ICE503 DSP-Homework#10

1. Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follow:

$$x[n] = \sin\left(\frac{\pi n}{2}\right), n = 0,1,2,3$$

$$h[n] = 2^n, \qquad n = 0,1,2,3$$

   (a) Calculate the four-point DFT $X[k]$.

   (b) Calculate the four-point DFT $H[k]$.

   (c) Calculate $y[n] = x[n] \,④\, h[n]$ by doing the circular convolution directly.

   (d) Calculate $y[n]$ of Part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

2. MATLAB simulation:

   The idea of a spectrogram is plotting a sequence of short DFTs of the input signal using overlapping windows. If the signal is real, then one typically plots only the positive frequencies $k = 0,1,\ldots,\frac{N}{2} - 1$.

   (a) Download guitar4.wav from cyber university（網路大學）and use audioread function to obtain the sampled data $x[n]$ and the sample rate F_s.

   (b) Create a Hann window as the overlapping window

$$w[n] = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N}\right)\right), n = 0,1,\ldots,N-1$$

   where N is the DFT length. Here you need to choose a suitable $N = 2^m$ so that the bandwidth of the DFT frequency bins is around 20Hz. Plot $w[n]$.

   (c) Let $M = \frac{N}{4}$ be the number of samples to shift after each DFT. The energy in the k-th frequency bin of the i-th window is given by

$$X_i[k] = \left| \sum_{n=0}^{N-1} x[iM+n]w[n]e^{-\frac{j2\pi k}{N}} \right|^2, k = 0,1,\ldots,N-1$$

Write a MATLAB function "myspectrogram.m" that computes the short DFTs of the input signal for each window.

```
X = myspectrogram(x,N,w,M)
% x is the sampled data x[n]
% N is the DFT point
% w is the overlapping window
% M the number of samples to shift after each DFT
```

(d) Use the following code to plot the spectrogram

```
image(t,f,X(1:floor(N/2),:));  % t is time and f is frequency
colormap(hot(256));
colorbar;
```

ICE503 DSP-Homework#10

1. Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follow:

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   (a) Calculate the four-point DFT $X[k]$.

   (b) Calculate the four-point DFT $H[k]$.

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

   (c) Calculate $y[n] = x[n] \circledast_4 h[n]$ by doing the circular convolution directly.

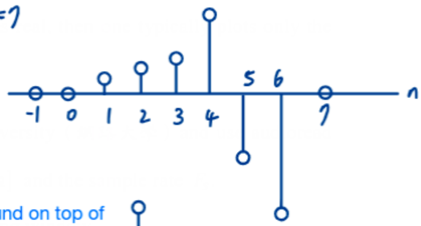
   (d) Calculate $y[n]$ of Part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

(a) $X[k] = \sum_{n=0}^{3} \sin\left(\frac{\pi n}{2}\right) W_4^{kn} = W_4^{k} - W_4^{3k}, k=0,1,2,3$

(b) $H[k] = \sum_{n=0}^{3} 2^n W_4^{kn} = 2^0 + 2^1 W_4^{k} + 2^2 W_4^{k2} + 2^3 W_4^{k3}, k=0,1,2,3$

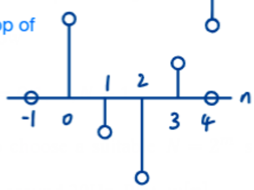
(c) To avoiding aliasing, we need: $N \ge L+M-1 = 4+4-1 = 7$
If we let N=4, we expect to happen aliasing,
First, we can find out: $y[n] = x[n] * h[n]$

$$\Rightarrow \begin{bmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&1&0&0\\ -1&0&1&0\\ 0&-1&0&1\\ 0&0&-1&0\\ 0&0&0&-1 \end{bmatrix} \begin{bmatrix} 1\\2\\4\\8 \end{bmatrix} = \begin{bmatrix} 0\\1\\2\\3\\6\\-4\\-8 \end{bmatrix} \Rightarrow$$



Cause aliasing, the last three points will wrap around on top of first three points giving: $y[n] = x[n] \circledast_4 h[n]$

$$\Rightarrow \begin{bmatrix} 0&-1&0&1\\ 1&0&-1&0\\ 0&1&0&-1\\ -1&0&1&0 \end{bmatrix} \begin{bmatrix} 1\\2\\4\\8 \end{bmatrix} = \begin{bmatrix} 6\\-3\\-6\\3 \end{bmatrix} \Rightarrow$$



(d) $y[n] = x[n] \circledast_4 h[n] \xrightarrow{DFT} Y[k] = X[k] \cdot H[k]$

$$= (W_4^{k} - W_4^{3k})(2^0 + 2^1 W_4^{k} + 2^2 W_4^{k2} + 2^3 W_4^{k3})$$

$$= (W_4^{k} - W_4^{3k})(1 + 2W_4^{k} + 4W_4^{k2} + 8W_4^{k3})$$

$$= (W_4^{k} + 2W_4^{2k} + 4W_4^{3k} + 8W_4^{4k}) - (W_4^{3k} + 2W_4^{4k} + 4W_4^{5k} + 8W_4^{6k})$$

$$= 6\underbrace{W_4^{4k}}_{1} - 3W_4^{k} - 6W_4^{2k} + 3W_4^{3k}, \quad 0 \le k \le 3$$

$$\xrightarrow{IDFT} \Rightarrow y[n] = 6\delta[n] - 3\delta[n-1] - 6\delta[n-2] + 3\delta[n-3], \quad 0 \le n \le 3$$

Same result as (c)

2. MATLAB simulation:

The idea of a spectrogram is plotting a sequence of short DFTs of the input signal using overlapping windows. If the signal is real, then one typically plots only the positive frequencies $k = 0,1,\ldots,\frac{N}{2}-1$.

(a) Download guitar4.wav from cyber university（網路大學）and use audioread function to obtain the sampled data $x[n]$ and the sample rate F_s.

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where N is the DFT length. Here you need to choose a suitable $N = 2^m$ so that the bandwidth of the DFT frequency bins is around 20Hz. Plot $w[n]$.

