ICE503 DSP-Homework#2

1. For each of the following systems, determine whether the system is (1) linear, (2) time invariant, and (3) causal.

   (a) $y[n] = ax[n] + b$, a and b are non-zero constant

   (b) $y[n] = x[an + b]$, a and b are non-zero positive constant

   (c) $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

   (d) $y[n] = \log_{10}(|x[n]|)$

2. The system T in Figure 1 is known to be time-invariant. When the inputs to the system are $x_1[n]$, $x_2[n]$, and $x_3[n]$, the responses of the system are $y_1[n]$, $y_2[n]$, and $y_3[n]$ as shown. Determine whether the system T is linear or nonlinear.

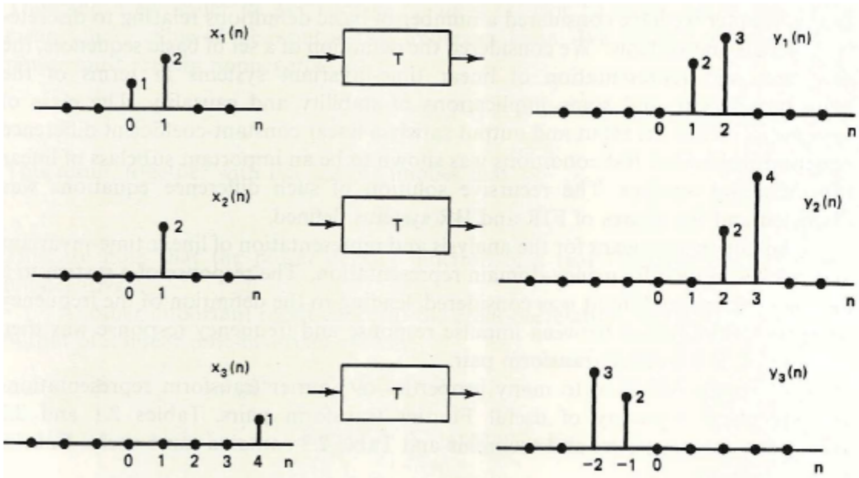


Figure 1: The time-invariant system T

3. In order to determine the impulse response of an unknown causal, linear time-invariant (LTI) system, Kai feeds the following input x[n] to the system:

$$x[n] = 0, \text{ if } n < 0; \; x[n] = 1, \text{ if } n \geq 0.$$

The corresponding output y[n] is given by the following: y[n] = 0, if n < 0; y[n] = 8, 12, 14, 15, 15.5, for n = 0, 1, 2, 3, 4, respectively; y[n] = 15.75, if n ≥ 5.

(a) Find the impulse response of this system.

(b) Let y = $[y[0],\ldots,y[5]]^T$ and x=$[x[0],\ldots,x[5]]^T$. The input-output relationship of this system can be written as y = **H**x, Determine the matrix **H**.

4. MATLAB simulation:

The input signal is

$$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] + 6\delta[n-3] + 7\delta[n-4] + 5\delta[n-5] + 4\delta[n-6]$$

and the output signal of a 3-point moving average is

$$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$$

(a) Use stem function to plot $x[n]$.

(b) Use for loop to calculate $y[n]$.

(c) Use convolution function to calculate $y[n]$.

(The result of $y[n]$ in (b) and (c) should be the same.)

(d) Use stem function to plot $y[n]$.

① (a) $y = a x(n) + b$

(1) Linear.

Let $x_1(n) \rightarrow y_1(n) = a x_1(n) + b$
$x_2(n) \rightarrow y_2(n) = a x_2(n) + b$

$$\alpha y_1(n) + \beta y_2(n) = \alpha\{a x_1(n) + b\} + \beta\{a x_2(n) + b\}$$
$$= \alpha a x_1(n) + \beta a x_2(n) + \alpha b + \beta b$$

$$\alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = a\{\alpha x_1(n) + \beta x_2(n)\} + b$$
$$= \alpha a x_1(n) + \beta a x_2(n) + b$$
$$\neq \alpha y_1(n) + \beta y_2(n)$$

$y(n)$ is NOT LINEAR.

