Symmetry in Probabilistic Databases

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KU Leuven

Joint work with

Dan Suciu, Paul Beame, Eric Gribkoff, Wannes Meert, Adnan Darwiche

Based on NIPS 2011, KR 2014, and upcoming PODS 2015 paper

Overview

- Motivation and convergence of
  - The artificial intelligence story (*recap*)
  - The machine learning story (*recap*)
  - The probabilistic database story
  - The database theory story
- Main theoretical results and proof outlines
- Discussion and conclusions
- Dessert

Overview

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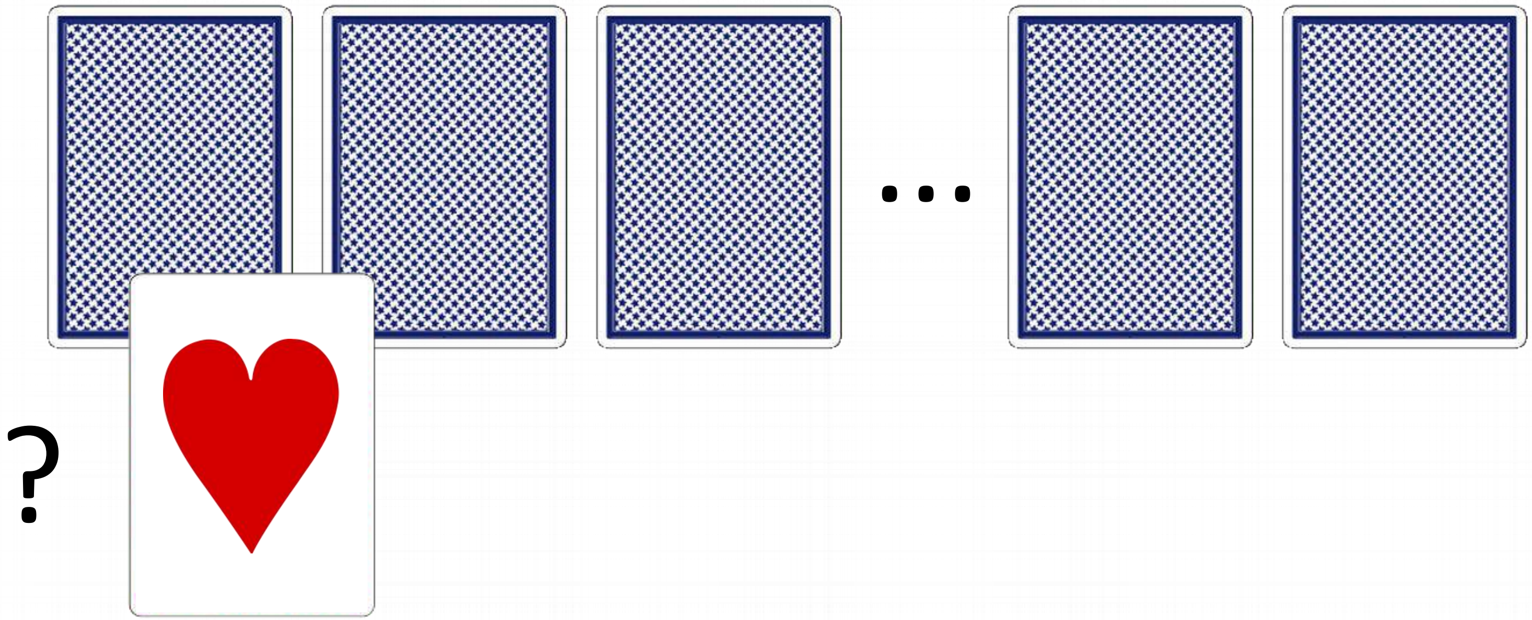
A Simple Reasoning Problem

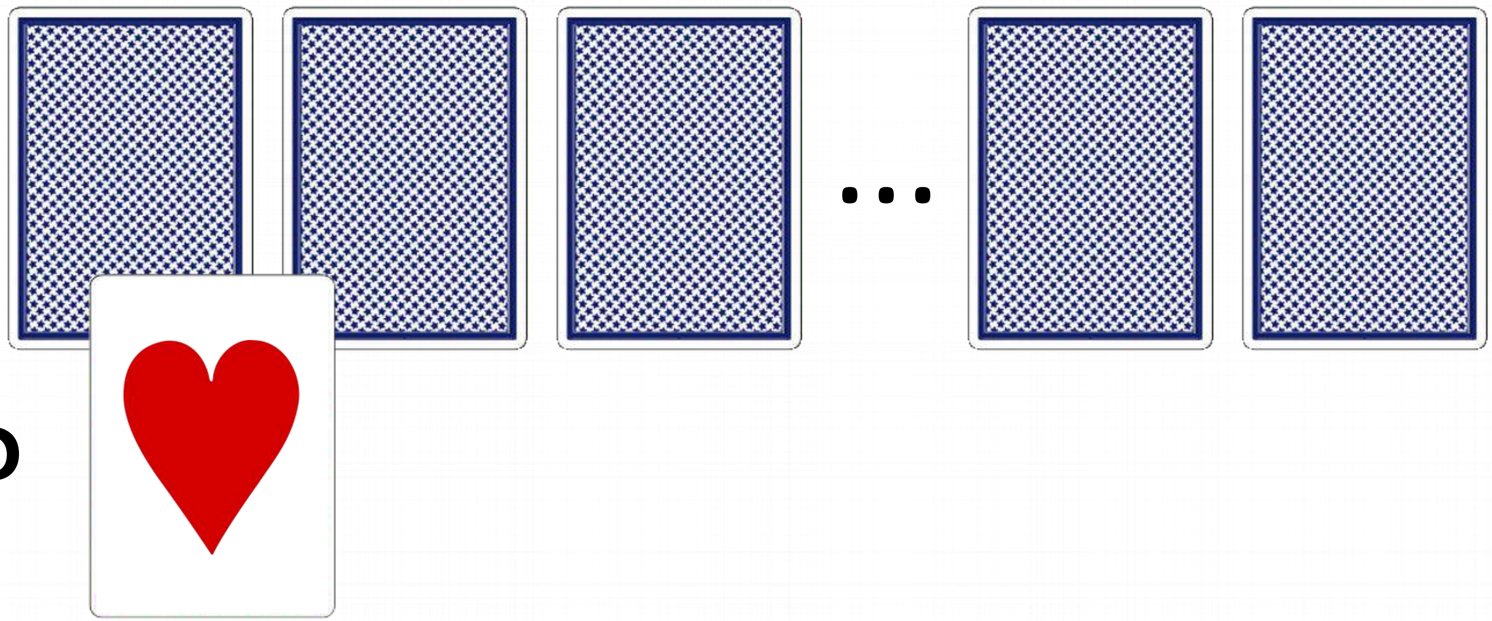


Probability that Card1 is Hearts?

[Van den Broeck; AAAI-KRR'15]

A Simple Reasoning Problem

?



Probability that Card1 is Hearts? 1/4

[Van den Broeck; AAAI-KRR'15]

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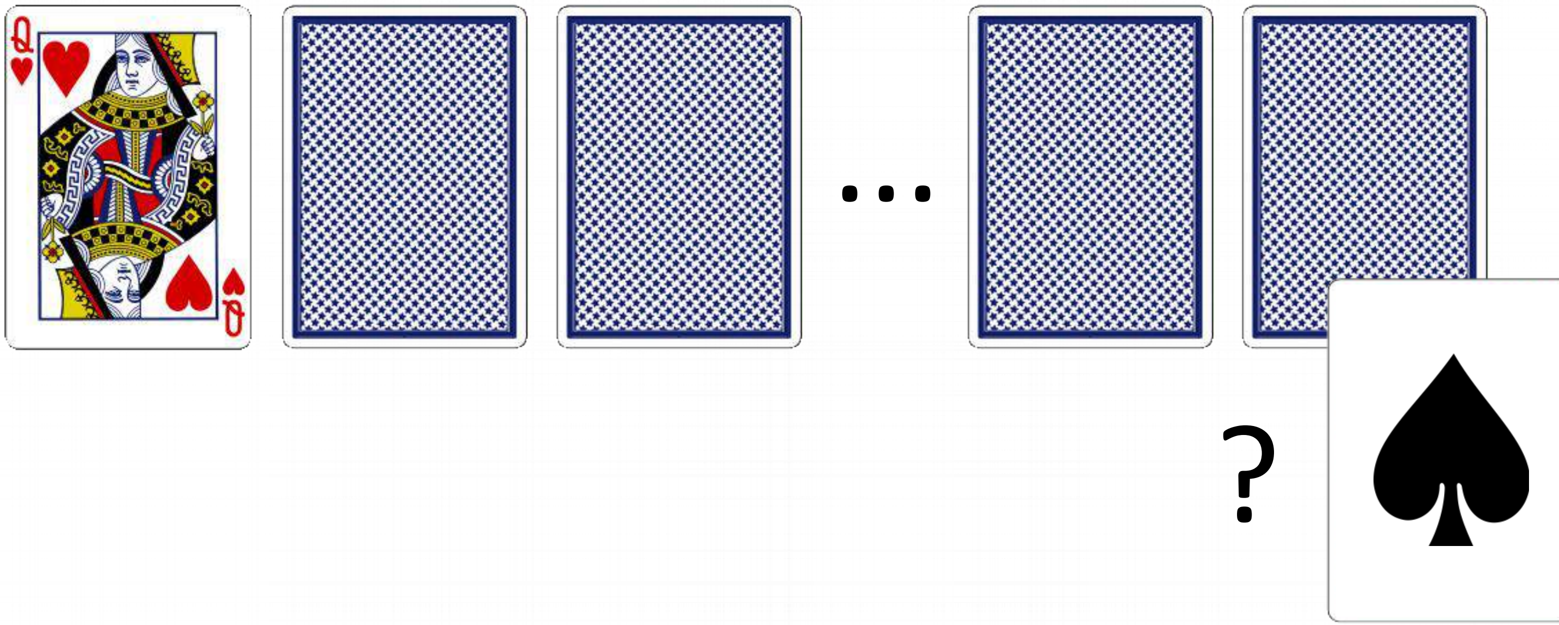

Probability that Card52 is Spades given that Card1 is QH?

[Van den Broeck; AAAI-KRR'15]

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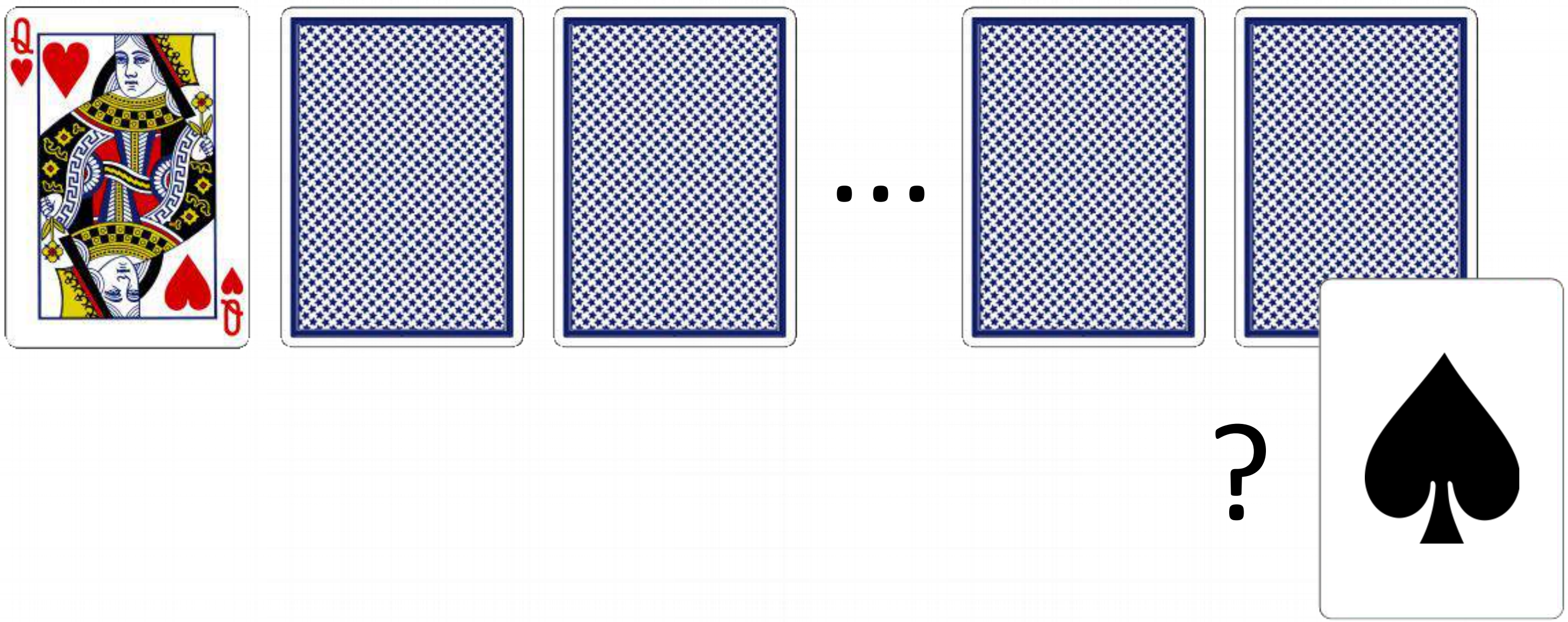


Probability that Card52 is Spades given that Card1 is QH?