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$$X_i[k] = \left|\sum_{n=0}^{N-1} x[iM+n]w[n]e^{-\frac{j2\pi k}{N}}\right|^2, k = 0,1,\ldots,N-1$$

Write a MATLAB function "myspectrogram.m" that computes the short DFTs of the input signal for each window.

```
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% w is the overlapping window
% M the number of samples to shift after each DFT
```
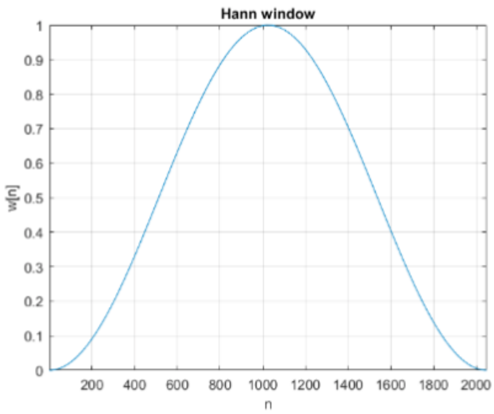

(d) Use the following code to plot the spectrogram

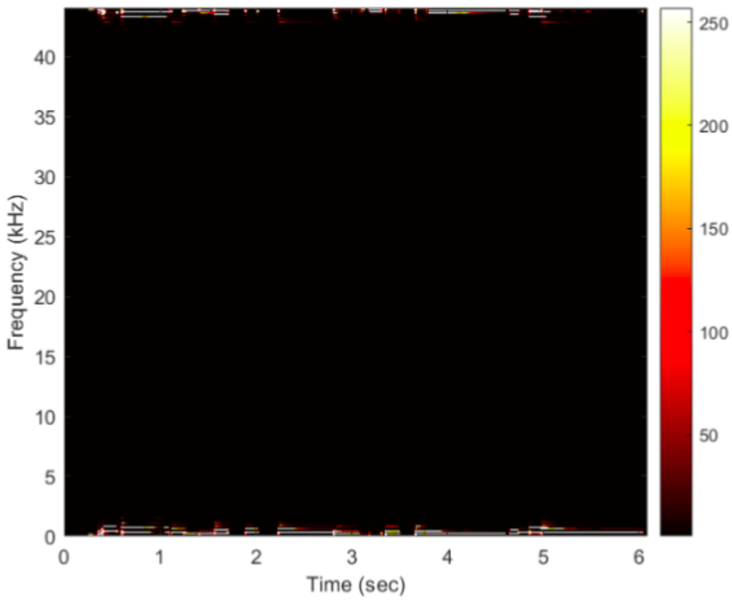
```
image(t,f,X(1:floor(N/2),:));  % t is time and f is frequency
colormap(hot(256));
colorbar;
```

ICE503 DSP-MATLAB#10

```
function X = myspecgram(x,N,w,M)
x = x(:); % make sure it's a column
w = w(:); % make sure it's a column
for i = 1 :floor((length(x)-(N-1))/M)
    X(:,i) = abs(fft(x((1:N)+(i-1)*M).*w)).^2;
end
end
=====================================================================

clear
clc

%% (a) load data
[x,Fs] = audioread('guitar4.wav'); % Read WAV file
Ts = 1/Fs;

%% (b) Hann window
m = round( log2(Fs/20) );  % 20Hz for each frequency bin
N = 2^m;  % DFT length or block size
n = [0:N-1];  % Hann window
w = 1/2*(1 - cos(2*pi*n'/N));

figure(1)
plot(w); grid on;
title('Hann window');
xlabel('n');
ylabel('w[n]');
axis([-inf,inf,-inf,inf])
```

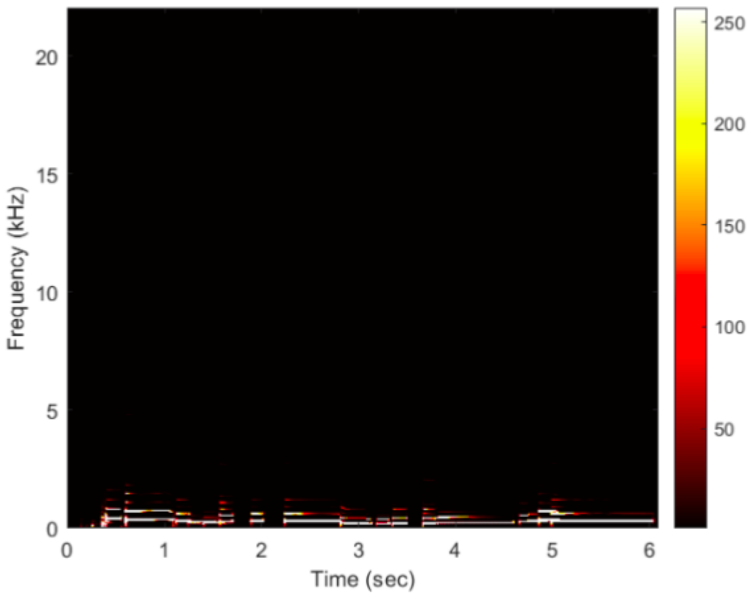


```
%% plot
M = N/4;  % hop size
X = myspecgram(x,N,w,M);
block_number = floor((length(x)-(N-1))/M);

t = (0:block_number-1)*M*Ts;   % time of spectrogram (sec)
f = (0:N-1)/N*Fs/1000;    % frequency of spectrogram (kHz)

figure(2)
image(t,f,X);
colormap(hot(256));
colorbar;
axis('xy');
xlabel('Time (sec)');
ylabel('Frequency (kHz)');
```



You can see that the spectrogram of a real-valued signal is symmetric, so we only need to plot half of the spectrogram.

```
%% plot half
f = (0:N/2-1)/N*Fs/1000;    % frequency of half spectrogram (kHz)

figure(3)
image(t,f,X(1:floor(N/2),:));
colormap(hot(256));
colorbar;
axis('xy');
xlabel('Time (sec)');
ylabel('Frequency (kHz)');
```



ICE503 DSP-Homework#11

1. The convolution of discrete-time system with an impulse response $\mathrm{h[n]}$ is given by:

$$\mathrm{y[n]} = \sum_{k=-\infty}^{\infty} h[k]x[n-k],$$

derive the z-transforms of transfer function $\mathrm{Y(z)} = \mathrm{H(z)X(z)}$ step by step.

2. A causal linear time-invariant system has the system function

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$$

(a) Write the difference equation that characterizes the system with $x[n]$ and $y[n]$.

(b) Plot the pole-zero diagram and indicate the region of convergence for the system function.