(2) Time Invariance;
$$x(n-\tau) \longrightarrow y'(n) = a x(n-\tau) + b$$

$$y''(n-\tau) = a x(n-\tau) + b = y'(n)$$

$\therefore$ $y(n)$ is TIME INVARIANT

(3) Causal.

$y(n)$ depends on present value of $x(n)$
$\Rightarrow$ Causal system.

(b) $y(n) = x(an+b)$ where a, b are +ve.

(1) Linearity.

Let $x_1(n) \longrightarrow y_1(n) = x_1(an+b)$
$x_2(n) \longrightarrow y_2(n) = x_2(an+b)$

$\therefore \alpha y_1(n) + \beta y_2(n) = \alpha x_1(an+b) + \beta x_2(an+b)$

$\therefore \alpha x_1(n) + \beta x_2(n) \longrightarrow y(n) = \alpha x_1(an+b) + \beta x_2(an+b)$
$= \alpha y_1(n) + \beta y_2(n)$

Hence, $y(n)$ is LINEAR SYSTEM.

(2) Time Invariance.

$x(a(n-\tau)+b) \longrightarrow y_1(n) = x(a(n-\tau)+b)$
$= x(an+b-a\tau)$

$y_2(n-\tau) = x(an+b-\tau) \neq y_1(n)$

Hence, $y(n)$ is NOT Time Invariant

(3) Causal

$y(0) = x(a \times 0 + b) = x(b)$

Since b is +ve $\Rightarrow y(0)$ depends on future value of $x(n)$ at $n=b$.

Hence, $y(n)$ is NOT Causal.

ⓒ $y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k)$

(1) Linear.

Let $x_1(n) \longrightarrow y_1(n) = \frac{1}{M}\sum_{k=0}^{M-1} x_1(n-k)$

$x_2(n) \longrightarrow y_2(n) = \frac{1}{M}\sum_{k=0}^{M-1} x_2(n-k)$

$\therefore \alpha y_1(n) + \beta y_2(n) = \frac{1}{M}\sum_{k=0}^{M-1}\{\alpha x_1(n-k) + \beta x_2(n-k)\}$

$\therefore \alpha x_1(n) + \beta x_2(n) \longrightarrow y(n) = \frac{1}{M}\sum_{k=0}^{M-1}\{\alpha x_1(n-k) + \beta x_2(n-k)\}$

$= \alpha y_1(n) + \beta y_2(n)$

Hence, $y(n)$ is LINEAR system.

(2) Time Invariance.

$x(n-\tau) \longrightarrow y_1(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-\tau-k)$

$= \frac{1}{M}\sum_{k=0}^{M-1} x(n-k-\tau)$

$y_2(n-\tau) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k-\tau) = y_1(n)$

Hence, $y(n)$ is Time Invariant.

(3) Causal.

$y(0) = \frac{1}{M}\sum_{k=0}^{M-1} x(0-k) = \frac{1}{M}\sum_{k=0}^{M-1} x(-k)$

$= \frac{x(0) + x(-1) + \dots + x(-(M-1))}{M}$

Thus, $y(n)$ depend on present and past values of $x(n)$.
Hence, system is causal.

(d) $y(n) = \log_{10}(|x(n)|)$

(1) Linearity.

$\Rightarrow$ Presence of log function makes it nonlinear.
Hence, $y(n)$ is NOT LINEAR.

Alt $x_1(n) \longrightarrow y_1(n) = \log_{10}(|x_1(n)|)$

$x_2(n) \longrightarrow y_2(n) = \log_{10}(|x_2(n)|)$

$$\alpha y_1(n) + \beta y_2(n) = \alpha \log_{10}(|x_1(n)|) + \beta \log_{10}(|x_2(n)|)$$
$$= \log_{10}\left(|x_1(n)|^{\alpha} |x_2(n)|^{\beta}\right)$$

$$\alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = \log_{10}\left(\alpha |x_1(n)| + \beta |x_2(n)|\right)$$
$$\neq \alpha y_1(n) + \beta y_2(n)$$

$\therefore y(n)$ is NOT LINEAR system.

