ICE503 Homework-11

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Q. 2

Design of Filter

Consider a given input signal $x(n)$ and output $y(n)$. From the given symmetric filter diagram, we can write

$$y(n) = \alpha x(n) + \beta x(n-1) + \alpha x(n-1)$$

Taking z-transform and manipulating, we have the transfer function as

$$H(z) = \alpha\left(1 + z^{-2}\right) + \beta z^{-1}$$

The filter response function is written by substituting $z = e^{j\omega}$ as

$$H\left(e^{j\omega}\right) = \alpha\left(1 + e^{-j2\omega}\right) + \beta e^{-j\omega} \tag{1}$$

The filter response is given as:

$$H\left(e^{j\omega}\right) = \begin{cases} 1, & \omega = \omega_1 \\ 0, & \omega = \omega_2 \end{cases}$$

Hence by substitution in the filter response function (1) we have two equations:

$$H\left(e^{j\omega}\right) = \alpha\left(1 + e^{-j2\omega_1}\right) + \beta e^{-j\omega_1} = 1$$
$$H\left(e^{j\omega_2}\right) = \alpha\left(1 + e^{-j2\omega_2}\right) + \beta e^{-j\omega_2} = 0$$

Solving for α and β, we have the relations

$$\alpha = \frac{1}{\left(1 + e^{-j2\omega_1}\right) - \left(e^{j\omega_1} + e^{-j\omega_1}\right)}$$
$$\beta = \frac{-\left(e^{j\omega_2} + e^{-j\omega_2}\right)}{\left(1 + e^{-j2\omega_1}\right) - \left(e^{j\omega_2} + e^{-j\omega_2}\right)}$$

Hence, the 3-point filter response is given by $\mathbf{h} = [\alpha, \beta, \alpha]$. The plot below shows the given response frequencies $\omega_1 = 0.1$ and $\omega_2 = 0.2$.

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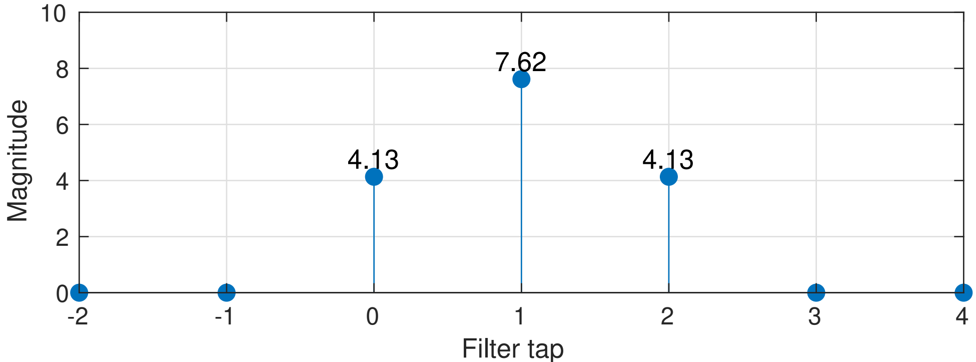
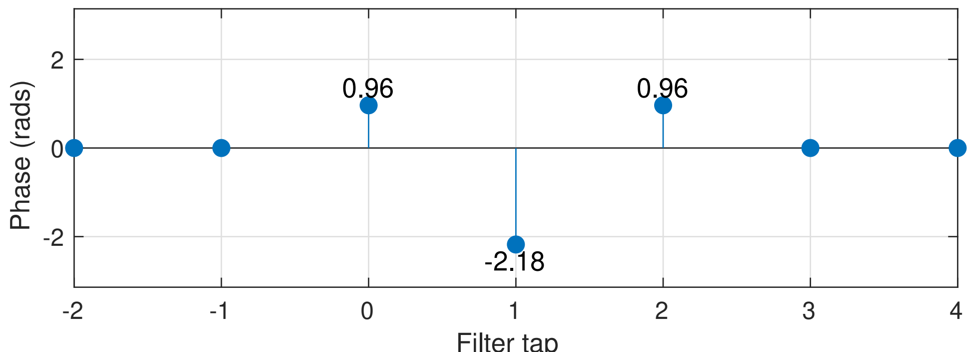
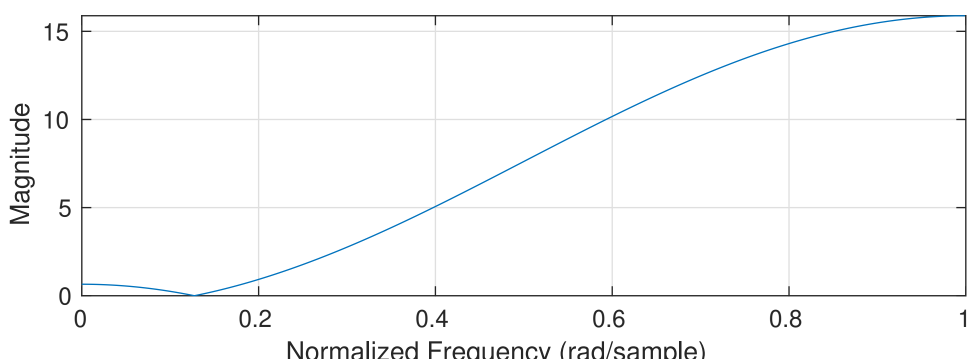


