CSCI 2270: Data Structures

Lecture 21: Hash Tables

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Key1	Value1
Key2	Value2
Key3	Value3

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Dictionary

Definition.

1. *A book or electronic resource that lists the words of a language (typically in alphabetical order) and gives their meaning.*
2. *A reference work on a particular subject, the items of which are typically arranged in alphabetical order.*

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Data Structure.

- A general-purpose data structure for storing a group of objects.
- A set of keys and each key has a single associated value.
- Given a key, the dictionary will return the associated value.
- Also known as associative array, map, symbol table.
- Example:
  - Students and their grades,
  - book titles and shelf number,
  - movie title and other information, etc.
- Three key operations are INSERT, DELETE, and SEARCH.
- Can be implemented as an array (sorted or unsorted), as a linked list, or as a binary search tree.

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DELETE	$O(n)$	$O(n)$	$O(n)$	$O(\log(n))$
SEARCH	$O(n)$	$O(\log(n))$	$O(n)$	$O(\log(n))$

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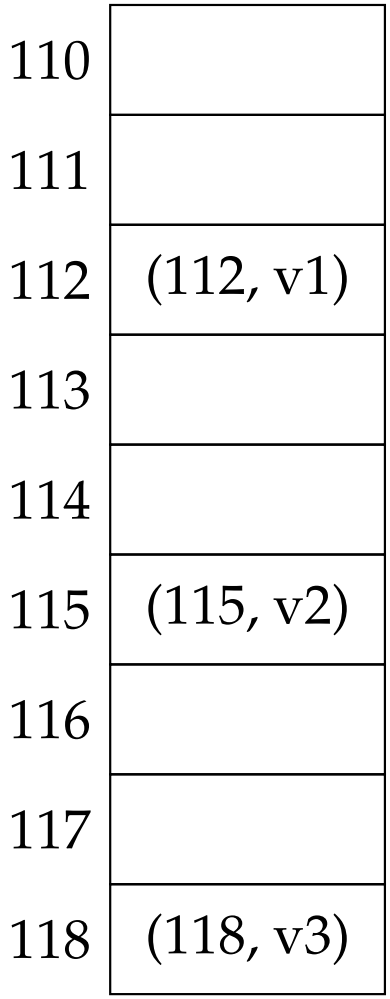
Is it possible to get insert, delete, and search operations with $O(1)$ complexity?

Direct-Address Tables

When the universe of key values is small:



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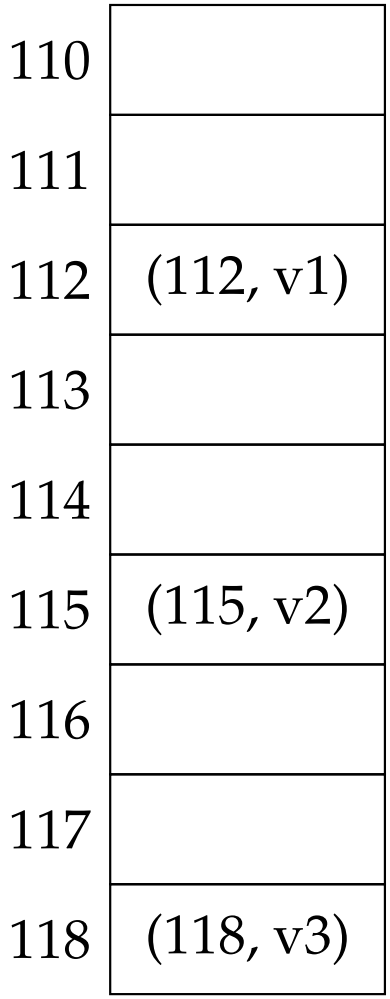

INSERT : $O(1)$, DELETE : $O(1)$, and SEARCH : $O(1)$.

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Solution. Hash-Tables!

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  - `hash(int key) = key % 20`
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- When perfect, it gives the same advantages of direct-address tables, with modest memory requirements.

Perfect/Imperfect Hash functions

Design a hash function to store up to 100 phone numbers in the range 7207079600 and 7207079699.

- a perfect hash function (k%100)

0	
1	
2	7207079602
3	
4	
5	7207079605
6	
7	
8	7207079608
9	

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- Cuckoo hashing, Hopscotch hashing, Robin-hood hashing, and many more.