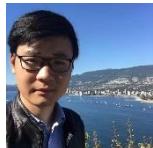




# Motion Planning for Mobile Robots



主讲人 Fei Gao

Ph.D. in Robotics  
Hong Kong University of Science and Technology  
Assistant Professor, Zhejiang University





# Outline



1. Introduction



2. Course Outline



3. Typical Planning Methods Overview



4. Map Representation



5. Pre-requirement



6. Homework



# Introduction



# About this Course

- **This course is about:**
  - Academism (old school) planning pipeline
  - Path finding algorithm
  - Trajectory generation/optimization
  - Real-time software implementation
- **This course is NOT about:**
  - Dynamics Modelling ☹
  - Robot design ☹
  - End-to-end navigation ☹
  - Learning-based method ☹
  - Mathematical/theoretical proof ☹
- **This course may also cover:**
  - Autonomy for mobile robots
  - Paper recommendations
  - Cutting-edge research direction/ethics



# Basic Expectation

- **Discipline:**

- Every researcher has his own taste/style. Since you choose this course, please follow my style.
- Q&A only at appointed office time.
- Finish your homework by yourself.

- **Project:**

- Basic algorithm validation (MATLAB)
- Sophisticated engineering implementation (ROS/C++)



# Teaching Team

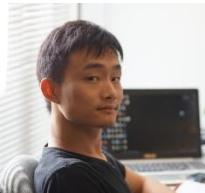


Instructor 1: **Fei Gao**

Assistant Professor, ZJU

Ph.D, HKUST

Email: fgaooa@connect.ust.hk



Instructor 2: **Chaoqun Wang**

Research Fellow, CUHK

Ph.D, CUHK

Email: zychaoqun@gmail.com



Instructor 3: **Shupeng Lai**

Research Fellow, NUS

Ph.D, NUS

Email: shupenglai@gmail.com



Instructor 4: **Delong Zhu**

Ph.D candidate, CUHK

Email: dlzhu@ee.cuhk.edu.hk



# Information

- **Lecture time:**
  - Every Friday night 19:00, except October 4th
- **Course Community:**
  - [www.shenlanxueyuan.com/course/188/threads/show](http://www.shenlanxueyuan.com/course/188/threads/show)
- **Course Website:**
  - [www.shenlanxueyuan.com/course/188](http://www.shenlanxueyuan.com/course/188)
- **Course Wechat Group:**
  - Connect the class teacher(Wechat: shenlancas)



# Workload

- Expected Student Background:
  - Linear algebra
  - Probability
  - MATLAB programming skills
  - C++ programming skills (VERY IMPORTANT)
  - Linux
  - Love robots ☺
- Workload:
  - Attend lectures
  - Lots of project work
  - Have fun with robots ☺

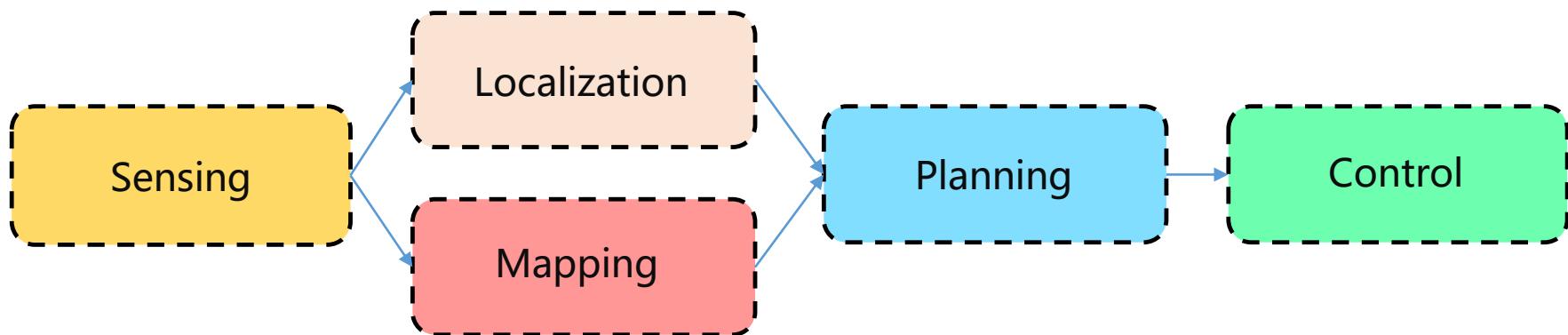


# What is autonomous robot

Definition: an autonomous robot is a robot that performs behaviors or tasks with a high degree of autonomy (without external influence).

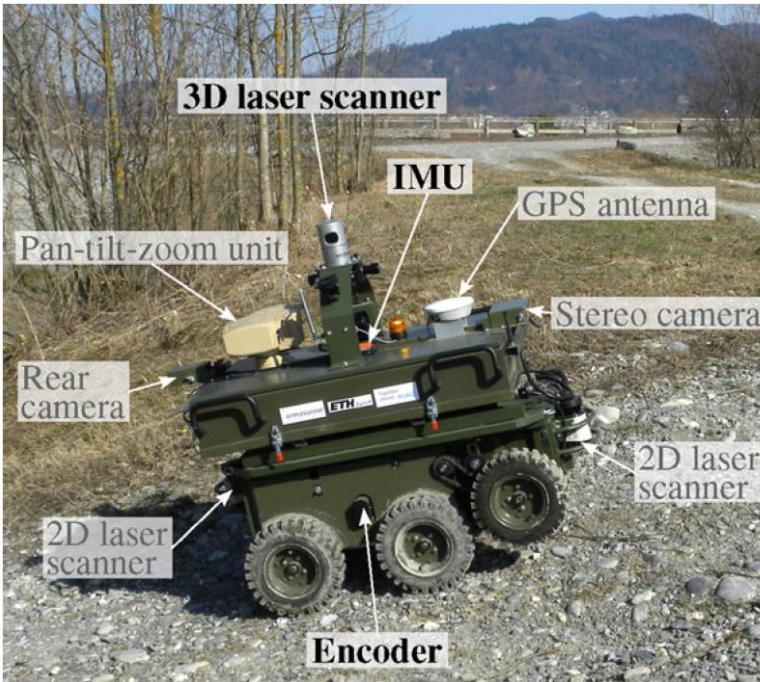
Subjects: Computer Science, Automation, Mechanism, Electronics ...

