1 Harris Corner Detector

Theory :

The idea is to consider a neighbourhood (Gaussian window) centred on this pixel, shift it slightly in several directions, and then calculate the intensity variation for each shift. This translates mathematically into the following function:

Equation 1 :
$$\boxed{E_{m,n}(u,v) = \sum_{(x,y)} w(x,y)[I(x+u,y+v) - I(x,y)]^2}$$

$E_{m,n}(u,v)$ represents the intensity difference between the neighbourhood $w(x,y)$ centered on pixel (m,n) and the neighbourhood $w(x,y)$ offset from (u,v).

In the Harris Corner Detector, windows are Gaussians thus: $w(x,y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}}$

Intensity: $I(x,y)$

Shifted Intensity: $I(x+u,y+v)$

$I(x+u,y+v)$ is approximated by a Taylor expansion in the neighbourhood (x,y) :

Equation 2 :
$$\boxed{I(x+u,y+v) \approx I(x,y) + u\frac{\partial I(x,y)}{\partial x} + v\frac{\partial I(x,y)}{\partial y}}$$

Thus :

Equation 3 :
$$\begin{aligned} E_{m,n}(u,v) &\approx \sum_{(x,y)} w(x,y)\left[u\frac{\partial I(x,y)}{\partial x} + v\frac{\partial I(x,y)}{\partial y}\right]^2 \\ &\approx [u \quad v]\, M \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

With :

Equation 4 :
$$\boxed{M = \sum_{x,y} w(x,y) \begin{bmatrix} \left(\frac{\partial I(x,y)}{\partial x}\right)^2 & \frac{\partial I(x,y)}{\partial x}\cdot\frac{\partial I(x,y)}{\partial y} \\ \frac{\partial I(x,y)}{\partial x}\cdot\frac{\partial I(x,y)}{\partial y} & \left(\frac{\partial I(x,y)}{\partial y}\right)^2 \end{bmatrix}}$$

The resulting matrix M describes the local behaviour of $E_{m,n}$. So, to detect the corners, it is necessary to determine the eigenvalues λ_1 et λ_2 of M. Finding the eigenvalues of M can be tedious. An alternative is to analyse the values of the Harris operator $H_{m,n}$ at the point (m, n) :

$$H_{m,n} = \det(M) - k \times trace(M)^2$$

with k, a constant chosen between 0.04 and 0.06.

We can therefore deduce that a point is a corner or not, depending on the value of $H_{m,n}$. The point (m, n) is a corner if $H_{m,n} > s$ with s a threshold to be chosen.

Questions :

1/ Is the Harris corner detector robust with respect to intensity shifts and intensity scaling ?

-Intensity shifting implies : $I'(x, y) = I(x, y) + c$ with c the constant shift thus when we switch to partial derivatives, c will disappear $\frac{\partial I'(x,y)}{\partial x} = \frac{\partial I(x,y)}{\partial x}$ thus $M' = M$. Therefore, the Harris Corner detector is invariant to intensity shifts: as it relies on the local gradient (because shifts affect the whole image uniformly).

-Intensity scaling implies : $I"(x, y) = s \times I(x, y)$ where s is a scaling factor greater than 1 (for amplification) or less than 1 (for attenuation).

Thus, partial derivatives will be impacted by the intensity scalling : $\frac{\partial I"(x,y)}{\partial x} = s \times \frac{\partial I(x,y)}{\partial x}$

So

$$S_{xx} = \sum w(x, y) \left(\frac{\partial I"(x,y)}{\partial x}\right)^2 = s^2 \times \sum w(x, y) \left(\frac{\partial I(x,y)}{\partial x}\right)^2$$

$$S_{yy} = \sum w(x, y) \left(\frac{\partial I"(x, y)}{\partial y}\right)^2 = s^2 \times \sum w(x, y) \left(\frac{\partial I(x, y)}{\partial y}\right)^2$$

$$S_{xy} = \sum w(x, y) \frac{\partial^2 I"(x, y)}{\partial x \partial y} = s^2 \times \sum w(x, y) \frac{\partial^2 I(x, y)}{\partial x \partial y}$$

Thus, M will change and the Harris coefficients H too. So, Harris Corner detector is sensitive to invariant to intensity scaling as it changes the local gradient.

2/ Is the Harris corner detector robust with respect to translation?

Image translation implies : $I'(x, y) = I(x + \Delta x, y + \Delta y)$ representing a translation by Δx pixels in the x-direction and Δy pixels in the y-direction.

$$S_{xx} = \sum w(x, y) \left(\frac{\partial I'(x, y)}{\partial x}\right)^2$$

$$S_{yy} = \sum w(x,y)\left(\frac{\partial I'(x,y)}{\partial y}\right)^2$$

$$S_{xy} = \sum w(x,y)\ \frac{\partial^2 I'(x,y)}{\partial x \partial y}$$

These sums are over the shifted coordinates $x + \Delta x, y + \Delta y$, which means the contribution of gradients from pixels within the local window will change because of the translation.