3. Matlab Simulation

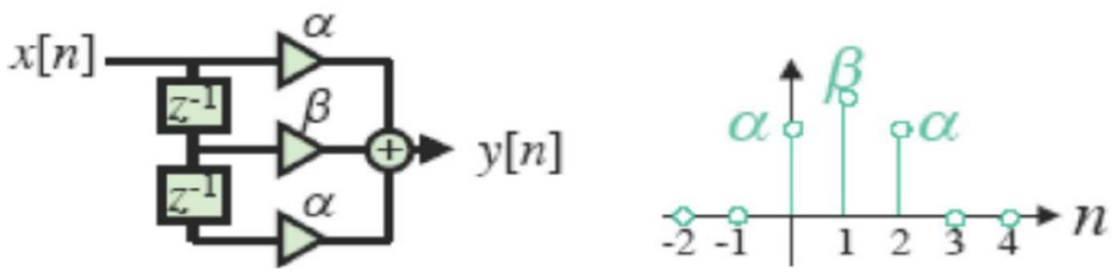
Separate the following information in frequency.

$$x[n] = Acos(\omega 1n) + Bcos(\omega 2n)$$

with construct $H(e^{j\omega})$

$$H(e^{j\omega}) = \begin{cases} |H(e^{j\omega_1})| & \sim \quad 1, \\ |H(e^{j\omega_2})| & \sim \quad 0, \end{cases}$$

Where $\omega_1 = 0.1$ and $\omega_2 = 0.4$. Consideer a 3 pt FIR filters with $h[n] = \{\alpha \ \ \beta \ \ \alpha\}$. Sketch the frequency response and compare the output signal with input signals.



HW 11

1. $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \right) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} h[k] \left(\sum_{n=-\infty}^{\infty} x[n-k] z^{-n} \right)$$

$l = n-k \Rightarrow n = l+k$

$$= \sum_{k=-\infty}^{\infty} h[k] \left(\sum_{l=-\infty}^{\infty} x[l] z^{-(l+k)} \right)$$

$$= \sum_{k=-\infty}^{\infty} h[k] \underbrace{\left(\sum_{l=-\infty}^{\infty} x[l] z^{-l} \right)}_{X[z]} z^{-k}$$

$$= X[z] \sum_{k=-\infty}^{\infty} h[k] z^{-k}$$

$$= X[z]\, H[z]$$

2. (a)

$$H(z) = \frac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7z^{-1})(1-0.7z^{-1})}$$

$$= \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}} = \frac{Y(z)}{X(z)}$$

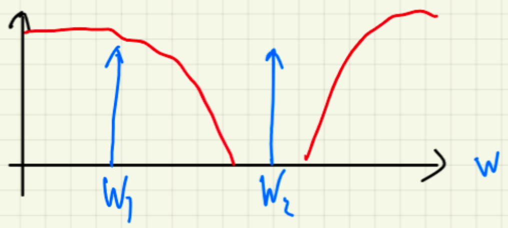
$$\Rightarrow X(z)(1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3})$$

$$= (1-z^{-1}+0.49z^{-2}-0.49z^{-3})Y(z)$$

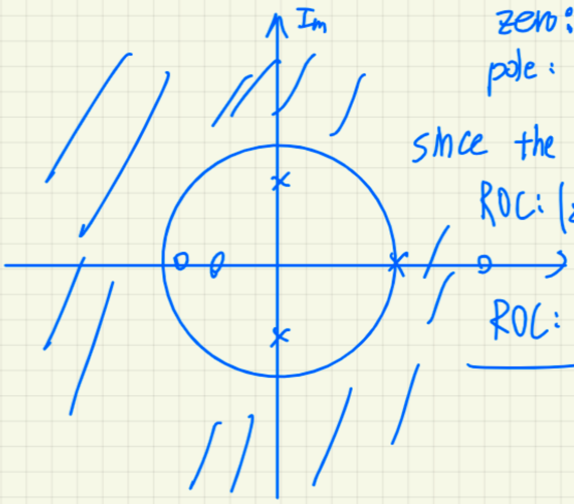
$$\downarrow$$

$$x[n]-0.6x[n-1]-2.35x[n-2]-0.9x[n-3]$$

$$= y[n]-y[n-1]+0.49y[n-2]-0.49y[n-3]$$



(b)



zero: $2, -\frac{1}{2}, -0.9$

pole: $1, -0.7j, 0.7j$

since the system is causal

ROC: $|z| >$ max pole

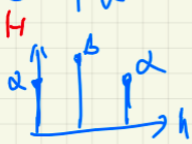
ROC: $|z| > 1$

3. Matlab Lec. 10 (P.29 – P.30)

$$H(e^{j\omega}) = \sum_{\forall n} h[n] e^{-j\omega n} = \alpha + \beta e^{-j\omega} + \alpha e^{-j2\omega}$$

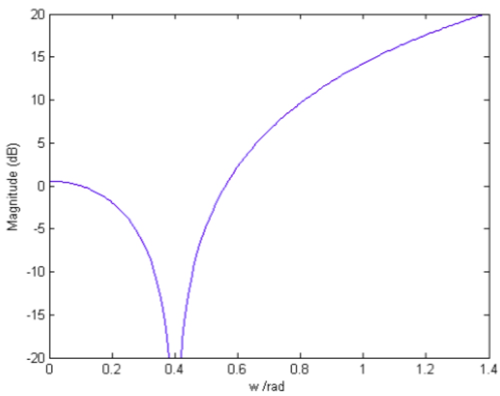
$$= e^{-j\omega}\left(\beta + \alpha\left(e^{j\omega} + e^{-j\omega}\right)\right)$$

$$= e^{-j\omega}\left(\beta + 2\alpha\cos\omega\right)$$



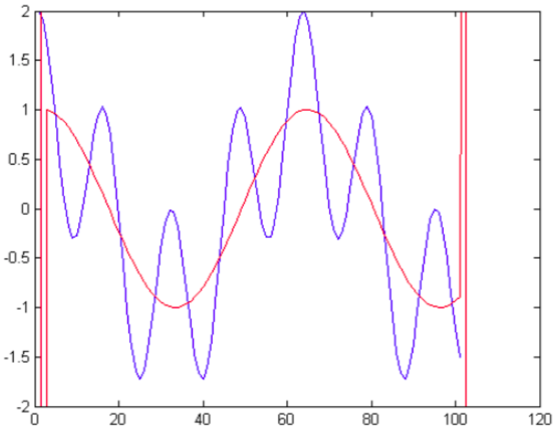
$$\left|H(e^{j\omega})\right| = \left|\beta + 2\alpha\cos\omega\right| = \begin{cases} 1, & \omega = \omega_1 \\ 0, & \omega = \omega_2 \end{cases}$$

```
clear all
clc

fs = 0:100;
x = cos(0.1*fs) + cos(0.4*fs);
FIR = [6.7619 -12.4563 6.7619];          %%L10   p30.    please explain
output = conv(x,FIR);
[h,w] = freqz(FIR);

figure(1)
plot(w,20*log10(abs(h)))
ylim([-20 20])
xlabel('w /rad')
ylabel('Magnitude (dB)')
```




```
figure(2)
plot(x)
hold on
plot(output,'r-')
```

```
ylim([-2 2 ])
hold off
```



ICE503 DSP-Homework#12

1. Figure 1 shows the impulse response for several different LTI systems. Determine the group delay associated with each systems.

