ICE503 Homework-06

Arnav Mukhopadhyay (D123070002)
EMAIL: gudduarnav@gmail.com

Q. 3
(a)

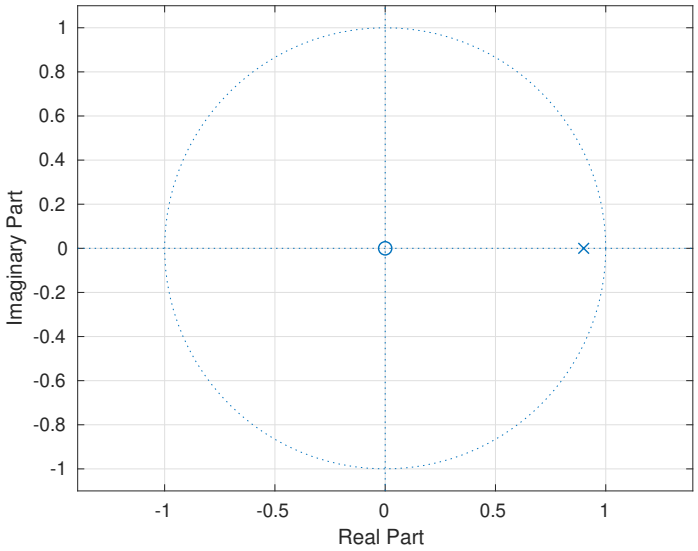

Fig. 1: Plot of poles and zeros of the transfer function $H(z)$.

The given function is

$$y(n) = 0.9y(n-1) + x(n) \implies y(n) - 0.9y(n-1) = x(n)$$

Taking the Z-transform of above, we have

$$(1 - 0.9z^{-1})Y(z) = X(z)$$

Hence, the transform is written as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.9z^{-1}}$$

The zero and poles are located on the z-plane at $(0, 0)$ and $(0.9, 0)$, respectively. It is obtained by equating the numerator and denominator of the transfer function above to zero. These points are shown on the z-plane in figure 1.

(b)

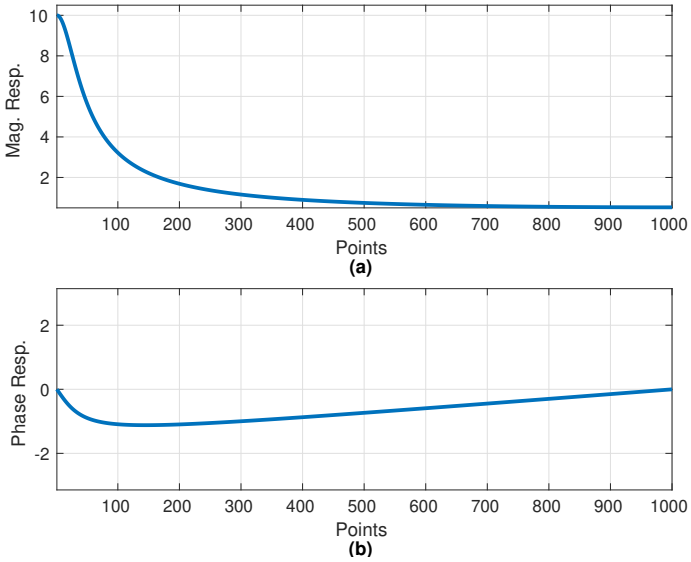

Fig. 2: Plot of magnitude and phase response of $H(z)$ as $H(\omega)$.

In the transfer function, replace z by $e^{j\omega}$. Then the corresponding response function is written as

$$H(\omega) = \frac{1}{(1 + 0.9\cos\omega) - 0.9\sin\omega}$$

Then the magnitude and phase response is written as

$$|H(\omega)| = \frac{1}{\sqrt{1.81 + 1.8\cos\omega}},$$
$$\angle H(\omega) = -\tan^{-1}\left(\frac{0.9\sin\omega}{1 + 0.9\sin\omega}\right)$$

The plot in figure 2 shows the response plot corresponding to response function above.