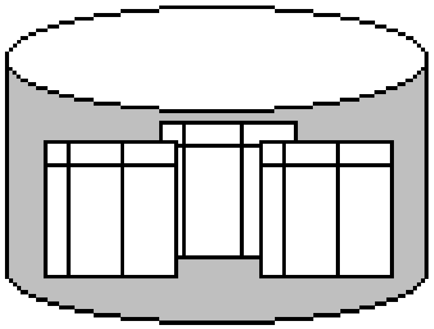
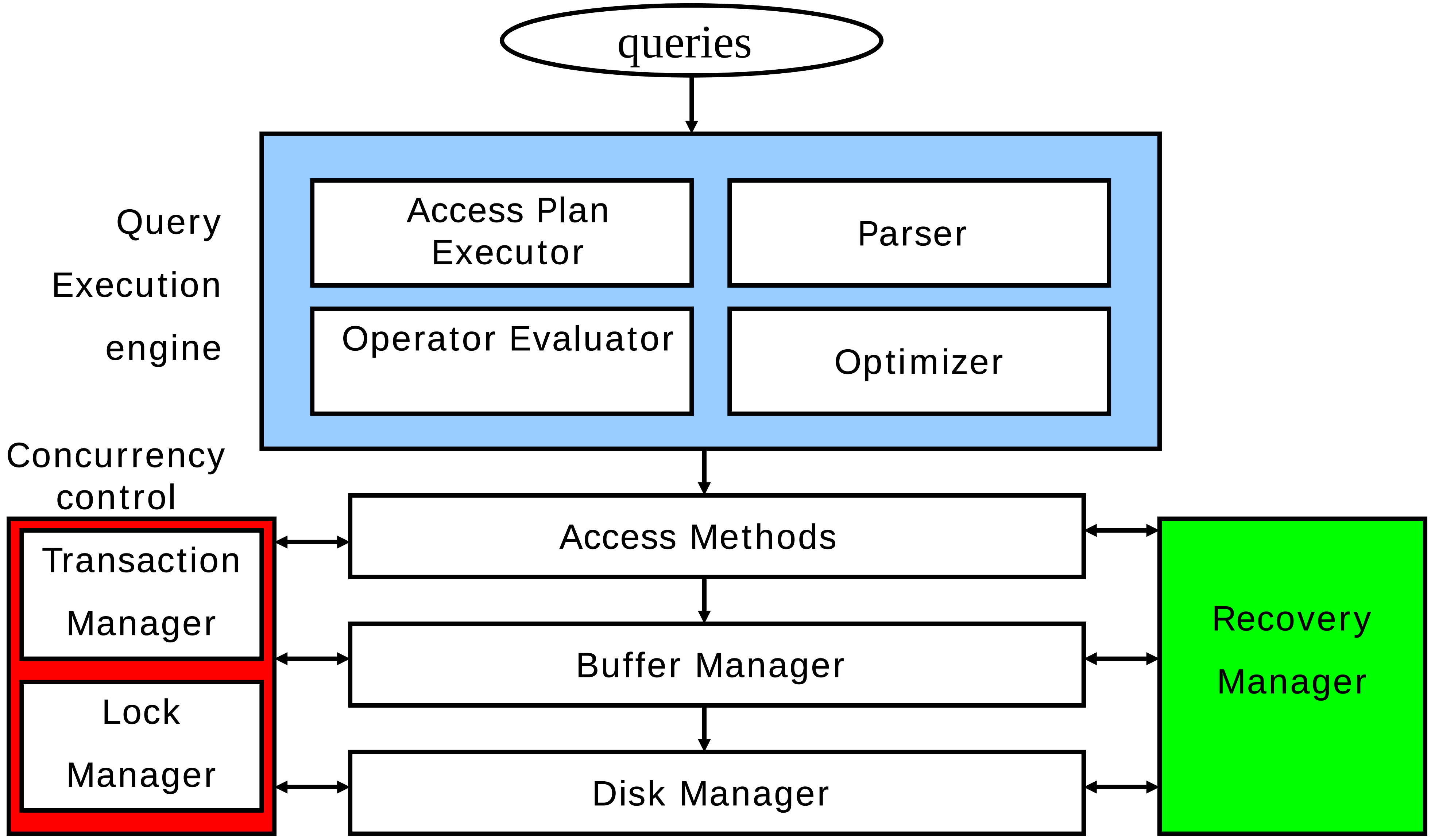
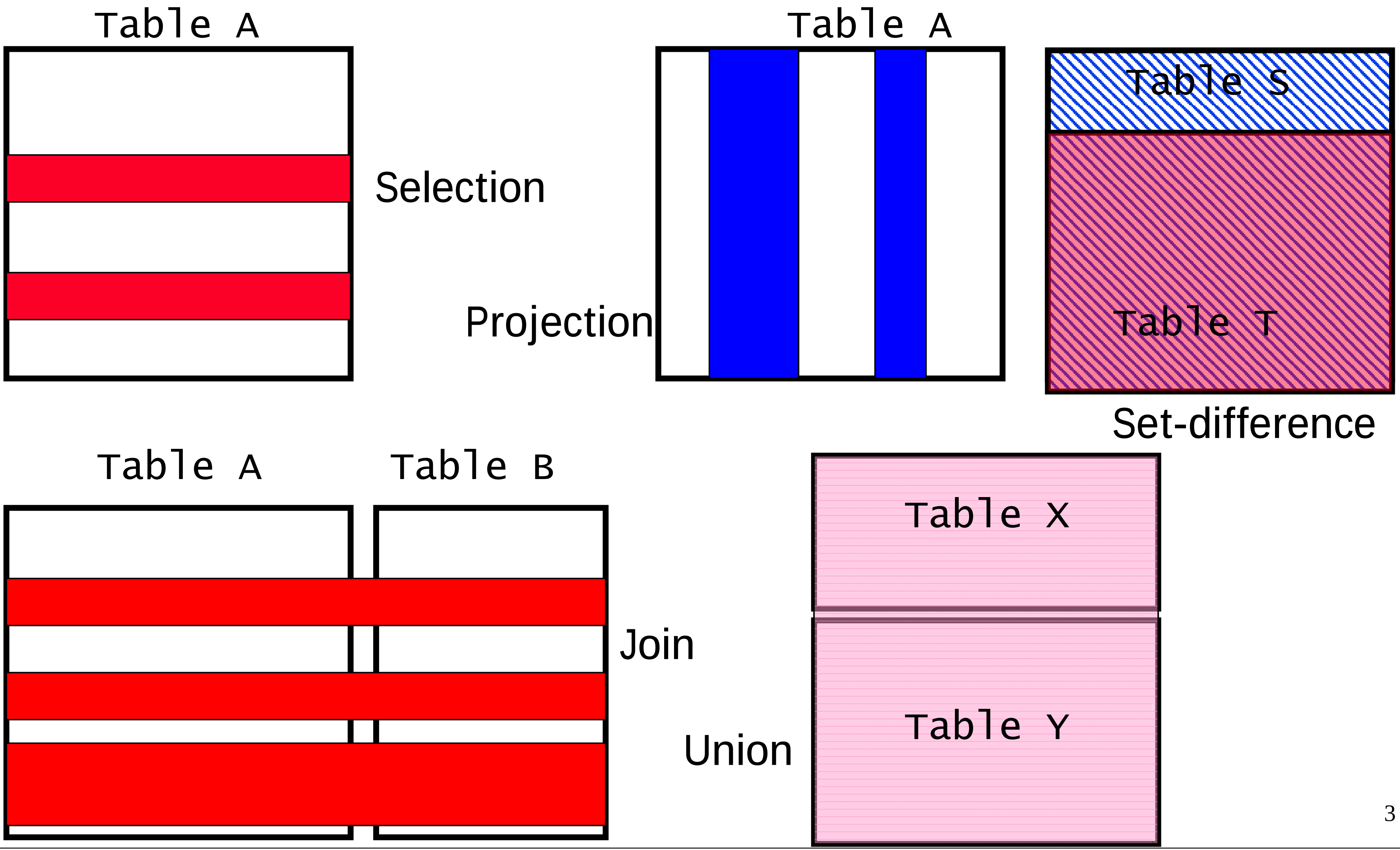
TUTORIAL ON QUERY OPTIMIZATION



DB Logical Architecture



Relational Ops



Measures of Query Cost

- Cost is generally measured as total elapsed time for answering query
  - ♦Many factors contribute to time cost: disk accesses, CPU, or even network communication
- Typically disk access is the predominant cost, and is also relatively easy to estimate
  - ♦Measured by taking into account
    - Number of blocks read * average-block-read-cost
    - Number of blocks written * average-block-write-cost
  - ♦Cost to write a block is greater than cost to read a block
    - data is read back after being written to ensure that the write was successful