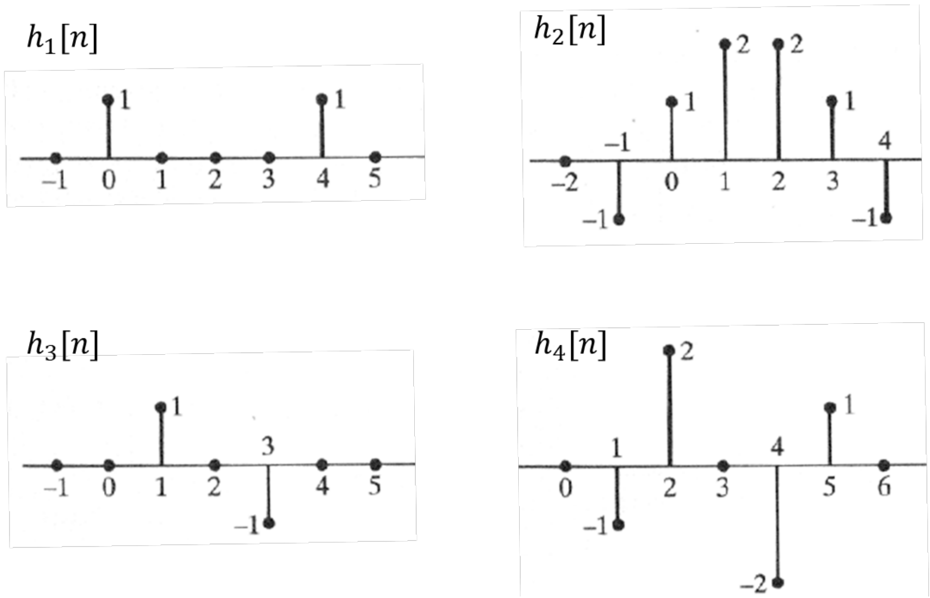


Figure 1: Impulse response for several different LTI systems

2. Figure 2 shows two different interconnections of three systems. The impulse responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are as shown in Figure 3. Determine whether system A and/or system B is a generalized linear-phase system.

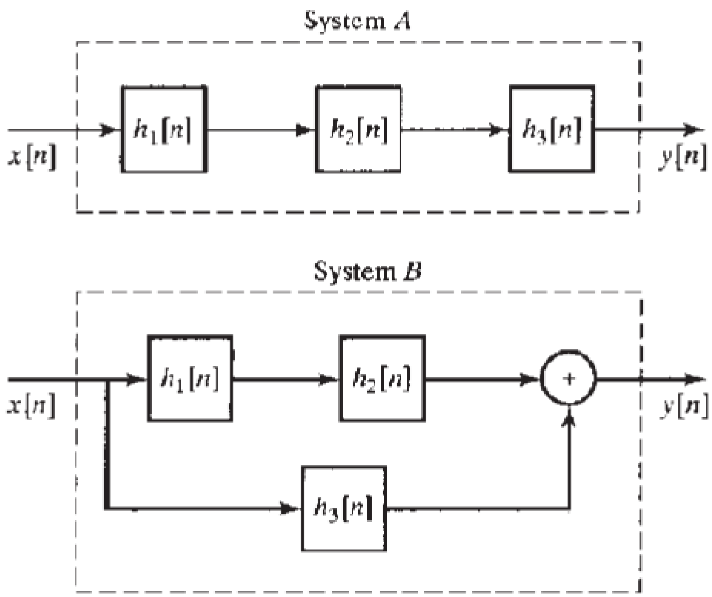


Figure 2: Two different interconnections of three systems

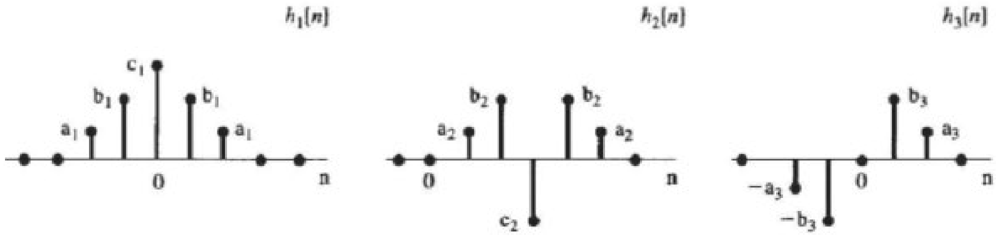


Figure 3 Impulse responses of the three systems

3. MATLAB simulation:

   Using iirnotch function to design a second order IIR notch filter with the notch located at $\omega c = 0.1\pi$ and with the 3 dB bandwidth of 0.001π and use fvtool function sketch the magnitude of the filter in dB and the group delay.

ICE DSP - Answer # 12

1. $h_1[n] = \delta[n] + \delta[n-4]$

$H_1(e^{jw}) = 1 + e^{-j4w} = e^{-j2w}(e^{j2w} + e^{-j2w}) = e^{-j2w}(2\cos 2w)$

$\theta(w) = -2w$

$\tau_g(w) = -\frac{d\theta(w)}{dw} = 2$ ※

$h_2[n] = -\delta[n+1] + \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3] - \delta[n-4]$

$H_2(e^{jw}) = -e^{jw} + 1 + 2e^{-jw} + 2e^{-j2w} + e^{-j3w} - e^{-j4w}$

$= e^{-j1.5w}(-e^{j2.5w} + e^{j1.5w} + 2e^{j0.5w} + 2e^{-j0.5w} + e^{-j1.5w} - e^{-j2.5w})$

