# Beyond 60/40: Optimizing Portfolios for a Changing Macro Landscape

A Multi-Period, Multi-Asset Class Analysis (1971-2024)

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# Contents

## Section 1: Data Collection & Preparation

**Objective**

The initial phase of this research focused on gathering, cleaning, and formatting the necessary macroeconomic and asset return data. This prepared the foundation for all subsequent portfolio analysis.

**Data Sources**

- **FRED API:** Used to pull annual Real GDP and Consumer Price Index (CPI) data from 1970 to 2025.

- **Historical Asset Dataset:** Included monthly prices for:
    - Gold (source: macrotrends.net)
    - S&P 500 (source: macrotrends.net)
    - Nikkei 225 (source: Nikkei.co.jp)
    - US 10-Year Treasury Yields (source: FRED)
    - UK 10-Year Treasury Yields (source: FRED)
    - USD/JPY Exchange Rates (source: macrotrends.net)
    - GBP/USD Exchange Rates (source: macrotrends.net)

**Data Processing Steps**

1. **Macroeconomic Calculations:**
    - Calculated year-over-year GDP growth.
    - Calculated year-over-year CPI inflation.

2. **Asset Return Calculations:**
    - Calculated monthly returns for Gold, S&P 500, and Nikkei.
    - Adjusted Nikkei returns for currency effects using USD/JPY exchange rates.
    - Calculated currency-adjusted bond returns for US and UK 10-Year Treasuries using an approximation formula.

3. **Bond Return Approximation:**
    - Developed a basic bond return calculator using yield changes, coupon payments, and price adjustments.
    - Adjusted UK bond returns for GBP/USD currency movements.

4. **Final Return Dataset:**
    - Removed non-essential columns and prepared a return-only dataset with the following assets:
        - Gold
        - S&P 500

- Nikkei (FX-adjusted)
- US 10-Year Treasury
- UK 10-Year Treasury (FX-adjusted)
  - The final dataset contains 651 monthly observations from January 1971 to March 2025.

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**Summary**

This structured and cleaned dataset serves as the core input for all portfolio optimization and risk analysis performed in the following sections. Additional assets with shorter histories will be incorporated in later stages of the project with similar processing techniques.

## Section 2: Portfolio Optimization Across Historical Periods

### Executive Summary

This section outlines the development of analytical tools and the preliminary macro trading research conducted to assess the performance of optimized investment portfolios across distinct historical periods.

We constructed a flexible Python-based portfolio optimization framework and systematically applied it to five major asset classes: Gold, S&P 500, Nikkei, US 10-Year Treasury, and UK 10-Year Treasury. Using 10-year and 5-year rolling periods, we analyzed the evolution of optimal asset allocations, asset correlations, and macroeconomic trends such as GDP growth and inflation.

The analysis demonstrates that gold provides critical diversification during financial turmoil, while equities and bonds dominate in stable growth environments. We also propose a custom all-weather portfolio based on our findings and benchmark its performance against both an optimized portfolio and a traditional 60/40 portfolio.

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### Methodology

### Portfolio Optimization Tools

We developed two core Python classes:

- **PortfolioOptimizer**
    - Calculates minimum variance, maximum return, and maximum Sharpe ratio portfolios.
    - Uses constrained optimization (SLSQP) assuming long-only portfolios and no leverage.

- **ReturnAnalyzer**
    - Allows slicing return data into customizable time blocks.
    - Calculates risk, return, and Sharpe ratios on an annualized and total basis.

Key formulas used:

- Portfolio Return: $R_p = w'\mu$

- Portfolio Variance: $\sigma_p^2 = w'\Sigma w$

- Sharpe Ratio: $S = \frac{R_p - R_f}{\sigma_p}$

(Full Python implementation provided in Appendix A)

### Analytical Approach

We segmented the data into 10-year blocks:

- 1971-1980
- 1981-1990

- 1991-2000
- 2001-2010
- 2011-2020

For each period, we:

- Optimized the portfolio for maximum Sharpe ratio.
- Calculated asset correlations.
- Summarized macroeconomic indicators (average GDP growth and CPI inflation).

We then compared the performance of:

- The optimized portfolio.
- A traditional 60/40 stock-bond portfolio.
- A custom-designed all-weather portfolio (15% Gold, 35% Stocks, 42% US Bonds, 8% UK Bonds).

## Summary Performance Comparison

**Performance across 10-year periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1971-1980 | 14.54% | 9.19% | 1.58 | 3.96% | 8.40% | 0.47 | 8.48% | 7.26% | 1.17 |
| 1981-1990 | 12.93% | 8.89% | 1.45 | 11.12% | 9.71% | 1.15 | 9.39% | 7.94% | 1.18 |
| 1991-2000 | 11.42% | 5.67% | 2.02 | 12.53% | 6.57% | 1.91 | 9.22% | 4.84% | 1.91 |
| 2001-2010 | 11.90% | 8.49% | 1.40 | 1.84% | 8.52% | 0.22 | 5.28% | 6.22% | 0.85 |
| 2011-2020 | 6.98% | 4.33% | 1.61 | 8.63% | 6.14% | 1.41 | 6.56% | 4.54% | 1.44 |

**Performance across 5-year periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1971-1975 | 13.65% | 7.81% | 1.75 | 1.76% | 9.35% | 0.19 | 6.86% | 7.11% | 0.96 |
| 1976-1980 | 15.18% | 9.24% | 1.64 | 6.29% | 7.35% | 0.86 | 10.21% | 7.42% | 1.38 |
| 1981-1985 | 16.17% | 10.15% | 1.59 | 12.53% | 9.88% | 1.27 | 9.57% | 9.23% | 1.04 |
| 1986-1990 | 9.75% | 6.01% | 1.62 | 9.75% | 9.60% | 1.02 | 9.35% | 6.48% | 1.44 |
| 1991-1995 | 10.24% | 4.49% | 2.28 | 12.36% | 5.73% | 2.16 | 10.06% | 4.51% | 2.23 |
| 1996-2000 | 10.86% | 5.82% | 1.87 | 12.73% | 7.35% | 1.73 | 8.42% | 5.18% | 1.63 |
| 2001-2005 | 11.05% | 7.63% | 1.45 | 1.60% | 7.75% | 0.21 | 4.70% | 5.75% | 0.82 |
| 2006-2010 | 13.18% | 8.96% | 1.47 | 2.08% | 9.28% | 0.22 | 5.88% | 6.70% | 0.88 |
| 2011-2015 | 6.88% | 4.01% | 1.72 | 7.99% | 5.08% | 1.57 | 5.02% | 3.92% | 1.28 |
| 2016-2020 | 7.97% | 4.53% | 1.76 | 9.27% | 7.08% | 1.31 | 8.19% | 5.08% | 1.61 |

**Key Insights**

- **Gold's Role:** The optimizer significantly allocated to gold during periods of high inflation and financial distress.
    - In 1971-1980, our optimizer recommends:
        - 24.08% Gold, 23.67% Nikkei, 43.74% US 10Y, 8.52% UK 10Y
    - In 2001-2010, our optimizer recommends:
        - 53.8% Gold, 4.14% SP500, 40.19% US Bonds, 1.87% UK Bonds
- **Limit Equity Exposure**: The optimizer, even in times of strong equity market performance, always constrains the total equity exposure
    - We find that in "Goldilocks" types of market environments like the 1980s, 1990s, and 2010s, the optimizer favored a portfolio around 40% equities and 60% bonds (split between US and UK)
    - Because we optimized for Sharpe ratio, the model prioritized risk-adjusted returns over raw performance, leveraging diversification to improve outcomes.

- **Optimizer Constraints:** We begin to see when we break our data into more granular time periods (10-year to 5-year) that it changes assets more frequently as we begin to "curve fit" the data more
  - For example in the 10-year block between 1991-2000, the recommended portfolio was:
    - 41.70% SP500, 53.48% US 10Y, and 4.82% UK 10Y
  - While in the two, 5-year blocks, the optimizer recommended:
    - **1991-1995:** 15.97% Gold, 43.14% SP500, 37.79% US 10Y, 3.10% UK 10Y
    - **1996-2000:** 38.88% SP500, 53.06% US 10Y, 8.06% UK 10Y
    - The optimizer essentially holds gold for a more optimum amount of time before its selloff from 1996-2000
  - This sensitivity to smaller time windows emphasizes the need for balance between responsiveness and overfitting in portfolio design.
- **Custom Portfolio:** The all-weather custom portfolio demonstrated robust performance, particularly improving returns during stressed periods while maintaining comparable Sharpe ratios during stable periods.
  - The optimizer's repeated preference for gold in downturns and a higher bond allocation in bull markets directly informed the construction of our custom weights.

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**Conclusion**

This second phase of research offers a data-driven blueprint for building resilient portfolios across economic cycles. The results directly challenge the traditional 60/40 structure, showing that dynamic, diversified allocations—particularly with meaningful gold exposure—can significantly improve performance during market downturns without sacrificing returns in stable environments.

Drawing from the optimizer's recommendations across different macroeconomic regimes and market cycles, we constructed a custom all-weather portfolio that blends the optimizer's preferences:

- Approximately 40% stocks and 60% bonds during "boom" periods
- A gold- and bond-heavy allocation during "panic" periods

The resulting portfolio—15% Gold, 35% Stocks, 42% US Bonds, and 8% UK Bonds—delivers 60/40-type returns in bull markets while strictly dominated the 60/40 portfolio during market crisis (3-4x the Sharpe ratio)

(Refer to Appendix A for detailed Python code and Appendix B for additional data tables.)

## Section 3: Portfolio Expansion to 9 Assets and Recent Period Analysis

### Executive Summary

In this section, we expand our portfolio universe to include nine assets by adding Oil, Copper, Japanese 10-Year Bonds, and the FTSE 100. Due to data limitations, our analysis now starts from 1990 and extends through March 2025. This is the first segment of our research that incorporates the most recent market environments, including the bond bear market of the 2020s. Importantly, we observe that bonds—historically a reliable hedge—begin to fail in this recent period, exposing a critical flaw in over-relying on fixed income for diversification.

We follow our established methodology of breaking down the analysis into 7-year and 5-year periods to capture more granular trends. This step reveals that as timeframes shorten, the optimizer dynamically shifts allocations, emphasizing the importance of regime-aware strategies.

### Methodology

### Data Expansion and Preparation

We enhanced the dataset to include nine assets

- Gold
- Oil (source: macrotrends.net)
- Copper (source: macrotrends.net)
- S&P 500
- Nikkei
- FTSE 100 (source: yahoo finance)
- US 10-Year Treasury
- UK 10-Year Treasury
- Japan 10-Year Treasury (source: FRED)

Monthly return calculations, currency adjustments, and bond return approximations were applied consistently with prior methodology.

### Optimization Structure

The following analytical steps were employed:

- Rolling optimizations over 7-year and 5-year periods.
- Calculation of maximum Sharpe portfolios for each block.
- Detailed correlation matrix reviews.
- Evaluation of average GDP and CPI growth rates in each period.
- Performance comparisons of the previously defined custom portfolio (15% Gold, 35% Stocks, 42% US Bonds, 8% UK Bonds) against the optimized portfolio and a traditional 60/40 portfolio.

## Summary Performance Comparison
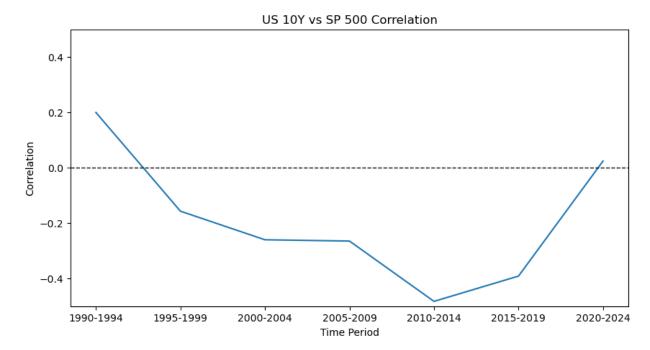
**Performance Across 7-Year Periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1990-1996 | 9.60% | 4.82% | 1.99 | 10.38% | 6.59% | 1.58 | 8.55% | 4.93% | 1.73 |
| 1997-2003 | 7.57% | 5.85% | 1.29 | 6.41% | 8.48% | 0.76 | 6.13% | 5.83% | 1.05 |
| 2004-2010 | 12.06% | 8.05% | 1.50 | 3.25% | 8.27% | 0.39 | 6.06% | 6.12% | 0.99 |
| 2011-2017 | 7.30% | 4.20% | 1.74 | 8.24% | 4.90% | 1.68 | 5.48% | 3.98% | 1.38 |
| 2018-2024 | 11.61% | 8.98% | 1.29 | 7.34% | 8.34% | 0.88 | 5.68% | 6.78% | 0.84 |

**Performance Across 5-Year Periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1990-1994 | 7.90% | 5.24% | 1.51 | 6.31% | 6.72% | 0.94 | 5.82% | 4.92% | 1.18 |
| 1995-1999 | 16.22% | 5.61% | 2.89 | 18.71% | 6.97% | 2.68 | 12.61% | 5.06% | 2.49 |
| 2000-2004 | 9.72% | 6.42% | 1.51 | 1.28% | 7.97% | 0.16 | 4.51% | 5.88% | 0.77 |
| 2005-2009 | 12.55% | 9.22% | 1.36 | 0.97% | 9.01% | 0.11 | 4.74% | 6.56% | 0.72 |
| 2010-2014 | 8.62% | 4.22% | 2.05 | 10.03% | 5.39% | 1.86 | 7.49% | 4.08% | 1.83 |
| 2015-2019 | 5.74% | 4.03% | 1.42 | 6.53% | 4.86% | 1.34 | 4.97% | 3.97% | 1.25 |
| 2020-2024 | 12.81% | 9.77% | 1.31 | 7.37% | 9.31% | 0.79 | 5.35% | 7.68% | 0.70 |

## Key Insights

- **Bonds' Fading Hedge Role:** In the 2020-2024 block, portfolios with over 40% bond exposure underperformed the 60/40, primarily due to sustained negative bond returns and rising correlations with equities.

