1. (a)

$$v[n] = x[n] + \frac{2}{3} v[n-1] - \frac{1}{9} v[n-2]$$

$$\Rightarrow V(z) = X(z) + \frac{2}{3}\frac{1}{z} V(z) - \frac{1}{9}\frac{1}{z^2} V(z)$$

$$\Rightarrow X(z) = \left(1 - \frac{2}{3}\frac{1}{z} + \frac{1}{9}\frac{1}{z^2}\right) V(z) \quad \text{—— ①}$$

$$y[n] = v[n] - 6\,v[n-1] + 8\,v[n-2]$$

$$\Rightarrow Y(z) = V(z) - 6\cdot\frac{1}{z} V(z) + 8\cdot\frac{1}{z^2} V(z)$$

$$\Rightarrow Y(z) = \left(1 - 6\cdot\frac{1}{z} + 8\cdot\frac{1}{z^2}\right) V(z) \quad \text{—— ②}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{②}{①} = \frac{1 - \frac{6}{z} + \frac{8}{z^2}}{1 - \frac{2}{3z} + \frac{1}{9z^2}} \quad \text{—— ③}$$

$$\Rightarrow Y(z)\cdot\left(1 - \frac{2}{3z} + \frac{1}{9z^2}\right) = X(z)\left(1 - \frac{6}{z} + \frac{8}{z^2}\right)$$

$$\Rightarrow y[n] - \frac{2}{3} y[n-1] + \frac{1}{9} y[n-2] = x[n] - 6x[n-1] + 8x[n-2]$$ ※

(b) By ③,

$$H(z) = \frac{z^2 - 6z + 8}{z^2 - \frac{2}{3}z + \frac{1}{9}} = \frac{(z-4)(z-2)}{\left(z-\frac{1}{3}\right)^2} \Rightarrow \begin{cases} \text{zeros}: z = 2, 4 \\ \text{poles}: z = \frac{1}{3}, \frac{1}{3} \end{cases}$$ ※

(c)

ROC: $|z| > \frac{1}{3} \rightarrow$ stable. ※

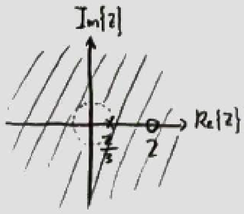
2.

(a)

$$x[n] = \left(\tfrac{1}{3}\right)^n u[n] + (2)^n u[-n-1]$$

$$\because \text{ if } x[n] = \lambda^n u[n], \text{ then } X(z) = \sum_{n=0}^{\infty} \lambda^n z^{-n} = \frac{1}{1-\lambda z^{-1}}, \text{ ROC}: |z| > |\lambda|$$

$$\therefore X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}}, \text{ ROC}: \tfrac{1}{3} < |z| < 2$$



$$y[n] = 5\left(\tfrac{1}{3}\right)^n u[n] - 5\left(\tfrac{2}{3}\right)^n u[n]$$

$$\Rightarrow Y(z) = \frac{5}{1-\frac{1}{3}z^{-1}} - \frac{5}{1-\frac{2}{3}z^{-1}}, \text{ ROC}: |z| > \tfrac{2}{3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{5\times\frac{(1-\frac{2}{3}z^{-1})-(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}}{\frac{(1-2z^{-1})-(1-\frac{1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}} = \frac{5\times\frac{(\frac{-1}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}}{\frac{(\frac{-5}{3}z^{-1})}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \Rightarrow \begin{cases} \text{ROC}: |z| > \frac{2}{3} \quad (\because \text{causal}) \\ \text{pole}: z = \frac{2}{3} \\ \text{zero}: z = 2 \end{cases}$$

※

(b) $H(z) = \dfrac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} = 3 - \dfrac{2}{1-\frac{2}{3}z^{-1}}$. ROC: $|z| > \frac{2}{3}$

$$\rightarrow h[n] = 3\delta[n] - 2\left(\tfrac{2}{3}\right)^n u[n]$$ ※

(c)

$$H(z) = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} = \frac{Y(z)}{X(z)} \Rightarrow Y(z) - \tfrac{2}{3}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

$$\Rightarrow y[n] - \tfrac{2}{3}y[n-1] = x[n] - 2x[n-1]$$ ※

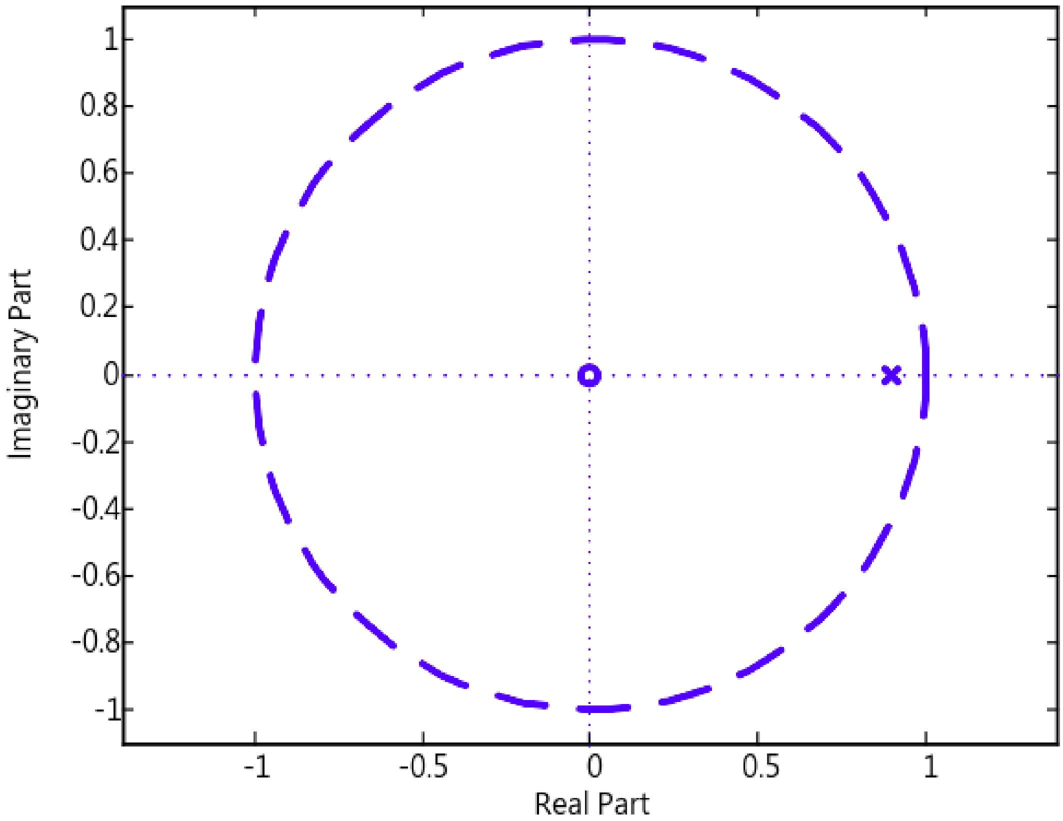
(d)