$= e^{-j1.5w}(-2\cos 2.5w + 2\cos 1.5w + 4\cos 0.5w)$

$\theta(w) = -1.5w$

$\tau_g(w) = -\frac{d\theta(w)}{dw} = 1.5$ ※

$h_3[n] = \delta[n-1] - \delta[n-3]$

$H_3(e^{jw}) = e^{-jw} - e^{-j3w} = e^{-j2w}(e^{jw} - e^{-jw}) = e^{-j2w}(2j\sin w) = e^{-j(2w - \frac{\pi}{2})}(2\sin w)$

$\theta(w) = -(2w - \frac{\pi}{2})$

$\tau_g(w) = -\frac{d\theta(w)}{dw} = 2$ ※

$h_4[n] = -\delta[n-1] + 2\delta[n-2] - 2\delta[n-4] + \delta[n-5]$

$H_4(e^{jw}) = -e^{-jw} + 2e^{-j2w} - 2e^{-j4w} + e^{-j5w}$

$= e^{-j3w}(-e^{j2w} + 2e^{jw} - 2e^{-jw} + e^{-j2w})$

$= e^{-j3w}(-2j\sin 2w + 4j\sin w)$

$= e^{-j(3w - \frac{\pi}{2})}(-2\sin 2w + 4\sin w)$

$\theta(w) = -(3w - \frac{\pi}{2})$

$\tau_g(w) = -\frac{d\theta(w)}{dw} = 3$ ※

2. $H_1(e^{jw}) = a_1 e^{j2w} + b_1 e^{jw} + c_1 + b_1 e^{-jw} + a_1 e^{-j2w}$

$= c_1 + 2b_1 \cos w + 2a_1 \cos 2w$

$H_2(e^{jw}) = a_2 e^{-jw} + b_2 e^{-j2w} - c_2 e^{-j3w} + b_2 e^{-j4w} + a_2 e^{-j5w}$

$= e^{-j3w}(-c_2 + 2b_2 \cos w + 2a_2 \cos 2w)$

$H_3(e^{jw}) = -a_3 e^{j2w} - b_3 e^{jw} + b_3 e^{-jw} + a_3 e^{-j2w}$

$= e^{-j\frac{\pi}{2}}(2b_3 \sin w + 2a_3 \sin 2w)$

System A: $h_A[n] = h_1[n] * h_2[n] * h_3[n]$

$\Rightarrow H_A(e^{jw}) = H_1(e^{jw}) H_2(e^{jw}) H_3(e^{jw})$

$= e^{-j(3w+\frac{\pi}{2})}\left[(c_1 + 2b_1 \cos w + 2a_1 \cos 2w)(c_2 + 2b_2 \cos w + 2a_2 \cos 2w)\right.$

$\left.\cdot (2b_3 \sin w + 2a_3 \sin 2w)\right]$

System A is a generalized linear-phase system.

System B: $h_B[n] = h_1[n] * h_2[n] + h_3[n]$

$\Rightarrow H_B(e^{jw}) = H_1(e^{jw}) H_2(e^{jw}) + H_3(e^{jw})$

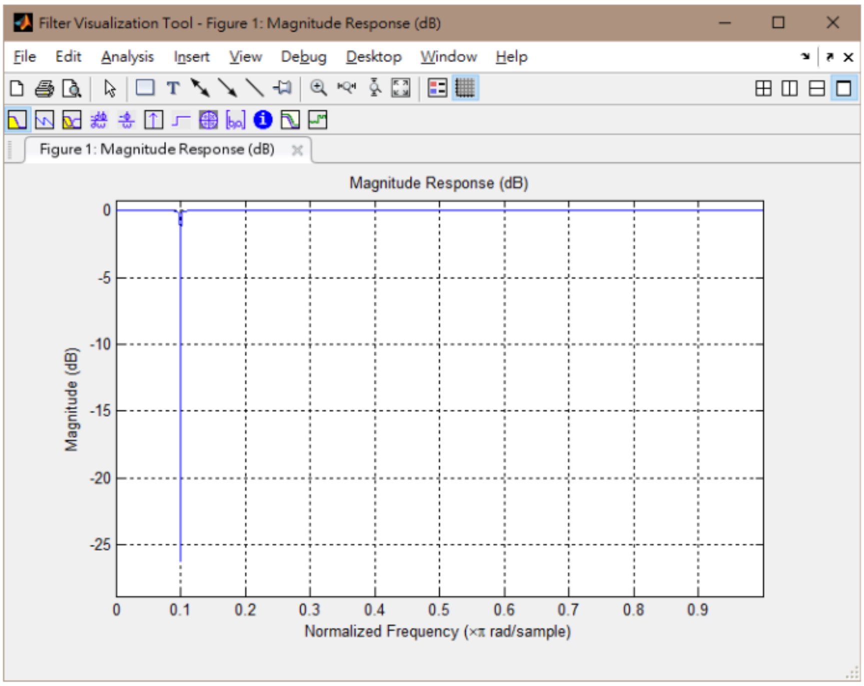
$= e^{-j3w}\left[(c_1 + 2b_1 \cos w + 2a_1 \cos 2w)(c_2 + 2b_2 \cos w + 2a_2 \cos 2w)\right]$

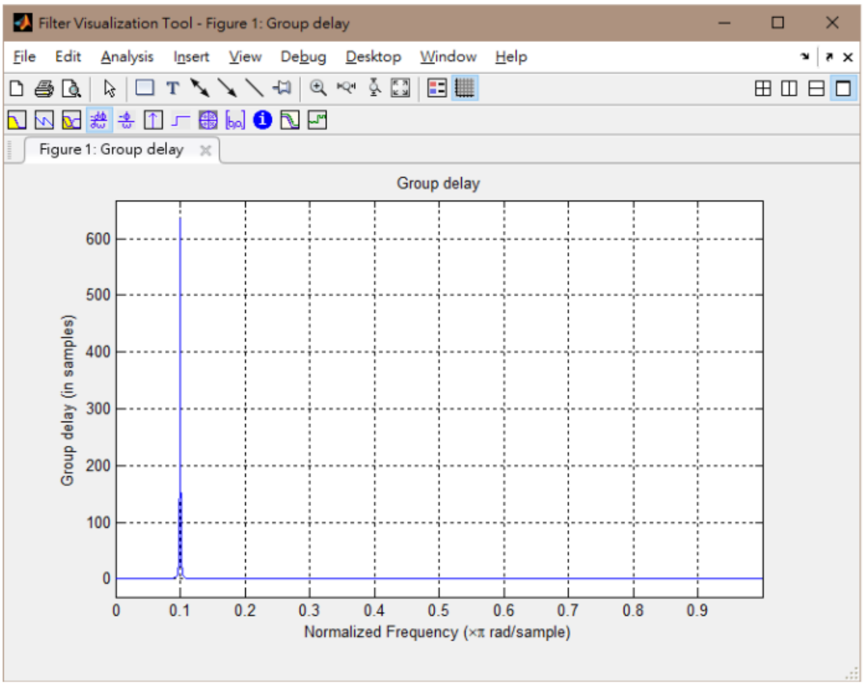
$+ e^{-j\frac{\pi}{2}}(2b_3 \sin w + 2a_3 \sin 2w)$

System B is not a generalized linear-phase system.