US 10Y vs SP 500 Correlation

- **Gold's Persistent Value:** Gold consistently reappears as a dominant allocation in stressed environments (e.g., 2005-2009 and 2020-2024), supporting our decision to maintain meaningful gold exposure in the custom portfolio.

- **Custom Portfolio Resilience:** The all-weather custom portfolio continued to outperform the 60/40 in stressed periods but showed reduced advantage in recent years, particularly during the 2020s bond bear market.

- **Granularity Trade-Off:** The optimizer finds higher Sharpe ratios in shorter periods, but this could reflect curve fitting rather than persistent, reliable patterns. The 7-year block remains a strong balance between sensitivity and stability.

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**Conclusion**

This third phase of our research highlights an urgent consideration: the macro landscape has shifted, and traditional portfolio constructs like 60/40 may no longer be sufficient. Bonds have lost their defensive strength, particularly in inflationary and rising rate regimes, as clearly observed in the post-2018 period. While the custom all-weather portfolio has maintained its edge in many periods, its margin of outperformance has narrowed in recent years. This suggests we should consider other assets for diversification, or otherwise develop a new strategy to better fit the changing macro landscape of recent times.

Future work will extend our analysis to 11 assets, adding US Investment Grade and High Yield Bonds to strengthen the diversification profile away from US Treasuries.

(Refer to Appendix A for detailed Python code and Appendix B for additional data tables.)

**Section 4: Portfolio Expansion to 11 Assets and Credit Bond Analysis**

**Executive Summary**

In this phase, we expand our portfolio universe to 11 assets by introducing US Investment Grade (IG) Bonds and US High Yield (HY) Bonds. This dataset starts in 1997 to incorporate the availability of corporate bond data.

This section continues the methodology from prior analyses, using 7-year and 4-year optimization periods to capture evolving asset dynamics and macroeconomic influences. The inclusion of IG and HY bonds introduces credit sensitivity to our portfolio construction, which significantly impacts the optimizer's selections, particularly in stable or low-rate environments.

Monthly return calculations, currency adjustments, and bond return approximations were applied consistently with prior methodology.

**Optimization Structure**

The following analytical steps were employed:

- Rolling optimizations over 7-year and 4-year periods.

- Calculation of maximum Sharpe portfolios for each block.

- Detailed correlation matrix reviews.

- Evaluation of average GDP and CPI growth rates in each period.

- **We also propose a new custom portfolio consisting of:**
  - **20% Gold**
  - **35% SP 500**
  - **30% US 10Y**
  - **10% US IG**
  - **5% US HY**

- We compare our new custom portfolio against the optimized portfolio and a traditional 60/40 portfolio.

## Summary Performance Comparison

**Performance Across 7-Year Periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1997-2003 | 10.10% | 4.65% | 2.17 | 6.41% | 8.48% | 0.76 | 6.09% | 5.81% | 1.05 |
| 2004-2010 | 11.50% | 7.21% | 1.60 | 3.25% | 8.27% | 0.39 | 7.15% | 6.69% | 1.07 |
| 2011-2017 | 7.31% | 4.17% | 1.75 | 8.24% | 4.90% | 1.68 | 5.58% | 4.11% | 1.36 |
| 2018-2024 | 11.61% | 8.98% | 1.29 | 7.34% | 8.34% | 0.88 | 6.97% | 6.93% | 1.01 |

**Performance Across 4-Year Periods**

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1997-2000 | 8.70% | 4.13% | 2.11 | 12.82% | 7.65% | 1.68 | 7.51% | 5.20% | 1.44 |
| 2001-2004 | 16.62% | 5.52% | 3.01 | 0.95% | 8.37% | 0.11 | 5.58% | 6.15% | 0.91 |
| 2005-2008 | 10.62% | 7.49% | 1.42 | -0.99% | 8.48% | -0.12 | 3.20% | 7.30% | 0.44 |
| 2009-2012 | 17.21% | 4.86% | 3.55 | 9.19% | 7.48% | 1.23 | 12.18% | 5.16% | 2.36 |
| 2013-2016 | 8.25% | 5.26% | 1.57 | 7.50% | 4.85% | 1.55 | 3.18% | 4.13% | 0.74 |
| 2017-2020 | 8.52% | 3.84% | 2.22 | 10.20% | 7.33% | 1.39 | 9.87% | 5.89% | 1.68 |
| 2021-2024 | 11.57% | 8.37% | 1.38 | 5.81% | 8.33% | 0.70 | 4.91% | 7.07% | 0.69 |

**Key Insights**

- **Corporate Bonds as Alpha Source:** The optimizer consistently recommended adding IG and HY corporate bonds across several periods, reflecting their dual characteristics: IG bonds behaving similarly to Treasuries and HY bonds exhibiting stock-like risk profiles.

  o **1997-2003:** The optimizer allocated nearly 60% to IG bonds and 30% to HY bonds, highlighting the strong performance of corporate credit during the post-tech bubble environment and amid Greenspan's rate cuts.

  o **2004-2010:** The optimizer maintained a 20% HY bond allocation as interest rates approached zero and credit losses remained relatively controlled through the Great Financial Crisis (GFC).

  o **2009-2012 (4-Year Period):** In the immediate GFC aftermath, the optimizer recommended 11% in IG bonds, 35% in HY bonds, and 27% in US 10Y Treasuries.

  o These consistent allocations validate the inclusion of HY and IG bonds in the bond sleeve of our custom portfolio going forward.

- **Persistent Regime Shifts:** Over the past 28 years, asset class behavior has been shaped by successive macro regimes and crises, including the 2000 tech bubble, the 2008 financial crisis, the 2020 COVID rally, and the collapse of the bond bull market (1980–2022).

  o **1997-2000:** The optimizer recommended a portfolio of 19% SP500, 11% US 10Y Treasuries, and 69% IG bonds.

  o **2021-2024:** The optimizer shifted to a completely different allocation: 30% Gold, 11% Oil, and 59% SP500, with no bond exposure.

  o These shifts confirm the importance of regime-aware asset allocation.

- **Diminishing Bond Exposure Over Time:** As interest rates trended toward the zero lower bound after the GFC, the optimizer steadily reduced exposure to Treasuries and corporate bonds.

  o **1997-2010:** Bond allocations consistently exceeded 70% of the portfolio.

  o **2013-2016:** Bond exposure dropped below 40%, with equities (SP500) climbing to over 60%.

  o This structural reduction in bond allocations highlights the declining utility of bonds as a diversifier in ultra-low-rate environments.

- **Equity Weighting – Boom or Bust:** The optimizer's equity allocation fluctuated significantly across periods, reflecting market cycles and crises.

  o Some periods saw maximum equity exposure (~60%), while others showed minimal or no equity exposure.

  o This dynamic illustrates the value of market timing, macro-driven allocation, and tactical flexibility to capture equity upside while managing downside risks.

**Conclusion**

With the inclusion of IG and HY bonds to the investible universe, we found that they are a worthy addition to a diversified investment portfolio. We redesigned our custom portfolio which performed admirably in the periods analyzed. As we break our analysis down into shorter periods, we however see that the custom portfolio does not consistently outperform the traditional 60/40 in all periods.

We also see that as the investible universe grows and the time frames shorten, that the optimal portfolio tends to "curve fit" and offer high risk-adjusted returns that are unlikely to be fully captured or replicable in the future by a diversified portfolio. However the optimal portfolios give an idea of what is possible if one can be exceptionally good at market timing.

## Section 5: Research Summary and Final Thoughts

This multi-phase research offers a comprehensive exploration of dynamic portfolio construction across more than five decades of macroeconomic cycles, asset class evolutions, and shifting market regimes. Through a structured Python-based optimization framework, we systematically analyzed the risk-adjusted performance of diversified portfolios, evaluating both long-term structural allocations and shorter-term tactical shifts.

**Key Conclusions:**

- **Dynamic Allocations Consistently Outperform Static Portfolios:** Optimized portfolios, when allowed to adapt allocations across different macro regimes, consistently offered significant outperformance compared to the traditional 60/40 benchmark in most historical windows—particularly in volatile and inflationary periods.

- **Gold's Enduring Value:** Gold consistently provided diversification benefits in stressed environments and inflationary regimes, particularly in the post-2000 and post-2018 periods when government bond hedging power deteriorated.

- **Corporate Credit as a Persistent Alpha Source:** High Yield and Investment Grade corporate bonds emerged as favored assets in optimized portfolios, especially during recovery periods following major crises (e.g., post-tech bubble, post-GFC). This validates their inclusion in diversified bond sleeves going forward.

- **Breakdown of Traditional Bond Diversification:** Government bonds, once a reliable diversifier, began to lose effectiveness after the Global Financial Crisis as rates approached the zero lower bound and bond-equity correlations increased, particularly during the 2020s bond bear market.

- **Custom Portfolio Strength:** The all-weather custom portfolio—featuring diversified allocations to stocks, bonds, gold, and corporate credit—delivered strong long-term risk-adjusted performance and consistently outperformed the 60/40 in stressed periods. This research reinforces the portfolio's resilience and strategic value, with recommended target weights of:

    - **20% Gold, 35% SP500, 25% US 10Y, 10% IG Bonds, 5% HY Bonds**

- **The Importance of Regime-Awareness:** The optimizer's shifting preferences across time—sometimes rapidly reducing exposure to bonds or equities—highlight the need for regime-aware strategies that can dynamically respond to evolving market conditions.

- **Tactical Overlays Can Unlock Additional Value:** The research strongly supports the case for adding tactical overlays to dynamically adjust portfolio tilts in response to real-time market signals, macroeconomic changes, and risk regime shifts.

**Final Thoughts and Next Steps:**

While the custom portfolio offers a robust, simplified allocation structure for long-term investors, a more sophisticated solution may involve:

- **Incorporating Additional Asset Classes:** Future research should explore adding emerging market equities, commodities, and inflation-linked securities to further diversify exposures.

- **Developing Tactical Overlays:** Building rule-based or discretionary tactical frameworks that allow portfolios to tilt toward favored assets during specific macro regimes.

- **Machine Learning-Based Regime Detection:** Integrating statistical or machine learning models to systematically identify regime shifts and recommend portfolio adjustments with improved responsiveness.

This work provides a foundational framework for constructing adaptable, regime-aware portfolios and offers a scalable path forward toward a modern, macro-driven investment strategy. This framework can be directly applied to macro-driven portfolio management strategies and adapted for professional investment workflows.

(Refer to Appendix A for detailed Python code and Appendix B for additional data tables.)

## Appendix A: Select Python Code

## A.1 Data Collection and Preparation

```python
In [1]:  # Import packages
         import pandas as pd
         import numpy as np
         import numpy_financial as npf
         from fredapi import Fred
         from scipy.optimize import minimize
         from matplotlib import pyplot as plt
```

## Let's collect GDP and CPI Data for our Macroeconomic Analysis

```python
In [2]:  # initialize Fred API
         # Note: To use FRED API you must register an account and receive an api_key
         fred = Fred(api_key = 'a1e14ac266190134fdbc578b6966ef59')
```

```python
In [3]:  # Set Date index for Fred Pull
         start = "01/01/1970"
         end = "03/31/2025"
```

```python
In [4]:  # Pull GDP data
         gdp_data = fred.get_series("GDPC1", observation_start = start, observation_end = end, frequency="a")
```

```python
In [5]:  # Calculate growth rate and format dataframe
         GDP = pd.DataFrame(gdp_data)
         GDP["GDP YoY"] = (GDP[0] - GDP[0].shift(1)) / GDP[0].shift(1)
         GDP.drop(columns=0, inplace=True)
         GDP.rename(columns={"GDP YoY" : "GDP"})
         GDP.index = GDP.index.year # switch to only show year
         GDP = GDP.dropna() # drop first row
```

```python
In [6]:  # Pull CPI Data
         cpi_data = fred.get_series("CPIAUCSL", observation_start = start, observation_end = end, frequency="a")
```

```python
In [7]:  CPI = pd.DataFrame(cpi_data)
         CPI["CPI YoY"] = (CPI[0] - CPI[0].shift(1)) / CPI[0].shift(1)
         CPI.drop(columns=0, inplace=True)
         CPI.rename(columns={"CPI YoY" : "CPI"})
         CPI.index = CPI.index.year # switch to only show year
         CPI = CPI.dropna() # drop first row
```

## Get Return Data

```
In [8]:   # Import Data Set
          data = pd.read_csv("monthly_macro_data_1971.csv")
          data = data.set_index("Date")
          data = data.dropna()
          original_columns = data.columns.to_list() # export column names for later use
```

```
In [9]:   data
```

