ICE503 DSP-Homework#12

1. Figure 1 shows the impulse response for several different LTI systems. Determine the group delay associated with each systems.

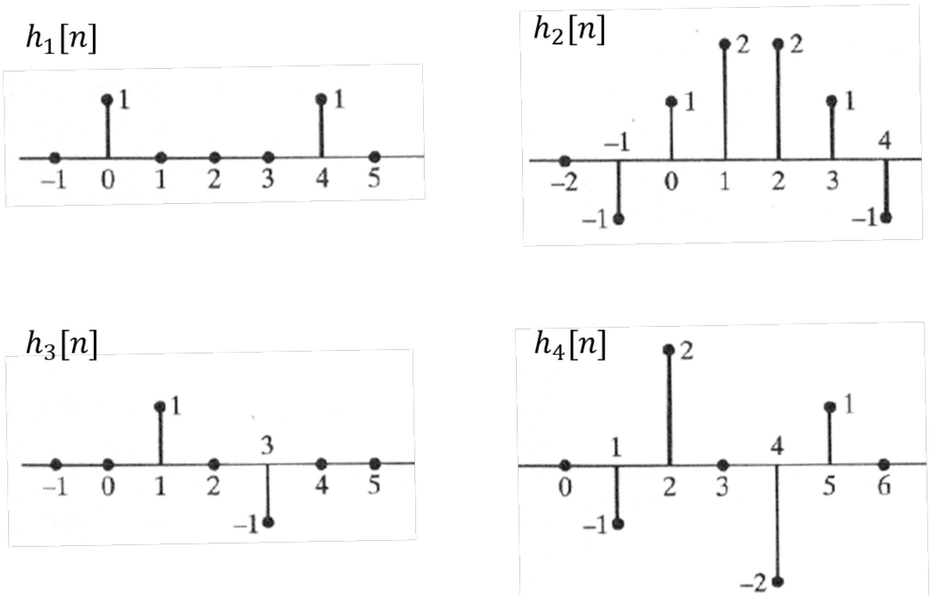

Figure 1: Impulse response for several different LTI systems

2. Figure 2 shows two different interconnections of three systems. The impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are as shown in Figure 3. Determine whether system A and/or system B is a generalized linear-phase system.

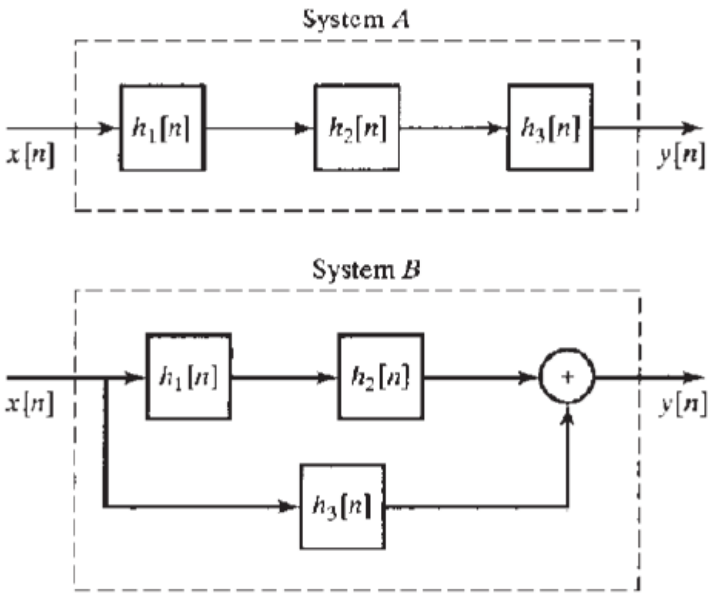

Figure 2: Two different interconnections of three systems

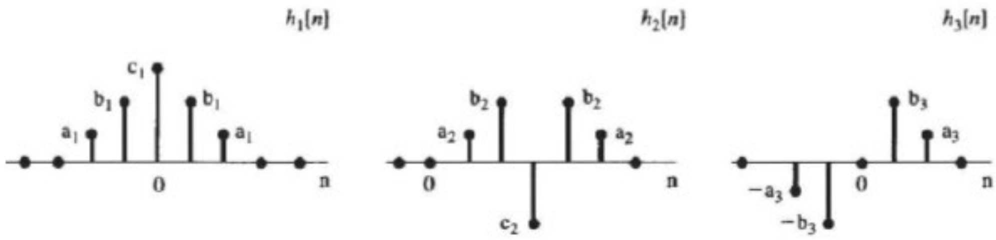

Figure 3 Impulse responses of the three systems

3. MATLAB simulation:

   Using iirnotch function to design a second order IIR notch filter with the notch located at $\omega c = 0.1\pi$ and with the 3 dB bandwidth of 0.001π and use fvtool function sketch the magnitude of the filter in dB and the group delay.

