homework 2

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1 2.2

$A(g_a)$ is a representation.

$$A(g_1)A(g_2) = A(g_1g_2) \tag{1}$$

So we can get :

$$A^T(g_1)^{-1}A^T(g_2)^{-1} = [[A(g_1)A(g_2)]^T]^{-1} = A^T(g_1g_2)^{-1} \tag{2}$$

$$A^\dagger(g_1)^{-1}A^\dagger(g_2)^{-1} = [[A(g_1)A(g_2)]^\dagger]^{-1} = A^\dagger(g_1g_2)^{-1} \tag{3}$$

$A^T(g_a)^{-1}, A^\dagger(g_a)^{-1}$ are also representation.

$A^T(g_a), A^\dagger(g_a)$ are not representation,but Abel group. if $A(g_a)$ is a Unitary representation:

$$\begin{aligned} &A^\dagger(g) = A(g^{-1}) \\ &A^{T\dagger}(g)^{-1} = A^T(g) \\ &A^{\dagger\dagger}(g)^{-1} = A(g)^{-1} \end{aligned} \tag{4}$$

So $A^T(g)^{-1}, A^\dagger(g)^{-1}$ are unitary representation.It is esay to prove they are irreducible representation.

2 2.3

It is easy to find the matrix commute with all elements in group $A(g)$:

$$[A(g_a), \sum_C A(g_b)] = 0 \tag{5}$$

Using shur lemma :

$$\sum_C A(g_b) = \lambda E \tag{6}$$

3 2.7

We choose natural basic :

$$e = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad a = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad a^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad a^3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

So we can get the left representation:

$$L(e) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad L(a) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{8}$$

The right presentation:

$$R(e) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad R(a) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

4 2.8

5 2

In the active view:

$$SO(2) = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

In the passive view:

$$SO(2) = \begin{pmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$