1. We conjecture that the sum follows the formula $S(n) = \frac{n}{n+1}$. Writing the given series in summation notation, we have the following equation:

$$S(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

Proof. When $n = 1$, the LHS of $S(1)$ is $\frac{1}{1(1+1)} = \frac{1}{2}$, and the RHS is $\frac{1}{2}$, so S(1) is true.

Without loss of generality, suppose that k is an arbitrary integer with $k \geq 1$ such that

$$S(k) = \sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$$

Assuming that this is true for $S(k)$, we must show that $S(k+1)$ is true:

$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{1}{(i+1)(i+2)} &= \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} \\
&= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \\
&= \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)} \\
&= \frac{k^2+2k+1}{(k+1)(k+2)} \\
&= \frac{(k+1)^2}{(k+1)(k+2)} \\
&= \frac{k+1}{k+2} \\
&= \frac{k+1}{(k+1)+1}
\end{aligned}$$

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2. *Proof.* We proceed by strong induction on n.

When $n = 1$, $F_1 = 1 < 2^1$.

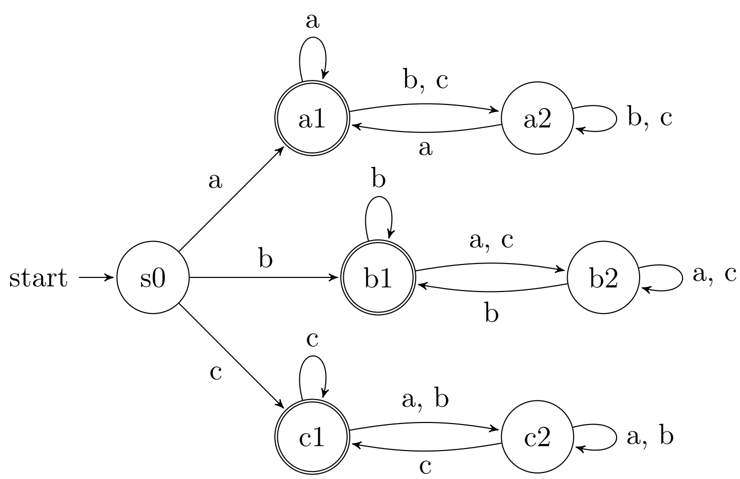
When $n = 2$, $F_2 = 1 < 2^2$.

Suppose that k is an arbitrary integer with $k \geq 2$ such that $F_k < 2^k$. Assuming that this is true for all k from 1 to k, we must show that $F_{k+1} < 2^{k+1}$:

$$\begin{aligned}
F_{k+1} &= F_k + F_{k-1} \\
F_k + F_{k-1} &< 2^k + 2^{k-1} \\
2^k + 2^{k-1} &= 2^k + \frac{2^k}{2} \\
2^k + \frac{2^k}{2} &= \frac{3}{2} \cdot 2^k \\
\frac{3}{2} \cdot 2^k &< 2 \cdot 2^k \\
2 \cdot 2^k &= 2^{k+1}
\end{aligned}$$

■

3. A DFA that takes in a string over the alphabet {a, b, c} and accepts exactly the strings that start and end with the same letter is shown below:



4. (a) A*B+A(A+B+A)*B[AB]*
   (b) c*(ac|bc|c)*[ab]?

5.