Out[9]:

| Date | Gold | SP500 | US 10Y | UK 10Y | USDJPY | GBPUSD | NIKKEI |
|---|---|---|---|---|---|---|---|
| 12/31/1970 | 35.17 | 91.110000 | 6.50 | 8.9300 | 357.72 | 2.4174 | 1987.14 |
| 1/31/1971 | 37.88 | 93.490000 | 6.09 | 8.8000 | 357.72 | 2.4174 | 2100.90 |
| 2/28/1971 | 38.74 | 97.110000 | 6.14 | 8.5900 | 357.56 | 2.4160 | 2232.42 |
| 3/31/1971 | 38.87 | 99.600000 | 5.53 | 8.3400 | 357.42 | 2.4168 | 2403.30 |
| 4/30/1971 | 39.01 | 103.000000 | 6.08 | 8.2100 | 357.40 | 2.4193 | 2468.18 |
| ... | ... | ... | ... | ... | ... | ... | ... |
| 11/30/2024 | 2651.13 | 5929.916000 | 4.18 | 4.4164 | 150.41 | 1.2699 | 38208.03 |
| 12/31/2024 | 2648.01 | 6010.908571 | 4.58 | 4.4345 | 157.37 | 1.2521 | 39894.54 |
| 1/31/2025 | 2709.69 | 5979.515500 | 4.58 | 4.6627 | 154.91 | 1.2447 | 39572.49 |
| 2/28/2025 | 2894.73 | 6038.690000 | 4.24 | 4.5063 | 150.64 | 1.2591 | 37155.50 |
| 3/31/2025 | 3122.80 | 5683.983333 | 4.23 | 4.5063 | 149.90 | 1.2896 | 35617.56 |

652 rows × 7 columns

```
In [10]:  # Generate Currency Performance
          data["Yen Return"] = (-1) * ((data.USDJPY / data.USDJPY.shift(1)) - 1) # we multiply by negative 1 to get the yen returns
          data["Pound Return"] = (data.GBPUSD / data.GBPUSD.shift(1)) - 1
```

```
In [11]:  # Calculate Returns
          data["Gold Return"] = (data.Gold  / data.Gold.shift(1)) - 1
          data["SP500 Return"] = (data.SP500 / data.SP500.shift(1)) - 1
          data["Nikkei Return_Nom"] = ((data.NIKKEI / data.NIKKEI.shift(1)) - 1)
          data["Nikkei Return_FX_Adj"] = data["Nikkei Return_Nom"] + data["Yen Return"] # add currency performance to return
```

```
In [12]:  # Copy Bond Data for Return Calculation
          US_10Y = data["US 10Y"].copy()
          US_10Y = pd.DataFrame(US_10Y)
          UK_10Y = data["UK 10Y"].copy()
          UK_10Y = pd.DataFrame(UK_10Y)
```

```
In [13]:  # Basic Bond Return Calculator for 10Y Bond with monthly data (Note: we should be able to create a better approximation)
          def Bond_Return_Calculator(yields):
              Returns = yields.copy()
              Returns['Return'] = 0.01 # placeholder
              for i in range(0, len(yields)-1):
                  old_yield = yields.iloc[i, 0] / 100
                  new_yield = yields.iloc[i+1, 0] /100
                  pmt_old = (old_yield/2)*100
                  new_price = -npf.pv(new_yield/2, 20, pmt_old, 100)
                  bond_return = (new_price - 100 + old_yield*100/12) / 100
                  Returns.iloc[i+1, 1] = bond_return
              return Returns
```

```
In [14]:  # Calculate UK and US 10Y Returns
          US_10Y_Returns = Bond_Return_Calculator(US_10Y)
          UK_10Y_Returns = Bond_Return_Calculator(UK_10Y)
```

```
In [15]:  # Import Bond Return Data
          data["US 10Y Return"] = US_10Y_Returns["Return"]
          data["UK 10Y Return_Nom"] = UK_10Y_Returns["Return"]
          data["UK 10Y Return_FX_Adj"] = data["UK 10Y Return_Nom"] + data["Pound Return"] # add currency performance to return
```

```
In [16]:  # Delete first row because no return data available
          data.drop(data.index[0], axis=0, inplace=True)
```

19

## Format Return Data

```
In [17]: # Create Returns only DataFrame
         returns = data.copy()
         returns.drop(columns=original_columns, inplace=True)
         returns.drop(columns=["Yen Return", "Pound Return", "UK 10Y Return_Nom", "Nikkei Return_Nom"], inplace=True)
         returns.rename(columns={'Gold Return' : 'Gold', 'SP500 Return' : 'SP500', 'US 10Y Return' : 'US 10Y', "UK 10Y Return_FX_Adj" : "U
                         , "Nikkei Return_FX_Adj" : "Nikkei"}, inplace=True)
         returns.index = pd.to_datetime(returns.index) # set index to datetime
         returns
```

Out[17]:

| Date | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| 1971-01-31 | 0.077054 | 0.026122 | 0.057248 | 0.035789 | 0.015971 |
| 1971-02-28 | 0.022703 | 0.038721 | 0.063049 | 0.001380 | 0.020658 |
| 1971-03-31 | 0.003356 | 0.025641 | 0.076936 | 0.051495 | 0.024224 |
| 1971-04-30 | 0.003602 | 0.034137 | 0.027052 | -0.036154 | 0.016737 |
| 1971-05-31 | 0.038708 | -0.013592 | -0.004801 | 0.005067 | 0.014963 |
| ... | ... | ... | ... | ... | ... |
| 2024-11-30 | -0.014479 | 0.023755 | -0.009610 | 0.011672 | -0.026111 |
| 2024-12-31 | -0.001177 | 0.013658 | -0.002133 | -0.028322 | -0.011786 |
| 2025-01-31 | 0.023293 | -0.005223 | 0.007559 | 0.003817 | -0.020289 |
| 2025-02-28 | 0.068288 | 0.009896 | -0.033513 | 0.031295 | 0.027935 |
| 2025-03-31 | 0.078788 | -0.058739 | -0.036480 | 0.004342 | 0.027979 |

651 rows × 5 columns

## A.2 Portfolio Optimization Framework

```python
In [18]: class PortfolioOptimizer:
    '''
    Takes a cleaned returns dataframe and allows us to find minimium risk, maximum return and maximium sharpe ratio portfolios
    '''
    def __init__ (self, returns: pd.DataFrame, risk_free_rate =0.0):
        self.assets = returns.columns.to_list()
        self.n_years = len(returns)/12
        self.mu = ((1 + returns).prod())**(1/self.n_years) - 1  # Annualized geometric mean return
        self.Sigma = returns.cov().values * 12  # Annualized covariance matrix
        self.n = len(self.assets)
        self.rfr = risk_free_rate
        self.start_year = returns.index[0].year
        self.end_year = returns.index[-1].year
        self.min_var_w, self.min_var_ret, self.min_var_vol, self.min_var_sharpe = self.optimize_min_variance()
        self.max_ret_w, self.max_ret_ret, self.max_ret_vol, self.max_ret_sharpe = self.optimize_max_return()
        self.max_sharpe_w, self.max_sharp_ret, self.max_sharpe_vol, self.max_sharpe_sharpe = self.optimize_max_sharpe()

    def portfolio_variance(self, w):
        return np.dot(w.T, np.dot(self.Sigma, w))

    def portfolio_return(self, w):
        return np.dot(w, self.mu)

    def negative_return(self, w):
        return -np.dot(w, self.mu)

    def negative_sharpe(self, w):
        ret = self.portfolio_return(w)
        vol = np.sqrt(self.portfolio_variance(w))
        sharpe = (ret - self.rfr) / vol
        return -sharpe

    def optimize_min_variance(self):
        initial_weights = np.array([1/self.n] * self.n)
        bounds = [(0,1)] * self.n # Long only
        constraints = {"type" : "eq", "fun" : lambda w: np.sum(w) -1} # No leverage

        result = minimize(self.portfolio_variance, initial_weights, method = 'SLSQP', bounds=bounds, constraints = constraints)
        w_opt = result.x
        ret = self.portfolio_return(w_opt)
        vol = np.sqrt(self.portfolio_variance(w_opt))
        sharpe = (ret - self.rfr) / vol

        return w_opt, ret, vol, sharpe

    def optimize_max_return(self):
        initial_weights = np.array([1/self.n] * self.n)
        bounds = [(0,1)] * self.n # Long only
        constraints = {"type" : "eq", "fun" : lambda w: np.sum(w) -1} # No leverage

        result = minimize(self.negative_return, initial_weights, method = 'SLSQP', bounds=bounds, constraints = constraints)
        w_opt = result.x
        ret = self.portfolio_return(w_opt)
        vol = np.sqrt(self.portfolio_variance(w_opt))
        sharpe = (ret - self.rfr) / vol

        return w_opt, ret, vol, sharpe
```

```python
def optimize_max_sharpe(self):
    initial_weights = np.array([1/self.n] * self.n)
    bounds = [(0,1)] * self.n # Long only
    constraints = {"type" : "eq", "fun" : lambda w: np.sum(w) -1} # No Leverage

    result = minimize(self.negative_sharpe, initial_weights, method = 'SLSQP', bounds=bounds, constraints = constraints)
    w_opt = result.x
    ret = self.portfolio_return(w_opt)
    vol = np.sqrt(self.portfolio_variance(w_opt))
    sharpe = (ret - self.rfr) / vol

    return w_opt, ret, vol, sharpe

def summary(self, show = "All"):
    """
    Prints optimal portfolio results.
    Select show options = "Minimum Variance", "Maximum Return", "Maximum Sharpe", "All"
    """

    min_var_w, min_var_ret, min_var_vol, min_vol_sharpe = self.optimize_min_variance()
    max_ret_w, max_ret_ret, max_ret_vol, max_ret_sharpe = self.optimize_max_return()
    max_sharpe_w, max_sharpe_ret, max_sharpe_vol, max_sharpe_sharpe = self.optimize_max_sharpe()

    if show == "All":

        print("\nMinimum Variance Portfolio:")
        for name, weight in zip(self.assets, min_var_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {min_var_ret:.2%}, Volatility: {min_var_vol:.2%}, Sharpe: {min_vol_sharpe:.2f}")

        print("\nMaximium Return Portfolio:")
        for name, weight in zip(self.assets, max_ret_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {max_ret_ret:.2%}, Volatility: {max_ret_vol:.2%}, Sharpe: {max_ret_sharpe:.2f}")

        print("\nMaximum Sharpe Portfolio:")
        for name, weight in zip(self.assets, max_sharpe_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {max_sharpe_ret:.2%}, Volatility: {max_sharpe_vol:.2%}, Sharpe: {max_sharpe_sharpe:.2f}"

    elif show == "Minimum Variance":

        print("\nMinimum Variance Portfolio:")
        for name, weight in zip(self.assets, min_var_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {min_var_ret:.2%}, Volatility: {min_var_vol:.2%}, Sharpe: {min_vol_sharpe:.2f}")

    elif show == "Maximum Return":

        print("\nMaximium Return Portfolio:")
        for name, weight in zip(self.assets, max_ret_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {max_ret_ret:.2%}, Volatility: {max_ret_vol:.2%}, Sharpe: {max_ret_sharpe:.2f}")

    elif show == "Maximum Sharpe":

        print("\nMaximum Sharpe Portfolio:")
        for name, weight in zip(self.assets, max_sharpe_w):
            print(f"{name}: {weight:.2%}")
        print(f" Annualized Return: {max_sharpe_ret:.2%}, Volatility: {max_sharpe_vol:.2%}, Sharpe: {max_sharpe_sharpe:.2f}"

    else:
        print(f"incorrect option selected. Please select 'Minimum Variance', 'Maximum Return', 'Maximum Sharpe' or 'All' ")
```

