ICE503 DSP-Homework#1

1. Consider a discrete-time system which can develop the output sequence:

$$y[n] = 3x[n] + 4x[n-1] - x[n-2] + 2x[n-4]$$

   (a) Plot the block diagram for this system.

   (b) The input sequence $x[n]$ is shown in Figure 1, sketch and label $y[n]$.

   (c) Following (b), sketch and label the down sampling sequence $y[3n]$.

   (d) Following (b), sketch and label the up sampling sequence $y[\frac{1}{2}n]$.

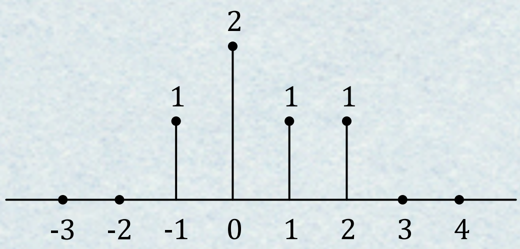


Figure 1: The input sequence $x[n]$

2. Determine whether each of the following signals is periodic. If the signal is periodic, state its fundamental period.

   (a) $x[n] = 6\cos\left(\frac{\pi}{2}n\right)$

   (b) $x[n] = n\sin\left(\frac{\pi}{12}n\right)$

   (c) $x[n] = e^{j\frac{3}{4}\pi n}$

3. MATLAB simulation:

   (a) Generate the complex-valued signal.

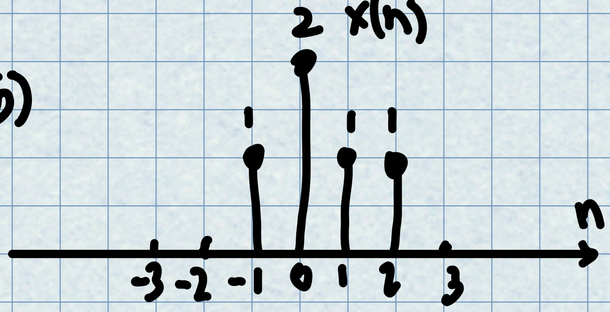
$$x[n] = e^{j\frac{1}{10}\pi n}, \qquad n = -10, \ldots, -1, 0, 1, \ldots 10$$

   (b) Use `stem` function to plot the real part and the imaginary part of $x[n]$.

   (c) Determine whether $x[n]$ is a conjugate symmetric sequence or a conjugate antisymmetric sequence, and explain the reason.

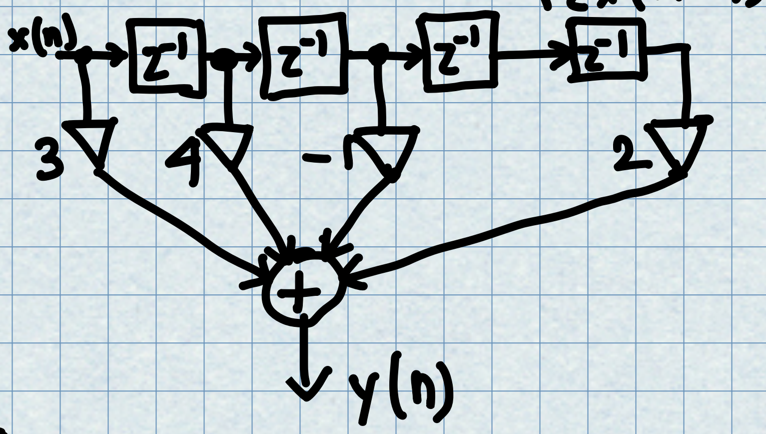
(1)

(b)

