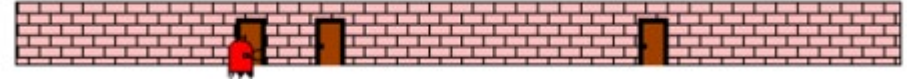


# lab-Particle filter

ENGR 509

# Problem Setup

# Problem Setup



- Robot localization
  - Given the map, where am I (on the map)?

$$Bel(x_t) = \eta p(z_t | x_t) \int p(x_t | x_{t-1}, u_t) Bel(x_{t-1}) dx_{t-1}$$

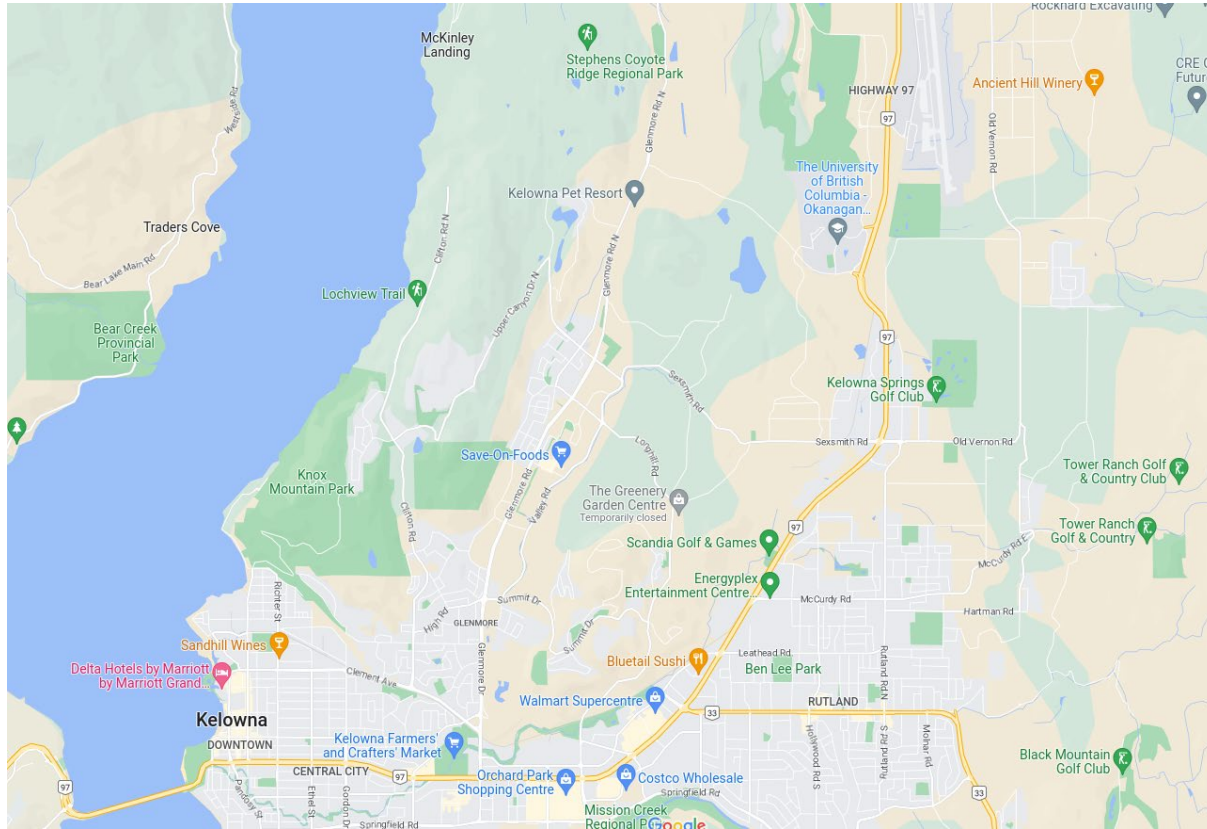
↓  
Posterior:  
current  
location  
estimate

↓  
observation  
model

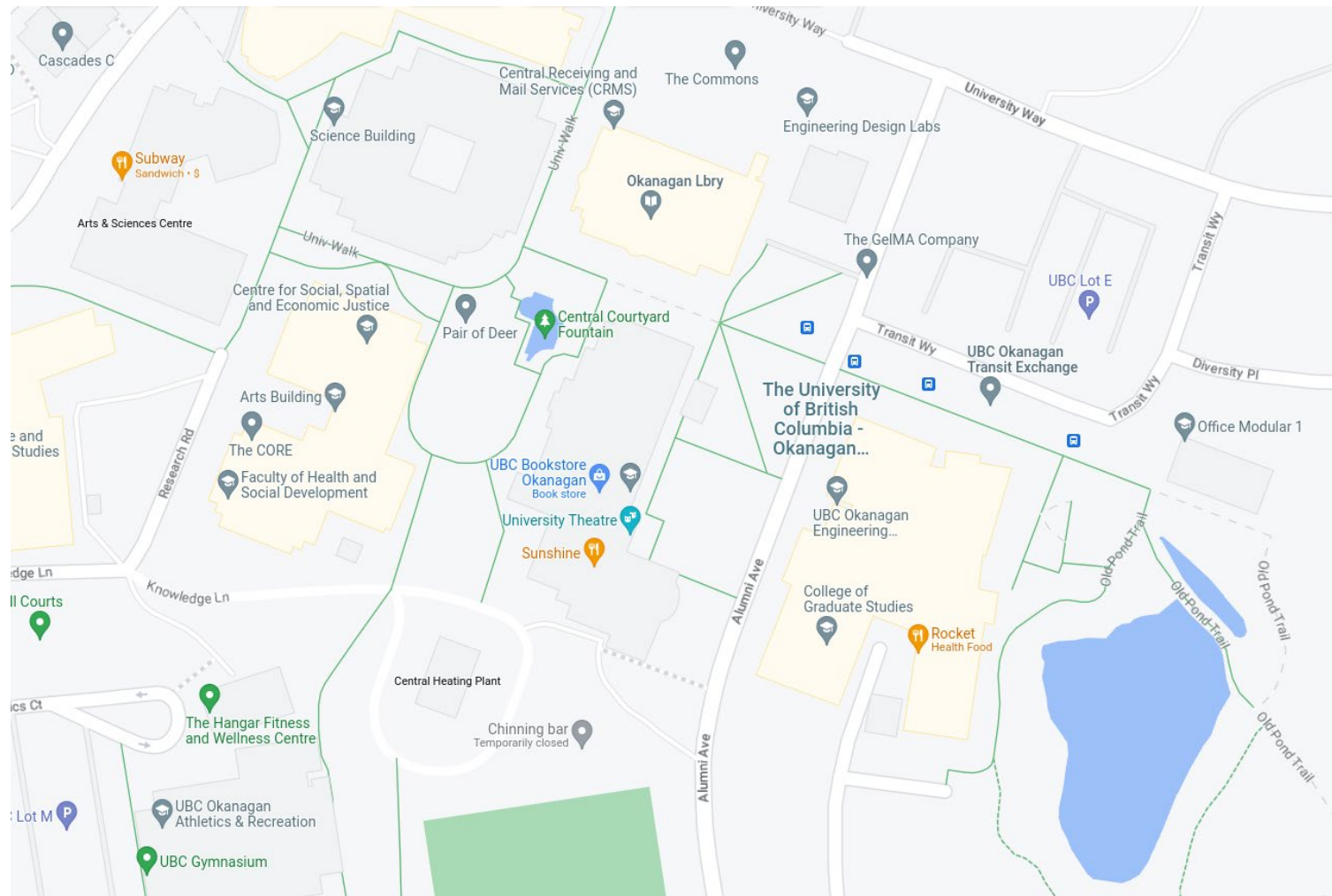
↓  
motion  
model

↓  
Prior:  
previous  
location  
estimate

# Examples



# Map



What are the information on the map that we need to do localization?

