chapter 6

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1 6.1

(a) We can get the likelihood function:

$$logL(\mu,\sigma^2) = \sum_{i=1}^{n}(log\frac{1}{\sqrt{2\pi}} + \frac{1}{2}log\frac{1}{\sigma^2} + \frac{(x_i-\mu)^2}{2\sigma^2}) \tag{1}$$

So we can get the maximum likelihood variation.

$$\begin{aligned} \hat{\mu} &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ \hat{\sigma^2} &= \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2 \end{aligned} \tag{2}$$

(b)

$$\begin{aligned} E[\hat{\mu}] = \frac{1}{n}\sum_{i=1}^{n}(\int \frac{x_i}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)}{2\sigma^2}} dx_i \times_{j<>i} \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_j-\mu)}{2\sigma^2}} dx_j) \\ = \mu \end{aligned} \tag{3}$$

$$E[\hat{\sigma^2}] = \frac{n-1}{n}\sigma^2 \tag{4}$$

$$\begin{aligned} V[\hat{\mu}] &= E[\hat{\mu}^2] - E[\hat{\mu}]^2 \\ &= \frac{\sigma^2}{n} \end{aligned} \tag{5}$$

$$\begin{aligned} V[\hat{\sigma^2}] &= E[\hat{\sigma^2}^2] - E[\hat{\sigma^2}]^2 \\ &= \frac{(n-1)^2}{n^3}(3\sigma^4 - \frac{n-3}{n-1}\sigma^3) \end{aligned} \tag{6}$$

(c)

$$V^{-1} = \begin{matrix} -\frac{n}{\sigma^2} & -\frac{1}{\sigma^4}\sum_{i=1}^{n}(x_i-\mu) \\ -\frac{1}{\sigma^4}\sum_{i=1}^{n}(x_i-\mu) & -\sum_{i=1}^{n}[\frac{(x_i-\mu)^2}{\sigma^6} - \frac{1}{2\sigma^4}] \end{matrix} \tag{7}$$

when the $n->\infty$, the answer is same.

2 6.2

The likelihood function:

$$L = C_N^n p^n (1-p)^{N-n} \tag{8}$$

$$\hat{p} = \frac{n}{N} \tag{9}$$

$$E(\hat{p}) = \frac{E(n)}{N} = p \tag{10}$$

$$V(\hat{p}) = \frac{p(1-p)}{N} \tag{11}$$

According to the 6.16:

$$V(\hat{p}) > \frac{1}{E[-\frac{\partial^2 logL}{\partial p^2}]} = \frac{N}{p(1-p)} \tag{12}$$

3 6.3

(a)

$$\hat{\alpha} = \frac{2n}{N} - 1 \tag{13}$$

$$\sigma_{\hat{\alpha}} = \sqrt{\frac{1-\alpha^2}{N}} \tag{14}$$

(b) $N > 9 * 10^6$

4 6.4

$$\hat{\nu} \tag{15}$$