DSP-MATLAB#12

```
clear
clc

Wc = 0.1;   % the notch at frequency Wc = 0.1*pi
BW = 0.001; % 3dB bandwidth: BW = 0.001*pi
[num,den] = iirnotch(Wc,BW);
fvtool(num,den);
```


Filter Visualization Tool - Figure 1: Magnitude Response (dB)
File Edit Analysis Insert View Debug Desktop Window Help
Figure 1: Magnitude Response (dB)
Magnitude Response (dB)
0
-5
-10
-15
-20
-25
Magnitude (dB)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)


Filter Visualization Tool - Figure 1: Group delay
File Edit Analysis Insert View Debug Desktop Window Help
Figure 1: Group delay
Group delay
600
500
400
300
200
100
0
Group delay (in samples)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)

ICE503 DSP-Homework#13

1. Figure 1 shows the pole-zero plots for eight different causal LTI systems with real impulse responses. Indicate which of the following properties apply to each of the systems pictured: stable, IIR, FIR, all-pass, generalized linear phase (which type).

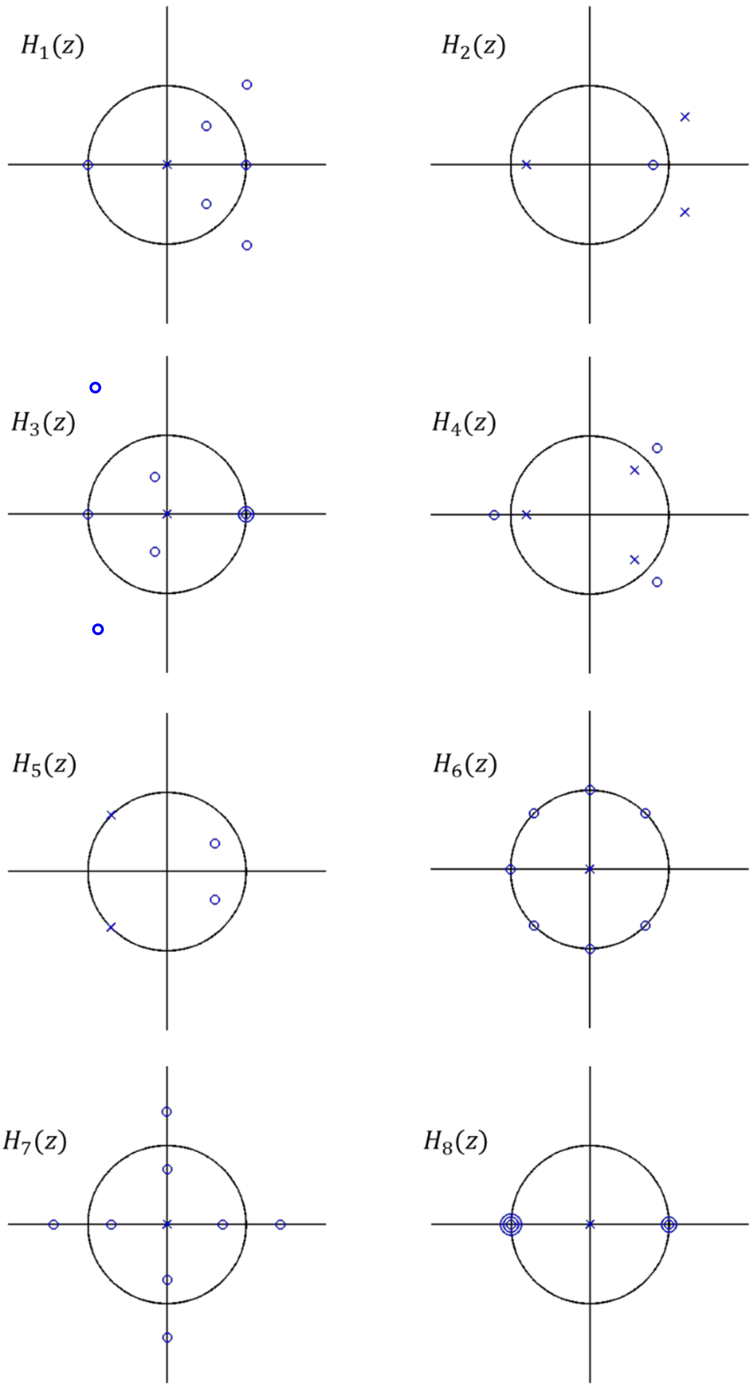
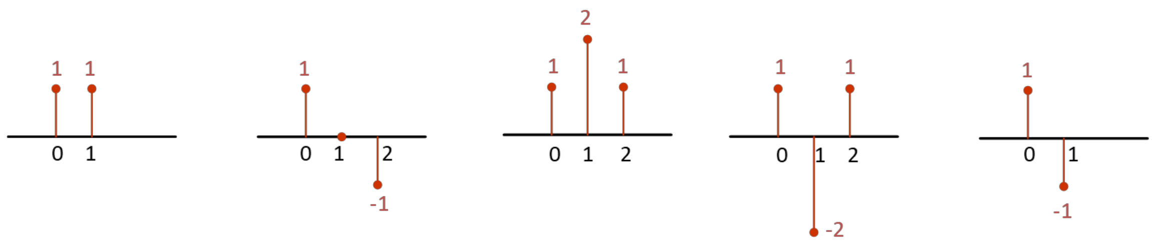


Figure 1: The pole-zero plots for eight different causal LTI systems

2. Given the following impulse responses



(1) (2) (3) (4) (5)

(a) Determine their phase delay.

(b) Determine the types of the FIR filters.

(c) Sketch the zeros of the corresponding system.

3. MATLAB simulation:

Using the impulse response for two different causal LTI systems in Figure 2 and sketch the magnitude of the filter in dB, group delay, pole-zero diagram and discuss the result.

