chapter 2

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1 2.1

$$\begin{aligned}
\sigma_k^2 &= \sum_{i,j=1}^{M} V_{ij} \\
&= \sum_{i,j=1}^{M} (\delta_{ij} n_{tot} p_i (1-p_i) + (\delta_{ij}-1) p_i p_j n_{tot}) \\
&= \sum_{i=1}^{M} (n_{tot} p_i (1-p_i) - p_i (p_0 - p_i) n_{tot}) \\
&= n_{tot}(p_0 - p_0^2)
\end{aligned} \tag{1}$$

While $p_0 = \sum_{i=1}^{M} p_i$

2 2.2

$$\begin{aligned}
E[\frac{1}{x}] &= \int_{\alpha}^{\beta} \frac{In(\beta) - In(\alpha)}{\beta - \alpha} = In(2) \\
\frac{1}{E[x]} &= \frac{2}{\beta + \alpha} = \frac{2}{3}
\end{aligned} \tag{2}$$

3 2.3

(a)

$$F(x) = \int_0^x f(x)dx = 1 - e^{-\frac{x}{\xi}} \tag{3}$$

(b)

$$P(x \leq x^{'}) = F(x^{'}) = 1 - e^{-\frac{x^{'}}{\xi}} \tag{4}$$

$$P(x \leq x^{'} + x_0 | x \geq x_0) = \frac{P(x_0 \leq x \leq x^{'} + x_0)}{P(x \geq x_0)} = \frac{e^{-\frac{x_0}{\xi}} - e^{-\frac{x_0 + x^{'}}{\xi}}}{e^{-\frac{x_0}{\xi}}} = 1 - e^{-\frac{x^{'}}{\xi}} \tag{5}$$

(c)

$$P(DecayInDetector|EnterDetector) = P(t \leq t_0) \tag{6}$$

So the time before the detector doesn't affect the lifetime.

4 2.4

(a)It is easy to say $\phi(x)$ is a Gaussian distribution.

$$\begin{aligned} E[x] &= E[\frac{y-\mu}{\sigma}] = \frac{E[y]-\mu}{\sigma} = 0 \\ V[x] &= V[\frac{y-\mu}{\sigma}] = \frac{V[y]}{\sigma^2} = 1 \end{aligned} \tag{7}$$

(b)

$$F(y) = \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy \tag{8}$$

$$\Phi(x) = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{9}$$

We could do variable substitution $x = \frac{y=\mu}{\sigma}$, equation(7),(8) are same.

5 2.5

(a)

$$g(y)dy = f(x)dx \tag{10}$$

$$f(x) = g(y)\frac{dy}{dx} = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(y-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(In(x)-\mu)^2}{2\sigma^2}} \tag{11}$$

(b)

$$\begin{aligned} E[x] &= \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(In(x)-\mu)^2}{2\sigma^2}} dx \\ &= \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} e^y dy \\ &= \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu-\sigma^2)^2}{2\sigma^2}} e^{\mu+\sigma^2/2} dy \\ &= e^{\mu+\sigma^2/2} \end{aligned} \tag{12}$$

$$\begin{aligned} V[x] &= \int (x-E[x])^2 f(x)dx \\ &= \int x^2 f(x)dx - E[x]^2 \\ &= \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu-2\sigma^2)^2}{2\sigma^2}} e^{2\mu+2\sigma^2} dy - e^{2\mu+\sigma^2} \\ &= e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} \end{aligned} \tag{13}$$

(c) According to error transfer formula:

$$\sigma_x^2 = e^{2\mu}\sigma^2 \tag{14}$$

When $\sigma^2 \ll 1$ and $\sigma^2 \ll \mu$, the formula is great.

6 2.6

$$\begin{aligned}
F_{\chi^2}(z;n) &= \int_0^z \frac{1}{2^{n/2}\Gamma(n/2)} z^{n/2-1} e^{-z/2} dz \\
let \quad x &= z/2 \\
&= \int_0^{\frac{z}{2}} \frac{1}{\Gamma(n/2)} x^{n/2-1} e^{-x} dx \\
&= P(\frac{z}{2}, \frac{n}{2})
\end{aligned} \tag{15}$$