# CSCI 2270: Data Structures

## Lecture 21: Hash Tables

Ashutosh Trivedi

| Key1 | Value1 |
|---|---|
| Key2 | Value2 |
| Key3 | Value3 |

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# Dictionary

**Definition.**

1. *A book or electronic resource that lists the words of a language (typically in alphabetical order) and gives their meaning.*
2. *A reference work on a particular subject, the items of which are typically arranged in alphabetical order.*

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**Data Structure.**

- A general-purpose data structure for storing a group of objects.
- A set of keys and each key has a single associated value.
- Given a key, the dictionary will return the associated value.
- Also known as associative array, map, symbol table.
- Example:
  - Students and their grades,
  - book titles and shelf number,
  - movie title and other information, etc.
- Three key operations are INSERT, DELETE, and SEARCH.
- Can be implemented as an array (sorted or unsorted), as a linked list, or as a binary search tree.

# Motivation

*How efficiently we can perform search?*

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| | Array | Sorted-array | Linked Lists | BST |
|---|---|---|---|---|
| INSERT | $O(1)$ | $O(n)$ | $O(1)$ | $O(\log(n))$ |
| DELETE | $O(n)$ | $O(n)$ | $O(n)$ | $O(\log(n))$ |
| SEARCH | $O(n)$ | $O(\log(n))$ | $O(n)$ | $O(\log(n))$ |

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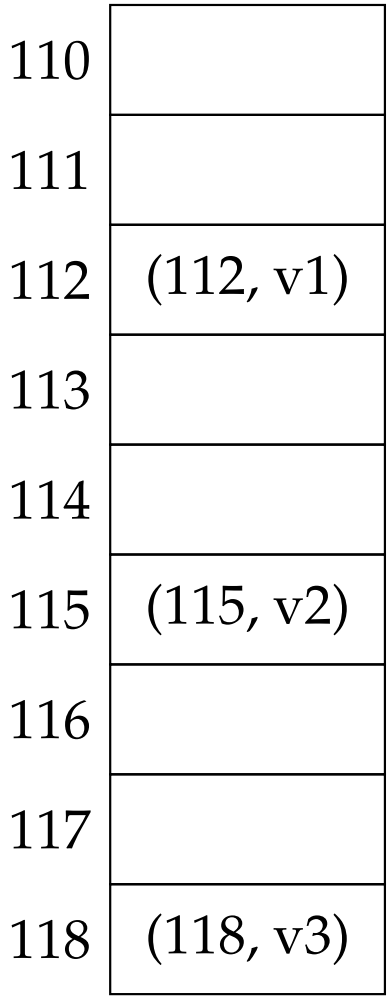
| SEARCH | $O(n)$ | $O(\log(n))$ | $O(n)$ | $O(\log(n))$ |

Is it possible to get insert, delete, and search operations with $O(1)$ complexity?

# Direct-Address Tables

When the universe of key values is small:



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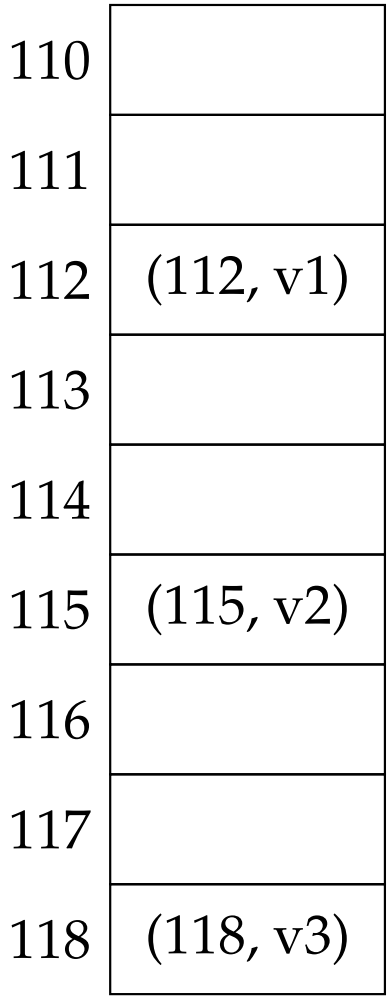


INSERT : $O(1)$, DELETE : $O(1)$, and SEARCH : $O(1)$.

# Storage Requirements

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**Solution.** Hash-Tables!

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- Examples of hash functions:
  - `hash(int key) = key % 20`
  - `hash(string key) = sum-of-ASCII-values % 50`

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- When perfect, it gives the same advantages of direct-address tables, with modest memory requirements.

