ICE503 DSP-Homework#2

1. For each of the following systems, determine whether the system is (1) linear, (2) time invariant, and (3) causal.

   (a) $y[n] = ax[n] + b$, a and b are non-zero constant

   (b) $y[n] = x[an + b]$, a and b are non-zero positive constant

   (c) $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

   (d) $y[n] = \log_{10}(|x[n]|)$

2. The system T in Figure 1 is known to be time-invariant. When the inputs to the system are $x_1[n]$, $x_2[n]$, and $x_3[n]$, the responses of the system are $y_1[n]$, $y_2[n]$, and $y_3[n]$ as shown. Determine whether the system T is linear or nonlinear.

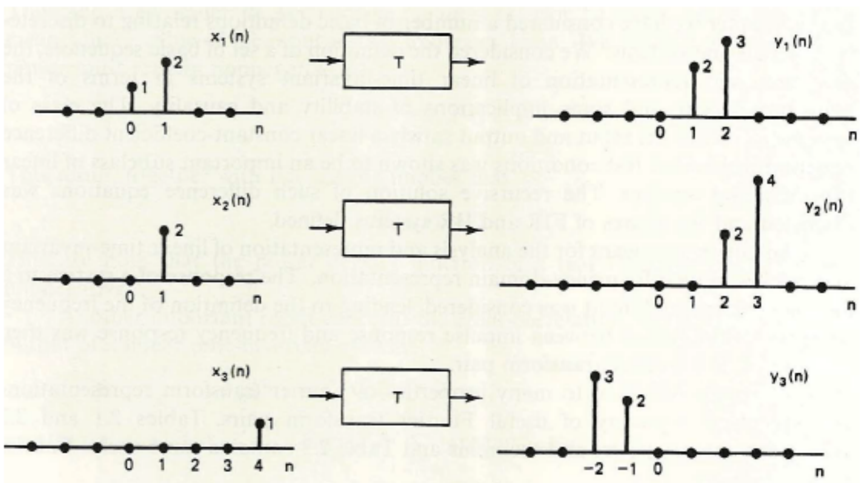

Figure 1: The time-invariant system T

3. In order to determine the impulse response of an unknown causal, linear time-invariant (LTI) system, Kai feeds the following input x[n] to the system:

$$x[n] = 0, \text{ if } n < 0;\ x[n] = 1, \text{ if } n \geq 0.$$

The corresponding output y[n] is given by the following: y[n] = 0, if n < 0; y[n] = 8, 12, 14, 15, 15.5, for n = 0, 1, 2, 3, 4, respectively; y[n] = 15.75, if n ≥ 5.

(a) Find the impulse response of this system.

(b) Let $y = [y[0],\ldots,y[5]]^T$ and $x=[x[0],\ldots,x[5]]^T$. The input-output relationship of this system can be written as $y = \mathbf{H}x$, Determine the matrix **H**.

4. MATLAB simulation:

The input signal is

$$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] + 6\delta[n-3] + 7\delta[n-4] + 5\delta[n-5] + 4\delta[n-6]$$

and the output signal of a 3-point moving average is

$$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$$

(a) Use stem function to plot $x[n]$.

(b) Use for loop to calculate $y[n]$.

(c) Use convolution function to calculate $y[n]$.

(The result of $y[n]$ in (b) and (c) should be the same.)

(d) Use stem function to plot $y[n]$.

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(b) $y[n] = x[an + b]$, a and b are non-zero positive constant

(c) $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

(d) $y[n] = \log_{10}(|x[n]|)$

(a) $y[n] = ax[n]+b$, $a \,\&\, b \in \mathbb{Z} \cap a \,\&\, b \neq 0$

(1) Linear

$x_1[n] \longrightarrow y_1[n] = a x_1[n] + b$

$x_2[n] \longrightarrow y_2[n] = a x_2[n] + b$

若 $x[n] = \alpha x_1[n] + \beta x_2[n]$

$\overset{?}{\longrightarrow} y[n] = \alpha y_1[n] + \beta y_2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n]$

$\longrightarrow y[n] = a(\alpha x_1[n] + \beta x_2[n]) + b$

$= a\alpha x_1[n] + a\beta x_2[n] + b$

$\neq a\alpha x_1[n] + a\beta x_2[n] + (\alpha+\beta)b$

$= \alpha y_1[n] + \beta y_2[n]$ ∴ Nonlinear

(2) T.I.

① $x[n] \longrightarrow y[n] = ax[n]+b$

if $x[n-a] \rightarrow o/p = ax[n-a]+b$

$\|$ ∴ T.I.

