chapter 1

Chenxi Gu
2017311017

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1 Problem 1

The energy of a Boeing 747:

$$E = \frac{p^2}{2m} = 1.11 * 10^{10} J$$

The energy of mosquito–antimosquito annihilation:

$$E = 2mc^2 = 3.6 * 10^{11} J$$

2 Problem 2

$$\begin{aligned} s &= (3E)^2 - 0^2 = 9(m^2 + p^2) = 88.9 GeV^2 \\ m &= \sqrt{s} = 9.43 GeV \end{aligned} \tag{1}$$

3 Problem 3

We know the decay width $\Gamma = \frac{\hbar}{\tau}$.

$$\Gamma_\pi = 2.538 * 10^{-8} eV, \quad \Gamma_K = 5.5 * 10^{-8} eV, \quad \Gamma_\Lambda = 2.538 * 10^{-6} eV$$

4 Problem 4

Same reason with Problem 3

$$\tau_\rho = 4.429 * 10^{-24} s, \quad \tau_\omega = 7.765 * 10^{-23} s, \quad \tau_\phi = 1.535 * 10^{-22} s$$
$$\tau_{K^*} = 1.294 * 10^{-23} s, \quad \tau_{J/\psi} = 7.097 * 10^{-21} s, \quad \tau_\Delta = 5.593 * 10^{-24} s$$

5 Problem 5

$$E = \sqrt{p^2 + m^2} \tag{2}$$

The momentum of electron beam are:$p = 20GeV/c$ The angular change of the electron beam is $6°$. So momentum transfer is $\Delta p = 2.094GeV/c$. According to uncertainty principle:

$$\Delta x \Delta p \sim \hbar \tag{3}$$

$$\Delta x = 9.453 * 10^{-17}$$

6 problem 6

(a) Because of energy conservation law:

$$\sqrt{p^2c^2 + m_p^2c^4} + m_pc^2 = \sqrt{(2m_p + m)^2c^4 + p^2c^2} \tag{4}$$

So we can solve the threshold energy and momentum:

$$\begin{aligned} E_p &= \frac{(2m_p^2 + 4m_pm + m^2)c^2}{2m_p} \\ p &= \frac{\sqrt{E_p^2 - m_p^2c^4}}{c} \end{aligned} \tag{5}$$

(b)

$$\begin{aligned} E_p^* &= \frac{(2m_p + m)c^2}{2} \\ p &= \frac{\sqrt{E_p^2 - m_p^2c^4}}{c} \end{aligned} \tag{6}$$

(c) We know the pion is π^0, and it's mass is $135MeV/c^2$.

$$E_p = 1217.7MeV, \quad E_p^* = 1005.5MeV \tag{7}$$

The kinetic energy in case (a):

$$K = E_p - m_pc^2 = 279.7MeV \tag{8}$$

7 Problem 7

(a) Because of energy conservation law:

$$pc + m_pc^2 = \sqrt{p^2c^2 + (m_p + m_{\pi^0})^2c^4} \tag{9}$$

We solve the threshold energy:

$$E_\gamma = 144.7MeV \tag{10}$$

(b) The collision between photon and proton must head to head.

$$p_\gamma c + \sqrt{p^2c^2 + m_p^2c^4} = \sqrt{(pc - p_\gamma c)^2 + (m_p + m_{\pi^0})^2c^4} \tag{11}$$

using Mathematica we can calculate $E_p = 6.78 * 10^{13} MeV$.

(c) The attenuation length$L = \frac{1}{\sigma\rho} = 1.67 * 10^{23} m$, So we compare it with light-year:$L = 1.76 * 10^7 l.y.$.