## A.3 Return Analysis Framework

```python
In [19]: class ReturnAnalyzer:
             '''
             class which takes a monthly return data frame and performs certain analysis and transformations with it
             1) Splice data by year, so we can have 1 year worth of monthly returns, 2 years of returns, etc.
             2) Perform summary analysis on this spliced data such as return, vol, sharpe on a total and annualized basis
             '''
             def __init__(self, returns: pd.DataFrame):
                 self.returns = returns
                 self.assets = self.returns.columns.to_list()
                 self.n_assets = len(self.assets)
                 self.n_returns = len(self.returns) # grab number of monthly returns datapoints
                 self.n_fyears = self.n_returns // 12 # number of full years

             def splice_years(self, number_years, start_year = None, drop_orphans = True):
                 '''
                 Method to allow us to quickly splice years from the dataframe, dropping data which doesn't fit neatly
                 '''
                 adj_returns = self.returns.copy() # make copy to manipulate

                 # drop rows until the first is January and the last is December
                 if drop_orphans == True:
                     while adj_returns.index[0].month != 1:
                         adj_returns = adj_returns.iloc[1:]
                     while adj_returns.index[-1].month != 12:
                         adj_returns = adj_returns.iloc[:-1]

                 # splice data
                 if start_year == None:
                     start = adj_returns.index[0].year
                 else:
                     start = start_year
                 end = start + number_years # end year
                 spliced = adj_returns[(start <= adj_returns.index.year) & (adj_returns.index.year < end)] # use boolean vectorized filte

                 if number_years*12 > len(spliced):
                     print("Requested more data than available... returning partial splice:\n")

                 return spliced

             def roll_up_returns(self, number_years = 1, drop_orphans=True, annualized_only = True):
                 '''
                 takes a return dataframe and rolls it up into annual data with the number of years specified by the user
                 summarizes data with return, risk, shapre on a total and annualized basis
                 '''

                 adj_returns = self.returns.copy()

                 # drop rows until the first is January and the  last is December
                 if drop_orphans == True:
                     while adj_returns.index[0].month != 1:
                         adj_returns = adj_returns.iloc[1:]
                     while adj_returns.index[-1].month != 12:
                         adj_returns = adj_returns.iloc[:-1]

                 # create column names lists
                 ret_names = [f"{asset}_return" for asset in self.assets]
                 risk_names = [f"{asset}_risk" for asset in self.assets]
                 sharpe_names =[f"{asset}_sharpe" for asset in self.assets]

                 ann_ret_names = [f"{asset}_ann_return" for asset in self.assets]
                 ann_risk_names = [f"{asset}_ann_risk" for asset in self.assets]
                 ann_sharpe_names = [f"{asset}_ann_sharpe" for asset in self.assets]

                 # create some empty lists to store asset data (we will store lists in this list to effectively create a matrix)
                 asset_returns = []
                 asset_risks = []
                 asset_sharpes = []

                 asset_ann_returns = []
                 asset_ann_risks = []
                 asset_ann_sharpes =[]

                 years_list = []

                 n_periods = len(adj_returns) // (12*number_years) # only uses full-year data
```

```python
for i in range(n_periods):
    # splice the dataset, then calculate risk, return data and store it for use in a summary table
    spliced = adj_returns[(12*number_years*i):(12*number_years*(1+i))]

    if number_years == 1:
        # create name of years
        year = spliced.index[0].year
        years_list.append(year)

    elif number_years != 1:
        year_0 = spliced.index[0].year
        year_1 = spliced.index[-1].year
        year_concat = str(year_0) + "-" + str(year_1)
        years_list.append(year_concat)

    # calculate total by-asset return for the period
    returns = []
    for asset in self.assets:
        ret = np.prod([1 + r for r in spliced[asset]]) - 1 # compound all the returns for each asset
        returns.append(ret)

    asset_returns.append(returns)

    # calculate total by-asset risk for the period
    risks = []
    for asset in self.assets:
        risk = spliced[asset].std() * np.sqrt(12*number_years) # multiply by sqrt(12) to get total risk
        risks.append(risk)

    asset_risks.append(risks)

# calculate each sharpe by dividing each element in the arrays
asset_sharpes = np.array(asset_returns) / np.array(asset_risks)

temp_returns = pd.DataFrame(asset_returns, columns= ret_names)
temp_risks = pd.DataFrame(asset_risks, columns = risk_names)
temp_sharpes = pd.DataFrame(asset_sharpes, columns = sharpe_names)

# calculate "annualized" metrics for multi-year periods
if number_years != 1:
    asset_ann_returns = (np.array(asset_returns)+1) ** (1/number_years) -1
    asset_ann_risks = np.array(asset_risks) / np.sqrt(number_years)
    asset_ann_sharpes = np.array(asset_ann_returns) / np.array(asset_ann_risks)

    temp_ann_returns = pd.DataFrame(asset_ann_returns, columns= ann_ret_names)
    temp_ann_risks = pd.DataFrame(asset_ann_risks, columns = ann_risk_names)
    temp_ann_sharpes = pd.DataFrame(asset_ann_sharpes, columns = ann_sharpe_names)

    rolled = pd.concat([temp_ann_returns, temp_ann_risks, temp_ann_sharpes],
                       axis=1)

else: rolled = pd.concat([temp_returns, temp_risks, temp_sharpes], axis=1)

rolled["Time Frame"] = years_list
rolled.set_index("Time Frame", inplace=True)
return rolled
```

## A.4 Rolling Optimization Execution

## Analyze Data

```
In [20]:  # Set a copy of the formatted return data to work on
          returns_copy = returns.copy()
```

```
In [21]:  # create an analyzer to analyze the data
          analyzer = ReturnAnalyzer(returns_copy)
```

```
In [22]:  # Break Data into 10 Year Blocks for Optimization and create optimizers as well as correlation matrices
          # and average economic data figures  for each time period
          block = 10 # specify number of years to put into each block
          start_year = returns_copy.index.year[0] # get first year
          n_blocks = (returns_copy.index.year[-1] - returns_copy.index.year[0]) // block # calculate number of blocks
          year_list = []
          start_list = []
          optimizer_list = []
          corr_list = []
          econ_list = []

          for i in range(n_blocks):
              start_i = start_year+i*block
              end_i = start_i+block -1
              start_list.append(start_i)
              start_end = f"{start_i}_{end_i}"
              year_list.append(start_end)

              # splice data for each block using our splice method from the analyzer class
              block_returns = analyzer.splice_years(block, start_year-start_i)

              # create optimizer for each block using our optimizer class
              globals()[f"optimize_{start_end}"] = PortfolioOptimizer(block_returns)
              optimizer_list.append(globals()[f"optimize_{start_end}"])

              # create correlation matrix for each block
              corr = returns_copy.loc[(returns_copy.index.year >= start_i) & (returns_copy.index.year < end_i)].corr()
              corr_list.append(corr)

              # create tuple with average GDP and CPI for the time period
              avg_gdp = GDP.loc[(GDP.index >= start_i) & (GDP.index < end_i)].mean()
              avg_cpi = CPI.loc[(CPI.index >= start_i) & (CPI.index < end_i)].mean()
              econ = avg_gdp, avg_cpi
              econ_list.append(econ)
```

## The 10-year by asset returns and risks as follows:

```
In [23]:  rolled_returns = analyzer.roll_up_returns(block)
          rolled_returns
```

Out[23]:

| Time Frame | Gold_ann_return | SP500_ann_return | Nikkei_ann_return | US 10Y_ann_return | UK 10Y_ann_return | Gold_ann_risk | SP500_ann_risk | Nikkei_ann_risk | 10Y_ann_r |
|---|---|---|---|---|---|---|---|---|---|
| 1871-1880 | 0.313657 | 0.038943 | 0.188740 | 0.040665 | 0.087553 | 0.276554 | 0.130554 | 0.186245 | 0.0805 |
| 1881-1890 | -0.034996 | 0.094305 | 0.152429 | 0.136493 | 0.110036 | 0.165574 | 0.128594 | 0.240722 | 0.1018 |
| 1891-2000 | -0.032300 | 0.150082 | -0.050734 | 0.088170 | 0.093407 | 0.100270 | 0.101147 | 0.252056 | 0.0672 |
| 2001-2010 | 0.177470 | -0.006929 | -0.000969 | 0.056311 | 0.061765 | 0.137446 | 0.149284 | 0.191372 | 0.0847 |
| 2011-2020 | 0.029427 | 0.115243 | 0.082800 | 0.042878 | 0.035279 | 0.118362 | 0.113369 | 0.143736 | 0.0633 |

## Print optimized portfolios, asset correlations, and economic data for each time period

```
In [24]: for optimizer, corr, econ in zip(optimizer_list, corr_list, econ_list):
             print(f"\nTime Period: {optimizer.start_year} - {optimizer.end_year}")
             optimizer.summary(show="Maximum Sharpe")
             print(f"\nAsset Correlations:")
             print(corr)
             print(f"\nEconomic Data:")
             print(f"Average Annual Real GDP Growth: {econ[0].values[0]:.2%}")
             print(f"Average CPI Growth: {econ[1].values[0]:.2%}")
```

```
Time Period: 1971 - 1980

Maximum Sharpe Portfolio:
Gold: 24.08%
SP500: 0.00%
Nikkei: 23.67%
US 10Y: 43.74%
UK 10Y: 8.52%
 Annualized Return: 14.54%, Volatility: 9.19%, Sharpe: 1.58

Asset Correlations:
            Gold      SP500     Nikkei    US 10Y    UK 10Y
Gold    1.000000 -0.023952  0.128914 -0.077173 -0.008456
SP500  -0.023952  1.000000  0.421974  0.097206  0.197447
Nikkei  0.128914  0.421974  1.000000  0.013634  0.234572
US 10Y -0.077173  0.097206  0.013634  1.000000 -0.009357
UK 10Y -0.008456  0.197447  0.234572 -0.009357  1.000000
```

## Compare Optimized Portfolios against 60/40

```
In [25]:  # Get stock and bond return data
          stocks_bonds_only = returns_copy[["SP500", "US 10Y"]].copy()
          stocks_bonds_only_analyze = ReturnAnalyzer(stocks_bonds_only)
          # Initialize lists to store the results for each period
          rfr = 0
          time_frames = []
          opt_returns = []
          opt_risks = []
          opt_sharpes = []

          bench_returns = []
          bench_risks = []
          bench_sharpes = []

          for i in range(n_blocks):
              # Define year range
              start_i = start_year + i * block
              end_i = start_i + block - 1
              time_label = f"{start_i}-{end_i}"
              time_frames.append(time_label)

              # Get optimized max sharpe portfolio info
              optimizer = optimizer_list[i]
              w_opt, ret_opt, risk_opt, sharpe_opt = optimizer.optimize_max_sharpe()

              opt_returns.append(ret_opt)
              opt_risks.append(risk_opt)
              opt_sharpes.append(sharpe_opt)

              # Compute benchmark 60/40 portfolio
              sb_block_returns = stocks_bonds_only_analyze.splice_years(block, start_year=start_i)

              weights_6040 = np.array([0.6, 0.4])
              mu_6040 = (1 + sb_block_returns).prod() ** (1 / block) - 1  # annualized
              Sigma_6040 = sb_block_returns.cov().values * 12  # annualized

              ret_6040 = np.dot(weights_6040, mu_6040)
              var_6040 = np.dot(weights_6040.T, np.dot(Sigma_6040, weights_6040))
              vol_6040 = np.sqrt(var_6040)
              sharpe_6040 = (ret_6040 - rfr) / vol_6040

              bench_returns.append(ret_6040)
              bench_risks.append(vol_6040)
              bench_sharpes.append(sharpe_6040)

          # Create DataFrame for comparison
          comparison_df = pd.DataFrame({
              "Time Period": time_frames,
              "Optimized Return": opt_returns,
              "Optimized Risk": opt_risks,
              "Optimized Sharpe": opt_sharpes,
              "60/40 Return": bench_returns,
              "60/40 Risk": bench_risks,
              "60/40 Sharpe": bench_sharpes
          })

          # Format percentages and print
          comparison_formatted = comparison_df.copy()
          for col in ["Optimized Return", "Optimized Risk", "60/40 Return", "60/40 Risk"]:
              comparison_formatted[col] = comparison_formatted[col].apply(lambda x: f"{x:.2%}")

          comparison_formatted["Optimized Sharpe"] = comparison_formatted["Optimized Sharpe"].apply(lambda x: f"{x:.2f}")
          comparison_formatted["60/40 Sharpe"] = comparison_formatted["60/40 Sharpe"].apply(lambda x: f"{x:.2f}")
          comparison_formatted.set_index("Time Period", inplace=True)
          comparison_formatted
```

Out[25]:

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe |
|---|---|---|---|---|---|---|
| 1871-1880 | 14.54% | 9.19% | 1.58 | 3.96% | 8.40% | 0.47 |
| 1881-1890 | 12.93% | 8.89% | 1.45 | 11.12% | 9.71% | 1.15 |
| 1891-2000 | 11.42% | 5.67% | 2.02 | 12.53% | 6.57% | 1.91 |
| 2001-2010 | 11.90% | 8.49% | 1.40 | 1.84% | 8.52% | 0.22 |
| 2011-2020 | 6.98% | 4.33% | 1.61 | 8.63% | 6.14% | 1.41 |

**Let's design a custom portfolio of 15% Gold, 35% Stocks, 42% US Bonds and 8% UK Bonds and see how it performs compared to the optimized portfolio and 60/40**

In [26]:
```
# Asset list
optimizer_list[0].assets
```

Out[26]: ['Gold', 'SP500', 'Nikkei', 'US 10Y', 'UK 10Y']

In [27]:
```
# Build custom portfolio
custom_weights = np.array([0.15, 0.35, 0, 0.42, 0.08])
# Gold, SP500, Nikkei, US Bonds, UK Bonds

# Get risk and return metrics for each time period
custom_returns = []
custom_risks = []
custom_sharpes = []
for i in range(len(optimizer_list)):
    ret = optimizer_list[i].portfolio_return(custom_weights)
    risk = np.sqrt(optimizer_list[i].portfolio_variance(custom_weights))
    sharpe = (ret - rfr) / (risk)
    custom_returns.append(ret)
    custom_risks.append(risk)
    custom_sharpes.append(sharpe)

# Append summary table with custom portfolio performance
comparison_formatted["Custom Return"] = custom_returns
comparison_formatted["Custom Risk"] = custom_risks
comparison_formatted["Custom Sharpe"] = custom_sharpes

for col in ["Custom Return", "Custom Risk"]:
    comparison_formatted[col] = comparison_formatted[col].apply(lambda x: f"{x:.2%}")
comparison_formatted["Custom Sharpe"] = comparison_formatted["Custom Sharpe"].apply(lambda x: f"{x:.2f}")
comparison_formatted
```

Out[27]:

| Time Period | Optimized Return | Optimized Risk | Optimized Sharpe | 60/40 Return | 60/40 Risk | 60/40 Sharpe | Custom Return | Custom Risk | Custom Sharpe |
|---|---|---|---|---|---|---|---|---|---|
| 1871-1880 | 14.54% | 9.19% | 1.58 | 3.96% | 8.40% | 0.47 | 8.48% | 7.26% | 1.17 |
| 1881-1890 | 12.93% | 8.89% | 1.45 | 11.12% | 9.71% | 1.15 | 9.39% | 7.94% | 1.18 |
| 1891-2000 | 11.42% | 5.67% | 2.02 | 12.53% | 6.57% | 1.91 | 9.22% | 4.84% | 1.91 |
| 2001-2010 | 11.90% | 8.49% | 1.40 | 1.84% | 8.52% | 0.22 | 5.28% | 6.22% | 0.85 |
| 2011-2020 | 6.98% | 4.33% | 1.61 | 8.63% | 6.14% | 1.41 | 6.56% | 4.54% | 1.44 |