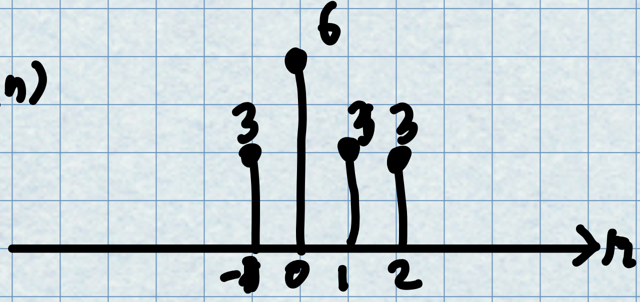


(a) Block diagram

$y(n) = 3x(n) + 4x(n-1) - x(n-2) + 2x(n-4)$



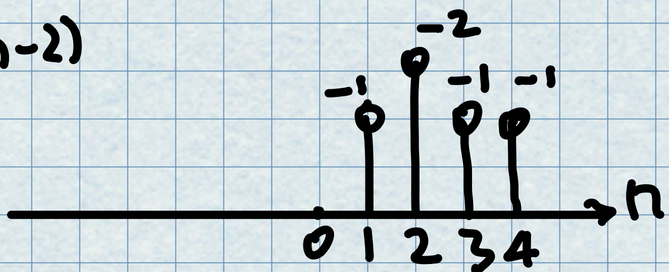
3x(n)



4x(n-1)

x(n-1) means shift x(n) to 1 pos. right

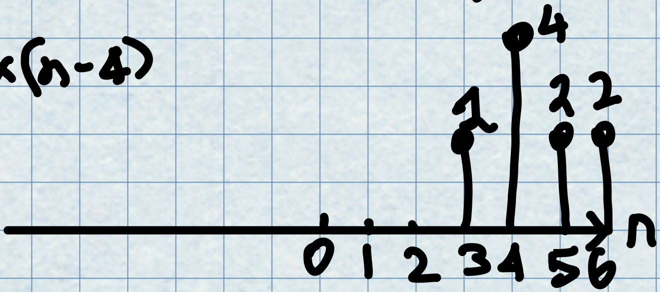
-x(n-2)



x(n-2) shift x(n) 2 pos. right

-x(n-2) means ampl. is -ve

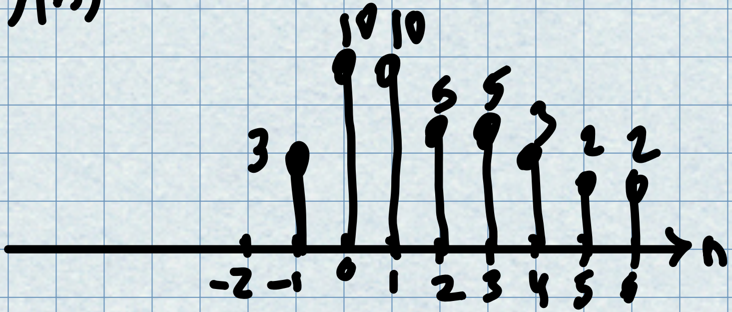
+2x(n-4)



x(n-4) → shift 4 pos. to right

2x(n-4) → ampl. made 2 times

=> y(n)



$y(n) = \{ 3, \underset{\uparrow}{\dot{10}}, 10, 5, \dot{5}, 3, 2, \dot{2}\}$

(c) $y(3n)$

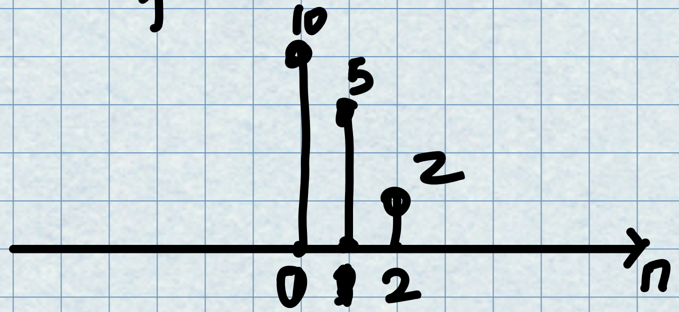
$= \{0, \underset{\uparrow}{10}, 5, 2\}$

$= \{\underset{\uparrow}{10}, 5, 2\}$

Keep 0^{th} element.

Across 0^{th} element, drop every $(3-1)=2$ elements.

Then keep every 3 elements.