8 Problem 8

(a)

$$E_\gamma = 2.61 * 10^8 MeV$$

(b) The EBL photons energy is $E = 1.24eV$

$$E_\gamma = 2.61 * 10^5 MeV$$

9 Problem 9

The minimum energy is the head to head collision situation.

$$E_p = 2m_pc^2 = 1.876GeV \tag{12}$$

10 Problem 10

Consider Lorenz invariant s:

$$\begin{aligned} &s = (E_{\gamma_1} + E_{\gamma_2})^2 - (p_{\gamma_1} + p_{\gamma_2})^2 \\ &E_p = 1.044 * 10^{11} MeV \end{aligned} \tag{13}$$

11 Problem 11

$$\sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} = M \tag{14}$$

We can infer:

$$\begin{aligned} &E_{m_1} = \frac{M^2 + m_1^2 - m_2^2}{2M} \\ &E_{m_2} = \frac{M^2 + m_2^2 - m_1^2}{2M} \\ &p = \sqrt{E_{m_1}^2 - m_1^2} \end{aligned} \tag{15}$$

12 Problem 12

Decay $\Lambda \to p\pi^-$:

$$\begin{aligned} M &= 1115.6MeV \\ m_1 &= 938.3MeV \\ m_2 &= 139.5MeV \end{aligned} \tag{16}$$

$$\begin{aligned} E_{m_1} &= 943.6MeV \\ E_{m_2} &= 171.9MeV \\ p &= 100.4MeV/c \end{aligned} \tag{17}$$

Decay $\Xi^- \to \Lambda\pi^-$

$$\begin{aligned} M &= 1321.7MeV \\ m_1 &= 1115.6MeV \\ m_2 &= 139.5MeV \end{aligned} \tag{18}$$

$$\begin{aligned} E_{m_1} &= 1124.3MeV \\ E_{m_2} &= 197.4MeV \\ p &= 139.6MeV/c \end{aligned} \tag{19}$$

13 Problem 13

We could use the result in Problem 11.

$$\begin{aligned} E_{m_1} &= \frac{M^2 + m_1^2}{2M} \\ E_{m_2} &= \frac{M^2 - m_1^2}{2M} \\ p &= \frac{M^2 - m_1^2}{2Mc} \end{aligned} \tag{20}$$

14 Problem 14

We consider critical situation:μ at rest, neutrino carry all momentum.Because mass of Neutrino is very small, we treat it like photon.

$$\begin{aligned} \sqrt{p_\pi^2 + m_\pi^2} &= m_\mu + p_\pi \\ p_\pi &= 39.3MeV/c \end{aligned} \tag{21}$$

15 problem 15

(a) We can use result from Problem 12:

$$\begin{aligned} E_\Lambda &= 1115.6MeV \\ E_\pi &= 171.9MeV \\ p_\Lambda &= 0MeV/c \\ p_\pi &= 100.4MeV/c \end{aligned} \tag{22}$$

(b)

$$\begin{aligned} E_\Lambda &= \sqrt{p_\Lambda^2 + m_\Lambda^2} = 2.29GeV \\ \gamma_\Lambda &= \frac{E_\Lambda}{m_\Lambda} = 2.05 \\ \beta_\Lambda &= \frac{p_\Lambda}{E_\Lambda} = 0.873 \end{aligned} \tag{23}$$

(c) We use the Lorenz transformation:

$$\begin{aligned} & E^{'} = \gamma\beta p_x + \gamma E \\ & E_{lab,\pi} = 196.8MeV, \quad p_{lab,\pi} = 138.8MeV/c \end{aligned} \tag{24}$$

similarly, we can calculate the proton momentum.

$$\begin{aligned} E_{lab,p} &= 2089.9MeV \\ p_{lab,p} &= 1867.7MeV/c \end{aligned} \tag{25}$$

we know the p_y is a Lorenz invariant.

$$\begin{aligned} p_y &= p_y^{'} sin(\theta) = 50.2MeV/c \\ \theta &= arcsin(\frac{p_y}{p}) = 1.54° \end{aligned} \tag{26}$$

16 Problem 16

The angular between two final directions is 90°

17 Problem 17

$$\begin{aligned} & E_p = \sqrt{p^2 + m^2} = 3.14GeV \\ & s = (E_1 + E_2)^2 - (p_1 + p_2)^2 = 7.66GeV^2 \end{aligned} \tag{27}$$

