

Optimization Theory Lab – CVX Project

Lab Homework Questions

November 17, 2025

Problem 1 – Bound-Constrained Least-Squares

Consider the bound-constrained least-squares problem

$$\min_{\mathbf{x} \in \mathbb{R}^n} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 \quad (1)$$

$$\text{s.t. } \boldsymbol{\ell} \leq \mathbf{x} \leq \mathbf{u}, \quad (2)$$

where $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$, and the vectors $\boldsymbol{\ell}, \mathbf{u} \in \mathbb{R}^n$ denote the lower and upper bounds on \mathbf{x} .

Task: Formulate and solve this problem using CVX (or a similar convex optimization toolbox).

Problem 2 – Average Sidelobe Energy Minimization

Consider a uniform linear array (ULA) beamformer with complex weight vector $\mathbf{w} \in \mathbb{C}^M$. Let $\mathbf{P} \in \mathbb{C}^{M \times M}$ be a positive semidefinite matrix that characterizes the average sidelobe energy in a given angular region, and let $\mathbf{a}(\theta_0)$ denote the steering vector of the desired look direction θ_0 . We want to find beamforming weights that minimize the average sidelobe energy:

$$\min_{\mathbf{w} \in \mathbb{C}^M} \mathbf{w}^H \mathbf{P} \mathbf{w} \quad (3)$$

$$\text{s.t. } \mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (4)$$

Hint: In CVX, you can use `quad_form(w, P)` to represent $\mathbf{w}^H \mathbf{P} \mathbf{w}$.

- Implement the above problem in CVX and obtain the optimal beamforming vector \mathbf{w} .
- Plot the angle spectrum (e.g., `beampattern`) and comment on the sidelobe behavior.

Problem 3 – Worst-Case Sidelobe Minimization

Now consider robust beamforming where we minimize the *worst-case* sidelobe level over a set of sidelobe angles $\{\theta_1, \dots, \theta_L\}$. Let $\mathbf{a}(\theta)$ denote the steering vector at angle θ .

We introduce an auxiliary scalar t and solve

$$\min_{\mathbf{w}, t} t \quad (5)$$

$$\text{s.t. } |\mathbf{w}^H \mathbf{a}(\theta_\ell)|^2 \leq t, \quad \ell = 1, \dots, L, \quad (6)$$

$$\mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (7)$$

- For a given number of antennas M and sidelobe region, solve the above problem and plot the resulting angle spectrum. Compare it with the result of Problem 2.
- Repeat the design for $M = 40$ antennas and plot the angle spectrum. Comment on how increasing M affects the mainlobe width and sidelobe levels.
- Consider two different desired mainlobe regions:

- (i) $\theta_1 = 15^\circ, \theta_u = 25^\circ$,
- (ii) $\theta_1 = 0^\circ, \theta_u = 40^\circ$.

For each case, design the beamformer, plot the angle spectrum, and comment on the trade-offs in mainlobe width and sidelobe behavior.

Problem 4 – Downlink Beamforming with SINR Constraints

A base station (BS) equipped with M antennas serves N mobile stations (MSs) simultaneously in the downlink. Each MS has a single antenna. Let $\mathbf{h}_k \in \mathbb{C}^M$ denote the channel vector from the BS to user k , and let $\mathbf{w}_k \in \mathbb{C}^M$ be the beamforming vector for user k , $k = 1, \dots, N$.

Assume the received signal at user k is affected by multiuser interference and noise with variance σ_k^2 , and the SINR requirement for user k is $\gamma_k > 0$. The downlink SINR of user k is

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{\substack{j=1 \\ j \neq k}}^N |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma_k^2}. \quad (8)$$

We want to minimize the total transmit power while satisfying all SINR constraints:

$$\min_{\{\mathbf{w}_k\}_{k=1}^N} \sum_{k=1}^N \|\mathbf{w}_k\|_2^2 \quad (9)$$

$$\text{s.t.} \quad \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{\substack{j=1 \\ j \neq k}}^N |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma_k^2} \geq \gamma_k, \quad k = 1, \dots, N. \quad (10)$$

Task: Reformulate this problem as a convex (e.g., second-order cone) optimization problem suitable for CVX. Implement it numerically and verify that all users meet their SINR targets with minimum total transmit power.