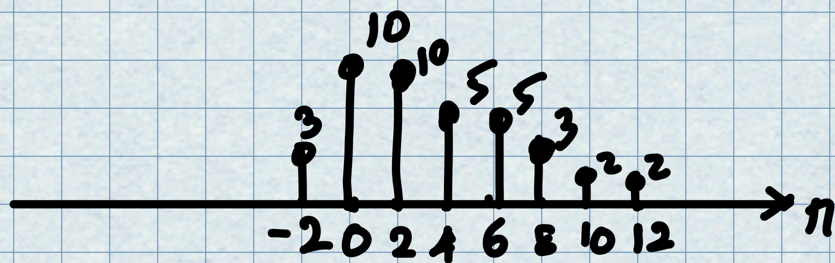


(d) $y(\frac{1}{2}n)$

$= \{3, 0, \underset{\uparrow}{10}, 0, 10, 0, 5, 0, 5, 0, 3, 0, 2, 0, 2\}$

Keep 0^{th} element.

Insert $(2-1)=1$ zero in between every element of $x(n)$



(2)

(a) $x[n] = 6\cos\left(\frac{\pi}{2}n\right)$

(b) $x[n] = n\sin\left(\frac{\pi}{12}n\right)$

(c) $x[n] = e^{j\frac{3}{4}\pi n}$

(a) $x(n) = 6\cos\left(\frac{\pi}{2}n\right)$

$$x(n+mT_0) = 6\cos\frac{\pi}{2}(n+mT_0)$$

If they are periodic, then

$$x(n) = x(n+mT_0)$$

$$\Rightarrow 6\cos\left(\frac{\pi}{2}n\right) = 6\cos\frac{\pi}{2}(n+mT_0)$$

$$\Rightarrow 6\cos\left(\frac{\pi}{2}n+2\pi p\right) = 6\cos\left(\frac{\pi}{2}n+\frac{\pi}{2}mT_0\right)$$

for any integer p.

$$\Rightarrow \frac{\pi}{2}n+2\pi p = \frac{\pi}{2}n+\frac{\pi}{2}mT_0$$

$$\Rightarrow 2\pi p = \frac{\pi}{2}mT_0$$

$$\Rightarrow 4p = mT_0$$

where p, m, T_0 are all integers.

Set $m=1$, $p=1$ then

time period $T_0 = 4$.

(b) $x(n) = n \sin\left(\frac{\pi}{12} n\right)$

$x(n+mT_0) = (n+mT_0) \sin \frac{\pi}{12}(n+mT_0)$

If they are periodic, then

$$x(n) = x(n+mT_0)$$

$$\Rightarrow n \sin\left(\frac{\pi}{12} n + 2\pi p\right) = (n+mT_0) \sin \frac{\pi}{12}(n+mT_0)$$

$\Rightarrow$ There is no unique rational solution to this problem. Hence, $x(n)$ is NOT periodic

(c) $x(n) = e^{j\frac{3}{4}\pi n}$

$x(n+mT_0) = e^{j\frac{3}{4}\pi(n+mT_0)}$

If $x(n)$ is periodic, then

$$x(n) = x(n+mT_0)$$

$$\Rightarrow e^{j\frac{3}{4}\pi n} = e^{j\frac{3}{4}\pi(n+mT_0)}$$

$$\Rightarrow e^{j\left(\frac{3}{4}\pi n + 2\pi p\right)} = e^{j\frac{3}{4}\pi(n+mT_0)}$$

$$\Rightarrow e^{j\frac{3}{4}\pi n} \times e^{j2\pi p} = e^{j\frac{3}{4}\pi n} \times e^{j\frac{3}{4}\pi mT_0}$$

$$\Rightarrow 2\pi p = \frac{3}{4}\pi mT_0 \quad \Rightarrow 2p = \frac{3}{4} mT_0$$

$$\Rightarrow 8p = 3mT_0$$

The least integral solution is $p=3, m=1$ then $T_0 = 8$, which is integer. Therefore, $x(n)$ is periodic.

$\frac{8p}{3m} = T_0$

$m=1 \Rightarrow T_0 = \frac{8p}{3}$

$= \frac{8\times3}{3}$

$= 8$

(4) $x(n) = e^{j\frac{1}{10}\pi n}$ $\quad n \in \{-10, \dots, 10\}$

(c) Conjugate symmetry or antisymmetry.

$$x(n) = e^{j\frac{1}{10}\pi n}$$

$$x^*(-n) = \left(e^{j\frac{1}{10}\pi(-n)}\right)^* = \left(e^{-j\frac{1}{10}\pi n}\right)^*$$

$$= e^{j\frac{1}{10}\pi n}$$

$$= x(n)$$

=> the sequence is conjugate symmetric.

