chapter 5

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1 5.1

(a)

$$\begin{aligned} & P((x-\mu)>a)*a^2 \\ =& a^2 * \int_{|x-\mu|>a} f(x)dx \\ <& \int_{|x-\mu|>a} (x-\mu)^2 f(x)dx \\ <& \int (x-\mu)^2 f(x)dx \\ =& \sigma^2 \end{aligned} \tag{1}$$

(b)

$$P\left(\left|\frac{\sum(x_i-\mu)}{n}\right|>a\right) < P(|x_i-\mu|>a)^n < \left(\frac{\sigma^2}{a^2}\right)^n \tag{2}$$

2 5.2

(a)

$$E[\hat{s^2}] = limE(\frac{n}{n-1}(\bar{x^2}-\bar{x}^2)) = \sigma^2 \tag{3}$$

(b)

$$E[\hat{S^2}] = E(x^2-\mu^2) = \mu^2+\sigma^2-\mu^2 = \sigma^2 \tag{4}$$

3 5.3

(a)

$$s^2 = \frac{1}{n-1}\sum x_i^2 - \frac{1}{n(n-1)}\sum x_i x_j \tag{5}$$

$$E(s^4) = \frac{1}{(n-1)^2}\sum E[x_i x_j] - \frac{2}{n(n-1)^2}\sum E[x_i x_j x_k^2] + \frac{1}{n^2(n-1)^2}\sum E[x_i x_j x_k x_l] \tag{6}$$

$$E(s^4) = \frac{n(n-1)\mu_2^2 + n\mu_4}{(n-1)^2} - \frac{2(n\mu_4 + n(n-1)(n-2)\mu_1^2\mu_2 + n(n-1)\mu_2^2 + 2n(n-1)\mu_1\mu_3)}{n(n-1)^2} + \frac{n(n-1)(n-2)(n-3)\mu_1^4 + 6n(n-1)(n-2)\mu_1^2\mu_2 + 3n(n-1)\mu_2^2 + 4n(n-1)\mu_1\mu_3 + n\mu_4}{n^2(n-1)^2} \tag{7}$$

final we get :

$$V[s^2] = \frac{1}{n}\left(\mu_4 - \frac{n-3}{n-1}\mu_2^2\right) \tag{8}$$

(b)

$$V[s^2] = \frac{2}{n-1}\sigma^4 \tag{9}$$