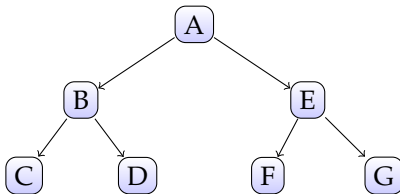


CSCI 2270: Data Structures

Lecture 14: Trees (introduction)

Ashutosh Trivedi



Department of Computer Science
UNIVERSITY OF COLORADO BOULDER

Stacks and Queues: Recap

Trees

Stacks: Recap

1. Lists with LIFO (last-in first-out structure)
2. Insertions and deletions happen from the same side, called the *top* of the stack.
3. An *insertion* is called a **push** to the stack
4. A *deletion* is called a **pop** from the stack
5. You can implement them using array or linked lists
6. When implementing as an array, keep the top of the stack as the **highest occupied index**
7. When implementing as a linked list, keep the top of the stack as the **head** of the list.

Some applications of stacks

- Reverse a string How?
- Check for balanced parenthesis:
 - $(5 + 3) * ((8 * 2) / 7)$
 - `if(x < 4){if(y < 3){cout << "hello"; }}{ }`
 - `((()())())` and `(((((())((()())))))` versus `))(((` or `((()((()()))))`
- Check if a string is palindrome (Able was I ere I saw Elba). How?
- Infix, prefix, and postfix expressions:

| | | | |
|--------------|-----------------|---------------------|-----------------|
| - (infix): | $A + B * C + D$ | $(A + B) * (C + D)$ | $A * B + C * D$ |
| - (prefix): | $++ A * B C D$ | $* + A B + C D$ | $+ * A B * C D$ |
| - (postfix): | $A B C * + D +$ | $A B + C D + *$ | $A B * C D * +$ |
- Tree and Graph Traversal. Later in this course

Queues: Recap

1. Lists with FIFO (first-in first-out structure)
2. Insertions and deletions happen from the opposite side of the queue, called the head and tail of the queue, respectively.
3. An *insertion* is called a **enqueue** to the “tail” end of the queue
4. A *deletion* is called a **dequeue** from the “head” end of the queue.
5. You can implement them using array or linked lists
6. When implementing as a linked list, keep the “head” of the queue as the **first** node of the list and “tail” as the “end” of the list. Why?
7. When implementing as an array, both options were equally bad!
8. Solution: Circular array based queue implementation.

Queue implemented as a Circular Array

| | | | | | | | |
|------|----|----|------|---|---|---|---|
| head | 1 | 2 | tail | 4 | 5 | 6 | 7 |
| 44 | 33 | 22 | 11 | B | B | B | B |

- enqueue(55);
- dequeue();
- enqueue(66); enqueue(77); enqueue(88);
- dequeue();
- enqueue(99);

Queue implemented as a Circular Array

| | | | | | | | |
|------|----|----|----|------|----------|----------|----------|
| head | 1 | 2 | 3 | tail | 5 | 6 | 7 |
| 44 | 33 | 22 | 11 | 55 | <i>B</i> | <i>B</i> | <i>B</i> |

- **enqueue(55);**
- dequeue();
- enqueue(66); enqueue(77); enqueue(88);
- dequeue();
- enqueue(99);

Queue implemented as a Circular Array

| | | | | | | | |
|---|------|----|----|------|---|---|---|
| 0 | head | 2 | 3 | tail | 5 | 6 | 7 |
| B | 33 | 22 | 11 | 55 | B | B | B |

- enqueue(55);
- **dequeue()**;
- enqueue(66); enqueue(77); enqueue(88);
- dequeue();
- enqueue(99);

Queue implemented as a Circular Array

| | | | | | | | |
|---|------|----|----|----|----|----|------|
| 0 | head | 2 | 3 | 4 | 5 | 6 | tail |
| B | 33 | 22 | 11 | 55 | 66 | 77 | 88 |

- enqueue(55);
- dequeue();
- **enqueue(66); enqueue(77); enqueue(88);**
- dequeue();
- enqueue(99);

Queue implemented as a Circular Array

| | | | | | | | |
|---|---|------|----|----|----|----|------|
| 0 | 1 | head | 3 | 4 | 5 | 6 | tail |
| B | B | 22 | 11 | 55 | 66 | 77 | 88 |

- enqueue(55);
- dequeue();
- enqueue(66); enqueue(77); enqueue(88);
- **dequeue()**;
- enqueue(99);

Queue implemented as a Circular Array

| | | | | | | | |
|------|---|------|----|----|----|----|----|
| tail | 1 | head | 3 | 4 | 5 | 6 | 7 |
| 99 | B | 22 | 11 | 55 | 66 | 77 | 88 |

- enqueue(55);
- dequeue();
- enqueue(66); enqueue(77); enqueue(88);
- dequeue();
- **enqueue(99);**

CircularArrayQueue: Enqueue

```
void CircularArrayQueue::enqueue(int element) {
    if (isFull()) {
        std::cerr << "Queue Overflow!! Enqueue failed" << std::endl;
    }
    else {
        if (head == -1) {
            //first element to insert
            head = 0;
            tail = 0;
            items[tail] = element;
        }
        else {
            if (tail == capacity-1) {
                items[0] = element;
                tail = 0;
            }
            else {
                tail = tail + 1;
                items[tail] = element;
            }
        }
    }
}
```