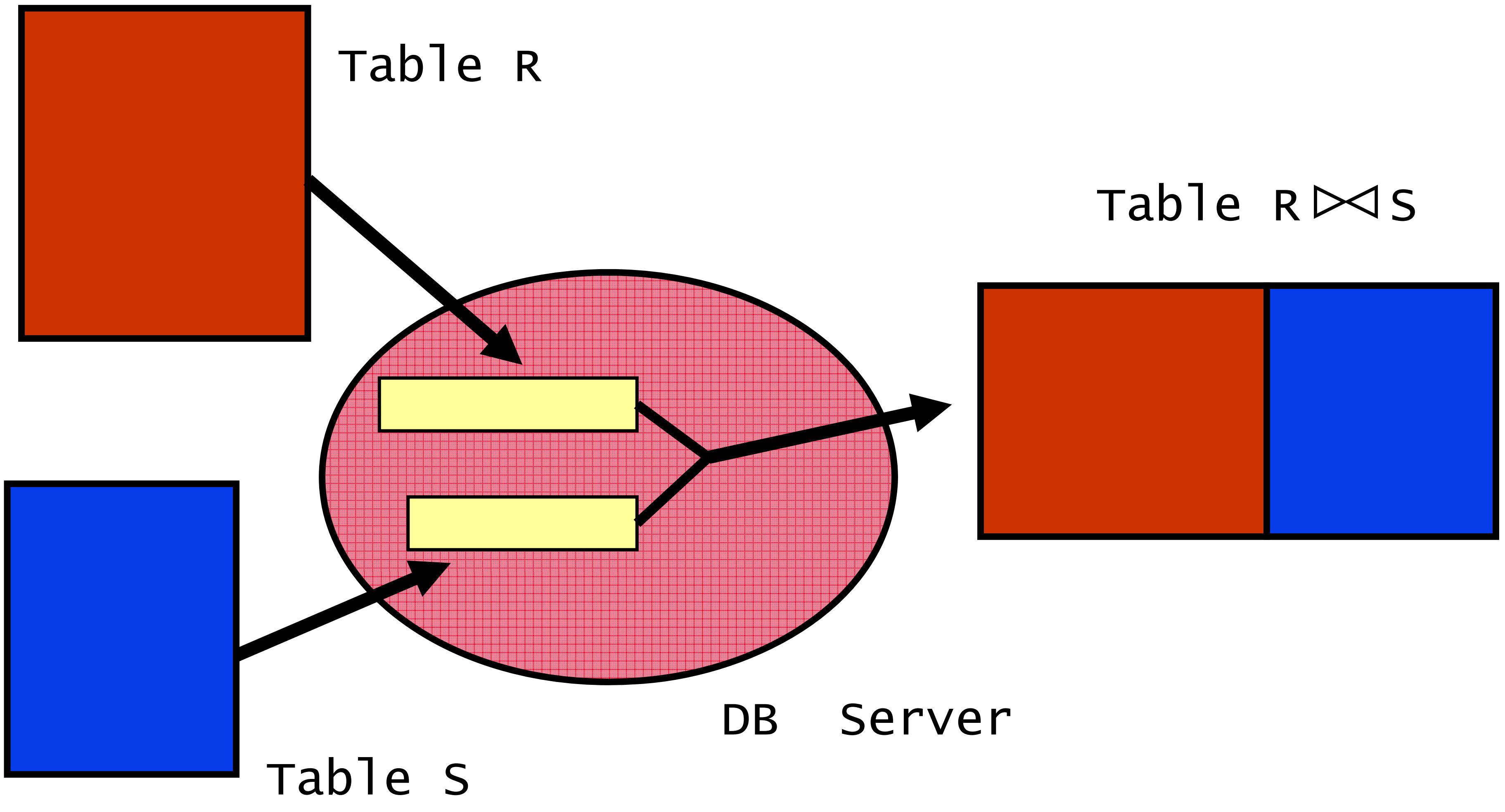
Measures of Query Cost

- For simplicity we just use number of block transfers from disk as the cost measure
  - ♦We ignore the difference in cost between sequential and random I/O for simplicity
  - ♦We also ignore CPU costs for simplicity
  - ♦We do not include cost to writing output to disk in our cost formulae
- Costs depends on the size of the buffer in main memory
  - ♦Having more memory reduces need for disk access
  - ♦Amount of real memory available to buffer depends on other concurrent OS processes, and hard to determine ahead of actual execution
  - ♦We often use worst case estimates, assuming only the minimum amount of memory needed for the operation is available
- Real systems take CPU cost into account, differentiate between sequential and random I/O, and take buffer size into account

Nested-Loop Join

- Read in outer relation R block by block
  - ♦ Then, for each tuples in R, we scan the entire inner relation S (scan means read in S block by block)
- n_R : no. of record for R
- b_R : no. of block for R
- Worst Cost: $b_R + n_R * b_S$
- Best Cost: $b_R + b_S$ (if smaller relation can fit in memory)
- Use small relation as outer relation
- Buffer: 3 pages (1 for R, 1 for S, 1 for output)

Nested Loops Join



Exercise

- Relations: `R1(A,B,C)` and `R2(C,D,E)`
- `R1` has 20,000 tuples
- `R2` has 45,000 tuples
- 25 tuples of `R1` fit on one block (blocking factor)
- 30 tuples of `R2` fit on one block
- `R1 JOIN R2`
- `R1` need 800 blocks (20000/25)
- `R2` need 1500 blocks (45000/30)
- Assume M pages in memory
- If `M > 800`, cost = $b_R + b_S$ = 1500 + 800

- Consider only `M <=800`
- Using R1 as outer relation
  - ♦Cost: 20000*1500 + 800 = 30000800 I/Os

- If R2 as outer relation
  - ♦Cost: 45000*800 + 1500 = 36001500 I/Os

Block Nested Loop Join

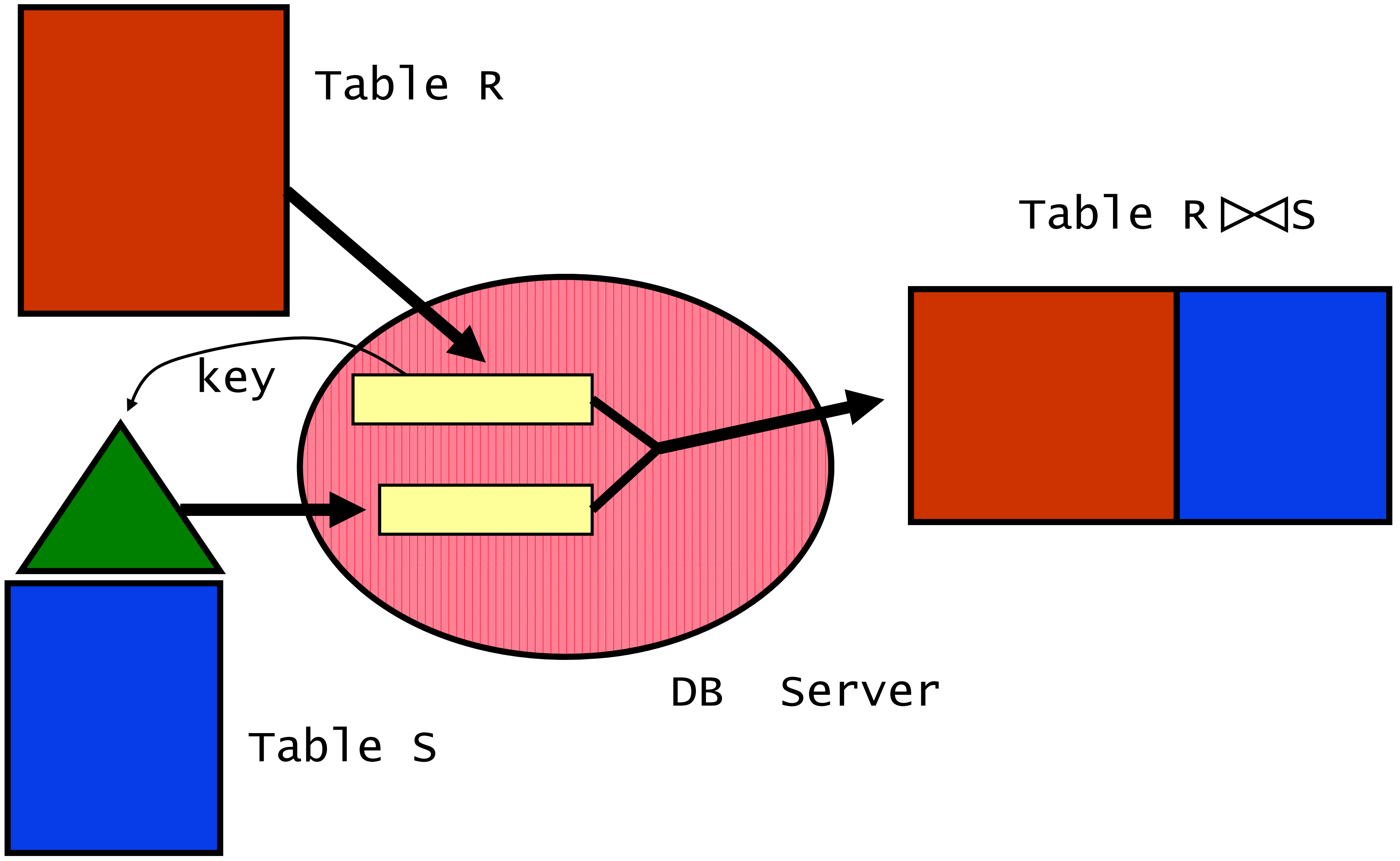
- Cost: $b_R + b_R * b_S$

- If M buffer pages available
  - ♦Cost: $b_R + \lceil b_R /(M-2) \rceil * b_S$
  - ♦M buffer pages (1 for inner S, 1 for output and all remaining M-2 pages to hold “block” of outer R