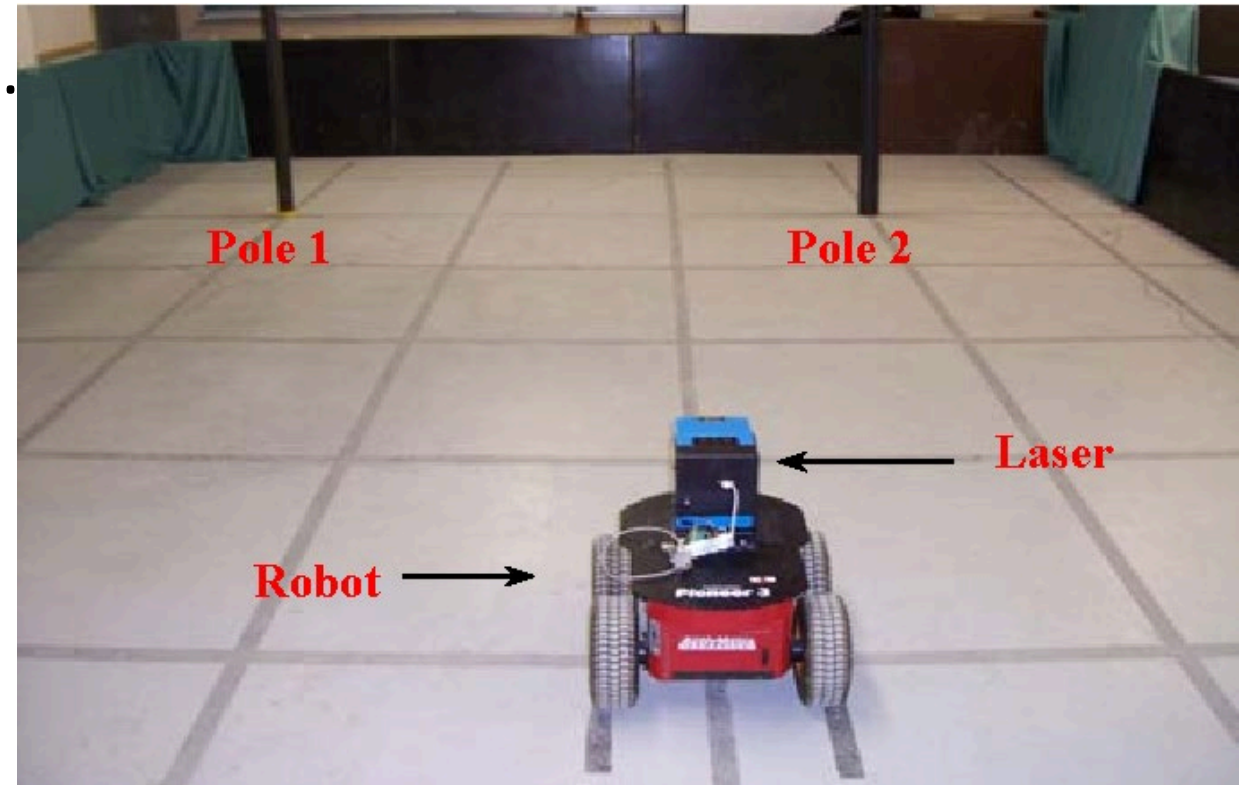
- locations of landmarks

As long as we have enough landmarks, and relative location to the landmarks, we can localize yourself.



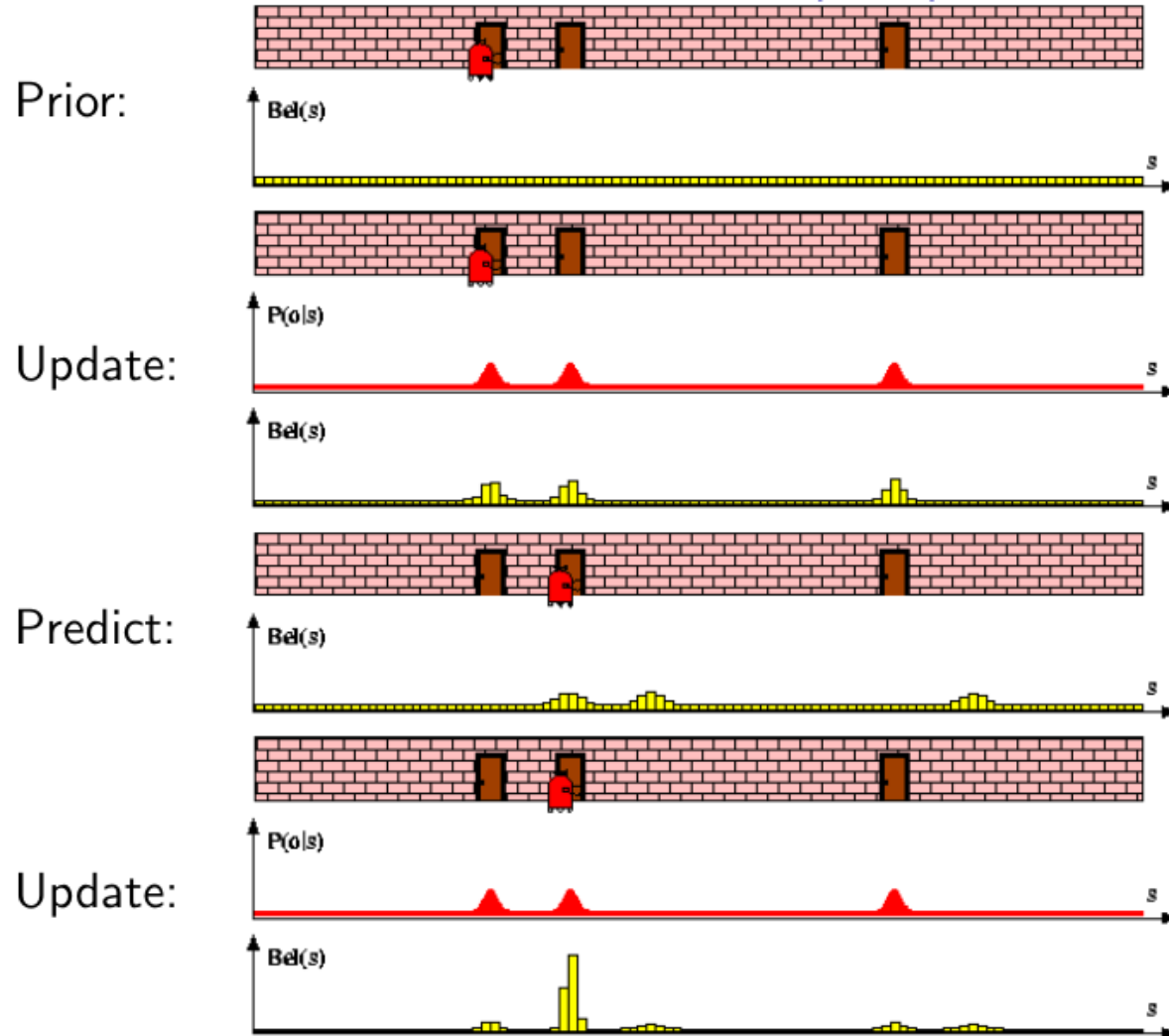
# Our implementation

- landmark: poles
- A range sensor tells how far to a pole.
- Remember: robot knows the map.



