chapter 3

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1 3.1

	I	I_z	S	B	L	T	C	P	J	J_3
S	Y	Y	Y	Y	Y	N	Y	Y	Y	Y
EM	N	Y	Y	Y	Y	N	Y	Y	Y	Y
W	N	N	N	Y	Y	N	N	N	Y	Y

2 3.4

- $\pi^- + p \to \pi^0 + n$: strong interaction
- $\pi^+ \to \mu^+ + \nu_\mu$: weak interaction
- $\pi^+ \to \mu^+ + \bar{\nu}_\mu$: violate L_μ conservation law
- $\pi^0 \to \gamma + \gamma$: electromagnetic interaction
- $\pi^0 \to \gamma + \gamma + \gamma$: violate charge conjugation.
- $e^+e^- \to \gamma$: violate momentum and energy conservation law
- $p(uud) + \bar{p}(\bar{u}\bar{u}\bar{d}) \to \Lambda(uds) + \Lambda(uds)$: violate B and S
- $p(uud) + p(uud) \to \Sigma^+(uus) + \pi^+(u\bar{d})$: violate B and S
- $n \to p + e^-$: violate L_e and J and J_3 conservation law
- $n \to p + \pi^-$: violate energy conservation law

3 3.5

- $\mu^+ \to e^+ + \gamma$: violate L_e and L_μ conservation law
- $e^- \to \nu_e + \gamma$: violate charge conservation
- $p + p \to \Sigma^+ + K^+$: violate B

- $p + p \to p + \Sigma^+ + K^-$: violate charge conservation and S
- $p \to e^+ + \nu_e$: violate baryon conservation
- $p + p \to \Lambda + \Sigma^+$: violate charge and S conservation
- $p + n \to \Lambda + \Sigma^+$: violate S conservation
- $p + n \to \Xi^0(uss) + p$: violate S conservation
- $p \to n + e^+ + \nu_e$: violate energy conservation
- $n \to p + e^- + \nu_e$: violate L_e conservation

4 3.6

- $n \to p + e^-$: violate L_e conservation
- $n \to \pi^+ + e^-$: violate L_e conservation
- $n \to p + \pi^-$: violate energy conservation
- $n \to p + \gamma$: violate charge conservation

5 3.7

- $\pi^- + p \to K^- + p$: forbidden by S conservation
- $\pi^- + p \to K^+ + \Sigma^-$: allowed
- $K^- + p \to K^+ + \pi^- + \Xi^0$: allowed
- $K^+ + p \to K^- + \pi^- + \Xi^0$: forbidden by charge and S conservation

6 3.8

- $p \to n + e^+$: violate L_e and energy conservation
- $\mu^+ \to \nu_\mu + e^+$: violate L_e and L_μ conservation
- $e^+ + e^- \to \nu_\mu + \bar{\nu}_\mu$: allowed
- $\nu_\mu + p \to \mu^+ + n$: violate L_μ conservation
- $\nu_\mu + n \to \mu^- + p$: allowed
- $\nu_\mu + n \to e^- + p$: violate L_μ and L_e conservation
- $e^+ + n \to p + \nu_e$: violate L_e conservation
- $e^- + p \to n + \nu_e$: allowed

7 3.9

$$\begin{aligned}
\pi^+ p &= \left|\frac{3}{2},+\frac{3}{2}\right\rangle \\
\pi^- p &= \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \\
K^0\Sigma^0 &= \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \\
K^+\Sigma^- &= \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \\
K^+\Sigma^+ &= \left|\frac{3}{2},+\frac{3}{2}\right\rangle
\end{aligned} \tag{1}$$

With a proportionality constant N equal for all we obtain

$$\begin{aligned}
\sigma(\pi^- p \to K^0\Sigma^0) &= N\left|\frac{\sqrt{2}}{3}A_{3/2} - \frac{\sqrt{2}}{3}A_{1/2}\right|^2 \\
\sigma(\pi^- p \to K^+\Sigma^-) &= N\left|\frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2}\right|^2 \\
\sigma(\pi^+ p \to K^+\Sigma^+) &= N\left|A_{3/2}\right|^2
\end{aligned} \tag{2}$$

they proceed only through the $I = \frac{3}{2}$ channel:

$$\sigma(\pi^- p \to K^0\Sigma^0) : \sigma(\pi^- p \to K^+\Sigma^-) : \sigma(\pi^+ p \to K^+\Sigma^+) = 2:1:9 \tag{3}$$

8 3.10

Using the result in 3.9.