- Perception-Planning-Control action loop





# Autonomous robot: applications

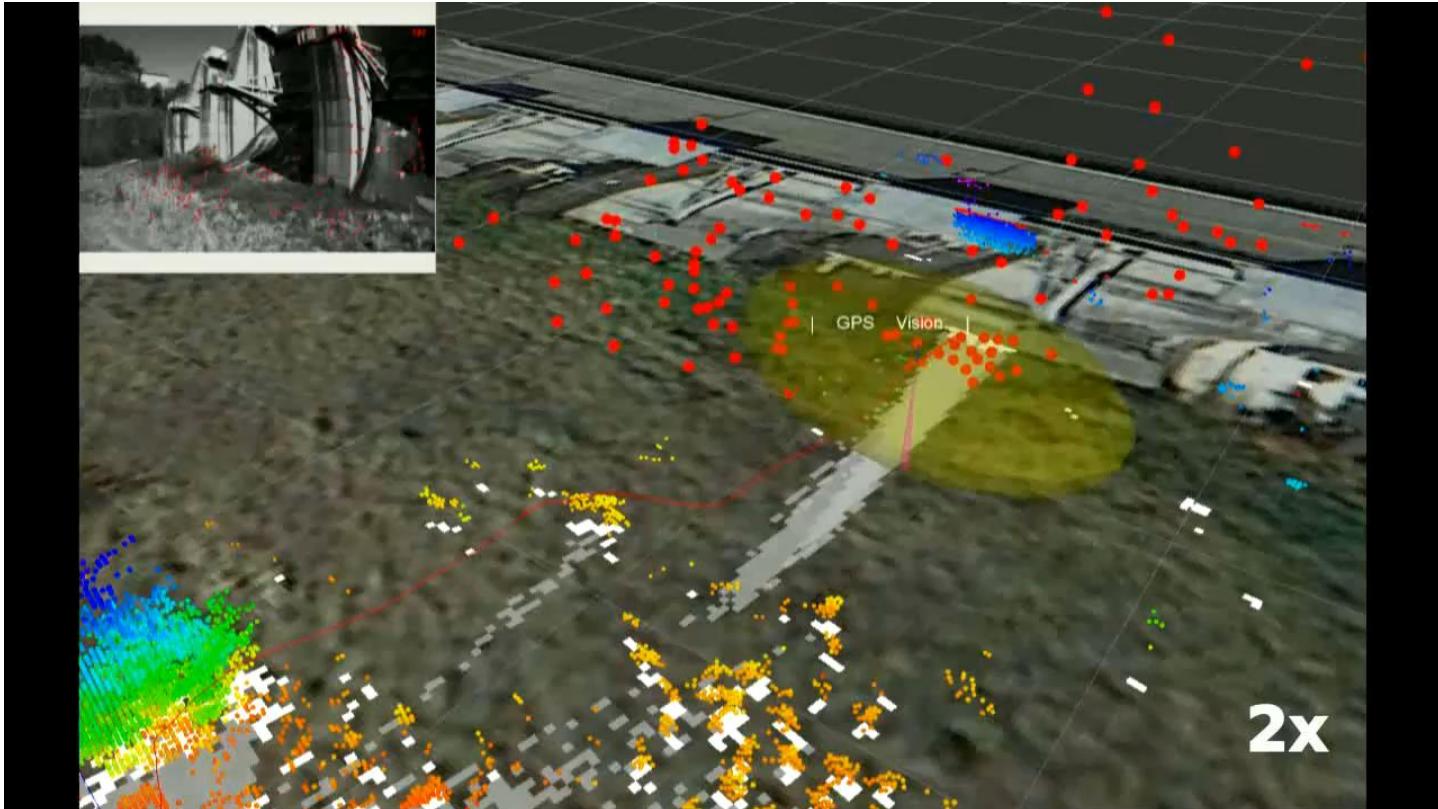




# Aerial Robots



# Dam Inspection



Shen, et al, 2014



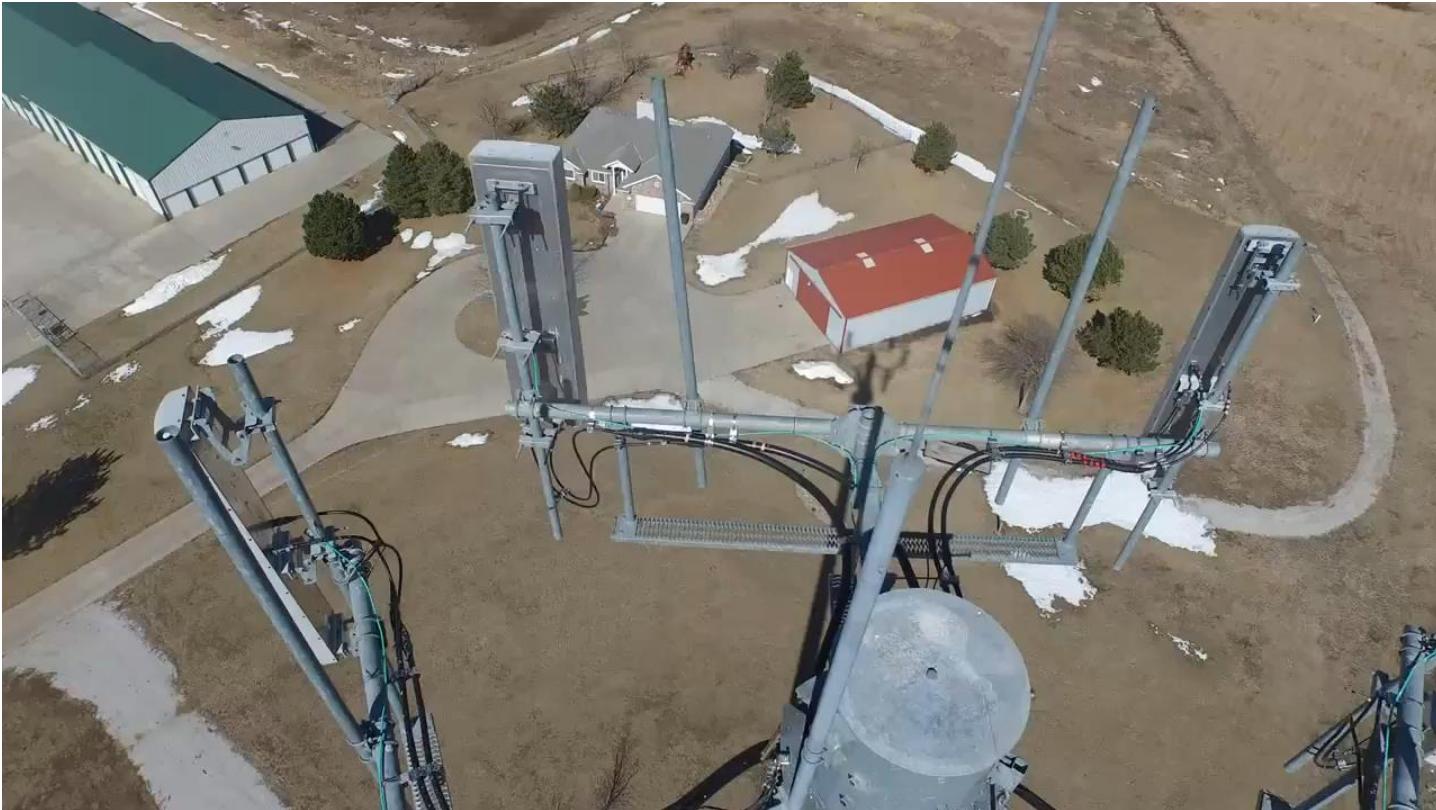
# Dam Inspection



Ozaslan, et al, 2014



# Cellular Tower Inspection





# Precision Farming

**2X**



# Delivery





# Ground Robots



# Search and Rescue





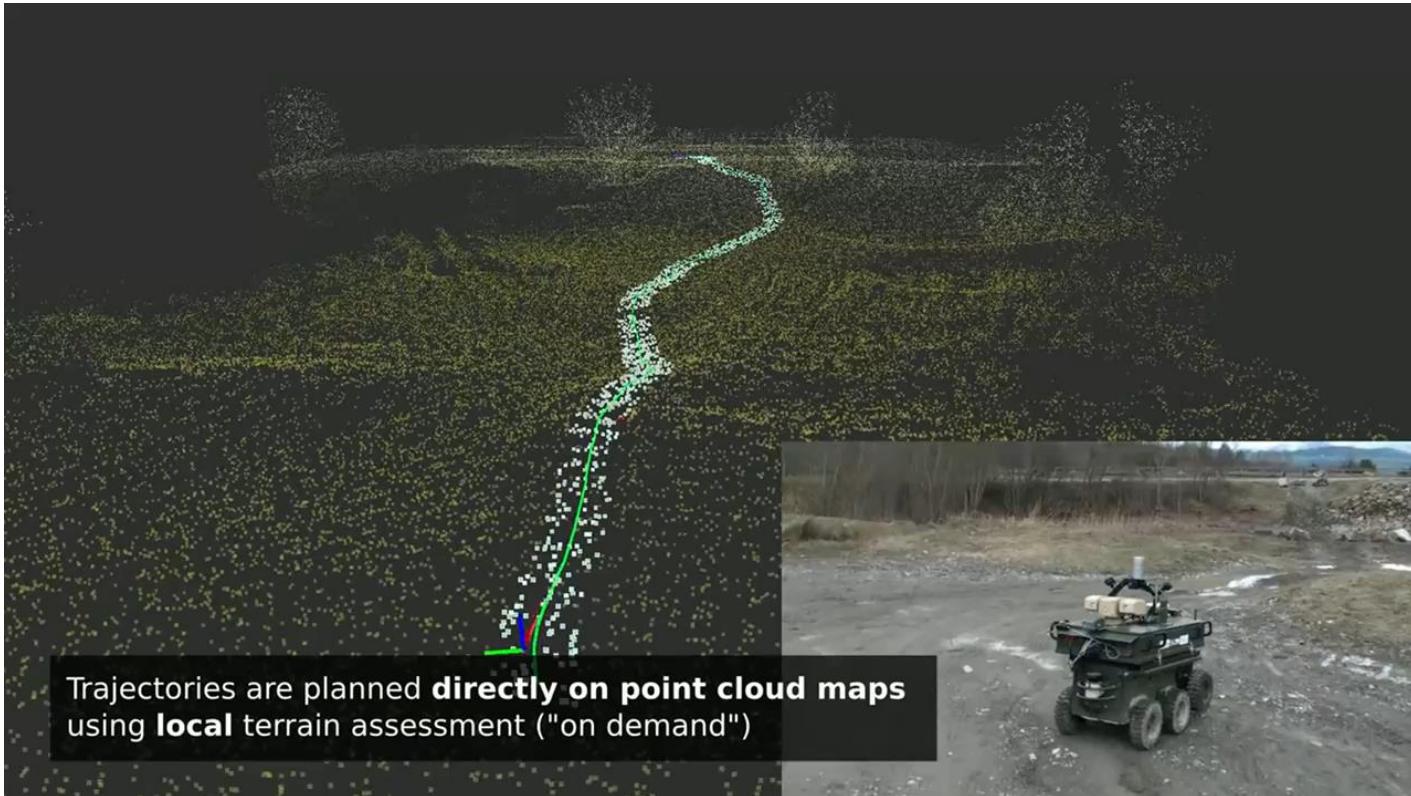
# Housekeeping



**iRobot®**

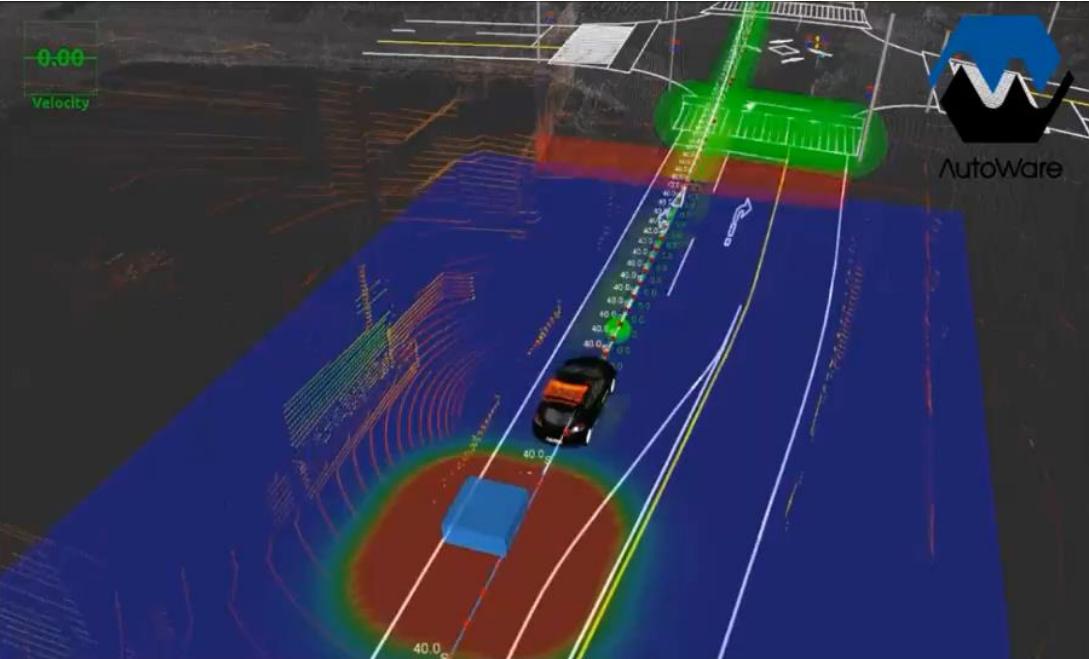


# Hills Driving





# Urban Driving



**Tier IV**  
Intelligent Vehicle

アイサンテクノロジー株式会社 **AXE**



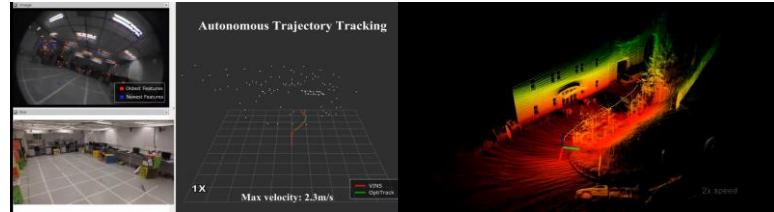
# Crowd Driving



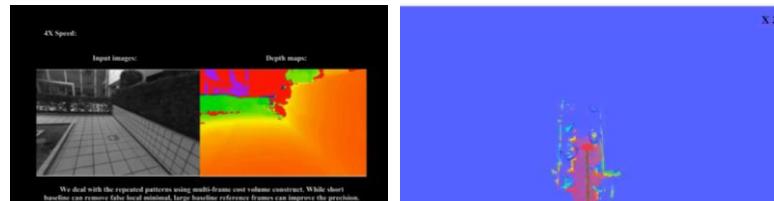


# What is autonomous robot

- Estimation
  - Low latency
  - High accuracy & consistency



- Perception
  - 3D sensing & dense perception
  - Map fusion & integration for planning



- Planning
  - Complex & unknown environments
  - Safety & dynamical feasibility
  - Limited sensing & computation



- Control
  - Aggressive maneuvers
  - Smooth trajectory tracking





# What is motion planning

- Basic requirements
  - Safety: collision avoidance
  - Smoothness: energy saving, comfort
  - Kinodynamic feasibility: executable, controllable
- Old-school pipeline
  - Front-end path finding
    - Search for an initial safe path
    - Low dimensional
    - Discrete space
  - Back-end trajectory generation
    - Search for an executable trajectory
    - High dimensional
    - Continuous space



# How to do robotics research

- **Find a problem**

- ✓ A problem only in your imagination is not a problem at all.
- ✓ A robotician must be an engineer first.
- ✓ Hot topic is just hot.
- ✓ Be honest.

- **Solve a problem**

- ✓ Don't intentionally making things complicated.
- ✓ Simple but effective solution is always preferable.
- ✓ Simulation tells nothing, show everyone real robots.
- ✓ Solve real problems.
- ✓ Solve it 100%, or not.



# How to do motion planning

- Overall knowledge of planning
  - ✓ Choose suitable methods for different scenarios.
  - ✓ Design customized strategy.
- Dirty hands
  - ✓ Don't wait, don't just read papers. Do it yourself.
  - ✓ A lot of coding work.
  - ✓ A lot of field work.
- Know the whole system well
  - ✓ Take care every component in your robot.



