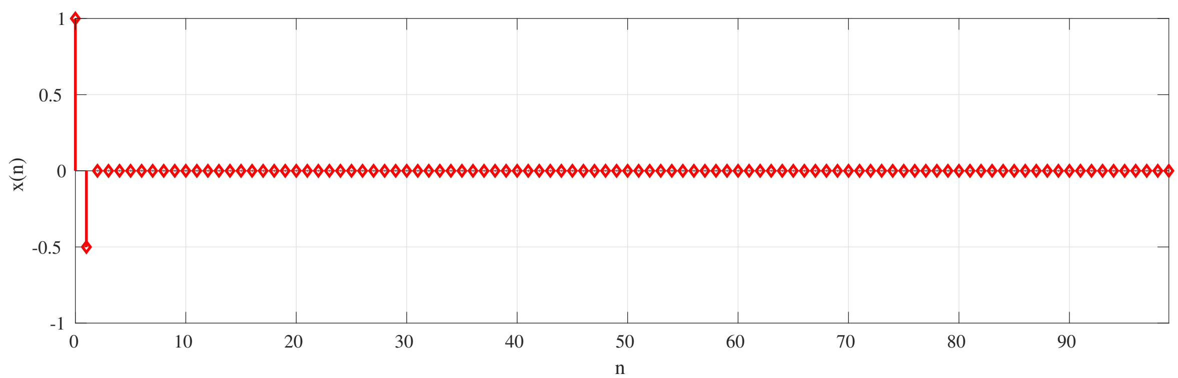
ICE503 Homework-04

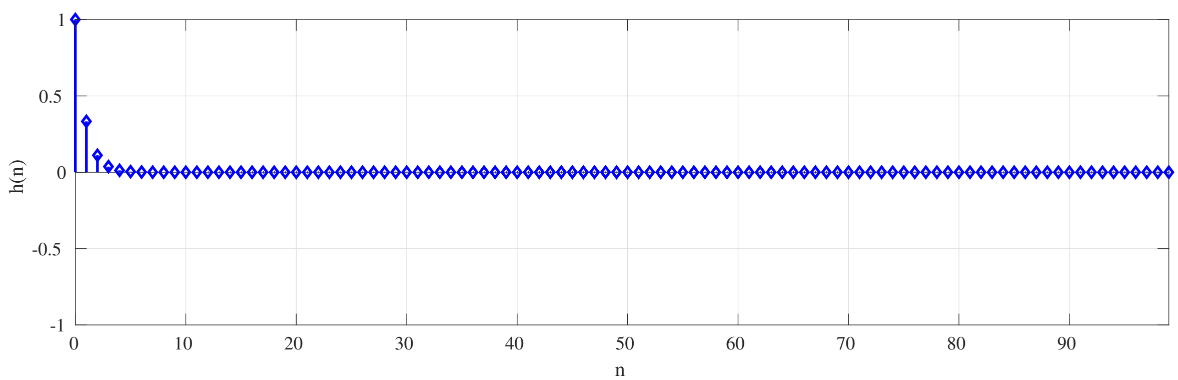
Arnav Mukhopadhyay (D123070002)
EMAIL: gudduarnav@gmail.com

Q. 3

(a)



(a)



(b)

Fig. 1: Plot of (a) $x(n)$ and (b) $h(n)$ for $0 \leq n \leq 99$.

The given functions are:

$$x(n) = \delta(n) - \frac{1}{2}\delta(n-1)$$
$$h(n) = (\frac{1}{3})^n \mu(n)$$

The corresponding plots are shown in figure 1 (a) and (b), respectively.