$$\begin{aligned}
&\sigma(\pi^- p \to K^0\Sigma^0) : \sigma(\pi^- p \to K^+\Sigma^-) : \sigma(\pi^+ p \to K^+\Sigma^+) \\
=&\left|\frac{\sqrt{2}}{3}A_{3/2} - \frac{\sqrt{2}}{3}A_{1/2}\right|^2 : \left|\frac{1}{3}A_{3/2} + \frac{2}{3}A_{1/2}\right|^2 : \left|A_{3/2}\right|^2
\end{aligned} \tag{4}$$

9 3.11

$$\begin{aligned}
\pi^- p &= \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \\
\pi^+ n &= \sqrt{\frac{1}{3}}\left|\frac{3}{2},+\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},+\frac{1}{2}\right\rangle \\
\Lambda K^0 &= \left|\frac{1}{2},-\frac{1}{2}\right\rangle \\
\Lambda K^+ &= \left|\frac{1}{2},+\frac{1}{2}\right\rangle
\end{aligned} \tag{5}$$

With a proportionality constant N equal for all we obtain

$$\begin{aligned} \sigma(\pi^- p \to \Lambda K^0) &= N \left| \sqrt{\frac{2}{3}} A_{1/2} \right|^2 \\ \sigma(\pi^+ n \to \Lambda K^+) &= N \left| \sqrt{\frac{2}{3}} A_{1/2} \right|^2 \end{aligned} \tag{6}$$

So the ratio of cross-sections is 1 : 1

10 3.12

Same analysis as 3.9

$$\sigma(p + d \to {}^3\mathrm{He} + \pi^0) : \sigma(p + d \to {}^3\mathrm{H} + \pi^+) = 1 : 2 \tag{7}$$

11 3.13

Same analysis as 3.9

$$\frac{\sigma(pp \to d\pi^+)}{\sigma(pn \to d\pi^0)} = 2 \tag{8}$$

12 3.14

$$\frac{\sigma(K^- + {}^4\mathrm{He} \to \Sigma^0 + {}^3\mathrm{H})}{\sigma(K^- + {}^4\mathrm{He} \to \Sigma^- + {}^3\mathrm{He})} = 2 \tag{9}$$

13 3.15

$$K^- p = \sqrt{\frac{1}{2}} |1, 0\rangle - \sqrt{\frac{1}{2}} |0, 0\rangle \tag{10}$$

$$\pi^+ \Sigma^- = \sqrt{\frac{1}{6}} |2, 0\rangle + \sqrt{\frac{1}{2}} |1, 0\rangle + \sqrt{\frac{1}{3}} |0, 0\rangle \tag{11}$$

$$\pi^0 \Sigma^0 = \sqrt{\frac{2}{3}} |2, 0\rangle - \sqrt{\frac{1}{3}} |0, 0\rangle \tag{12}$$

$$\pi^- \Sigma^+ = \sqrt{\frac{1}{6}} |2, 0\rangle - \sqrt{\frac{1}{2}} |1, 0\rangle + \sqrt{\frac{1}{3}} |0, 0\rangle \tag{13}$$

$$\begin{aligned} &\sigma(K^- p \to \pi^+ \Sigma^-) : \sigma(K^- p \to \pi^0 \Sigma^0) : \sigma(K^- p \to \pi^- \Sigma^+) \\ &= \left| \frac{1}{2} A_1 - \sqrt{\frac{1}{6}} A_0 \right|^2 : \left| \frac{1}{6} A_0 \right|^2 : \left| \frac{1}{2} A_1 + \sqrt{\frac{1}{6}} A_0 \right|^2 \end{aligned} \tag{14}$$

14 3.16

Using the result of 3.11:

$$\sigma(\pi^- p \to \pi^- p) = N \left| \frac{1}{3} A_{3/2} + \frac{2}{3} A_{1/2} \right|^2 \tag{15}$$

And the spin for $\pi^0 n$ system.

$$\begin{aligned} \pi^0 n &= \sqrt{\frac{2}{3}} \left| \frac{3}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{1}{3}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \\ \sigma(\pi^- p \to \pi^0 n) &= N \left| \frac{\sqrt{2}}{3} A_{3/2} - \frac{\sqrt{2}}{3} A_{1/2} \right|^2 \end{aligned} \tag{16}$$

So

$$\frac{\sigma(\pi^- p \to \pi^- p)}{\sigma(\pi^- p \to \pi^0 n)} = \frac{\left| \frac{1}{3} A_{3/2} + \frac{2}{3} A_{1/2} \right|^2}{\left| \frac{\sqrt{2}}{3} A_{3/2} - \frac{\sqrt{2}}{3} A_{1/2} \right|^2} \tag{17}$$

15 3.17

(a)For the S wave : $P_{\pi^- d} = -1$, the nn system orbital momentum is 1. And the nn system wave function must be antisymmetric. Since the spatial part is antisymmetric, the spin function is symmetric. We can get S=1
(b)For the P wave : $P_{\pi^- d} = +1$, the nn system orbital momentum is 0 or 2. The total spin is 0.

