Lesson 07: numpy $3 \sim 5$

2023-01-30

ref

https://guebin.github.io/IP2022/2022/04/11/(6)-4 11 .ht ml

https://guebin.github.io/IP2022/2022/04/18/(7)-4 18 .html #matplotlib

imports

import numpy as np

3

- : numpy

2 1

 $\begin{cases} y+z+w=3\\ x+z+w=3\\ x+y+w=3\\ x+y+z=3 \end{cases}$

```
\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
   A = np.array([[0,1,1,1],[1,0,1,1],[1,1,0,1],[1,1,1,0]])
   Α
array([[0, 1, 1, 1],
         [1, 0, 1, 1],
         [1, 1, 0, 1],
         [1, 1, 1, 0]])
   b= np.array([3,3,3,3]).reshape(4,1)
   b
array([[3],
         [3],
         [3],
         [3]])
   np.linalg.inv(A) @ b
array([[1.],
         [1.],
         [1.],
         [1.]])
b
   b=np.array([3,3,3,3])
   b
```

```
array([3, 3, 3, 3])
  b.shape # b.shape
                      1
(4,)
  np.linalg.inv(A) @ b
array([1., 1., 1., 1.])
0
             . - (2,2) @ (2,1) => (2,1) - (1,2) @ (2,2) =>
(1,2)
  A = np.array([1,2,3,4]).reshape(2,2)
  b = np.array([1,2]).reshape(2,1)
  A@b
array([[ 5],
       [11]])
  A.shape, b.shape, (A@b).shape
((2, 2), (2, 1), (2, 1))
  A = np.array([1,2,3,4]).reshape(2,2)
  b = np.array([1,2]).reshape(1,2)
  b@A
array([[ 7, 10]])
```

```
A.shape, b.shape, (b@A).shape
((2, 2), (1, 2), (1, 2))
  A = np.array([1,2,3,4]).reshape(2,2)
  b = np.array([1,2]).reshape(2,1)
  b@A
ValueError: matmul: Input operand 1 has a mismatch in its core dimension 0, with gufunc signat
  A = np.array([1,2,3,4]).reshape(2,2)
  b = np.array([1,2]).reshape(1,2)
  A@b
ValueError: matmul: Input operand 1 has a mismatch in its core dimension 0, with gufunc signat
           ? \rightarrow
                   ! - (2,) @ (2,2) = (2,) - (2,2) @ (2,) =
(2,)
  A = np.array([1,2,3,4]).reshape(2,2)
  b = np.array([1,2])
  A@b
array([ 5, 11])
  A.shape, b.shape, (A@b).shape
((2, 2), (2,), (2,))
  • b (2,1)
                       (2,) .
```

```
b@A
array([ 7, 10])
  A.shape, b.shape, (b@A).shape
((2, 2), (2,), (2,))
         (1,2)
                         (2,)
      ?
  b1 = np.array([1,2,3,4])
  b2 = np.array([1,2,3,4])
  b1@b2
30
  b1.shape, b2.shape, (b1@b2).shape
((4,), (4,), ())
  • (1,4) @ (4,1) = (1,1)
  b1 = np.array([1,2,3,4]).reshape(1,4)
  b2 = np.array([1,2,3,4]).reshape(4,1)
  b1@b2
array([[30]])
  b1.shape, b2.shape, (b1@b2).shape
```

((1, 4), (4, 1), (1, 1))

(4,1) @ (1,4)

```
b1 = np.array([1,2,3,4]).reshape(4,1)
  b2 = np.array([1,2,3,4]).reshape(1,4)
  b1@b2
array([[ 1, 2, 3, 4],
       [2, 4, 6, 8],
      [3, 6, 9, 12],
       [4, 8, 12, 16]])
       .shape
  a=np.array(3.14) # , Od array
  a, a.shape
(array(3.14), ())
  a=np.array([3.14]) # , 1d array
  a, a.shape
(array([3.14]), (1,))
  a=np.array([[3.14]]) # , 2d array
  a, a.shape
(array([[3.14]]), (1, 1))
  a=np.array([[[3.14]]]) # , 3d array
  a, a.shape
(array([[[3.14]]]), (1, 1, 1))
```

```
4
```

```
-: axis .
```

