

# Optimization Theory Lab – CVX Project

## Lab Homework Questions

November 17, 2025

### Problem 1 – Bound-Constrained Least-Squares

Consider the bound-constrained least-squares problem

$$\min_{\mathbf{x} \in \mathbb{R}^n} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 \quad (1)$$

$$\text{s.t. } \boldsymbol{\ell} \leq \mathbf{x} \leq \mathbf{u}, \quad (2)$$

where  $\mathbf{A} \in \mathbb{R}^{m \times n}$ ,  $\mathbf{b} \in \mathbb{R}^m$ , and the vectors  $\boldsymbol{\ell}, \mathbf{u} \in \mathbb{R}^n$  denote the lower and upper bounds on  $\mathbf{x}$ .

**Task:** Formulate and solve this problem using CVX (or a similar convex optimization toolbox).

### Problem 2 – Average Sidelobe Energy Minimization

Consider a uniform linear array (ULA) beamformer with complex weight vector  $\mathbf{w} \in \mathbb{C}^M$ . Let  $P \in \mathbb{C}^{M \times M}$  be a positive semidefinite matrix that characterizes the average sidelobe energy in a given angular region, and let  $\mathbf{a}(\theta_0)$  denote the steering vector of the desired look direction  $\theta_0$ .

We want to find beamforming weights that minimize the average sidelobe energy:

$$\min_{\mathbf{w} \in \mathbb{C}^M} \mathbf{w}^H P \mathbf{w} \quad (3)$$

$$\text{s.t. } \mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (4)$$

*Hint:* In CVX, you can use `quad_form(w, P)` to represent  $\mathbf{w}^H P \mathbf{w}$ .

- (a) Implement the above problem in CVX and obtain the optimal beamforming vector  $\mathbf{w}$ .
- (b) Plot the angle spectrum (e.g., beampattern) and comment on the sidelobe behavior.

### Problem 3 – Worst-Case Sidelobe Minimization

Now consider robust beamforming where we minimize the *worst-case* sidelobe level over a set of sidelobe angles  $\{\theta_1, \dots, \theta_L\}$ . Let  $\mathbf{a}(\theta)$  denote the steering vector at angle  $\theta$ .

We introduce an auxiliary scalar  $t$  and solve

$$\min_{\mathbf{w}, t} t \quad (5)$$

$$\text{s.t. } |\mathbf{w}^H \mathbf{a}(\theta_\ell)|^2 \leq t, \quad \ell = 1, \dots, L, \quad (6)$$

$$\mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (7)$$

- (a) For a given number of antennas  $M$  and sidelobe region, solve the above problem and plot the resulting angle spectrum. Compare it with the result of Problem 2.
- (b) Repeat the design for  $M = 40$  antennas and plot the angle spectrum. Comment on how increasing  $M$  affects the mainlobe width and sidelobe levels.
- (c) Consider two different desired mainlobe regions:

- (i)  $\theta_1 = 15^\circ, \theta_u = 25^\circ,$
- (ii)  $\theta_1 = 0^\circ, \theta_u = 40^\circ.$

For each case, design the beamformer, plot the angle spectrum, and comment on the trade-offs in mainlobe width and sidelobe behavior.

### Problem 4 – Downlink Beamforming with SINR Constraints

A base station (BS) equipped with  $M$  antennas serves  $N$  mobile stations (MSs) simultaneously in the downlink. Each MS has a single antenna. Let  $\mathbf{h}_k \in \mathbb{C}^M$  denote the channel vector from the BS to user  $k$ , and let  $\mathbf{w}_k \in \mathbb{C}^M$  be the beamforming vector for user  $k$ ,  $k = 1, \dots, N$ .

Assume the received signal at user  $k$  is affected by multiuser interference and noise with variance  $\sigma_k^2$ , and the SINR requirement for user  $k$  is  $\gamma_k > 0$ . The downlink SINR of user  $k$  is

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{\substack{j=1 \\ j \neq k}}^N |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma_k^2}. \quad (8)$$

We want to minimize the total transmit power while satisfying all SINR constraints:

$$\min_{\{\mathbf{w}_k\}_{k=1}^N} \quad \sum_{k=1}^N \|\mathbf{w}_k\|_2^2 \quad (9)$$

$$\text{s.t.} \quad \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{\substack{j=1 \\ j \neq k}}^N |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma_k^2} \geq \gamma_k, \quad k = 1, \dots, N. \quad (10)$$

**Task:** Reformulate this problem as a convex (e.g., second-order cone) optimization problem suitable for CVX. Implement it numerically and verify that all users meet their SINR targets with minimum total transmit power.