CPSC 121: Models of Computation
Assignment #1

Due: Friday January 24, 500 pm **Total: 36 Marks**

<u>Submission Instructions</u>-- read carefully

All assignments should be done in groups of 2. It is very important to work with another student and exchange ideas. **Each group should submit ONE assignment**. Type or write your assignment on clean sheets of paper with question numbers prominently labelled. Answers that are difficult to read or locate may lose marks. We recommend working problems on a draft copy then writing a separate final copy to submit.

Your submission must be **STAPLED** and include the **CPSC 121 assignment cover page** – located at the Assignments section of the course web page. Additionally, include your names at the top of each page. We are not responsible for lost pages from unstapled submissions.

Submit your assignment to the appropriately marked box in room **ICCS X235** by the due date and time listed above. **Late submissions are not accepted**.

Note: the number of marks allocated to a question appears in square brackets after the question number.

<u>A Note on the Marking Scheme</u>

Most items (i.e., question or, for questions divided into parts, part of a question) will be worth 3 marks with the following general marking scheme:

- **3 marks:** correct, complete, legible solution.
- **2 marks**: legible solution contains some errors or is not quite complete but shows a clear grasp of how the concepts and techniques required apply to this problem.
- **1 mark**: legible solution contains errors or is not complete but shows a clear grasp of the concepts and techniques required, although not their application to this problem or the solution is somewhat difficult to read but otherwise correct.
- **0 marks**: the solution contains substantial errors, is far from complete, does not show a clear grasp of the concepts and techniques required, or is illegible.

This marking scheme reflects our intent for you to learn the key concepts and techniques underlying computation, determine where they apply, and apply them correctly to interesting problems. It also reflects a practical fact: we have insufficient time to decipher illegible answers. At the instructor's discretion, some items may be marked on a different scale. TAs may very occasionally award a bonus mark for exceptional answers.

Question 1 [12]

One way to better understand a computational system is to look at the minimum set of primitives (simple operations) that are sufficient to express all the tasks performed by the system. In this question, you will prove that every truth table can be implemented by a circuit that uses only 2-input NAND gates. To do that you need to show the following steps:

[3] a. Show that ~ can be simulated using a NAND gate. That is, design a circuit of NAND gates, that takes as input a signal x, and whose output is ~x.

[3] b. Show that ^ can be simulated using NAND gates. That is, design a circuit whose only gates are NAND gates, that takes as inputs two signals x and y, and whose output is x ^ y. (Hint: take advantage of what you learned in the previous part!)

[3] c. Show that v can be simulated using NAND gates. That is, design a circuit whose only gates are NAND gates, that takes as inputs two signals x and y, and whose output is x v y. (Hint: take advantage of what you learned in the previous parts!)

[3] d. Now argue that any logic function that is represented by a truth table over k atomic propositions can be implemented with a circuit that uses only 2-input NAND gates.

Question 2 [6]

The truth table below defines the truth value of s for each combination of truth values of p, q and r.

p	**q**	**r**	**s**
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	F

a. [3] Find a logic formula for **s** that uses each variable name at most twice. Then verify the correctness of your formula by drawing a truth table corresponding to this formula, including the truth values of all the subformulas.

Hint: try to divide the rows of the truth table which contribute to the formula into two groups based on p*, find a logical formula for each group, and then combine them appropriately.*

b. [3] Draw a circuit with inputs **p**, **q** and **r**, whose output is the value **s** described by the truth table.

Question 3 [9]

Consider the following logical expressions and circuits:

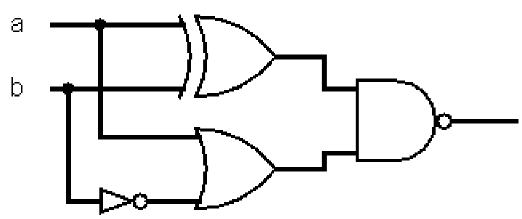
a) $(c \vee a) \rightarrow (c \vee b)$

b) $\sim a \vee b$

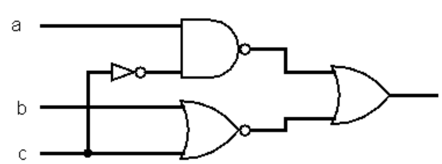
c) $c \vee (a \rightarrow b)$

d) $(a \wedge (b \vee c)) \rightarrow (a \wedge c)$

e)



f)



Each proposition or circuit is logically equivalent to exactly one other proposition or circuit. For each of them, write down which one it is equivalent to and provide a proof for that equivalency. Each proof of the three logical equivalences must use a sequence of the logical equivalences that are listed in the “Logical Equvalence Laws”and “Implication” sections of the 121 “official” formula list shown at http://www.ugrad.cs.ubc.ca/~cs121/2013W2/handouts/formulasheet/formulasheet.pdf , not a truth table. (see Epp-4 theorem 2.1.1, Epp-3 theorem 1.1.1, Rosen-6 table 6 in section 1.2, Rosen-7 table 6 in section 1.3; you can also assume that $x \rightarrow y \equiv \sim x \vee y$). You may also use the abbreviations for the rule names which are shown on this handout.

Question 4 [6]

Design a circuit that takes three inputs a, b, and c and returns the value that the majority of inputs have (this is called a majority vote).

a. [3] Show the formula for this problem and explain how did you find it.
b. [3] Show the circuit.

Question 5 [3]

Let's do a puzzle now for fun. This is an adaptation from Shakespeare's *The Merchant of Venice*. Portia, wants to find a very intelligent husband. She does not care about his fortune, but she cares about his brains. So, she asks her boyfriend to play the following game with her. She puts on the table a gold, a silver and a lead box, inside one of which is an engagement ring, and asks her boyfriend to choose one box. If he chooses the one with the ring, she will marry him. Otherwise, they will break up.

The three boxes have the following statements on top:

- GOLD: The ring is in this box.
- SILVER: The ring is not in this box.
- LEAD: The ring is not in the gold box.

Portia explains to her boyfriend that at most one of the three statements is true. Which box should her boyfriend choose if he wants to marry Portia?
[Hint: Think about which statement is consistent with all the facts.]