## Appendix B: Additional Data Tables

### B.1 Optimized Portfolio Weights per Period

#### 5 Assets, 10-year periods

| Time Period | Annualized Return | Annualized Risk | Sharpe Ratio | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|---|---|---|
| 1971-1980 | 14.5% | 9.2% | 1.58 | 24% | 0% | 24% | 44% | 9% |
| 1981-1990 | 12.9% | 8.9% | 1.45 | 0% | 14% | 7% | 70% | 9% |
| 1991-2000 | 11.4% | 5.7% | 2.02 | 0% | 42% | 0% | 53% | 5% |
| 2001-2010 | 11.9% | 8.5% | 1.40 | 54% | 4% | 0% | 40% | 2% |
| 2011-2020 | 7.0% | 4.3% | 1.61 | 0% | 37% | 1% | 58% | 4% |

#### 5 Assets, 5-year periods

| Time Period | Annualized Return | Annualized Risk | Sharpe Ratio | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|---|---|---|
| 1971-1975 | 13.6% | 7.8% | 1.75 | 24% | 0% | 15% | 61% | 0% |
| 1976-1980 | 15.2% | 9.2% | 1.64 | 16% | 21% | 31% | 18% | 14% |
| 1981-1985 | 16.2% | 10.1% | 1.59 | 0% | 7% | 12% | 80% | 0% |
| 1986-1990 | 9.8% | 6.0% | 1.62 | 20% | 20% | 0% | 42% | 18% |
| 1991-1995 | 10.2% | 4.5% | 2.28 | 16% | 43% | 0% | 38% | 3% |
| 1996-2000 | 10.9% | 5.8% | 1.87 | 0% | 39% | 0% | 53% | 8% |
| 2001-2005 | 11.0% | 7.6% | 1.45 | 57% | 0% | 0% | 13% | 30% |
| 2006-2010 | 13.2% | 9.0% | 1.47 | 48% | 7% | 0% | 45% | 0% |
| 2011-2015 | 6.9% | 4.0% | 1.72 | 0% | 42% | 0% | 46% | 12% |
| 2016-2020 | 8.0% | 4.5% | 1.76 | 16% | 23% | 8% | 53% | 0% |

#### 9 Assets, 7-year periods

| Time Period | Annualized Return | Annualized Risk | Sharpe Ratio | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1990-1996 | 9.6% | 4.8% | 1.99 | 4% | 9% | 0% | 45% | 0% | 0% | 27% | 11% | 4% |
| 1997-2003 | 7.6% | 5.9% | 1.29 | 0% | 2% | 0% | 19% | 0% | 0% | 55% | 24% | 0% |
| 2004-2010 | 12.1% | 8.0% | 1.50 | 40% | 2% | 7% | 3% | 1% | 0% | 48% | 0% | 0% |
| 2011-2017 | 7.3% | 4.2% | 1.74 | 0% | 0% | 0% | 49% | 0% | 0% | 48% | 3% | 0% |
| 2018-2024 | 11.6% | 9.0% | 1.29 | 60% | 0% | 0% | 40% | 0% | 0% | 0% | 0% | 0% |

**9 Assets, 5-year periods**

| Time Period | Annualized Return | Annualized Risk | Sharpe Ratio | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1990-1994 | 7.9% | 5.2% | 1.51 | 4% | 3% | 9% | 24% | 0% | 0% | 30% | 6% | 24% |
| 1995-1999 | 16.2% | 5.6% | 2.89 | 0% | 4% | 0% | 35% | 0% | 15% | 35% | 10% | 0% |
| 2000-2004 | 9.7% | 6.4% | 1.51 | 11% | 3% | 21% | 0% | 0% | 0% | 42% | 23% | 0% |
| 2005-2009 | 12.6% | 9.2% | 1.36 | 45% | 1% | 5% | 0% | 1% | 0% | 49% | 0% | 0% |
| 2010-2014 | 8.6% | 4.2% | 2.05 | 2% | 0% | 0% | 42% | 0% | 0% | 45% | 11% | 0% |
| 2015-2019 | 5.7% | 4.0% | 1.42 | 6% | 0% | 0% | 46% | 0% | 0% | 48% | 0% | 0% |
| 2020-2024 | 12.8% | 9.8% | 1.31 | 63% | 0% | 0% | 37% | 0% | 0% | 0% | 0% | 0% |

**11 Assets, 7-year periods**

| Time Period | 1997-2003 | 2004-2010 | 2011-2017 | 2018-2024 |
|---|---|---|---|---|
| Annualized Return | 10.1% | 11.5% | 7.3% | 11.6% |
| Annualized Risk | 4.7% | 7.2% | 4.2% | 9.0% |
| Sharpe Ratio | 2.17 | 1.60 | 1.75 | 1.29 |
| Gold | 0% | 34% | 0% | 60% |
| Oil | 1% | 1% | 0% | 0% |
| Copper | 0% | 3% | 0% | 0% |
| SP500 | 0% | 0% | 44% | 40% |
| Nikkei | 0% | 0% | 0% | 0% |
| FTSE | 0% | 0% | 0% | 0% |
| US 10Y | 0% | 42% | 46% | 0% |
| UK 10Y | 9% | 0% | 0% | 0% |
| JP 10Y | 0% | 0% | 0% | 0% |
| US IG | 59% | 0% | 2% | 0% |
| US HY | 30% | 20% | 8% | 0% |

**11 Assets, 4-year periods**

| Time Period | 1997-2000 | 2001-2004 | 2005-2008 | 2009-2012 | 2013-2016 | 2017-2020 | 2021-2024 |
|---|---|---|---|---|---|---|---|
| Annualized Return | 8.7% | 16.6% | 10.6% | 17.2% | 8.3% | 8.5% | 11.6% |
| Annualized Risk | 4.1% | 5.5% | 7.5% | 4.9% | 5.3% | 3.8% | 8.4% |
| Sharpe Ratio | 2.11 | 3.01 | 1.42 | 3.55 | 1.57 | 2.22 | 1.38 |
| Gold | 0% | 15% | 25% | 26% | 0% | 14% | 30% |
| Oil | 0% | 3% | 1% | 0% | 0% | 0% | 11% |
| Copper | 0% | 4% | 0% | 0% | 0% | 3% | 0% |
| SP500 | 19% | 0% | 0% | 0% | 62% | 14% | 59% |
| Nikkei | 0% | 0% | 0% | 0% | 0% | 11% | 0% |
| FTSE | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| US 10Y | 11% | 0% | 73% | 27% | 30% | 57% | 0% |
| UK 10Y | 0% | 18% | 0% | 0% | 0% | 2% | 0% |
| JP 10Y | 1% | 0% | 0% | 0% | 0% | 0% | 0% |
| US IG | 69% | 12% | 0% | 12% | 0% | 0% | 0% |
| US HY | 0% | 49% | 0% | 35% | 8% | 0% | 0% |

# B.2 Sharpe Ratio, Returns, and Risk Tables

### 5 Assets, 10-year periods

| Time Frame | 1971-1980 | 1981-1990 | 1991-2000 | 2001-2010 | 2011-2020 |
|---|---|---|---|---|---|
| Gold_ann_return | 31.4% | -3.5% | -3.2% | 17.7% | 2.9% |
| SP500_ann_return | 3.9% | 9.4% | 15.0% | -0.7% | 11.5% |
| Nikkei_ann_return | 18.9% | 15.2% | -5.1% | -0.1% | 8.3% |
| US 10Y_ann_return | 4.1% | 13.6% | 8.8% | 5.6% | 4.3% |
| UK 10Y_ann_return | 8.8% | 11.0% | 9.3% | 6.2% | 3.5% |
| Gold_ann_risk | 27.7% | 16.6% | 10.0% | 13.7% | 11.8% |
| SP500_ann_risk | 13.1% | 12.9% | 10.1% | 14.9% | 11.3% |
| Nikkei_ann_risk | 18.6% | 24.1% | 25.2% | 19.1% | 14.4% |
| US 10Y_ann_risk | 8.1% | 10.2% | 6.7% | 8.5% | 6.3% |
| UK 10Y_ann_risk | 15.9% | 16.7% | 11.1% | 9.2% | 8.5% |
| Gold_ann_sharpe | 1.13 | -0.21 | -0.32 | 1.29 | 0.25 |
| SP500_ann_sharpe | 0.30 | 0.73 | 1.48 | -0.05 | 1.02 |
| Nikkei_ann_sharpe | 1.01 | 0.63 | -0.20 | -0.01 | 0.58 |
| US 10Y_ann_sharpe | 0.50 | 1.34 | 1.31 | 0.66 | 0.68 |
| UK 10Y_ann_sharpe | 0.55 | 0.66 | 0.84 | 0.67 | 0.42 |

### 5 Assets, 5-year periods

| Time Frame | 1971-1975 | 1976-1980 | 1981-1985 | 1986-1990 | 1991-1995 | 1996-2000 | 2001-2005 | 2006-2010 | 2011-2015 | 2016-2020 |
|---|---|---|---|---|---|---|---|---|---|---|
| Gold_ann_return | 31.7% | 31.0% | -9.8% | 3.2% | 0.6% | -6.9% | 13.4% | 22.2% | -5.0% | 11.6% |
| SP500_ann_return | -0.5% | 8.5% | 9.2% | 9.7% | 13.3% | 16.7% | -1.1% | -0.3% | 10.6% | 12.5% |
| Nikkei_ann_return | 19.9% | 17.9% | 11.4% | 19.2% | 0.8% | -10.6% | 1.8% | -2.0% | 5.6% | 11.0% |
| US 10Y_ann_return | 5.2% | 2.9% | 17.5% | 9.9% | 10.9% | 6.8% | 5.6% | 5.7% | 4.1% | 4.5% |
| UK 10Y_ann_return | 1.4% | 16.7% | 5.6% | 16.6% | 9.1% | 9.6% | 8.9% | 3.5% | 4.4% | 2.7% |
| Gold_ann_risk | 24.1% | 31.0% | 19.4% | 13.1% | 7.9% | 11.7% | 10.1% | 16.6% | 12.6% | 10.7% |
| SP500_ann_risk | 14.7% | 11.2% | 12.3% | 13.5% | 7.7% | 12.1% | 13.4% | 16.4% | 9.4% | 13.1% |
| Nikkei_ann_risk | 21.6% | 15.3% | 18.4% | 28.8% | 26.1% | 24.4% | 19.8% | 18.6% | 13.6% | 15.2% |
| US 10Y_ann_risk | 6.5% | 9.4% | 11.4% | 8.7% | 6.8% | 6.6% | 8.1% | 8.9% | 6.5% | 6.2% |
| UK 10Y_ann_risk | 13.4% | 17.8% | 15.9% | 17.4% | 12.4% | 9.7% | 8.6% | 9.8% | 7.6% | 9.3% |
| Gold_ann_sharpe | 1.32 | 1.00 | -0.50 | 0.25 | 0.07 | -0.59 | 1.33 | 1.34 | -0.40 | 1.08 |
| SP500_ann_sharpe | -0.04 | 0.76 | 0.75 | 0.72 | 1.73 | 1.38 | -0.08 | -0.02 | 1.13 | 0.95 |
| Nikkei_ann_sharpe | 0.92 | 1.17 | 0.62 | 0.67 | 0.03 | -0.43 | 0.09 | -0.11 | 0.41 | 0.72 |
| US 10Y_ann_sharpe | 0.80 | 0.31 | 1.54 | 1.13 | 1.60 | 1.02 | 0.69 | 0.64 | 0.62 | 0.73 |
| UK 10Y_ann_sharpe | 0.10 | 0.93 | 0.36 | 0.95 | 0.74 | 0.99 | 1.03 | 0.35 | 0.58 | 0.29 |

