chapter 4

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1 4.1

(a) Significance level $\alpha = 0.1737$
(b) Power $1 - \beta = 0.8263$

$$P_{\pi \to e} = 0.1737 \tag{1}$$

(c)

$$P = \frac{0.01 * 0.8263}{0.01 * 0.8263 + 0.99 * 0.1737} = 0.046 \tag{2}$$

(d)

$$\frac{0.01 * \Phi(t)}{0.01 * \Phi(t) + 0.99 * \Phi(t-2)} = 0.95 \tag{3}$$

$$\begin{aligned} t &= -2.5155 \\ \epsilon_e &= 0.005943 \\ \alpha &= 1 - \epsilon_e = 0.994057 \end{aligned} \tag{4}$$

2 4.2

(a) When $J(\mathbf{a})$ have max value, satisfy $\frac{\partial J}{\partial a_i} = 0$. We can get n equations such as:

$$\frac{\sum_j a_j w_{ij}}{(\mu_0 - \mu_1)_i} = \frac{\sum_j a_j (\mu_0 - \mu_1)_j \sum_{i,j} a_i a_j w_{ij}}{\sum_{i,j} a_i a_j (\mu_0 - \mu_1)_i (\mu_0 - \mu_1)_j} \tag{5}$$

We find the right of the equation is a constant.So we make it equal k.

$$\mathbf{a} = k\mathbf{W}^{-1}(\boldsymbol{\mu}_0 - \boldsymbol{\mu}_1) \tag{6}$$

(b)

$$P(H_0|\mathbf{x}) = \frac{f(\mathbf{x}|H_0)\pi_0}{f(\mathbf{x}|H_0)\pi_0 + f(\mathbf{x}|H_1)\pi_1} = \frac{1}{1 + \frac{\pi_1}{\pi_0 r}} \tag{7}$$

while $r = exp[(\boldsymbol{\mu}_0 - \boldsymbol{\mu}_1)^T V^{-1}\mathbf{x} - \frac{1}{2}\boldsymbol{\mu}_0^T V^{-1}\boldsymbol{\mu}_0 + \frac{1}{2}\boldsymbol{\mu}_1^T V^{-1}\boldsymbol{\mu}_1]$
(c) using (b) result :

$$\begin{aligned} t &= In(\frac{\pi_0}{\pi_1}) + In(r) \\ a_0 &= In(\frac{\pi_0}{\pi_1}) - \frac{1}{2}\boldsymbol{\mu}_0^T V^{-1}\boldsymbol{\mu}_0 + \frac{1}{2}\boldsymbol{\mu}_1^T V^{-1}\boldsymbol{\mu}_1 \end{aligned} \tag{8}$$

3 4.3

$$P = 1 - Poisson_{cdf}(15, 3.9) = 0.0000035797 \tag{9}$$

4 4.4

(a) For two different theory:

$$\begin{aligned} \chi_1^2 &= 15.8193 \\ \chi_2^2 &= 35.9653 \end{aligned} \tag{10}$$

