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## Motivation

General form of subspace clustering:

$$\min_{\mathbf{C}} \underbrace{\mathcal{L}(\mathbf{Z} - \mathbf{Z}\mathbf{C})}_{\text{① self-expression}} + \lambda_1 \sum_i \underbrace{\mathcal{P}(\mathbf{X}_i, \hat{\mathbf{X}}_i)}_{\text{② data reconstruction}} + \lambda_2 \underbrace{\mathcal{R}(\mathbf{C})}_{\text{③ structure prior}}$$

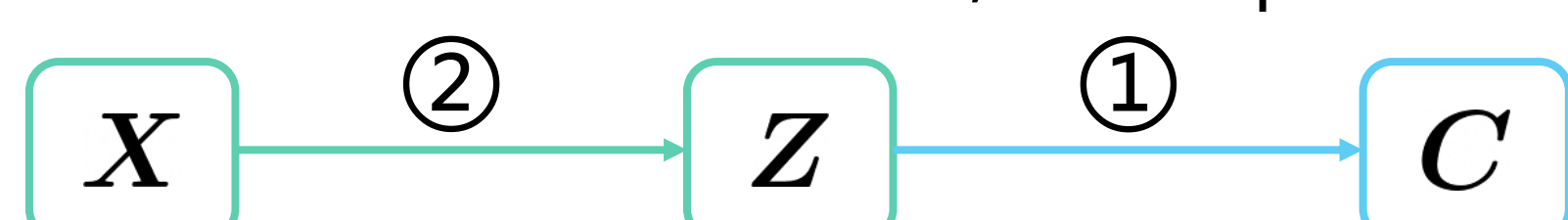
$$s.t. \text{diag}(\mathbf{C}) = 0$$

When facing corruptions:

- Influence via ①: directly distort the estimated subspace structures



- Influence via ②: induce an unfaithful  $\mathbf{Z}$ , then spread to  $\mathbf{C}$



...most existing work only focuses on reduce the influence on ①

## Framework

Double-assurance for robustness with **two-fold explicit noise modeling**:

- Robust reconstruction with  $\mathcal{P}(\cdot)$

$$\mathcal{P}(\mathbf{X}_i, \hat{\mathbf{X}}_i) = \frac{1}{2} \|\mathbf{X}_i - \hat{\mathbf{X}}_i - \mathbf{E}_i\|_F^2 + \eta_1 \underbrace{\|\mathbf{E}_i\|_F}_{\text{corruption indicator for each sample}}$$

F-norm to prevent the loss from being dominated by the corruptions

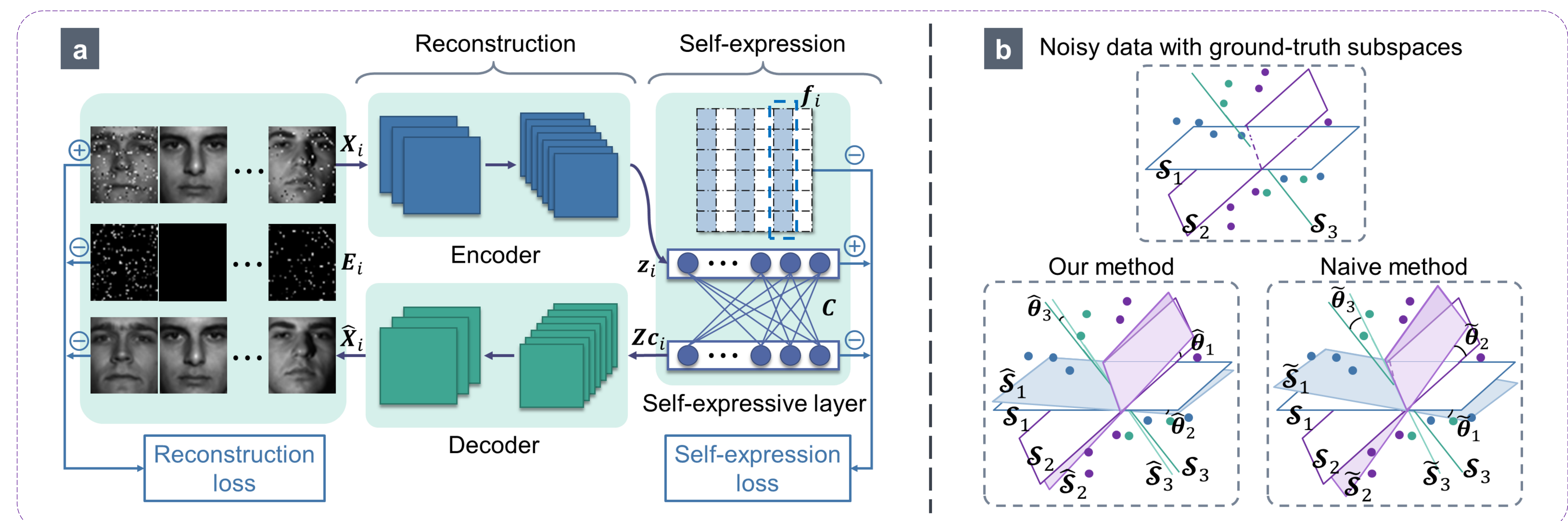
- Robust self-expression with  $\mathcal{L}(\cdot)$

$$\mathcal{L}(\mathbf{Z} - \mathbf{Z}\mathbf{C}) = \sum_{i=1}^n \frac{1}{2} \|z_i - \mathbf{Z}c_i - \mathbf{f}_i\|_2^2 + \eta_2 \|\mathbf{f}_i\|_2$$