(2) Time Invariance.

$x(n-\tau) \rightarrow y_1(n) = \log_{10}(|x(n-\tau)|)$

$y_2(n-\tau) = \log_{10}(|x(n-\tau)|) = y_1(n)$

Hence, $y(n)$ is TIME INVARIANT.

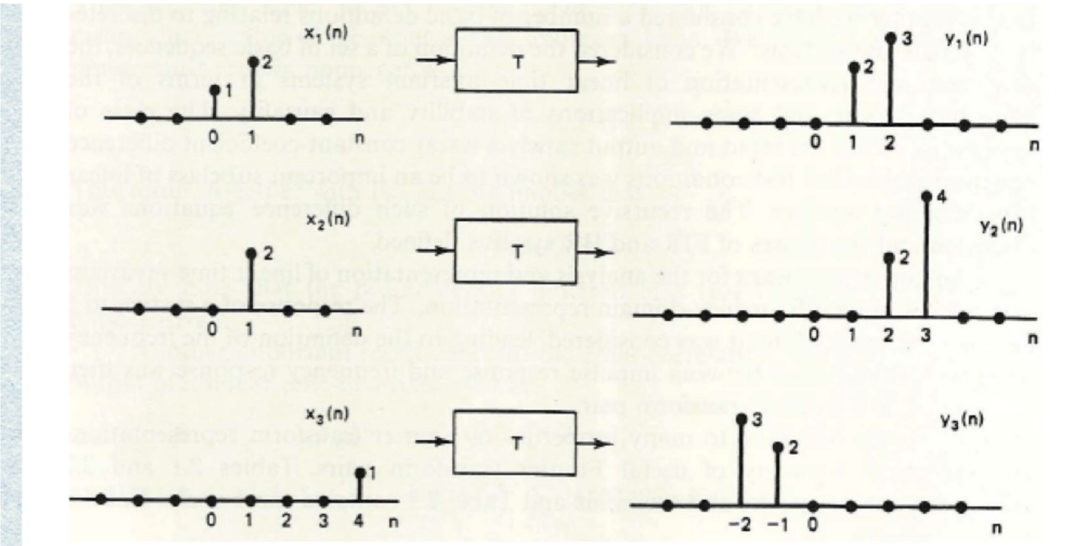
(3) Causality

$y(0) = \log(|x(0)|)$

Present output depends on present input.

Hence, $y(n)$ is CAUSAL system.

②



From LHS side of figure,

$x_1(0) = 1 = x_3(4)$

$x_1(1) = 2 = x_2(1)$

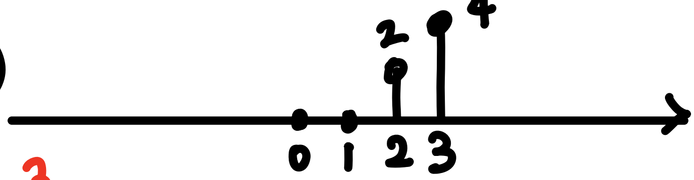
$\therefore\ x_1(n) = x_2(n) + x_3(n+4)$

- $x(n+\tau)$ means shift $x(n)$ LEFT by τ times.
- $x(n-\tau)$ means shift $x(n)$ RIGHT by τ times.

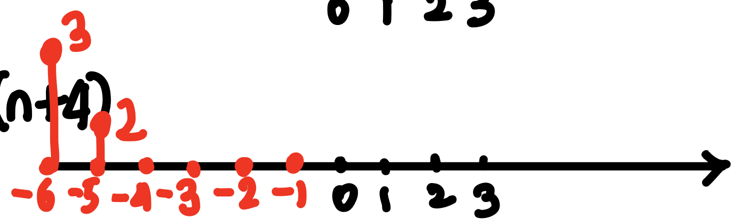
If the system is linear, then we must have,