np.concatenate

```
a=np.array([1,2])
  b=-a
  np.concatenate([a,b])
array([ 1, 2, -1, -2])
  a=np.array([1,2])
  c=np.array([3,4,5])
  np.concatenate([a,b,c])
array([ 1, 2, -1, -2, 3, 4, 5])
      a+b+c ?
- 2d array .
  a=np.arange(4).reshape(2,2)
  b=-a
  np.concatenate([a,b])
```

```
array([[ 0, 1],
       [2, 3],
       [ 0, -1],
       [-2, -3]])
      ?
  np.concatenate([a,b],axis=1)
array([[ 0, 1, 0, -1],
       [2, 3, -2, -3]
      axis=1 ? axis=0,2
  np.concatenate([a,b],axis=0)
array([[ 0, 1],
       [2, 3],
       [ 0, -1],
       [-2, -3]])
      np.concatenate([a,b])
  • np.concatenate([a,b])
                            np.concatenate([a,b],axis=0)
        ?
  np.concatenate([a,b],axis=2)
AxisError: axis 2 is out of bounds for array of dimension 2
- axis
  a=np.array(range(2*3*4)).reshape(2,3,4)
  a
```

```
array([[[ 0, 1, 2, 3],
       [4, 5, 6, 7],
       [8, 9, 10, 11]],
      [[12, 13, 14, 15],
       [16, 17, 18, 19],
       [20, 21, 22, 23]])
  b=-a
  b
array([[[ 0, -1, -2, -3],
       [-4, -5, -6, -7],
       [-8, -9, -10, -11]
      [[-12, -13, -14, -15],
       [-16, -17, -18, -19],
       [-20, -21, -22, -23]]])
  np.concatenate([a,b],axis=0)
array([[[ 0,
                    2,
                         3],
               1,
       [ 4,
               5,
                    6,
                        7],
       [ 8,
               9,
                  10,
                       11]],
      [[ 12, 13, 14,
                       15],
       [ 16, 17,
                  18, 19],
       [ 20, 21,
                  22,
                       23]],
      [[0, -1, -2, -3],
       [-4, -5, -6, -7],
       [-8, -9, -10, -11]],
      [[-12, -13, -14, -15],
       [-16, -17, -18, -19],
       [-20, -21, -22, -23]]])
```

```
np.concatenate([a,b],axis=1)
array([[[ 0,
                1,
                     2,
                          3],
        [ 4,
                5,
                    6,
                         7],
        [ 8,
               9, 10,
                        11],
        [ 0,
              -1,
                   -2,
                         -3],
               -5, -6,
        [-4,
                       -7],
               -9, -10, -11]],
        [ -8,
       [[ 12, 13, 14, 15],
        [ 16, 17, 18, 19],
        [ 20, 21, 22, 23],
        [-12, -13, -14, -15],
        [-16, -17, -18, -19],
        [-20, -21, -22, -23]]])
  np.concatenate([a,b],axis=2)
                         3, 0, -1, -2, -3],
array([[[ 0,
                1,
                     2,
                    6,
                         7, -4, -5, -6, -7],
        4,
                5,
                        11, -8, -9, -10, -11]],
        [ 8,
                9,
                   10,
                    14, 15, -12, -13, -14, -15],
               13,
              17,
                   18, 19, -16, -17, -18, -19],
                   22, 23, -20, -21, -22, -23]]])
        [ 20, 21,
       axis=2
  np.concatenate([a,b],axis=3)
AxisError: axis 3 is out of bounds for array of dimension 3
  • axis=3
               ?
(1) np.concatenate([a,b],axis=0)
```

```
a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  a.shape, b.shape, np.concatenate([a,b],axis=0).shape
((2, 3, 4), (2, 3, 4), (4, 3, 4))
                    => axis=0 (0 !)
           =>
(2) np.concatenate([a,b],axis=1)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  a.shape, b.shape, np.concatenate([a,b],axis=1).shape
((2, 3, 4), (2, 3, 4), (2, 6, 4))
           =>
                    => axis=1
(3) np.concatenate([a,b],axis=2)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  a.shape, b.shape, np.concatenate([a,b],axis=2).shape
((2, 3, 4), (2, 3, 4), (2, 3, 8))
           =>
               => axis=2
(4) np.concatenate([a,b],axis=3)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  a.shape, b.shape, np.concatenate([a,b],axis=3).shape
```

```
AxisError: axis 3 is out of bounds for array of dimension 3
         =>
                 => axis=3
(1)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  np.concatenate([a,b],axis=-1)
array([[[ 0,
                   2,
                       3, 0, -1, -2, -3],
               1,