# Bayes filter on 1D localization (Homework)

# Bayes filter on 1D localization



PR book Figure 8.1

You can implement a discrete bayes filter for the 1D robot localization.

```

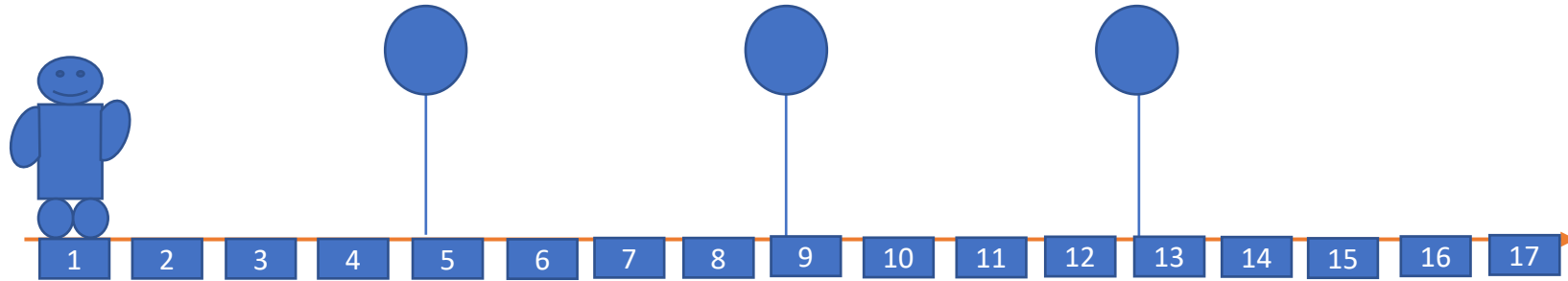
1:  Algorithm Discrete_Bayes_filter( $\{p_{k,t-1}\}, u_t, z_t$ ):
2:    for all  $k$  do
3:       $\bar{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i) p_{i,t-1}$ 
4:       $p_{k,t} = \eta p(z_t \mid X_t = x_k) \bar{p}_{k,t}$ 
5:    endfor
6:    return  $\{p_{k,t}\}$ 
    
```

**Table 4.1** The discrete Bayes filter. Here  $x_i, x_k$  denote individual states.

PR book pp. 87



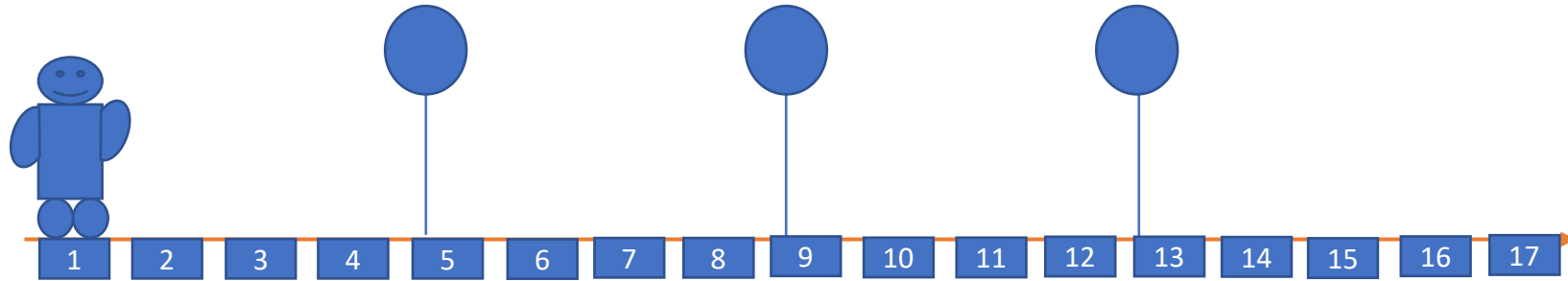
# Bayes filter on simplified 1D localization (Homework)



We further simplify the scenario as follows:

- world: discrete grids denoted as spots #1 ~ #40
- control command: move forward one grid
- landmarks: three poles with known locations
- sensor observations: pole detected/not detected in front of the robot

# Bayes filter on simplified 1D localization (Homework)



Notations:

$P(L_i)$  = The probability the robot is located in location  $i$

$P(D)$  = The probability that a pole is detected

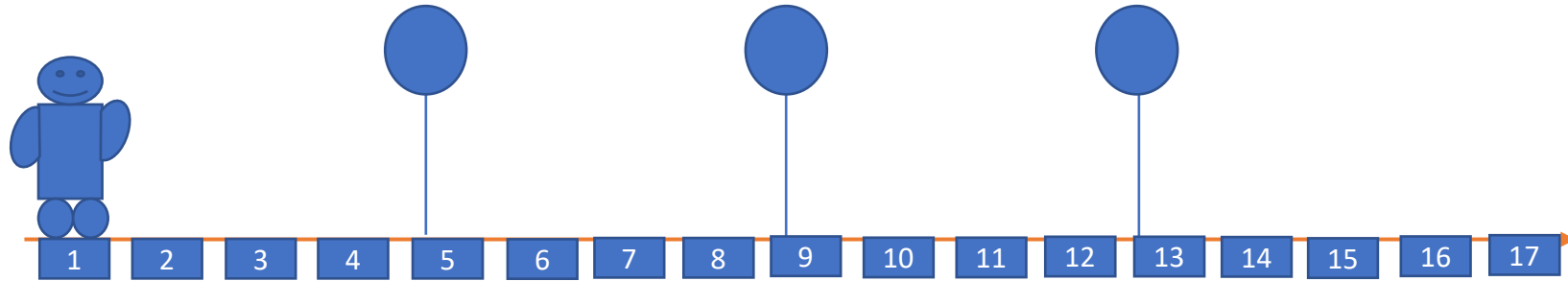
$P(!D)$  = The probability that a pole is not detected

$P(D \mid L_i)$  = The probability of a pole being detected, given the robot is located at the location  $i$