# Groups and Researchers

## University of Pennsylvania

- GRASP Lab,Vijay Kumar  
Research Interests: planning, control, swarm  
Homepage: [www.kumarrobotics.org](http://www.kumarrobotics.org)

## Massachusetts Institute of Technology

- Jonathan How  
Research Interests: modelling, control, planning  
Homepage: [www.mit.edu/~jhow](http://www.mit.edu/~jhow)
- Nicholas Roy  
Research Interests: perception, learning  
Homepage: [groups.csail.mit.edu/rrg](http://groups.csail.mit.edu/rrg)

## Carnegie Mellon University

- Nathan Michael  
Homepage: [www.rislab.org](http://www.rislab.org)
- Sebastian Scherer  
Research Interests: perception, planning  
Homepage: [theairlab.org](http://theairlab.org)

## University of California, Berkeley

- Markus Mueller  
Research Interests: control, planning

## ETH Zurich

- ASL Team, Roland Siegwart  
Research Interests: perception, control  
Homepage: [asl.ethz.ch](http://asl.ethz.ch)
- Raffaello D'Andrea  
Research Interests : control, swarm  
Homepage: [raffaello.name](http://raffaello.name)

## University of Zurich

- Davide Scaramuzza  
Research Interests : perception, control  
Homepage: [rpg.ifi.uzh.ch](http://rpg.ifi.uzh.ch)

## Hong Kong University of Science Technology

- Shaojie Shen  
Research Interests : UAV  
Homepage: [uav.ust.hk](http://uav.ust.hk)
- Ming Liu  
Research Interests : UGV  
Homepage: [ram-lab.com](http://ram-lab.com)



# Young researchers who most related to me



## Helen Oleynikova

[helenol.github.io](https://helenol.github.io)

Graduate at Jan. 2019, from ASL team, ETH

Supervised by Roland Siegwart

Author of **VoxBlox**

Planning and mapping



## Sikang Liu

[github.com/sikang](https://github.com/sikang)

Graduate at Aug. 2018, from GRASP Lab, UPenn

Supervised by Vijay Kumar

Planning



# Course Outline



# Front-end: Path finding

- SEARCH-BASED PATH FINDING

- Graph Search Basis
- Dijkstra and A\*
- Jump Point Search
- Homework

- SAMPLING-BASED PATH FINDING

- Probabilistic Road Map
- Rapidly-exploring Random Tree (RRT)
- Optimal Sampling-based Methods
- Advanced Sampling-based Methods
- Homework

- KINODYNAMIC PATH FINDING

- Introduction
- State-state Boundary Value Optimal Control Problem
- State Lattice Search
- Kinodynamic RRT\*
- Hybrid A\*
- Homework



# Back-end: Trajectory Generation

- MINIMUM SNAP TRAJECTORY GENERATION
  - Differential Flatness
  - Minimum Snap Optimization
  - Closed-form Solution to Minimum Snap
  - Time Allocation
  - Implementation in Practice
  - Homework
- SOFT AND HARD CONSTRAINED TRAJECTORY OPTIMIZATION
  - Soft Constrained Trajectory Optimization
  - Hard Constrained Trajectory Optimization
  - Homework



# MDP & MPC

- MARKOV DECISION PROCESS-BASED PLANNING
  - Uncertainties in Planning and MDP
  - Minimax Cost Planning and Expected Cost Minimal Planning
  - Value Iteration and Real-Time Dynamic Programming
  - Homework
- MODEL PREDICTIVE CONTROL FOR ROBOTICS PLANNING
  - Introduction
  - Linear MPC
  - Non-linear MPC
  - Homework

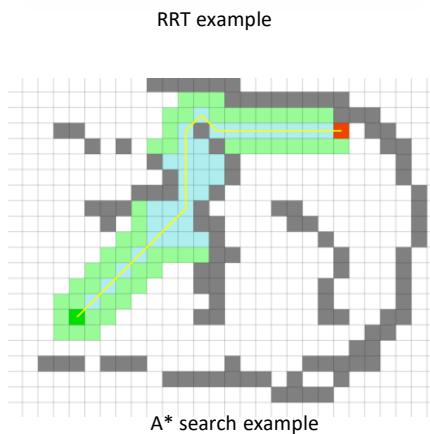


# Overview



# Front-end: Path Finding

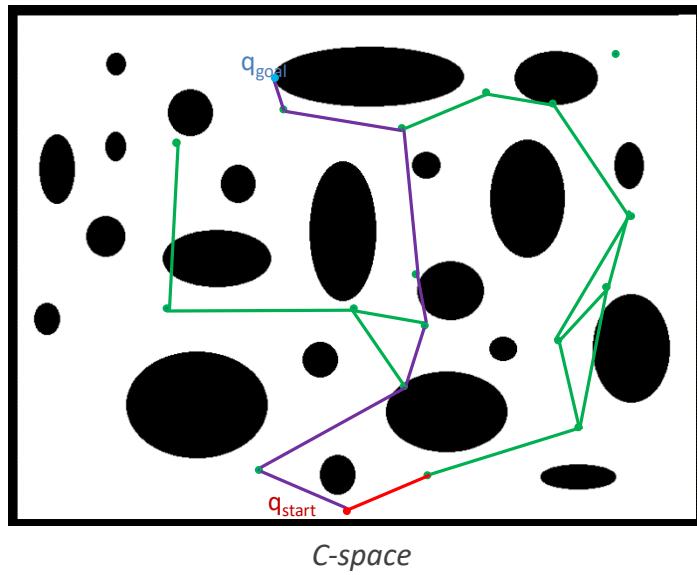
- Sampling-based methods
  - Probabilistic roadmap (PRM)
  - Rapidly exploring random tree (RRT)
  - RRT\*, informed RRT\*
- Search-based methods
  - General graph search: DFS, BFS
  - Dijkstra and A\* search
  - Jump point search





# Sampling-based Method

Probabilistic Roadmap (PRM)

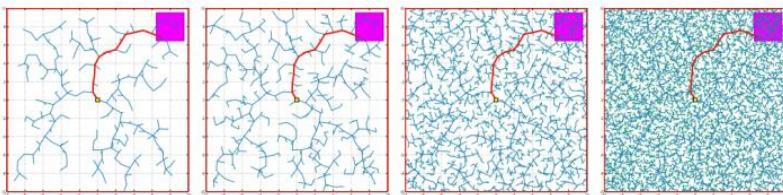




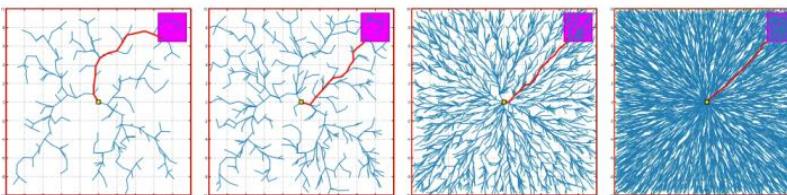
# Sampling-based Method

RRT\* vs RRT

RRT

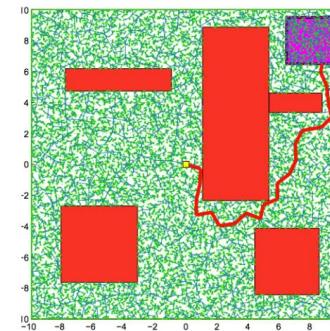


RRT\*

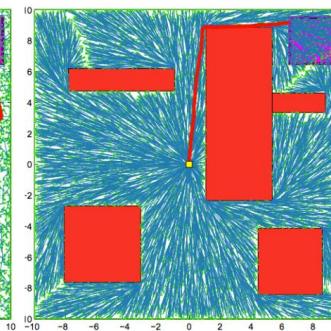


Source: Karaman and Frazzoli

RRT



RRT\*

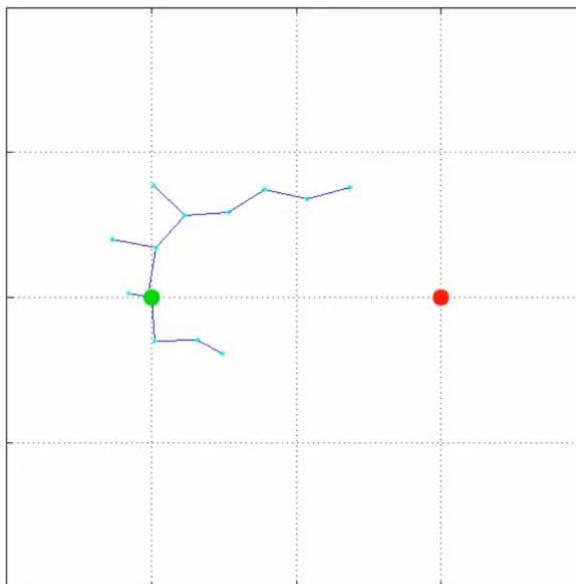




# Sampling-based Method

Informed RRT\*

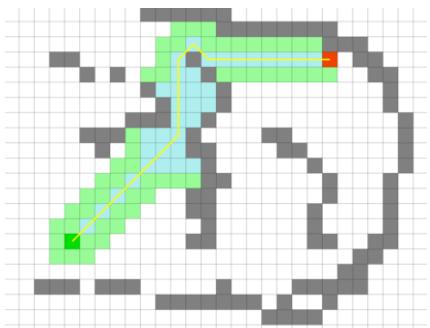
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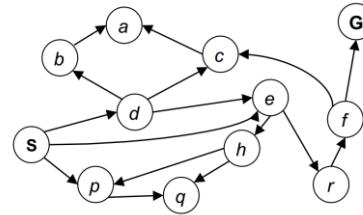


# Search-based Method

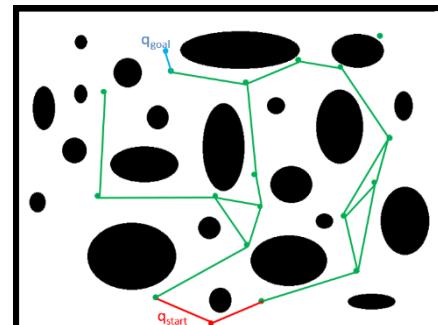
- For every search problem, there's a corresponding state space graph
- Connectivity between nodes in the graph is represented by (directed or undirected) edges



Grid-based graph: use grid as vertices and grid connections as edges



*Ridiculously tiny search graph  
for a tiny search problem*



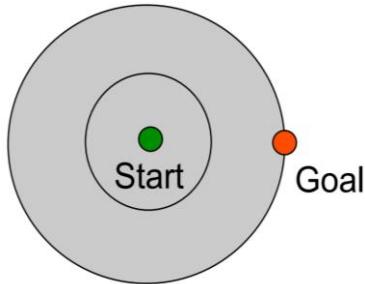
The graph generated by probabilistic roadmap (PRM)