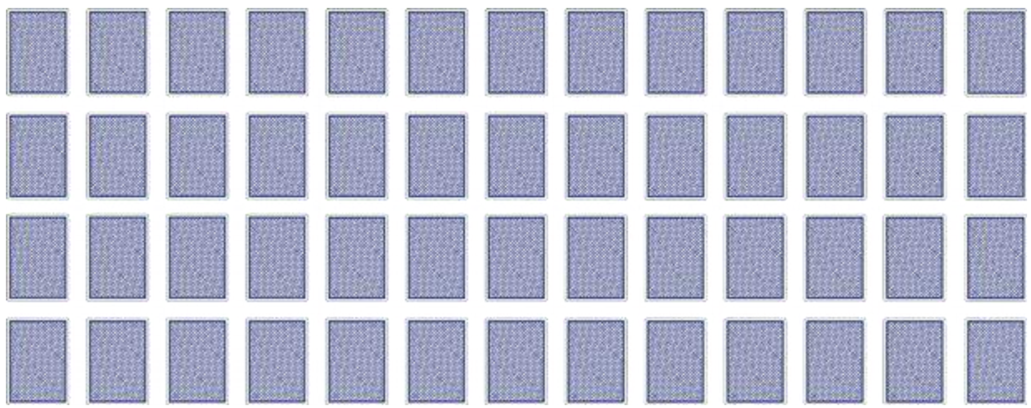
13/51

[Van den Broeck; AAAI-KRR’15]

Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph)

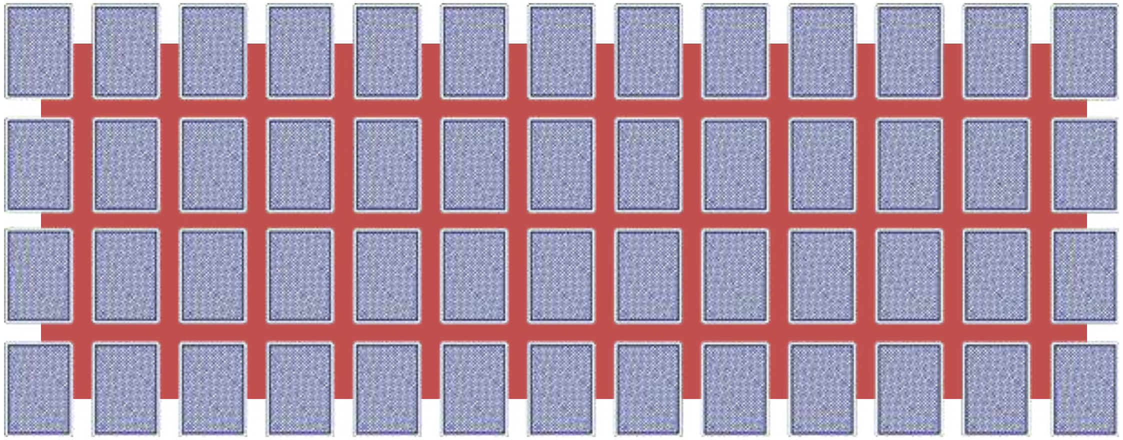


2. Probabilistic inference algorithm
   (e.g., variable elimination or junction tree)

Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph) is fully connected!

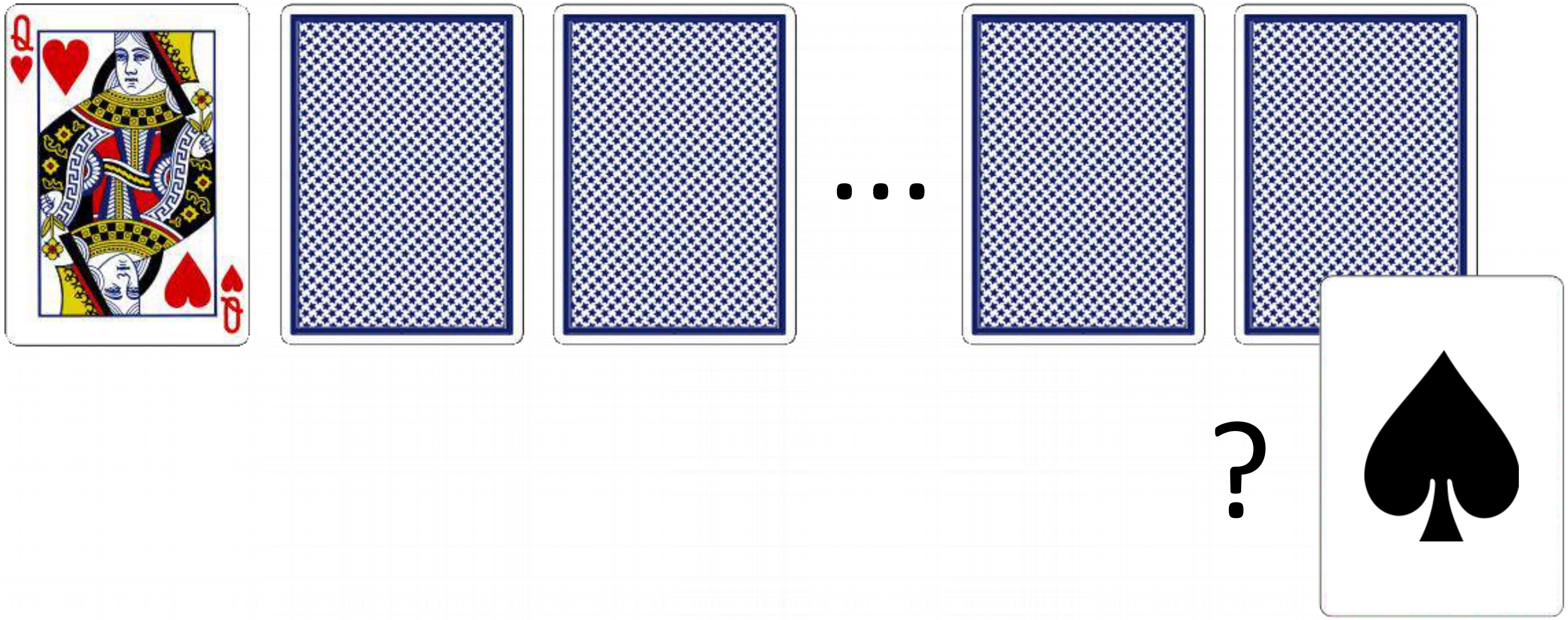


(artist's impression)

2. Probabilistic inference algorithm (e.g., variable elimination or junction tree) builds a table with $\mathbf{52}^{52}$ rows

[Van den Broeck; AAAI-KRR'15]

What's Going On Here?



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[Van den Broeck; AAAI-KRR’15]

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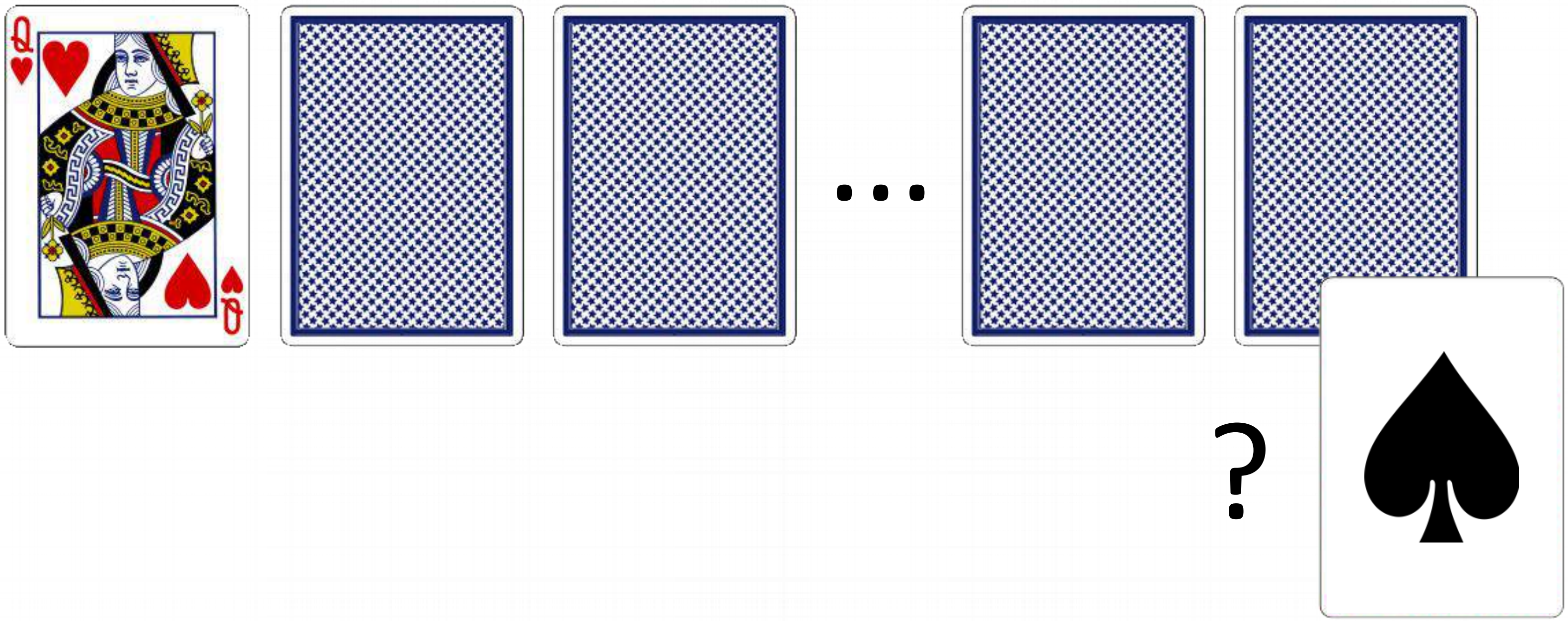

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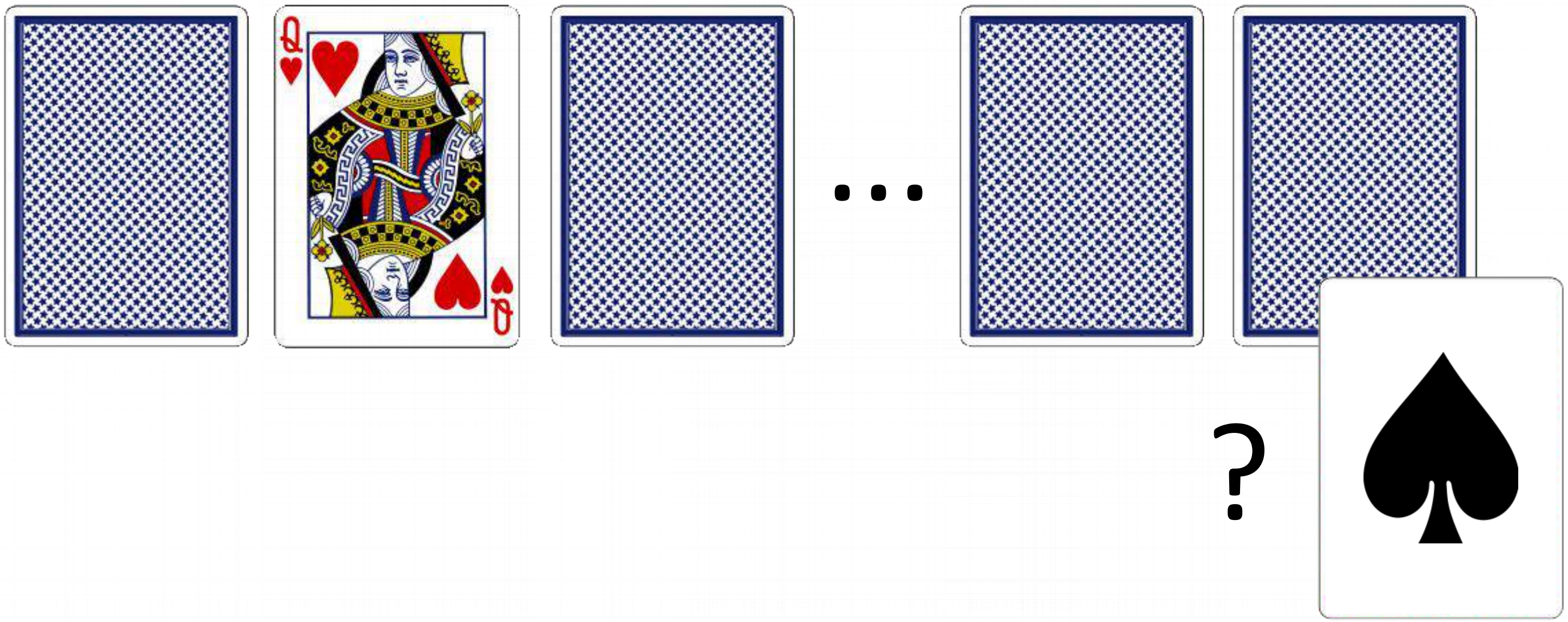

Probability that Card52 is Spades given that Card2 is QH?