$\because$ ROC contain unit circle , $\therefore$ the system is stable.

```
clear
clc

% y(n) = 0.9 * y(n-1) + x(n)
% y(n) - 0.9 * y(n-1) = x(n)
% Y(z) = H(z)X(z)
% H(z) = Y(z)/X(z)
% H(z) = 1/(1-0.9z^-1)    |z|>0.9
a=[1,0];                                    %H numerator part
b=[1,-0.9];                                 %H denominator part
zplane(a,b);                                %figure zero and pole
%returns the 100-point frequency response vector, h,
%and the corresponding angular frequency vector, w,
[H,w] = freqz(a,b,100);
%mgnitude of H and phase of H
mgnitude_H = abs(H); phase_H = angle(H);
figure(2)
subplot(2,1,1);plot(w/pi,mgnitude_H);grid on%plot Magnitude
xlabel('frequency'); ylabel('Magnitude');          %x and y label
title('Magnitude Response')                          %title
subplot(2,1,2);plot(w/pi,phase_H/pi) ;grid on     %plot phase
xlabel('frequency'); ylabel('Phase');              %x and y label
-
title('Phase Response')                              %title
-
```

(a)



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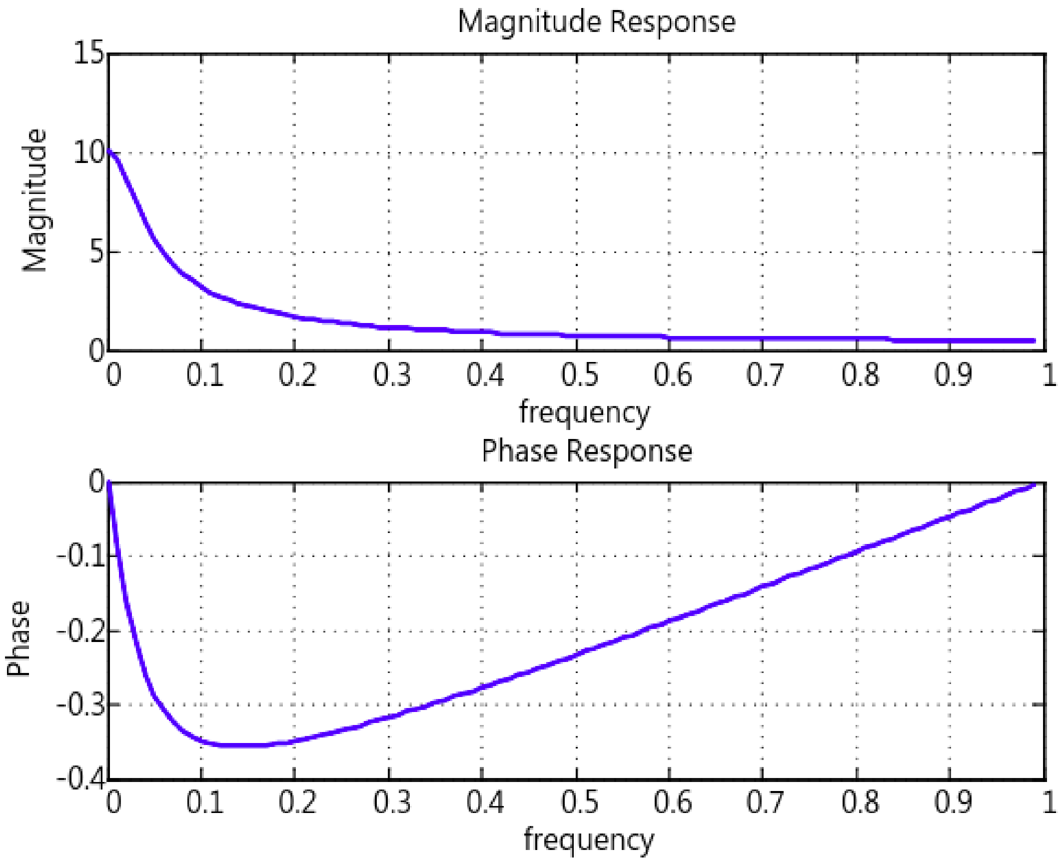
(b)


Magnitude Response
Magnitude
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
frequency
Phase Response
Phase
0
-0.1
-0.2
-0.3
-0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
frequency

-