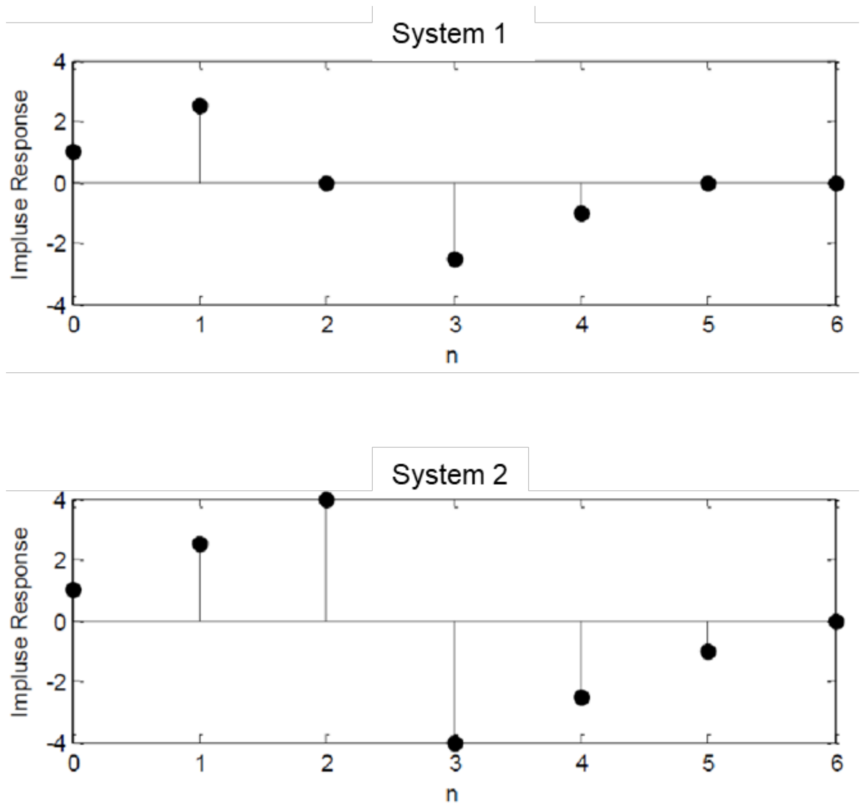


Figure 2: The impulse response for two different causal LTI systems

1. Figure 1 shows the pole-zero plots for eight different causal LTI systems with real impulse responses. Indicate which of the following properties apply to each of the systems pictured: stable, IIR, FIR, all-pass, generalized linear phase (which type).

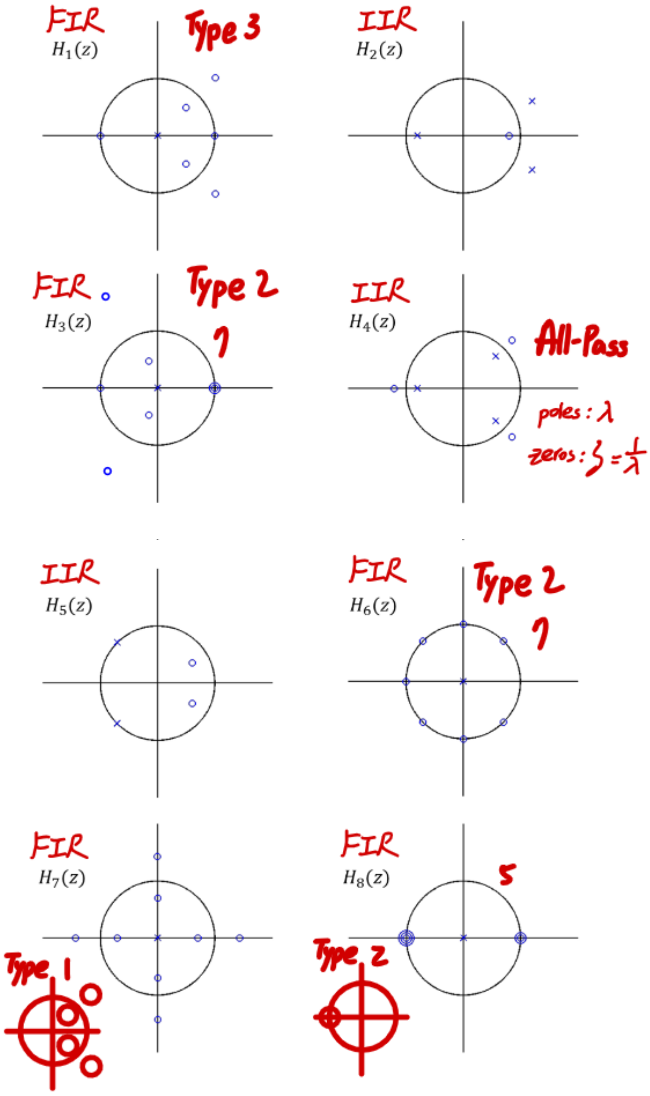
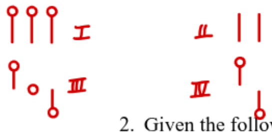
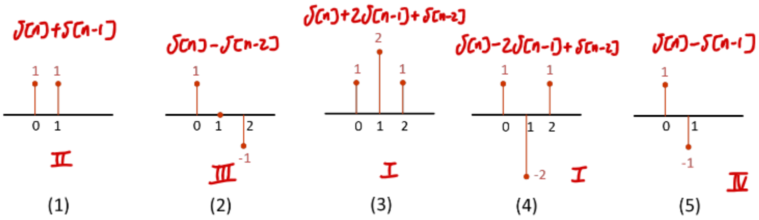


Figure 1: The pole-zero plots for eight different causal LTI systems



2. Given the following impulse responses



(a) Determine their phase delay.

(b) Determine the types of the FIR filters.

(c) Sketch the zeros of the corresponding system.

1\. ① If ROC $|z| > |\lambda|$ includes unit circle, the system is stable.

Stable : $H_1(z), H_3(z), H_4(z), H_6(z), H_7(z), H_8(z)$

Unstable : $H_2(z), H_5(z)$

② FIR has no poles (only zeros), IIR has poles (and often zeros)

FIR : $H_1(z), H_3(z), H_6(z), H_7(z), H_8(z)$

IIR : $H_2(z), H_4(z), H_5(z)$

③ Allpass filter has poles λ and zeros $\zeta = \frac{1}{\lambda}$ $\Rightarrow H_4(z)$

④ Generalized linear phase : determine the type for FIR

$H_1(z)$: Type 3 , $H_3(z)$: Type 2 , $H_6(z)$: Type 2

$H_7(z)$: Type 1 , $H_8(z)$: Type 2

2\.