- If R1 is outer
  - ♦Cost = $\lceil 800 /(M-2) \rceil * 1500 + 800$

- If R2 is outer
  - ♦Cost + $\lceil 1500 /(M-2) \rceil * 800 + 1500$

Index Nested-Loop Join



Index Nested-Loop Join

- Primary B+tree index on the join attribute of R2:
  - ♦ $b_{R1} + n_{R1} * (x_{R2} + 1)$

  where:
    - n_{R1} (n_{R2}) is the number of R_1 (R_2) tuples
    - x_{R2} is the height of the B+-tree index on the join attribute
    - $n_{R1} * (x_{R2} + 1)$ is the cost of using B+-tree index to find matching tuple in R2

- Secondary B+tree index on the join attribute of R2:
  - ♦ $b_{R1} + n_{R2} * (x_{R2} + 1)$
  - ♦ where $n_{R2} * (x_{R2} + 1)$ is the cost of using B+-tree index to find matching tuple in R2

Index Nested loop join

- Hash index on the join attribute of R2:
  - ♦ $b_{R1} + n_{R1} * H$
  - ♦ Where H is the average number of page accesses necessary to retrieve a tuple from R2 with a given key

- We use:
  - ♦ H = 1.2 for a primary hash index and
  - ♦ H = 2.2 for a secondary hash index

External Sorting

- File has b_R pages
- Buffer M page
- No. of runs in the first pass $R = b_R / M$
- No. of passes to sort file completely

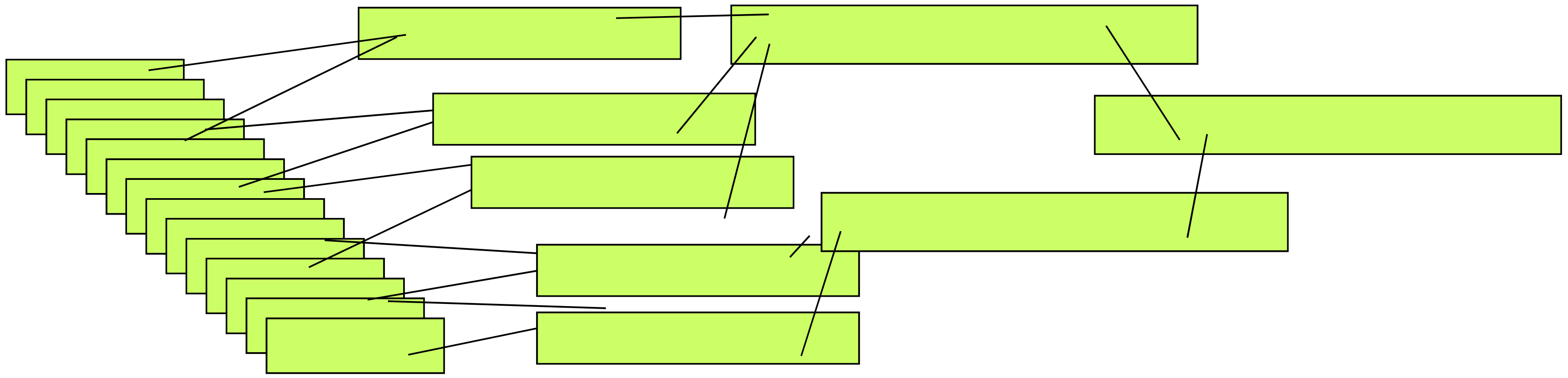
  $P = \lceil \log_{M-1} (b_R / M) \rceil + 1$

  $P = \lceil \log_{M-1} R \rceil + 1$
- Total cost for sorting

  $= b_R * (2 * \lceil \log_{M-1} R \rceil + 1)$

  $= b_R * 2 * \lceil \log_{M-1} R \rceil + b_R$

Multi-step Merging Runs



Merge Join

- Assuming R1 and R2 are not initially sorted on the join key
- Cost = Sorting + b_R + b_s
- Sorting = $1500 * (2 * \lceil \log_{M-1} (1500/M) \rceil + 1) + 800 * (2 * \lceil \log_{M-1} (800/M) \rceil + 1)$
- Assuming all tuples with same values for join attributes fit in memory (each block needs to be read only once)

Merge Join

- Assuming that there is a secondary B+tree on Rx

- Cost = $C_{R1} + C_{R2}$

- where $C_{Rx} = (n_{Rx} * ps)/(0.69 * bs) + b_{Rx}$ for the R which has the index on the join attribute
  - ♦*ps* : the size of the tuple reference (tuple identifier, `rid`)
  - ♦*bs* : the size of the block

- i.e.: the leaf nodes of the index tree (assumed to be 69% full) have to be scanned for pointers to the tuples of the relation and the blocks containing the tuples itself must be read at least once

Hash join

- Hash both relations on the join attribute using the same hash function
- Since R1 is smaller, we use it as the build relation and R2 as probe relation
- Assume no overflow occurs
- If M >= 800/M, no need for recursive partitioning, cost = 3(1500 + 800) = 6900 disk access = $3(b_R + b_s)$
- Else, cost = $2(1500 + 800)\lceil \log_{M-1}(800) - 1 \rceil + 1500 + 800$ disk access $= 2(b_R + b_s)\lceil \log_{M-1}(b_s) - 1 \rceil + b_R + b_s$

Why Optimize?

- Given a query of size *n* and a database of size *m*, how big can the output of applying the query to the database be?

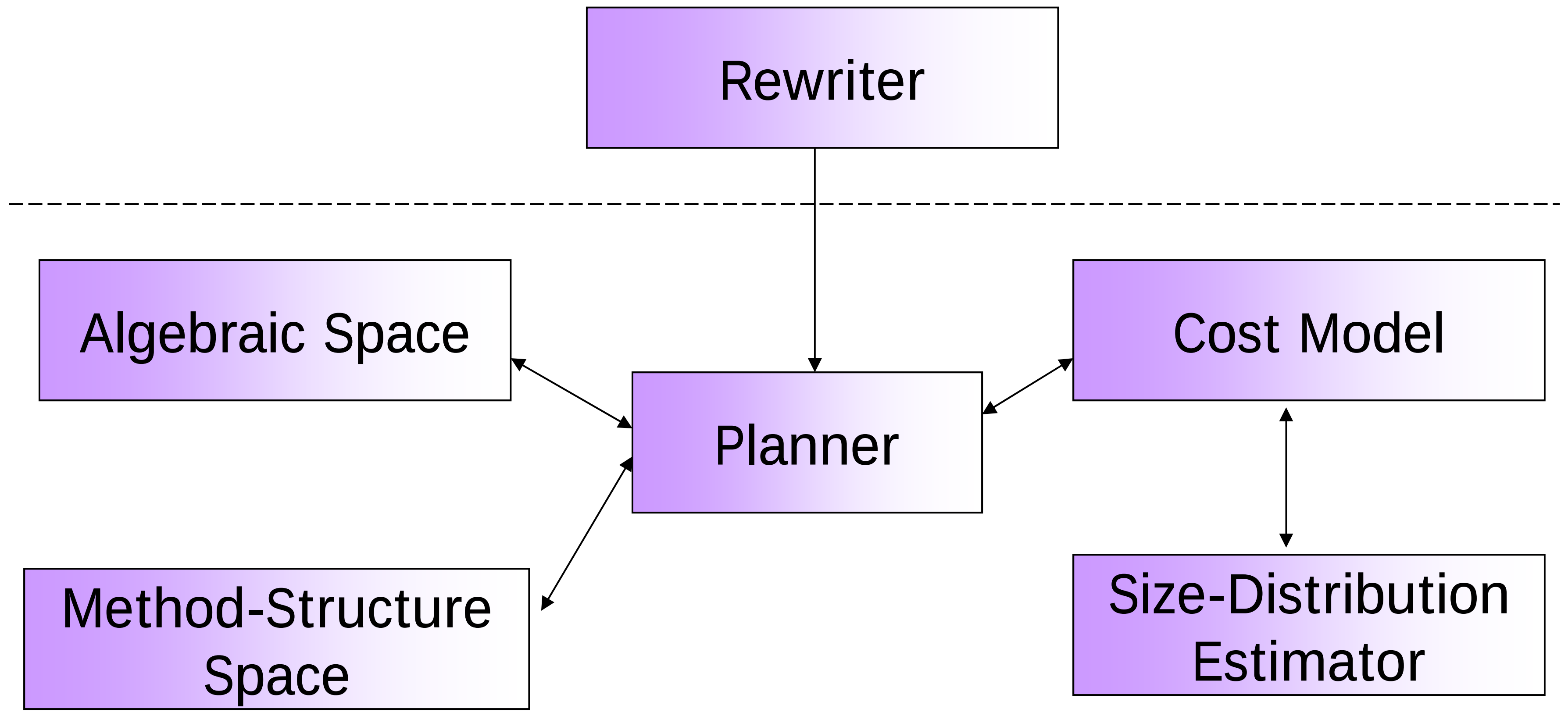
- Example: R(A) with 2 rows. One row has value 0. One row has value 1.
  - ♦How many rows are in R x R?
  - ♦How many in R x R x R?

