This folder has the Matlab codes and a Macaulay2 script used to produce the paper "Bounds on the Index of an Umbilic Point" by John Guckenheimer. There are two subdirectories Mng9 with codes for Figures $1-5$ and Mng4 with codes for Figure 6. The programs in Mng9 compute information about the principal foliations of surfaces S_a that are the graphs of the functions

$$h_a(x,y) = 1-(1-x^2-y^2)^{(1/2)}+a1x^9+a2x^8y+a3x^6y^2+a4x^5y^3+a5x^4y^4+a6x^3y^5+a7x^2y^6+a8xy^7+a9y^8$$

- mng9_z.m

  This function evaluates

  $$h_a(x,y) = 1-(1-x^2-y^2)^{(1/2)}+a1x^9+a2x^8y+a3x^6y^2+a4x^5y^3+a5x^4y^4+a6x^3y^5+a7x^2y^6+a8xy^7+a9y^8$$

- mng9_pv_sym.m

  This script uses the symbolic toolbox to generate the expressions used in computing the principal directions.

- mng9_pv_xy.m

  This function uses the expressions produced by mng9_pv_sym.m to compute the principal directions of S_a at $(x, y, h_a(x, y))$.

- mng9_fig_map_x.m

  These scripts plot the figures in the paper. More extensive documentation is included in mng9_fig_map_1 than the remainder of these scripts since they are all similar.

- mng9_umbilic_figures.m

  This script is a "driver" to plot all the figures.

- mng9_lcurv2.m

  This function determines the orientation of a principal vector compatible with that of an adjacent point along a line of curvature.

- mng9_step.m

  This function computes one step along a line of curvature using the "standard" Runge-Kutta algorithm rk4.

- mng9_traj2.m

  This function computes a line of curvature using Runge-Kutta steps.

- mng9_umb_a13_sym.m

  This script uses the symbolic toolbox to find $a3(x)$ that gives approximate umbilics in the subfamily with $a1 = 1$,$a2 = 0$, $y = 0$ and $a_j = 0, j > 3$.

- mng9_umb_a3x.m

  This script uses the output of mng9_umb_a13_sym.m to locate approximation of curve of umbilics emanating from origin along x axis.

The programs in Mng4 compute information about the principal foliations of surfaces S_a that are the graphs of the functions

$$h(x,y) = x^2/2+y^2/2+l_d(x^2-y^2)+2l_oxy+a_3x^3+a_2X^2y+a_1xy^2+a_0y^3+b_4x^4+b_3x^3y+b_2x^2y^2+b_1xy^3+b_0y^4$$

Most of the files in this directory are similar to the ones in Mng9:

- mng4_z.m

  This function evaluates h(x,y).

- mng4_pv_sym.m

  This script uses the symbolic toolbox to generate the expressions used in computing the principal directions.

- mng4_pv_xy.m

  This function uses the expressions produced by mng9_pv_sym.m to compute the principal directions of the graph of $h(x, y)$.

- mng4_hcoeffs.m

  This function computes derivatives of $h(x, y)$.

- mng4_lcurv2.m

  This function determines the orientation of a principal vector compatible with that of an adjacent point along a line of curvature.

- mng4_step.m

  This function computes one step along a line of curvature using the "standard" Runge-Kutta algorithm rk4.

- mng4_traj_yc.m

  This function computes a line of curvature using Runge-Kutta steps. It uses step sizes adapted to proximity to an umbilic and computes "events" that are intersections of the line of curvature with a cross-section.

- mng4_cd.m

  Function that measures proximity to an umbilic.

- mng4_umbnewton.m

  This function employs Newton's method to locate an umbilic.

- mng4_umbnewtonx.m

  Driver to locate umbilics close to the x-axis.

- mng4_umbnewtony.m

  Driver to locate umbilics close to the y-axis.

- mng4_pp0.m

  Generate figure displaying principal foliations when parameters a_i and b_i are all zero.

- mng4_pp0.m

  Generate figure displaying principal foliations without umbilic connections. The middle panel illustrates that lines of curvature are not closed when $l_d = l_o = 0$.

There are also three Matlab codes and one Macaulay2 script in this directory:

- h_remove_z.m

  This script uses the symbolic toolbox to compute the Monge form of a surface defined as the zero level set of a function $h(x, y, z)$ at a point $(xu, 0, zu)$. It assumes that $dh(0, 0, 0) = (0, 0, az)$ and that h is even in y.

- mngdof_monge.m

  This function computes Monge coordinates of an umbilic on the surface which is the zero level set of

  $$z - h(x, y) = z - 1 + (1 - x^2 - y^2)^{(1/2)} - a1 * x^d - a3 * x^{d-2} * y^2 - a5 * x^{d-4} * y^4$$

  located at $(xu, 0, h(xu, 0))$. It then evaluates an expression $i(d)$ from a paper of Berry and Hannay that whose sign determines the index of the umbilic points which emerge from the origin as the parameter is varied.

- mngd_tind.m

  This script evaluates the function $i(d)$ from mngdof_monge.m for even degrees $d \in [4, 20]$. It discovers that $i(d)$ is a polynomial of degree 5.

- mng9_matrix_macaulay.m2

  Macaulay 2 script used to analyze the determinant appearing in the paper.