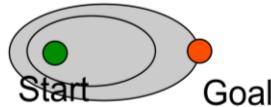
# Search-based Method

## Dijkstra's vs. A\*

- Dijkstra's algorithm expanded in all directions



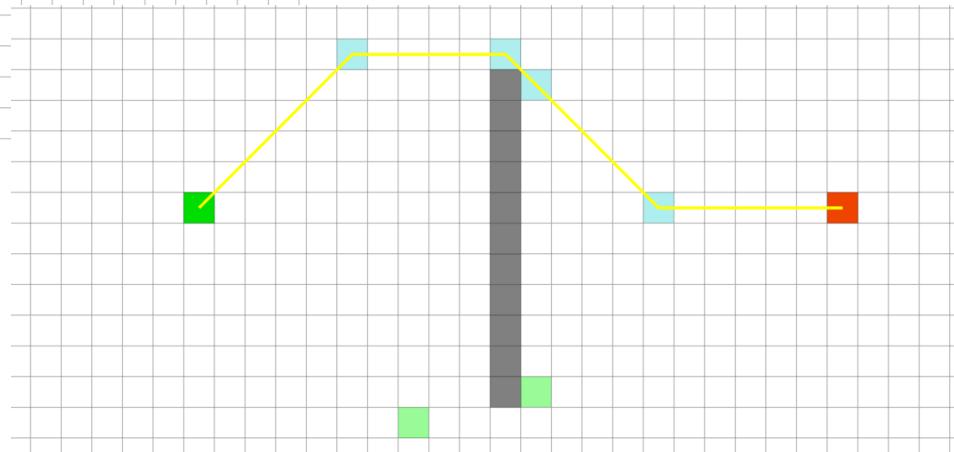
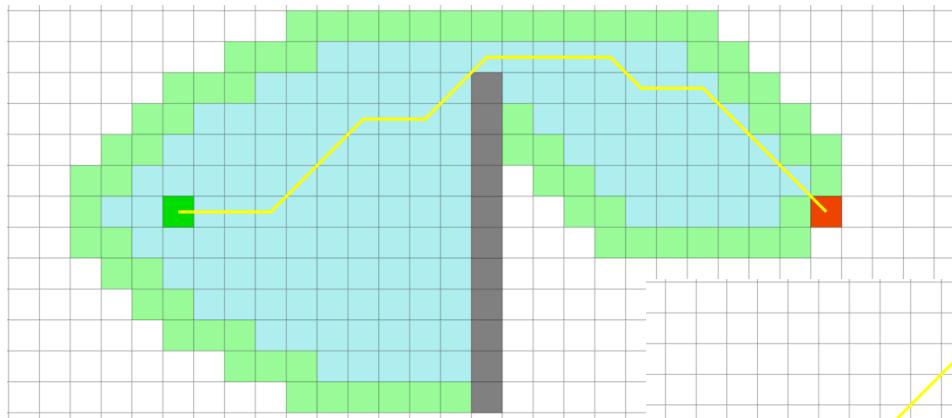
- A\* expands mainly towards the goal, but does not hedge its bets to ensure optimality





# Search-based Method

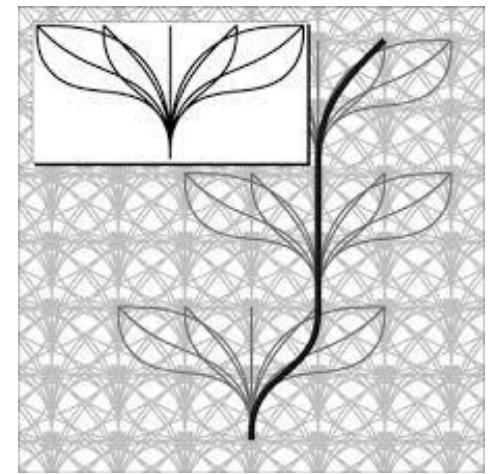
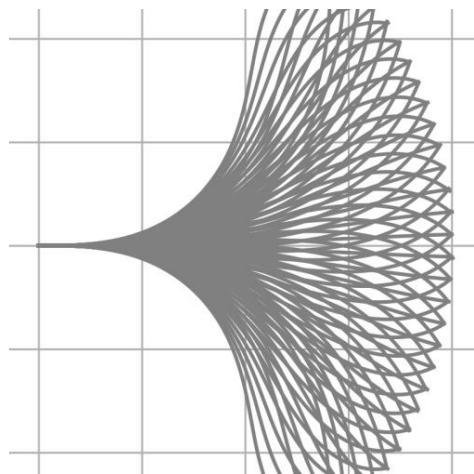
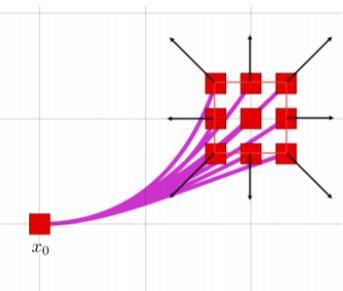
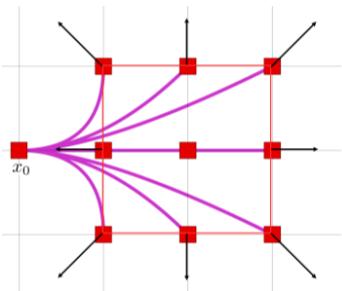
A\* vs. JPS





# Kinodynamic Path Finding

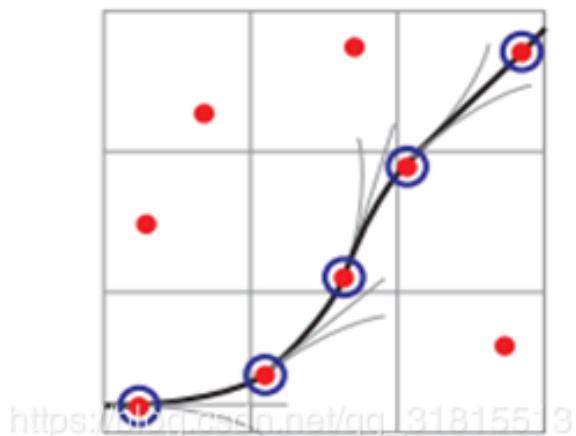
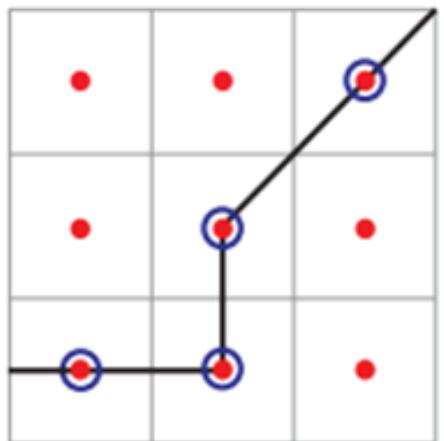
State Lattice Search





# Kinodynamic Path Finding

## Hybrid A\*



<https://blog.csail.mit.edu/cjq/318155/> 3

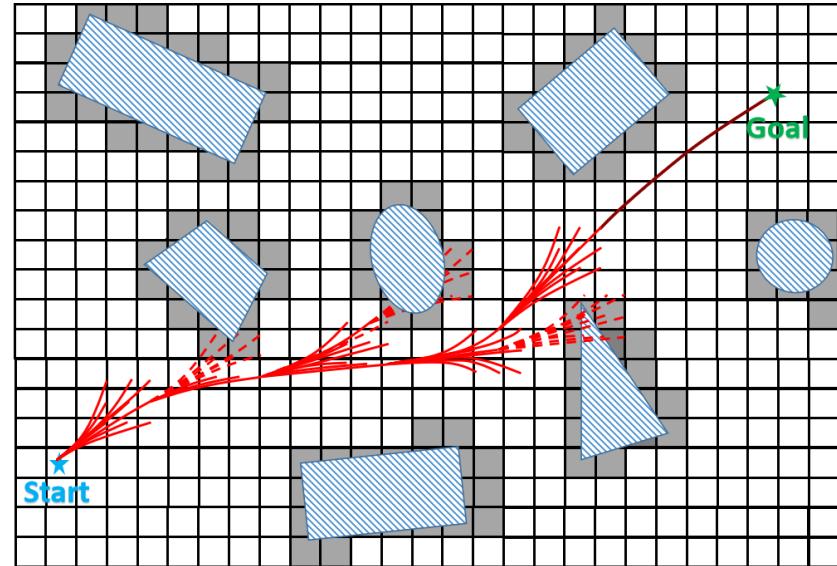
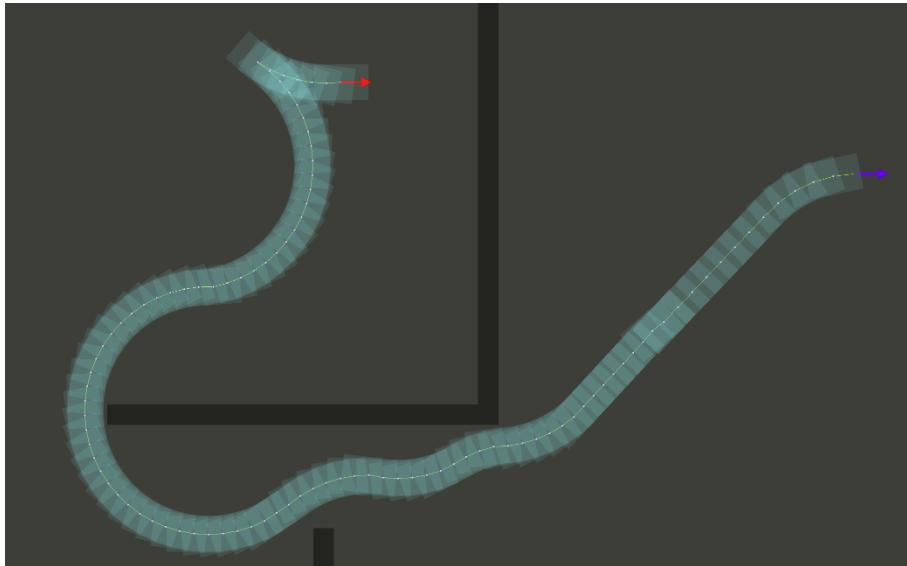
1. Follow A\* algorithm
2. Forward simulate states with different discrete control inputs
3. Keep only 1 state in each grid