                       7, -4, -5, -6, -7],
       [ 4,
               5,
                   6,
               9, 10,
       [ 8,
                      11, -8, -9, -10, -11]],
      [[ 12, 13, 14, 15, -12, -13, -14, -15],
       [ 16, 17, 18, 19, -16, -17, -18, -19],
                  22, 23, -20, -21, -22, -23]]])
       [ 20, 21,
  a.shape, b.shape, np.concatenate([a,b],axis=-1).shape
((2, 3, 4), (2, 3, 4), (2, 3, 8))
           => => axis = -1
(2)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  np.concatenate([a,b],axis=-2)
array([[[ 0,
             1,
                   2,
                        3],
       [ 4,
               5, 6,
                        7],
       [ 8,
             9, 10,
                      11],
       [0, -1, -2,
                      -3],
       [-4, -5, -6, -7],
```

```
[-8, -9, -10, -11]],
       [[ 12, 13, 14, 15],
        [ 16, 17, 18, 19],
        [ 20, 21,
                    22, 23],
        [-12, -13, -14, -15],
        [-16, -17, -18, -19],
        [-20, -21, -22, -23]]])
  a.shape, b.shape, np.concatenate([a,b],axis=-2).shape
((2, 3, 4), (2, 3, 4), (2, 6, 4))
        2
                \Rightarrow 2 \Rightarrow axis \Rightarrow -2
(3)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  b=-a
  np.concatenate([a,b],axis=-3)
array([[[ 0,
                1,
                     2,
                          3],
                          7],
                     6,
        [ 4,
                5,
        [ 8,
                   10,
                         11]],
                9,
       [[ 12, 13, 14,
                        15],
        [ 16, 17,
                    18,
                         19],
                   22,
        [ 20, 21,
                        23]],
       [[0, -1, -2, -3],
        [-4, -5, -6, -7],
        [-8, -9, -10, -11]],
       [[-12, -13, -14, -15],
        [-16, -17, -18, -19],
        [-20, -21, -22, -23]]])
```

```
a.shape, b.shape, np.concatenate([a,b],axis=-3).shape
((2, 3, 4), (2, 3, 4), (4, 3, 4))
       3 = 3 = 3
(3)
  a=np.array(range(2*3*4)).reshape(2,3,4)
  np.concatenate([a,b],axis=-4)
AxisError: axis -4 is out of bounds for array of dimension 3
                         => axis = -4
        concatenate
  a= np.array(1)
  b= np.array(-1)
  a.shape, b.shape
((), ())
  np.concatenate([a,b])
ValueError: zero-dimensional arrays cannot be concatenated
- a,b
  a=np.array(range(4)).reshape(2,2)
  b=np.array(range(2)).reshape(2,1)
```

```
np.concatenate([a,b],axis=1)
array([[0, 1, 0],
       [2, 3, 1]])
  a.shape, b.shape, np.concatenate([a,b],axis=1).shape
((2, 2), (2, 1), (2, 3))
np.stack
  • (3,) (3,) => (3,2)
  a=np.array([1,2,3])
  a,b
(array([1, 2, 3]), array([-1, -2, -3]))
  np.concatenate([a,b],axis=1)
AxisError: axis 1 is out of bounds for array of dimension 1
  a=np.array([1,2,3]).reshape(3,1)
  a,b
```

```
(array([[1],
        [2],
        [3]]),
 array([[-1],
        [-2],
        [-3]]))
  np.concatenate([a,b],axis=1)
array([[ 1, -1],
       [2,-2],
       [3, -3]])
  • : (3)(3) => (3,1)(3,1) => (3,1) concat(3,1)
  a=np.array([1,2,3])
  b=-a
  np.stack([a,b],axis=1)
array([[ 1, -1],
       [2,-2],
       [ 3, -3]])
  np.stack([a,b],axis=0)
array([[ 1, 2, 3],
       [-1, -2, -3]])
(1)
```

```
a=np.array([1,2,3])
  b=-a
  a.shape, b.shape, np.stack([a,b],axis=0).shape
((3,), (3,), (2, 3))
  • (3) (3) => (axis=0) => (1,3) (1,3) => (2,3)
(2)
  a=np.array([1,2,3])
  b=-a
  a.shape, b.shape, np.stack([a,b],axis=1).shape
((3,), (3,), (3, 2))
   • (3) (3) => (axis=1) => (3,1) (3,1) => (3,2)
  a=np.arange(3*4*5).reshape(3,4,5)
  b=-a
  a.shape, b.shape
((3, 4, 5), (3, 4, 5))
  np.stack([a,b],axis=0).shape # (3,4,5) => (1,3,4,5) //
(2, 3, 4, 5)
  np.stack([a,b],axis=1).shape # (3,4,5) => (3,1,4,5) //
(3, 2, 4, 5)
```

```
np.stack([a,b],axis=2).shape # (3,4,5) => (3,4,1,5) //
(3, 4, 2, 5)
  np.stack([a,b],axis=3).shape # (3,4,5) => (3,4,5,1) //
(3, 4, 5, 2)
  np.stack([a,b],axis=-1).shape # axis=-1 <=> axis=3
(3, 4, 5, 2)
  np.stack([a,b],axis=-2).shape # axis=-2 <=> axis=2
(3, 4, 2, 5)
np.concate nate \\
                          , np.stack
sum
- 1
  a = np.array([1,2,3])
  a
array([1, 2, 3])
  a.sum()
```