$$y_1(n) = y_2(n) + y_3(n+4)$$

$y_2(n)$

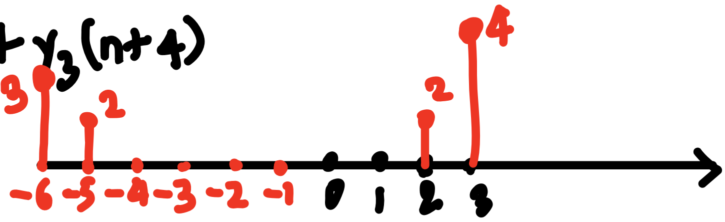


$y_3(n+4)$

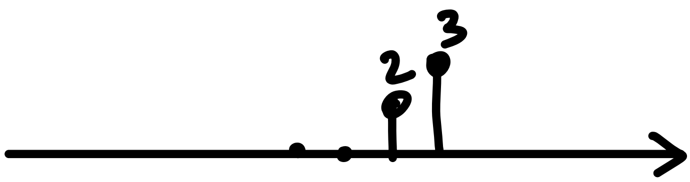


move $y_3(n)$ by 4 times towards left side.

$y_2(n) + y_3(n+4)$



$y_1(n)$



But from the figure, $y_1(n) \neq y_2(n) + y_3(n+4)$

Hence the system is NOT LINEAR.

③
(a) $x(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} = u(n)$ is unit step function

$$y(n) = \begin{cases} 8 & n=0 \\ 12 & n=1 \\ 14 & n=2 \\ 15 & n=3 \\ 15.5 & n=4 \\ 15.75 & n \geq 5 \end{cases}$$

Since the system is causal LTI system, we can assume that

$$\frac{d}{dt}x(n) \longrightarrow \frac{d}{dt}y(n)$$

$$\Rightarrow \frac{d}{dt}u(n) \xrightarrow{\frac{d}{dt}u(n) = \nabla u(n)} \frac{d}{dt}y(n)$$

Because $\delta(n) = \frac{d}{dt}u(n)$

$$\Rightarrow \delta(n) \equiv \frac{d}{dt}u(n) \longrightarrow h(n) \triangleq \frac{d}{dt}y(n)$$

But $\frac{d}{dt}y(n) \triangleq y(n) - y(n-1) = \nabla y(n)$

Hence, $h(n) = y(n) - y(n-1) = \nabla y(n)$

$h(0) = y(0) - y(-1) = 8 - 0 = 8$
$h(1) = y(1) - y(0) = 12 - 8 = 4$
$h(2) = y(2) - y(1) = 14 - 12 = 2$
$h(3) = y(3) - y(2) = 15 - 14 = 1$
$h(4) = y(4) - y(3) = 15.5 - 15 = 0.5$
$h(5) = y(5) - y(4) = 15.75 - 15.5 = 0.25$
$h(6) = y(6) - y(5) = 15.75 - 15.75 = 0$

$$h(n) = \{\underset{\uparrow}{8}, 4, 2, 1, 0.5, 0.25\}$$
$$= 8\delta(n) + 4\delta(n-1) + 2\delta(n-2) + \delta(n-3) + 0.5\delta(n-4) + 0.25\delta(n-5)$$

is the Impulse Response of the system.

(b) $Hx = y$

$$\begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ a & b & 0 & 0 & 0 & 0 \\ a & b & c & 0 & 0 & 0 \\ a & b & c & d & 0 & 0 \\ a & b & c & d & e & 0 \\ a & b & c & d & e & f \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 14 \\ 15 \\ 15.5 \\ 15.75 \end{bmatrix}$$

The above matrix is assumed in Echelon form which can be used to directly obtain the solution for coefficient matrix. Furthermore, any linear combination of Echelon matrix is also a valid solution.

$$\therefore a = 8$$
$$b = 12 - a = 4$$
$$c = 14 - a - b = 14 - 8 - 4 = 14 - 12 = 2$$
$$d = 15 - a - b - c = 15 - 8 - 4 - 2 = 1$$
$$e = 15.5 - a - b - c - d = 15.5 - 8 - 4 - 2 - 1 = 0.5$$
$$f = 15.75 - a - b - c - d - e = 15.75 - 8 - 4 - 2 - 1 - 0.5 = 0.25$$

$$\therefore H = \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 \\ 8 & 4 & 0 & 0 & 0 & 0 \\ 8 & 4 & 2 & 0 & 0 & 0 \\ 8 & 4 & 2 & 1 & 0 & 0 \\ 8 & 4 & 2 & 1 & 0.5 & 0 \\ 8 & 4 & 2 & 1 & 0.5 & 0.25 \end{bmatrix}$$

