4. Use simplex method to solve the LPP.

max $z = 4x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 50$, $2x_1 + 5x_2 \leq 100$, $2x_1 + 3x_2 \leq 90$ and $x_1, x_2 \geq 0$

solution:-

$\max z = 4x_1 + 10x_2$

subject to

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

and $x_1, x_2 \geq 0$;

Introducing slack variable.

$\text{Max } z = 4x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$2x_1 + x_2 + S_1 + 0S_2 + 0S_3 = 50$

$2x_1 + 5x_2 + 0S_1 + S_2 + 0S_3 = 100$

$2x_1 + 3x_2 + 0S_1 + 0S_2 + S_3 = 90$

and $x_1, x_2, S_1, S_2, S_3 \geq 0$

Iteration-0

		C_j	4	10	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	Ratio
S_1	0	50	2	1	1	0	0	$\frac{50}{1} = 50$
S_2	0	100	2	5	0	1	0	$\frac{100}{5} = 20$
S_3	0	90	2	3	0	0	1	$90/3 = 30$
		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-4	-10	0	0	0	

Negative min $z_j - c_j$ is 10, entering variable is x_2

leaving basis variable is S_2

new pivot eq = old pivot eq % Pivot element

new variable eq = old variable eq - column coefficient × new pivot eq

Iteration 1

B	C_B	C_j / X_B	4 / x_1	10 / x_2	0 / S_1	0 / S_2	0 / S_3	Ratio
S_1	0	30	8/5	0	1	-1/5	0	
x_2	10	20	2/5	1	0	1/5	0	
S_3	0	30	4/5	0	0	-3/5	1	
		z_j	4	10	0	2	0	
		$z_j - c_j$	0	0	0	2	0	

since $z_j - c_j \geq 0$

Hence optimal solution is arrived with value of variable as $x_1 = 0$, $x_2 = 20$

Max $z = 200$

8) min $z = 8x_1 - 2x_2$, $-4x_1 + 2x_2 \leq 1$, $5x_1 - 4x_2 \leq 3$ and $x_1, x_2 \geq 0$.

solution :-

min $z = 8x_1 - 2x_2$

subject to

$-4x_1 + 2x_2 \leq 1$

$5x_1 - 4x_2 \leq 3$

and $x_1, x_2 \geq 0$;

Introducing slack variable

min $z = 8x_1 - 2x_2 + 0S_1 + 0S_2$

subject to

$-4x_1 + 2x_2 + S_1 + 0S_2 = 1$

$5x_1 - 4x_2 + 0S_1 + S_2 = 3$

and $x_1, x_2, S_1, S_2 \geq 0$

Iteration

		C_j	8	-2	0	0	
B	CB	XB	x_1	x_2	S_1	S_2	ratio
S_1	0	1	-4	(2)	1	0	$\frac{1}{2} = 0.5$
S_2	0	3	5	-4	0	1	
		Z_j	0	0	0	0	
		$Z_j - C_j$	-8	2	0	0	

$Z_j - C_j = 2$, entering variable is x_2

Min ratio = 0.5, leaving variable is S_1

Pivot value = 2

new pivot eq = old pivot eq % pivot value

new variable eq = old variable eq - column coefficient × new pivot eq

Iteration 1

		C_j	8	-2	0	0	
B	CB	XB	x_1	x_2	S_1	S_2	Ratio
x_2	-2	1/2	-2	1	1/2	0	
S_2	0	5	-3	0	2	1	
		Z_j	4	-2	-1	0	
		$Z_j - C_j$	-4	0	-1	0	

$Z_j - C_j \leq 0$

Hence optimal solution is arrived with value of variable as $x_1 = 0, x_2 = \frac{1}{2}$

min $z = -1$

9) max $z = 3x_1 + 5x_2 + 4x_3$, $2x_1 + 3x_2 \le 8$, $2x_2 + 5x_3 \le 10$, $3x_1 + 2x_2 + 4x_3 \le 15$ and $x_1, x_2, x_3 \ge 0$

sol:-

$\max z = 3x_1 + 5x_2 + 4x_3$

subject to

$2x_1 + 3x_2 + 0x_3 \le 8$

$0x_1 + 2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$

Introducing slack variable s_1, s_2, s_3

$\max z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

subject to

$2x_1 + 3x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 8$

$0x_1 + 2x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 10$

$3x_1 + 2x_2 + 4x_3 + 0s_1 + 0s_2 + s_3 = 15$

and $x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

Iteration

	c_j		3	5	4	0	0	0	
B	c_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	ratio
s_1	0	0	2	3	0	1	0	0	$8/3 = 2.667$
s_2	0	10	0	2	0	1	0	0	$10/2 = 5$
s_3	0	15	3	2	4	0	0	1	$15/2 = 7.5$
		z_j	0	0	0	0	0	0	
		$z_j - c_j$	-3	-5	-4	0	0	0	

