정규방정식

$$\begin{cases} \frac{\partial}{\partial \beta_0}\sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)^2=0 & \cdots ① \\ \frac{\partial}{\partial \beta_1}\sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)^2=0 & \cdots ② \end{cases}$$

① $\Leftrightarrow \sum_{i=1}^{n}(y_i-\beta_0-\beta_1 x_i)=0 \Leftrightarrow \sum y_i - n\beta_0 - \beta_1\sum x_i=0.$

$\Leftrightarrow \beta_0=\frac{1}{n}\sum y_i-\beta_1\frac{1}{n}\sum x_i. \Leftrightarrow \beta_0=\bar{y}-\beta_1\bar{x}.$

② $\Leftrightarrow \sum_{i=1}^{n}x_i(y_i-\beta_0-\beta_1 x_i)=0 \Leftrightarrow \sum x_i y_i-\beta_0 n\bar{x}-\beta_1\sum x_i^2=0.$

$\Leftrightarrow \sum x_i y_i-(\bar{y}-\beta_1\bar{x})n\bar{x}-\beta_1\sum x_i^2=0.$

$\Leftrightarrow \beta_1\left(n(\bar{x})^2-\sum x_i^2\right)+\sum x_i y_i-\bar{y}n\bar{x}=0.$

$\Leftrightarrow \beta_1=\dfrac{\sum x_i y_i-n\bar{x}\bar{y}}{\sum x_i^2-n(\bar{x})^2}=\dfrac{\beta_1\text{의 분자}}{\beta_1\text{의 분모}}.$

(예비학습)

$S_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})=\sum_{i=1}^{n}(x_i-\bar{x})y_i.$

pf. $\sum_{i=1}^{n}(x_i-\bar{x})\bar{y}=0$ 임을 보이면된다.

$LHS=\sum_{i=1}^{n}x_i\bar{y}-n\bar{x}\bar{y}=\bar{y}\sum_{i=1}^{n}x_i-n\bar{x}\bar{y}=0$

(β_1의 분자 $=S_{xy}$ 임을보이자)

$S_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})y_i=\sum_{i=1}^{n}x_i y_i-\bar{x}\sum_{i=1}^{n}y_i=\sum x_i y_i-\bar{x}n\bar{y}.$

$=\beta_1$의 분자.

(B_1의 분모 = S_{xx} 임을 보이자).

$$S_{xx} = \sum_{i=1}^{n}(x_i-\bar{x})^2 = \Sigma\left(x_i^2-2x_i\bar{x}+(\bar{x})^2\right) = \Sigma x_i^2 - 2n\bar{x}\,\bar{x}+n(\bar{x})^2$$

$$= \Sigma x_i^2 - n(\bar{x})^2 = B_1\text{의 분모}.$$