➔Size of output as a function of input: O(?)

- Usually, queries are small
  - ♦Therefore, it is usually assumed that queries are of a fixed size
  - ♦Use term data complexity when we analyze time, assuming that query is constant

- What is the size of the output in this case?

Optimizer Architecture



Optimizer Architecture

- Rewriter: Finds equivalent queries that, perhaps can be computed more efficiently; all such queries are passed on to the Planner
  - ♦Examples of Equivalent queries: Join orderings
- Planner: Examines all possible execution plans and chooses the cheapest one, i.e., fastest one
  - ♦Uses other modules to find best plan
- Algebraic Space: Determines which types of queries will be examined
  - ♦Example: Try to avoid Cartesian Products
- Method-Structure Space: Determines what types of indexes are available and what types of algorithms for algebraic operations can be used
  - ♦Example: Which types of join algorithms can be used
- Cost Model: Estimates the cost of execution plans
  - ♦Uses Size-Distribution Estimator for this
- Size-Distribution Estimator: Estimates size of tables, intermediate results, frequency distribution of attributes and size of indexes

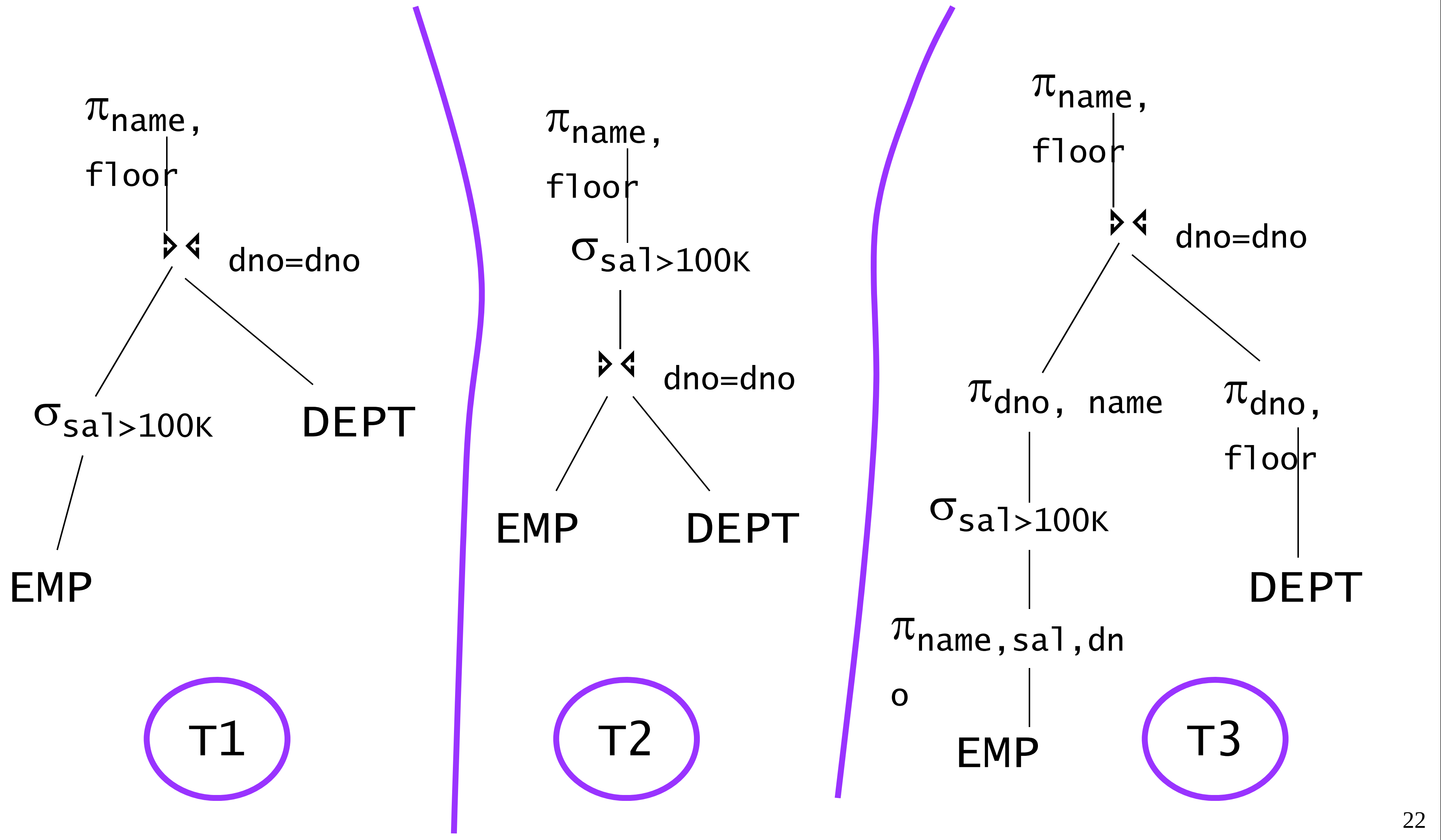
Algebraic Space

- We consider queries that consist of select, project and join (Cartesian product is a special case of join)
- Such queries can be represented by a tree.
- Example:

```
emp(name, age, sal, dno)
dept(dno, dname, floor, mgr, ano)
act(ano, type, balance, bno)
bank(bno, bname, address)
```

```
select name, floor
from emp, dept
where emp.dno=dept.dno and sal > 100K
```

3 Trees



π name, floor
⋈ dno=dno
σ sal>100K
DEPT
EMP
T1
π name, floor
σ sal>100K
⋈ dno=dno
EMP
DEPT
T2
π name, floor
⋈ dno=dno
π dno, name
π dno, floor
σ sal>100K
DEPT
π name,sal,dno
EMP
T3

Restriction 1 of Algebraic Space

- Algebraic space may contain many equivalent queries
- Important to restrict space – Why?
- Restriction (heuristic) 1: Only allow queries for which selection and projection:
    - are processed as early as possible
    - are processed on the fly
- Which trees in our example conform to Restriction 1?

Performing Selection and Projection "On the Fly"

- Selection and projection are performed as part of other actions
- Projection and selection that appear one after another are performed one immediately after another
  - ♦Projection and Selection do not require writing to the disk
- Selection is performed while reading relations for the first time
- Projection is performed while computing answers from previous action

Processing Selection/Projection as Early as Possible

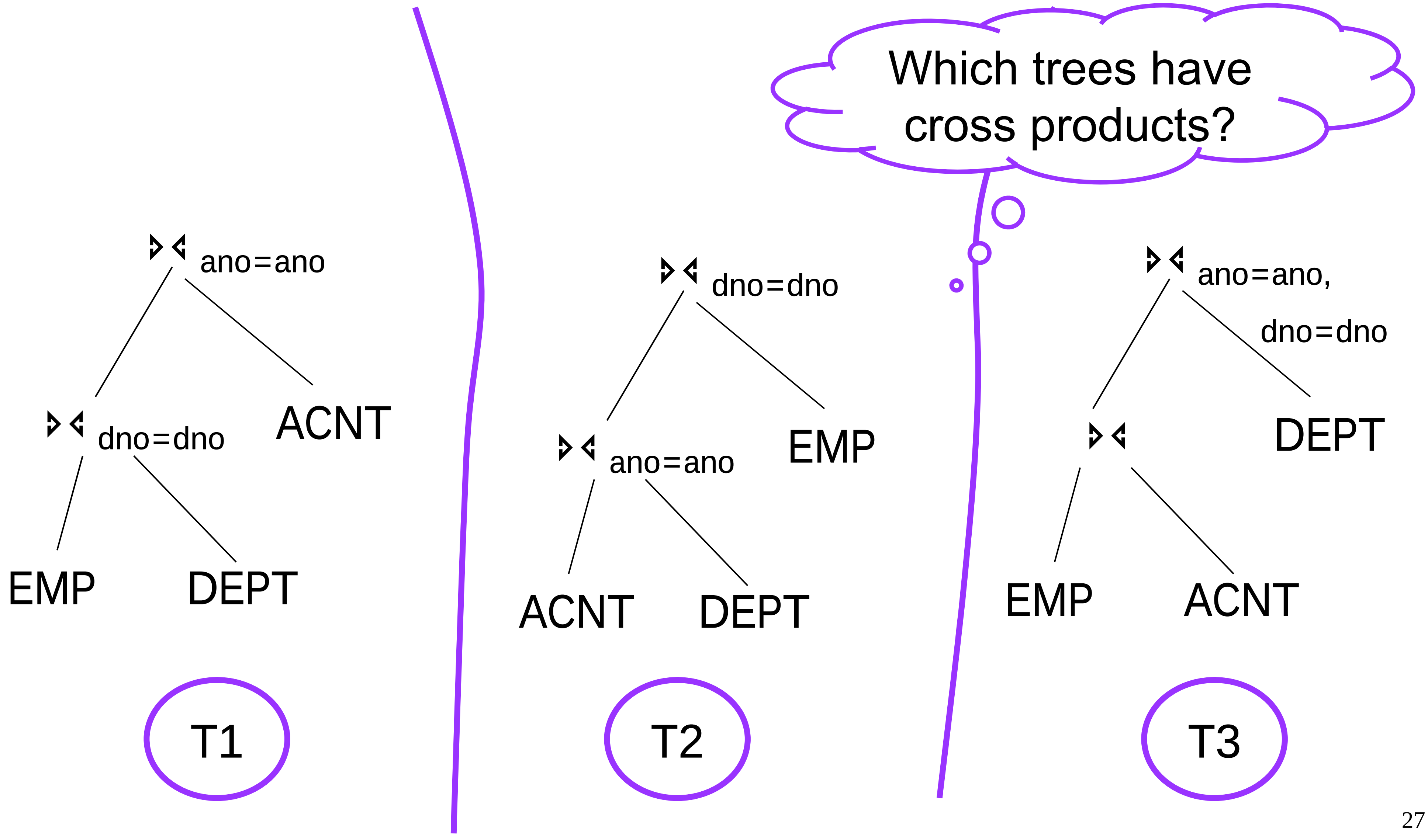
- The three trees differ in the way that selection and projection are performed
- In T3, there is "maximal pushing of selection and projection"
  - Rewriter finds such expressions
- Why is it good to push selection and projection?

