**Birla Institute of Technology & Science, Pilani**

**Work Integrated Learning Programmes Division**

**M. Tech (Software Engineering) at Wipro Technologies (WASE)**

**Comprehensive Examination (Regular)**

No. of Pages:- 4

No. of Questions:- 6

Course Number : SEWP ZC132

Course Title : LINEAR ALGEBRA & OPTIMIZATION

Type of Exam : Open Book

Weightage : 60 Marks

Duration : 3 Hours

Date of Exam : Session : FN( 9 AM to 12 Noon)

**Note:**

1. Please read and follow all instructions given on the cover page of the question paper & answerscript.
2. Start each answer from a fresh page. All parts of a question should be answered consecutively.
3. Use of calculators is allowed
4. If a question is attempted more than once, the first occurrence will be graded and the rest will be ignored. So, carefully cancel the answers which are not required.

**(a) (4 Marks)** A population of Kangaroos has the following features:

* Half of the Kangaroos survive their first year. Of these, half survive the second year. The maximum life span of Kangaroos is three years.
* During the first year, the Kangaroos produce no offspring. The average offspring is 6 during the second year and 8 during the third year.

The population of Kangaroos at present is 24 in the first age class, 24 in the second age class and 20 in the third age class. Answer the following questions through application of matrices.

(i) How many Kangaroos will be there in each age class after one year? Provide the age distribution vector.

(ii) Find an age distribution vector for the population of Kangaroos with stable growth pattern. A stable growth pattern is one in which the percent of Kangaroos in each age class remains the same year after year.

The age transition matrix is

**(b) (3 Marks)**Show that is a linear combination of and .

**(c) (3 Marks)**Calculate the Rank of the following matrix:

**(a) (5 Marks)**Using Gauss Seidel method, solve the following system of equations:

with initial values. If necessary, rearrange equations to achieve convergence. Perform four iterations. Take approximations upto 2 decimal places in each iteration.

**(b) (3 Marks)**Find the Dual of the following LPP:

+

**(c) (2 Marks)** Transform the following LPP to standard form:

+3

1. **(10 Marks)**A Dietician in a hospital is to arrange special diet using three foods named P, Q, and R. Each gram of food P contains 20 units of Calcium, 10 units of Iron, 10 units of Vitamin A and 20 units of Cholesterol. Each gram of food Q contains 10 units of Calcium, 10 units of Iron, 20 units of Vitamin A and 24 units of Cholesterol. Each gram of food R contains 10 units of Calcium, 10 units of Iron, 10 units of Vitamin A and 18 units of Cholesterol. If the minimum daily requirements are 300 units of Calcium, 200 units of Iron and 240 units of Vitamin A. How many grams of each food should we use to meet the minimum requirements and at the same time minimize the cholesterol intake? What is the minimum cholesterol intake? Formulate the LPP and use appropriate Simplex method to solve the problem.

**(a) (8 marks)** Solve the following LPP Graphically:

Subject to:

Find the Optima and also the optimal value.

If the RHS of the first constraint is changed to 7, find the new optima and new optimum value. Also find the dual price for the same.

**(b) (2 marks)**Consider the following LPP:

Subject to:

Write down the initial basic feasible solution by inspection and also calculate the optimum value from them.

1. **(10 marks)**A firm produces three products A,B,C. Unit contributions of the products are $5,$10 and $8 respectively. Each unit of product A requires 3 kgs of material, 4 machine hours and 2 labor hours. Each unit of Product B requires 5 Kgs of material, 4 machine hours and 4 labor hours. Each unit of product C requires 2 Kg of material, 4 machine hours, and 5 labor hours. Everyday 60 Kgs of material, 72 machine hours and 100 labor hours are available.
2. Find out the best production strategy.
3. Investigate the effect on the solution if available machine hours are increased by 12 hours.
4. How will the solution obtained in (i) be affected if amount of material available is decreased by 6 Kgs.

**(a) (7 marks)**Solve the following LPP by the Dual Simplex method. Write down the initial basic solution and mention whether it is feasible or not.

Subject to :

**(b) (3marks)** Consider the following LPP:

Subject to:

Given that the artificial variable and the slack variable form the starting basic variables and that M was set equal to 100 when solving the problem, the last iteration(optimal) of the Simplex table is:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | 5 | 2 | 3 | -100 | 0 |  |
|  |  |  |  |  |  |  | **RHS** |
| 5 |  | 1 | 5 | 2 | 1 | 0 | 30 |
| 0 |  | 0 | -10 | -8 | -1 | 1 | 10 |
|  | Cj-zj | 0 | -23 | -7 | -105 | 0 | 150 |

Solution is and . Optimal value .

Determine the associated Dual of the problem and its optimal solution from the above table. Do not solve the Dual problem explicitly.

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