6

```
a.sum(axis=0)
6
- 2
  a=np.array(range(6)).reshape(2,3)
array([[0, 1, 2],
       [3, 4, 5]])
  a.sum() #
15
  a.sum(axis=0)
array([3, 5, 7])
  a.sum(axis=1)
array([ 3, 12])
- 2
  a.shape, a.sum(axis=0).shape
((2, 3), (3,))
          => axis=0
```

```
a.shape, a.sum(axis=1).shape
((2, 3), (2,))
  • => axis=1
  a=np.array(range(10)).reshape(5,2)
  a
array([[0, 1],
      [2, 3],
      [4, 5],
      [6, 7],
      [8, 9]])
(1) 1 , 2 ?
() (5,2) => (2,) . (
                           ?)
 a.sum(axis=0)
array([20, 25])
(2) 1, 2, \dots, 5
() (5,2) => (5,) . (
                              ?)
  a.sum(axis=1)
array([ 1, 5, 9, 13, 17])
(3) a ?
() (5,2) => () . ( ,
                              ?)
```

```
a.sum(axis=(0,1))
45
  a.sum() # a.sum(axis=(0,1))
45
mean, std, max, min, prod
   \operatorname{sum}
  a=np.array(range(10)).reshape(5,2)
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7],
       [8, 9]])
  a.mean(axis=0), a.std(axis=0), a.max(axis=0), a.min(axis=0), a.prod(axis=0)
(array([4., 5.]),
array([2.82842712, 2.82842712]),
array([8, 9]),
 array([0, 1]),
 array([ 0, 945]))
  a.mean(axis=1), a.std(axis=1), a.max(axis=1), a.min(axis=1), a.prod(axis=1)
```

```
(array([0.5, 2.5, 4.5, 6.5, 8.5]),
array([0.5, 0.5, 0.5, 0.5, 0.5]),
array([1, 3, 5, 7, 9]),
 array([0, 2, 4, 6, 8]),
 array([ 0, 6, 20, 42, 72]))
  \operatorname{std} \operatorname{n} .
  a=np.array([1,2,3,4])
  a.std()
1.118033988749895
  np.sqrt(sum((a-a.mean())**2)/4)
1.118033988749895
- n-1 ?
  a=np.array([1,2,3,4])
  a.std(ddof=1)
1.2909944487358056
  np.sqrt(sum((a-a.mean())**2)/3)
1.2909944487358056
argmax, argmin
- 1
  a = np.array([1,-2,3,10,4])
  a
array([ 1, -2, 3, 10, 4])
```

```
a.argmax() #
3
  a.argmin() #
1
- 2
  np.random.seed(43052)
  a=np.random.randn(4*5).reshape(4,5)
  a
array([[ 0.38342049, 1.0841745, 1.14277825, 0.30789368, 0.23778744],
       [0.35595116, -1.66307542, -1.38277318, -1.92684484, -1.4862163],
       [0.00692519, -0.03488725, -0.34357323, 0.70895648, -1.55100608],
       [1.34565583, -0.05654272, -0.83017342, -1.46395159, -0.35459593]])
  a.argmin(), a.min()
(8, -1.9268448358915802)
  a.argmax(), a.max()
(15, 1.3456558341738827)
  a.argmin(axis=0), a.argmin(axis=1)
(array([2, 1, 1, 1, 2]), array([4, 3, 4, 3]))
```

```
a.argmax(axis=0), a.argmax(axis=1)
(array([3, 0, 0, 2, 0]), array([2, 0, 3, 0]))
cumsum, cumprod
- 1
  a=np.array([1,2,3,4])
array([1, 2, 3, 4])
  a.cumsum()
array([ 1, 3, 6, 10])
  a.cumprod()
array([ 1, 2, 6, 24])
- 2
  a=np.array(range(3*4)).reshape(3,4)
  a
array([[ 0, 1, 2, 3],
       [4, 5, 6, 7],
      [8, 9, 10, 11]])
  a.cumsum(axis=0), a.cumsum(axis=1)
```

