CPSC 121: Models of Computation
Assignment #2

Due: Friday February 7, 5:00 pm **Total: 33 Marks**

Submission Instructions-- read carefully

All assignments should be done in groups of 2. It is very important to work with another student and exchange ideas. **Each group should submit ONE assignment**. Type or write your assignment on clean sheets of paper with question numbers prominently labelled. Answers that are difficult to read or locate may lose marks. We recommend working problems on a draft copy then writing a separate final copy to submit.

Your submission must be **STAPLED** and include the **CPSC 121 assignment cover page** – located at the Assignments section of the course web page. Additionally, include your names at the top of each page. We are not responsible for lost pages from unstapled submissions.

Submit your assignment to the appropriately marked box in room **ICCS X235** by the due date and time listed above. **Late submissions are not accepted**.

Note: the number of marks allocated to a question appears in square brackets after the question number.

A Note on the Marking Scheme

Most items (i.e., question or, for questions divided into parts, part of a question) will be worth 3 marks with the following general marking scheme:

- **3 marks:** correct, complete, legible solution.
- **2 marks**: legible solution contains some errors or is not quite complete but shows a clear grasp of how the concepts and techniques required apply to this problem.
- **1 mark**: legible solution contains errors or is not complete but shows a clear grasp of the concepts and techniques required, although not their application to this problem or the solution is somewhat difficult to read but otherwise correct.
- **0 marks**: the solution contains substantial errors, is far from complete, does not show a clear grasp of the concepts and techniques required, or is illegible.

This marking scheme reflects our intent for you to learn the key concepts and techniques underlying computation, determine where they apply, and apply them correctly to interesting problems. It also reflects a practical fact: we have insufficient time to decipher illegible answers. At the instructor's discretion, some items may be marked on a different scale. TAs may very occasionally award a bonus mark for exceptional answers.

Question 1 [6]

a) Translate the following to (a) unsigned binary notation, and (b) hexadecimal notation. In each case, use the minimum number of digits necessary to represent the value.
   i. 72
   ii. 549
   iii. 4422

b) Consider the following binary numbers. For each number, translate it to a decimal number (a) first assuming it is unsigned, then (b) assuming it is a signed 8-bit number. (As is typical, "zero-extend" values with too few digits: adding zeroes to the left of the value until it has enough digits.)
   i. 10101010
   ii. 11100011
   iii. 101010

Question 2 [9]

We want to represent "polar coordinates" in a compact form. A polar coordinate is an angle (which we represent in degrees, where there are 360 degrees in a full circle, 0 degrees is "east" along the positive x-axis, and 90 degrees is "north" along the positive y axis) and a radius or distance from the origin (which we represent in arbitrary units that you can think of as meters).

Consider the following representation scheme: A polar coordinate is a 32-bit value with a 9-bit unsigned binary integer angle and a 23-bit radius. The radius is divided into a 16-bit unsigned binary number plus a 7-bit fraction (with a $\frac{1}{2}$'s place, a $\frac{1}{4}$'s, a $\frac{1}{8}$'s, …, and a $\frac{1}{128}$'s or $\frac{1}{2^7}$'s place).

Now, solve the following problems:

a) Find **two** very **different** examples of a single value that can be represented with at least two different bit patterns. (Give two values and, for each one, give two example bit patterns that represent that same value. In each case, briefly explain what about the representation leads to this redundancy.)

b) **[PRACTICE ONLY, unmarked]** What is the maximum distance from the origin that is representable with this scheme?

c) What is the minimum distance from the origin (greater than 0) that is representable with this scheme?