$y[n-a] = ax[n-a]+b$

(3) Causal

$y[0] = ax[0]+b$

$y[1] = ax[1]+b$

$y[-1] = ax[-1]+b$

∴ Causal

(b) $y[n] = x[an+b]$, $a \,\&\, b \in \mathbb{N} \cap a \,\&\, b \neq 0$

(1) Linear

$x_1[n] \longrightarrow y_1[n] = x_1[an+b]$

$x_2[n] \longrightarrow y_2[n] = x_2[an+b]$

若 $x[n] = \alpha x_1[n] + \beta x_2[n]$

$\overset{?}{\longrightarrow} y[n] = \alpha y_1[n] + \beta y_2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n]$

$\longrightarrow y[n] = \alpha x_1[an+b] + \beta x_2[an+b]$

$= \alpha y_1[n] + \beta y_2[n]$ ∴ Linear

(2) T.I.

① $x[n] \longrightarrow y[n] = x[an+b]$

$x[n-a] \rightarrow o/p = x[an-a+b]$

$\nparallel$ ∴ Not T.I.

$y[n-a] = x[a(n-a)+b]$

(3) Causal

$y[0] = x[b]$, $b \in \mathbb{N}$, $b \geq 0$

∴ Non-causal

(c) $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

(1) Linear

$x_1[n] \longrightarrow y_1[n] = \frac{1}{M}\sum_{k=0}^{M-1} x_1[n-k]$

$x_2[n] \longrightarrow y_2[n] = \frac{1}{M}\sum_{k=0}^{M-1} x_2[n-k]$

若 $x[n] = \alpha x_1[n] + \beta x_2[n]$

$\overset{?}{\longrightarrow} y[n] = \alpha y_1[n] + \beta y_2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n]$

$\longrightarrow y[n] = \frac{\alpha}{M}\sum_{k=0}^{M-1} x_1[n-k]$

$+ \frac{\beta}{M}\sum_{k=0}^{M-1} x_2[n-k]$

$= \alpha y_1[n] + \beta y_2[n]$ ∴ Linear

(2) T.I.

① $x[n] \longrightarrow y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

$x[n-a] \longrightarrow o/p = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k-a]$

$\|$ ∴ T.I.

$y[n-a] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-a-k]$

(3) Causal

$y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$, $k \in \mathbb{N}$

$y[0] = \frac{1}{M}\sum_{k=0}^{M-1} x[-k]$

$y[1] = \frac{1}{M}\sum_{k=0}^{M-1} x[1-k]$

$y[-1] = \frac{1}{M}\sum_{k=0}^{M-1} x[-1-k]$

∴ Causal

(d) $y[n] = \log_{10}(|x[n]|)$

(1) Linear

$x_1[n] \longrightarrow y_1[n] = \log_{10}(|x_1[n]|)$

$x_2[n] \longrightarrow y_2[n] = \log_{10}(|x_2[n]|)$

若 $x[n] = \alpha x_1[n] + \beta x_2[n]$

$\not\rightarrow y[n] = \alpha y_1[n] + \beta y_2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n]$

$\longrightarrow y[n] = \log_{10}(|\alpha x_1[n] + \beta x_2[n]|)$

$\neq \alpha \log_{10}(|x_1[n]|) + \beta \log_{10}(|x_2[n]|) = \alpha y_1[n] + \beta y_2[n]$: Nonlinear

(2) T.I.

① $x[n] \longrightarrow y[n] = \log_{10}(|x[n]|)$

$x[n-a] \longrightarrow o/p = \log_{10}(|x[n-a]|)$

$\parallel$: T.I.

$y[n-a] = \log_{10}(|x[n-a]|)$

(3) Causal

$y[n] = \log_{10}(|x[n]|)$

$y[0] = \log_{10}(|x[0]|)$

$y[1] = \log_{10}(|x[1]|)$

$y[-1] = \log_{10}(|x[-1]|)$

: Causal

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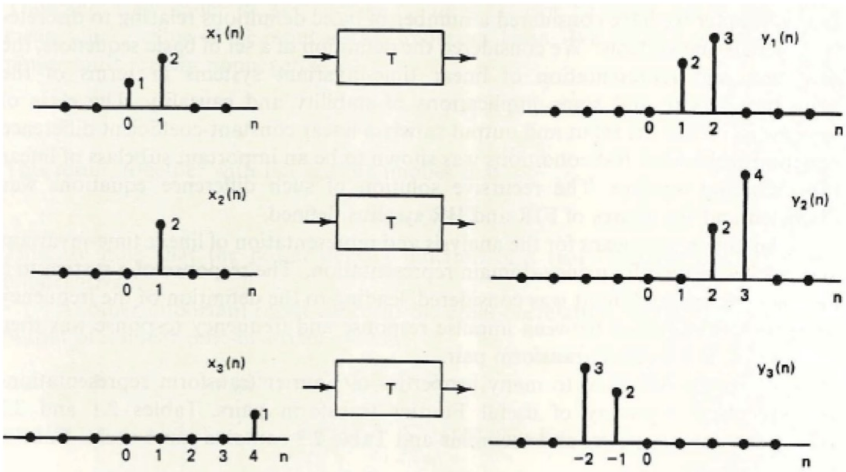