$P(L_i \mid D) = ?$

$P(L_i \mid !D) = ?$

# Bayes filter on simplified 1D localization (Homework)



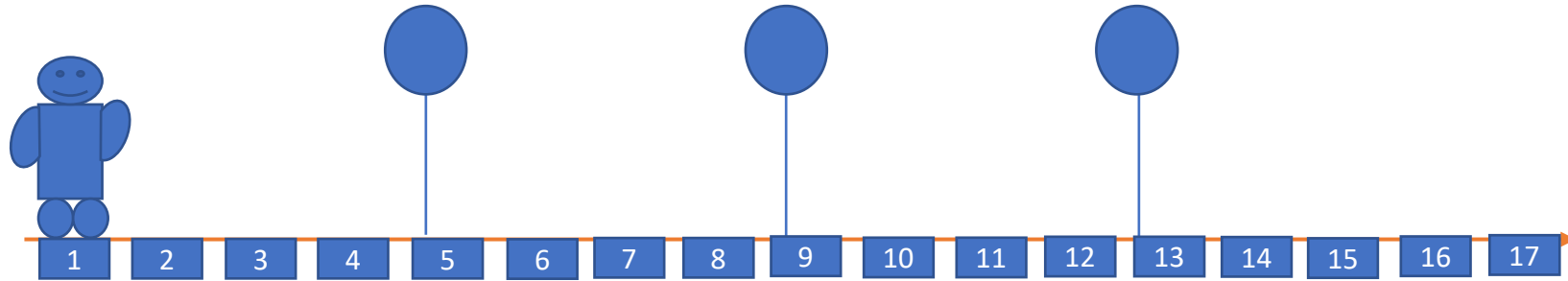
$P(L4)?$

$P(D \mid L4)?$

$P(D)?$

$P(L4 \mid D)?$

# Bayes filter on simplified 1D localization (Homework)



$P(L_3)?$

$P(D \mid L_3)?$

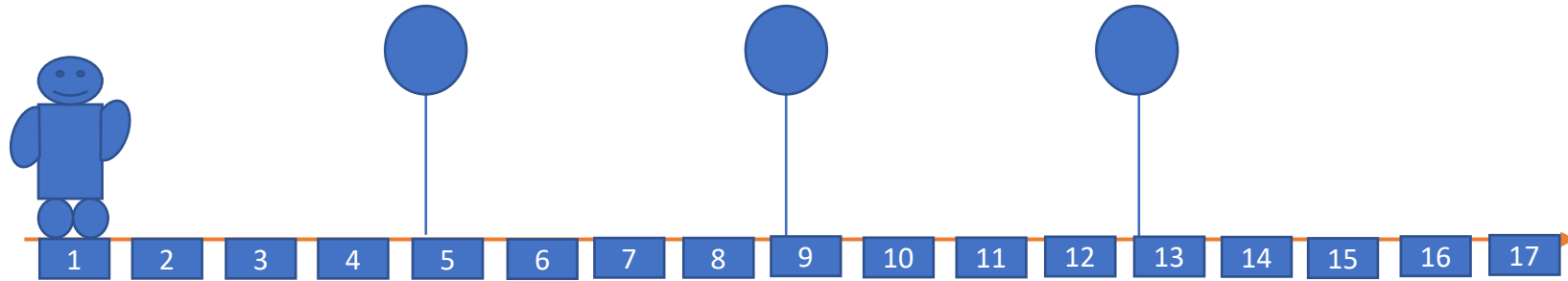
$P(D)?$

$P(L_3 \mid D)?$

$P(L_5 \mid D)?$   $P(L_6 \mid D)?$   $P(L_{10} \mid D)?$   $P(L_{17} \mid D)?$

Similarly, you can compute  $P(L_i \mid !D)$  for each location

# Bayes filter on simplified 1D localization (Homework)



$P(L3)?$

$P(D \mid L3)?$

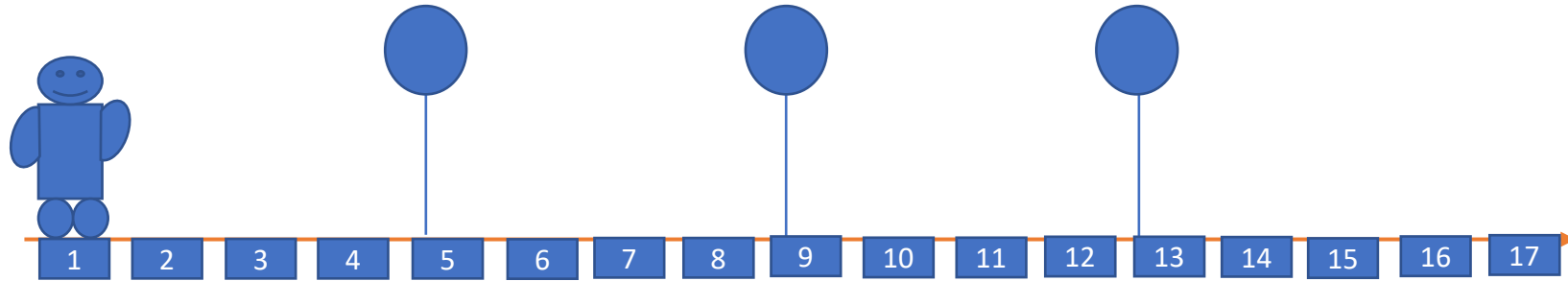
$P(D)?$

$P(L3 \mid D)?$

$P(L5 \mid D)? P(L6 \mid D)? P(L10 \mid D)? P(L17 \mid D)?$

What happens when the robot moves?

# Bayes filter on simplified 1D localization (Homework)

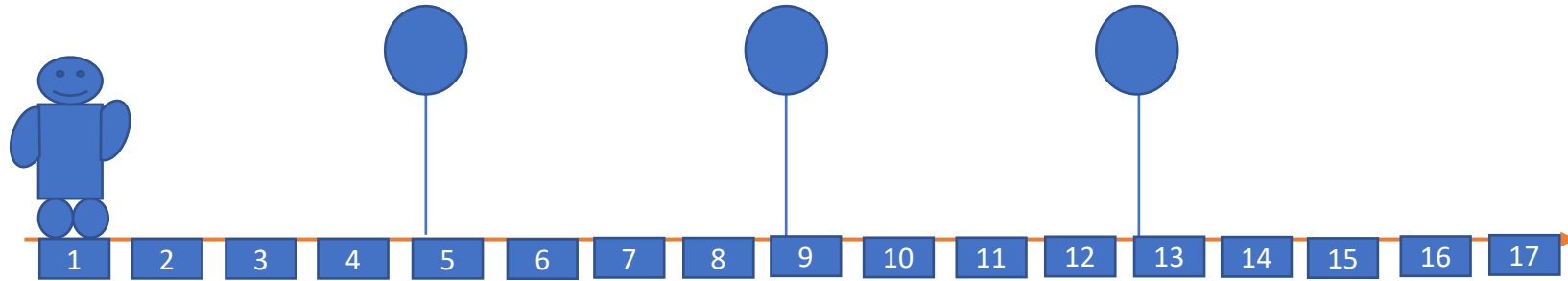


What happens when the robot moves?

1. Shift the  $P(L_i)$  think: what is  $P(L_i)$  now? e.g.,  $P(L_5) \leftarrow P(L_4 | D)$
2. Do Bayes rule for all  $P(L_i | D)$  again

Complete the assignment 2-1

# Bayes filter on simplified 1D localization (Homework)



What if robot moves with uncertainties:

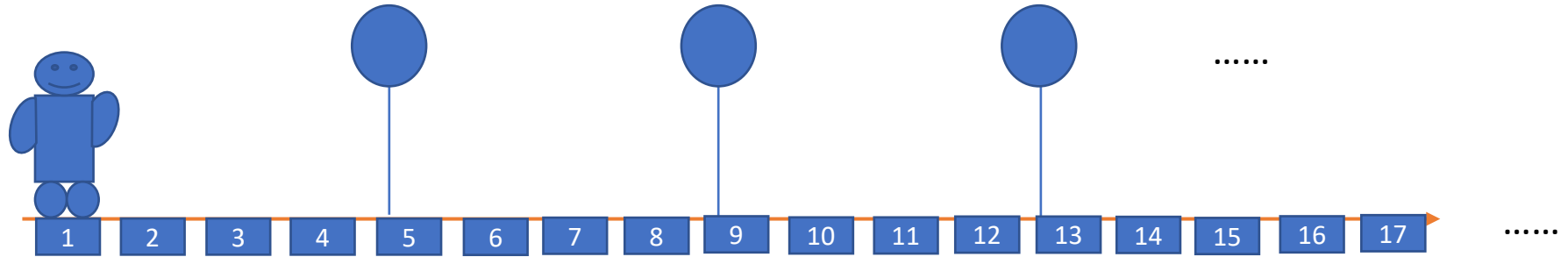
- sometimes, the control command asks the robot to move one unit ahead, but the robot can accidentally move two units ahead with the probability 10%.

Complete the assignment 2-2



# Particle Filter on 1D localization

# Particle Filter for 1D localization



Now, we change the rule:

- The robot movement is not perfect. Although the control command is moving forward by 1.0 unit, the robot can move 1.0 unit plus some errors.
- The robot measurements are not perfect. For example, the measurement is 2.5 units to a pole, meaning distance could be 2.5 units plus some errors.
- For simplicity, we model the errors follow zero-mean Normal distributions.

# Particle Filter for 1D localization

$$Bel(x_t) = \eta p(z_t | x_t) \int p(x_t | x_{t-1}, u_t) Bel(x_{t-1}) dx_{t-1}$$

draw  $x_{t-1}^i$  from  $Bel(x_{t-1})$

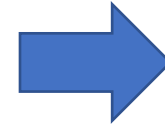
draw  $x_t^i$  from  $p(x_t | x_{t-1}^i, u_t)$

Importance factor for  $x_t^i$ :

$$\begin{aligned} w_t^i &= \frac{\text{target distribution}}{\text{proposal distribution}} \\ &= \frac{\eta p(z_t | x_t) p(x_t | x_{t-1}^i, u_t) Bel(x_{t-1}^i)}{p(x_t | x_{t-1}^i, u_t) Bel(x_{t-1}^i)} \\ &\propto p(z_t | x_t) \end{aligned}$$

# Particle Filter for 1D localization

- Sample the next generation for particles using the proposal distribution
- Compute the importance weights :  
 $weight = target\ distribution / proposal\ distribution$
- Resampling: “Replace unlikely samples by more likely ones”



**Algorithm Particle\_filter( $\mathcal{X}_{t-1}, u_t, z_t$ ):**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

*for*  $m = 1$  *to*  $M$  *do*

*sample*  $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p(z_t \mid x_t^{[m]})$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