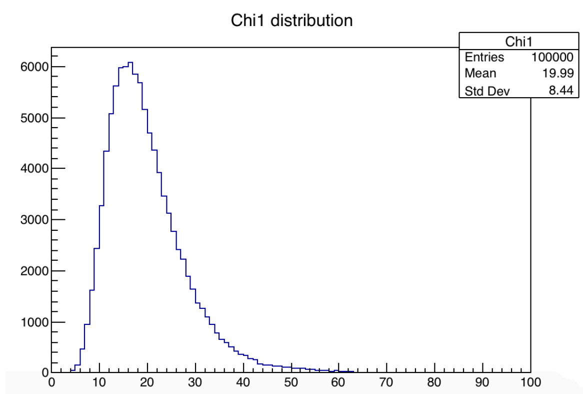

Figure 1: Chi1 distribution

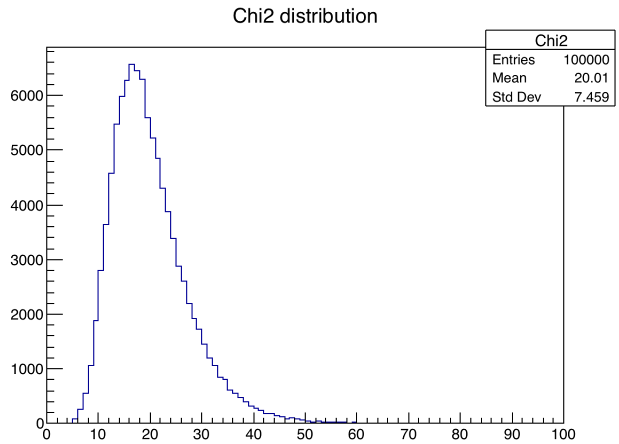

Figure 2: Chi2 distribution

We can calculate P value using Chi1 and Chi2 distribution:

$$\begin{aligned} P_1 &= 0.65251 \\ P_2 &= 0.0353 \end{aligned} \tag{11}$$

using χ^2 distribution:

$$\begin{aligned} P_1^* &= 0.727769 \\ P_2^* &= 0.015526 \end{aligned} \tag{12}$$

5 4.5

(a)

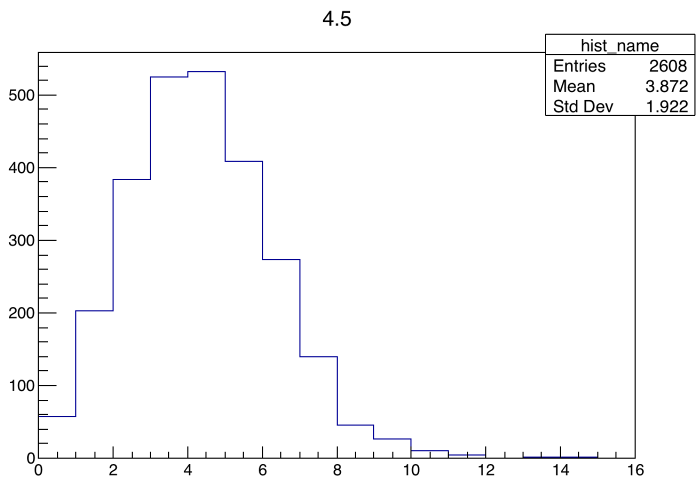

Figure 3: Data

$$t = \frac{s^2}{\bar{m}} = 0.954 \tag{13}$$

(b)If we use gaussian distribution:

$$P = 0.951749 \tag{14}$$

We should let $t = 1$ represent whether observations consistent with Poisson hypothesis.
(c)

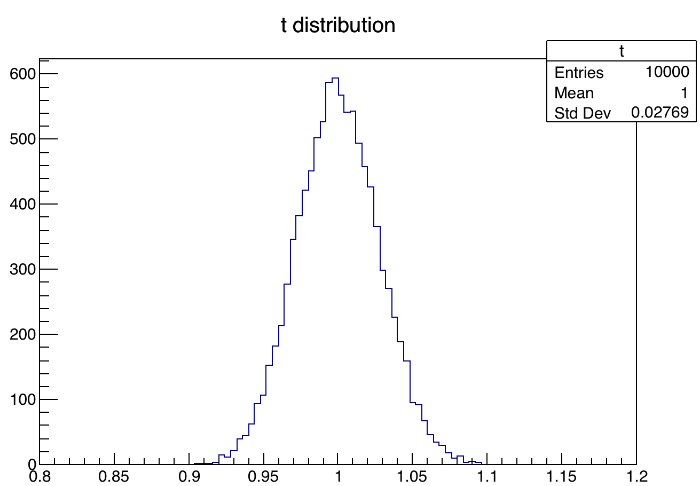

Figure 4: t distribution

$$P = 0.95268 \tag{15}$$