Matlab programs to compute lines of curvature on the surfaces

$$\mathbf{h_a(x, y, z) = x^2 + \frac{1}{3}y^2 + \frac{1}{5}z^2 + axyz = 0}$$

The lines of curvature on the ellipsoid $h_0 = 0$ are intersections of a triply orthogonal family of quadric surfaces. All of them are closed curves except for those that lie in the plane $y = 0$. In this plane, there are four umbilic points, two maximal lines of curvature that connect the umbilic points and two minimal lines of curvature that connect the umbilic points. The programs described here compute (approximate) maximal lines of curvature in the family $h_a = 0$ for the two values $a = 0.11, 0.1857$. Both parameter values still yield four umbilic points, each with a single separatrix. When $a = 0.11$, each line of curvature, including the separatrices, appears to be dense in the surface $h_a = 0$. When $a = 0.1857$ there appear to be three closed lines of curvature that are the limits of all of the other lines of curvature, including the separatrices.

This a short archive contains the following Matlab scripts and functions:

- `qp_coeffs.m`: function that computes Taylor polynomial of degree 2 of h.

- `qp_lcurv.m`: function that evaluates oriented maximal principal directions for steps along a line of curvature.

- `qp_nml.m`: function that computes normal vector to the surface $h_a = 0$.

- `qp_pframe.m`: function that computes an orthonormal frame aligned with the principal directions and normal to the surface $h_a = 0$. It also computes normal curvatures and "regular" auxiliary information in computing the normal curvatures.

- `qp_po.m`: script that visualizes maximal principal foliation with two closed lines of curvature.

- `qp_proj.m`: function that projects points to surface $h_a = 0$ along gradient of h_a.

- `qp_qp.m`: script that visualizes maximal principal foliation with a dense line of curvature.

- `qp_step.m`: function that performs an Runge-Kutta step along a line of curvature.

- `qp_traj.m`: function that steps along a line of curvature.

- `qp_umbilic.m`: function that computes umbilic points of surface $h_a = 0$.

The data file `qp_umb` has the locations of the four umbilic points on the surface $h_0 = 0$.