$z_j - c_j = -5$, Entering variable is s_1

min ratio = 2.667, the leaving variable is s_1

Pivot element is 3

Linear Algebra &

Iteration 1

B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
	C_j		3	5	4	0	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	
s_2	0	14/3	-4/3	0	5	-2/3	1	0	0.933
s_3	0	29/3	5/3	0	4	-2/3	0	1	2.4167
		Z_j	10/3	5	0	5/3	0	0	
		$Z_j - C_j$	1/3	0	-4	5/3	0	0	

$Z_j - c_j = -4$ Leaving variable is x_3

min ratio is 0.933 & Leaving variable is s_2

Iteration 2

B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
	C_j		3	5	4	0	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	4
x_3	4	14/15	4/15	0	1	-3/15	1/5	0	
S_3	0	89/15	41/15	0	0	-2/15	-4/5	1	2.1707
		Z_j	34/15	5	4	17/15	4/5	0	
		$Z_j - C_j$	-11/15	0	0	17/15	4/5	0	

$Z_j - C_j$ is $-\frac{11}{15}$, Entering variable is x_1 & Leaving variable is S_3

The min ratio 2.1707

Iteration 3

B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
	C_j		3	5	4	0	0	0	
x_2	5	$\frac{50}{41}$	0	1	0	15/41	$\frac{8}{41}$	$-\frac{10}{41}$	
x_3	4	$\frac{62}{41}$	0	0	1	-6/41	5/41	4/41	
x_1	3	89/41	1	0	0	2/41	-12/41	15/41	
		Z_j	3	5	0	45/41	24/41	11/41	
		$Z_j - C_j$	0	0	0	$\frac{45}{41}$	24/41	11/41	

Since all $z_j - c_j \geq 0$

Hence optimal solution is derived with value of variable as

$x_1 = \frac{80}{41}$, $x_2 = \frac{50}{41}$, $x_3 = \frac{62}{41}$

Max $z = \frac{765}{41}$

15. max $z = x_1 + 2x_2 + x_3$, $2x_1 + x_2 - x_3 \leq 2$, $-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$ and $x_1, x_2\ x_3 \geq 0$

Sol:-

$$\text{Max } z = x_1 + 2x_2 + x_3$$

subject to

$$2x_1 + x_2 - x_3 \leq 2$$

$$-2x_1 + x_2 - 5x_3 \geq -6$$

Here $b_2 = -6 \leq 0$, so multiply by -1

$$2x_1 + x_2 + 5x_3 \leq 6$$

$$4x_1 + x_2 + x_3 \leq 6$$

and $x_1, x_2\ x_3 \geq 0$

Introducing slack variable

$$\text{max } z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$$

subject to

$$2x_1 + x_2 - x_3 + s_1 + 0s_2 + 0s_3 = 2$$

$$2x_1 - x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 6$$

$$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Iteration

		c_j	1	2	1	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
s_1	0	2	2	1	-1	1	0	0	2
s_2	0	6	2	-1	5	0	1	0	
s_3	0	6	4	1	1	0	0	1	6
		z_j	0	0	0	0	0	0	
		$z_j - c_j$	-1	-2	-1	0	0	0	

$z_j - c_j$ is -2, entering variable is s_1

min ratio is 2, leaving variable is s_1

pivot element is 1

Iteration 1

B	CB	C_j / XB	1 x_1	2 x_2	1 x_3	0 s_1	0 s_2	0 s_3	Ratio
x_2	2	2	2	1	-1	1	0	0	
s_2	0	8	4	0	4	1	1	0	2
s_3	0	4	2	0	2	-1	0	1	2
		Z_j	4	2	-2	2	0	0	
		$Z_j - C_j$	3	0	-3	2	0	0	

$Z_j - C_j = -3$, the entering variable is s_3, min ratio 2

Leaving variable is s_3

Iteration 2 :

B	CB	C_j / XB	1 x_1	2 x_2	1 x_3	0 s_1	0 s_2	0 s_3	Ratio
x_2	2	4	3	1	0	1/2	0	1/2	
s_2	0	0	0	0	0	3	1	-2	
x_3	1	2	1	0	1	-1/2	0	3/2	
		Z_j	7	2	1	1/2	0	3/2	
		$Z_j - C_j$	6	0	0	1/2	0	3/2	

$Z_j - C_j \geq 0$

Hence optimal solution is arraived with value of Variable

$x_1 = 0, x_2 = 4, x_3 = 2$

Max $Z = 10$.