(b)

The DTFT of $x(n)$ and $h(n)$ are $X(e^{j\omega}$ and $H(e^{j\omega})$ written respectively as,

$$X(e^{j\omega}) = 1 - \frac{1}{2}e^{-j\omega}$$
$$H(e^{j\omega}) = \frac{\frac{1}{3}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}}$$

(c)

The DTFT of the function $y(n) = x(n) * h(n)$ is written as $Y(e^{j\omega})$ given by,

$$Y(e^{j\omega}) = \frac{\frac{1}{3}e^{-\omega}}{1 - \frac{1}{3}e^{-j\omega}} - \frac{1}{2}\frac{\frac{1}{3}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}}e^{-j\omega}$$

The corresponding plot of magnitude and argument of $X(e^{j\omega})$, $H(e^{j\omega})$ and $Y(e^{j\omega})$ is shown in the plot in figure 2.

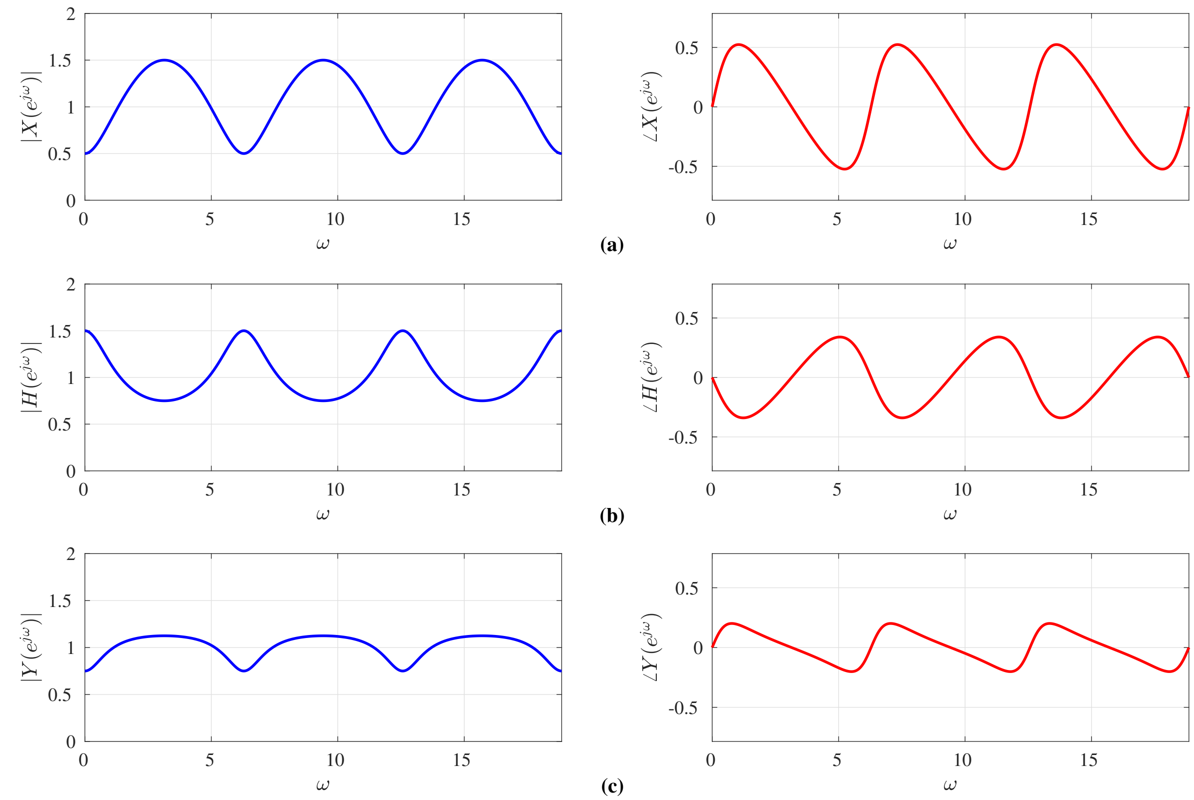


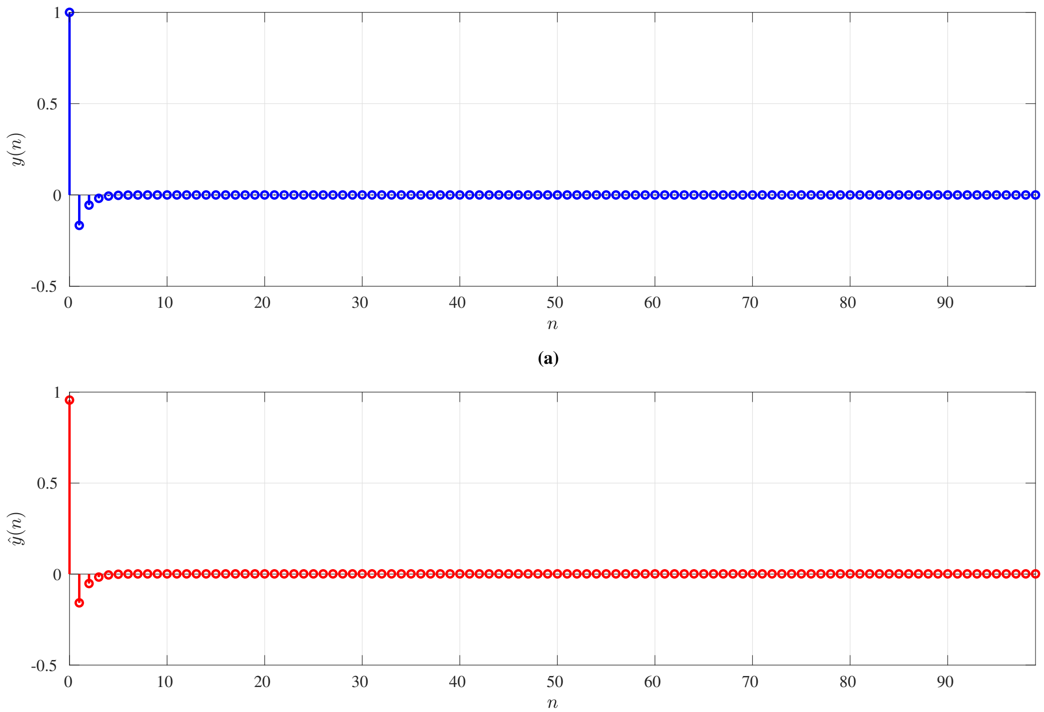
Fig. 2: Plot of the magnitude and argument (angle) of complex (a) $X(e^{j\omega})$, (b) $H(e^{j\omega})$ and, (c) $Y(e^{j\omega})$

(d)

The value of $y(n) = x(n) * h(n)$ is deduced as follows:

$$\begin{aligned} y(n) &= h(n) * x(n) \\ &= [\delta(n) - \frac{1}{2}\delta(n-1)] * [(\frac{1}{2})^n \mu(n)] \\ &= (\frac{1}{3})^n \mu(n) - \frac{1}{2}(\frac{1}{3})^n \mu(n-1) \end{aligned}$$

The above relation considers the identity that, $x(n) * \delta(n - N) = x(n - N)$.



(b)

Fig. 3: Plot of (a) $y(n)$ and (b) $\hat{y}(n)$.

The $\hat{y}(n)$ is obtained from IDFT of $Y(e^{j\omega})$, as:

$$
\begin{aligned}
\hat{y}(n) &= IDFT[Y(e^{j\omega})] \\
&= \frac{1}{2\pi}\int_{\pi}^{\pi}[\frac{\frac{1}{3}e^{-\omega}}{1-\frac{1}{3}e^{-j\omega}} - \frac{1}{2}\frac{\frac{1}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}e^{-j\omega}]e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\frac{1}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}e^{j\omega n}d\omega - \frac{1}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\frac{1}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}e^{j\omega(n-1)}d\omega \\
&= IDFT[\frac{\frac{1}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}] - \frac{1}{2}IDFT[\frac{\frac{1}{3}e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}]_{n\leftarrow(n-1)} \\
&= (\frac{1}{3})^{n}\mu(n) - \frac{1}{2}(\frac{1}{3})^{(n-1)}\mu(n-1) \\
&= y(n)
\end{aligned}
$$

The above result is proven by the plot in figure 3.