac{8}{3}\pi^3 b^2). ...(3 分)$$

4. 计算曲面积分 $\iint_{\Sigma} \frac{2x^2 + 2y^2 + z}{\sqrt{4x^2 + 4y^2 + 1}} dS$, 其中 Σ 为曲面 $z = 9 - x^2 - y^2 (z \ge 0)$.

解:
$$\sqrt{1+{z_x'}^2+{z_y'}^2} = \sqrt{1+4x^2+4y^2}$$
, $D_{xy}: x^2+y^2 \le 9$...(3 分)

原式=
$$\iint_{D_{xy}} \frac{2x^2 + 2y^2 + (9 - x^2 - y^2)}{\sqrt{4x^2 + 4y^2 + 1}} \cdot \sqrt{4x^2 + 4y^2 + 1} dxdy \qquad \dots (2 分)$$

$$= \iint_{D} (9 + x^{2} + y^{2}) dxdy = \int_{0}^{2\pi} d\theta \int_{0}^{3} (9 + r^{2}) r dr = \frac{243\pi}{2}. \dots (3 \%)$$

5. 求平面3x+4y-z-26=0上距原点最近的点.

解:
$$F(x, y, z, \lambda) = x^2 + y^2 + z^2 + \lambda(3x + 4y - z - 26) \dots (2 分)$$

$$\begin{cases} \frac{\partial F}{\partial x} = 2x + 3\lambda = 0\\ \frac{\partial F}{\partial y} = 2y + 4\lambda = 0\\ \frac{\partial F}{\partial z} = 2z - \lambda = 0\\ 3x + 4y - z - 26 = 0 \end{cases} \dots (4 \%)$$

 $\lambda = -2, x = 3, y = 4, z = -1,$ 该点为(3,4,-1). ...(2 分)

四.解下列各题: (每小题 10 分,共 30 分)

1. 计算曲线积分 $I = \int_{L} (x+y)^{2} dx - (x^{2} + y^{2} \sin y) dy$, 其中 L 是抛物线 $y = x^{2}$ 上从点 ...(2 分)

(-1,1)到点(1,1)的那一段.

解:补上有向直线段 $y = 1(-1 \le x \le 1)$,

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -4x - 2y,$$

利用格林公式

$$I = \iint_{D} (-4x - 2y) dxdy + \int_{-1}^{1} (x+1)^{2} dx \qquad \dots (3 \%)$$

$$= -2\iint_{D} y dx dy + \frac{8}{3} = \frac{16}{15}.$$
 ...(3 \(\frac{1}{2}\))

2. 计算曲面积分 $\iint_{\Sigma} x dy dz + 2y dz dx + 3(z-1) dx dy$, 其中 Σ 是锥面 $z = \sqrt{x^2 + y^2}$ (0 \le z \le 1) 的下侧.

解: 补曲面
$$\Sigma_1$$
: $\begin{cases} x^2 + y^2 \le 1 \\ z = 1 \end{cases}$, 上侧 ...(2 分)

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 6, \qquad ...(2 \%)$$

$$\iint_{\Sigma + \Sigma_1} x dy dz + 2y dz dx + 3(z - 1) dx dy = \iiint_{\Omega} 6 dx dy dz = 2\pi ...(3 \%)$$

$$\mathbb{X} \iint_{\Sigma_1} x dy dz + 2y dz dx + 3(z - 1) dx dy = 0 \qquad ...(2 \%)$$
从而,原式==2 π ...(1 $\%$)

3. 求幂级数 $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} x^{n-1}$ 的和函数,并写出收敛区间.

$$\int_0^x f(x) dx = \sum_{n=1}^\infty \int_0^x \frac{n}{(n+1)!} x^{n-1} dx = \sum_{n=1}^\infty \frac{x^n}{(n+1)!}$$
 (2 分)

$$= \frac{1}{x} \sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)!} = \frac{1}{x} \left[\sum_{n=0}^{\infty} \frac{x^n}{n!} - (1+x) \right] = \frac{1}{x} \left[e^x - (1+x) \right] (x \neq 0) \dots (2 / 7)$$

$$f(x) = \left\{ \frac{1}{x} [e^x - (1+x)] \right\}' = \frac{xe^x - (e^x - 1)}{x^2} (x \neq 0)$$
 (2 \(\frac{1}{2}\))

$$= \frac{1}{x} \sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)!} = \frac{1}{x} \left[\sum_{n=0}^{\infty} \frac{x^n}{n!} - (1+x) \right] = \frac{1}{x} \left[e^x - (1+x) \right] (x \neq 0) \dots (2 / \pi)$$

$$f(x) = \left\{ \frac{1}{x} \left[e^x - (1+x) \right] \right\}' = \frac{xe^x - (e^x - 1)}{x^2} (x \neq 0) \qquad (2 / \pi)$$

$$f(x) = \begin{cases} \frac{xe^x - (e^x - 1)}{x^2}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$

$$\dots \dots (2 / \pi)$$