Optimization Theory Course

Optimization Theory Lab – CVX Project

Lab Homework Questions

Last date of submission: 02-Dec-2025

Source code (MATLAB/CVX and Python/CVXPY):

https://github.com/gudduarnav/project-optimization-beamforming

Problem 1 – Bound-Constrained Least-Squares

Consider the bound-constrained least-squares problem

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \ \|A\boldsymbol{x}-b\|_2^2 \quad \text{s.t.} \quad 0.1 \le \boldsymbol{x} \le 3.14, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, and the vectors $\ell, u \in \mathbb{R}^n$ denote generic lower and upper bounds on $\boldsymbol{x}$, and in this homework each component of $\boldsymbol{x}$ is constrained to lie between 0.1 and 3.14.

Task: Formulate and solve this problem using CVX (or a similar convex optimization toolbox), report the optimal vector $\boldsymbol{x}^\star$, and briefly discuss what you can infer from its entries given the bounds $0.1 \le x_i \le 3.14$ (e.g., which components are active at the bounds and how this affects the fit $\|A\boldsymbol{x}-b\|_2^2$).

Problem 2 – Average Sidelobe Energy Minimization

Consider a uniform linear array (ULA) beamformer with complex weight vector $\boldsymbol{w} \in \mathbb{C}^M$. Let $\boldsymbol{P} \in \mathbb{C}^{M\times M}$ be a positive semidefinite matrix that characterizes the average sidelobe energy in a given angular region, and let $\boldsymbol{a}(\theta_0)$ denote the steering vector of the desired look direction θ_0 (for this homework, take $\theta_0 = +10^\circ$). We want to find beamforming weights that minimize the average sidelobe energy:

$$\min_{\boldsymbol{w}\in\mathbb{C}^M} \ \boldsymbol{w}^H\boldsymbol{P}\boldsymbol{w} \quad \text{s.t.} \quad \boldsymbol{w}^H\boldsymbol{a}(\theta_0) = 1. \tag{2}$$

Hint: In CVX, you can use `quad_form(w, P)` to represent $\boldsymbol{w}^H\boldsymbol{P}\boldsymbol{w}$.

(a) Implement the above problem in CVX and obtain the optimal beamforming vector $\boldsymbol{w}$ for $\theta_0 = +10^\circ$.
(b) Plot the angle spectrum (e.g., beampattern) and comment on the sidelobe behavior. In particular, discuss how the beampattern behaves around $\theta_0 = +10^\circ$ and what you can infer about mainlobe sharpness and sidelobe suppression.

Problem 3 – Worst-Case Sidelobe Minimization

Now consider robust beamforming where we minimize the worst-case sidelobe level over a set of sidelobe angles $\{\theta_1, \dots, \theta_L\}$. Let $\boldsymbol{a}(\theta)$ denote the steering vector at angle θ. We introduce an auxiliary scalar t and solve

$$\min_{\boldsymbol{w},t} \ t \quad \text{s.t.} \quad |\boldsymbol{w}^H\boldsymbol{a}(\theta_\ell)|^2 \le t, \quad \ell = 1,\dots,L, \qquad \boldsymbol{w}^H\boldsymbol{a}(\theta_0) = 1. \tag{3}$$

(a) For a given number of antennas M and sidelobe region, solve the above problem and plot the resulting angle spectrum. Compare it with the result of Problem 2.
(b) Repeat the design for $M = 32$ antennas and plot the angle spectrum. Comment on how increasing M (relative to part (a)) affects the mainlobe width and sidelobe levels.
(c) Consider two different desired mainlobe regions:

(i) $\theta_\ell = -10^\circ, \theta_u = +10^\circ$,

(ii) $\theta_\ell = 45^\circ, \theta_u = 55^\circ$.

For each case, design the beamformer, plot the angle spectrum, and comment on the trade-offs in mainlobe width and sidelobe behavior.

Problem 4 – Downlink Beamforming with SINR Constraints

A base station (BS) equipped with M antennas serves N mobile stations (MSs) simultaneously in the downlink. Each MS has a single antenna. Let $\boldsymbol{h}_k \in \mathbb{C}^M$ denote the channel vector from the BS to user k, and let $\boldsymbol{w}_k \in \mathbb{C}^M$ be the beamforming vector for user k, $k = 1, \ldots, N$. Assume the received signal at user k is affected by multiuser interference and noise with variance σ_k^2, and the SINR requirement for user k is $\gamma_k > 0$. The downlink SINR of user k is

$$\mathrm{SINR}_k = \frac{\left|\boldsymbol{h}_k^H \boldsymbol{w}_k\right|^2}{\sum\limits_{\substack{j=1 \\ j\neq k}}^{N} \left|\boldsymbol{h}_k^H \boldsymbol{w}_j\right|^2 + \sigma_k^2}. \tag{4}$$

We want to minimize the total transmit power while satisfying all SINR constraints:

$$\min_{\{\boldsymbol{w}_k\}_{k=1}^N} \sum_{k=1}^{N} \|\boldsymbol{w}_k\|_2^2 \quad \text{s.t.} \quad \frac{\left|\boldsymbol{h}_k^H \boldsymbol{w}_k\right|^2}{\sum\limits_{\substack{j=1 \\ j\neq k}}^{N} \left|\boldsymbol{h}_k^H \boldsymbol{w}_j\right|^2 + \sigma_k^2} \geq \gamma_k, \quad k = 1, \ldots, N. \tag{5}$$

(a) Reformulate this problem as a convex (e.g., second-order cone) optimization problem suitable for CVX.

(b) Implement the problem numerically for a system with $M = 32$ antennas and $N = 5$ mobile stations, assuming a noise variance corresponding to $-70\,\mathrm{dBm}$ for each user and SINR targets $\gamma_k = 10\,\mathrm{dB}$, $k = 1, \ldots, 5$. Verify that all users meet their SINR targets with minimum total transmit power, and briefly discuss the resulting power allocation across users.