Lecture 3: Discrete Time Systems (Part II)

Impulse response of LCCDEs

- Impulse response: $\delta[n] \rightarrow \boxed{\text{LCCDE}} \rightarrow h[n]$

  i.e. solve with $x[n] = \delta[n] \rightarrow y[n] = h[n]$ (zero ICs)

- With $x[n] = \delta[n]$, 'form' of $y_p[n] = \beta\delta[n]$

  → solve $y[n]$ for $n = 0, 1, 2\ldots$ to find α_is

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Impulse Response:

The **impulse response** is the system's output when the input is an impulse signal, denoted $\delta[n]$. Mathematically, if we input $\delta[n]$ into the LCCDE, the output is the impulse response, denoted as $h[n]$:

$$\delta[n] \quad \rightarrow \quad \text{LCCDE} \quad \rightarrow h[n]$$

This is a key function that characterizes the system's behavior completely.

How to Solve for the Impulse Response:

To find the impulse response, we solve the LCCDE with **zero initial conditions** (ICs), meaning no pre-existing output or memory in the system, and use $x[n] = \delta[n]$ as the input.

1. Set $x[n] = \delta[n]$.

2. The **particular solution** $y_p[n]$ will take the form $\beta\delta[n]$.
3. Solve the LCCDE for $y[n]$, determining the **complementary solution** $y_c[n]$ and the constants α_i.

Solving for α_i:

To determine the constants α_i in the complementary solution, you substitute values of $n = 0, 1, 2, \ldots$ into the LCCDE. The resulting solution gives us the complete impulse response $h[n]$, which describes how the system responds to the impulse input.

4. Correlation

- Correlation ~ identifies similarity between sequences:

Cross correlation of x against y $\quad r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell]$

"lag"

- Note: $r_{yx}[\ell] = \sum_{n=-\infty}^{\infty} y[n]x[n-\ell]$ $\quad$ call $m = n - \ell$

$$= \sum_{m=-\infty}^{\infty} y[m+\ell]x[m] = r_{xy}[-\ell]$$

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What is Correlation?

Correlation helps us **identify the similarity between two sequences**. It shows how much one signal resembles another signal as you shift or "lag" one of the signals.

Cross-Correlation:

The **cross-correlation** between two signals]$\mathrm{x[n]}$ and $\mathrm{y[n]}$ is denoted as $\mathrm{r_{xy}}[\ell]$, where ℓ represents the **lag**. This gives us a measure of how similar $\mathrm{x[n]}$ is to a **shifted version** of $\mathrm{y[n]}$. Mathematically, cross-correlation is expressed as:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell]$$

This means that we sum the product of the sequences $\mathrm{x[n]}$]and a lagged version of $\mathrm{y[n]}$, where ℓ controls the amount of shift.

Symmetry in Correlation:

