

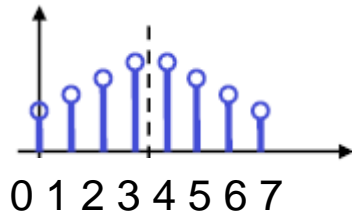
Type 1

$$\begin{aligned}
 H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad + h[4]e^{-j4\omega} + h[5]e^{-j5\omega} + h[6]e^{-j6\omega} \\
 &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad + h[2]e^{-j4\omega} + h[1]e^{-j5\omega} + h[0]e^{-j6\omega} \\
 &= e^{-j3\omega} (h[3] + h[0](e^{j3\omega} + e^{-j3\omega}))
 \end{aligned}$$

$$= 2e^{-j3\omega} (h[3] + h[0] \cos 3\omega + h[1] \cos 2\omega + h[2] \cos \omega)$$

pure-real

$$\theta(\omega) = -3\omega$$



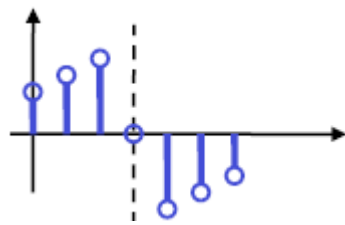
Type 2

$$\begin{aligned}
 H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad + h[4]e^{-j4\omega} + h[5]e^{-j5\omega} + h[6]e^{-j6\omega} + h[7]e^{-j7\omega} \\
 &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad + h[3]e^{-j4\omega} + h[2]e^{-j5\omega} + h[1]e^{-j6\omega} + h[0]e^{-j7\omega} \\
 &= e^{-j7/2\omega} \left( h[0](e^{j7/2\omega} + e^{-j7/2\omega}) + h[1](e^{j5/2\omega} + e^{-j5/2\omega}) \right.
 \end{aligned}$$

$$\left. + h[2](e^{j3/2\omega} + e^{-j3/2\omega}) \right)$$

pure-real

$$\theta(\omega) = -7/2\omega$$



Type 3

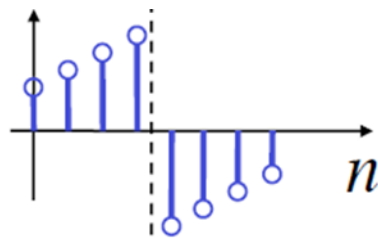
0 1 2 3 4 5 6

$$\begin{aligned}
 H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + \cancel{h[3]e^{-j3\omega}} \\
 &\quad + h[4]e^{-j4\omega} + h[5]e^{-j5\omega} + h[6]e^{-j6\omega} \\
 &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + \cancel{h[3]e^{-j3\omega}} \\
 &\quad - h[2]e^{-j4\omega} - h[1]e^{-j5\omega} - h[0]e^{-j6\omega} \\
 &= e^{-j3\omega} (\cancel{h[3]} + h[0](e^{j3\omega} - e^{-j3\omega}))
 \end{aligned}$$

$$= 2je^{-j3\omega} (\cancel{h[3]} + h[0] \sin 3\omega + h[1] \sin 2\omega + h[2] \sin \omega)$$

pure-real

$$\theta(\omega) = -3\omega + \pi/2$$



Type 4

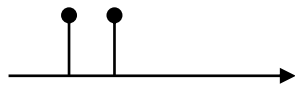
0 1 2 3 4 5 6 7

$$\begin{aligned}
 H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad + h[4]e^{-j4\omega} + h[5]e^{-j5\omega} + h[6]e^{-j6\omega} + h[7]e^{-j7\omega} \\
 &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} \\
 &\quad - h[3]e^{-j4\omega} - h[2]e^{-j5\omega} - h[1]e^{-j6\omega} - h[0]e^{-j7\omega} \\
 &= e^{-j7/2\omega} \left( h[0](e^{j7/2\omega} - e^{-j7/2\omega}) + h[1](e^{j5/2\omega} - e^{-j5/2\omega}) \right. \\
 &\quad \left. + h[2](e^{j3/2\omega} - e^{-j3/2\omega}) \right) \\
 &= 2je^{-j7/2\omega} (h[0] \sin 7/2\omega + h[1] \sin 5/2\omega + h[2] \sin 3/2\omega)
 \end{aligned}$$

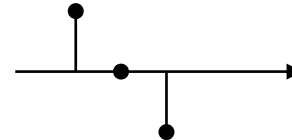
pure-real

$$\theta(\omega) = -7/2\omega + \pi/2$$

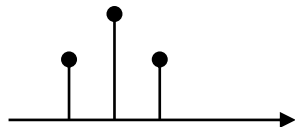
Given the following impulse responses, please 1) determine their phase delay, 2) determine the types of the FIR filters, 3) sketch the zeros and poles of the corresponding systems.



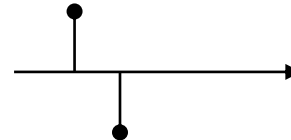
(a)



(c)



(b)



(d)