**9 Assets, 7-year periods**

| Time Frame | 1990-1996 | 1997-2003 | 2004-2010 | 2011-2017 | 2018-2024 |
|---|---|---|---|---|---|
| Gold_ann_return | -1.5% | 1.4% | 19.2% | -1.3% | 11.1% |
| Oil_ann_return | 2.6% | 3.5% | 15.7% | -6.0% | 2.8% |
| Copper_ann_return | -0.6% | -0.4% | 22.6% | -4.1% | 3.9% |
| SP500_ann_return | 11.4% | 5.5% | 2.0% | 11.5% | 12.3% |
| Nikkei_ann_return | -7.7% | -8.6% | 3.0% | 7.6% | 2.8% |
| FTSE_ann_return | 9.1% | 2.2% | 1.8% | 1.7% | -0.6% |
| US 10Y_ann_return | 8.8% | 7.8% | 5.1% | 3.3% | -0.1% |
| UK 10Y_ann_return | 13.3% | 9.1% | 4.1% | 3.1% | -3.2% |
| JP 10Y_ann_return | 10.6% | 2.9% | 4.2% | -3.2% | -6.7% |
| Gold_ann_risk | 8.6% | 12.3% | 14.9% | 12.3% | 10.5% |
| Oil_ann_risk | 29.6% | 28.3% | 32.1% | 28.1% | 45.9% |
| Copper_ann_risk | 19.3% | 15.1% | 31.3% | 15.3% | 16.3% |
| SP500_ann_risk | 8.9% | 14.5% | 14.5% | 8.9% | 13.2% |
| Nikkei_ann_risk | 28.8% | 24.1% | 18.1% | 13.1% | 17.1% |
| FTSE_ann_risk | 16.0% | 15.1% | 18.8% | 14.3% | 17.2% |
| US 10Y_ann_risk | 7.0% | 7.6% | 8.3% | 6.3% | 7.9% |
| UK 10Y_ann_risk | 12.5% | 9.5% | 9.3% | 8.4% | 11.5% |
| JP 10Y_ann_risk | 13.4% | 12.6% | 11.7% | 10.3% | 10.2% |
| Gold_ann_sharpe | -0.17 | 0.11 | 1.28 | -0.11 | 1.06 |
| Oil_ann_sharpe | 0.09 | 0.12 | 0.49 | -0.21 | 0.06 |
| Copper_ann_sharpe | -0.03 | -0.03 | 0.72 | -0.27 | 0.24 |
| SP500_ann_sharpe | 1.28 | 0.38 | 0.14 | 1.29 | 0.93 |
| Nikkei_ann_sharpe | -0.27 | -0.36 | 0.16 | 0.58 | 0.16 |
| FTSE_ann_sharpe | 0.57 | 0.15 | 0.09 | 0.12 | -0.03 |
| US 10Y_ann_sharpe | 1.27 | 1.03 | 0.62 | 0.53 | -0.02 |
| UK 10Y_ann_sharpe | 1.07 | 0.95 | 0.44 | 0.37 | -0.28 |
| JP 10Y_ann_sharpe | 0.79 | 0.23 | 0.36 | -0.31 | -0.66 |

**9 Assets, 5-year periods**

| Time Frame | 1990-1994 | 1995-1999 | 2000-2004 | 2005-2009 | 2010-2014 | 2015-2019 | 2020-2024 |
|---|---|---|---|---|---|---|---|
| Gold_ann_return | -1.5% | -5.7% | 9.3% | 20.7% | 1.1% | 4.3% | 12.4% |
| Oil_ann_return | -4.0% | 8.7% | 10.6% | 11.5% | -4.5% | 0.2% | 3.2% |
| Copper_ann_return | 4.7% | -10.0% | 12.2% | 17.3% | -1.6% | -1.2% | 8.0% |
| SP500_ann_return | 5.5% | 25.7% | -3.4% | -1.5% | 13.1% | 9.1% | 13.6% |
| Nikkei_ann_return | -7.5% | -2.7% | -10.8% | 0.5% | 5.4% | 8.5% | 2.2% |
| FTSE_ann_return | 4.6% | 17.7% | -2.6% | -1.3% | 2.9% | -0.4% | 0.0% |
| US 10Y_ann_return | 7.6% | 8.2% | 8.4% | 4.7% | 5.4% | 2.7% | -2.0% |
| UK 10Y_ann_return | 11.9% | 12.6% | 10.0% | 2.3% | 5.6% | 0.3% | -5.3% |
| JP 10Y_ann_return | 14.2% | 4.7% | 1.0% | 2.6% | -3.3% | 1.9% | -9.9% |
| Gold_ann_risk | 9.8% | 11.1% | 10.1% | 16.7% | 12.7% | 10.5% | 11.0% |
| Oil_ann_risk | 32.9% | 25.8% | 27.8% | 35.0% | 22.6% | 30.5% | 52.1% |
| Copper_ann_risk | 18.3% | 19.0% | 15.0% | 34.6% | 16.5% | 15.2% | 17.7% |
| SP500_ann_risk | 9.2% | 11.1% | 13.7% | 15.7% | 10.1% | 9.0% | 14.3% |
| Nikkei_ann_risk | 31.8% | 22.2% | 23.2% | 18.5% | 14.1% | 12.6% | 18.6% |
| FTSE_ann_risk | 18.1% | 11.4% | 15.1% | 19.5% | 16.5% | 13.8% | 18.3% |
| US 10Y_ann_risk | 6.8% | 6.9% | 8.1% | 8.3% | 7.2% | 6.0% | 8.5% |
| UK 10Y_ann_risk | 14.1% | 8.7% | 9.3% | 9.0% | 8.3% | 9.1% | 12.6% |
| JP 10Y_ann_risk | 12.0% | 15.8% | 9.9% | 11.7% | 10.4% | 9.8% | 11.1% |
| Gold_ann_sharpe | -0.15 | -0.51 | 0.92 | 1.24 | 0.09 | 0.40 | 1.12 |
| Oil_ann_sharpe | -0.12 | 0.34 | 0.38 | 0.33 | -0.20 | 0.01 | 0.06 |
| Copper_ann_sharpe | 0.26 | -0.52 | 0.81 | 0.50 | -0.09 | -0.08 | 0.45 |
| SP500_ann_sharpe | 0.59 | 2.32 | -0.25 | -0.10 | 1.30 | 1.01 | 0.95 |
| Nikkei_ann_sharpe | -0.23 | -0.12 | -0.47 | 0.02 | 0.39 | 0.68 | 0.12 |
| FTSE_ann_sharpe | 0.25 | 1.56 | -0.17 | -0.07 | 0.17 | -0.03 | 0.00 |
| US 10Y_ann_sharpe | 1.11 | 1.20 | 1.03 | 0.56 | 0.76 | 0.44 | -0.23 |
| UK 10Y_ann_sharpe | 0.84 | 1.44 | 1.08 | 0.25 | 0.67 | 0.03 | -0.42 |
| JP 10Y_ann_sharpe | 1.18 | 0.30 | 0.10 | 0.22 | -0.31 | 0.19 | -0.89 |

**11 Assets, 7-year periods**

| Time Frame | 1997-2003 | 2004-2010 | 2011-2017 | 2018-2024 |
|---|---|---|---|---|
| Gold_ann_return | 1.4% | 19.2% | -1.3% | 11.1% |
| Oil_ann_return | 3.5% | 15.7% | -6.0% | 2.8% |
| Copper_ann_return | -0.4% | 22.6% | -4.1% | 3.9% |
| SP500_ann_return | 5.5% | 2.0% | 11.5% | 12.3% |
| Nikkei_ann_return | -8.6% | 3.0% | 7.6% | 2.8% |
| FTSE_ann_return | 2.2% | 1.8% | 1.7% | -0.6% |
| US 10Y_ann_return | 7.8% | 5.1% | 3.3% | -0.1% |
| UK 10Y_ann_return | 9.1% | 4.1% | 3.1% | -3.2% |
| JP 10Y_ann_return | 2.9% | 4.2% | -3.2% | -6.7% |
| US IG_ann_return | 8.9% | 5.9% | 4.0% | 2.0% |
| US HY_ann_return | 13.1% | 9.7% | 8.3% | 5.3% |
| Gold_ann_risk | 12.3% | 14.9% | 12.3% | 10.5% |
| Oil_ann_risk | 28.3% | 32.1% | 28.1% | 45.9% |
| Copper_ann_risk | 15.1% | 31.3% | 15.3% | 16.3% |
| SP500_ann_risk | 14.5% | 14.5% | 8.9% | 13.2% |
| Nikkei_ann_risk | 24.1% | 18.1% | 13.1% | 17.1% |
| FTSE_ann_risk | 15.1% | 18.8% | 14.3% | 17.2% |
| US 10Y_ann_risk | 7.6% | 8.3% | 6.3% | 7.9% |
| UK 10Y_ann_risk | 9.5% | 9.3% | 8.4% | 11.5% |
| JP 10Y_ann_risk | 12.6% | 11.7% | 10.3% | 10.2% |
| US IG_ann_risk | 4.8% | 6.4% | 3.4% | 6.8% |
| US HY_ann_risk | 8.3% | 13.8% | 8.2% | 12.1% |
| Gold_ann_sharpe | 0.11 | 1.28 | -0.11 | 1.06 |
| Oil_ann_sharpe | 0.12 | 0.49 | -0.21 | 0.06 |
| Copper_ann_sharpe | -0.03 | 0.72 | -0.27 | 0.24 |
| SP500_ann_sharpe | 0.38 | 0.14 | 1.29 | 0.93 |
| Nikkei_ann_sharpe | -0.36 | 0.16 | 0.58 | 0.16 |
| FTSE_ann_sharpe | 0.15 | 0.09 | 0.12 | -0.03 |
| US 10Y_ann_sharpe | 1.03 | 0.62 | 0.53 | -0.02 |
| UK 10Y_ann_sharpe | 0.95 | 0.44 | 0.37 | -0.28 |
| JP 10Y_ann_sharpe | 0.23 | 0.36 | -0.31 | -0.66 |
| US IG_ann_sharpe | 1.86 | 0.92 | 1.20 | 0.29 |
| US HY_ann_sharpe | 1.57 | 0.70 | 1.01 | 0.44 |

**11 Assets, 4-year periods**

| Time Frame | 1997-2000 | 2001-2004 | 2005-2008 | 2009-2012 | 2013-2016 | 2017-2020 | 2021-2024 |
|---|---|---|---|---|---|---|---|
| Gold_ann_return | -7.4% | 13.0% | 16.6% | 19.9% | -9.0% | 12.6% | 9.3% |
| Oil_ann_return | 3.0% | 11.0% | -1.2% | 20.9% | -12.3% | -2.5% | 10.5% |
| Copper_ann_return | -4.9% | 14.1% | -0.3% | 26.6% | -8.2% | 8.2% | 3.5% |
| SP500_ann_return | 15.7% | -2.6% | -7.5% | 12.8% | 12.1% | 13.2% | 12.9% |
| Nikkei_ann_return | -9.9% | -2.7% | -2.8% | 5.2% | 8.3% | 13.0% | -2.2% |
| FTSE_ann_return | 7.6% | 0.1% | -8.6% | 9.7% | -2.0% | -0.2% | 3.4% |
| US 10Y_ann_return | 8.5% | 6.2% | 8.8% | 3.7% | 0.6% | 5.6% | -4.9% |
| UK 10Y_ann_return | 7.5% | 12.4% | -0.1% | 10.2% | -3.8% | 6.4% | -8.6% |
| JP 10Y_ann_return | 2.4% | 3.7% | 4.2% | 2.1% | -6.3% | 2.9% | -13.2% |
| US IG_ann_return | 6.9% | 9.5% | 1.4% | 12.4% | 2.2% | 5.9% | -1.3% |
| US HY_ann_return | 5.0% | 21.5% | -5.2% | 27.2% | 6.8% | 8.9% | 2.4% |
| Gold_ann_risk | 12.9% | 10.1% | 17.3% | 13.2% | 12.0% | 9.6% | 10.8% |
| Oil_ann_risk | 29.9% | 26.8% | 34.3% | 26.9% | 32.3% | 55.5% | 26.8% |
| Copper_ann_risk | 16.5% | 15.3% | 35.9% | 22.2% | 15.2% | 14.9% | 16.6% |
| SP500_ann_risk | 12.8% | 14.6% | 14.6% | 13.7% | 8.4% | 13.7% | 11.1% |
| Nikkei_ann_risk | 26.0% | 20.7% | 16.5% | 18.2% | 13.1% | 15.4% | 17.0% |
| FTSE_ann_risk | 13.9% | 15.3% | 17.3% | 20.8% | 13.9% | 17.9% | 14.8% |
| US 10Y_ann_risk | 6.5% | 8.5% | 7.8% | 8.4% | 6.5% | 5.9% | 8.7% |
| UK 10Y_ann_risk | 10.0% | 8.9% | 8.6% | 8.8% | 9.4% | 8.2% | 13.2% |
| JP 10Y_ann_risk | 14.7% | 9.5% | 10.6% | 11.9% | 11.8% | 5.9% | 12.1% |
| US IG_ann_risk | 4.0% | 5.7% | 6.5% | 4.9% | 3.5% | 5.8% | 6.8% |
| US HY_ann_risk | 6.0% | 9.0% | 13.6% | 12.8% | 7.9% | 12.1% | 10.7% |
| Gold_ann_sharpe | -0.57 | 1.29 | 0.96 | 1.51 | -0.75 | 1.30 | 0.86 |
| Oil_ann_sharpe | 0.10 | 0.41 | -0.03 | 0.78 | -0.38 | -0.04 | 0.39 |
| Copper_ann_sharpe | -0.30 | 0.92 | -0.01 | 1.20 | -0.54 | 0.55 | 0.21 |
| SP500_ann_sharpe | 1.22 | -0.18 | -0.51 | 0.93 | 1.44 | 0.97 | 1.16 |
| Nikkei_ann_sharpe | -0.38 | -0.13 | -0.17 | 0.28 | 0.64 | 0.84 | -0.13 |
| FTSE_ann_sharpe | 0.55 | 0.00 | -0.50 | 0.47 | -0.14 | -0.01 | 0.23 |
| US 10Y_ann_sharpe | 1.32 | 0.73 | 1.12 | 0.45 | 0.09 | 0.96 | -0.56 |
| UK 10Y_ann_sharpe | 0.75 | 1.40 | -0.01 | 1.16 | -0.40 | 0.79 | -0.65 |
| JP 10Y_ann_sharpe | 0.16 | 0.39 | 0.40 | 0.18 | -0.53 | 0.49 | -1.09 |
| US IG_ann_sharpe | 1.75 | 1.67 | 0.21 | 2.53 | 0.62 | 1.02 | -0.18 |
| US HY_ann_sharpe | 0.82 | 2.38 | -0.38 | 2.12 | 0.86 | 0.73 | 0.23 |