(a) Magnitude

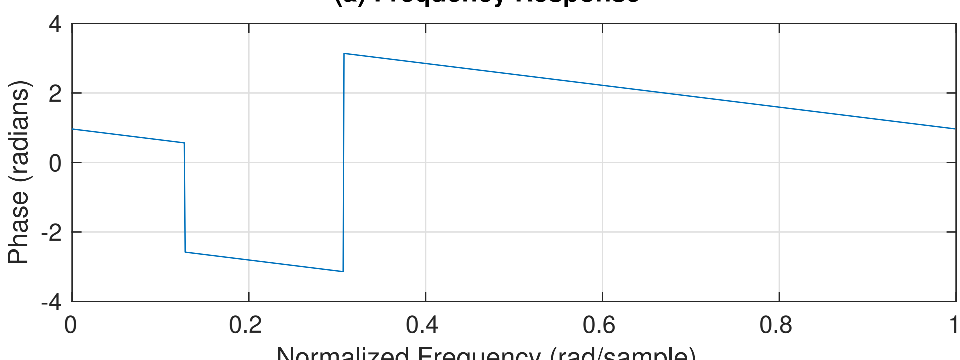


(b) Phase

Fig. 1: Plot of filter coefficients.



(a) Frequency Response



(b) Frequency Response

Fig. 2: Plot of filter response for the 3-point filter whose coefficients are shown in Fig. 1 above.

Comparison of Input-Output Response

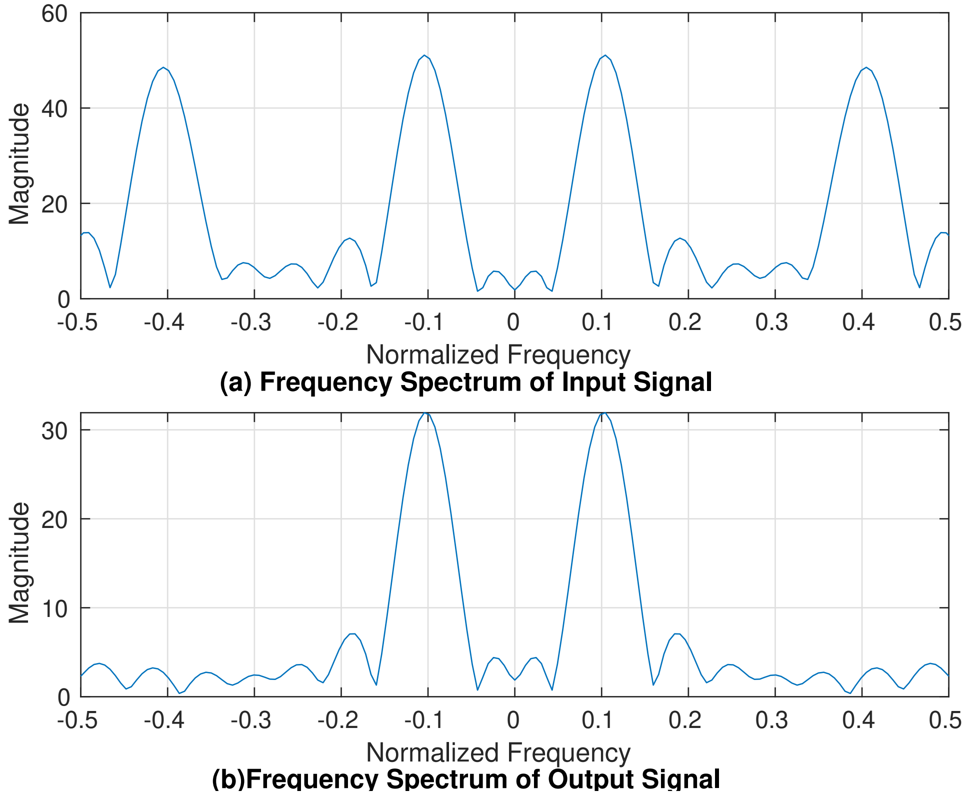


Fig. 3: Plot of input and output signal.

The input signal comprises frequency components at $\omega_1 = 0.1$ and $\omega_2 = 0.4$. However, the low pass filter designed retains the lower frequency $\omega_1 = 0.1$ and eliminates the higher frequency $\omega_2 = 0.4$. This is evident from the plots above.