CPSC 121: Models of Computation
Assignment #3

Due: Friday March 7, 5:00 pm **Total: 52 Marks**

<u>Submission Instructions</u>-- read carefully

All assignments should be done in groups of 2. It is very important to work with another student and exchange ideas. **Each group should submit ONE assignment**. Type or write your assignment on clean sheets of paper with question numbers prominently labelled. Answers that are difficult to read or locate may lose marks. We recommend working problems on a draft copy then writing a separate final copy to submit.

Your submission must be **STAPLED** and include the **CPSC 121 assignment cover page** – located at the Assignments section of the course web page. Additionally, include your names at the top of each page. We are not responsible for lost pages from unstapled submissions.

Submit your assignment to the appropriately marked box in room **ICCS X235** by the due date and time listed above. **Late submissions are not accepted**.

Note: the number of marks allocated to a question appears in square brackets after the question number.

<u>A Note on the Marking Scheme</u>

Most items (i.e., question or, for questions divided into parts, part of a question) will be worth 3 marks with the following general marking scheme:

- **3 marks:** correct, complete, legible solution.
- **2 marks**: legible solution contains some errors or is not quite complete but shows a clear grasp of how the concepts and techniques required apply to this problem.
- **1 mark**: legible solution contains errors or is not complete but shows a clear grasp of the concepts and techniques required, although not their application to this problem or the solution is somewhat difficult to read but otherwise correct.
- **0 marks**: the solution contains substantial errors, is far from complete, does not show a clear grasp of the concepts and techniques required, or is illegible.

This marking scheme reflects our intent for you to learn the key concepts and techniques underlying computation, determine where they apply, and apply them correctly to interesting problems. It also reflects a practical fact: we have insufficient time to decipher illegible answers. At the instructor's discretion, some items may be marked on a different scale. TAs may very occasionally award a bonus mark for exceptional answers.

Question 1 [12 marks]

Consider the following predicates over the given sets :

- **W** the set of Winter Olympic Games (like downhill, ski jumping, etc.)
- **A** the set of Athletes
- **P** the set of medals ("p" for prizes: gold, silver, bronze)
- ***C(x)***: x is Canadian
- ***U(x)***: x is from USA
- ***R(x)***: x is Russian
- ***F(x)***: x is Female
- ***M(x)***: x is Male
- ***Comp(x, y)***: athlete x competes in sport y
- ***Won(x, y, z)***: athlete x won medal y in game z
- ***Coach(x, y)***: athlete x coaches athlete y

Rewrite each of the following statements in Predicate Logic using only the given predicates (and any helper predicates you define, as long as they're **very clearly** indicated and correctly defined using predicate logic in terms of the predicates above) and the operators = and ≠. If you use a constant like "free-skating", put it in quotation marks to distinguish it from variables like **x**.

a) [2] Male athletes who won a medal in free-skating are either from Canada or US.

b) [2] No female athlete from Russia has won a metal in bobsleigh or ski-jumping.

c) [2] Every Canadian athlete is coached by some US athlete who has won some metal.

d) [2] Some female Russian athletes do not coach any athlete who competes in the same sport in which they compete.

e) [2] Mary is coaching at least two female athletes who are not in any of the sport Mary competes in.

f) [2] Every US athlete who is competing in luge is coached by exactly one Canadian luge athlete.

Question 2 [10 marks, 2 marks per question]

Using the definitions of question 1, translate each of the following predicate logic statements into English. Try to make your English translations as natural as possible.

Note: We use typical operator precedence (as the text does): ~ has the highest precedence, then $\wedge$ and $\vee$, and $\rightarrow$ has the lowest precedence.

a) $\forall x \in A,\ C(x) \rightarrow \sim\exists\ y \in A, \text{Coach}(x, y) \vee \text{Coach}(y, x)$

b) $\forall x \in A, \text{Coach}(x, x) \rightarrow \sim\exists\ y \in A, (\text{Coach}(x, y) \vee \text{Coach}(y, x)) \wedge x \neq y$

c) $\forall x \in A, \forall y \in A,\ (U(x) \wedge \text{Comp}(x, \text{“skeleton”}) \wedge \text{Coach}(y, x)) \rightarrow (\forall z \in A, \text{Coach}(z, x) \rightarrow y = z)$

d) $\forall x \in A, (C(x) \wedge \exists y \in W, \exists z \in P, \text{Won}(x, z, y)) \rightarrow \sim\exists u \in A, \text{Coach}(u, x)$

e) $\forall x \in A, \forall y \in A,\ \forall\ w \in A, \forall\ z \in A, (x \neq y \wedge \text{Coach}(x, w) \wedge \text{Coach}(y, z)) \rightarrow (w \neq y \wedge z \neq x)$

Question 3[12 marks]

Consider again the domain and predicates of question 1. For each of the following arguments translate them into Predicate Logic (using quantifiers and predicates) and provide a formal proof for them.

a) [6]
Neither US nor Russian athletes have a coach.
Mary is Paul’s coach.

Paul is not a Russian athlete.

b) [6]
Every Canadian athlete is a medal winner
Medal winners are not coached
Skeleton athletes are coached

There are no Canadian skeleton athletes

Question 4 [9 marks]

Consider the following theorem:

If *x* is a positive odd integer that is not divisible by *3*, then $x^2 - 1$ is divisible by *3*.

a) [3] Rewrite the theorems using quantifiers and predicates. You can use predicates for oddness and divisibility, like
   - *Odd(x)* : x is odd
   - *Divisible(x,y)* : x is divisible by y

   or for any other numerical properties you need to define.

b) [6] Give a proof for the theorem. Your proof should be informal (in natural language), but it must have a similar structure of a formal proof, with clear steps and correct application of the rules.

   Recall that
   - to prove a universal statement we start by instantiating the statement with a generic element of the domain and prove it for that item
   - to prove a existential statement we need to find an element of the domain and prove that the statement is true for that element.

   Hint for this proof: if an integer is not divisible by `3`, then it can be written as either `3k+1` or as `3k+2` for some integer `k`. Also, recall that an integer `x` is divisible by non-zero integer `y` exactly when there's a third integer `z` such that `zy = x`.

Question 5 [9 marks]

Suppose that an algorithm A executes $3n^2 + 60n + 100$ steps when run with inputs of size n. Prove that the algorithm is in $O(n^2)$ (i.e. A is in big-O of n^2) . Recall that for any two functions **f(n)** and **g(n)** we have define

$$\textbf{f is in O(g)} \equiv \exists c \in R+, \exists n_0 \in N, \forall n \in N, n \geq n_0 \rightarrow f(n) \leq cg(n)$$

a) [3] Using this predicate for the big-O notation, translate this statement into predicate logic .

b) [6] Give a proof for this statement. Your proof can be informal (in natural language), but it must have a similar structure of a formal proof.