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BITS Pilani presentation

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Vector Space

- Let **V** be a non-empty set.
- Let R be the set of real numbers.
- Define vector addition '+' and scalar multiplication '**.**'

such that :

I. UNDER VECTOR ADDITION:

- (A) u + v € **V** for all u, v € **V:**
- **V is closed under vector addition. Moreover,**
- A1. u + v = v + u for all u, v € **V**
- A2. (u +v) + w = u + (v + w)
- for all u, v, w € **V**

- A3. There is an element 0 € **V** such that

  0 + u = u for all u € **V**

- A4. For each u € V there is -u € **V**

  such that u + (-u) = 0

NOTE: V is a commutative group (abelian group) under vector addition

II. UNDER SCALAR MULTIPLICATION:

(B) $\alpha u \in V$ for all u in **V** , α is real: **V is closed under scalar multiplication.**

Moreover

B1. $\alpha(u + v) = \alpha u + \alpha v$

for all $u, v \in V$, for all real α

B2. $(\alpha + \beta)\, u = \alpha\, u + \beta\, u$

for all u € V, for all real α, β

B3. $(\alpha\, \beta)\, u = \alpha\, (\beta\, u)$

for all u € V, for all real α, β

B4. $1 \,.\, u = u$ for all u € V,

Note: B1-B4: scalar multiplication with vectors

Example 1.

- Rn is a vector space with usual addition of vectors and multiplication of vectors by scalars.

Example 2

- Let Pn be the set of all polynomials of degree $\leq$ n in the variable x with coefficients in R.
- Then Pn is a vector space,

with addition of polynomials and scalar multiplication of a polynomial

Some important results in a vector space

Let V be a vector space. Then

- 1. $0.u = \mathbf{0}$ for every $u \in V$, $0 \in R$.
- 2. $\alpha.\mathbf{0} = \mathbf{0}$ for every scalar α, $\mathbf{0} \in V$.
- 3. $(-1).u = -u$ for every $u \in V$.
- 4. $\alpha . u = \mathbf{0} \Rightarrow \alpha = 0$ or $u = \mathbf{0}$

SUBSPACE

Let V be a vector space and W be non-empty subset of V.

If W is a vector space with respect to operations in V, then W is called a ***subspace*** of V.

Every vector space has at least two subspace: {0} and V itself.

- These are known as trivial subspace & {0} is known as zero subspace

Theorem: Let V be a vector space.

Let W be non-empty subset of V.

Then, W is ***subspace*** of V ***iff***

1. x, y € W => x + y € W

(Closed under vector addition)

2. α is scalar, x € W => α x € W.

(Closed under scalar multiplication)

Ex-1: Let V = R3

***Then* W = {(x, y, 0)| x, y are $\in$R } is a subspace of V.**

(First show W is a non-emty set)

Ex2: Let V = P (Set of all polynomials p)

W = {p $\in$P| degree of p=3 }

Then W is NOT a subspace of P.

Example 3. Let W be the set of points inside and on the unit circle in the $xy-plane$, that is,

$$W = \{(x, y) : x^2 + y^2 \leq 1\}.$$

Then W is not a subspace of V_2 as

$$u = \left(\frac{1}{2}, \frac{1}{2}\right) \in W \text{ and for } \alpha = 4,$$

$$\alpha u = (2,2) \notin W.$$

Linear Combination

Let V be a vector space.

Let $v1, v2, \ldots\ldots, vk$ € V.

Then, v € V is linear combination of

$v1, v2, \ldots\ldots, vk$ **if**

$v = c1\ v1 + c2\ v2 + \ldots\ldots + ck\ \ vk$, ci € R

Span of a set:

Let V be a vector space and

S = { v1, v2, ... , vk} be a subset of V.

Then, span S = [S]

$=\{c_1 v_1 + c_2 v_2 + \ldots + c_k v_k : c_i \in R\}$

Ex: Let V = R3

Let S = {(1, 0, 0), (0, 0, 1) }

Then [S] = { (a, 0, b) | a, b € R }

Theorem : Let V be a vector space.

Let S be a non-empty subset of V. Then [S] is a subspace of V

Proof: Let u, v € [S]. Then

$u = c_1 u_1 + c_2 u_2 + \ldots\ldots + c_m u_m$

for some scalar c_i, for some u_i's € S

and a positive integer m

v = d1 v1 + d2 v2 + ...+ dn vn

for some scalars di, for some vi's € S

and a positive integer n

- Now u+v= c1 u1 + c2 u2 + ...+ cm um +

d1 v1 + d2 v2 + ...+ dn vn

is a finite linear combination of S

Therefore u+v € [S]

- Simlarly,

au = (ac1) u1 + (ac2)u2 + ...+ (acm) um

for any scalar a

is a finite linear combination of S

So a u € [S]

Hence [S] is subspace of V

Ex: Let **V = P2**

S = { x2 + 1, x – 1, x2 + x }

Does S spans V ?? i.e. V = [S] ??

Sol:

Let any element p(t) = ax2 + bx + c € V ; a, b, c are real number

Now p(x) € [S] if

ax2 + bx + c = **α(x2 + 1)+β(x – 1)+γ(x2 + x) for any a, b & c for some α, β & γ**

System consistent if $c - a + b = 0$

p(t) does not belongs to [S]

V is not a subset of [S]

Hence $V \neq [S]$