Discrete control



# Kinodynamic Path Finding

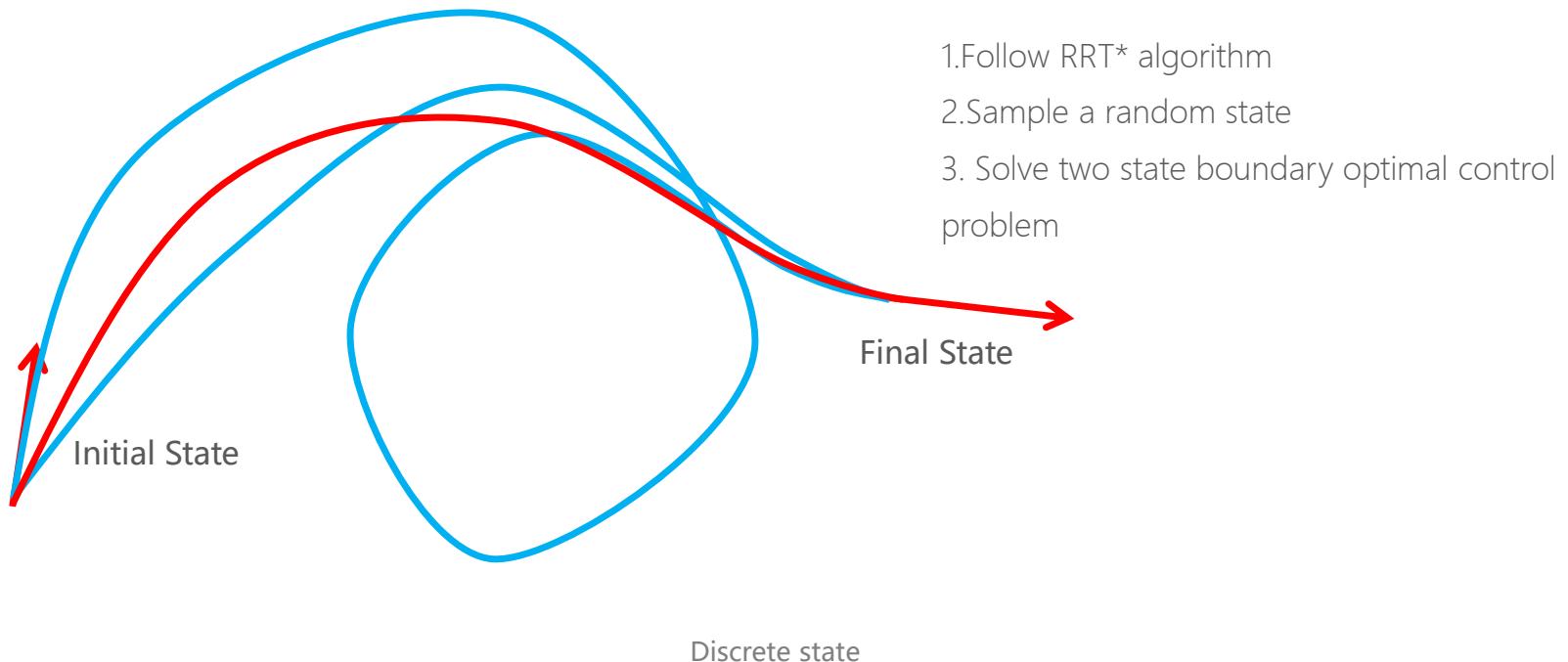
Hybrid A\*





# Kinodynamic Path Finding

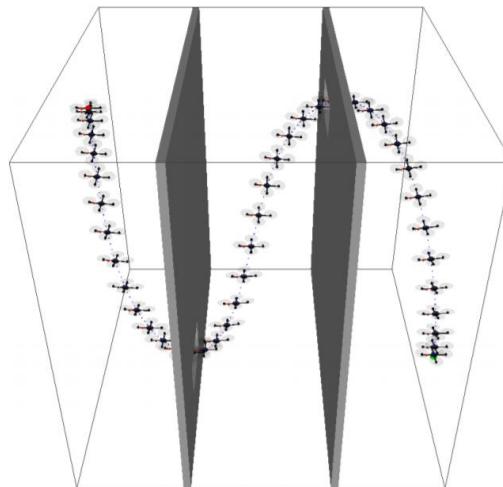
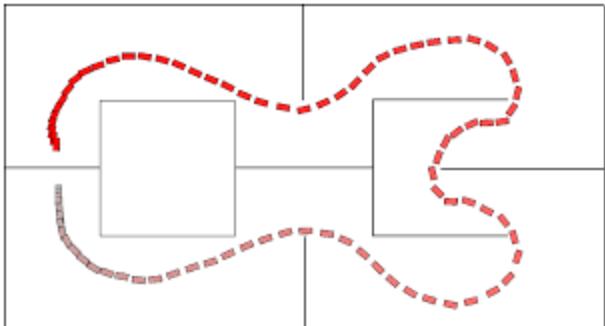
## Kinodynamic RRT\*





# Kinodynamic Path Finding

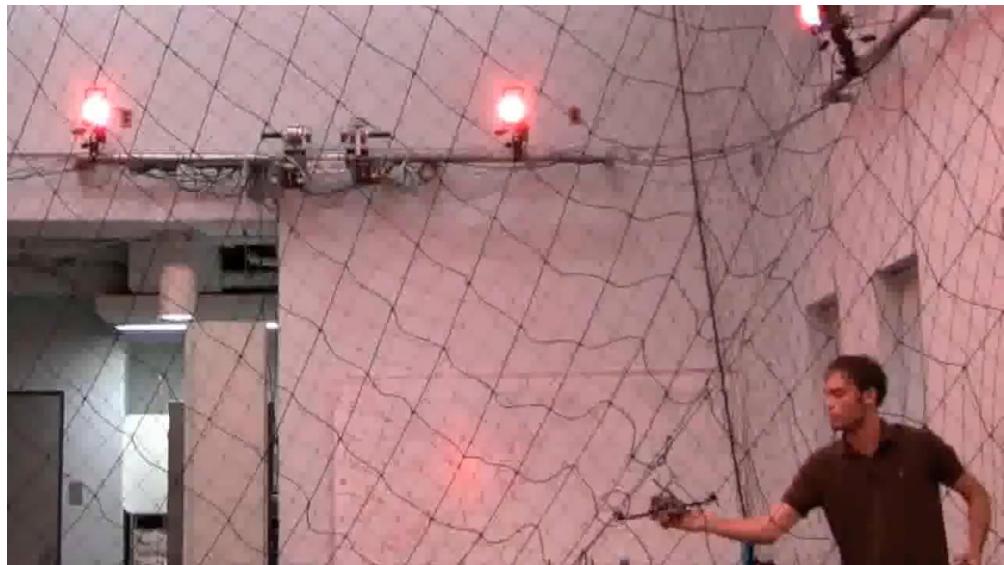
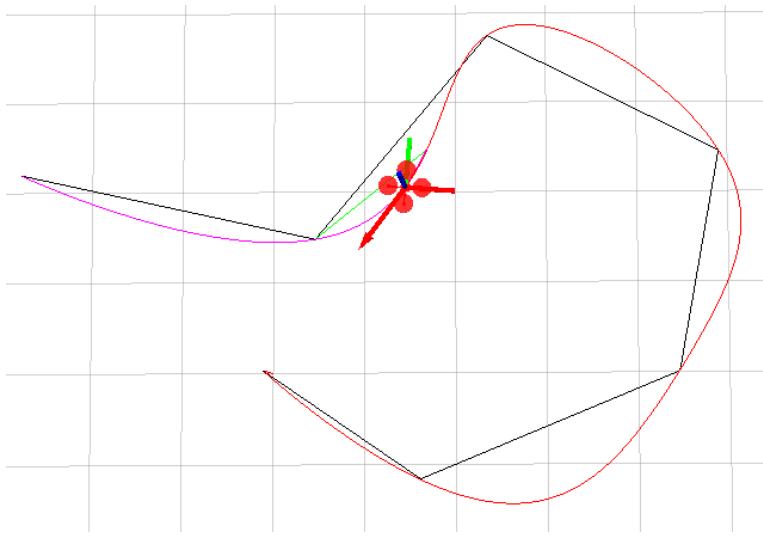
Kinodynamic RRT\*





# Back-end: Trajectory Optimization

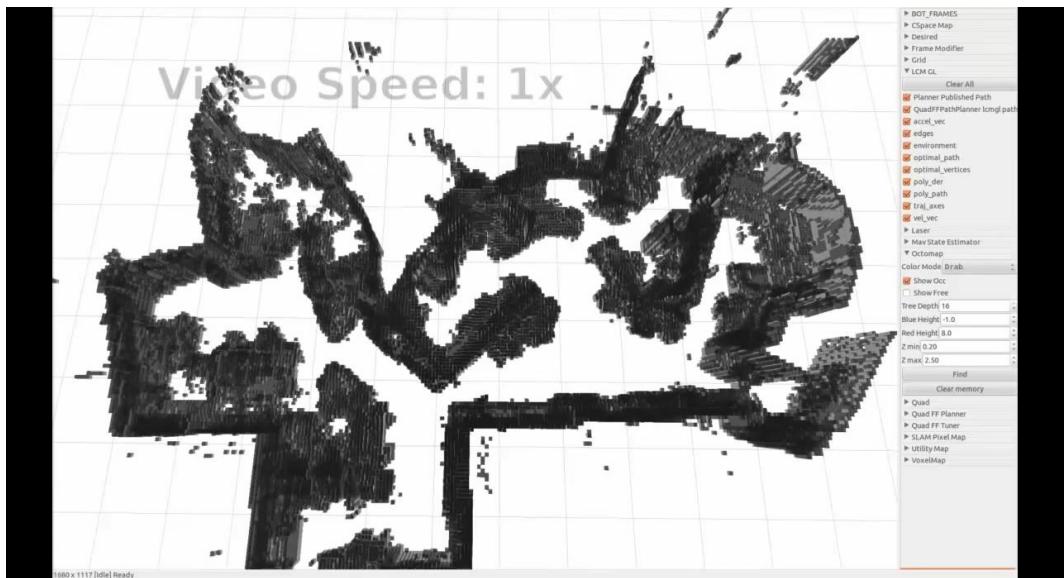
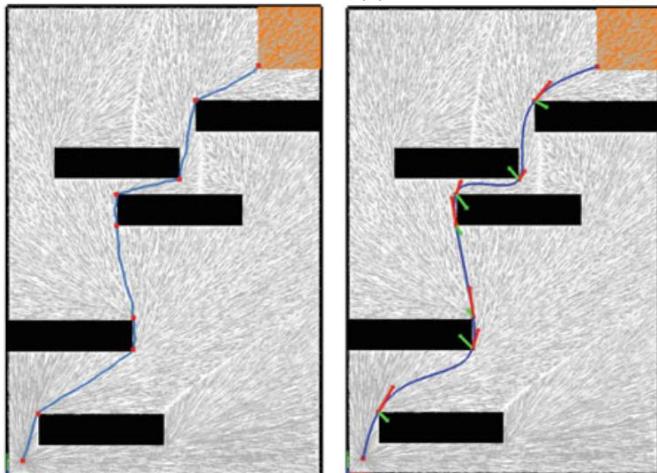
Basic Minimum-snap