16 3.18

(1)

$$C = (-1)^{l+s} \tag{18}$$

(2)

$$(-1)^{l+s} = (-1)^n \tag{19}$$

(3)For the ortho-positronium minimum number photon is 3,para-positronium minimum number photons are 2.

17 3.19

(1)

$$C(\bar{p}p) = (-1)^{l+s} = C(n\pi^0) = +1 \tag{20}$$

So the state are ${}^0S_1, {}^3P_0, {}^3P_1, {}^3P_2, {}^1D_2$

(2) $P(2\pi^0)$ must be symmetric the L is oven, for the $2\pi^0$ system $J = L$ is oven.We also know $P(\bar{p}p) = (-1)^{L+1}$, L is odd. ${}^3P_2$ and ${}^3P_0$ satisfy the condition.

18 3.20

Because the $I = 0$ is symmetric, $P(\pi^+\pi^-) = (-1)^l$ l is even.
For $I = 1$ is antisymmetric, l is odd.

19 3.21

We know $\bar{p}p$ system P and C:

$$\begin{aligned} P(\bar{p}p) &= (-1)^{l+1} \\ C(\bar{p}p) &= (-1)^{l+s} \end{aligned} \tag{21}$$

(a)S wave we can get:

$$\begin{aligned} &P(\pi^+\pi^-) = +1 \\ &C(\pi^+\pi^-) = +1 \\ &J_{\pi^+\pi^-} = 0 \end{aligned} \tag{22}$$

So only ${}^3P_0$ is allowed. (b)Follow the analysis in (a).${}^3S_1, {}^3D_1$ are allowed. (c)${}^3P_2$ is allowed.

20 3.22

- $\pi^+p \to D^+p$: is allowed.
- $\pi^+p \to D^-\Lambda_c\pi^+\pi^+$: is allowed.
- $\pi^+p \to D^+\Lambda_c$: is allowed.
- $\pi^+p \to D^-\Lambda_c$: is not allowed.

21 3.23

- $\pi^-p \to D^0\Lambda_b$: is allowed.
- $\pi^-p \to B^0\Lambda_b$: is allowed.
- $\pi^-p \to B^+\Lambda_b\pi^-$: is allowed.
- $\pi^-p \to B^-\Lambda_b\pi^+$: is allowed.
- $\pi^-p \to B^-B^+$: is not allowed.

22 3.25

(a)If $I = \frac{3}{2}$

$$\frac{\sigma(\Delta^0 \to p\pi^-)}{\sigma(\Delta^0 \to n\pi^0)} = 1:2 \tag{23}$$

(b)If $I = \frac{1}{2}$

$$\frac{\sigma(\Delta^0 \to p\pi^-)}{\sigma(\Delta^0 \to n\pi^0)} = 2:1 \tag{24}$$

23 3.27

- $\mu^- \to e^- + \gamma$: forbidden by L_e and L_μ conservation.
- $\pi^+ \to \mu^+ + \nu_\mu + \bar{\nu}_\mu$: forbidden by L_μ conservation.
- $\Sigma^0 \to \Lambda + \gamma$: allowed.
- $\eta \to \gamma + \gamma + \gamma$: forbidden by C conservation.
- $\gamma + p \to \pi^0 + p$: is allowed.
- $p \to \pi^0 + e^+$: forbidden by L_e conservation.
- $\pi^- \to \mu^- + \gamma$: forbidden by L_μ conservation.

24 3.28

- $\pi^- + p \to \Sigma^0 + K^0$
- $e^+ + n \to p + \bar{\nu}_e$
- $\Xi^0 \to \Lambda + \bar{K}^0$

25 3.29

We consider the I_z:

$$1 + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + I_{z_{\Xi^0}} \tag{25}$$

So $I_{z_{\Xi^0}} = \frac{1}{2}$.

Second,$\pi^+ p = \left|\frac{3}{2}, \frac{3}{2}\right\rangle$ and $K^+K^+ = |1, 1\rangle$.
The I_{Ξ^0} could be $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$.