Figure 1: The time-invariant system T

2. $x_1[n] = \delta[n] + 2\delta[n-1] \xrightarrow{T} y_1[n] = 2\delta[n-1] + 3\delta[n-2]$

$x_2[n] = 2\delta[n-1] \xrightarrow{T} y_2[n] = 2\delta[n-2] + 4\delta[n-3]$

$x_3[n] = \delta[n-4] \xrightarrow{T} y_3[n] = 2\delta[n+1] + 3\delta[n+2]$

$x_1[n] - x_2[n] = \delta[n]$

$x_3[n] = \delta[n-4] = x_1[n-4] - x_2[n-4]$

If system is linear, $y_3[n]$ should be $y_1[n-4] - y_2[n-4]$

$y_3[n] = 2\delta[n+1] + 3\delta[n+2]$

However $y_1[n-4] - y_2[n-4] = 2\delta[n-5] + \delta[n-6] - 4\delta[n-7]$

$\neq y_3[n]$

→ The system is Nonlinear

3. In order to determine the impulse response of an unknown causal, linear time-invariant (LTI) system, Kai feeds the following input x[n] to the system:

$$x[n] = 0, \text{ if } n < 0; \; x[n] = 1, \text{ if } n \geq 0.$$

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(a) Find the impulse response of this system.

(b) Let y = $[y[0],\ldots,y[5]]^T$ and x=$[x[0],\ldots,x[5]]^T$. The input-output relationship of this system can be written as y = **H**x, Determine the matrix **H**.

(a) $h[n] = ?$

Assume $h[n] = a_0\delta[n] + a_1\delta[n-1] + a_2\delta[n-2] \cdots = \sum_{k=0}^{n} a_k \delta[n-k]$

$\because y[n] = x[n] * h[n] = \sum_{k=0}^{n} x[k]h[n-k] = x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + \cdots + x[n]h[0]$

$\downarrow$ $x[n]=1;$ if $n \geq 0$

$= h[n] + h[n-1] + h[n-2] + \cdots + h[0]$

$\downarrow$

$$\begin{array}{llllllll}
n= 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\
 & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\
 & & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \;\cdot\cdot \\
 & & & a_0 & a_1 & a_2 & a_3 & a_4 \\
 & & & & a_0 & a_1 & a_2 & a_3 \\
 & & & & & a_0 & a_1 & a_2 \\
+) & & & & & & \vdots & \\
\hline
y[n]= 8 & 12 & 14 & 15 & 15.5 & 15.75 & 15.75 & 15.75 \cdots
\end{array}$$

$a_0=8, a_1=4, a_2=2, a_3=1, a_4=0.5, a_5=0.25, a_6=a_7=\cdots=0$

$\therefore h[n] = 8\delta[n] + 4\delta[n-1] + 2\delta[n-2] + \delta[n-3] + 0.5\delta[n-4] + 0.25\delta[n-5]$

(b) $H=?$

$\because y[n] = x[n] * h[n] = \sum_{k=0}^{n} x[k]h[n-k]$

$$\Rightarrow \begin{cases}
y[0] = x[0]h[0] = 8x[0] \\
y[1] = x[0]h[1] + x[1]h[0] = 4x[0] + 8x[1] \\
y[2] = 2x[0] + 4x[1] + 8x[2] \\
y[3] = 1x[0] + 2x[1] + 4x[2] + 8x[3] \\
y[4] = 0.5x[0] + 1x[1] + 2x[2] + 4x[3] + 8x[4] \\
y[5] = 0.25x[0] + 0.5x[1] + 1x[2] + 2x[3] + 4x[4] + 8x[5]
\end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \end{bmatrix}}_{\vec{y}} = \underbrace{\begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 \\ 4 & 8 & 0 & 0 & 0 & 0 \\ 2 & 4 & 8 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 0 & 0 \\ 0.5 & 1 & 2 & 4 & 8 & 0 \\ 0.25 & 0.5 & 1 & 2 & 4 & 8 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \end{bmatrix}}_{\vec{x}}$$

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