4

ICE503 Homework-02

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Q. 4 (a)

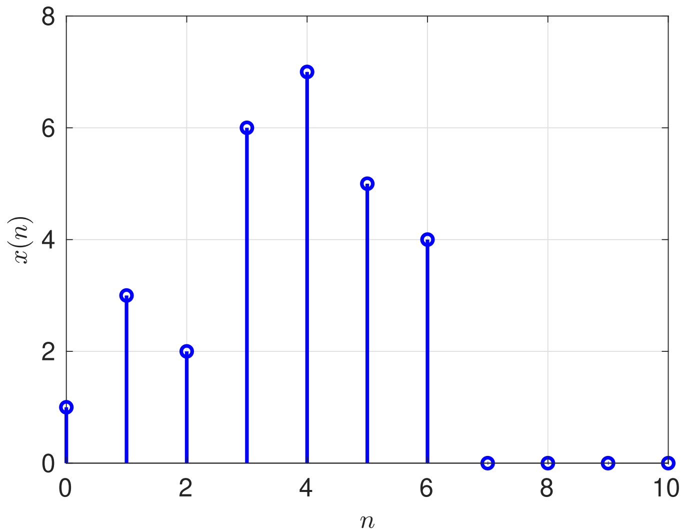

Fig. 1: 4(a) Plot of $x(n)$.

(b)
3-point moving average (MA) is defined as follows,

$$y(n) = \frac{1}{M}\sum_{k=0}^{2} x(n-k)$$

Since the data for $x(n)$ is considered from $n = 0$ to 10, while other values are 0. Therefore, the MA for the *for-loop* follows the formula,

$$\begin{aligned}
y(n<0) &= 0\\
y(0) &= \frac{1}{3}x(0) = \frac{1}{3}\times 1\\
y(1) &= \frac{1}{3}(x(1)+x(0)) = \frac{1}{3}\times 4\\
y(2) &= \frac{1}{3}(x(2)+x(1)+x(0)) = \frac{1}{3}\times 6\\
y(3) &= \frac{1}{3}(x(3)+x(2)+x(1)) = \frac{1}{3}\times 11\\
y(4) &= \frac{1}{3}(x(4)+x(3)+x(2)) = \frac{1}{3}\times 15\\
y(5) &= \frac{1}{3}(x(5)+x(4)+x(3)) = \frac{1}{3}\times 18\\
y(6) &= \frac{1}{3}(x(6)+x(5)+x(4)) = \frac{1}{3}\times 16\\
y(7) &= \frac{1}{3}(x(6)+x(5)) = \frac{1}{3}\times 9\\
y(8) &= \frac{1}{3}x(6) = \frac{1}{3}\times 4\\
y(n\geq 9) &= 0
\end{aligned}$$

(c)
The 3-point MA formula is written as,

$$\begin{aligned} y(n) &= \frac{1}{3}\sum_{k=0} 2x(n-k) \\ &= \frac{1}{3}x(n) + \frac{1}{3}x(n-1) + \frac{1}{3}x(n-2) \end{aligned}$$

The impulse response of the 3-point MA is found when $x(n) = \delta(n)$, where $\delta(n)$ is the Dirac Delta function. Then, the corresponding output from this system is written as,

$$\begin{aligned} h(n) &= \frac{1}{3}\delta(n) + \frac{1}{3}\delta(n-1) + \frac{1}{3}\delta(n-2) \\ &= \frac{1}{3}\sum_{k=0}^{2}\delta(n) \end{aligned}$$

Hence, the output of the system $y(n)$ with input $x(n)$ can be equivalently written with *conv*olution operator ($\circledast$) as,

$$y(n) = h(n) \circledast x(n)$$

The resultant output is same as that of the 3-point MA definition $y(n) = \frac{1}{3}\sum_{k=0} 2x(n)$.
(d)