Restriction 2 of Algebraic Space

- Since the order of selection and projection is determined, we can write trees only with joins
- Restriction (heuristic) 2: Cross products are never formed, unless the query asks for them
- Why this restriction?
- Example:

```
select name, floor, balance
from emp, dept, acnt
where emp.dno=dept.dno and
     dept.ano = acnt.ano
```

3 Trees



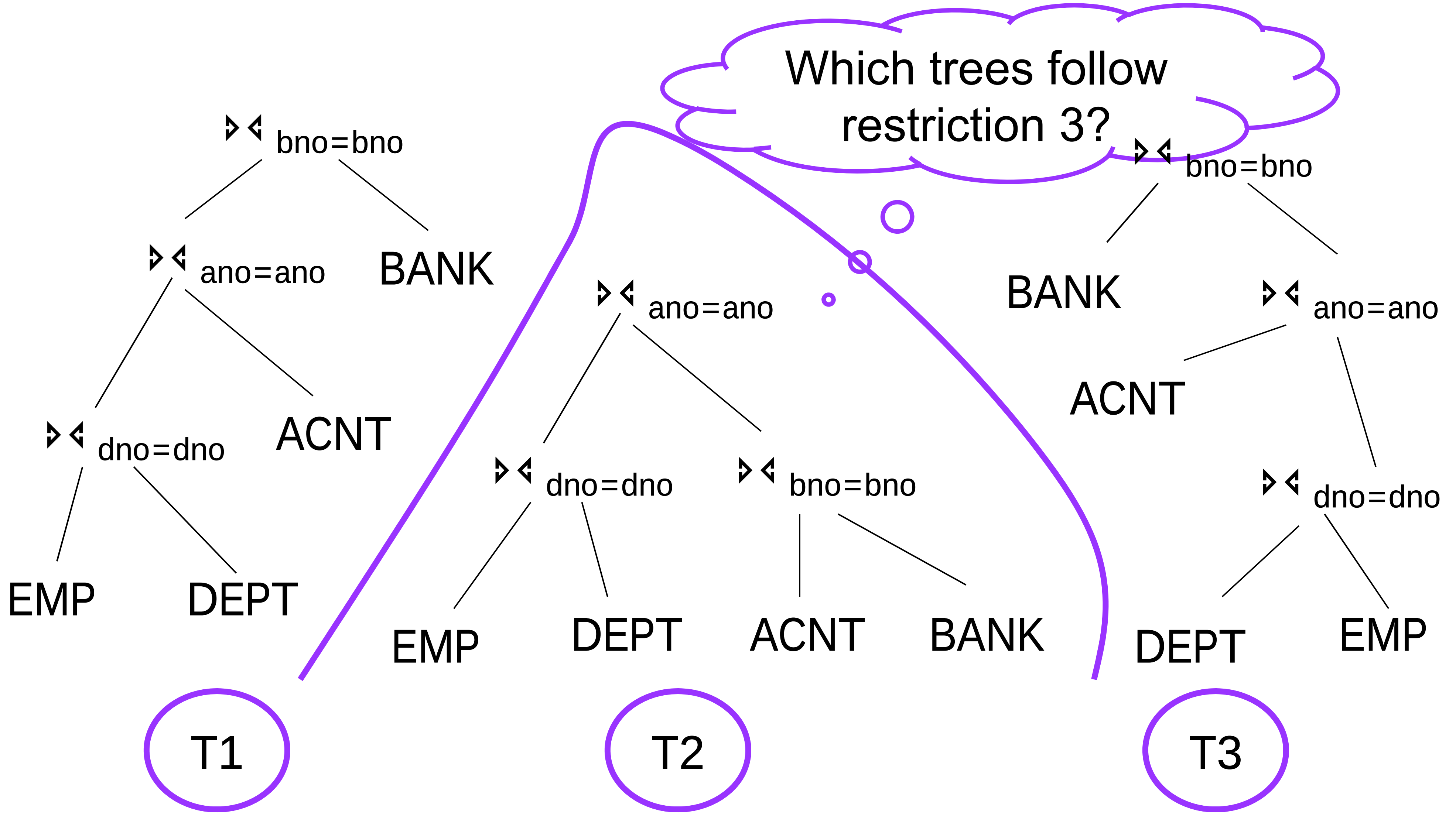
Which trees have cross products?
⋈ ano=ano
⋈ dno=dno
EMP
DEPT
ACNT
T1
⋈ dno=dno
⋈ ano=ano
ACNT
DEPT
EMP
T2
⋈ ano=ano, dno=dno
⋈
EMP
ACNT
DEPT
T3

Restriction 3 of Algebraic Space

- The left relation is called the outer relation in a join and the right relation is the inner relation (as in terminology of nested loops algorithms)
- Restriction (heuristic) 3: The inner operand of each join is a database relation, not an intermediate result (left-deep plans)
- Example:

```
select name, floor, balance
from emp, dept, acnt, bank
where emp.dno=dept.dno and dept.ano=acnt.ano
     and acnt.bno = bank.bno
```

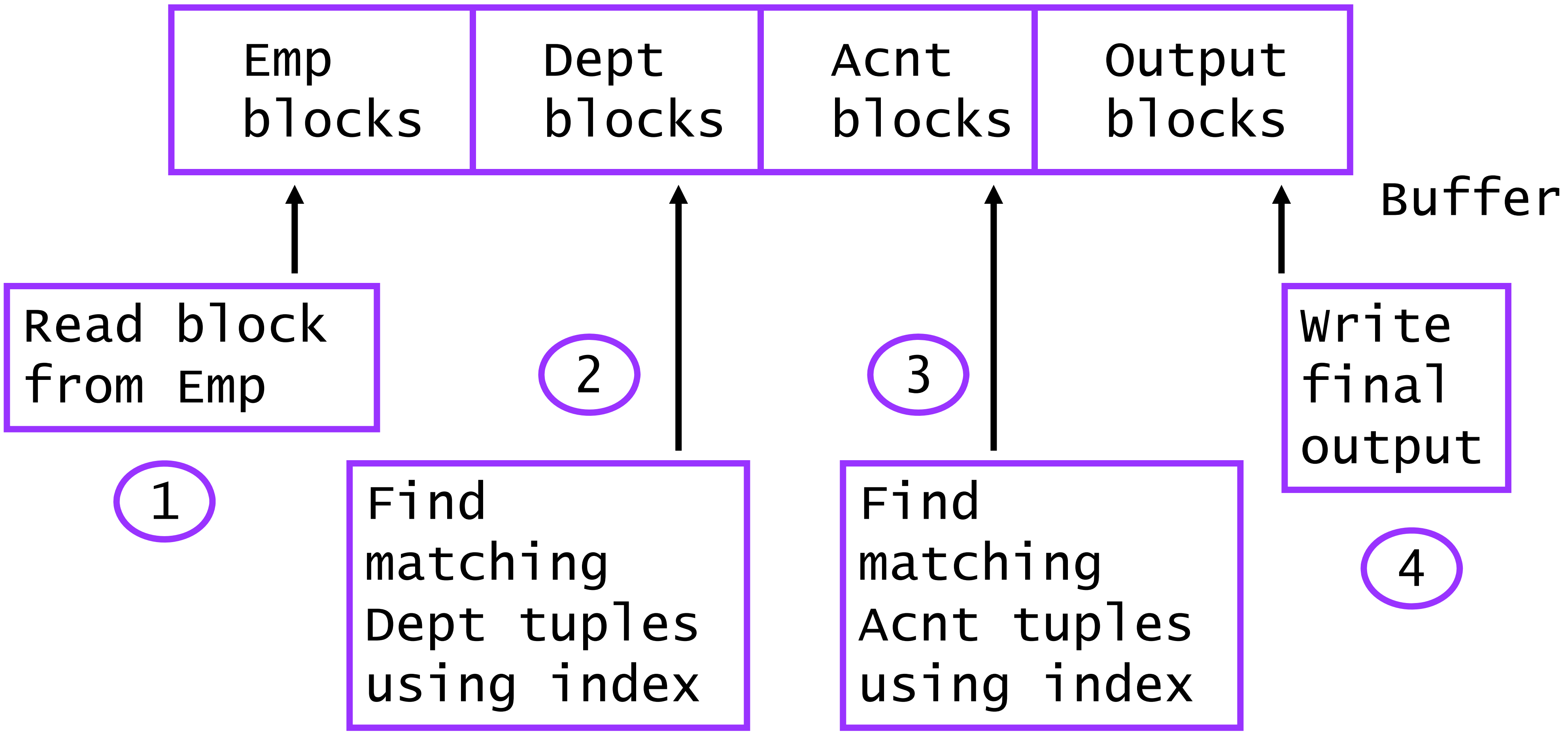
3 Trees


Which trees follow restriction 3?
⋈ bno=bno
⋈ ano=ano
BANK
⋈ dno=dno
ACNT
EMP
DEPT
⋈ ano=ano
⋈ dno=dno
⋈ bno=bno
EMP
DEPT
ACNT
BANK
⋈ bno=bno
BANK
⋈ ano=ano
ACNT
⋈ dno=dno
DEPT
EMP
T1
T2
T3