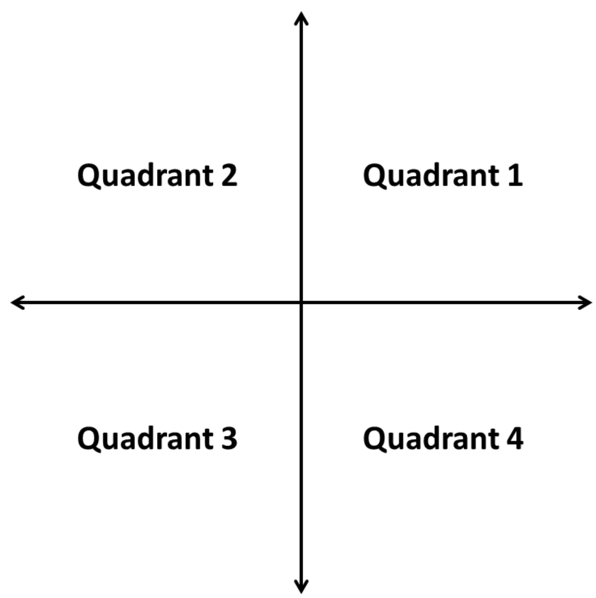
d) **[PRACTICE ONLY, unmarked]** Propose an alternate way to use the 9 bits of the angle that makes better use of the 9 bits. Ensure that your representation choice allows expressing the angles 0^o, 45^o, 90^o, and 180^o, and explain how it does so.

e) Now consider this representation scheme that uses many fewer bits and focuses on the angles from the previous part: a 3-bit unsigned integral radius $r = r_1 r_2 r_3$ and a 3-bit angle

$a = a_1a_2a_3$, where we interpret the angle as an unsigned binary number multiplied by 45°. So, $a = 000$ means 0°, while 101 means 225°.

i. **Design** a circuit (i.e., give a propositional logic formula for the output *o* but do not draw the circuit diagram) that determines whether a value is on the origin. Hint: you do **not** need a 6-variable propositional logic table!

ii. **[PRACTICE ONLY, unmarked] Design** a circuit (i.e., give propositional logic formulae for each bit of the output $o_1o_2o_3$ but do not draw the circuit diagram) that takes a value in this scheme that is **not** on the origin and determines which quadrant the value is in according to the following chart of the quadrants, where anything on the x- or y-axis counts as "quadrant 0". (Hint: again, you do **not** need a 6-variable propositional logic table!)



Question 3 [PRACTICE ONLY, unmarked]

Consider the following function:

```
;; Boolean Boolean Boolean -> Boolean
(define (is-it? a b c)
  (cond [a (not b)]
        [b false]
        [else (or c (not b))]))
```

Use logical equivalences to write a simplified version of the function.

Your answer must include the following circled and clearly labeled parts:

a) a propositional logic statement directly modeling the function above,
b) a simplified form of that statement, and
c) code corresponding directly to the simplified statement.

Show your work as you simplify from (a) to (b), but you do **not** need to formally prove their equivalence.

Question 4 [12]

Using any necessary rules of inference and equivalences, derive the conclusion from the premises. For each step, indicate on the right the inference rule or equivalence which has been used in the step:

a) $s \vee r$
p
$r \rightarrow \sim q$
$p \rightarrow q$

$\therefore\ s \vee t$

b) $p \rightarrow (q \rightarrow r)$
$p \vee s$
$t \rightarrow q$
$\sim s$

$\therefore\ \sim r \rightarrow \sim t$

c) $p \wedge q$
$p \rightarrow (r \wedge q)$
$r \rightarrow (s \vee t)$
$\sim s$

$\therefore\ t$

d) $(\sim p \vee q) \rightarrow r$
$r \rightarrow (s \vee t)$
$\sim s \wedge \sim u$
$\sim u \rightarrow \sim t$

$\therefore p$

Question 5 [PRACTICE ONLY, unmarked]

For each of the following arguments, first show whether the argument is valid. Then, for each valid argument, show a formal proof and for each invalid argument provide a counter example, i.e. an assignment of truth values that make the argument invalid.

a) $p \rightarrow (q \rightarrow r)$
$\sim q \rightarrow \sim p$
p

$\therefore\ r$

b) $p \leftrightarrow q$
$q \rightarrow r$
$r \vee \sim s$
$\sim s \rightarrow q$

$\therefore\ s$

<u>Question 6 [6]</u>

Write the following arguments in symbolic form. Then establish the validity of the argument by providing a proof, or give a counterexample to show that it is invalid:

a) If Dominic goes to the racetrack, then Helen will be upset. If Ralph plays cards all night, then Carmen will be upset. If either Helen or Carmen gets upset, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Therefore, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.

b) If Newton is not considered a great mathematician and Leibniz's work is not ignored, then calculus would not be the centerpiece of the math curriculum. Newton is considered a great mathematician only if Leibniz's work is ignored. Therefore, calculus is the centerpiece of the math curriculum and Leibniz's work is not ignored.