ICE503 DSP-Homework#6

1. Consider the system shown in Figure 1 for discrete-time processing of the continuous-time input signal $g_c(t)$. The input signal is of the form $g_c(t) = f_c(t) + e_c(t)$. The Fourier transform of $f_c(t)$ and $e_c(t)$ are shown in Figure 2. Since the total input signal is not bandlimited, a continuous-time antialiasing filter $H_{aa}(j\Omega)$ is used to combat aliasing distortion. The magnitude of the frequency response for $H_{aa}(j\Omega)$ is shown in Figure 3, and the phase response is $\angle H_{aa}(j\Omega) = -\Omega^3$.

   (a) If the sampling rate is $2\pi/T = 1600\pi$, determine the frequency response of the discrete-time system $H(e^{j\omega})$, so that the output is $y_c(t) = f_c(t)$.

   (b) Is it possible that $y_c(t) = f_c(t)$ if $2\pi/T < 1600\pi$? If so, what is the minimum value of $2\pi/T$? Determine $H(e^{j\omega})$ for this choice of $2\pi/T$.

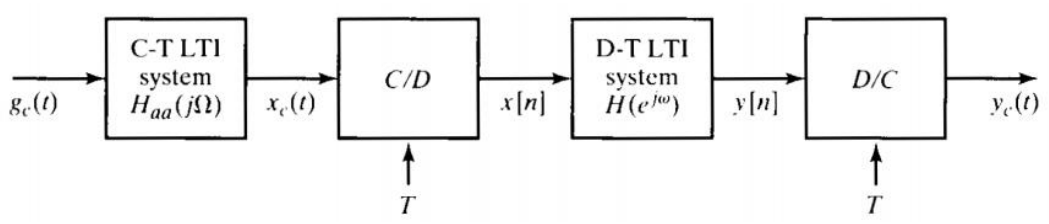

Figure 1: system

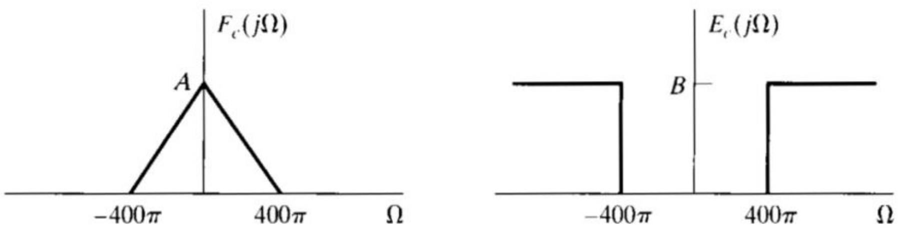

Figure 2: the Fourier transform of $f_c(t)$ and $e_c(t)$

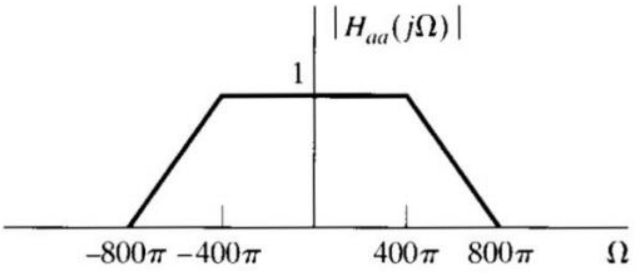

Figure 3: the frequency response of $H_{aa}(j\Omega)$

2. Figure 4 shows the overall system for filtering a continuous-time signal using a discrete-time filter. The frequency responses of the reconstruction filter $H_r(j\Omega)$ and the discrete-time filter $H(e^{jw})$ are shown in Figure 5.