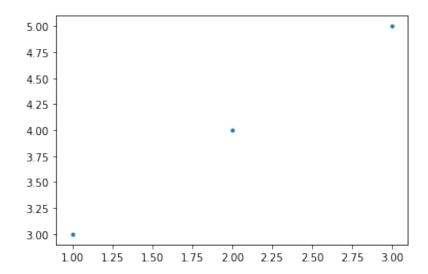
```
(array([[ 0, 1, 2, 3],
       [4, 6, 8, 10],
       [12, 15, 18, 21]]),
array([[ 0, 1, 3, 6],
       [4, 9, 15, 22],
       [8, 17, 27, 38]]))
  a.cumprod(axis=0), a.cumprod(axis=1)
(array([[ 0, 1, 2, 3],
       [ 0, 5, 12, 21],
       [ 0, 45, 120, 231]]),
array([[
         Ο,
               Ο,
                      0, 0],
           4, 20, 120, 840],
       72, 720, 7920]]))
           8,
diff
- 1
  a=np.array([1,2,4,6,7])
  a
array([1, 2, 4, 6, 7])
  np.diff(a)
array([1, 2, 2, 1])
- 2
  np.diff(np.diff(a))
array([ 1, 0, -1])
- prepend, append
```

```
a=np.array([1,2,4,6,7])
   a
array([1, 2, 4, 6, 7])
  np.diff(a,prepend=100)
   #np.diff(np.array([100]+a.tolist()) )
array([-99, 1, 2,
                          2,
                               1])
   • [1,2,4,6,7] \rightarrow [100,1,2,3,4,6] \rightarrow \text{np.diff}
   np.diff(a,append=100)
   #np.diff(np.array(a.tolist()+[100]) )
array([ 1, 2, 2, 1, 93])
() a=[1,2,4,6,7]
   np.diff(a,prepend=a[0])
   #np.diff(a,prepend=1)
array([0, 1, 2, 2, 1])
( ) a=[1,2,4,6,7] 7
   np.diff(a,append=a[-1])
   #np.diff(a,append=7)
array([1, 2, 2, 1, 0])
- 2 array
```