*endfor*

*for*  $m = 1$  *to*  $M$  *do*

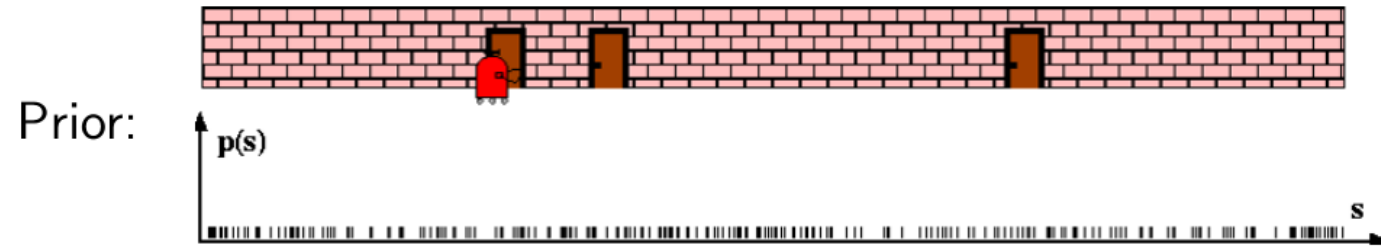
*draw*  $i$  *with probability*  $\propto w_t^{[i]}$