CircularArrayQueue: Dequeue

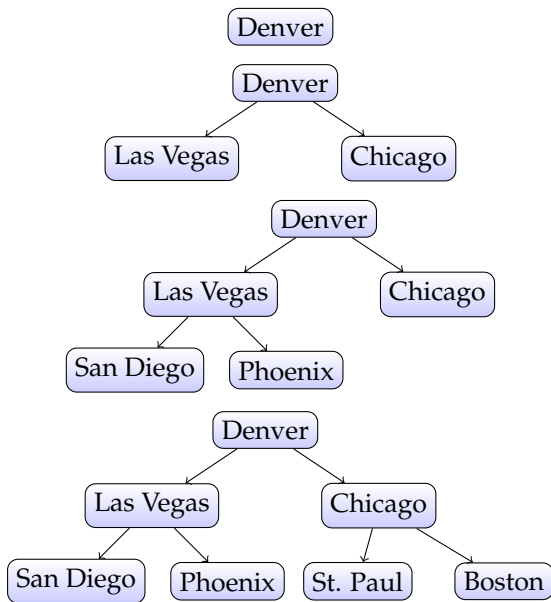
```
int CircularArrayQueue::dequeue() {
    if (isEmpty()) {
        std::cerr << "Queue Empty!! Returning garbage" << std::endl;
        return -1;
    }
    else {
        int result = items[head];

        if (head == tail) {
            // Only one element in the queue
            head = -1;
            tail = -1;
        }
        else {
            if (head == capacity - 1) {
                head = 0;
            }
            else {
                head = head + 1;
            }
        }
        return result;
    }
}
```

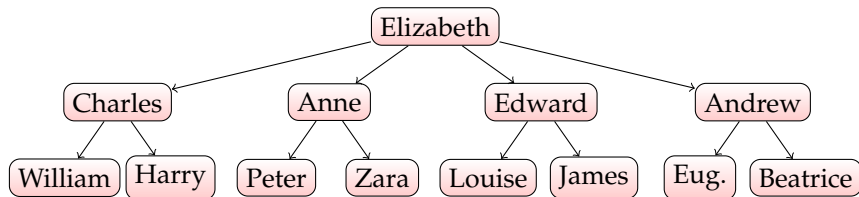
Stacks and Queues: Recap

Trees

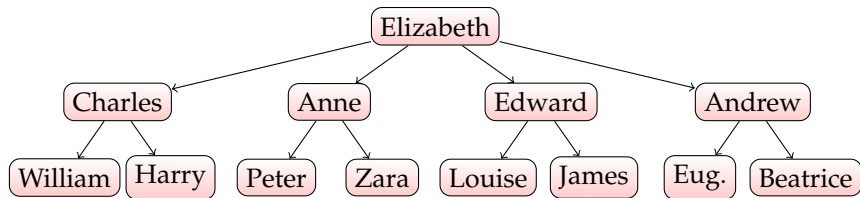
Trees: Terminology



Trees: Terminology

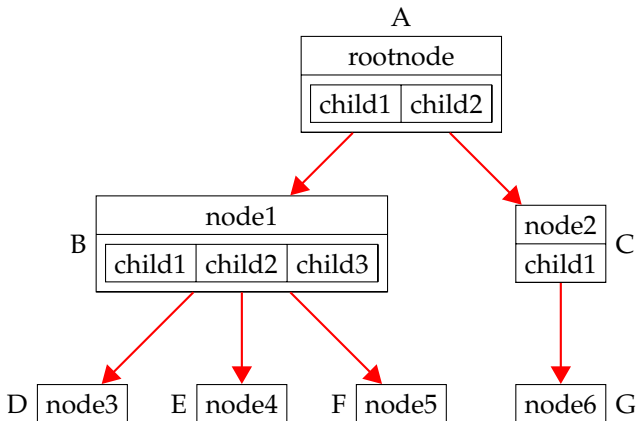


Trees: Terminology

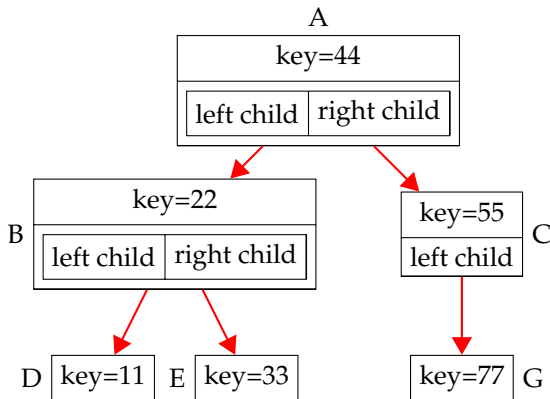


- *arity* of a tree: binary trees, ternary trees, k -ary trees, etc.
- *root* of a tree
- *leaf* of a tree
- *child* of a node
- *parent* of a node
- *ancestor* of a node
- *descendant* of a node
- *sibling* of a node

Trees as a linked structure

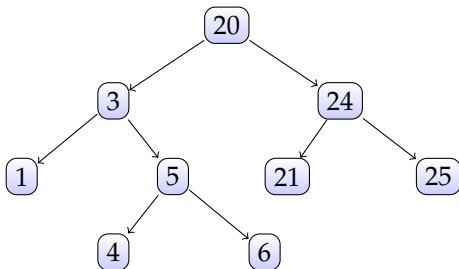


Binary Trees as a linked structure



- *left sub-tree* of a node
- *right sub-tree* of a node

Binary Trees



Properties: If x and y are nodes, and

1. y is in the left sub-tree of x , then

$$y.key < x.key$$

2. y is in the right sub-tree of x , then

$$y.key \geq x.key$$