Objective function

$$L = \sum_{i=1}^n \frac{1}{2} \|\mathbf{X}_i - \hat{\mathbf{X}}_i - \mathbf{E}_i\|_F^2 + \eta_1 \|\mathbf{E}_i\|_F + \lambda_1 \left( \frac{1}{2} \|z_i - \mathbf{Z}c_i - \mathbf{f}_i\|_2^2 + \eta_2 \|\mathbf{f}_i\|_2 \right) + \lambda_2 \|\mathbf{C}\|_p$$

non-smooth! (non-differentiable at zero)



## Smoothing of norms

**DEFINITION 2** ( $\frac{1}{\mu}$ -SMOOTH APPROXIMATION, [2]). For a convex function  $h: \mathbb{E} \rightarrow \mathbb{R}$ , a convex differentiable function  $h_\mu: \mathbb{E} \rightarrow \mathbb{R}$  is said to be its  $\frac{1}{\mu}$ -**smooth approximation** with parameters  $(\alpha, \beta)$  if for any  $\mu > 0$  the following holds:

- $h_\mu(\mathbf{x}) \leq h(\mathbf{x}) \leq h_\mu(\mathbf{x}) + \beta\mu$  for all  $\mathbf{x} \in \mathbb{E}$ ,
- $h_\mu$  is  $\frac{\alpha}{\mu}$ -smooth.

$\frac{1}{\mu}$ -smooth approximation of F-norm and  $\ell_2$ -norm:

$$\|\mathbf{X}\|_{F(\mu)} = \sqrt{\|\mathbf{X}\|_F^2 + \mu^2} - \mu$$

$$\|\mathbf{x}\|_{2(\mu)} = \sqrt{\|\mathbf{x}\|_2^2 + \mu^2} - \mu$$

balance the approximation accuracy and smoothness

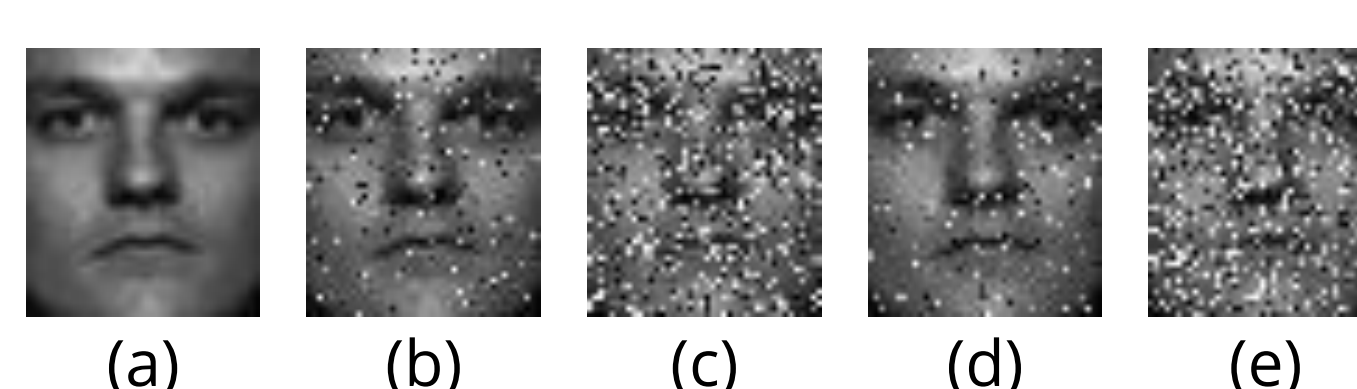
Smoothed objective function:

$$L = \sum_{i=1}^n \frac{1}{2} \|\mathbf{X}_i - \hat{\mathbf{X}}_i - \mathbf{E}_i\|_{F(\mu)}^2 + \eta_1 \|\mathbf{E}_i\|_{F(\mu)} + \lambda_1 \left( \frac{1}{2} \|z_i - \mathbf{Z}c_i - \mathbf{f}_i\|_{2(\mu)}^2 + \eta_2 \|\mathbf{f}_i\|_{2(\mu)} \right) + \lambda_2 \|\mathbf{C}\|_p$$

a totally differentiable network, simultaneously update all the params



## Experiments



Corrupted samples on Extended Yale B dataset. (a) Original image, (b/c) 10%/40% corrupted by random pixel corruption, (d/e) 10%/40% corrupted by Gaussian noise.

