1. (a) $\forall a \in A, (M(x) \wedge (\exists p \in P, \text{Won}(x, p, \text{“free-skating”}))) \rightarrow (C(a) \vee U(a))$
   (b) $\sim\exists a \in A, F(a) \wedge R(a) \wedge (\exists p \in P, \text{Won}(a, p, \text{“bobsleigh”}) \vee \text{Won}(a, p, \text{“ski-jumping”}))$
   (c) $\forall x \in A, C(x) \rightarrow (\exists y \in A, U(y) \wedge (\exists p \in P, \exists w \in W, \text{Won}(y, p, w)) \wedge \text{Coach}(y, x))$
   (d) $\exists x \in A, R(x) \wedge F(x) \wedge (\sim\exists y \in A, \text{Coach}(x, y) \wedge (\forall w \in W, \text{Comp}(x, w) \rightarrow \text{Comp}(y, w)))$
   (e) $\exists a \in A, \exists b \in A, a \neq b \wedge \text{Coach}(\text{“Mary”}, a) \wedge \text{Coach}(\text{“Mary”}, b) \wedge$
   $(\forall w \in W, \text{Comp}(\text{“Mary”}, w) \rightarrow (\sim\text{Comp}(a, w) \wedge \sim\text{Comp}(b, w)))$
   (f) $\forall a \in A, (U(a) \wedge \text{Comp}(a, \text{“luge”})) \rightarrow (\exists x \in A, C(x) \wedge \text{Comp}(x, \text{“luge”}) \wedge \text{Coach}(x, a) \wedge$
   $(\sim\exists y \in A, x \neq y \wedge C(y) \wedge \text{Comp}(y, \text{“luge”}) \wedge \text{Coach}(y, a)))$

2. (a) Canadian athletes neither coach athletes nor are coached by athletes.
   (b) Self-coached athletes neither coach other athletes nor are coached by other athletes.
   (c) American skeleton athletes are coached by at most one athlete.
   (d) Medal-winning Canadian athletes are not coached by any athlete
   (e) Every athlete is coached by one of two athlete coaches.

3. (a) *Proof.*

(1)	$\forall x \in A, (U(x) \vee R(x)) \rightarrow \sim\exists y \in A, \text{Coach}(y, x)$	*Premise*
(2)	$\text{Coach}(\text{“Mary”}, \text{“Paul”})$	*Premise*
(3)	$\forall x \in A, (U(x) \vee R(x)) \rightarrow \forall y \in A, \sim\text{Coach}(y, x)$	*1, Generalized De Morgan's*
(4)	$(U(\text{“Paul”}) \vee R(\text{“Paul”})) \rightarrow \sim\text{Coach}(\text{“Mary”}, \text{“Paul”})$	*3, Universal instantiation*
(5)	$\sim(U(\text{“Paul”}) \vee R(\text{“Paul”}))$	*4, 2, [M.TOL]*
(6)	$\sim U(\text{“Paul”}) \wedge \sim R(\text{“Paul”}))$	*5, [DM]*
	$\therefore \sim U(\text{“Paul”})$	*6, [SPEC]* ■

   (b) *Proof.*

(1)	$\forall a \in A, C(a) \rightarrow \exists x \in P, \exists y \in W, \text{Won}(a, x, y)$	*Premise*
(2)	$\forall a \in A, \exists x \in P, \exists y \in W, \text{Won}(a, x, y) \rightarrow \forall b \in A, \sim\text{Coach}(b, a)$	*Premise*
(3)	$\forall a \in A, \text{Comp}(a, \text{“skeleton”}) \rightarrow \exists b \in A, \text{Coach}(b, a)$	*Premise*
(4)	$\forall a \in A, C(a) \rightarrow \text{Won}(a, i, j)$	*1, Existential instantiation*
(5)	$C(p) \rightarrow \text{Won}(p, i, j)$	*4, Universal instantiation*
(6)	$\forall a \in A, \text{Won}(a, i, j) \rightarrow \forall b \in A, \sim\text{Coach}(b, a)$	*2, Existential instantiation*
(7)	$\text{Won}(p, i, j) \rightarrow \sim\text{Coach}(q, p)$	*6, Universal instantiation*
(8)	$C(p) \rightarrow \sim\text{Coach}(q, p)$	*5, 7, [TRANS]*
(9)	$\forall a \in A, \text{Comp}(a, \text{“skeleton”}) \rightarrow \text{Coach}(w, a)$	*3, Existential instantiation*
(10)	$\text{Comp}(p, \text{“skeleton”}) \rightarrow \text{Coach}(w, p)$	*9, Universal instantiation*
(11)	$\text{Coach}(w, p) \rightarrow \sim C(p)$	*8, [IMP]*
(12)	$\text{Comp}(p, \text{“skeleton”}) \rightarrow \sim C(p)$	*10, 11, [TRANS]*
(13)	$\forall x \in A, \text{Comp}(x, \text{“skeleton”}) \rightarrow \sim C(x)$	*12, Universal generalization*
(14)	$\sim\exists x \in A, \sim(\text{Comp}(x, \text{“skeleton”}) \rightarrow \sim C(x))$	*13, Generalized De Morgan's*
	$\therefore \sim\exists x \in A, C(x) \wedge \text{Comp}(x, \text{“skeleton”})$	*14, Negation of implication* ■

4. (a) $\forall x \in \mathbb{Z}^+, (Odd(x) \wedge \sim Divisible(x, 3)) \rightarrow Divisible(x^2 - 1, 3)$

(b) *Proof.* Let there be two general cases for x: $x = 3k + 1$ and $3k + 2$, where $k \in \mathbb{N}$.
In the first case, we find that $(3k+1)^2 - 1 = (9k^2 + 6k + 1) - 1 = 9k^2 + 6k = 3k(k+2)$.
For the second case, we can see that $(3k+2)^2 - 1 = (9k^2 + 12k + 4) - 1 = 9k^2 + 12k + 3 = 3(3k^2 + 4k + 1)$.
In both cases, $x^2 - 1$ produces a formula which is divisible by 3, and thus for all values of x that are not divisible by 3, $x^2 - 1$ must be divisble by 3. Given that the odd values of x that are not divisible by 3 is a subset of all values of x that are not divisible by 3, it is thus proven that if x is a positive odd integer that is not divisible by 3, then $x^2 - 1$ is divisible by 3. ■

5. (a) $\exists c \in \mathbb{R}^+, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \rightarrow (3n^2 + 60n + 100) \leq cn^2$

(b) *Proof.* Let $c = 4$ and $n_0 = 62$. Then, let n by any arbitrary natural number. If we assume that the antecedent, $n \geq n_0$, is true, then let us prove that $(3n^2 + 60n + 100) \leq 4n^2$:

$$\begin{aligned}
(3n^2 + 60n + 100) &\leq 4n^2 \\
-n^2 + 60n + 100 &\leq 0 \\
n^2 - 60n - 100 &\geq 0
\end{aligned}$$

Solving the quadratic equation, we find that $n \leq -1.62$ and $n \geq 61.62$. As the domain of n only includes natural numbers, we find that when $n \geq 62$ (thus also satisfying the initial assumption of $n \geq n_0$), the statement $(3n^2 + 60n + 100) \leq 4n^2$ holds true. Therefore, an algorithm that executes in $3n^2 + 60n + 100$ is in $O(n^2)$. ■