(1) $h[n] = \delta[n] + \delta[n-1]$

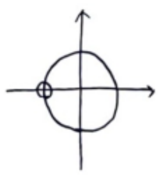
$H(e^{j\omega}) = 1 + e^{-j\omega} = e^{-j0.5\omega}(e^{j0.5\omega} + e^{-j0.5\omega}) = e^{-j0.5\omega}(2\cos 0.5\omega)$

$\theta(\omega) = -0.5\omega$

(a) Phase delay $\tau_p(\omega) = -\frac{(-0.5\omega)}{\omega} = 0.5$

(b) Symmetric, even length $\Rightarrow$ Type 2

(c) $H(z) = 1 + z^{-1}$, zero $= -1$



(2) $h[n] = \delta[n] - \delta[n-2]$

$H(e^{jw}) = 1 - e^{-j2w} = e^{-jw}(e^{jw} - e^{-jw}) = e^{-jw}(2j\sin w) = e^{-j(w-\frac{\pi}{2})}(2\sin w)$

$\theta(w) = -w + \frac{\pi}{2}$

(a) Phase delay $\tau_p(w) = -\frac{(-w+\frac{\pi}{2})}{w} = 1 - \frac{\pi}{2w}$

(b) Antisymmetric, odd length $\Rightarrow$ Type 3

(c) $H(z) = 1 - z^{-2} = (1+z^{-1})(1-z^{-1})$, zero $= 1, -1$

(3) $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$H(e^{jw}) = 1 + 2e^{-jw} + e^{-j2w} = e^{-jw}(e^{jw} + 2 + e^{-jw}) = e^{-jw}(2 + 2\cos w)$

$\theta(w) = -w$

(a) Phase delay $\tau_p(w) = -\left(\frac{-w}{w}\right) = 1$

(b) Symmetric, odd length $\Rightarrow$ Type 1

(c) $H(z) = 1 + 2z^{-1} + z^{-2} = (1+z^{-1})(1+z^{-1})$, zero $= -1, -1$

(4) $h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$

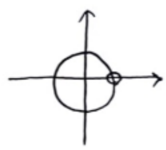
$H(e^{jw}) = 1 - 2e^{-jw} + e^{-j2w} = e^{-jw}(e^{jw} - 2 + e^{-jw}) = e^{-jw}(-2 + 2\cos w)$

$\theta(w) = -0.5w + \frac{\pi}{2}$

(a) Phase delay $\tau_p(w) = -\left(\frac{-0.5w + \frac{\pi}{2}}{w}\right) = 0.5 - \frac{\pi}{2w}$

(b) Antisymmetric, even length $\Rightarrow$ Type 4

(c) $H(z) = 1 - z^{-1}$, zero $= 1$



3. MATLAB simulation:

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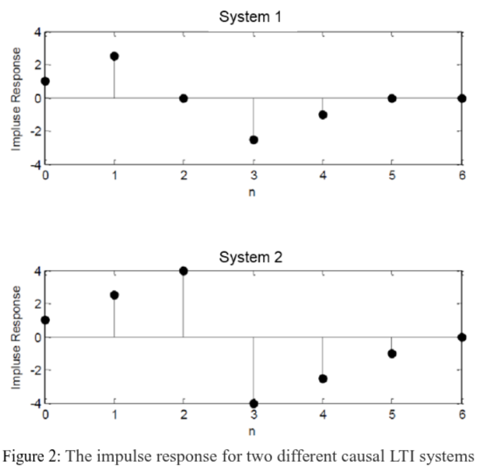

Figure 2: The impulse response for two different causal LTI systems

```
clear
clc

system_1 = [1 2.5 0 -2.5 -1];
[magnitude,w] = freqz(system_1, [1 0 0 0 0 ], 1024);
group_delay_1 = grpdelay(system_1, 1, 1024);

figure(1)
suptitle('System 1')

subplot(2,2,1);
plot(w/pi, 20*log10(abs(magnitude)));
grid on;
ylabel('Magnitude Response in dB')
xlabel('Frequency, \omega/\pi')

subplot(2,2,2);
plot(w/pi, angle(magnitude));
grid on;
ylabel('Phase Response')
xlabel('Frequency, \omega/\pi')

subplot(2,2,3)
plot(w/pi, group_delay_1);

grid on
ylabel('Group delay')
xlabel('Frequency, \omega/\pi')


subplot(2,2,4)
zplane(system_1,1);
grid on;
```

ICE503 DSP-Homework#13

1. We know that any rational system can be expressed as

$$H(z) = H_{\min}(z)H_{\mathrm{ap}}(z),$$

Where $H_{\min}(z)$ is minimum phase system and $H_{\mathrm{ap}}(z)$ is an all-pass system.

For each of the following system, please specify and plot pole and zero for the $H_{\min}(z)$, $H_{\mathrm{ap}}(z)$ and make sure $|H(z)| = |H_{\min}(z)|$.

(a) $H_1(z) = \frac{(1+3z^{-1})}{1+\frac{1}{2}z^{-1}}$

(b) $H_2(z) = \frac{(1+\frac{3}{2}e^{\frac{j\pi}{4}}z^{-1})(1+\frac{3}{2}e^{\frac{-j\pi}{4}}z^{-1})}{1+\frac{1}{3}z^{-1}}$

2. Consider the causal LTI system with the system function

$$H(z) = \frac{D - Mz^{-1}}{(C - Hz^{-1} + Iz^{-2})(A + Nz^{-1})},$$

Where $C = 1, H = \frac{1}{2}, I = \frac{1}{3}, A = 1, N = \frac{1}{4}, D = 1, M = \frac{1}{5}$.

(a) Draw the signal flow graphs in each of the following.

I. Direct form I

II. Direct form II

III. Cascade form with first- and second-order sections of direct form II

IV. Parallel form with first- and second-order sections of direct form II

V. Transposed direct form I

VI. Transposed direct form I

(b) Write the different each for the flow graph of (a)-VI, and show this system has the correct system function.

3. Matlab simulation

(a) Use matlab fvtool to plot and analyze the system 1.(a), $H_1(z), H_{1,\min}(z)$ *and* $H_{1,\mathrm{AP}}(z)$, respectively.

(b) Use matlab fvtool to plot and analyze comb filter of $H_1(z)$ for $L = 4$.