Pipelining Joins

- Consider computing: (Emp $\bowtie$ Dept) $\bowtie$ Acnt. In principle, we should
  - ♦compute Emp $\bowtie$ Dept, write the result to the disk
  - ♦then read it from the disk to join it with Acnt
- When using block and index nested loops join, we can avoid the step of writing to the disk
  - ♦Do you understand now restriction 3?
- We allow plans that
  - ♦Perform selection and projection early and on the fly
  - ♦Do not create cross products
  - ♦Use database relations as inner relations (also called left –deep trees)

Pipelining Joins - Example



Planner

- Dynamic programming algorithm to find best plan for performing join of *N* relations
- Intuition:
  - Find all ways to access a single relation
    - Estimate costs and choose best access plan(s)
  - For each pair of relations, consider all ways to compute joins using all access plans from previous step
    - Choose best plan(s)...
  - For each i-1 relations joined, find best option to extend to i relations being joined...
  - Given all plans to compute join of n relations, output the best

Reminder: Dynamic Programming

- To find an optimal plan for joining R_1, R_2, R_3, R_4, choose the best among:
  - ♦Optimal plan for joining R_2, R_3, R_4 + for reading R_1 + optimal join of R_1 with result of previous joins
  - ♦Optimal plan for joining R_1, R_3, R_4 + for reading R_2 + optimal join of R_2 with result of previous joins
  - ♦Optimal plan for joining R_1, R_2, R_4 + for reading R_3 + optimal join of R_3 with result of previous joins
  - ♦Optimal plan for joining R_1, R_2, R_3 + for reading R_4 + optimal join of R_4 with result of previous joins

Not Good Enough: Interesting Orders

- Example, suppose we are computing (R(A,B) $\bowtie$ S(B,C)) $\bowtie$ T(B,D)
  - ♦Maybe merge-sort join of R and S is not the most efficient, but the result is sorted on B
  - ♦If T is sorted on B, the performing a sort-merge join of R and S, and then of the result with T, maybe the cheapest total plan
- For some joins, such as sort-merge join, the cost is cheaper if relations are ordered
  - ♦Therefore, it is of interest to create plans where attributes that participate in a join are ordered on attributes in joins later on
- For each interesting order, save the best plan
  - ♦We save plans for non interesting order if it better than all interesting order costs

Example

- We want to compute the query:

```
select name, mgr
from emp, dept
where emp.dno=dept.dno and sal>30K and floor = 2
```

- Available Indexes: B+tree index on `emp.sal`, B+tree index on `emp.dno`, hashing index on `dept.floor`
- Join Methods: Block nested loops, index nested loops and sort-merge
- In the example, all cost estimations are fictional

Step 1 – Accessing Single Relations

Relation	**Interesting Order**	**Plan**	**Cost**
emp	`emp.dno`	Access through B+tree on `emp.dno`	700
		Access through B+tree on `emp.sal` Sequential scan	200 600
dept		Access through hashing on `dept.floor` Sequential scan	50 200

- Which do we save for the next step?

Step 2 – Joining 2 Relations

Join Method	Outer/Inner	Plan	Cost
nested loops	`empt/dept`	●For each `emp` tuple obtained through B+Tree on `emp.sal`, scan `dept` through hashing index on `dept.floor` to find tuples matching on dno	1800
		●For each `emp` tuple obtained through B+Tree on `emp.dno` and satisfying selection, scan `dept` through hashing index on `dept.floor` to find tuples matching on dno	3000

Step 2 – Joining 2 Relations

Join Method	**Outer/Inner**	**Plan**	**Cost**
nested loops	`dept/emp`	●For each `dept` tuple obtained through hashing index on `dept.floor`, scan emp through B+Tree on `emp.sal` to find tuples matching on dno	2500
		●For each dept tuple obtained through hashing index on dept.floor, scan emp through B+Tree on `emp.dno` to find tuples satisfying the selection on `emp.sal`	1500

Step 2 – Joining 2 Relations

Join Method	**Outer/ Inner**	**Plan**	**Cost**
sort merge		●Sort the emp tuples resulting from accessing the B+Tree on `emp.sal` into L1 ●Sort the `dept` tuples resulting from accessing the hashing index on `dept.floor` into L2 ●Merge L1 and L2	2300
		●Sort the `dept` tuples resulting from accessing the hashing index on `dept.floor` into L2 ●Merge L2 and the `emp` tuples resulting from accessing the B+Tree on `emp.dno` and satisfying the selection on `emp.sal`	2000

- Which plan will be chosen?

Picking a Query Plan

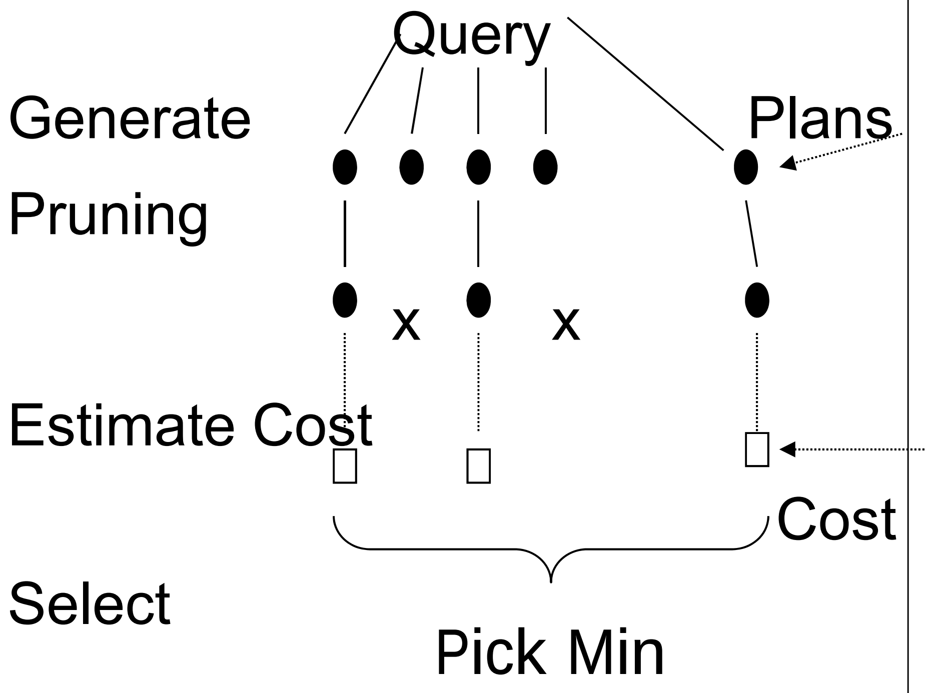
- Suppose we want to find the natural join of: `Reserves`, `Sailors`, `Boats`
- The 2 options that appear the best are (ignoring the order within a single join):

`(Sailors▷◁ Reserves) ▷◁ Boats`

`Sailors▷◁(Reserves ▷◁ Boats)`

- We would like intermediate results to be as small as possible
  - ♦Which is better?