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$2\cos^2\theta = 1 + \cos 2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

•

1. Figure 1 shows the impulse response for several different LTI systems. Determine the group delay associated with each systems.

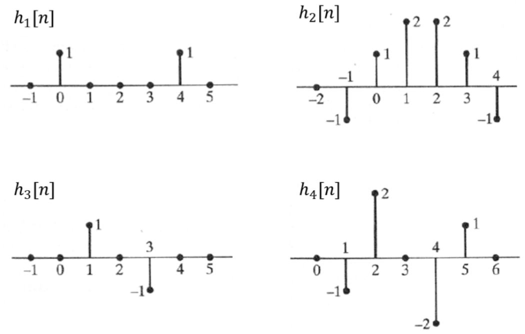

Figure 1: Impulse response for several different LTI systems

(1) $h_1(n) = \delta(n) + \delta(n-4)$

Take Fourier transform of above, we get,

$H_1(n) = \mathcal{F}[\delta(n)] + \mathcal{F}[\delta(n-4)]$ $\qquad \delta(n) \longleftrightarrow 1$

$= 1 + e^{-j4\omega}$ $\qquad \delta(n-k) \longleftrightarrow e^{-jk\omega}$

$\Rightarrow \delta(n-4) \longleftrightarrow e^{-j4\omega}$

$= (1 + \cos 4\omega) - j\sin 4\omega$

Hence phase $\varphi = \tan^{-1}\left(-\dfrac{\sin 4\omega}{1+\cos 4\omega}\right)$

$= \tan^{-1}\left(-\dfrac{2\sin 2\omega \cos 2\omega}{2\cos^2 2\omega}\right)$

$= \tan^{-1}\left(-\dfrac{\sin 2\omega}{\cos 2\omega}\right)$

$= -\tan^{-1}(\tan 2\omega)$

$= -2\omega$

$\therefore$ Group delay $\tau_g = \dfrac{d\varphi}{d\omega} = \dfrac{d}{d\omega}(-2\omega) = -2$ sec

- $h_2(n) = -\delta(n+1) + \delta(n) + 2\delta(n-1) + 2\delta(n-2) + \delta(n-3) - \delta(n-4)$

Shortcut

$-\delta(n+1) \longrightarrow$ coefficient $= -1$ $\qquad n = -1$

$\delta(n) \rightarrow$ coeff. $= 1$ $\qquad n = 0$

$2\delta(n-1) \rightarrow$ coeff. $= 2$ $\qquad n = 1$

$2\delta(n-2) \rightarrow$ coeff. $= 2$ $\qquad n = 2$

$\delta(n-3) \rightarrow$ coeff. $= 1$ $\qquad n = 3$

$\delta(n-4) \rightarrow$ coeff. $= 1$ $\qquad n = 4$

Σ (coeff) $= -1 + 1 + 2 + 2 + 1 + 1 = 6$

Σ (coeff)(n) $= (-1 \times -1) + (1 \times 0) + (2 \times 1) + (2 \times 2) + (1 \times 3) + (1 \times 4)$

$= 1 + 0 + 2 + 4 + 3 + 4$

$= 14$

- $\tau_g = \dfrac{\Sigma(\text{coeff})(n)}{\Sigma(\text{coeff})}$

$= \dfrac{14}{6} = \dfrac{7}{3} = 2.33$

- $h_3(n) = \delta(n-1) - 3\delta(n-3)$

$\delta(n-1) \rightarrow$ coeff $= 1$ $\qquad n = 1$

$-3\delta(n-3) \rightarrow$ coeff $= -3$ $\qquad n = 3$

$\therefore \Sigma$ (coeff) $= 1 - 3 = -2$

Σ (coeff)(n) $= (1 \times 1) + (-3 \times 3)$

$= 1 - 9 = -8$

$\therefore \tau_g = \dfrac{\Sigma(\text{coeff})(n)}{\Sigma\text{coeff}} = \dfrac{-8}{-2} = 4$

- $h_4(n) = -\delta(n-1) + 2\delta(n-2) - 2\delta(n-4) + \delta(n-5)$

$-\delta(n-1) \rightarrow$ coeff $= -1$ $\qquad n = 1$

$2\delta(n-2) \rightarrow$ coeff $= 2$ $\qquad n = 2$

$-2\delta(n-4) \rightarrow$ coeff $= -2$ $\qquad n = 4$

$\delta(n-5) \rightarrow$ coeff $= 1$ $\qquad n = 5$

$\therefore \Sigma$(coeff) $= 1 + 2 - 2 + 1 = 2$

Σ(coeff)$\times$(n) $= (-1 \times 1) + (2 \times 2) + (-2 \times 4) + (1 \times 5)$

$= -1 + 4 - 8 + 5$

$= 0$

$\tau_g = \dfrac{\Sigma(\text{coeff}) \times n}{\Sigma\text{coeff}} = \dfrac{0}{2} = 0$

2. Figure 2 shows two different interconnections of three systems. The impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are as shown in Figure 3. Determine whether system A and/or system B is a generalized linear-phase system.