[Van den Broeck; AAAI-KRR’15]

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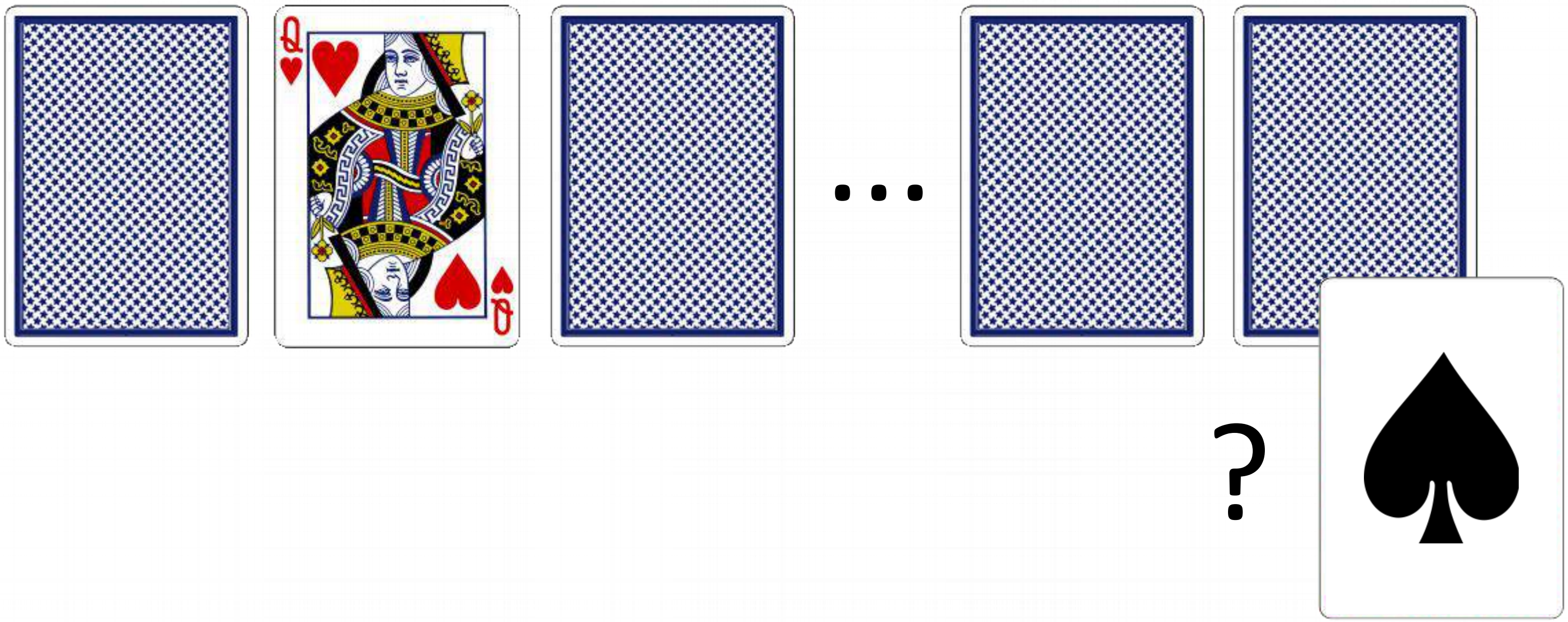

Probability that Card52 is Spades given that Card2 is QH?

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[Van den Broeck; AAAI-KRR’15]

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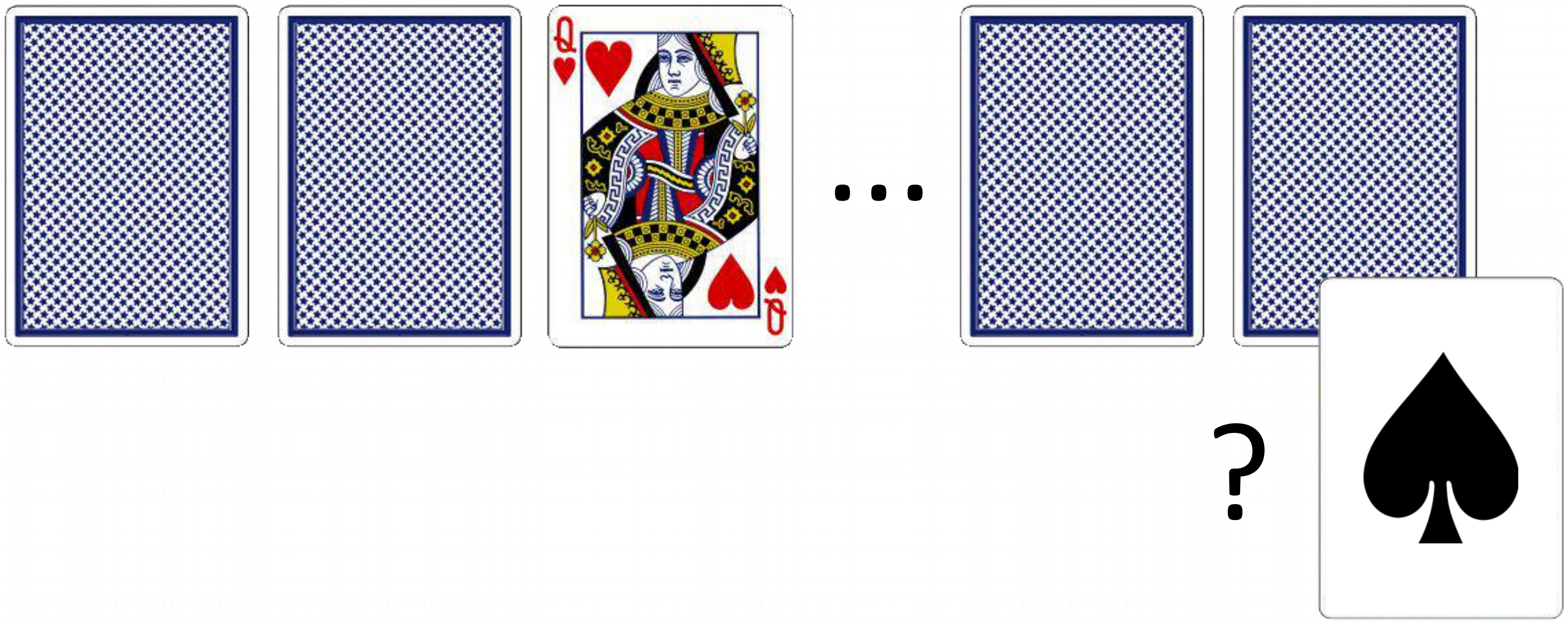

Probability that Card52 is Spades given that Card3 is QH?

[Van den Broeck; AAAI-KRR'15]

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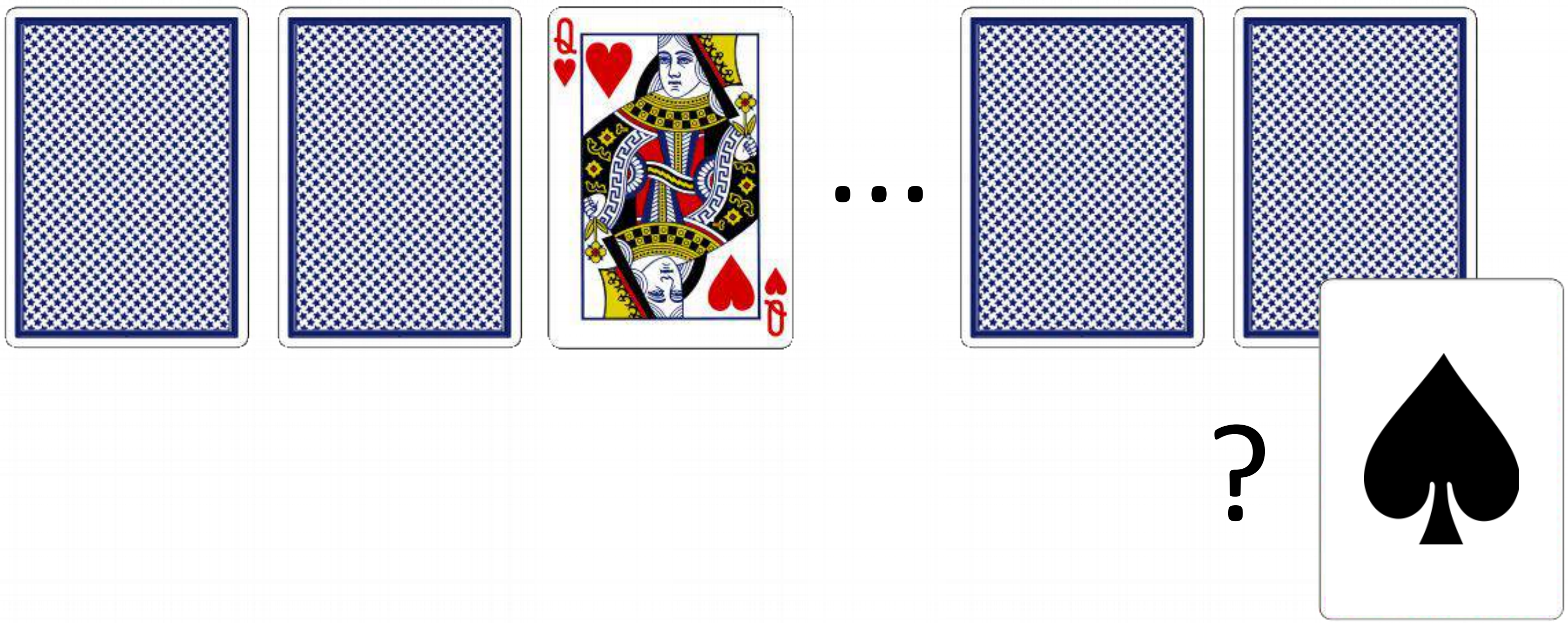
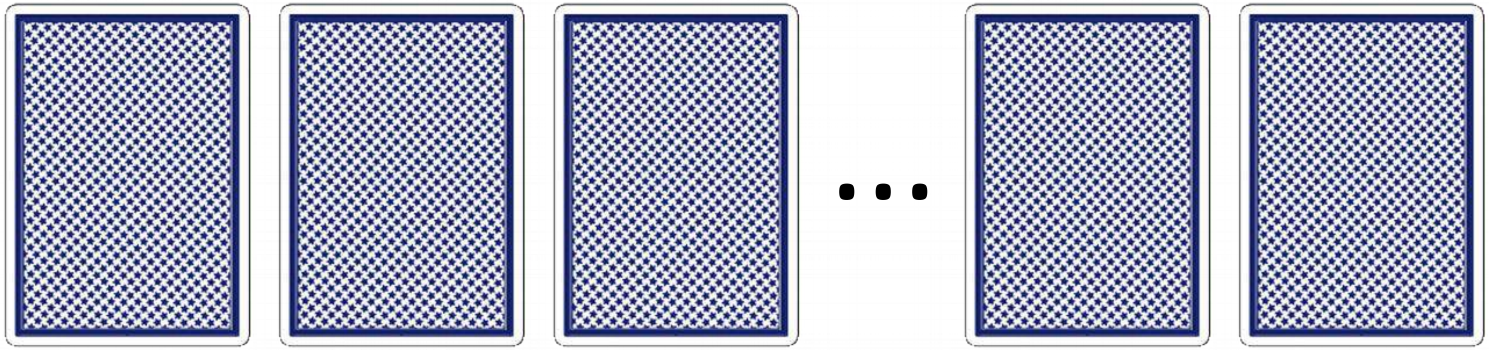


Probability that Card52 is Spades given that Card3 is QH?