# Perfect/Imperfect Hash functions

Design a hash function to store up to 100 phone numbers in the range 7207079600 and 7207079699.

- a perfect hash function ($k$%100)

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | |
| 4 | |
| 5 | 7207079605 |
| 6 | |
| 7 | |
| 8 | 7207079608 |
| 9 | |

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| | |
|---|---|
| 0 | $\rightarrow$ |
| 1 | $\rightarrow$ |
| 2 | 7207079602 $\rightarrow$ 7207079612 $\rightarrow$ 7207079622 |
| 3 | $\rightarrow$ |
| 4 | $\rightarrow$ |
| 5 | 7207079605 $\rightarrow$ 7207079655 |
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| 8 | 7207079608 $\rightarrow$ |
| 9 | $\rightarrow$ |

# Open Addressing

In case of a hash-collision:

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- Linear probing:
  - find the next free position in the array in the linear order, i.e. $h(x), h(x)+1, h(x)+2$

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  - find the next free position in the array in the quadratic order, i.e. $h(x), h(x)+1^2, h(x)+2^2$

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- Double hashing:
  - find the next free position in the array using double hash functions, i.e. $h_1(x) + i \cdot h_2(x)$.

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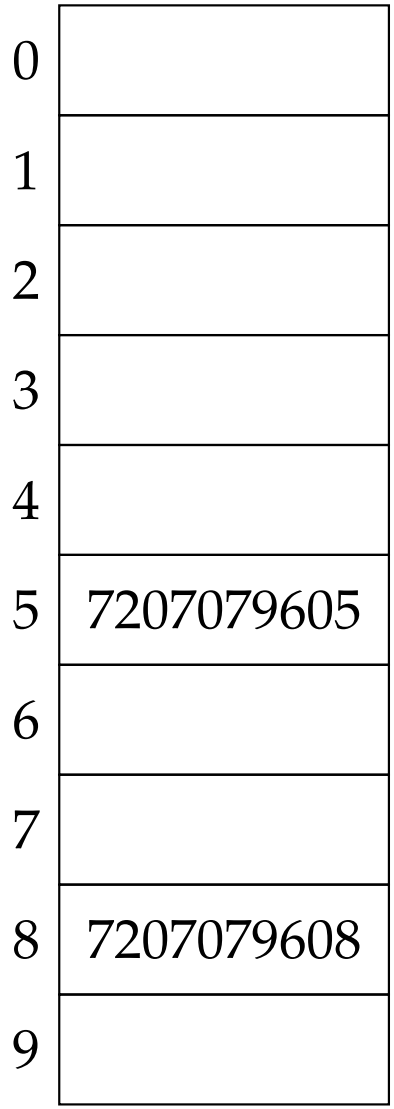
- Double hashing:
  - find the next free position in the array using double hash functions, i.e. $h_1(x) + i \cdot h_2(x)$.
- Cuckoo hashing, Hopscotch hashing, Robin-hood hashing, and many more.

# Animation: Open Addressing

# Open Addressing: Insertion, Search, and Delete

1. insert(7207079602) with hash(n) = n mod 10



# Open Addressing: Insertion, Search, and Delete

2. insert(7207079612) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | |
| 4 | |
| 5 | 7207079605 |
| 6 | |
| 7 | |
| 8 | 7207079608 |
| 9 | |

# Open Addressing: Insertion, Search, and Delete

3. insert(7207079622) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | 7207079612 |
| 4 | |
| 5 | 7207079605 |
| 6 | |
| 7 | |
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# Open Addressing: Insertion, Search, and Delete

4. insert(7207079632) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | 7207079612 |
| 4 | 7207079622 |
| 5 | 7207079605 |
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| 7 | |
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# Open Addressing: Insertion, Search, and Delete

5. search(7207079642) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | 7207079612 |
| 4 | 7207079622 |
| 5 | 7207079605 |
| 6 | |
| 7 | |
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| 9 | |

# Open Addressing: Insertion, Search, and Delete

6. delete(7207079622) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
| 3 | 7207079612 |
| 4 | 7207079622 |
| 5 | 7207079605 |
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# Open Addressing: Insertion, Search, and Delete

7. search(7207079642) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
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| 4 | |
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# Open Addressing: Insertion, Search, and Delete

8. search(7207079632) with hash(n) = n mod 10

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 7207079602 |
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# Open Addressing: Insertion, Search, and Delete

6. delete(7207079622) with **Lazy Delete**

| | |
|---|---|
| 0 | |
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# Open Addressing: Insertion, Search, and Delete

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