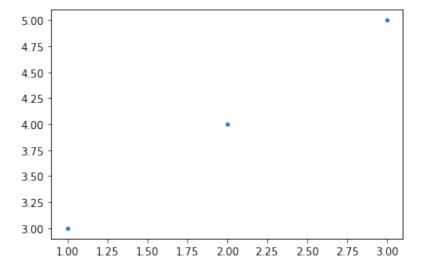
```
a=np.arange(24).reshape(4,6)
  a
array([[ 0, 1, 2, 3, 4, 5],
       [6, 7, 8, 9, 10, 11],
       [12, 13, 14, 15, 16, 17],
       [18, 19, 20, 21, 22, 23]])
  np.diff(a,axis=0)
array([[6, 6, 6, 6, 6, 6],
       [6, 6, 6, 6, 6, 6],
       [6, 6, 6, 6, 6, 6]])
  np.diff(a,axis=1)
array([[1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1]])
     5
- : maplotlib
  import matplotlib.pyplot as plt
```

plt.plot

plt.plot([1,2,3],[3,4,5],'.')

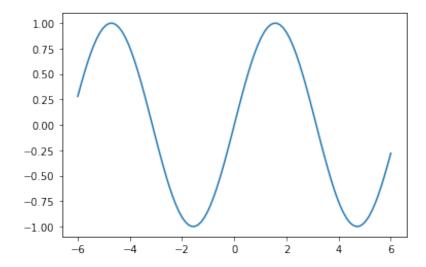


plt.plot(np.array([1,2,3]),np.array([3,4,5]),'.')

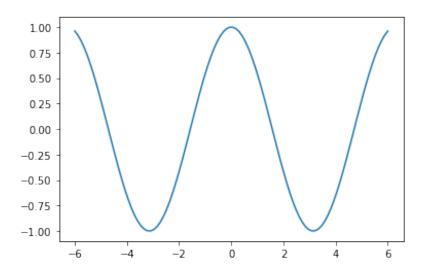


```
t=np.linspace(-6,6,100)
x=np.sin(t)
y=np.cos(t)

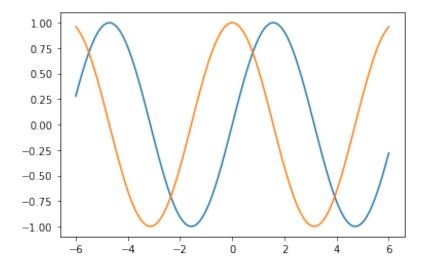
plt.plot(t,x)
```



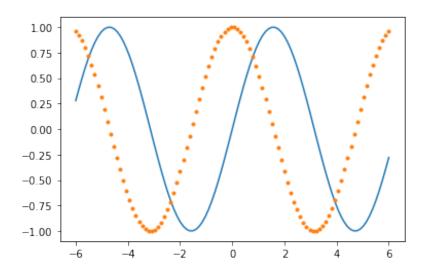
plt.plot(t,y)



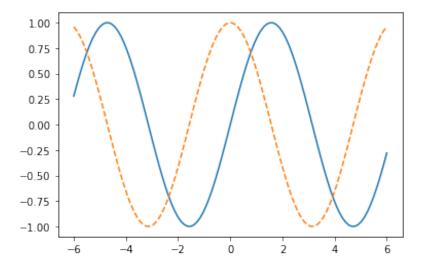
plt.plot(t,x)
plt.plot(t,y)



plt.plot(t,x)
plt.plot(t,y,'.')



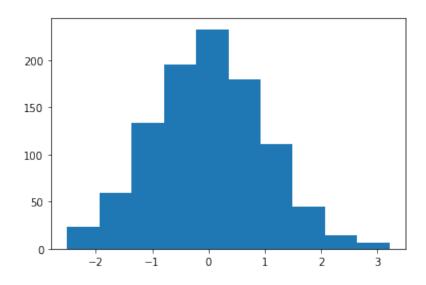
plt.plot(t,x)
plt.plot(t,y,'--')



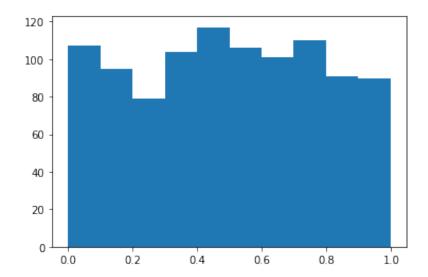
plt.hist

```
X = np.random.randn(1000)
Y = np.random.rand(1000)
plt.hist(X)
```

<BarContainer object of 10 artists>)



plt.hist(Y)



Quiz

```
( )
  a=np.arange(24).reshape(4,6)
  a
array([[ 0, 1, 2, 3, 4, 5],
       [6, 7, 8, 9, 10, 11],
       [12, 13, 14, 15, 16, 17],
       [18, 19, 20, 21, 22, 23]])
          np.diff
                   (4,5) array
                                 1
                                      column
 axis=1
array([[1, 1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1, 1]])
```