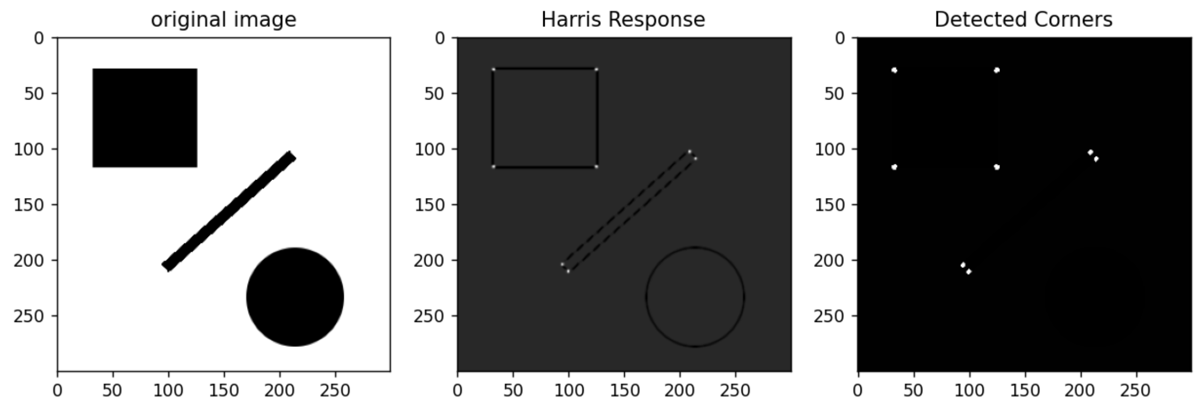
So, Harris Corner detector is not robust to translation: corner's location will change even if local structures and intensity gradients will remain the same.

3/ Is the Harris corner detector robust with respect to rotation?

Image translation implies : $I'(x,y) = I(x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta))$ with θ the angle of rotation.

S_{xx}, S_{yy}, S_{xy} will evolve but with an adapted to rotation w, the Harris Corner detector is invariant to rotation: this is its most robustness quality.

Results :



2 Canny Edge Detection

Theory :

Since edge detection is susceptible to noise in the image, first step is to remove the noise in the image with a 5x5 Gaussian filter.

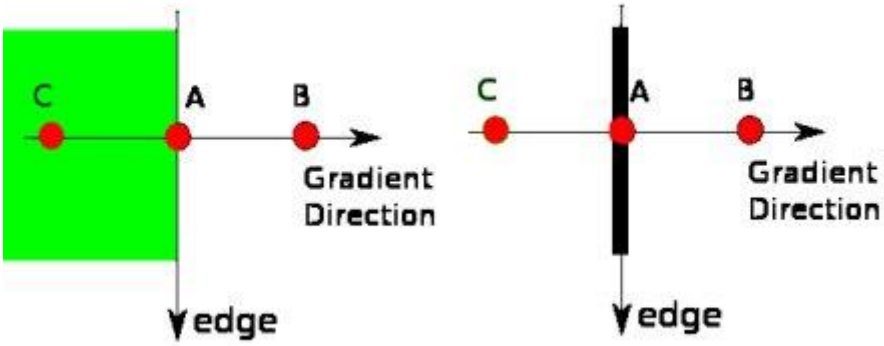
1. **Finding Intensity Gradient of the Image**

Smoothened image is then filtered with a Sobel kernel in both horizontal and vertical direction to get first derivative in horizontal direction (G_x) and vertical direction (G_y). From these two images, we can find edge gradient and direction for each pixel as follows:

$$||\nabla I(x,y)|| = \sqrt{(S_x * I(x,y))^2 + (S_y * I(x,y))^2}$$
$$\Theta = \arctan\left(\frac{S_y * I(x,y)}{S_x * I(x,y)}\right)$$

2. **Non-maximum Suppression**

After getting gradient magnitude and direction, a full scan of image is done to remove any unwanted pixels which may not constitute the edge. For this, at every pixel, pixel is checked if it is a local maximum in its neighbourhood in the direction of gradient. Check the image below:



Point A is on the edge (in vertical direction). Gradient direction is normal to the edge. Point B and C are in gradient directions. So point A is checked with point B and C to see if it forms a local maximum. If so, it is considered for next stage, otherwise, it is suppressed (put to zero).

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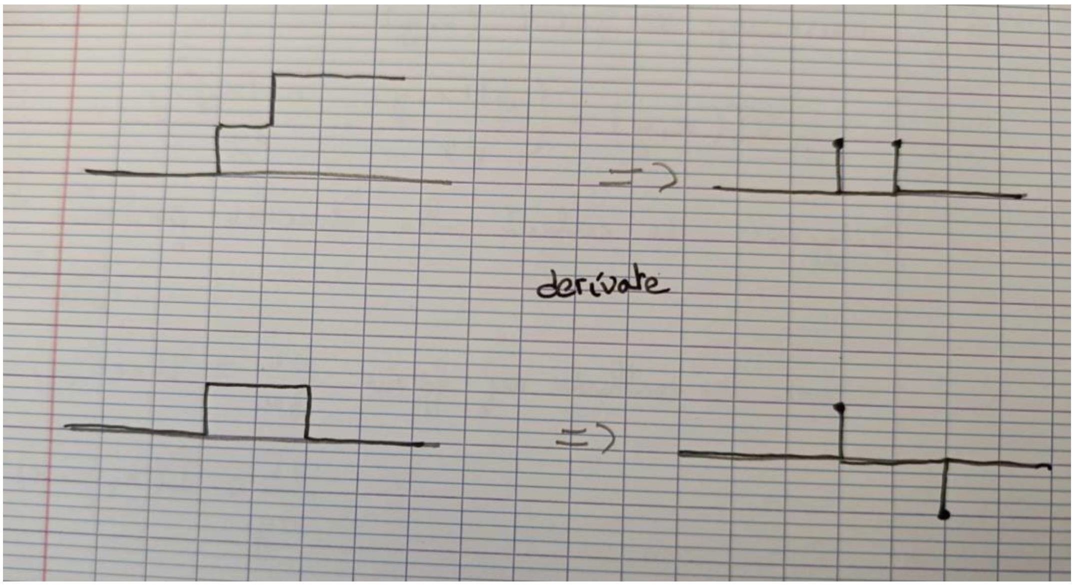
Question :



Results :