13/51

Tractable Probabilistic Inference



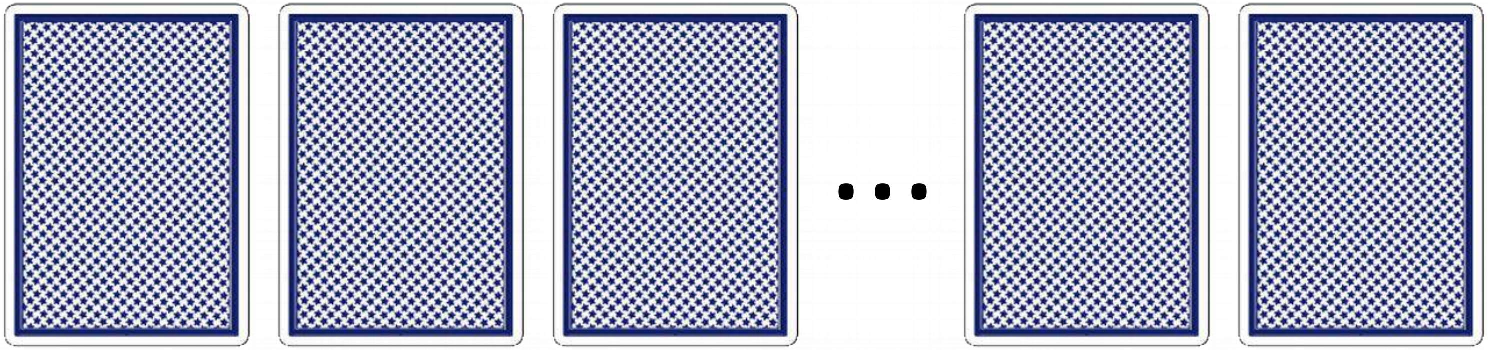
Which property makes inference tractable?

~~Traditional belief: Independence~~

What's going on here?

[Niepert, Van den Broeck; AAAI'14], [Van den Broeck; AAAI-KRR'15]

Tractable Probabilistic Inference



Which property makes inference tractable?

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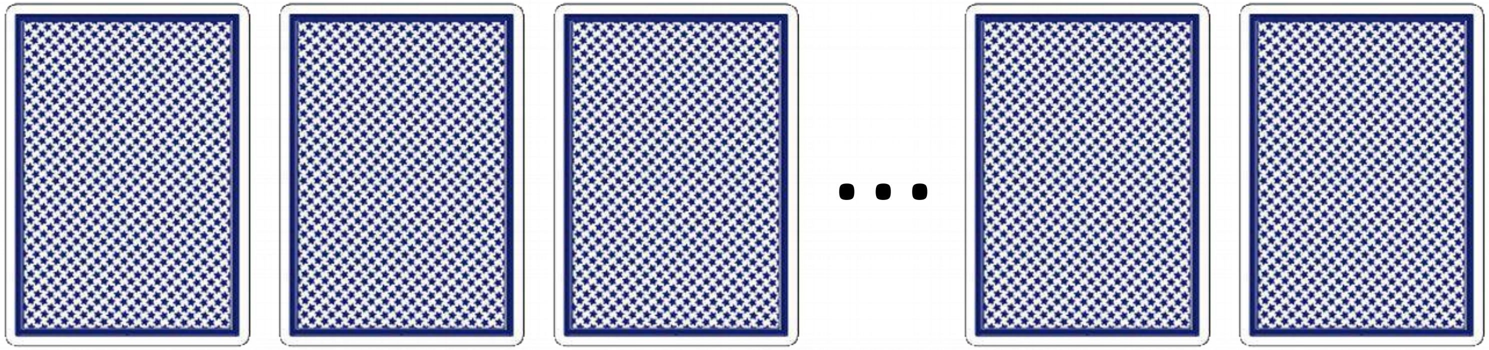
What's going on here?

- High-level (first-order) reasoning
- Symmetry
- Exchangeability

⇒ Lifted Inference

[Niepert, Van den Broeck; AAAI'14], [Van den Broeck; AAAI-KRR'15]



Let us automate this:

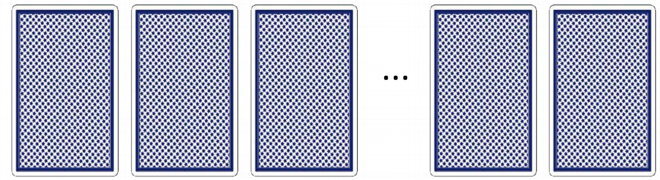
- **Relational** model

$$\forall p, \exists c, \mathrm{Card}(p,c)$$
$$\forall c, \exists p, \mathrm{Card}(p,c)$$
$$\forall p, \forall c, \forall c', \mathrm{Card}(p,c) \wedge \mathrm{Card}(p,c') \Rightarrow c = c'$$

- **Lifted** probabilistic inference algorithm

Playing Cards Revisited

Let us automate this:



$$\forall p, \exists c, \mathrm{Card}(p,c)$$
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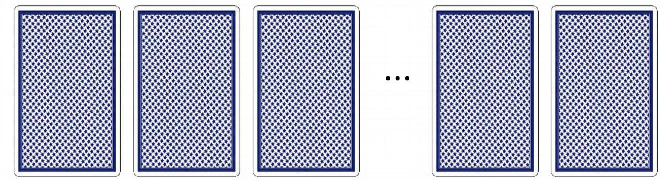
[Van den Broeck.; AAAI-KR'15]

Playing Cards Revisited

Let us automate this: 

∀p, ∃c, Card(p,c)
∀c, ∃p, Card(p,c)
∀p, ∀c, ∀c', Card(p,c) ∧ Card(p,c') ⇒ c = c'

$$\#SAT = \sum_{k=0}^{n} \binom{n}{k} \sum_{l=0}^{n} \binom{n}{l} (l+1)^{k} (-1)^{2n-k-l} = n!$$

[Van den Broeck.; AAAI-KR'15]

Playing Cards Revisited

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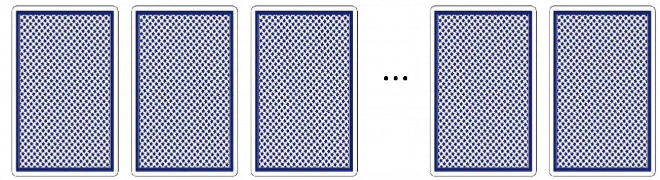

$\forall p, \exists c, \mathrm{Card}(p,c)$
$\forall c, \exists p, \mathrm{Card}(p,c)$
$\forall p, \forall c, \forall c', \mathrm{Card}(p,c) \wedge \mathrm{Card}(p,c') \Rightarrow c = c'$

$$\#SAT = \sum_{k=0}^{n} \binom{n}{k} \sum_{l=0}^{n} \binom{n}{l} (l+1)^k (-1)^{2n-k-l} = n!$$

Computed in time polynomial in n

[Van den Broeck.; AAAI-KR'15]

Model Counting

- Model = solution to a propositional logic formula Δ
- Model counting = #SAT

Δ = (Rain ⇒ Cloudy)

Rain	Cloudy	Model?
T	T	Yes
T	F	No
F	T	Yes
F	F	Yes

+ ______

#SAT = 3

[Valiant] #P-hard, even for 2CNF

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d)
⇒ Cloudy(d))

Days = {Monday}

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday}

Rain(M)	Cloudy(M)	Model?
T	T	Yes
T	F	No
F	T	Yes
F	F	Yes

+

FOMC = 3

First-Order Model Counting

Model = solution to first-order logic formula Δ

$\Delta = \forall d\ (\mathrm{Rain}(d) \Rightarrow \mathrm{Cloudy}(d))$

Days = {Monday **Tuesday**}

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday Tuesday}

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

+

FOMC = 9

FOMC Inference: Example 1

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3. $\Delta = \forall x, (Stress(x) \Rightarrow Smokes(x))$

Domain = {n people}

FOMC Inference: Example 1

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$

Domain = {n people}

→ 3^n models

FOMC Inference: Example 1

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$

Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (\text{ParentOf}(y) \wedge \text{Female} \Rightarrow \text{MotherOf}(y))$

D = {n people}

FOMC Inference: Example 1

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$

Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (\text{ParentOf}(y) \wedge \text{Female} \Rightarrow \text{MotherOf}(y))$

D = {n people}

If Female = true? $\Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y))$ $\rightarrow 3^n$ models

FOMC Inference: Example 1

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Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (\text{ParentOf}(y) \wedge \text{Female} \Rightarrow \text{MotherOf}(y))$

D = {n people}

If Female = true? $\Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y))$ $\rightarrow 3^n$ models

If Female = false? $\Delta = \text{true}$ $\rightarrow 4^n$ models

FOMC Inference: Example 1

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$

Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (\text{ParentOf}(y) \wedge \text{Female} \Rightarrow \text{MotherOf}(y))$

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If Female = true? $\Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y))$ $\rightarrow 3^n$ models

If Female = false? $\Delta = \text{true}$ $\rightarrow 4^n$ models

$\rightarrow 3^n + 4^n$ models

FOMC Inference: Example 1

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$ Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (\text{ParentOf}(y) \wedge \text{Female} \Rightarrow \text{MotherOf}(y))$ D = {n people}

If Female = true? $\Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y))$ $\rightarrow 3^n$ models

If Female = false? $\Delta = \text{true}$ $\rightarrow 4^n$ models

$\rightarrow 3^n + 4^n$ models

1. $\Delta = \forall x,y, (\text{ParentOf}(x,y) \wedge \text{Female}(x) \Rightarrow \text{MotherOf}(x,y))$ D = {n people}

FOMC Inference: Example 1

3. $\Delta = \forall x, (Stress(x) \Rightarrow Smokes(x))$ — Domain = {n people}

$\rightarrow 3^n$ models

2. $\Delta = \forall y, (ParentOf(y) \wedge Female \Rightarrow MotherOf(y))$ — D = {n people}

If Female = true? $\Delta = \forall y, (ParentOf(y) \Rightarrow MotherOf(y))$ $\rightarrow 3^n$ models

If Female = false? $\Delta = true$ $\rightarrow 4^n$ models

$\rightarrow 3^n + 4^n$ models

1. $\Delta = \forall x,y, (ParentOf(x,y) \wedge Female(x) \Rightarrow MotherOf(x,y))$ — D = {n people}

$\rightarrow (3^n + 4^n)^n$ models

FOMC Inference : Example 2

$\Delta = \forall x,y, (\text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y))$

Domain = {n people}

FOMC Inference : Example 2

$\Delta = \forall x,y, (\text{Smokes}(x) \wedge \text{Friends}(x,y) \Rightarrow \text{Smokes}(y))$

Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

Database:

Smokes(Alice) = 1
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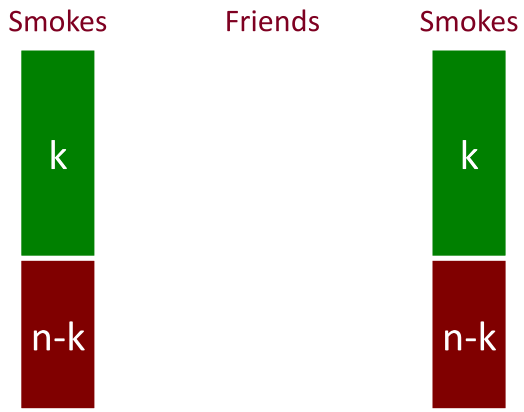
...