Because s is a lorentz invariant:

$$\begin{aligned} & E_{p,CM} = 1.383GeV \\ & \gamma_{CM} = 1.47, \beta_{CM} = 0.735 \end{aligned} \tag{28}$$

relativity momentum transformation:

$$\begin{aligned} p_y^{'} &= p_y = 0.176GeV/c \\ p_{1x}^{'} &= p_{1x}\gamma + p_{10}\gamma\beta = 2.96GeV/c \\ p_{2x}^{'} &= p_{2x}\gamma - p_{20}\gamma\beta = 0.0231GeV/c \end{aligned} \tag{29}$$

$$\theta = arctan(\frac{p_y^{'}}{p_{1x}^{'}}) + arctan(\frac{p_y^{'}}{p_{2x}^{'}}) = 85.9^\circ \tag{30}$$

18 Problem 18

We know $m_{D^0} = 1864.8MeV$.

$$\beta = \sqrt{1 - (\frac{M}{E})^2} = 0.9981 \tag{31}$$

And we know the proper time relation:

$$\begin{aligned} d &= \frac{ct\beta}{\sqrt{1-\beta^2}} \\ t &= 6.21 * 10^{-13} \end{aligned} \tag{32}$$

we could use the result from Problem 11:

$$E_{\pi^+} = 871.2MeV p_{\pi^+} = 859.9MeV/c \tag{33}$$

19 Problem 19

Consider relativity effect:

$$\begin{aligned} \frac{ct\beta}{\sqrt{1-\beta^2}} &= l \\ \beta &= 0.99915 \end{aligned} \tag{34}$$

use relativity mass-energy relation:

$$\begin{aligned} p_{\pi^-} &= 3385MeV/c \\ E_{\pi^-} &= 3388MeV \end{aligned} \tag{35}$$

20 Problem 20

We know $m_p = 938.3MeV, m_n = 939.5MeV, m_{\pi^-} = 139.5MeV, m_{\pi^0} = 134.9MeV$.
We could use result from Problem 11.

$$\begin{aligned} E_n &= \frac{M^2 + m_n^2 - m_{\pi^0}^2}{2M} = 939.9MeV \\ E_{\pi^0} &= \frac{M^2 - m_n^2 + m_{\pi^0}^2}{2M} = 137.8MeV \\ K_n &= E_n - m_n = 0.43MeV \end{aligned} \tag{36}$$

according to $E_{\pi^0} = \frac{m_{\pi^0}}{\sqrt{1-\beta^2}}$,we can get $\beta_{\pi^0} = 0.2, l_{\pi^0} = 5nm$

21 Problem 27

The relation is:

$$p = BqR \tag{37}$$

22 Problem 28

$$10^3 \sigma l \rho N_A = \frac{N_0 - N_H}{N_0} \tag{38}$$

So $\sigma = 2.22 * 10^{-30} m^2$

23 Problem 29

$$\begin{gathered} \beta = \sqrt{\frac{p^2}{p^2 + m^2}} \\ \beta = 0.787 \end{gathered} \tag{39}$$

Cherenkov threshold is that $n > \frac{1}{\beta} = 1.27$.
If the index is 1.5, the Cherenkov angle is $\theta = arccos(\frac{1}{n\beta}) = 32.1°$

24 Problem 30

According to the relativity mass-momentum relation:

$$\beta = \sqrt{\frac{p^2}{p^2 + m^2}} \tag{40}$$

So we can get Δt:

$$\Delta t = 2L(\sqrt{1 + \frac{m_1^2}{p^2}} - \sqrt{1 + \frac{m_2^2}{p^2}}) \tag{41}$$

$$L_{min} = 25.4m \tag{42}$$

25 Problem 32

We know the red light wavelength is 700nm, green light wavelength is 500nm. use the Doppler effect formula:

$$\begin{aligned} \frac{\lambda_r}{\lambda_g} &= \sqrt{\frac{1+\beta}{1-\beta}} \\ \beta &= 0.324 \end{aligned} \tag{43}$$

So our superman's speed is very high.