*add*  $x_t^{[i]}$  *to*  $\mathcal{X}_t$

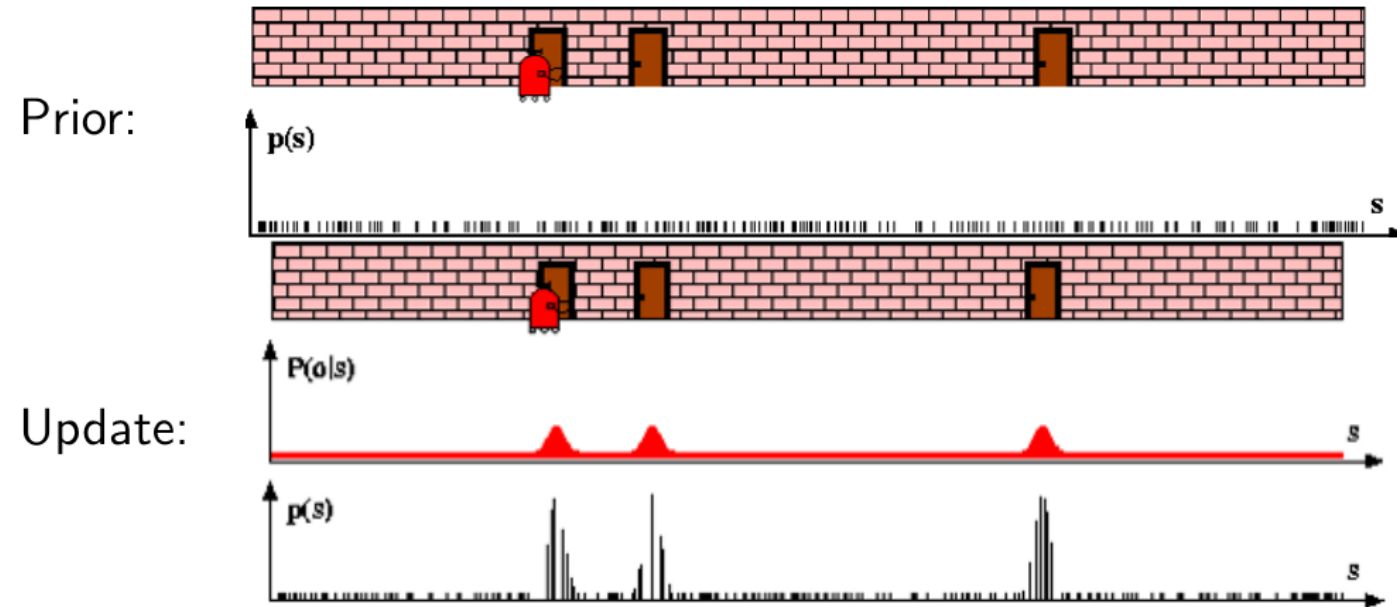
*endfor*

*return*  $\mathcal{X}_t$

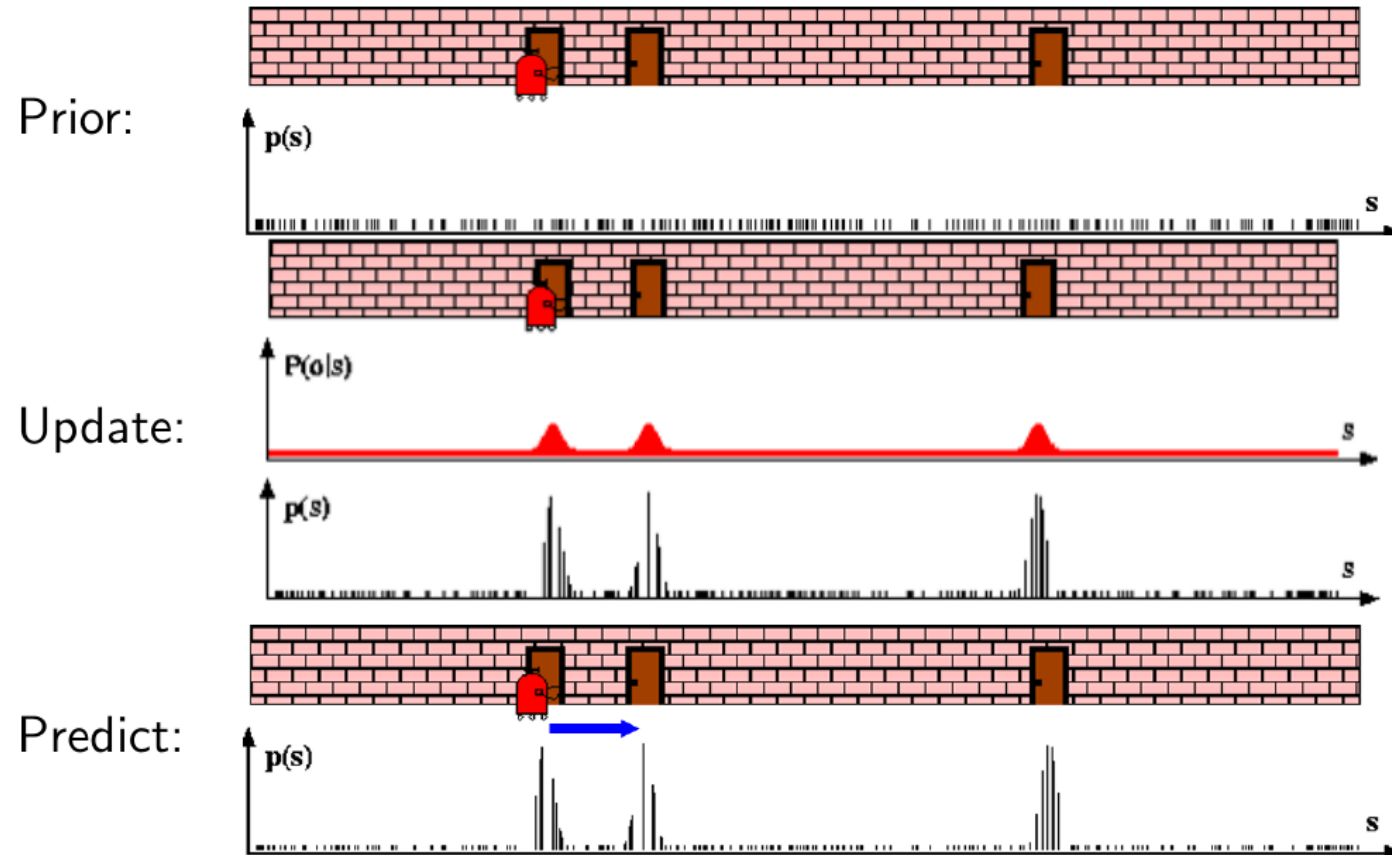
# Particle filter for 1D localization



# Particle filter for 1D localization

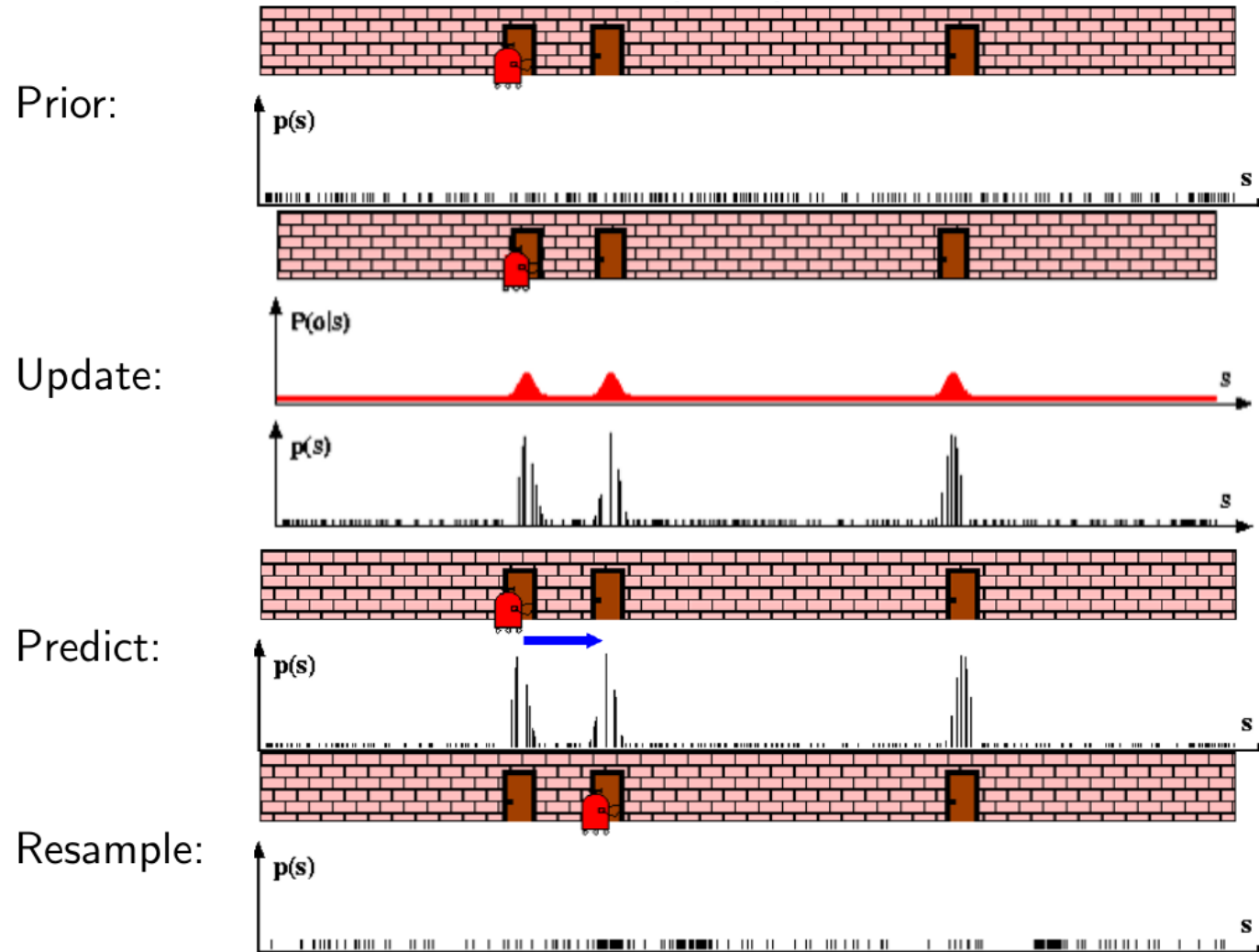


# Particle filter for 1D localization

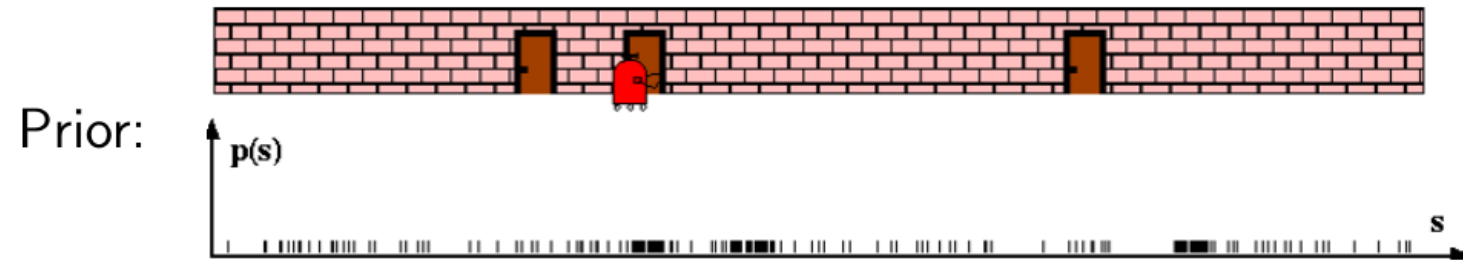




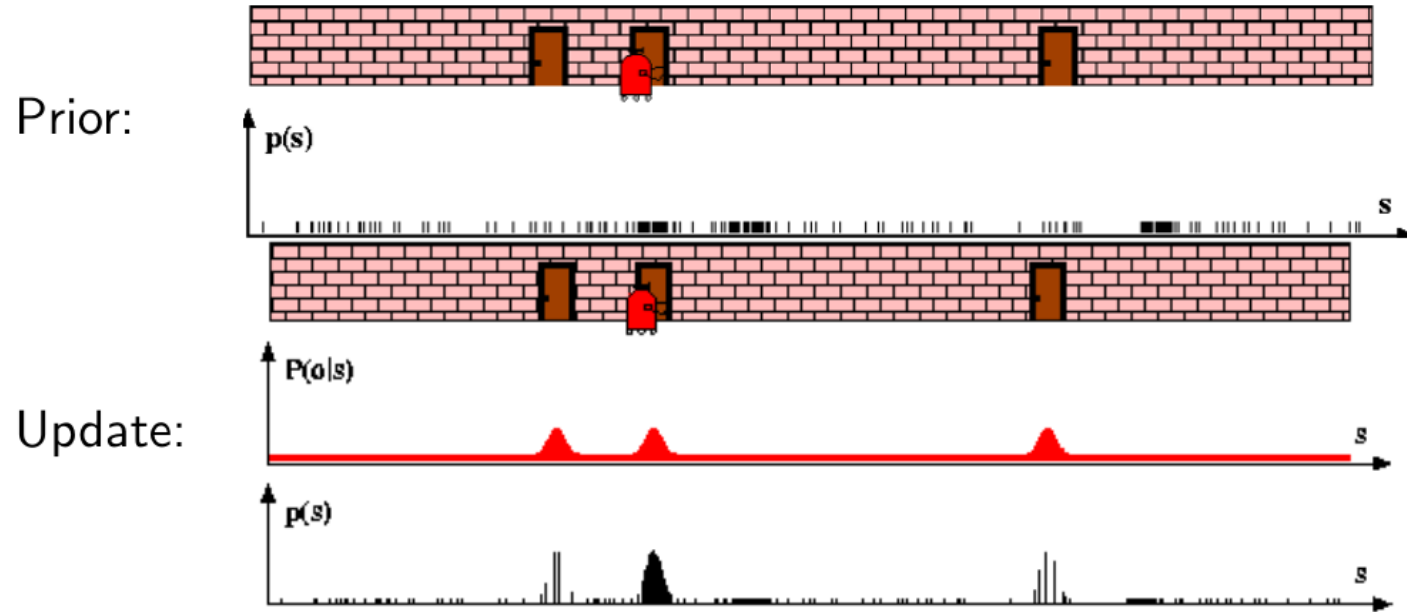
# Particle filter for 1D localization



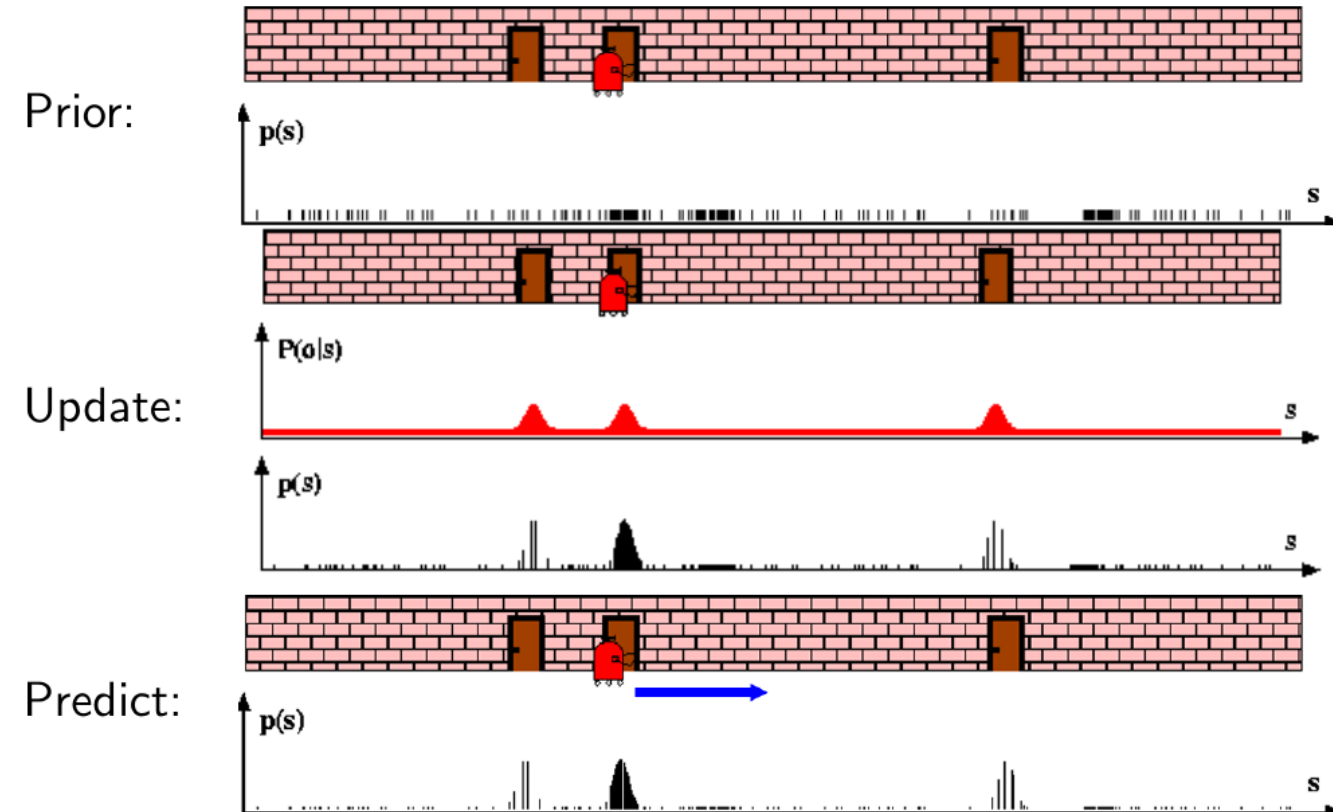
# Particle filter for 1D localization



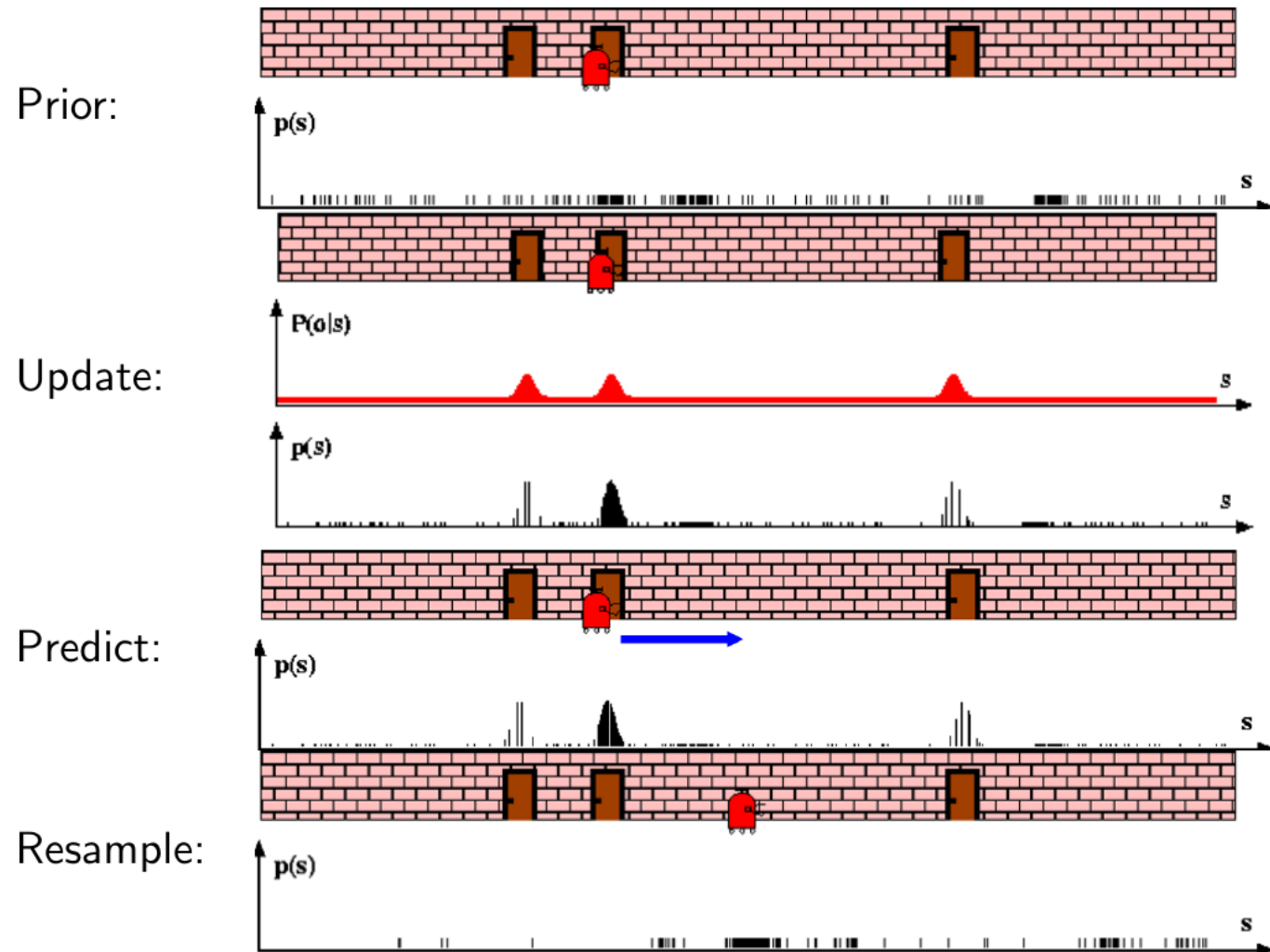
# Particle filter for 1D localization



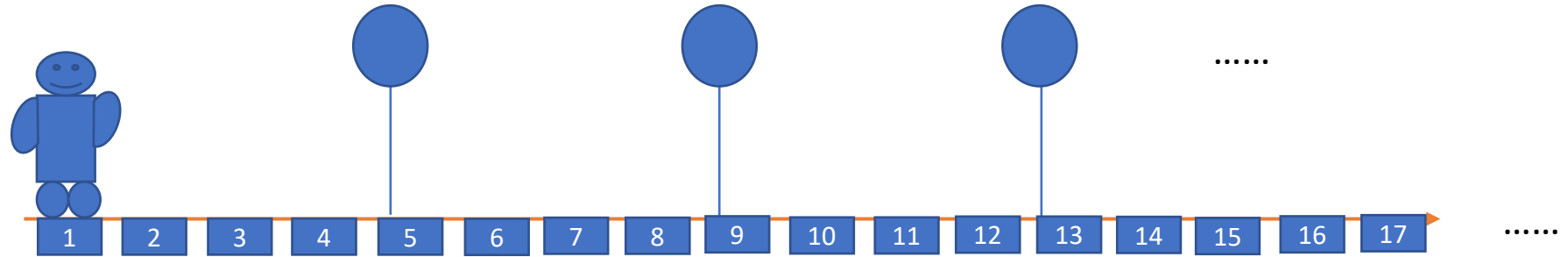
# Particle filter for 1D localization



# Particle filter for 1D localization



# Step by step implementing a particle filter



## Step 1: Generate particles based on our prior

- uniformly prior distributed (each spot has one particle)
- each particle moves following with robot moving, no uncertainties in their movements.
- at this step, we assume each particle holds their beliefs only to be true (belief=1) of false (belief=0)

# Step by step implementing a particle filter

## Step 2: Add uncertainties in particles' movements

- movement errors follow zero-mean Gaussian distribution with a pre-defined standard deviation
- create one particle, move 10 times, print and observe the measurements
- uncomment `# quit()` to see how distribution converges with more samples.