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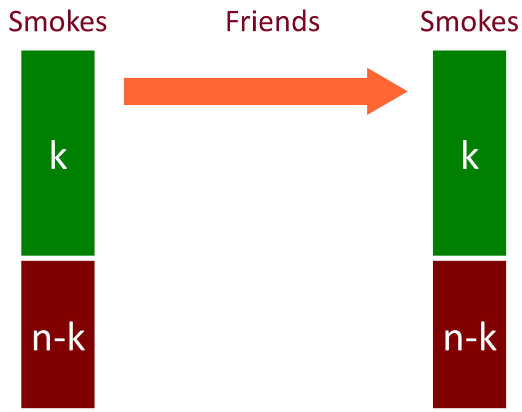

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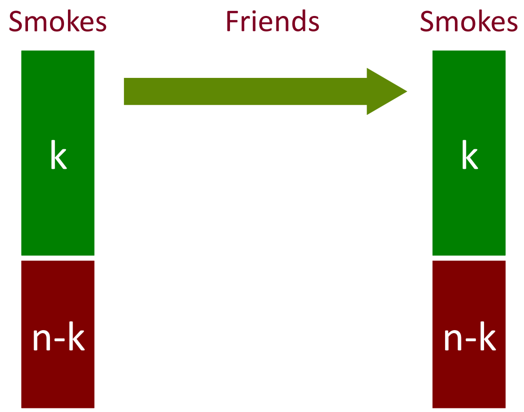

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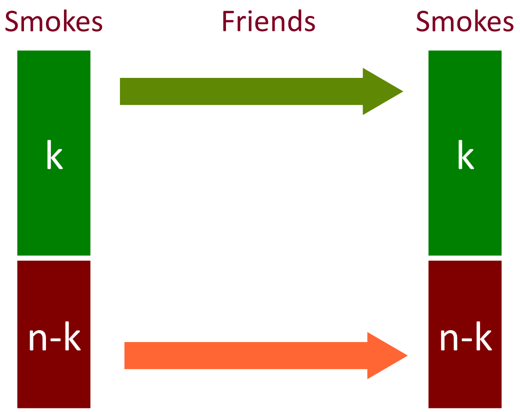

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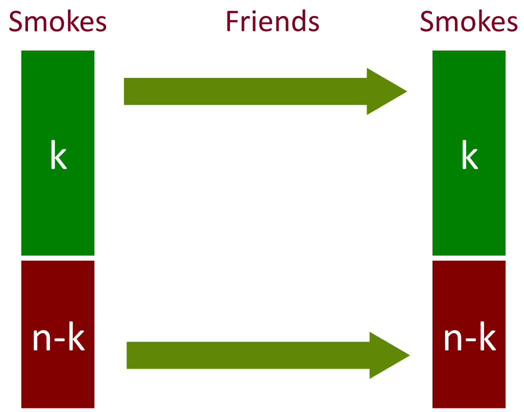

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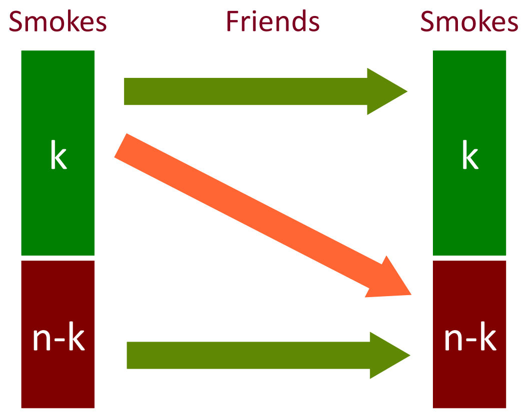

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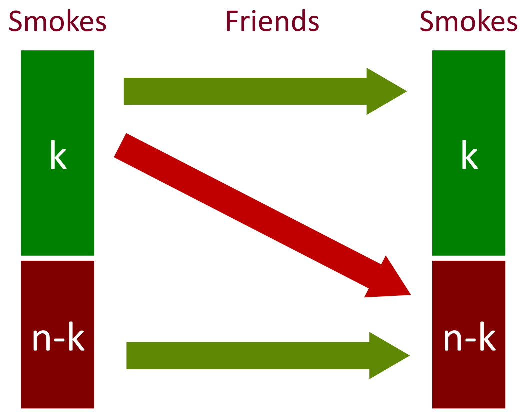

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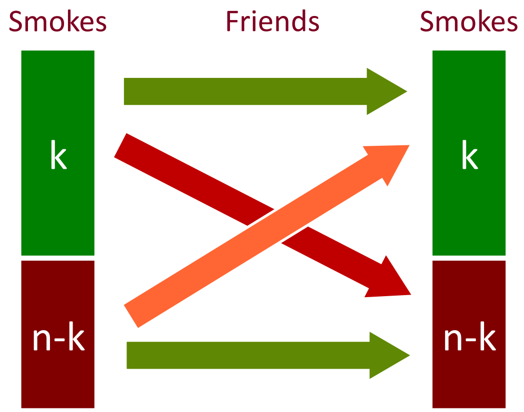

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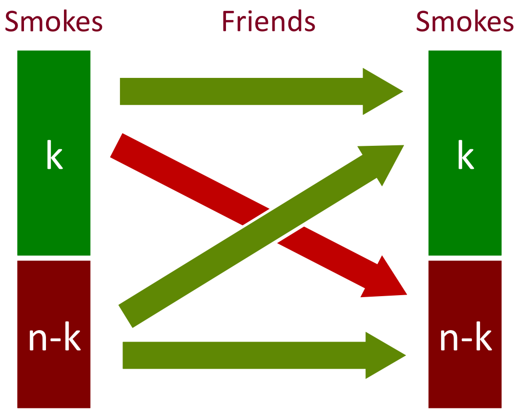

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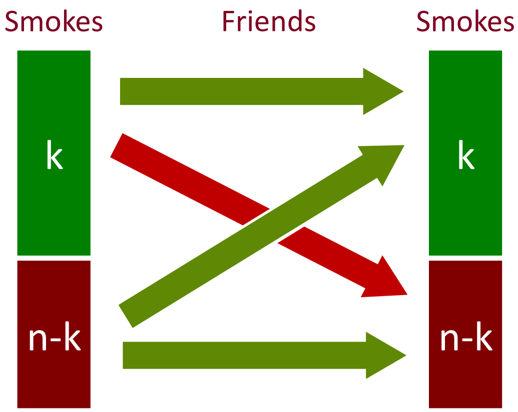
→ $2^{n^2-k(n-k)}$ models



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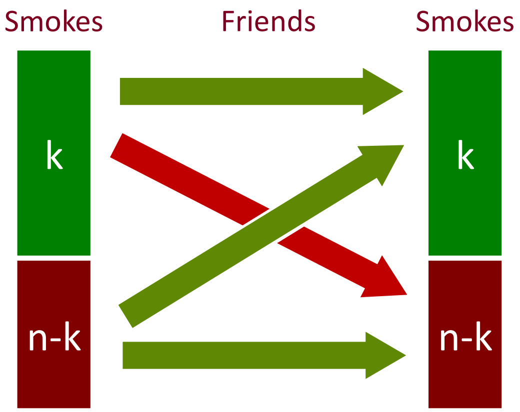

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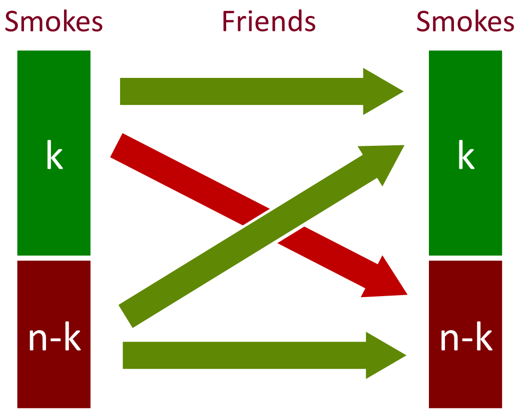


- If we know that there are *k* smokers?

→ $\binom{n}{k} 2^{n^2-k(n-k)}$ models

FOMC Inference : Example 2

Δ = ∀x,y, (Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y))

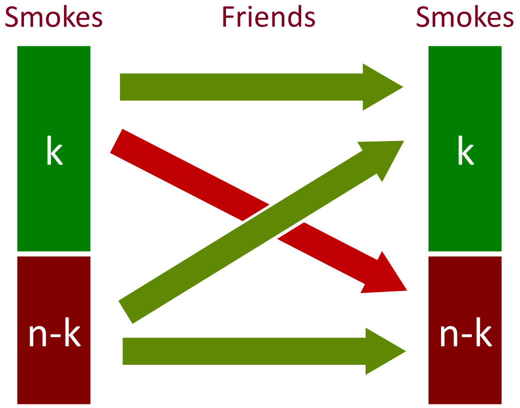
Domain = {n people}

- If we know precisely who smokes, and there are *k* smokers?

Database:

Smokes(Alice) = 1
Smokes(Bob) = 0
Smokes(Charlie) = 0
Smokes(Dave) = 1
Smokes(Eve) = 0
...

→ $2^{n^2 - k(n-k)}$ models



- If we know that there are *k* smokers? → $\binom{n}{k} 2^{n^2 - k(n-k)}$ models

- In total…

FOMC Inference : Example 2

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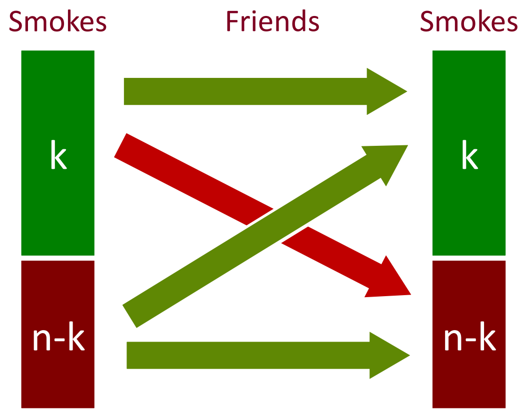

- If we know that there are *k* smokers? → $\binom{n}{k} 2^{n^2-k(n-k)}$ models

- In total... → $\sum_{k=0}^{n} \binom{n}{k} 2^{n^2-k(n-k)}$ models

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Statistical Relational Models

Hard constraint

∞ Smoker(x) ⇒ Person(x)

Soft constraint

3.75 Smoker(x) ∧ Friend(x,y) ⇒ Smoker(y)

- An MLN = set of constraints (w, $\Gamma(\mathbf{x})$)
- **Weight of a world** = product of w, for all rules (w, $\Gamma(\mathbf{x})$) and groundings $\Gamma(\mathbf{a})$ that hold in the world

$P_{MLN}(Q)$ = [sum of weights of models of Q] / Z

Applications: large KBs, e.g. DeepDive

Weighted Model Counting

- Model = solution to a propositional logic formula Δ
- Model counting = #SAT

Δ = (Rain ⇒ Cloudy)

Rain	Cloudy	Model?
T	T	Yes
T	F	No
F	T	Yes
F	F	Yes

+

#SAT = 3

Weighted Model Counting

- Model = solution to a propositional logic formula Δ
- Model counting = #SAT
- Weighted model counting (WMC)
  - Weights for assignments to variables
  - Model weight is product of variable weights w(.)

Δ = (Rain ⇒ Cloudy)

w(R)=1
w(¬R)=2
w(C)=3
w(¬C)=5

Rain	Cloudy	Model?	Weight
T	T	Yes	1 * 3 = 3
T	F	No	0
F	T	Yes	2 * 3 = 6
F	F	Yes	2 * 5 = 10

+ ———

#SAT = 3

Weighted Model Counting

- Model = solution to a propositional logic formula Δ
- Model counting = #SAT
- Weighted model counting (WMC)
  - Weights for assignments to variables
  - Model weight is product of variable weights w(.)

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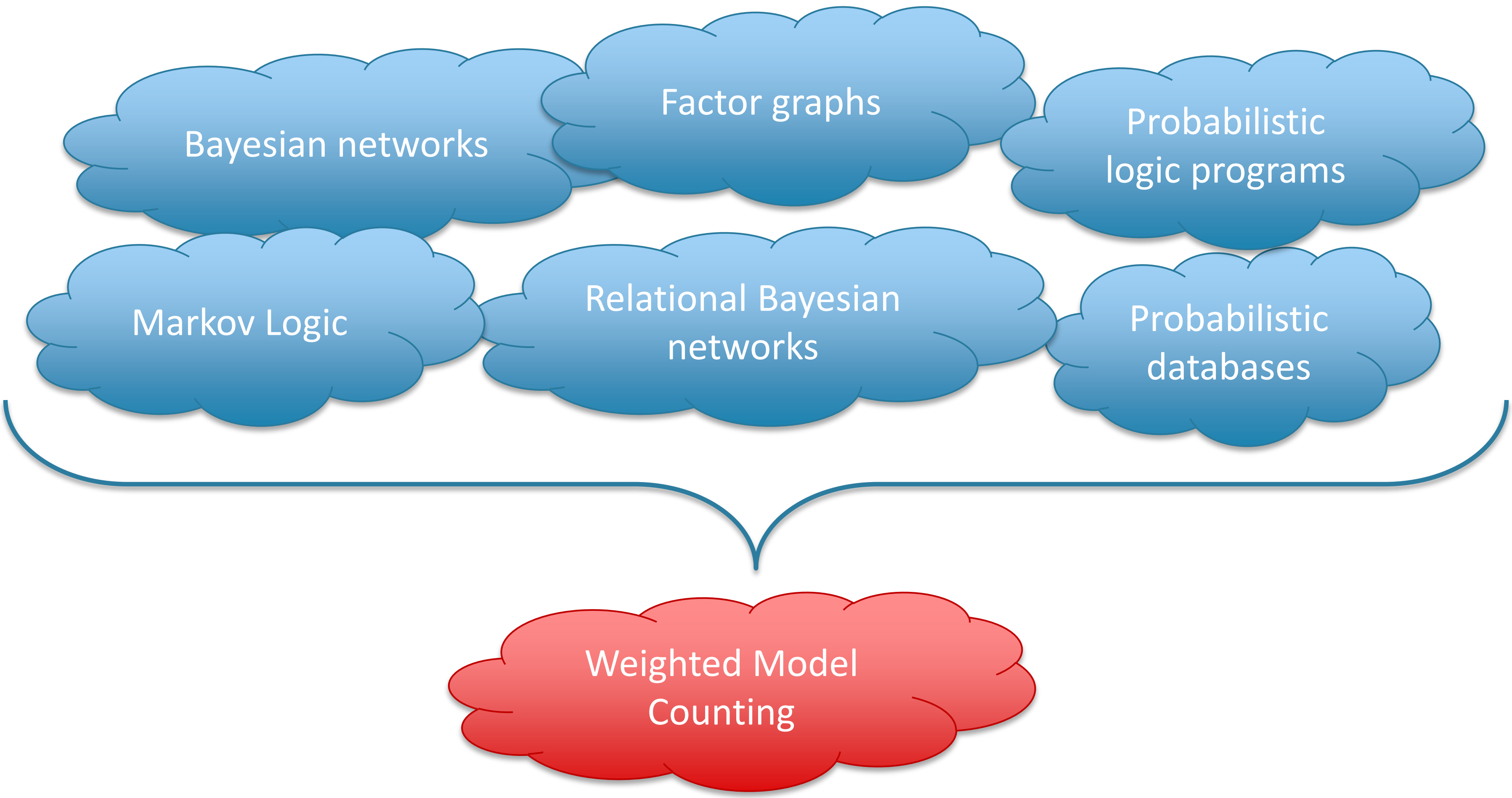
Rain	Cloudy	Model?	Weight
T	T	Yes	1 * 3 = 3
T	F	No	0
F	T	Yes	2 * 3 = 6
F	F	Yes	2 * 5 = 10
		+	+
		#SAT = 3	**WMC = 19**