# B.3 Correlation Matrices

**5 Assets, 10-year periods**

**1971-1980:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | -0.02 | 0.13 | -0.08 | -0.01 |
| SP500 | -0.02 | 1.00 | 0.42 | 0.10 | 0.20 |
| Nikkei | 0.13 | 0.42 | 1.00 | 0.01 | 0.23 |
| US 10Y | -0.08 | 0.10 | 0.01 | 1.00 | -0.01 |
| UK 10Y | -0.01 | 0.20 | 0.23 | -0.01 | 1.00 |

**1981-1990:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | 0.05 | 0.12 | 0.06 | 0.13 |
| SP500 | 0.05 | 1.00 | 0.32 | 0.26 | 0.06 |
| Nikkei | 0.12 | 0.32 | 1.00 | 0.10 | 0.41 |
| US 10Y | 0.06 | 0.26 | 0.10 | 1.00 | 0.24 |
| UK 10Y | 0.13 | 0.06 | 0.41 | 0.24 | 1.00 |

**1991-2000:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | -0.18 | 0.09 | -0.06 | 0.00 |
| SP500 | -0.18 | 1.00 | 0.19 | -0.02 | 0.09 |
| Nikkei | 0.09 | 0.19 | 1.00 | 0.04 | 0.22 |
| US 10Y | -0.06 | -0.02 | 0.04 | 1.00 | 0.39 |
| UK 10Y | 0.00 | 0.09 | 0.22 | 0.39 | 1.00 |

**2001-2010:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | 0.01 | 0.05 | 0.12 | 0.42 |
| SP500 | 0.01 | 1.00 | 0.56 | -0.30 | 0.18 |
| Nikkei | 0.05 | 0.56 | 1.00 | -0.07 | 0.22 |
| US 10Y | 0.12 | -0.30 | -0.07 | 1.00 | 0.28 |
| UK 10Y | 0.42 | 0.18 | 0.22 | 0.28 | 1.00 |

**2011-2020:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | -0.21 | -0.13 | 0.30 | 0.36 |
| SP500 | -0.21 | 1.00 | 0.59 | -0.43 | 0.10 |
| Nikkei | -0.13 | 0.59 | 1.00 | -0.29 | 0.31 |
| US 10Y | 0.30 | -0.43 | -0.29 | 1.00 | 0.07 |
| UK 10Y | 0.36 | 0.10 | 0.31 | 0.07 | 1.00 |

**5 Assets, 5-year periods**

**1971-1975:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | 0.07 | 0.13 | -0.12 | -0.07 |
| SP500 | 0.07 | 1.00 | 0.48 | 0.05 | 0.13 |
| Nikkei | 0.13 | 0.48 | 1.00 | -0.12 | 0.18 |
| US 10Y | -0.12 | 0.05 | -0.12 | 1.00 | 0.10 |
| UK 10Y | -0.07 | 0.13 | 0.18 | 0.10 | 1.00 |

**1976-1980:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | -0.04 | 0.07 | -0.08 | 0.04 |
| SP500 | -0.04 | 1.00 | 0.23 | 0.18 | 0.10 |
| Nikkei | 0.07 | 0.23 | 1.00 | 0.05 | 0.24 |
| US 10Y | -0.08 | 0.18 | 0.05 | 1.00 | -0.03 |
| UK 10Y | 0.04 | 0.10 | 0.24 | -0.03 | 1.00 |

**1981-1985:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|-------|------|-------|--------|--------|--------|
| Gold | 1.00 | 0.30 | 0.09 | 0.24 | 0.17 |
| SP500 | 0.30 | 1.00 | 0.41 | 0.37 | 0.33 |
| Nikkei | 0.09 | 0.41 | 1.00 | 0.13 | 0.15 |
| US 10Y | 0.24 | 0.37 | 0.13 | 1.00 | 0.45 |
| UK 10Y | 0.17 | 0.33 | 0.15 | 0.45 | 1.00 |

**1986-1990:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.26 | 0.13 | -0.24 | -0.01 |
| SP500 | -0.26 | 1.00 | 0.28 | 0.18 | -0.09 |
| Nikkei | 0.13 | 0.28 | 1.00 | 0.08 | 0.62 |
| US 10Y | -0.24 | 0.18 | 0.08 | 1.00 | 0.10 |
| UK 10Y | -0.01 | -0.09 | 0.62 | 0.10 | 1.00 |

**1991-1995:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.32 | -0.03 | -0.06 | -0.04 |
| SP500 | -0.32 | 1.00 | 0.19 | 0.08 | 0.01 |
| Nikkei | -0.03 | 0.19 | 1.00 | 0.25 | 0.39 |
| US 10Y | -0.06 | 0.08 | 0.25 | 1.00 | 0.39 |
| UK 10Y | -0.04 | 0.01 | 0.39 | 0.39 | 1.00 |

**1996-2000:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.10 | 0.16 | -0.07 | 0.03 |
| SP500 | -0.10 | 1.00 | 0.23 | -0.13 | 0.13 |
| Nikkei | 0.16 | 0.23 | 1.00 | -0.08 | -0.06 |
| US 10Y | -0.07 | -0.13 | -0.08 | 1.00 | 0.49 |
| UK 10Y | 0.03 | 0.13 | -0.06 | 0.49 | 1.00 |

**2001-2005:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | 0.04 | 0.26 | 0.29 | 0.48 |
| SP500 | 0.04 | 1.00 | 0.46 | -0.31 | -0.02 |
| Nikkei | 0.26 | 0.46 | 1.00 | -0.16 | 0.03 |
| US 10Y | 0.29 | -0.31 | -0.16 | 1.00 | 0.51 |
| UK 10Y | 0.48 | -0.02 | 0.03 | 0.51 | 1.00 |

**2006-2010:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.01 | -0.13 | 0.06 | 0.46 |
| SP500 | -0.01 | 1.00 | 0.67 | -0.32 | 0.37 |
| Nikkei | -0.13 | 0.67 | 1.00 | 0.02 | 0.45 |
| US 10Y | 0.06 | -0.32 | 0.02 | 1.00 | 0.07 |
| UK 10Y | 0.46 | 0.37 | 0.45 | 0.07 | 1.00 |

**2011-2015:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.33 | -0.28 | 0.31 | 0.29 |
| SP500 | -0.33 | 1.00 | 0.48 | -0.44 | -0.11 |
| Nikkei | -0.28 | 0.48 | 1.00 | -0.28 | 0.28 |
| US 10Y | 0.31 | -0.44 | -0.28 | 1.00 | 0.31 |
| UK 10Y | 0.29 | -0.11 | 0.28 | 0.31 | 1.00 |

**2016-2020:**

| Asset | Gold | SP500 | Nikkei | US 10Y | UK 10Y |
|---|---|---|---|---|---|
| Gold | 1.00 | -0.11 | -0.04 | 0.32 | 0.43 |
| SP500 | -0.11 | 1.00 | 0.68 | -0.44 | 0.24 |
| Nikkei | -0.04 | 0.68 | 1.00 | -0.33 | 0.37 |
| US 10Y | 0.32 | -0.44 | -0.33 | 1.00 | -0.07 |
| UK 10Y | 0.43 | 0.24 | 0.37 | -0.07 | 1.00 |

**9 Assets, 7-year periods**

**1990-1996:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.35 | 0.06 | -0.37 | -0.04 | -0.11 | -0.12 | 0.01 | 0.00 |
| Oil | 0.35 | 1.00 | 0.18 | -0.59 | -0.10 | -0.16 | -0.13 | 0.03 | -0.06 |
| Copper | 0.06 | 0.18 | 1.00 | -0.10 | -0.09 | -0.20 | -0.13 | -0.11 | -0.10 |
| SP500 | -0.37 | -0.59 | -0.10 | 1.00 | 0.21 | 0.34 | 0.28 | 0.05 | 0.14 |
| Nikkei | -0.04 | -0.10 | -0.09 | 0.21 | 1.00 | 0.50 | 0.19 | 0.41 | 0.48 |
| FTSE | -0.11 | -0.16 | -0.20 | 0.34 | 0.50 | 1.00 | 0.37 | 0.71 | 0.34 |
| US 10Y | -0.12 | -0.13 | -0.13 | 0.28 | 0.19 | 0.37 | 1.00 | 0.36 | 0.32 |
| UK 10Y | 0.01 | 0.03 | -0.11 | 0.05 | 0.41 | 0.71 | 0.36 | 1.00 | 0.43 |
| JP 10Y | 0.00 | -0.06 | -0.10 | 0.14 | 0.48 | 0.34 | 0.32 | 0.43 | 1.00 |

**1997-2003:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|-------|------|-----|--------|-------|--------|------|--------|--------|--------|
| Gold | 1.00 | 0.07 | 0.14 | -0.19 | 0.12 | -0.11 | 0.05 | 0.13 | 0.24 |
| Oil | 0.07 | 1.00 | 0.26 | 0.03 | 0.26 | -0.20 | 0.10 | -0.07 | 0.25 |
| Copper | 0.14 | 0.26 | 1.00 | 0.18 | 0.11 | -0.04 | -0.20 | -0.05 | 0.07 |
| SP500 | -0.19 | 0.03 | 0.18 | 1.00 | 0.39 | 0.51 | -0.30 | -0.09 | -0.02 |
| Nikkei | 0.12 | 0.26 | 0.11 | 0.39 | 1.00 | 0.42 | -0.12 | 0.02 | 0.48 |
| FTSE | -0.11 | -0.20 | -0.04 | 0.51 | 0.42 | 1.00 | -0.28 | 0.06 | 0.14 |
| US 10Y | 0.05 | 0.10 | -0.20 | -0.30 | -0.12 | -0.28 | 1.00 | 0.49 | 0.21 |
| UK 10Y | 0.13 | -0.07 | -0.05 | -0.09 | 0.02 | 0.06 | 0.49 | 1.00 | 0.38 |
| JP 10Y | 0.24 | 0.25 | 0.07 | -0.02 | 0.48 | 0.14 | 0.21 | 0.38 | 1.00 |

**2004-2010:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|-------|------|-----|--------|-------|--------|------|--------|--------|--------|
| Gold | 1.00 | 0.22 | 0.36 | 0.01 | -0.02 | 0.04 | 0.04 | 0.42 | 0.24 |
| Oil | 0.22 | 1.00 | 0.57 | 0.30 | 0.19 | 0.34 | -0.18 | 0.42 | -0.11 |
| Copper | 0.36 | 0.57 | 1.00 | 0.47 | 0.24 | 0.44 | -0.24 | 0.43 | -0.13 |
| SP500 | 0.01 | 0.30 | 0.47 | 1.00 | 0.61 | 0.75 | -0.30 | 0.34 | -0.25 |
| Nikkei | -0.02 | 0.19 | 0.24 | 0.61 | 1.00 | 0.71 | 0.04 | 0.41 | -0.04 |
| FTSE | 0.04 | 0.34 | 0.44 | 0.75 | 0.71 | 1.00 | -0.20 | 0.54 | -0.19 |
| US 10Y | 0.04 | -0.18 | -0.24 | -0.30 | 0.04 | -0.20 | 1.00 | 0.14 | 0.57 |
| UK 10Y | 0.42 | 0.42 | 0.43 | 0.34 | 0.41 | 0.54 | 0.14 | 1.00 | 0.19 |
| JP 10Y | 0.24 | -0.11 | -0.13 | -0.25 | -0.04 | -0.19 | 0.57 | 0.19 | 1.00 |

**2011-2017:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|-------|------|-----|--------|-------|--------|------|--------|--------|--------|
| Gold | 1.00 | 0.00 | 0.16 | -0.25 | -0.17 | -0.06 | 0.34 | 0.35 | 0.37 |
| Oil | 0.00 | 1.00 | 0.34 | 0.39 | 0.16 | 0.26 | -0.32 | -0.03 | -0.05 |
| Copper | 0.16 | 0.34 | 1.00 | 0.37 | 0.17 | 0.17 | -0.27 | -0.01 | -0.14 |
| SP500 | -0.25 | 0.39 | 0.37 | 1.00 | 0.59 | 0.59 | -0.41 | 0.09 | -0.25 |
| Nikkei | -0.17 | 0.16 | 0.17 | 0.59 | 1.00 | 0.62 | -0.26 | 0.29 | -0.19 |
| FTSE | -0.06 | 0.26 | 0.17 | 0.59 | 0.62 | 1.00 | -0.34 | 0.50 | -0.18 |
| US 10Y | 0.34 | -0.32 | -0.27 | -0.41 | -0.26 | -0.34 | 1.00 | 0.13 | 0.56 |
| UK 10Y | 0.35 | -0.03 | -0.01 | 0.09 | 0.29 | 0.50 | 0.13 | 1.00 | 0.13 |
| JP 10Y | 0.37 | -0.05 | -0.14 | -0.25 | -0.19 | -0.18 | 0.56 | 0.13 | 1.00 |