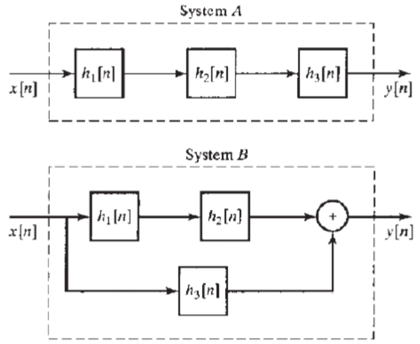

Figure 2: Two different interconnections of three systems

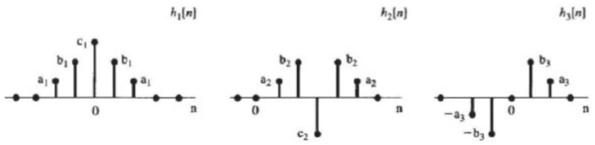

Figure 3 Impulse responses of the three systems

$$\left.\begin{array}{l} h_1(0) = h_1(0) \\ h_1(-1) = h_1(1) \\ h_1(-2) = h_1(2) \end{array}\right\} h_1(n) \text{ is symmetric.}$$

$$\left.\begin{array}{l} h_1(0) = h_2(0) \\ h_2(-1) \neq h_2(1) \end{array}\right\} h_2(n) \text{ is not symmetric.}$$

Hence both system will not be generalized linear phase,

3. MATLAB simulation:

Using iirnotch function to design a second order IIR notch filter with the notch located at $\omega c = 0.1\pi$ and with the 3 dB bandwidth of 0.001π and use fvtool function sketch the magnitude of the filter in dB and the group delay.