# Back-end: Trajectory Optimization

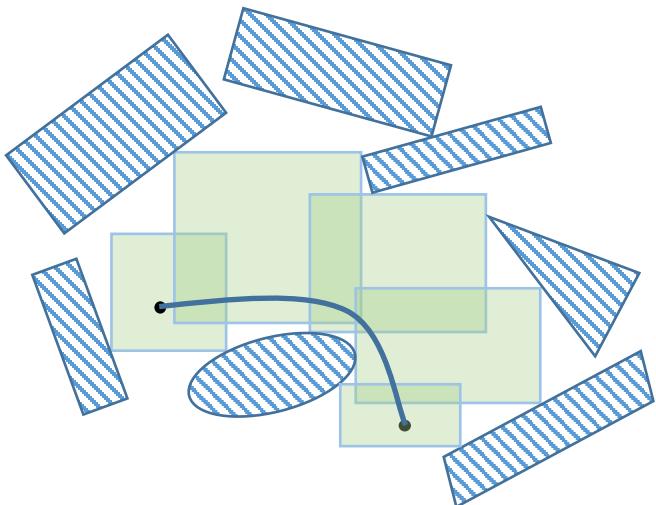
Basic Minimum-snap



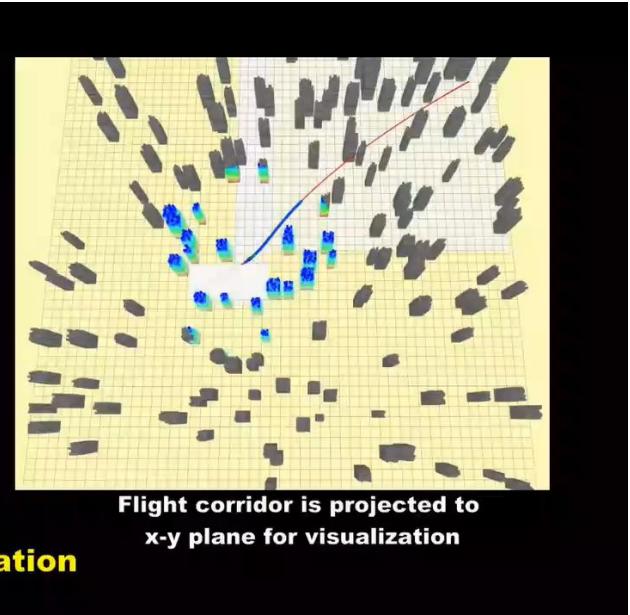


# Back-end: Trajectory Optimization

Hard constrained Minimum-snap



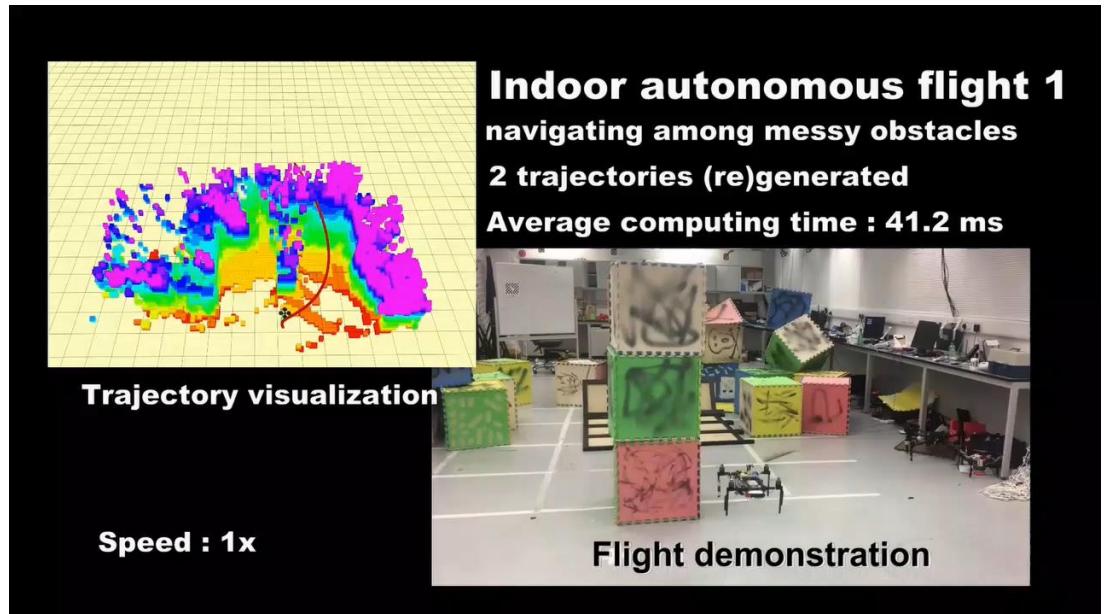
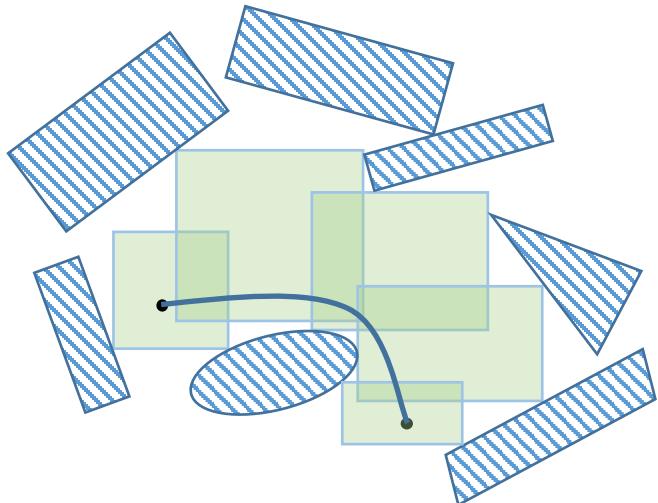
**Blue curve : trajectory  
in execution horizon**  
**Red curve :  
current trajectory**  
**White cube :  
flight corridor**  
**Colorful voxels :  
mapped obstacles**  
**Grey voxels :  
un-mapped obstacles**  
**Green arrow : velocity**  
**Yellow arrow : acceleration**