▫ Conjugate symmetry
$x^*(-n) = x(n)$

▫ Conjugate antisymmetric
$x^*(-n) = -x(n)$

$$|x(n)| = |e^{j\frac{1}{10}\pi n}|$$

$$= 1$$

This is a unit-modulus signal having phase conjugate symmetry. Thus, such data acquisition requires half of the data for the signal reconstruction, reducing storage and phase encoding steps by almost 50%. This further reduces computational load when processing such signals.

```
% Homework 1
% Q. 3

% ---- clear ----
close all;
clear all;
clc;

% ---- (a) ----
f = @(n) exp(1j*(pi/10)*n);

n = -10: 1: 10;
x = f(n);

% --- (b) ----
xr = real(x);
xi = imag(x);

f = figure(1);
f.Position = [10 10 800 640];
stem(n, xr, '-r', 'linewidth', 2);
hold on
stem(n, xi, '-.db', 'linewidth', 2);
hold off
grid on
legend('Re $x(n)$', 'Im $x(n)$', 'Location', 'Northwest');
xlabel('$n$')
ylabel('Re (Im) $x(n)$')
xlim([-10, 10])
ylim([-1.2, 1.2])
set(findall(f,'-property','FontSize'),'FontSize',14);
set(findall(f,'-property','Interpreter'), 'Interpreter', 'latex');

saveas(f, 'hw01.eps', 'epsc');
```

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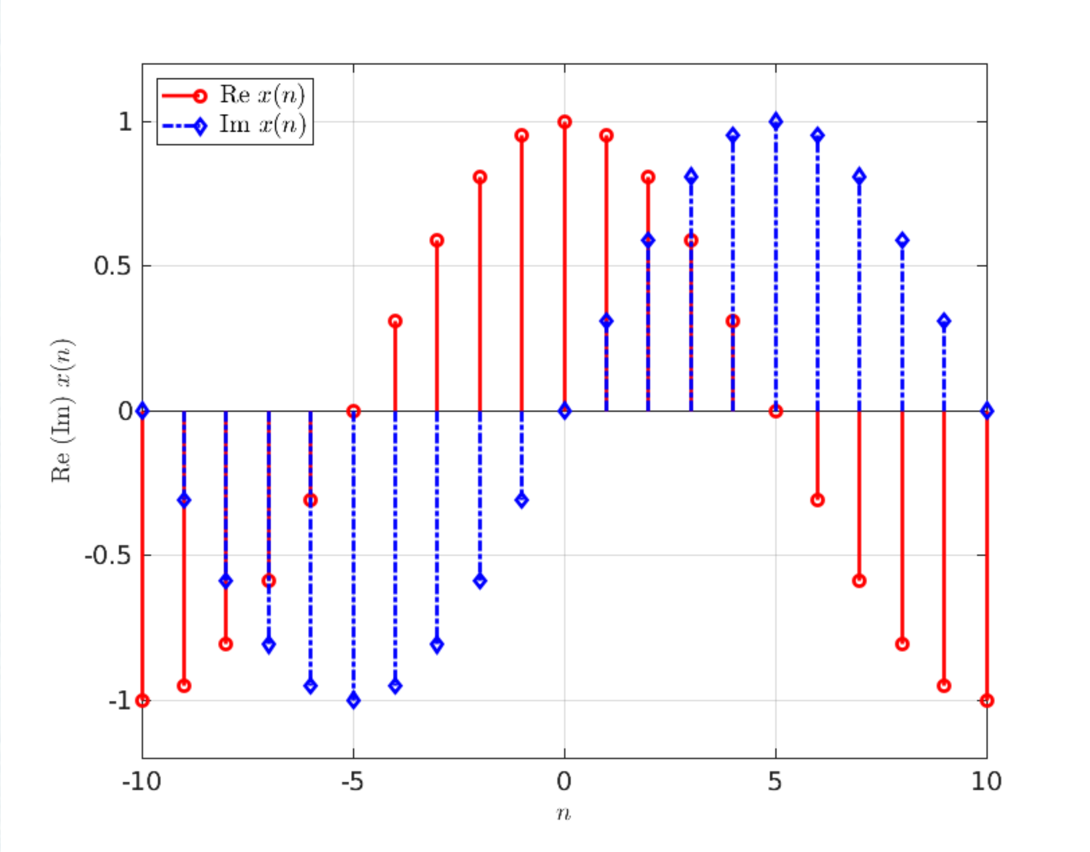