ICE503 Homework-12

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Q. 3

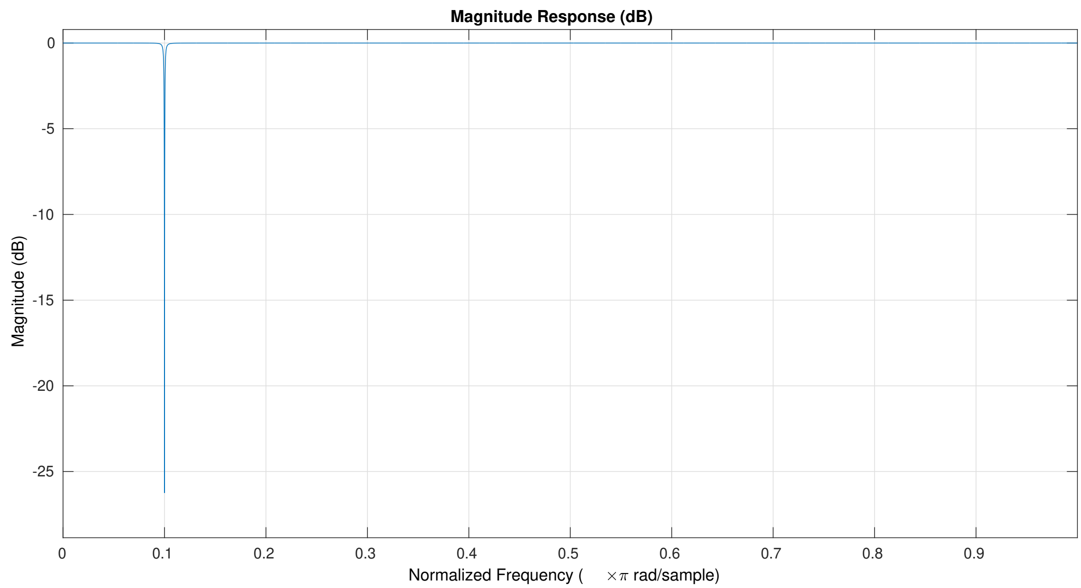

Fig. 1: Magnitude response of the IIR notch filter

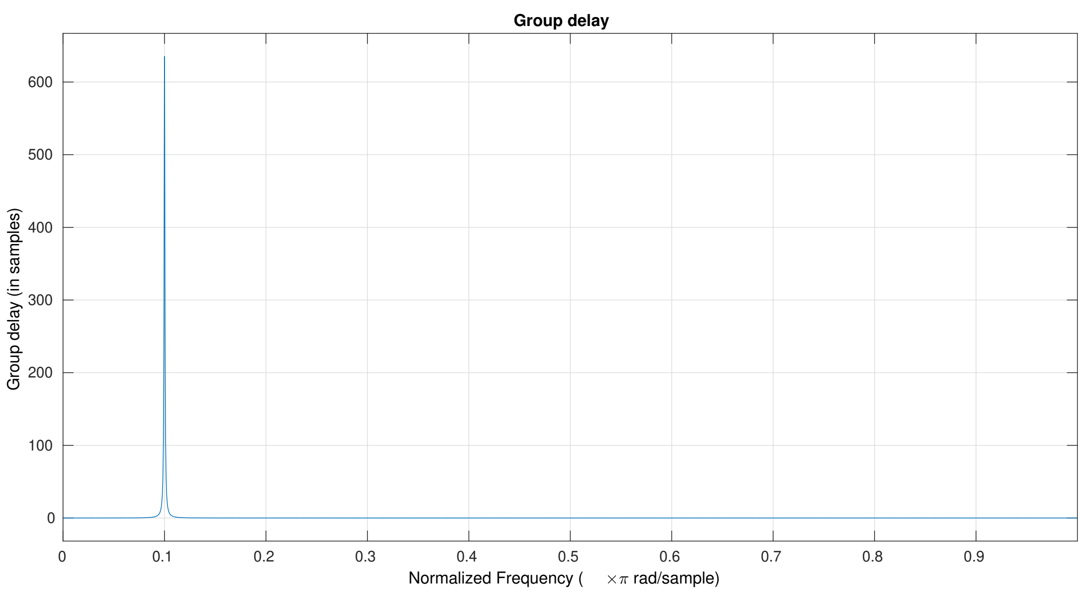

Fig. 2: Group delay response of the IIR notch filter

Designing an IIR Notch Filter Using MATLAB

The `iirnotch` function in MATLAB can be used to design a notch filter. The syntax is:

```
[b, a] = iirnotch(wo, bw);
```

where:

- wo $= \omega_c/\pi$: Normalized notch frequency (in the range [0, 1]),
- bw: Bandwidth in normalized frequency.

Given $\omega_c = 0.1\pi$ and bandwidth 0.001π, we calculate the normalized parameters as follows:

Normalized Parameters

$$\text{wo} = \frac{\omega_c}{\pi} = 0.1$$
$$\text{bw} = \frac{0.001\pi}{\pi} = 0.001$$

Generate Filter Coefficients

Use the `iirnotch` function to compute the filter coefficients b and a. The MATLAB code is:

```
% Design the notch filter
[b, a] = iirnotch(wo, bw);
```

Magnitude and Group Delay Plot

To plot the magnitude response and group delay, use the `fvtool` function. The MATLAB code is:

```
fvtool(b, a, 'Analysis', 'freq'); % Shows magnitude response and group delay
```

The resultant figures are shown in Fig. 1 and 2.

```
% HW 12
% q. 3

% --------- clear ---------------
close all;
clear all;
clc;

% --------- q.2 -----------------
% Parameters
wo = 0.1; % Normalized notch frequency (omega_c / pi)
bw = 0.001; % Normalized 3 dB bandwidth

% Design the notch filter
[b, a] = iirnotch(wo, bw);

% Visualize the filter properties using FVTool
fvtool(b, a, 'Analysis', 'freq'); % Shows magnitude response and group
 delay
```

```
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters
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```

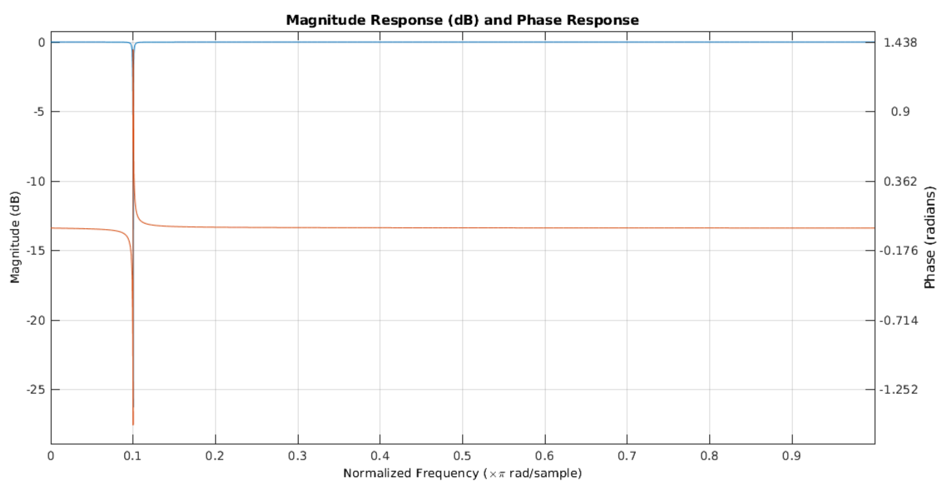

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