Assembly language for probabilistic reasoning and learning



Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday **Tuesday**}

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday, Tuesday}

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?
T	T	T	T	Yes
T	F	T	T	No
F	T	T	T	Yes
F	F	T	T	Yes
T	T	T	F	No
T	F	T	F	No
F	T	T	F	No
F	F	T	F	No
T	T	F	T	Yes
T	F	F	T	No
F	T	F	T	Yes
F	F	F	T	Yes
T	T	F	F	Yes
T	F	F	F	No
F	T	F	F	Yes
F	F	F	F	Yes

+ ______

#SAT = 9

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

Δ = ∀d (Rain(d) ⇒ Cloudy(d))

Days = {Monday **Tuesday**}

w(R)=1
w(¬R)=2
w(C)=3
w(¬C)=5

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?	Weight
T	T	T	T	Yes	1 * 1 * 3 * 3 = 9
T	F	T	T	No	0
F	T	T	T	Yes	2 * 1* 3 * 3 = 18
F	F	T	T	Yes	2 * 1 * 5 * 3 = 30
T	T	T	F	No	0
T	F	T	F	No	0
F	T	T	F	No	0
F	F	T	F	No	0
T	T	F	T	Yes	1 * 2 * 3 * 3 = 18
T	F	F	T	No	0
F	T	F	T	Yes	2 * 2 * 3 * 3 = 36
F	F	F	T	Yes	2 * 2 * 5 * 3 = 60
T	T	F	F	Yes	1 * 2 * 3 * 5 = 30
T	F	F	F	No	0
F	T	F	F	Yes	2 * 2 * 3 * 5 = 60
F	F	F	F	Yes	2 * 2 * 5 * 5 = 100

+

#SAT = 9

Weighted First-Order Model Counting

Model = solution to first-order logic formula Δ

$\Delta = \forall d\ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

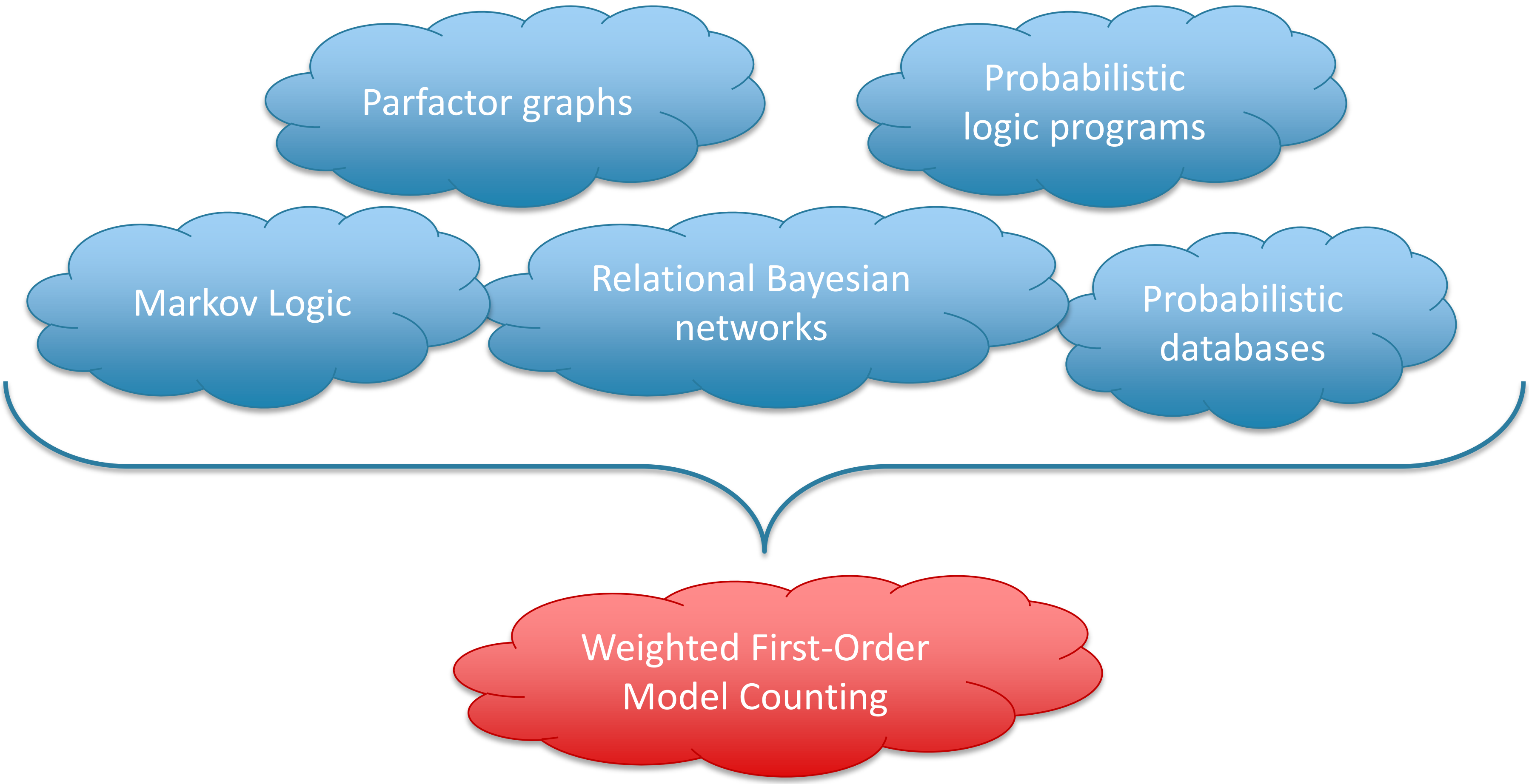
Days = {Monday **Tuesday**}

w(R)=1
w(¬R)=2
w(C)=3
w(¬C)=5

Rain(M)	Cloudy(M)	Rain(T)	Cloudy(T)	Model?	Weight
T	T	T	T	Yes	1 * 1 * 3 * 3 = 9
T	F	T	T	No	0
F	T	T	T	Yes	2 * 1* 3 * 3 = 18
F	F	T	T	Yes	2 * 1 * 5 * 3 = 30
T	T	T	F	No	0
T	F	T	F	No	0
F	T	T	F	No	0
F	F	T	F	No	0
T	T	F	T	Yes	1 * 2 * 3 * 3 = 18
T	F	F	T	No	0
F	T	F	T	Yes	2 * 2 * 3 * 3 = 36
F	F	F	T	Yes	2 * 2 * 5 * 3 = 60
T	T	F	F	Yes	1 * 2 * 3 * 5 = 30
T	F	F	F	No	0
F	T	F	F	Yes	2 * 2 * 3 * 5 = 60
F	F	F	F	Yes	2 * 2 * 5 * 5 = 100

#SAT = 9 **WFOMC = 361**

Assembly language for high-level probabilistic reasoning and learning



[VdB et al.; IJCAI'11, PhD'13, KR'14, UAI'14]

Symmetric WFOMC

Def. A weighted vocabulary is (**R**, **w**), where

- **R** = (R_1, R_2, …, R_k) = relational vocabulary
- **w** = (w_1, w_2, …, w_k) = weights

- Fix an FO formula Q, domain of size n
- The weight of a ground tuple t in R_i is w_i

This talk: complexity of FOMC / WFOMC(Q, n)

- Data complexity: fixed Q, input n / and **w**
- Combined complexity: input (Q, n) / and **w**

Example

$Q = \forall x \exists y\, R(x,y)$

$FOMC(Q,n) = (2^n-1)^n$ $WOMC(Q,n,w_R) = ((1+w_R)^n-1)^n$

Computable in PTIME in n

Example

$Q = \forall x \exists y\ R(x,y)$

$\text{FOMC}(Q,n) = (2^n-1)^n$ $\text{WOMC}(Q,n,w_R) = ((1+w_R)^n-1)^n$

$Q = \exists x \exists y\ [R(x) \wedge S(x,y) \wedge T(y)]$

$$\text{FOMC}(Q,n) = \sum_{i=0,n} \sum_{j=0,n} \binom{n}{i} \binom{n}{j} 2^{(n-i)(n-j)} \left(2^{ij} - 1\right)$$

Computable in PTIME in n

Example

$Q = \forall x \exists y\ R(x,y)$

$\text{FOMC}(Q,n) = (2^n-1)^n$ $\quad$ $\text{WOMC}(Q,n,w_R) = ((1+w_R)^n-1)^n$

$Q = \exists x \exists y\ [R(x) \wedge S(x,y) \wedge T(y)]$

$$\text{FOMC}(Q,n) = \sum_{i=0,n}\sum_{j=0,n}\binom{n}{i}\binom{n}{j}2^{(n-i)(n-j)}\left(2^{ij}-1\right)$$

$$\text{WFOMC}(Q,n,w_R,w_S,w_T) =$$

$$\sum_{i=0,n}\sum_{j=0,n}\binom{n}{i}\binom{n}{j}{w_R}^i{w_T}^j(1+w_S)^{(n-i)(n-j)}\left((1+w_S)^{ij}-1\right)$$

Computable in PTIME in n

Example

$Q = \exists x \exists y \exists z\ [R(x,y) \wedge S(y,z) \wedge T(z,x)]$

Can we compute FOMC(Q, n) in PTIME?

Open problem…

Conjecture FOMC(Q, n) not computable in PTIME in n

[Van den Broeck'2011, Gogate'2011]

From MLN to WFOMC

MLN:

∞	$\text{Smoker}(x) \Rightarrow \text{Person}(x)$
w	$\sim\text{Smoker}(x) \lor \sim\text{Friend}(x,y) \lor \text{Smoker}(y)$

→ MLN':

∞	$\text{Smoker}(x) \Rightarrow \text{Person}(x)$
∞	$R(x,y) \Leftrightarrow \sim\text{Smoker}(x) \lor \sim\text{Friend}(x,y) \lor \text{Smoker}(y)$
w	$R(x,y)$

Theorem $P_{MLN}(Q) = P(Q \mid \text{hard constraints in } MLN')$
$= WFOMC(Q \land MLN') / WFOMC(MLN')$

R is a symmetric relation

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Probabilistic Databases

- Weights or probabilities given explicitly, for each tuple
- Examples: Knowledge Vault, Nell, Yago
- Dichotomy theorem:
  for any query in UCQ/FO($\exists$,$\wedge$,$\vee$) (or FO($\forall$,$\wedge$,$\vee$), asymmetric WFOMC is in PTIME or #P-hard.