--> Generating and comparing plans



Analyzing Result Sizes

- In order to answer the question in the previous slide, we must be able to estimate the size of (`Sailors`$\triangleright\triangleleft$`Reserves`) and (`Reserves`$\triangleright\triangleleft$`Boats`)
- The DBMS stores statistics about the relations and indexes
  - ♦Cardinality: Num of tuples *NTuples(R)* in each relation *R*
  - ♦Size: Num of pages *NPages(R)* in each relation *R*
  - ♦Index Cardinality: Num of distinct key values *NKeys(I)* for each index *I*
  - ♦Index Size: Num of pages *INPages(I)* in each index *I*
  - ♦Index Height: Num of non-leaf levels *IHeight(I)* in each B+ Tree index *I*
  - ♦Index Range: The minimum *ILow(I)* and maximum value *IHigh(I)* for each index *I*
- They are updated periodically (*not* every time the underlying relations are modified)

Estimating Result Sizes

- Consider

```
SELECT attribute-list
FROM relation-list
WHERE term1 and ... and termn
```

- The maximum number of tuples is the product of the cardinalities of the relations in the FROM clause
- The WHERE clause is associating a reduction factor with each term
  - ♦***column = value:*** 1/NKeys(I) if there is an index I on *column*. This assumes a uniform distribution; otherwise, System R assumes 1/10
  - ♦***column1 = column2:*** 1/Max(NKeys(I1),NKeys(I2)) if there is an index I1 on *column1* and I2 on *column2.* If only one column has an index, we use it to estimate the value; otherwise, use 1/10
  - ♦***column > value:*** (High(I)-value)/(High(I)-Low(I)) if there is an index I on *column*
- Estimated result size is: maximum size times product of reduction factors

Example

```
SELECT *
FROM Reserves R, Sailors S
WHERE R.sid = S.sid and S.rating > 3
      and R.agent = ‘Joe’
```

- Cardinality(R) = 1,000 * 100 = 100,000
- Cardinality(S) = 500 * 80 = 40,000
- NKeys(Index on S.sid) = 40,000
- NKeys(Index on R.agent) = 100
- High(Index on Rating) = 10, Low = 0
- Maximum cardinality: 100,000 * 40,000
- Reduction factor of R.sid = S.sid: 1/40,000
- Reduction factor of S.rating > 3: (10–3)/(10-0) = 7/10
- Reduction factor of R.agent = ‘Joe’: 1/100
- Total Estimated size: 700

Second Example of Join Order Selection

- Consider the join of the four relations named R, S, T, U:

R(a,b), 800 total tuples	S(b,c), 10.000 total tuples	T(c,d), 4.000 total tuples	U(a,b,d), 2.500 total tuples
V(R,a) = 500			V(U,a) = 10
V(R,b) = 100	V(S,b) = 2.000		V(U,b) = 50
	V(S,c) = 1.000	V(T,c) = 1.000	
		V(T,d) = 1.000	V(U,d) = 500

Second Example of Join Order Selection

- For the singleton sets, the costs and best plans are given in the table below

	{R}	{S}	{T}	{U}
Size	800	10.000	4.000	2.500
Cost	0	0	0	0
Best plan	R	S	T	U

- As the costs for all relations are the same, the dynamic programming algorithm will consider them all. The greedy algorithm however, must choose one, or also consider them all. Let’s assume it takes the one with least cost, and if they are more present of these, take the one with smallest length. So, the plan called “R” is chosen.

Second Example of Join Order Selection

- Now, we consider the pairs of relations
  - ♦Again, the cost is 0 for each, as we do not have intermediate results

	{R,S}	{R,T}	{R,U}	{S,T}	{S,U}	{T,U}
Size	4.000	3.200.000	4.000	40.000	12.500	10.000
Cost	0	0	0	0	0	0
Best plan	RxS	RxT	RxU	SxT	SxU	TxU

- The dynamic programming algorithm again keeps them all for the next run, as the costs are 0. The greedy algorithm had chosen R in the previous run, so it must choose a plan based on this choice. Since there are two best plans here, it has to make a choice of them. Let’s assume it makes the wrong choice, and takes “RxU”.

Second Example of Join Order Selection

- Now, we consider the join of three out of these four relations:

	{R,S,T}	{R,S,U}	{R,T,U}	{S,T,U}
Size	16,000	5,000	16,000	50,000
Cost	4,000	4,000	4,000	10,000
Best plan	(RxS)xT	(RxS)xU	(RxU)xT	(TxU)xS

- As you can see, the best plan is clearly “(RxS)xU”, with the least cost and size. However, the greedy algorithm will not consider this plan, as it chose “RxU” as the best plan in previous run. Instead, it is forced to take “(RxU)xT” as best plan now.

Second Example of Join Order Selection

- Finally, we consider the join of all relations. We come to these four final results (for dynamic programming):

Plan	Cost
((RxS)xT)xU	20.000
((RxS)xU)xT	9.000
((RxU)xT)xS	20.000
((TxU)xS)xR	60.000

- The dynamic programming algorithm finds the optimal solution ((RxS)xU)xT, while the greedy algorithm, based on results of the previous run, comes to the more expensive solution ((RxU)xT)xS.

Selecting Algorithms for Plan Operators

- For each logical plan operator, select algorithms based on I/O cost estimation
- For selection operator, consider
  - ♦Index-scan algorithms that use single attribute indexes, multiple indexes, or multidimensional indexes
  - ♦Table-scan algorithm using no index
- For join operator, consider
  - ♦All types of join algorithms if enough statistics is available
  - ♦If statistics is in sufficient, follow some simple ideas
    - Try one-pass algorithm or nested-loops
    - Use sort-join if one or both arguments are already sorted
    - If index is available, use index-join
    - If sort and index are not available and multi-pass join is necessary, use a hash join

Pipelining Example

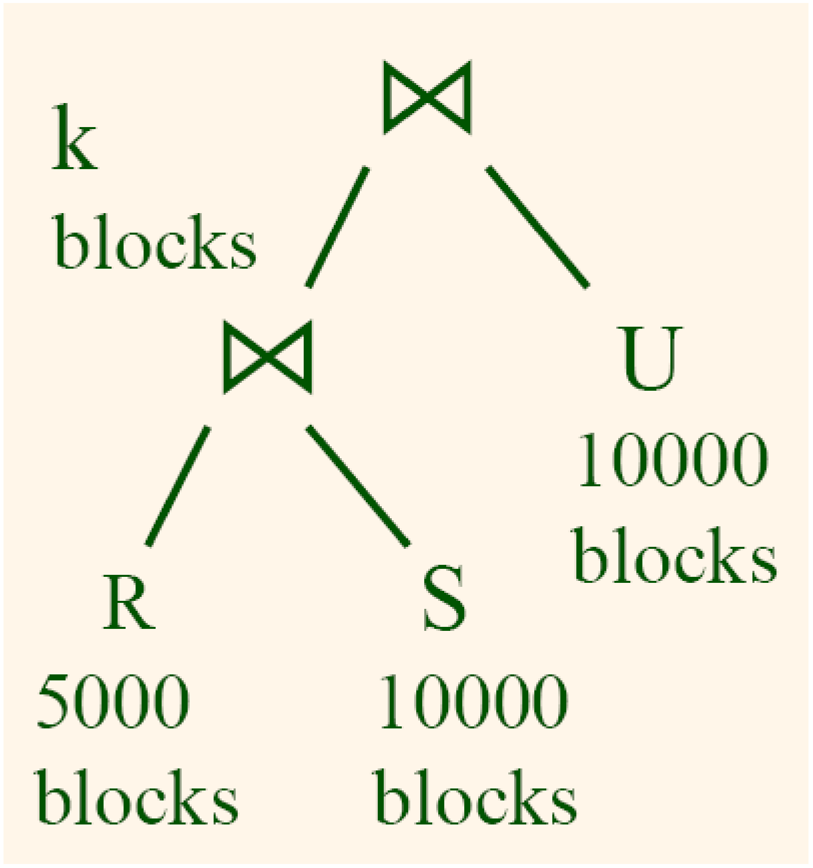
- Relations:

  $R(W,X),\ b_R = 5000$

  $S(X,Y),\ b_S = 10000$

  $U(Y,Z),\ b_U = 10000$

- Buffer: M = 101 blocks
- Both joins are hash join
- Size k is estimated, and used to choose join algorithms



Case 1: $k \leq 49$

- Can pipeline result of 1st join into 2nd join
- Two-pass hash join for $R \bowtie S$:
  - ♦Both R and S are hashed into 100 partitions, where each R partition has 50 blocks
  - ♦Join corresponding R & S partitions using 50 buffer blocks for R partition, 1 block for S partition, and store the result in 49 blocks as a hash table
- One-pass hash join for the 2nd join:
  - ♦Use 1 buffer block for U (no need to partition U), join with the intermediate result that is already in buffer
- Cost = 3(5000+10000) + 10000 = 55000

Case 2: $49 < k \leq 5000$

- Overlap the 1st join with the hash partitioning of the 2nd join

- Two-pass hash join for the 1st join:
  - ♦Partition R & S into 100 partitions, so that each R partition contains 50 blocks
  - ♦Join corresponding R & S partitions (using 51 buffer blocks)
  - ♦During the join, hash the result into 50 partitions (using the remaining 50 buffer blocks) & write the partitions to disk

- Two-pass hash join for the 2nd join:
  - ♦Partition U into 50 partitions
  - ♦Join corresponding partitions of intermediate result & U, using intermediate result partition as build relation (use 1 to 100 buffer blocks)

