homework 2

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1 2.2

$A(g_a)$ is a representation.

$$A(g_1)A(g_2) = A(g_1 g_2) \tag{1}$$

So we can get :

$$A^T(g_1)^{-1}A^T(g_2)^{-1} = [[A(g_1)A(g_2)]^T]^{-1} = A^T(g_1 g_2)^{-1} \tag{2}$$

$$A^\dagger(g_1)^{-1}A^\dagger(g_2)^{-1} = [[A(g_1)A(g_2)]^\dagger]^{-1} = A^\dagger(g_1 g_2)^{-1} \tag{3}$$

$A^T(g_a)^{-1}, A^\dagger(g_a)^{-1}$ are also representation.

$A^T(g_a), A^\dagger(g_a)$ are not representation,but Abel group. if $A(g_a)$ is a Unitary representation:

$$\begin{aligned} &A^\dagger(g) = A(g^{-1}) \\ &A^{T\dagger}(g)^{-1} = A^T(g) \\ &A^{\dagger\dagger}(g)^{-1} = A(g)^{-1} \end{aligned} \tag{4}$$

So $A^T(g)^{-1}, A^\dagger(g)^{-1}$ are unitary representation.It is esay to prove they are irreducible representation.

2 2.3

It is easy to find the matrix commute with all elements in group $A(g)$:

$$[A(g_a), \sum_C A(g_b)] = 0 \tag{5}$$

Using shur lemma :

$$\sum_C A(g_b) = \lambda E \tag{6}$$

3 2.7

We choose natural basic :

$$e = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad a = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad a^2 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad a^3 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{7}$$

So we can get the left representation:

$$L(e) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \quad L(a) = \begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix} \tag{8}$$

The right presentation:

$$R(e) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \quad R(a) = \begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{pmatrix} \tag{9}$$

4 2.8

In the representation $A^p(g_a)$, the character is $\chi^p(g_a)$.In the representation $A^r(g_a)$, the character is $\chi^r(g_a)$.
Since $A^p(g_a)$ and $A^r(g_a)$ are Irreducible unequivalent representation.

$$\sum_a \chi^p(g_a)\chi^{r*}(g_a) = 0 \tag{10}$$

this equation also mean the identical representation and $A^p(g_a) \otimes A^{r*}(g_a)$ are orthogonal.

$$\sum_a \chi^p(g_a)\chi^{p*}(g_a) = 1 \tag{11}$$

For the same reason, $A^p(g_a) \otimes A^{p*}(g_a)$ only have 1 identical representation.

5 2

In the active view:

$$SO(2) = \begin{pmatrix}cos(\theta) & -sin(\theta) & 0\\ sin(\theta) & cos(\theta) & 0\\ 0 & 0 & 1\end{pmatrix} \tag{12}$$

In the passive view:

$$SO(2) = \begin{pmatrix}cos(\theta) & sin(\theta) & 0\\ -sin(\theta) & cos(\theta) & 0\\ 0 & 0 & 1\end{pmatrix} \tag{13}$$

6 3

We can get the $D_{2n} = \{e, a, a^2..., a^{n-1}, b, ba, ba^2....., ba^{n-1}\}$
Secondly, we can find the class in the group:
(a)$n = 2k$

$$\begin{aligned} \{e\} \\ \{a, a^{n-1}\} \\ \{a^2, a^{n-2}\} \\ .... \\ \{a_k\} \\ \{b, b^2...., b^{2k}\} \\ \{ba, ba^3...b^{2k-1}\} \end{aligned} \tag{14}$$

So we can deduce there are 4 1-dimention representation.

$$\begin{pmatrix} a = 1 \\ b = 1 \end{pmatrix} \quad \begin{pmatrix} a = 1 \\ b = -1 \end{pmatrix} \quad \begin{pmatrix} a = -1 \\ b = 1 \end{pmatrix} \quad \begin{pmatrix} a = -1 \\ b = -1 \end{pmatrix} \tag{15}$$

And $\frac{n}{2} - 1$ 2-dimentions representation, the i_{th} 2-dimentions representation is:

$$A_i^{(2)} = \left\{ e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad a = \begin{pmatrix} cos(\frac{2\pi i}{n}) & -sin(\frac{2\pi i}{n}) \\ sin(\frac{2\pi i}{n}) & cos(\frac{2\pi i}{n}) \end{pmatrix} \quad b = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \tag{16}$$

(b)$n = 2k + 1$

$$\begin{aligned} \{e\} \\ \{a, a^{n-1}\} \\ \{a^2, a^{n-2}\} \\ .... \\ \{a_k, a_{k+1}\} \\ \{b, b^2...., b^n\} \end{aligned} \tag{17}$$

So we can deduce there are 2 1-dimention representation.

$$\begin{pmatrix} a = 1 \\ b = 1 \end{pmatrix} \quad \begin{pmatrix} a = 1 \\ b = -1 \end{pmatrix} \tag{18}$$

And $\frac{n-1}{2}$ 2-dimentions representation, the i_{th} 2-dimentions representation is:

$$A_i^{(2)} = \left\{ e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad a = \begin{pmatrix} cos(\frac{2\pi i}{n}) & -sin(\frac{2\pi i}{n}) \\ sin(\frac{2\pi i}{n}) & cos(\frac{2\pi i}{n}) \end{pmatrix} \quad b = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \tag{19}$$

7 4

We know the pauli group:

$$\begin{aligned} &[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \\ &\sigma_i^2 = 1 \end{aligned} \tag{20}$$

So we can get the group element:

$$G = \{\pm 1, \pm i, \pm\sigma_x, \pm i\sigma_x, \pm\sigma_y, \pm i\sigma_y, \pm\sigma_z, \pm i\sigma_z\} \tag{21}$$

It is easy to find there are 10 classes.8 1-dimention representation and 2 2-dimentions representation.
2-dimentions representation:

$$G_1^{(2)} = \left\{1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad i = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\} \tag{22}$$

$$G_2^{(2)} = \left\{1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad i = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\} \tag{23}$$