Motivation 2: Probabilistic Databases

Probabilistic database D:

x	y	P
a1	b1	p_1
a1	b2	p_2
a2	b2	p_3

Motivation 2: Probabilistic Databases

Probabilistic database D:

x	y	P
a1	b1	p_1
a1	b2	p_2
a2	b2	p_3

Possible worlds semantics:

x	y
a1	b1
a1	b2
a2	b2

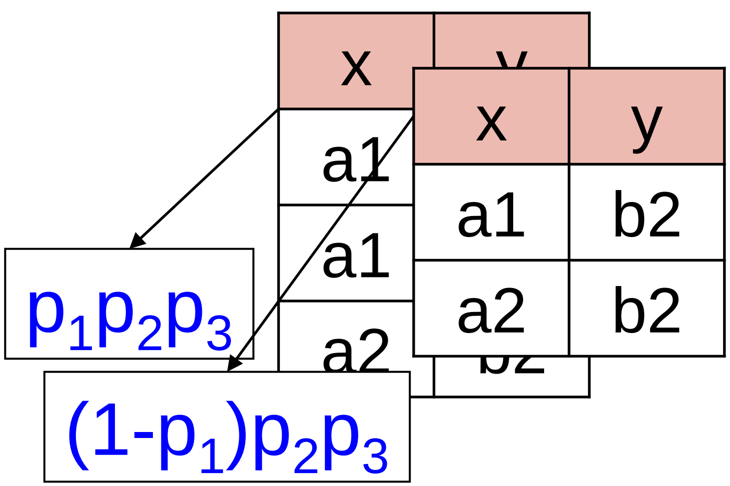
$p_1 p_2 p_3$

Motivation 2: Probabilistic Databases

Probabilistic database D:

x	y	P
a1	b1	p_1
a1	b2	p_2
a2	b2	p_3

Possible worlds semantics:

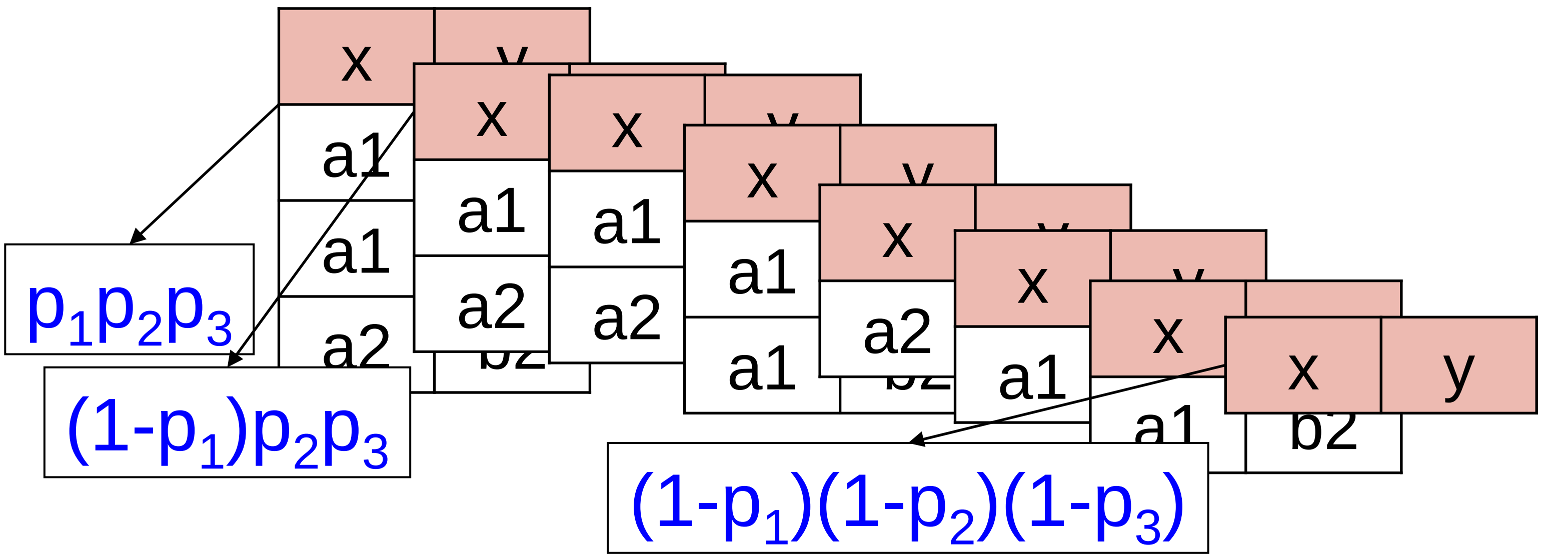


Motivation 2: Probabilistic Databases

Probabilistic database D:

x	y	P
a1	b1	p_1
a1	b2	p_2
a2	b2	p_3

Possible worlds semantics:



$Q = \exists x \exists y\ R(x) \wedge S(x,y)$

An Example

$P(Q) =$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\ R(x) \wedge S(x,y)$

An Example

$P(Q) =$ $1-(1-q_1)*(1-q_2)$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\ R(x) \wedge S(x,y)$

An Example

$$P(Q) = p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]$$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\, R(x) \wedge S(x,y)$

An Example

$$P(Q) = p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]$$

$$1-(1-q_3)*(1-q_4)*(1-q_5)$$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\ R(x) \wedge S(x,y)$

An Example

$$P(Q) = \begin{aligned} & p_1 * [\ 1-(1-q_1)*(1-q_2)\ ] \\ & p_2 * [\ 1-(1-q_3)*(1-q_4)*(1-q_5)\ ] \end{aligned}$$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\ R(x) \wedge S(x,y)$

An Example

$$P(Q) = 1 - \{1 - p_1 * [\ 1 - (1-q_1)*(1-q_2)\ ]\} *$$
$$\{1 - p_2 * [\ 1 - (1-q_3)*(1-q_4)*(1-q_5)\ ]\}$$

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\, R(x) \wedge S(x,y)$

An Example

$$P(Q) = 1 - \{1 - p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]\} *$$
$$\{1 - p_2 * [\ 1-(1-q_3)*(1-q_4)*(1-q_5)\ ]\}$$

One can compute P(Q) in PTIME in the size of the database D

R

x	P
a_1	p_1
a_2	p_2
a_3	p_3

S

x	y	P
a_1	b_1	q_1
a_1	b_2	q_2
a_2	b_3	q_3
a_2	b_4	q_4
a_2	b_5	q_5

$Q = \exists x \exists y\, R(x) \wedge S(x,y)$

An Example

$$P(Q) = 1 - \{1 - p_1 * [\ 1-(1-q_1)*(1-q_2)\ ]\} * \{1 - p_2 * [\ 1-(1-q_3)*(1-q_4)*(1-q_5)\ ]\}$$

One can compute P(Q) in PTIME in the size of the database D

R

x	P	
a_1	p_1	X_1
a_2	p_2	X_2
a_3	p_3	X_3

S

x	y	P	
a_1	b_1	q_1	Y_1
a_1	b_2	q_2	Y_2
a_2	b_3	q_3	Y_3
a_2	b_4	q_4	Y_4
a_2	b_5	q_5	Y_5

Probabilistic Database Inference

Preprocess Q (omitted from this talk; see book [S.'2011])

- $P(Q1 \wedge Q2) = P(Q1)P(Q2)$
  $P(Q1 \vee Q2) = 1 - (1 - P(Q1))(1 - P(Q2))$

Independent join / union

- $P(\exists z\, Q) = 1 - \prod_{a \in Domain} (1 - P(Q[a/z]))$
  $P(\forall z\, Q) = \prod_{a \in Domain} P(Q[a/z])$

Independent project

- $P(Q1 \wedge Q2) = P(Q1) + P(Q2) - P(Q1 \vee Q2)$
  $P(Q1 \vee Q2) = P(Q1) + P(Q2) - P(Q1 \wedge Q2)$

Inclusion/ exclusion

If rules succeed, WFOMC(Q,n) in PTIME; else, #P-hard

#P-hardness no longer holds for symmetric WFOMC

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Motivation: 0/1 Laws

Definition. $\mu_n(Q)$ = fraction of all structures over a domain of size n that are models of Q

$\mu_n(Q) = \text{FOMC}(Q, n) \; / \; \text{FOMC}(\text{TRUE}, n)$

Theorem.

For every Q in FO, $\lim_{n \to \infty} \mu_n(Q) = 0$ or 1

Example: $Q = \forall x \exists y\, R(x,y)$;

$\text{FOMC}(Q,n) = (2^n-1)^n$

$\mu_n(Q) = (2^n-1)^n / 2^{n\char`^2} \to 1$

Motivation: 0/1 Laws

In 1976 Fagin proved the 0/1 law for FO using a transfer theorem.

But is there an elementary proof? Find explicit formula for $\mu_n(Q)$, then compute the limit. [Fagin communicated to us that he tried this first]

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Class FO^2

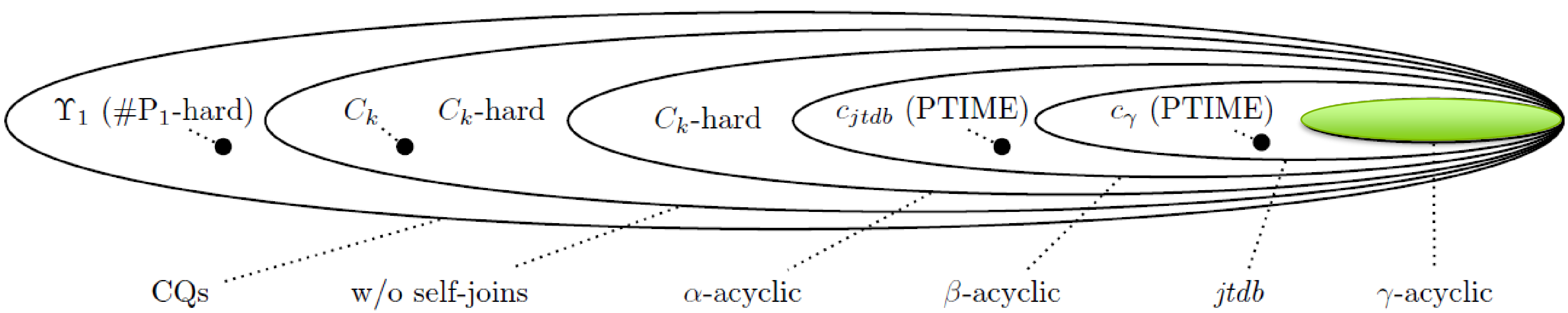
- FO^2 = FO restricted to two variables
- Intuition: SQL queries that have a plan where all temp tables have arity ≤ 2
- "The graph has a path of length 10":

$$\exists x \exists y (R(x,y) \wedge \exists x (R(y,x) \wedge \exists y (R(x,y) \wedge \ldots)))$$

Main Positive Results

Data complexity:

- for any formula Q in FO^2, WFOMC(Q, n) is in PTIME [see NIPS'11, KR'13]
- for any γ-acyclic conjunctive query w/o self-joins Q, WFOMC(Q, n) is in PTIME



Main Negative Results

Data complexity:

- There exists an FO formula Q s.t. symmetric FOMC(Q, n) is $\#P_1$ hard
- There exists Q in FO^3 s.t. FOMC(Q, n) is $\#P_1$ hard
- There exists a conjunctive query Q s.t. symmetric WFOMC(Q, n) is $\#P_1$ hard
- There exists a positive clause Q w.o. '=' s.t. symmetric WFOMC(Q, n) is $\#P_1$ hard

Combined complexity:

- FOMC(Q, n) is #P-hard

Review: $\#P_1$

- $\#P_1$ = class of functions in #P over a unary input alphabet
- Valiant 1979: there exists $\#P_1$ complete problems
- Bertoni, Goldwurm, Sabatini 1988:
  counting strings of a given length in some CFG is $\#P_1$ complete
- Goldberg: “no natural combinatorial problems known to be $\#P_1$ complete”

Main Result 1

Theorem 1. There exists an FO^3 sentence Q s.t. $FOMC(Q,n)$ is $\#P_1$-hard

Proof

- Step 1. Construct a Turing Machine U s.t.
  - U is in $\#P_1$ and runs in linear time in n
  - U computes a $\#P_1$ –hard function
- Step 2. Construct an FO^3 sentence Q s.t. $FOMC(Q,n) / n! = U(n)$

Main Result 2

Theorem 2 There exists a Conjunctive Query Q s.t. WFOMC(Q,n) is $\#P_1$-hard

- Note: the decision problem is trivial (Q has a model iff $n > 0$)
- *Unweighted* Model Counting for CQ: open

Proof Start with a formula Q that is $\#P_1$-hard for FOMC, and transform it to a CQ in five steps (next)

Start: Q s.t. FOMC(Q, n) is $\#P_1$-hard

Step 1: Remove ∃

Rewrite $Q = \forall x \exists y\ \psi(x,y)$

to $Q' = \forall x\ \forall y\ (\neg\psi(x,y) \lor \neg A(x))$

where A = new symbol with weight $w = -1$

Claim: WFOMC(Q, n) = WFOMC(Q', n)

Proof Consider a model for Q', and a constant $x=a$

- If $\exists b\ \psi(a,b)$, then $A(a)$=false; contributes w=1
- Otherwise, $A(a)$ can be either true or false, contributing either w=1 or w=-1, and 1 – 1 = 0.

