$$\underset{n\times p}{X} = UDV^T, \quad U = [u_1 \cdots u_p],\ D = \mathrm{diag}(d_1 \cdots d_p)$$

$n > p$

$$V = [v_1 \cdots v_p]$$

$$Z = \tilde{U}\tilde{D}. \quad \tilde{U} = [u_1 \cdots u_q],\ \tilde{D} = \mathrm{diag}(d_1 \cdots d_q),\ q < p.$$

목표: $Z = \tilde{U}\tilde{D} = X\tilde{V}.\quad \tilde{V} = [v_1 \cdots v_q]$

(증명) $X\tilde{V} = UDV^T\tilde{V} = UD \begin{bmatrix} v_1^T \\ \vdots \\ v_q^T \\ \vdots \\ v_p^T \end{bmatrix} [v_1 \cdots v_q]$

$$= UD \begin{bmatrix} v_1^T v_1 & \cdots & v_1^T v_q \\ \vdots & & \\ v_q^T v_1 & \cdots & v_q^T v_q \\ \vdots & & \\ v_p^T v_1 & \cdots & v_p^T v_q \end{bmatrix} = UD \begin{bmatrix} I_{q\times q} \\ 0 \end{bmatrix}$$

$$= U \begin{bmatrix} D_1 & D_2 \end{bmatrix} \begin{bmatrix} I_{q\times q} \\ 0 \end{bmatrix} = UD_1 = U \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_q \\ \hline & 0 & \end{bmatrix}$$

$$= U \begin{bmatrix} \tilde{D} \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{U} & ? \end{bmatrix} \begin{bmatrix} \tilde{D} \\ 0 \end{bmatrix} = \tilde{U}\tilde{D}$$