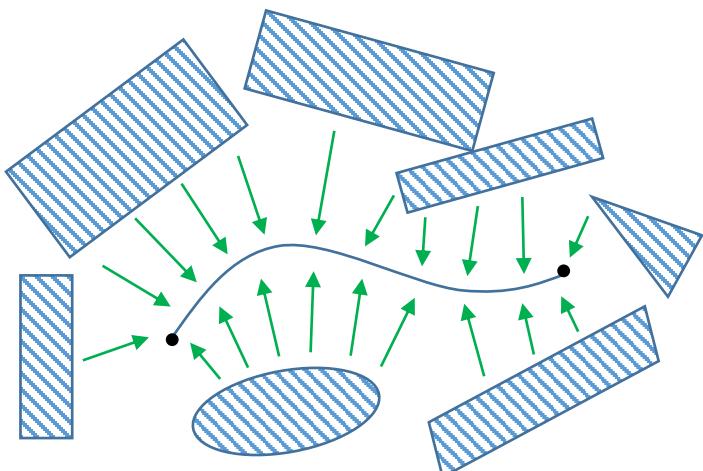
# Back-end: Trajectory Optimization

Hard constrained Minimum-snap

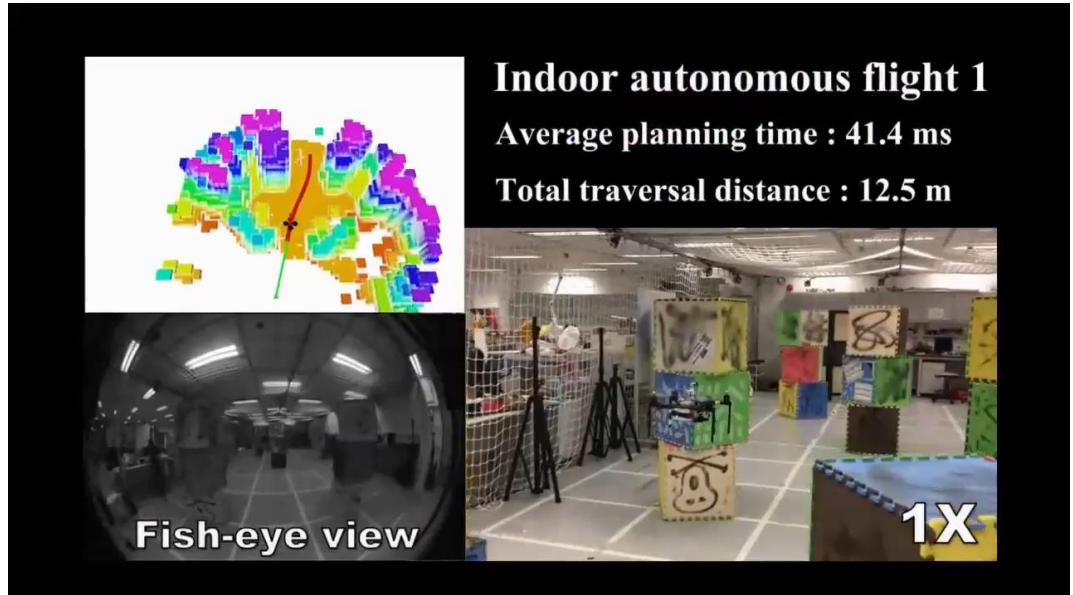




# Back-end: Trajectory Optimization

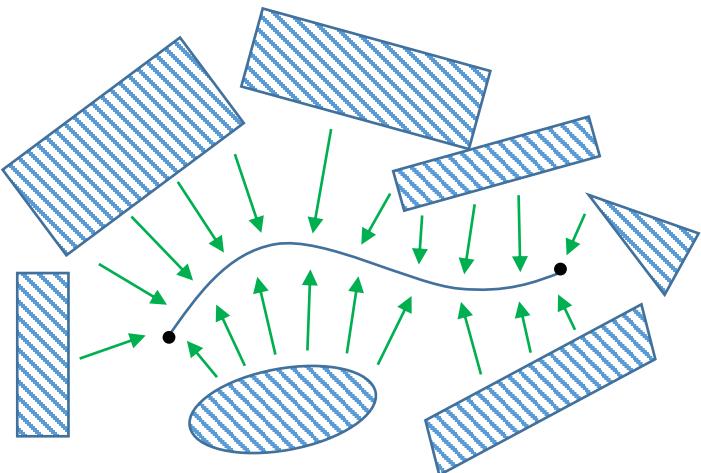


Soft constrained Minimum-snap

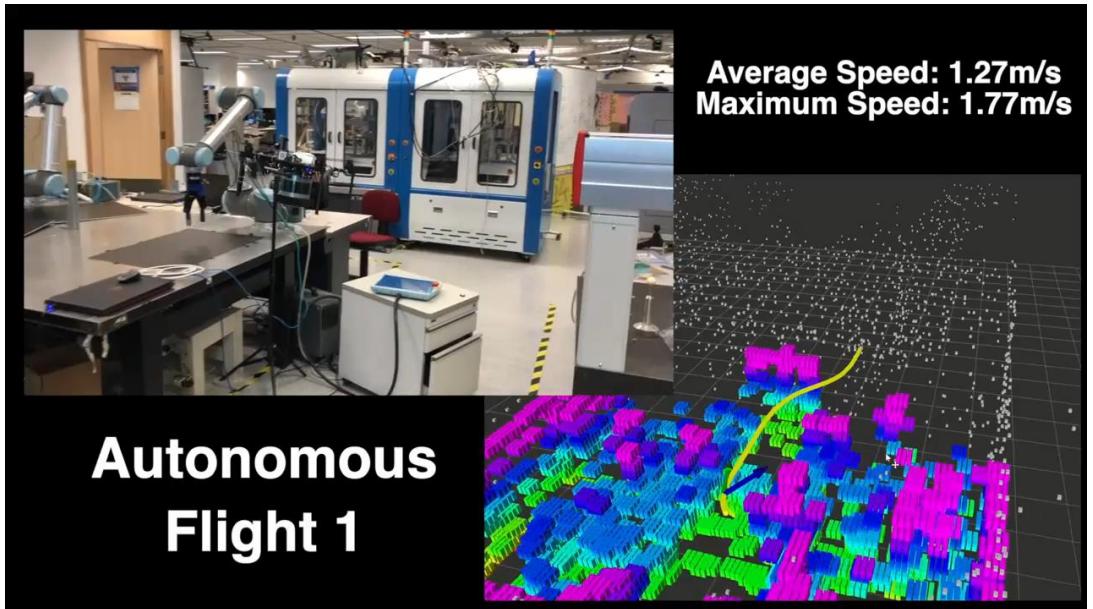




# Back-end: Trajectory Optimization

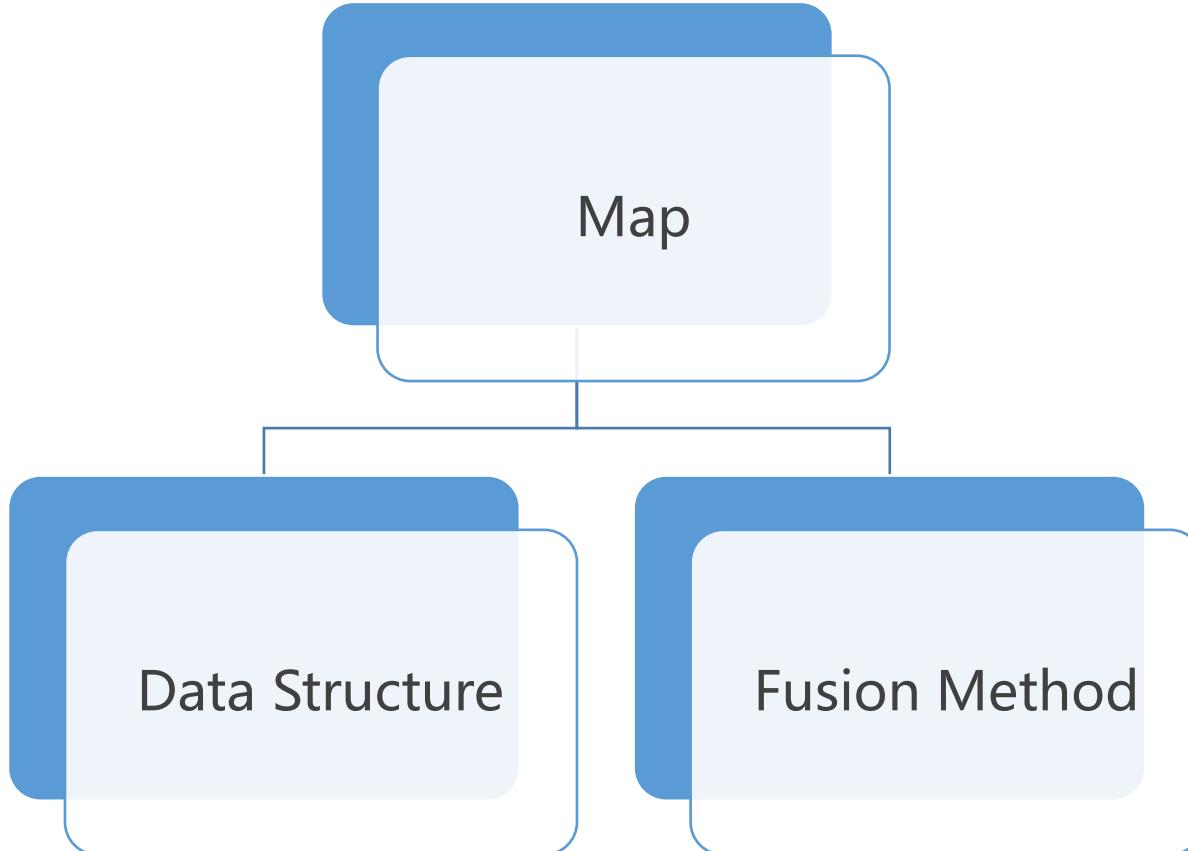


Soft constrained Minimum-snap



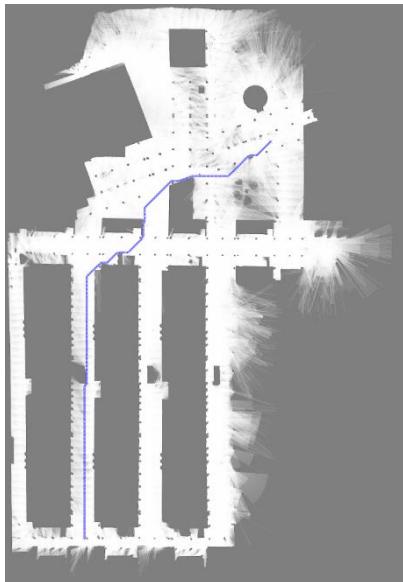
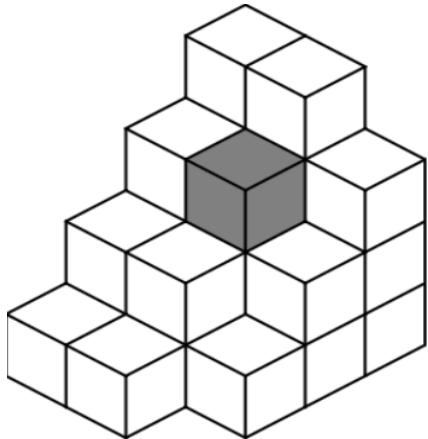


# Map Representation

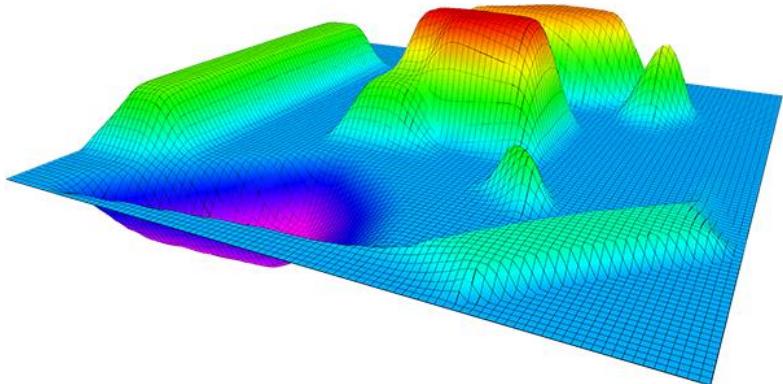




# Occupancy grid map



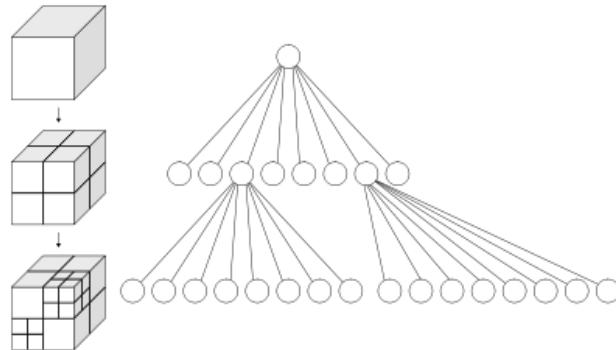
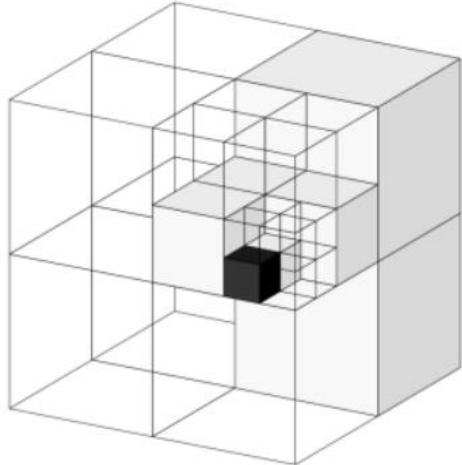
- Most Dense
- Structural
- Direct Index Query



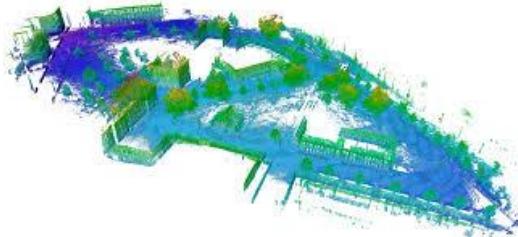
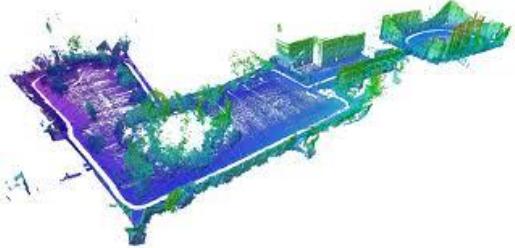
[https://github.com/ANYbotics/grid\\_map](https://github.com/ANYbotics/grid_map)



# Octo-map



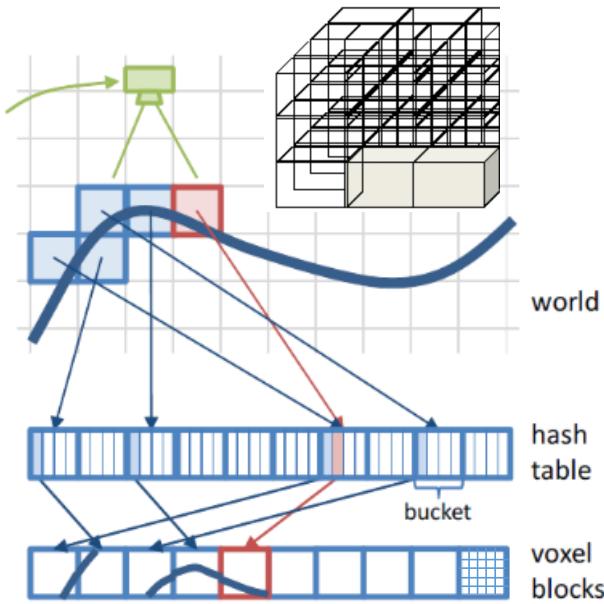
- Sparse
- Structural
- Indirect Index Query



<https://octomap.github.io/>



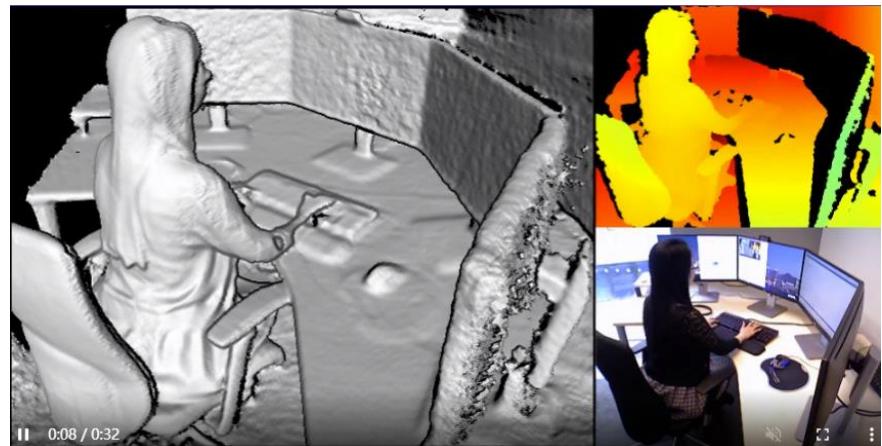
# Voxel hashing



Voxel Hashing:

<https://github.com/niessner/VoxelHashing>

- Most Sparse
- Structural
- Indirect Index Query

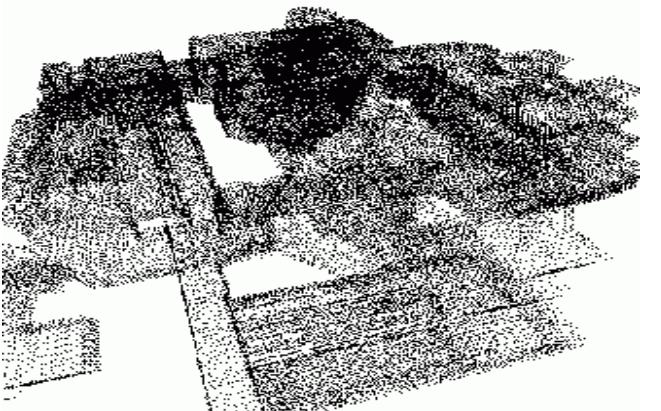


InfiniTAM:

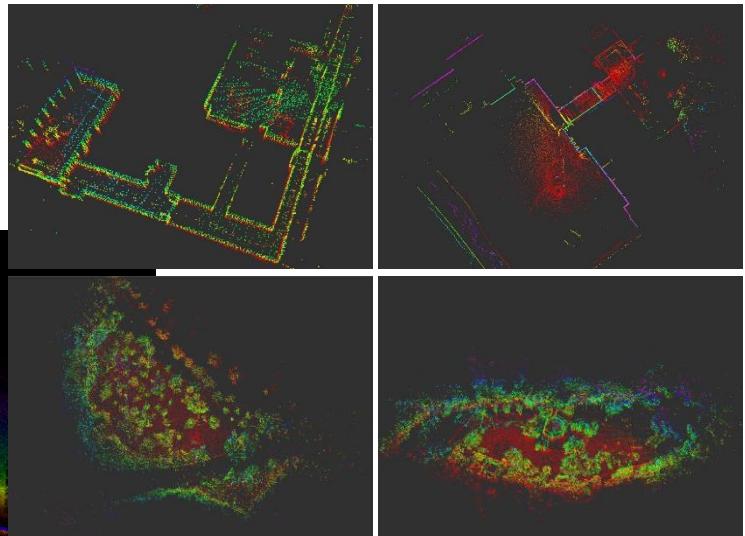
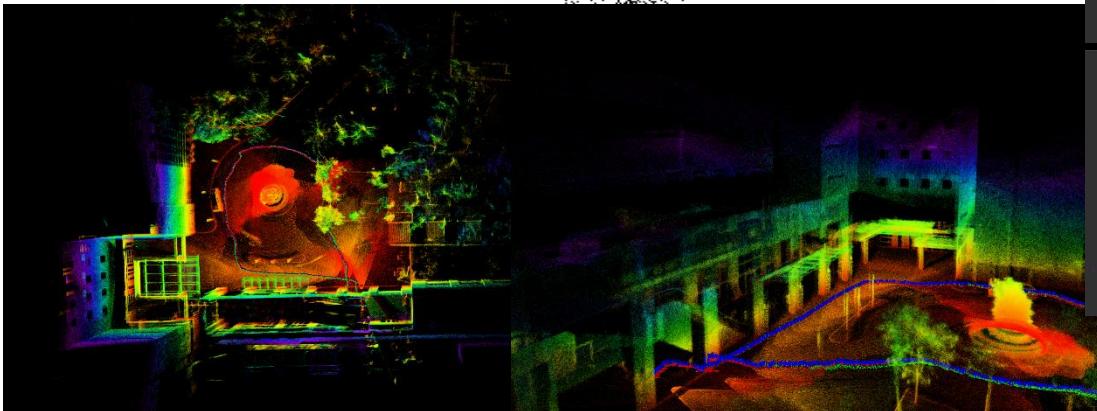
<http://www.robots.ox.ac.uk/~victor/infinitam/>