- Cost = 3(10000+5000) + k + 2(10000) + (k+10000) = 75000 + 2k

Case 3: k > 5000

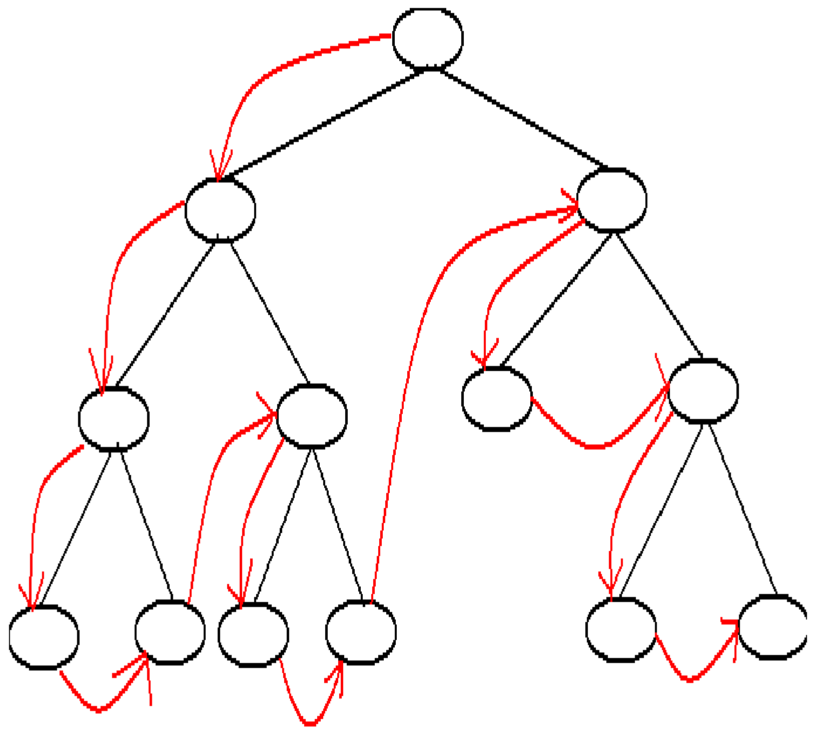
- Cannot use pipelining
- Two-pass hash join for the 1st join:
  - ♦Partition R & S into 51 partitions, so that each R partition has <100 blocks
  - ♦Join corresponding R & S partitions, write results to disk
- Two-pass hash join for the 2nd join:
  - ♦Partition intermediate result & U into more than 50 partitions
  - ♦Join corresponding partitions of U & intermediate result, using the smaller partition as the build relation
- Cost = 3(5000+10000) + k + 3(10000+k) = 75000 + 4k

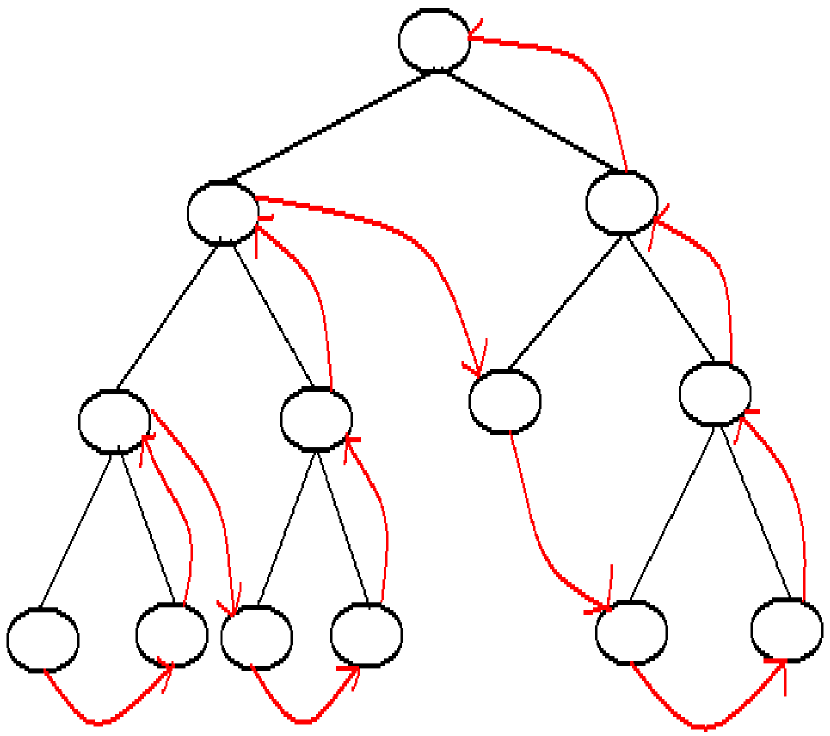
Pipelining vs. Materialization

- Advantages of 64 bit processors
  - ♦More main memory possible
    - And so, more pipelining operations possible without having to write intermediate results to disk
  - ♦Complex in-memory processing does not require intermediate results being temporarily written to disk
    - Saves costly disk I/O's and increases scalability
- Disadvantages of 64 bit processors
  - ♦Application must be fully supporting 64 bit to make full use of the speed advantages
  - ♦Upgrading to a 32 bit system with (more) parallel processors (using shared memory perhaps) might be cheaper to implement
- DBMS's implementing 64 bit are e.g. Oracle 10g

Ordering of Physical Operations

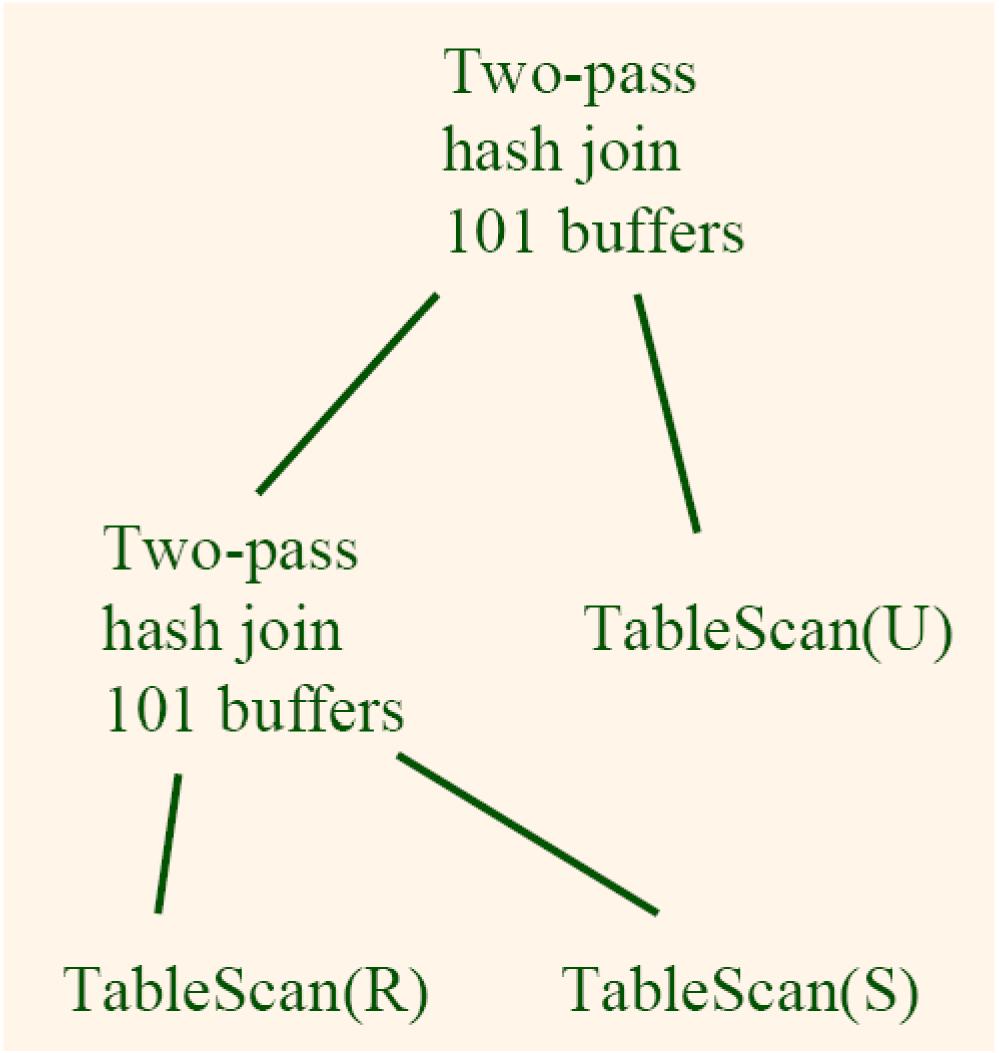
- Pre-order traversal
- Post-order traversal





Notation for Physical Query Plans

- Non-standard among DBMSs
- Typical physical plan operators include
  - For leave nodes: `TableScan(R)`, `SortScan(R,AttrList)`, `IndexScan(R,A)`, `IndexScan(R, AθC)`
  - For selection nodes: combination of `TableScan(R)`, `Filter(Cond)`, `SortScan(R, AttrList)`


Two-pass hash join 101 buffers
Two-pass hash join 101 buffers
TableScan(U)
TableScan(R)
TableScan(S)

Points to Remember

- Step 1: Choose a *logical* plan
  - ♦Involves choosing a query tree, which indicates the order in which algebraic operations are applied
  - ♦*Heuristic*: Pushed trees are good, but sometimes “nearly fully pushed” trees are better due to indexing (as we saw in the example)
  - ♦**So**: Take the initial “master plan” tree and produce a *fully pushed* tree plus several *nearly fully pushed* trees
- Step 2: Reduce *search space*
  - ♦Deal with *associativity* of binary operators (join, union, …)
  - ♦Choose a particular *shape* of a tree (left-deep trees)
    - Equals the number of ways to parenthesize N-way join – grows very rapidly
  - ♦Choose a particular permutation of the leaves
    - E.g., 4! permutations of the leaves A, B, C, D
- Step 3: Use a *heuristic search* to further reduce complexity
  - ♦The choice of left-deep trees still leaves open too many options
  - ♦A heuristic algorithm is used to get a ‘good’ plan