homework 2

Chenxi Gu 2017311017

April 25, 2018

1 2.2

 $A(g_a)$ is a representation.

$$A(g_1)A(g_2) = A(g_1g_2) (1)$$

So we can get:

$$A^{T}(g_{1})^{-1}A^{T}(g_{2})^{-1} = [[A(g_{1})A(g_{2})]^{T}]^{-1} = A^{T}(g_{1}g_{2})^{-1}$$
 (2)

$$A^{\dagger}(g_1)^{-1}A^{\dagger}(g_2)^{-1} = [[A(g_1)A(g_2)]^{\dagger}]^{-1} = A^{\dagger}(g_1g_2)^{-1}$$
(3)

 $A^{T}(g_a)^{-1}, A^{\dagger}(g_a)^{-1}$ are also representation.

 $A^T(g_a), A^\dagger(g_a)$ are not representation, but Abel group. if $A(g_a)$ is a Unitary representation:

$$A^{\dagger}(g) = A(g^{-1})$$

$$A^{T\dagger}(g)^{-1} = A^{T}(g)$$

$$A^{\dagger\dagger}(g)^{-1} = A(g)^{-1}$$
(4)

So $A^T(g)^{-1}, A^{\dagger}(g)^{-1}$ are unitary representation. It is esay to prove they are irreducible representation.

2 2.3

It is easy to find the matrix commute with all elements in group A(g):

$$[A(g_a), \sum_C A(g_b)] = 0 \tag{5}$$

Using shur lemma :

$$\sum_{C} A(g_b) = \lambda E \tag{6}$$

3 2.7

We choose natural basic:

$$e = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad a = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad a^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad a^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
 (7)

So we can get the left representation:

$$L(e) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad L(a) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
(8)

The right presentation:

$$R(e) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R(a) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$
(9)

4 2.8

5 2

In the active view:

$$SO(2) = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0\\ sin(\theta) & cos(\theta) & 0\\ 0 & 0 & 1 \end{pmatrix}$$
 (10)

In the passive view:

$$SO(2) = \begin{pmatrix} cos(\theta) & sin(\theta) & 0\\ -sin(\theta) & cos(\theta) & 0\\ 0 & 0 & 1 \end{pmatrix}$$
 (11)

6 3

We can get the $D_{2n} = \{e, a, a^2, ..., a^{n-1}, b, ba, ba^2, ..., ba^{n-1}\}$ Secondly, we can find the class in the group:

$$\{e\} \\ \{a, a^{n-1}\} \\ \{a^2, a^{n-2}\} \\ \dots$$

$$\{a_k\} \\ \{b, b^2, \dots, b^{2k}\} \\ \{ba, ba^3, \dots b^{2k-1}\}$$

$$(b) n = 2k + 1$$

$$\{e\} \\ \{a, a^{n-1}\} \\ \{a^2, a^{n-2}\} \\ \dots$$

$$\{a_k, a_{k+1}\} \\ \{b, b^2, \dots, b^n\}$$

$$(12)$$