CPSC 121: Models of Computation
Assignment #4

Due: Wednesday, March 26, 2014 at 17:00 **Total Marks: 30**

Submission Instructions-- read carefully

All assignments should be done in groups of 2. It is very important to work with another student and exchange ideas. **Each group should submit ONE assignment**. Type or write your assignment on clean sheets of paper with question numbers prominently labelled. Answers that are difficult to read or locate may lose marks. We recommend working problems on a draft copy then writing a separate final copy to submit.

Your submission must be **STAPLED** and include the **CPSC 121 assignment cover page** – located at the Assignments section of the course web page. Additionally, include your names at the top of each page. We are not responsible for lost pages from unstapled submissions.

Submit your assignment to the appropriately marked box in room **ICCS X235** by the due date and time listed above. **Late submissions are not accepted**.

Note: the number of marks allocated to a question appears in square brackets after the question number.

A Note on the Marking Scheme

Most items (i.e., question or, for questions divided into parts, part of a question) will be worth 3 marks with the following general marking scheme:

- **3 marks:** correct, complete, legible solution.
- **2 marks**: legible solution contains some errors or is not quite complete but shows a clear grasp of how the concepts and techniques required apply to this problem.
- **1 mark**: legible solution contains errors or is not complete but shows a clear grasp of the concepts and techniques required, although not their application to this problem or the solution is somewhat difficult to read but otherwise correct.
- **0 marks**: the solution contains substantial errors, is far from complete, does not show a clear grasp of the concepts and techniques required, or is illegible.

This marking scheme reflects our intent for you to learn the key concepts and techniques underlying computation, determine where they apply, and apply them correctly to interesting problems. It also reflects a practical fact: we have insufficient time to decipher illegible answers. At the instructor's discretion, some items may be marked on a different scale. TAs may very occasionally award a bonus mark for exceptional answers.

Question 1 [6]

Consider the following theorem:

Every odd positive integer can be written as the difference of two perfect squares.

[A number m is a perfect square, if $m = k^2$, for some number k.]

a. Translate this theorem into predicate logic. You may use predicates *Odd(x)* and *Even(x)* to indicate odd and even numbers,

b. Prove the theorem using a direct proof. *Hint*: What is $(x + 1)^2$

Question 2 [6]

Consider the following theorem:

For all integers a, b and c, if a does not divide (b – c) then a does not divide b or a does not divide c.

a. Translate this theorem into predicate logic. You may use the following predicate
Divides(x, y) : x divides y

b. Prove the theorem using an ***indirect*** proof.

Question 3 [6]

Prove the following statement:

For every integer n ≥ 3, the value n(n-1)(n-2)/6 is an integer.

Hint: You should first prove that ***for any integer m, if m/2 and m/3 are both integers, then m/6 is also an integer***. You may also use the following facts

- ***One of any three consecutive integers is divisible by 3.***
- ***The product of two odd integers is an odd integer.***

Question 4 [6]

A group of CPSC 121 students show up in a room for a tutorial. The TA is late, and so the students start talking to each other. Use a proof by contradiction to show that

If every student has talked to at least one other student, then two of the students talked to exactly the same number of people.

You don’t need to write the statement in logic, but make it clear which are your premises and which is the conclusion in your proof. For convenience, you can use

- **S** to denote the group of students, and
- **Talk(s, n)** = student s has talked to n other students

Question 5 [6]

Design a DFA for a vending machine that sells a number of three items (lemon juice, orange juice, etc.) for 35¢ each. It should accept 10¢ and 25¢ coins, and does not need to return change. Your machine should verify that the deposited amount is equal or larger than the item price.

a) Draw a DFA for the problem.
b) Draw a circuit that implements your DFA.