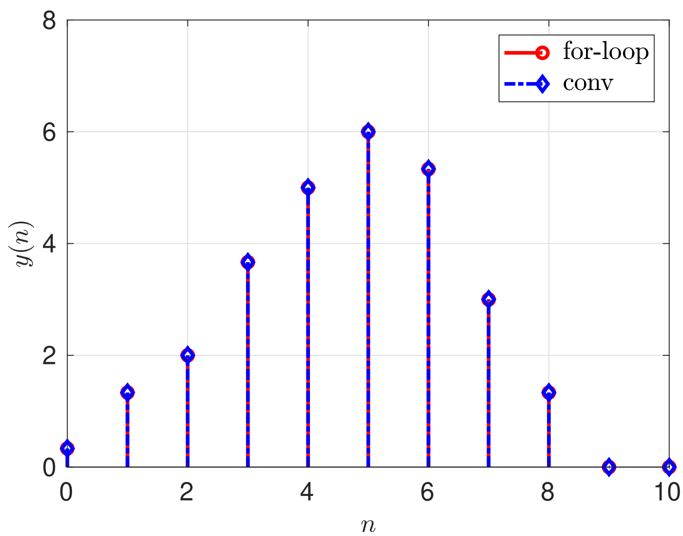

Fig. 2: 4(d) Plot of $y(n)$ by *for-loop* and *conv* method.

```
% Homework 2

% --- clear ---
close all;
clear all;
clc;

% --- function ---
dd = @(n) ((n==0)*1);
fx = @(n) dd(n) + 3*dd(n-1) + 2*dd(n-2) + 6*dd(n-3) + 7*dd(n-4) +
 5*dd(n-5) + 4*dd(n-6);

% --- series ---
n = 0: 10;
x = fx(n);

% --- (a) ---
f1 = figure(1);
stem(n, x, '-b', 'linewidth', 2)
xlabel('$n$')
ylabel('$x(n)$')
xlim([0, 10])
ylim([0, 8])
grid on
set(findall(f1,'-property','FontSize'),'FontSize',14);
set(findall(f1,'-property','Interpreter'), 'Interpreter', 'latex');
saveas(f1, 'hw02a.eps', 'epsc');

% --- (b) ---
y = [];
for k=1: length(x)
    k0 = max([1, k-2]);
    yb(k) = (1/3) * sum(x(k0:k));
end

% --- (c) ---
fma = @(n) (1/3) * (dd(n) + dd(n-1) + dd(n-2));

ma = fma(n)
yc = conv(x, ma,'full');
yc = yc(1: length(n));