26 Problem 33

(a)

$$v_{min} = \frac{c}{n} = 2.26 * 10^8 m/s \tag{44}$$

(b)

$$K_{min} = \frac{mc^2}{\sqrt{1-\beta^2}} - mc^2 \tag{45}$$

For the proton:

$$K_{min} = 484.7MeV \tag{46}$$

For the pion:

$$K_{min} = 72.0MeV \tag{47}$$

(c)

$$E_\pi = \frac{m_\pi c^2}{\sqrt{1-\beta^2}} \tag{48}$$

We can solve the $\beta = 0.937$.

$$\theta = arccos(\frac{c}{nv}) = arccos(\frac{1}{n\beta}) = 36.6° \tag{49}$$

27 Problem 35

Use formula $p(GeV) = 0.3B(T)R(m)$.
(a)In solar system:

$$p = 3000GeV/c \tag{50}$$

We need consider relativity effect:

$$E = \sqrt{p^2c^2 + m^2c^4} = 3000GeV \tag{51}$$

(b)In galaxy system:

$$\begin{aligned} p &= 1.5 * 10^{10} GeV/c \\ E &= pc = 1.5 * 10^{10} GeV \end{aligned} \tag{52}$$

28 Additional problem 1

We consider a δ function:

$$\begin{aligned} F(p) &= \int F(p^{'})\delta^{(3)}(p-p^{'})d^3p^{'} \\ &= \int F(p^{'})\sqrt{m^2+p^{'2}}\delta^{(3)}(p-p^{'})\frac{d^3p^{'}}{\sqrt{m^2+p^{'2}}} \end{aligned} \tag{53}$$

Because $\frac{d^3p^{'}}{\sqrt{m^2+p^{'2}}}$ is a lorentz invariant, $\sqrt{m^2+p^{'2}}\delta^{(3)}(p-p^{'})$ is also a lorentz invariant.

29 Additional problem 2

For two body decay:

$$\begin{aligned} \int d\Phi_2 &= \int (2\pi)^4\delta^{(4)}(p_i-p_f)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2} \\ &= \int (2\pi)^4\delta^{(3)}(p_1+p_2)\delta(E_1+E_2-\sqrt{s})\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2} \\ &= \int \frac{1}{4(2\pi)^2}\delta(\sqrt{p^2+m_1^2}+\sqrt{p^2+m_2^2}-\sqrt{s})\frac{d^3p}{\sqrt{p^2+m_1^2}\sqrt{p^2+m_2^2}} \\ &= \int \frac{1}{4(2\pi)^2}[\frac{\delta(p-p_0)E_1E_2}{p_0(E_1+E_2)}-\frac{\delta(p+p_0)E_1E_2}{p_0(E_1+E_2)}]\frac{d^3p}{\sqrt{p^2+m_1^2}\sqrt{p^2+m_2^2}} \\ &= \int \frac{\delta(p-p_0)-\delta(p+p_0)}{16\pi^2\sqrt{s}p_0}d^3p \\ &= \frac{p_0}{4\pi\sqrt{s}} \end{aligned} \tag{54}$$

and:

$$p_0 = \frac{(s^2+m_2^4+m_1^4-2sm_2^2-2sm_1^2-2m_1^2m_2^2)^{\frac{1}{2}}}{2\sqrt{s}} \tag{55}$$

For three body decay:

$$\begin{aligned} \int d\Phi_2 &= \int (2\pi)^4\delta^{(4)}(p_i-p_f)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3} \\ &= \int \frac{1}{8(2\pi)^5}\delta^{(3)}(p_1+p_2+p_3)\delta(\sqrt{s}-E_1-E_2-E_3)\frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\frac{d^3p_3}{E_3} \\ &= \int \frac{1}{8(2\pi)^5}\delta(\sqrt{s}-\sqrt{p_1^2+m_1^2}-\sqrt{p_2^2+m_2^2}-\sqrt{p_1^2+p_2^2+2p_1p_2cos(\theta)+m_3^2})\frac{d^3p_1d^3p_2}{E_1E_2E_3} \end{aligned} \tag{56}$$