# Step by step implementing a particle filter

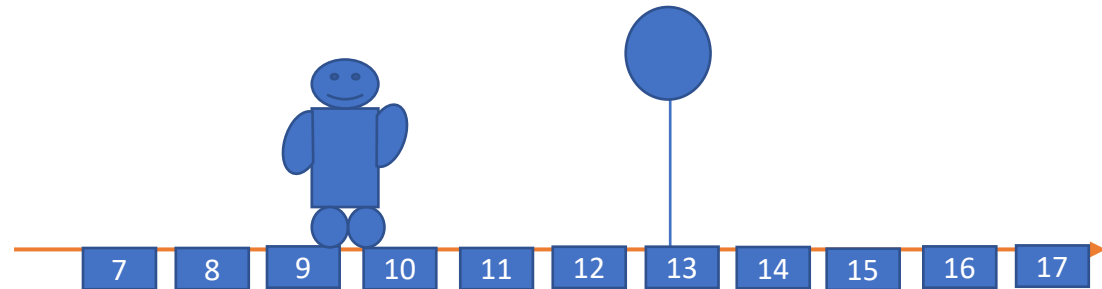
## Step 3: More realistic sensor model:

### Previous measurement

```
def detect_pole(self, poles):  
    if self.pos + 1 in poles:  
        self.pole_detected = True  
    else:  
        self.pole_detected = False
```

### More realistic measurement considering sensor specifications:

- Maximum measurement range: 3 units
- If object within the detection range, report the distance to the closest object
- If no object detected, output -1000



# Step by step implementing a particle filter

Your practice:

[complete the step-3.py](#), fill in the measure function so that

“Measurement should be XXX”

matches

“You Measured: XXX”

# Step by step implementing a particle filter

- Step 4: Update weights for each particle while taking into account measurement uncertainty in the sensor model.
  - For simplicity, we assume the sensor model: errors follow Gaussian distribution with a pre-defined standard deviation.
  - **Qs**: if robot got measurement 3.5 units to a pole, particle A is at the location distancing 2.0 units to the pole, particle B at the location distancing 3.5 units. Which particle should be assigned with higher weight? More generally, how do you assign the weights?
  - Complete the step-4.py: update weights by setting the particle.weight value based on the given Gaussian probability density function.

# Step by step implementing a particle filter

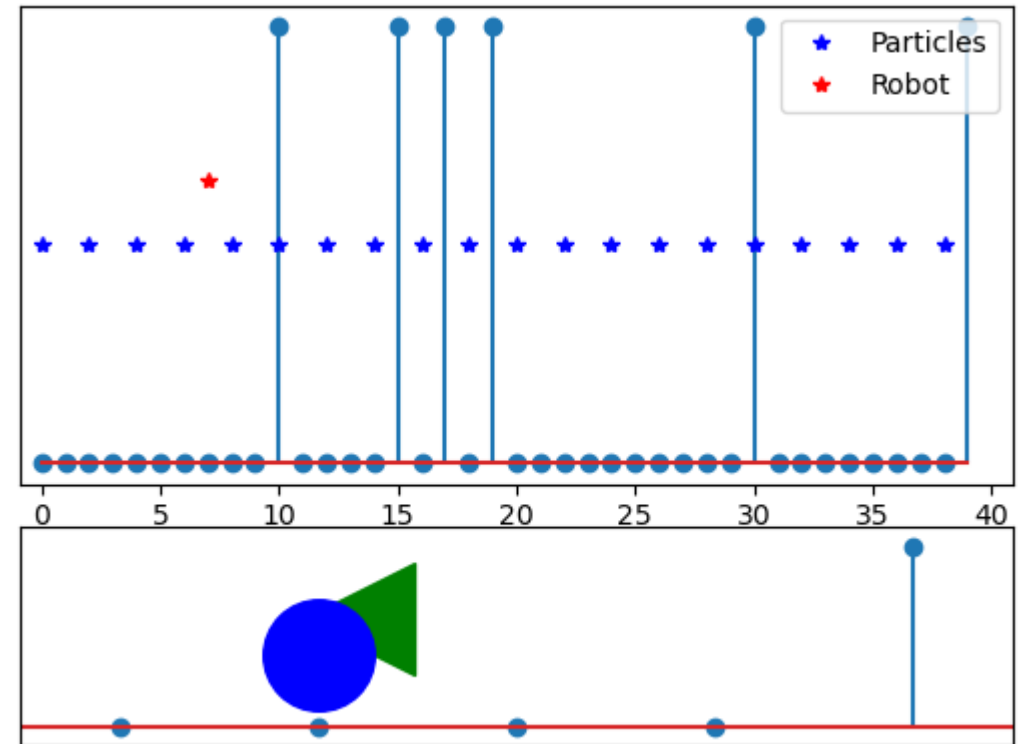
- So far, we've done move, measure and update weights for the particles. The last step is Resampling.
- **Step 5: Resample**
  - Complete the resample\_particles function
  - Change the particle number and weights, do experiment and Observe
    - e.g., change the number of particles to be 100, and weights to be either 1 and 0.05 (i.e., large number of particles, but most of them have really small likeliness)

# Step by step implementing a particle filter

- Step 6: Put them together

- You can reuse all the steps we implemented before
- Put them together to implement a complete particle filter for 1D localization

Any problems found?  
How do you fix it?



# Extend to 2D case

- Define the robot location representation and movement
- Add uncertainties to movement and measurement
- Generate the particle's and define the sampling motion function
- Define the weight update for the particles
- Define the resampling function

# Extend to 2D case—Not mandatory

- ~~• Define the robot location representation and movement~~
- ~~• Add uncertainties to movement and measurement~~
- I will give you files including the world definition and sensor readings, where the odometry motion model and a range-only sensor are used.

You only need to do the following:

- Generate the particle's and define the sampling motion function
- Define the weight update for the particles
- Define the resampling function

A code skeleton with the particle filter framework is provided for you.



# Extend to 2D case—Not mandatory

- **data** -This folder contains files representing the world definition and sensor readings used by the filter.
- **code** -This folder contains the particle filter framework.
- Run the simulation- in the terminal: `python particle_filter_st.py`. It will only work properly once you filled in the blanks in the file. The blanks include
  - **sample\_motion function**: **implementing the odometry motion model and sampling from it**, then return the new set of parameters after the motion update. The function samples new particle positions based on the old positions, the odometry measurements and the motion noise. The motion noise are
$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0.1, 0.1, 0.05, 0.05]$$
  - **weight\_update function**: using a **range-only sensor**, it takes the input including the landmarks positions and landmark observations (range), and returns a list of weights for the particle set. The standard deviation of the Gaussian zero-mean measurement noise is  $\sigma=0.2$ .
  - **resampling function**: same as the resampling function in 1D case.

# Extend to 2D case—Not mandatory

- Tips: To read in the sensor and landmark data, we have used dictionaries.

To access the sensor data from the `sensor_readings` dictionary, you can use

```
sensor_readings[timestamp, 'sensor']['id']  
sensor_readings[timestamp, 'sensor']['range']  
sensor_readings[timestamp, 'sensor']['bearing']
```

and for odometry you can access the dictionary as

```
sensor_readings[timestamp, 'odometry']['r1']  
sensor_readings[timestamp, 'odometry']['t']  
sensor_readings[timestamp, 'odometry']['r2']
```

To access the positions of the landmarks from `landmarks` dictionary , you can use

```
position_x = landmarks[id][0]  
position_y = landmarks[id][1]
```