% --- (d) ----
f2 = figure(2);
stem(n, yb, '-r', 'linewidth', 2)
hold on
stem(n, yc, '-.db', 'linewidth', 2)
hold off
grid on
xlabel('$n$')
ylabel('$y(n)$')
xlim([0, 10])
```

```
ylim([0, 8])
grid on
legend('for-loop', 'conv')
set(findall(f2,'-property','FontSize'),'FontSize',14);
set(findall(f2,'-property','Interpreter'), 'Interpreter', 'latex');
saveas(f2, 'hw02d.eps', 'epsc');
```

```
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters
Couldn't create JOGL canvas--using painters

ma =

  Columns 1 through 7

    0.3333    0.3333    0.3333         0         0         0         0

  Columns 8 through 11

         0         0         0         0

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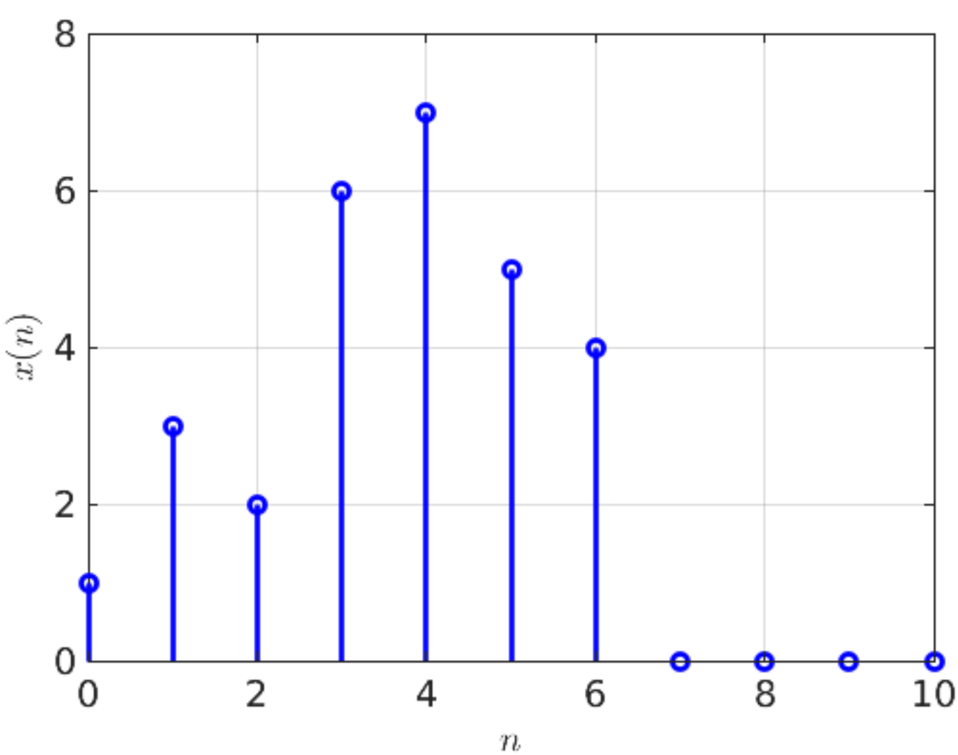

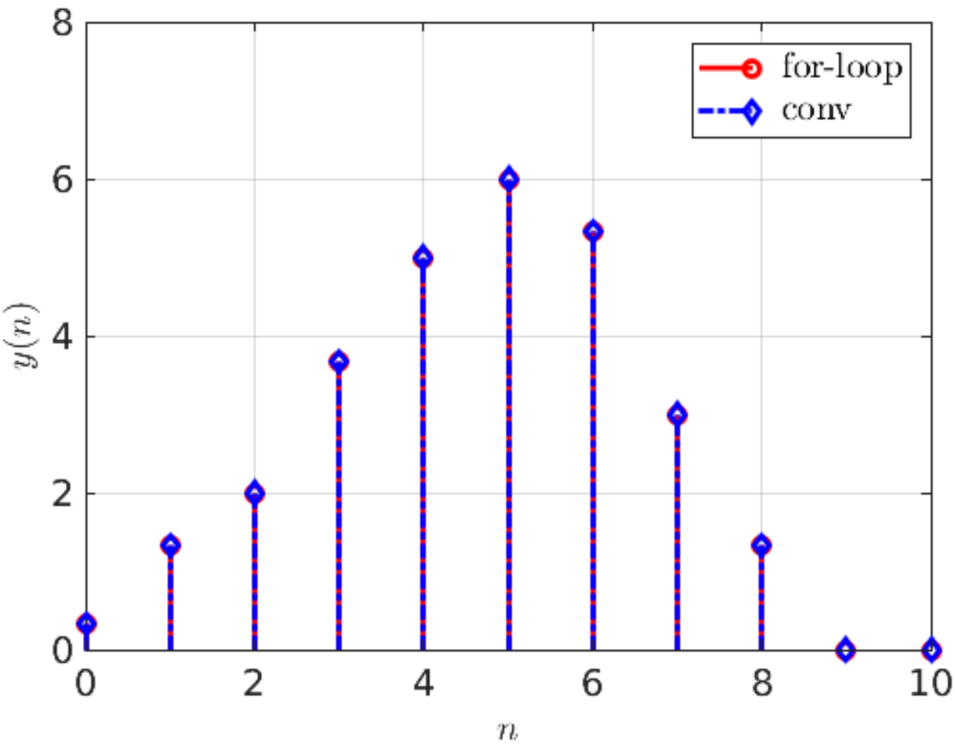

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