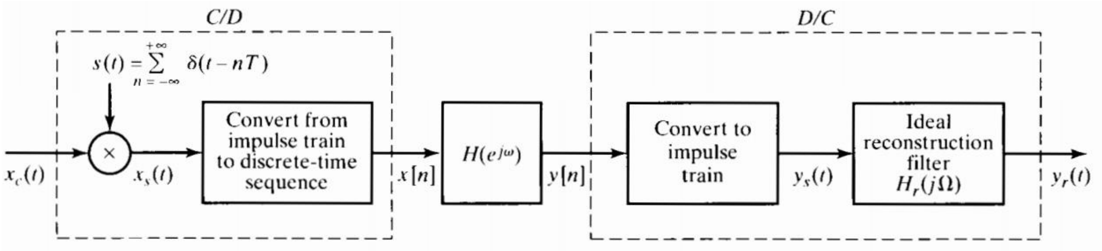

Figure 4: the system

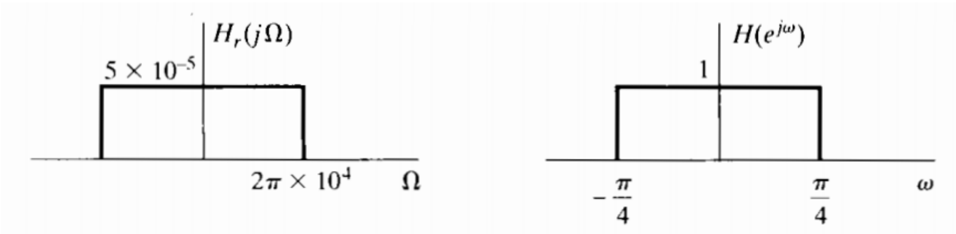

Figure 5: frequency response

(a) For $X_c(j\Omega)$ as shown in Figure 6 and 1/T=20kHz, sketch $X_s(j\Omega)$ and $X(e^{jw})$.

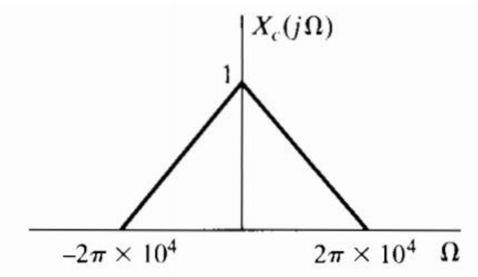

Figure 6: frequency response of input

(b) For a certain range of values of T, the overall system, with input $x_c(t)$ and output $y_c(t)$, is equivalent to a continuous-time lowpass filter with frequency response $H_{\text{eff}}(j\Omega)$ sketched in Figure 7.

Determine the range of values of T for which the information presented in (a) is true when $X_c(j\Omega)$ is bandlimited to $|\Omega| \leq 2\pi \times 10^4$ as shown in Figure 6.

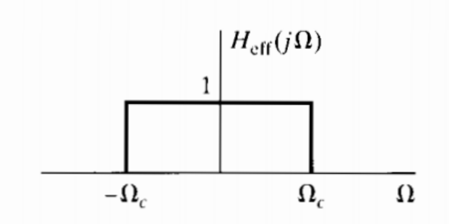

Figure 7: frequency response

3. MATLAB simulation:

(a) Generate a continuous-time signal $x_c(t) = \sin(2\pi t), 0 \leq \mathrm{t} \leq 1$. Plot $x_c(t)$ in figure (1). (Hint: since we can't generate a real continuous time signal with MATLAB, we generate $x_c(t)$ with $\mathrm{t} = 0{:}\,0.01{:}\,1$.)

(b) Generate three discrete-time signals $x[n]$ by sampling $x_c(t)$ with sampling period T=0.02 、0.05 and 0.1 second. Use stem function to plot these three $x[n]$ in subplot(3,2,1) 、subplot(3,2,3) and subplot(3,2,5) in figure (2),

(c) After sampling, use the following formula to reconstruct the continuous-time signal $y_c(t)$.

$$y_c(t) = \sum_{n=-\infty}^{\infty} x[nT] \frac{\sin \pi \left(\frac{t - nT}{T}\right)}{\pi \left(\frac{t - nT}{T}\right)}$$

Then, plot these three $y_c(t)$ in subplot(3,2,2)、subplot(3,2,4) and subplot(3,2,6) in figure (2).

(d) Calculate the mean square error between $x_c(t)$ and $y_c(t)$.

(e) When the sampling period T = 0.02, quantize the discrete-time signal $x[n]$ with 2-bit (4 levels) 、3-bit (8 levels) and 4-bit (16 levels), and round the quantized signal $x_q[n]$ with offset (midrise). Use stem function to plot these three $x_q[n]$ in subplot(3,1,1) 、subplot(3,1,2) and subplot(3,1,3) in figure (3).

(f) Calculate the mean square error between $x[n]$ and three $x_q[n]$, and discuss the advantages and disadvantages for quantizing with different numbers of bits.