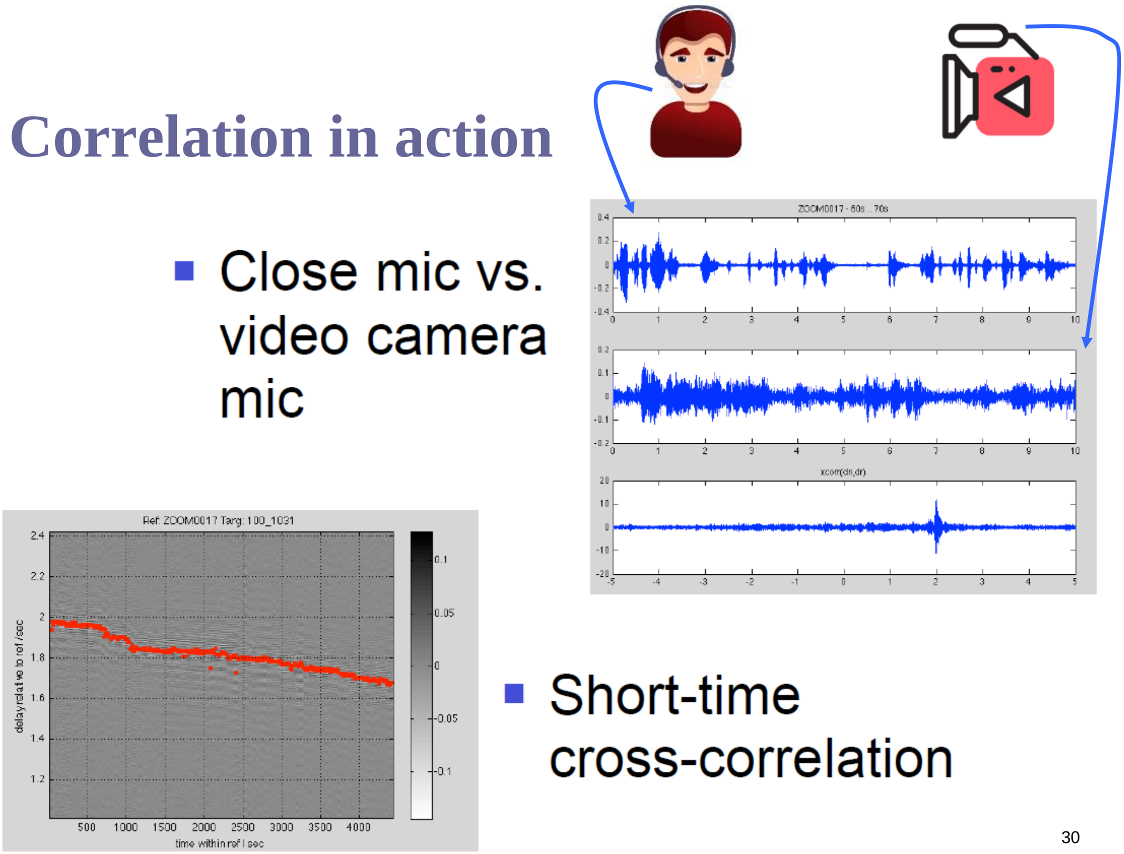
We also have an important symmetry relationship between the cross-correlations $\mathrm{r}_{xy}[\ell]$ and $\mathrm{r}_{yx}[\ell]$. If we flip the roles of x and y, we get:

$$r_{yx}[\ell] = \sum_{n=-\infty}^{\infty} y[n]x[n-\ell]$$

However, by changing the variable, we can show that:

$$\mathrm{r_{yx}}[\ell] = \mathrm{r_{xy}}[-\ell]$$

This means the cross-correlation of yyy against xxx is simply the cross-correlation of x against y, but with a **negative lag**.



In this slide, we're comparing two types of microphones: a **close mic** (placed near the sound source) and a **video camera mic** (which is generally farther from the source). The comparison highlights the correlation between the signals captured by these two microphones.

1. **Audio Signals**:
   - The waveforms on the right display the signals recorded by both microphones. The top two graphs show the raw signals from the close mic and the camera mic. The signals have similar shapes but are slightly delayed due to the difference in the microphones' distances from the sound source.
   - The third graph is the **cross-correlation** between the two signals, indicating the delay between them. This allows us to see the time shift between the two audio recordings.
2. **Short-time Cross-correlation**:
   - This analysis doesn't just compare the entire signals at once but breaks them into short-time segments. It computes the correlation

within these shorter segments, which is especially useful when signals change over time.

3. **Delay Analysis**:
    - In the graph at the bottom left, you can see the delay rate (time shift) between the close mic and the video camera mic signals over time. The red line shows the delay decreasing gradually, possibly due to changing positions of the sound source or the microphones.

Applications:

- **In practice**, short-time cross-correlation is valuable in applications like **speech recognition**, **audio synchronization** (e.g., syncing video and audio tracks), and **noise reduction** in audio recordings.

This is one example of how correlation helps in **analyzing, comparing, and aligning signals** in real-world scenarios.