# Point cloud map



- Un-ordered
- No Index Query

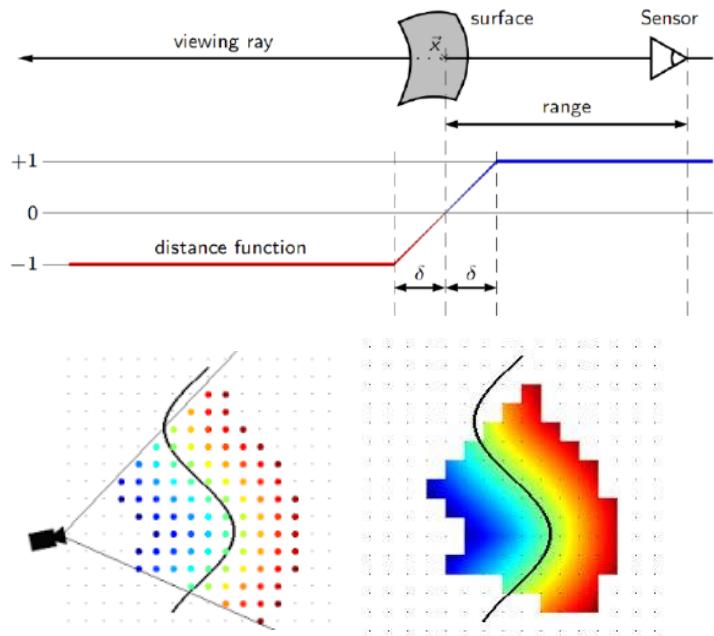


PCL  
<http://pointclouds.org/>



# TSDF map

Truncated Signed Distance Functions



OpenChisel

<https://github.com/personalrobotics/OpenChisel>



# ESDF map

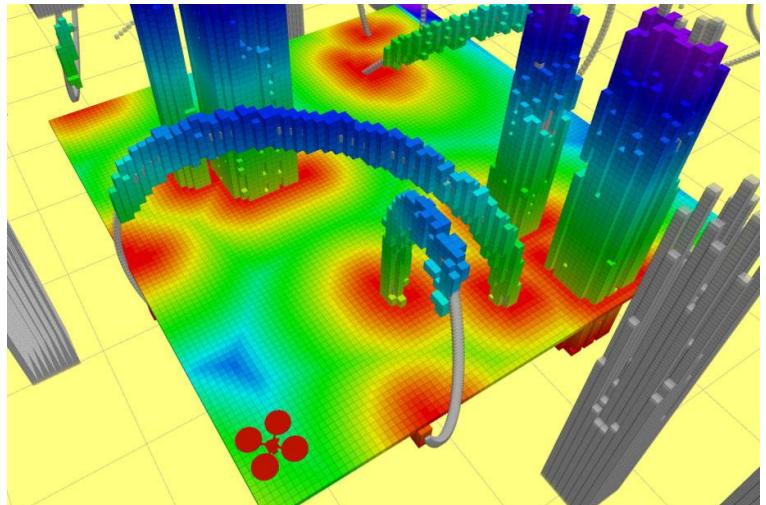
Euclidean Signed Distance Functions  
Incremental Update, Global Map

Running the Cow\_and\_Lady Dataset[1]  
Compare with Voxblox[2]

[1] <https://projects.asl.ethz.ch/datasets/doku.php?id=iros2017/>

[2] Helen Olynikova, Zachary Taylor, Marius Fehr, Juan Nieto, and Roland Siegwart, "Voxblox: Building 3D Signed Distance Fields for Planning", In IEEE Int. Conf. on Intelligent Robots and Systems (IROS), October 2017.

Batch Update, Local Map



Distance Transforms of Sampled Functions, PF Felzenswalb

VoxBlox

<https://github.com/ethz-asl/voxblox>

FIESTA

<https://github.com/HKUST-Aerial-Robotics/FIESTA>

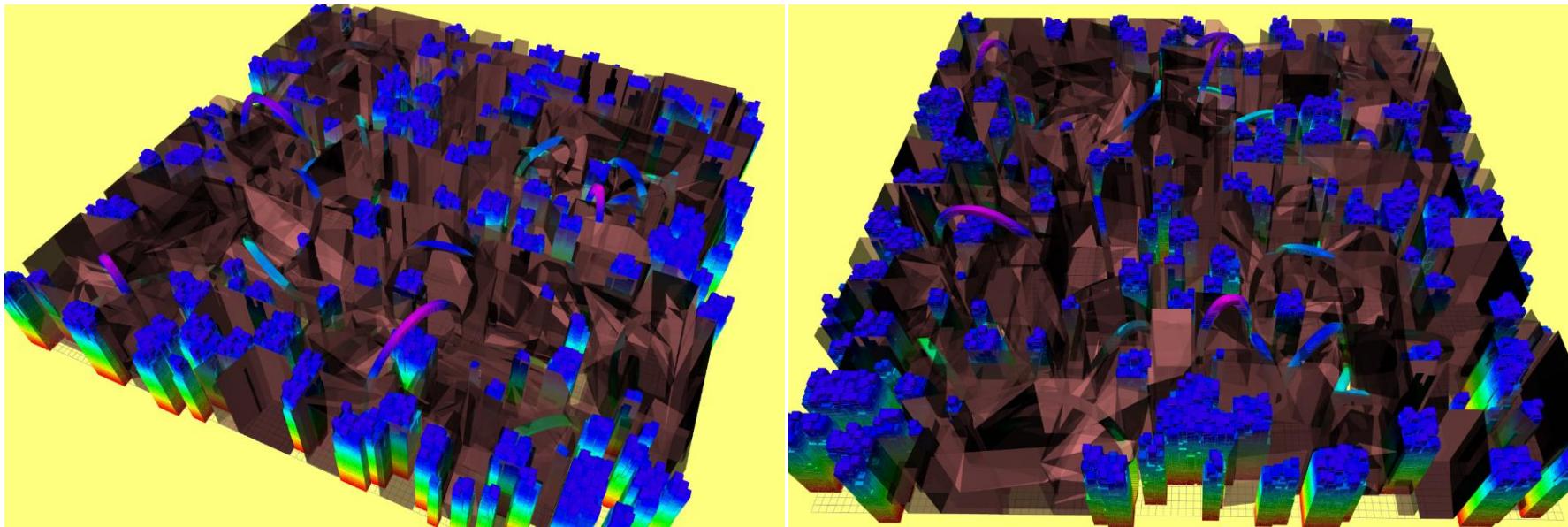
TRR's Local Map

<https://github.com/HKUST-Aerial-Robotics/Teach-Repeat-Replan>



# More ?

Free-space Roadmap

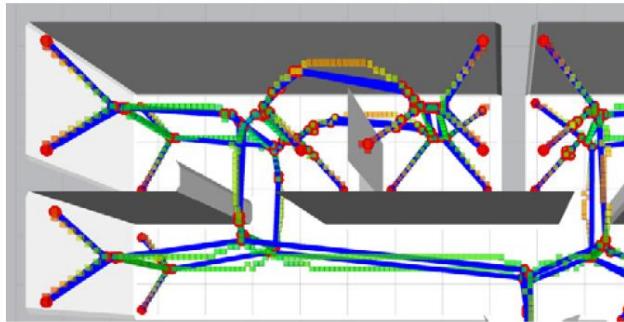


<https://github.com/HKUST-Aerial-Robotics/Teach-Repeat-Replan>

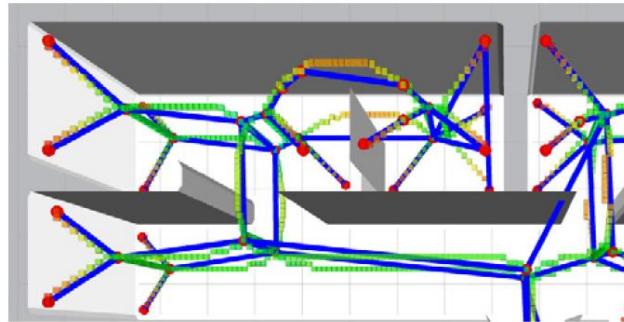


# More ?

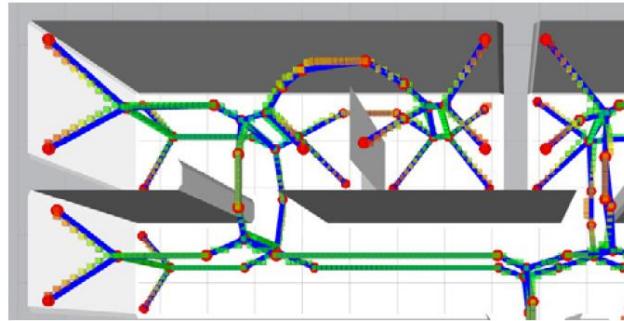
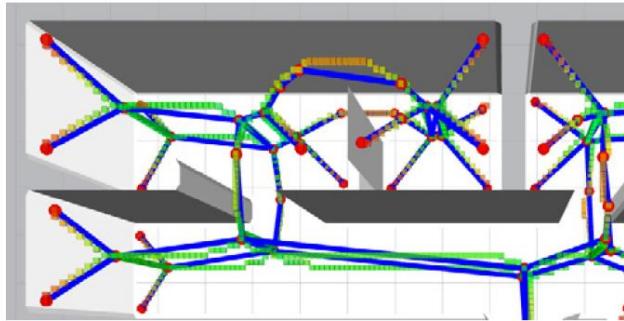
## Voronoi Diagram Map



(a)



(b)



[https://github.com/ethz-asl/mav\\_voxblox\\_planning](https://github.com/ethz-asl/mav_voxblox_planning)



# Pre-requirement



# Linux

- Linux file system
- How to install software in linux
- Useful commands



# C++ and GCC Toolchain

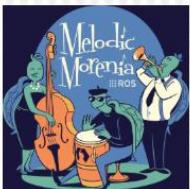
- C with class ?
- Gcc, Makefile, CMakeList
- Write CMakeList
- How to solve problems: google and document



# ROS

# ROS

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Melodic Morenia is the 12th official ROS release. It is supported on Ubuntu Artful and Bionic, along with Debian Stretch. Get Melodic Morenia now!

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Hear the latest discussions

- Follow ROS tutorial
- Ubuntu 16.04 + ros kinetic is recommended



# Matlab

- Please install Matlab



# Homework



Thanks for Listening!