$Q = \forall^* \ldots$, WFOMC(Q, n) is $\#P_1$-hard

Start: Q s.t. FOMC(Q, n) is $\#P_1$-hard

Step 2: Remove Negation

- Transform Q to Q' w/o negation s.t.
  WFOMC(Q, n) = WFOMC(Q', n)

- Similarly to step 1 and omitted

Q = ∀*[positive], WFOMC(Q, n) is $\#P_1$-hard

Start: Q s.t. FOMC(Q, n) is $\#P_1$-hard

Step 3: Remove "="

Rewrite Q to Q' as follows:

- Add new binary symbol E with weight w
- Define: $Q' = Q[\, E \,/\, \text{"="} \,] \wedge (\forall x\, E(x,x))$

Claim: WFOMC(Q,n) computable using oracle for WFOMC(Q', n)
(coefficient of w^n in polynomial WFOMC(Q', n)

Q = $\forall$*[positive, w/o =], WFOMC(Q, n) is $\#P_1$-hard

Start: Q s.t. FOMC(Q, n) is #P_1-hard

Step 4: To UCQ

- Write $Q = \forall^* (C_1 \wedge C_2 \wedge \ldots)$ where each C_i is a positive clause

- The dual $Q' = \exists^* (C_1' \vee C_2' \vee \ldots)$ is a UCQ

UCQ Q, WFOMC(Q, n) is #P_1-hard

Start: Q s.t. FOMC(Q, n) is #P_1-hard

Step 5: from UCQ to CQ

- UCQ: $Q = C_1 \vee C_2 \vee \ldots \vee C_k$
- $P(Q) = \ldots. + (-1)^S P(\wedge_{i \in S} C_i) + \ldots.$
- 2^k-1 CQs $P(Q_1), P(Q_2), \ldots P(Q_{2^\wedge k-1})$
- 1 CQ (using fresh copies of symbols): $P(Q'_1 Q'_2 \ldots Q'_{2^\wedge k-1}) = P(Q'_1) P(Q'_2) \ldots P(Q'_{2^\wedge k-1})$

CQ Q' $(=Q'_1 Q'_2 \ldots Q'_{2^\wedge k-1})$ WFOMC(Q', n) is #P_1-hard

Overview

- Motivation and convergence of
  - The artificial intelligence story (*recap*)
  - The machine learning story (*recap*)
  - The probabilistic database story
  - The database theory story
- Main theoretical results and proof outlines
- **Discussion and conclusions**
- Dessert

Motivation: 0/1 Laws

In 1976 Fagin proved the 0/1 law for FO using a transfer theorem.

But is there an elementary proof? Find explicit formula for $\mu_n(Q)$, then compute the limit. [Fagin communicated to us that he tried this first]

Motivation: 0/1 Laws

In 1976 Fagin proved the 0/1 law for FO using a transfer theorem.

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A: unlikely when FOMC(Q,n) is $\#P_1$-hard

Discussion

Fagin (1974) restated:

1. NP = ∃SO
   (Fagin's classical characterization of NP)
2. $NP_1 = \{Spec(\Phi) \mid \Phi \in FO\}$ in tally notation
   (less well known!)

We show: $\#P_1$ corresponds to $\{FOMC(Q,n) \mid Q \text{ in FO}\}$

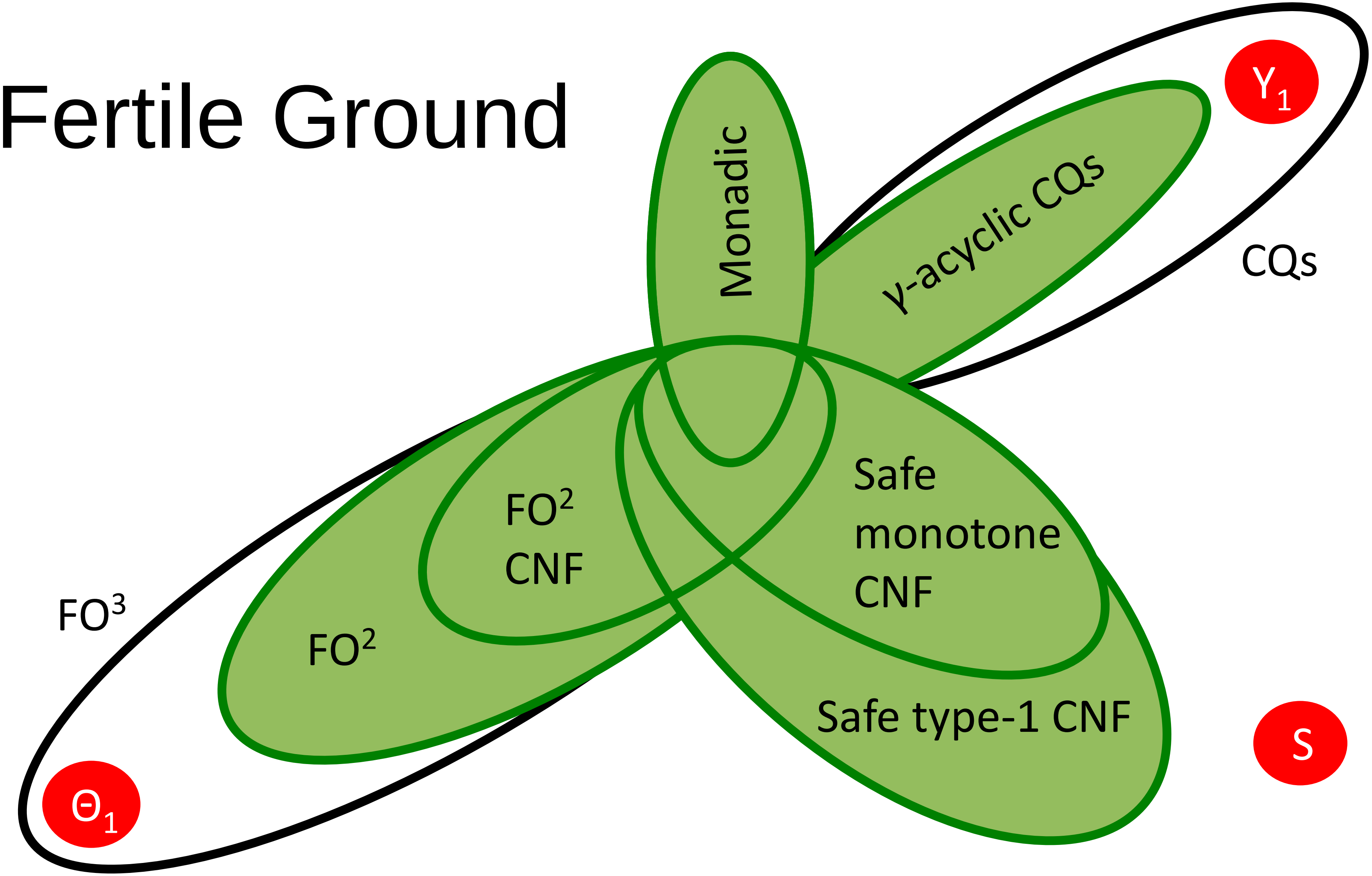
Discussion

- Convergence of AI/ML/DB/theory
- First-order model counting is a basic problem that touches all these areas
- Under-investigated
- Hardness proofs are more difficult than for #P

Open problems:

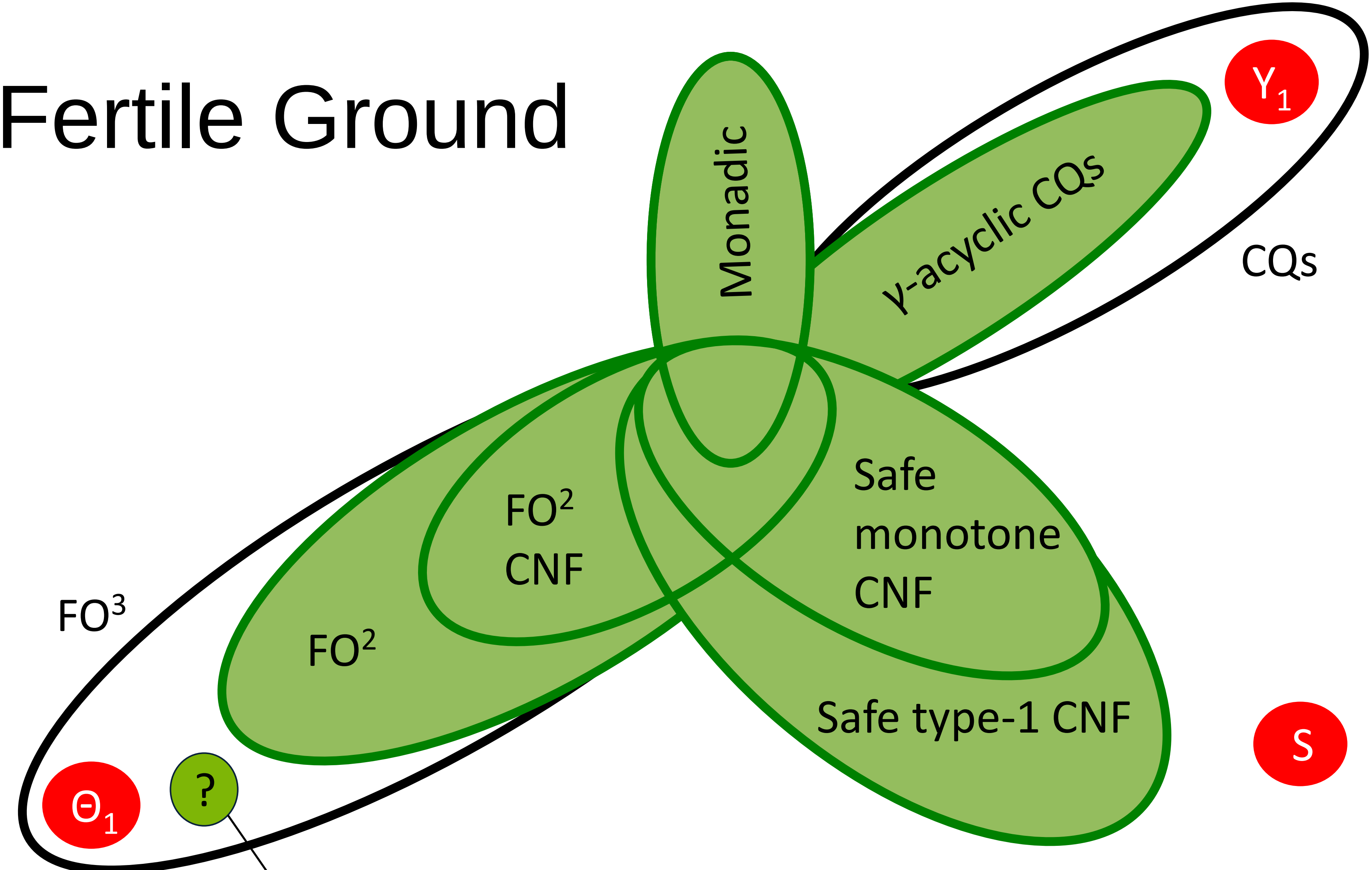
- New algorithm for symmetric model counting
- New hardness reduction techniques

Fertile Ground



[VdB; NIPS’11], [VdB et al.; KR’14], [Gribkoff, VdB, Suciu; UAI’15], [Beame, VdB, Gribkoff, Suciu; PODS’15], etc.

Fertile Ground



$\Delta = \forall x,y,z,\ Friends(x,y) \wedge Friends(y,z) \Rightarrow Friends(x,z)$

[VdB; NIPS'11], [VdB et al.; KR'14], [Gribkoff, VdB, Suciu; UAI'15], [Beame, VdB, Gribkoff, Suciu; PODS'15], etc.

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The Decision Problem

- Counting problem
  "count the number of XXX s.t…"

- Decision problem
  "does there exists an XXX s.t. …?"

- #3SAT and 3SAT:
  – counting is #P-complete, decision is NP-hard
- #2SAT and 2SAT:
  – counting is #P-hard, decision is in PTIME

Counting/Decision Problems for FO

- **Counting**: given Q,n, count the number of models of Q over a domain of size n

- **Decision**: given Q,n, does there exists a model of Q over a domain of size n?

- **Data complexity**: fix Q, input = n
- **Combined complexity**: input = Q, n

The Spectrum

Definition. [Scholz 1952]
$\mathrm{Spec}(Q) = \{n \mid Q$ has a model over domain $[n]\}$

Example: Q says "(D, +, *, 0, 1) is a field":

$$\mathrm{Spec}(Q) = \{p^k \mid p \text{ prime}, k \geq 1\}$$

Spectra studied intensively for over 50 years

The FO decision problem is precisely spectrum membership

The Data Complexity

Suppose n is given in binary representation:

- Jones&Selman'72: spectra = NETIME

$$\mathsf{NETIME} = \bigcup_{c\geq 0} \mathsf{NTIME}(2^{cn}) \qquad \mathsf{NEXPTIME} = \bigcup_{c\geq 0} \mathsf{NTIME}(2^{c^n})$$

Suppose n is given in unary representation:

- Fagin'74: spectra = NP_1

Combined Complexity

Consider the combined complexity for FO^2
"given Q, n, check if n ∈ Spec(Q)"

We prove its complexity:

- NP-complete for FO^2,
- PSPACE-complete for FO

Thanks!