chapter 7

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1 7.1

(a) For the hypothesis $d = \alpha h$

$$\begin{aligned} \alpha &= 1.66277 \\ \chi^2 &= 661.99 \\ P &= 2.7 * 10^{-142} \end{aligned} \tag{1}$$

For the hypothesis $d = \alpha h + \beta h^2$

$$\begin{aligned} \alpha &= 2.79292 \\ \beta &= -0.00135055 \\ \chi^2 &= 64.7418 \\ P &= 5.7 * 10^{-14} \end{aligned} \tag{2}$$

(b) For the hypothesis $d = \alpha h^\beta$

$$\begin{aligned} \alpha &= 43.7608 \\ \beta &= 0.511055 \\ \chi^2 &= 3.75593 \\ P &= 0.289 \end{aligned} \tag{3}$$

(c) For the hypothesis $d = \alpha h^{0.5}$

$$\begin{aligned} \alpha &= 47.0857 \\ \chi^2 &= 4.20755 \\ P &= 0.378 \end{aligned} \tag{4}$$

2 7.2

(a) if $\sigma_i^2 = \lambda_i$:

$$\chi^2(\theta, \nu) = \sum_{i=1}^{N} = \frac{(y_i - \lambda_i)^2}{\lambda_i} \tag{5}$$

We know $\partial\chi^2/\partial\nu = 0$:

$$\begin{aligned} \frac{2}{\nu}\sum_{i=0}^{N} y_i - 2 + \frac{1}{\nu}\sum_{i=0}^{N}\frac{(y_i-\lambda_i)^2}{\lambda_i} &= 0 \\ \nu_{LS} = n + \frac{\chi^2_{min}}{2} \end{aligned} \tag{6}$$

(b) if $\sigma_i^2 = y_i$:

$$\chi^2(\theta,\nu) = \sum_{i=1}^{N} = \frac{(y_i-\lambda_i)^2}{y_i} \tag{7}$$

We know $\partial\chi^2/\partial\nu = 0$:

$$\nu_{MLS} = n - \chi^2_{min} \tag{8}$$

3 7.3

(a)

$$V_{ij} = \begin{cases} -np_ip_j & (i \neq j) \\ np_i(1-p_i) & (i = j) \end{cases} \tag{9}$$

We find the N-matrix only have N-1 independent variables, so it doesn't has inverse matrix.
(b)

$$V_{ij}^{-1} = \frac{1}{np_N}\begin{cases} 1 & (i \neq j) \\ 1 + \frac{p_N}{p_i} & (i = j) \end{cases} \tag{10}$$

$$\chi^2 = \sum_{i=1}^{N}\frac{(y_i - np_i)^2}{np_i} \tag{11}$$

So considering the fit using only the first N-1 bins is same with N bins.

4 7.4

(a)

$$\rho_{ij} = \frac{V_{ij}}{\sigma_i\sigma_j} \propto 1 \tag{12}$$

The matrix of correlation coefficients is independent of the sample size (b)

$$\delta_{ij} = \sum_k (V^{-1})_{ik}\rho_{kj}\sigma_k\sigma_j \tag{13}$$

Both time $(\rho^{-1})_{jm}/\sigma_j$:

$$\begin{aligned} (\rho^{-1})_{im}/\sigma_i &= \sum_k (V^{-1})_{ik}\delta_{km}\sigma_k \\ &= (V^{-1})_{im}\sigma_m \end{aligned} \tag{14}$$

We prove it.

5 7.5

(a) We know x_i are independent variants.

$$cov[x_i, x_j] = 0 \quad (i \neq j) \tag{15}$$

There are c variants are commom.

$$cov[y_i, y_j] = \frac{c\sigma^2}{mn} \tag{16}$$

(b)

$$V = \begin{bmatrix} \frac{\sigma^2}{n} & \frac{c\sigma^2}{mn} \\ \frac{c\sigma^2}{mn} & \frac{\sigma^2}{m} \end{bmatrix} \tag{17}$$

$$V^{-1} = \frac{mn}{\sigma^2(mn - c^2)} \begin{bmatrix} n & -c \\ -c & m \end{bmatrix} \tag{18}$$

$$\hat{y} = \frac{n-c}{n+m-2c} y_1 + \frac{m-c}{n+m-2c} y_2 \tag{19}$$

$$V[\hat{y}] = V[y_1] + V[y_2] + 2\frac{(n-c)(n-c)}{(n+m-2c)^2} cov[y_1, y_2] \tag{20}$$

6 7.6

(a) For the hypothesis $\theta_r = \alpha\theta_i$

$$\begin{aligned} &\alpha = 0.666 \\ &\chi^2 = 134.6507 \\ &P = 6.7 * 10^{-26} \end{aligned} \tag{21}$$

For the hypothesis $\theta_r = \alpha\theta_i - \beta\theta_i^2$

$$\begin{aligned} &\chi^2 = 1.26 * 10^{-29} \\ &P = 1 \end{aligned} \tag{22}$$

(b)

$$\begin{aligned} &\hat{r} = 1.311 \\ &\chi^2 = 14.14 \\ &P = 0.0487 \end{aligned} \tag{23}$$

So the 0.5 is too small.

7 7.7

(a)

$$\chi^2 = \sum_{i=1}^{N} \frac{(n_i - \theta a(x_i))^2}{\theta a(x_i)} \tag{24}$$

We use $\partial\chi^2/\partial\theta = 0$

$$\sum_{i=1}^{N}[a(x_i) - \frac{n_i^2}{\theta^2 a(x_i)}] = 0 \tag{25}$$

So we can get

$$\hat{\theta} = (\frac{\sum_{i=1}^{N} a(x_i)}{\sum_{i=1}^{N} \frac{a(x_i)^2}{n_i}})^{0.5} \tag{26}$$

compute the bias

$$\begin{aligned} b &= E[\hat{\theta}] - \theta \\ &= \frac{N-1}{2\sum_{i=1}^{N} a(x_i)} \end{aligned} \tag{27}$$

(b)

$$\chi^2 = \sum_{i=1}^{N} \frac{(n_i - \theta a(x_i))^2}{n_i} \tag{28}$$

We use $\partial\chi^2/\partial\theta = 0$

$$\sum_{i=1}^{N}[a(x_i) - \frac{\theta a(x_i)^2}{n_i}] = 0 \tag{29}$$

So we can get

$$\hat{\theta} = \frac{\sum_{i=1}^{N} a(x_i)}{\sum_{i=1}^{N} \frac{a(x_i)^2}{n_i}} \tag{30}$$

compute the bias

$$\begin{aligned} b &= E[\hat{\theta}] - \theta \\ &= -\frac{N-1}{\sum_{i=1}^{N} a(x_i)} \end{aligned} \tag{31}$$

8 7.8

(a) Take the standard deviation σ_i^2 of n_i to be ν_i (the usual method of least squares).

$$\begin{aligned} &\nu_0 = 1845.52 + 39.4143 \\ &k = (1.19938 + 0.0327322) * 10^{-23} J/K \\ &\chi^2 = 4.56885 \end{aligned} \tag{32}$$

(b) Take the standard deviation σ_i^2 of n_i to be n_i (the usual method of least squares).

$$\begin{aligned} &\nu_0 = 1843.84 + 40.2043 \\ &k = (1.19702 + 0.0342872) * 10^{-23} J/K \\ &\chi^2 = 4.61863 \end{aligned} \tag{33}$$