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ICE503 Homework-01

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Q. 3

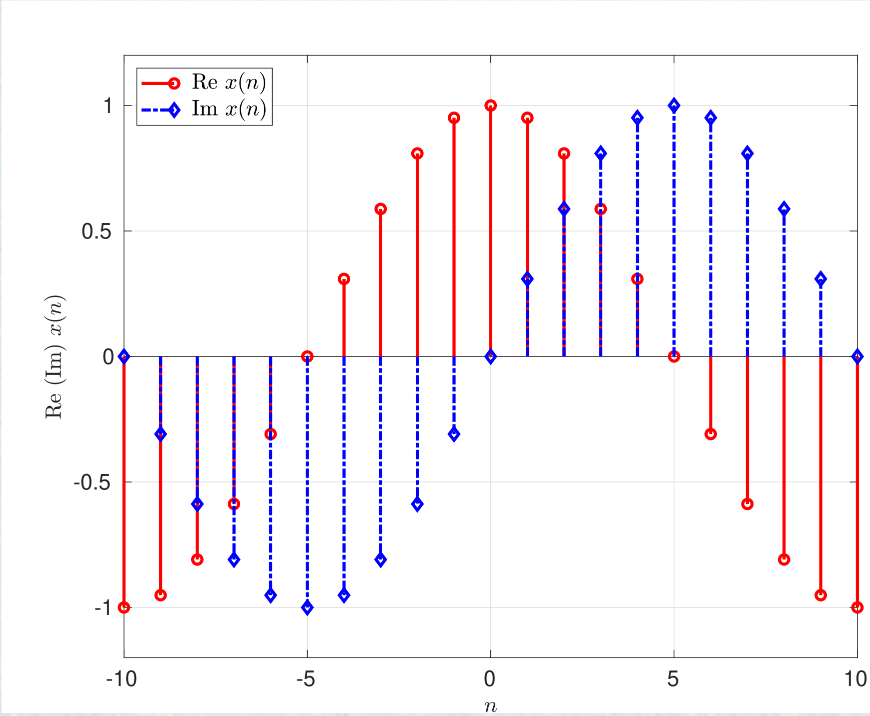

Fig. 1: 3(b) Plot of real and imaginary part of $x(n)$.

(b) The plot of the real and imaginary part is shown in Fig. 1
(c) The given series is:

$$x(n) = e^{j\frac{1}{10}\pi n} \quad n \in \{-10, -9, \cdots, -1, 0, 1, \cdots, 10\} \tag{1}$$

Then calculate the value of $x^*(-n)$, which is written from equation (1) as:

$$\begin{aligned} x^*(-n) &= (e^{j\frac{1}{10}\pi(-n)})^* \\ &= (e^{-j\frac{1}{10}\pi n})^* \\ &= e^{j\frac{1}{10}\pi n} \\ &= x(n) \end{aligned} \tag{2}$$

From the definition of conjugate symmetric and antisymmertric series,

$$\text{Conjugate Symmetry :} \quad x^*(-n) = x(n)$$
$$\text{Conjugate Anti-symmetry :} x^*(-n) = -x(n)$$

The series $x(n)$ is ***conjugate symmetric*** which can be inferred from the equation (2) and above identities.
Since $|x(n)| = 1$, which infers that the signal $x(n)$ is an unit-modulus signal and therefore, it exhibits phase conjugate symmetry. Thus, such data transmission requires half of the data for the signal reconstruction, reducing storage and phase encoding steps by almost 50 %. This further reduces the computational load by similar amount during processing of such signals.

Date: September 15, 2024