**2018-2024:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | -0.02 | 0.35 | 0.23 | 0.14 | 0.11 | 0.28 | 0.40 | 0.34 |
| Oil | -0.02 | 1.00 | 0.41 | 0.37 | 0.15 | 0.27 | -0.28 | -0.04 | -0.22 |
| Copper | 0.35 | 0.41 | 1.00 | 0.47 | 0.20 | 0.40 | -0.21 | 0.19 | 0.01 |
| SP500 | 0.23 | 0.37 | 0.47 | 1.00 | 0.51 | 0.54 | -0.08 | 0.34 | 0.09 |
| Nikkei | 0.14 | 0.15 | 0.20 | 0.51 | 1.00 | 0.74 | 0.20 | 0.61 | 0.35 |
| FTSE | 0.11 | 0.27 | 0.40 | 0.54 | 0.74 | 1.00 | 0.03 | 0.61 | 0.19 |
| US 10Y | 0.28 | -0.28 | -0.21 | -0.08 | 0.20 | 0.03 | 1.00 | 0.54 | 0.74 |
| UK 10Y | 0.40 | -0.04 | 0.19 | 0.34 | 0.61 | 0.61 | 0.54 | 1.00 | 0.64 |
| JP 10Y | 0.34 | -0.22 | 0.01 | 0.09 | 0.35 | 0.19 | 0.74 | 0.64 | 1.00 |

**9 Assets, 5-year periods**

**1990-1994:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.38 | 0.02 | -0.47 | -0.08 | -0.12 | -0.13 | -0.01 | -0.09 |
| Oil | 0.38 | 1.00 | 0.20 | -0.68 | -0.17 | -0.19 | -0.20 | 0.05 | -0.10 |
| Copper | 0.02 | 0.20 | 1.00 | -0.08 | -0.24 | -0.15 | 0.02 | -0.03 | -0.15 |
| SP500 | -0.47 | -0.68 | -0.08 | 1.00 | 0.25 | 0.35 | 0.20 | -0.01 | 0.15 |
| Nikkei | -0.08 | -0.17 | -0.24 | 0.25 | 1.00 | 0.53 | 0.36 | 0.46 | 0.65 |
| FTSE | -0.12 | -0.19 | -0.15 | 0.35 | 0.53 | 1.00 | 0.38 | 0.72 | 0.38 |
| US 10Y | -0.13 | -0.20 | 0.02 | 0.20 | 0.36 | 0.38 | 1.00 | 0.37 | 0.55 |
| UK 10Y | -0.01 | 0.05 | -0.03 | -0.01 | 0.46 | 0.72 | 0.37 | 1.00 | 0.48 |
| JP 10Y | -0.09 | -0.10 | -0.15 | 0.15 | 0.65 | 0.38 | 0.55 | 0.48 | 1.00 |

**1995-1999:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.22 | 0.17 | 0.13 | 0.32 | -0.03 | -0.18 | 0.16 | 0.34 |
| Oil | 0.22 | 1.00 | 0.07 | -0.15 | -0.01 | -0.33 | 0.09 | -0.02 | 0.11 |
| Copper | 0.17 | 0.07 | 1.00 | 0.28 | 0.11 | 0.08 | -0.06 | 0.10 | 0.02 |
| SP500 | 0.13 | -0.15 | 0.28 | 1.00 | 0.23 | 0.43 | -0.16 | 0.05 | -0.11 |
| Nikkei | 0.32 | -0.01 | 0.11 | 0.23 | 1.00 | 0.33 | -0.12 | 0.01 | 0.36 |
| FTSE | -0.03 | -0.33 | 0.08 | 0.43 | 0.33 | 1.00 | -0.13 | 0.17 | 0.19 |
| US 10Y | -0.18 | 0.09 | -0.06 | -0.16 | -0.12 | -0.13 | 1.00 | 0.33 | 0.12 |
| UK 10Y | 0.16 | -0.02 | 0.10 | 0.05 | 0.01 | 0.17 | 0.33 | 1.00 | 0.40 |
| JP 10Y | 0.34 | 0.11 | 0.02 | -0.11 | 0.36 | 0.19 | 0.12 | 0.40 | 1.00 |

**2000-2004:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.25 | 0.33 | -0.05 | 0.21 | -0.12 | 0.28 | 0.42 | 0.19 |
| Oil | 0.25 | 1.00 | 0.19 | -0.05 | 0.30 | -0.23 | 0.04 | -0.10 | 0.28 |
| Copper | 0.33 | 0.19 | 1.00 | 0.25 | 0.19 | 0.19 | -0.27 | 0.00 | 0.04 |
| SP500 | -0.05 | -0.05 | 0.25 | 1.00 | 0.44 | 0.54 | -0.26 | -0.08 | 0.17 |
| Nikkei | 0.21 | 0.30 | 0.19 | 0.44 | 1.00 | 0.42 | -0.06 | 0.00 | 0.42 |
| FTSE | -0.12 | -0.23 | 0.19 | 0.54 | 0.42 | 1.00 | -0.20 | 0.13 | 0.23 |
| US 10Y | 0.28 | 0.04 | -0.27 | -0.26 | -0.06 | -0.20 | 1.00 | 0.49 | 0.31 |
| UK 10Y | 0.42 | -0.10 | 0.00 | -0.08 | 0.00 | 0.13 | 0.49 | 1.00 | 0.40 |
| JP 10Y | 0.19 | 0.28 | 0.04 | 0.17 | 0.42 | 0.23 | 0.31 | 0.40 | 1.00 |

**2005-2009:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.33 | 0.47 | 0.05 | 0.12 | 0.14 | 0.09 | 0.59 | 0.33 |
| Oil | 0.33 | 1.00 | 0.63 | 0.42 | 0.17 | 0.49 | -0.37 | 0.44 | -0.29 |
| Copper | 0.47 | 0.63 | 1.00 | 0.47 | 0.19 | 0.52 | -0.32 | 0.48 | -0.22 |
| SP500 | 0.05 | 0.42 | 0.47 | 1.00 | 0.67 | 0.82 | -0.26 | 0.35 | -0.59 |
| Nikkei | 0.12 | 0.17 | 0.19 | 0.67 | 1.00 | 0.76 | 0.04 | 0.33 | -0.36 |
| FTSE | 0.14 | 0.49 | 0.52 | 0.82 | 0.76 | 1.00 | -0.26 | 0.53 | -0.52 |
| US 10Y | 0.09 | -0.37 | -0.32 | -0.26 | 0.04 | -0.26 | 1.00 | 0.16 | 0.63 |
| UK 10Y | 0.59 | 0.44 | 0.48 | 0.35 | 0.33 | 0.53 | 0.16 | 1.00 | 0.16 |
| JP 10Y | 0.33 | -0.29 | -0.22 | -0.59 | -0.36 | -0.52 | 0.63 | 0.16 | 1.00 |

**2010-2014:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | -0.01 | 0.22 | -0.23 | -0.28 | -0.17 | 0.26 | 0.23 | 0.20 |
| Oil | -0.01 | 1.00 | 0.45 | 0.54 | 0.09 | 0.20 | -0.26 | -0.22 | -0.05 |
| Copper | 0.22 | 0.45 | 1.00 | 0.60 | 0.32 | 0.31 | -0.22 | -0.02 | 0.00 |
| SP500 | -0.23 | 0.54 | 0.60 | 1.00 | 0.52 | 0.57 | -0.48 | -0.04 | -0.33 |
| Nikkei | -0.28 | 0.09 | 0.32 | 0.52 | 1.00 | 0.61 | -0.26 | 0.17 | -0.22 |
| FTSE | -0.17 | 0.20 | 0.31 | 0.57 | 0.61 | 1.00 | -0.46 | 0.43 | -0.22 |
| US 10Y | 0.26 | -0.26 | -0.22 | -0.48 | -0.26 | -0.46 | 1.00 | 0.25 | 0.60 |
| UK 10Y | 0.23 | -0.22 | -0.02 | -0.04 | 0.17 | 0.43 | 0.25 | 1.00 | 0.23 |
| JP 10Y | 0.20 | -0.05 | 0.00 | -0.33 | -0.22 | -0.22 | 0.60 | 0.23 | 1.00 |

**2015-2019:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.01 | 0.22 | -0.09 | 0.00 | 0.10 | 0.28 | 0.48 | 0.46 |
| Oil | 0.01 | 1.00 | 0.33 | 0.47 | 0.34 | 0.32 | -0.39 | 0.07 | -0.12 |
| Copper | 0.22 | 0.33 | 1.00 | 0.25 | 0.09 | 0.14 | -0.31 | 0.05 | -0.25 |
| SP500 | -0.09 | 0.47 | 0.25 | 1.00 | 0.77 | 0.67 | -0.39 | 0.29 | -0.20 |
| Nikkei | 0.00 | 0.34 | 0.09 | 0.77 | 1.00 | 0.65 | -0.25 | 0.27 | -0.15 |
| FTSE | 0.10 | 0.32 | 0.14 | 0.67 | 0.65 | 1.00 | -0.29 | 0.56 | -0.19 |
| US 10Y | 0.28 | -0.39 | -0.31 | -0.39 | -0.25 | -0.29 | 1.00 | -0.13 | 0.59 |
| UK 10Y | 0.48 | 0.07 | 0.05 | 0.29 | 0.27 | 0.56 | -0.13 | 1.00 | 0.06 |
| JP 10Y | 0.46 | -0.12 | -0.25 | -0.20 | -0.15 | -0.19 | 0.59 | 0.06 | 1.00 |

**2020-2024:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y |
|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.00 | 0.40 | 0.32 | 0.17 | 0.11 | 0.32 | 0.41 | 0.33 |
| Oil | 0.00 | 1.00 | 0.42 | 0.34 | 0.13 | 0.25 | -0.24 | -0.03 | -0.19 |
| Copper | 0.40 | 0.42 | 1.00 | 0.50 | 0.21 | 0.43 | -0.14 | 0.24 | 0.10 |
| SP500 | 0.32 | 0.34 | 0.50 | 1.00 | 0.49 | 0.54 | 0.02 | 0.38 | 0.23 |
| Nikkei | 0.17 | 0.13 | 0.21 | 0.49 | 1.00 | 0.74 | 0.34 | 0.65 | 0.52 |
| FTSE | 0.11 | 0.25 | 0.43 | 0.54 | 0.74 | 1.00 | 0.14 | 0.63 | 0.36 |
| US 10Y | 0.32 | -0.24 | -0.14 | 0.02 | 0.34 | 0.14 | 1.00 | 0.67 | 0.78 |
| UK 10Y | 0.41 | -0.03 | 0.24 | 0.38 | 0.65 | 0.63 | 0.67 | 1.00 | 0.77 |
| JP 10Y | 0.33 | -0.19 | 0.10 | 0.23 | 0.52 | 0.36 | 0.78 | 0.77 | 1.00 |

**11 Assets, 7-year periods**

**1997-2003**:

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.07 | 0.14 | -0.19 | 0.12 | -0.11 | 0.05 | 0.13 | 0.24 | 0.10 | -0.06 |
| Oil | 0.07 | 1.00 | 0.26 | 0.03 | 0.26 | -0.20 | 0.10 | -0.07 | 0.25 | 0.05 | 0.04 |
| Copper | 0.14 | 0.26 | 1.00 | 0.18 | 0.11 | -0.04 | -0.20 | -0.05 | 0.07 | -0.20 | 0.11 |
| SP500 | -0.19 | 0.03 | 0.18 | 1.00 | 0.39 | 0.51 | -0.30 | -0.09 | -0.02 | -0.13 | 0.55 |
| Nikkei | 0.12 | 0.26 | 0.11 | 0.39 | 1.00 | 0.42 | -0.12 | 0.02 | 0.48 | 0.03 | 0.30 |
| FTSE | -0.11 | -0.20 | -0.04 | 0.51 | 0.42 | 1.00 | -0.28 | 0.06 | 0.14 | -0.17 | 0.26 |
| US 10Y | 0.05 | 0.10 | -0.20 | -0.30 | -0.12 | -0.28 | 1.00 | 0.49 | 0.21 | 0.72 | -0.10 |
| UK 10Y | 0.13 | -0.07 | -0.05 | -0.09 | 0.02 | 0.06 | 0.49 | 1.00 | 0.38 | 0.37 | -0.09 |
| JP 10Y | 0.24 | 0.25 | 0.07 | -0.02 | 0.48 | 0.14 | 0.21 | 0.38 | 1.00 | 0.08 | -0.12 |
| US IG | 0.10 | 0.05 | -0.20 | -0.13 | 0.03 | -0.17 | 0.72 | 0.37 | 0.08 | 1.00 | 0.31 |
| US HY | -0.06 | 0.04 | 0.11 | 0.55 | 0.30 | 0.26 | -0.10 | -0.09 | -0.12 | 0.31 | 1.00 |

**2004-2010:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.22 | 0.36 | 0.01 | -0.02 | 0.04 | 0.04 | 0.42 | 0.24 | 0.13 | 0.10 |
| Oil | 0.22 | 1.00 | 0.57 | 0.30 | 0.19 | 0.34 | -0.18 | 0.42 | -0.11 | 0.13 | 0.26 |
| Copper | 0.36 | 0.57 | 1.00 | 0.47 | 0.24 | 0.44 | -0.24 | 0.43 | -0.13 | 0.20 | 0.42 |
| SP500 | 0.01 | 0.30 | 0.47 | 1.00 | 0.61 | 0.75 | -0.30 | 0.34 | -0.25 | 0.42 | 0.73 |
| Nikkei | -0.02 | 0.19 | 0.24 | 0.61 | 1.00 | 0.71 | 0.04 | 0.41 | -0.04 | 0.53 | 0.62 |
| FTSE | 0.04 | 0.34 | 0.44 | 0.75 | 0.71 | 1.00 | -0.20 | 0.54 | -0.19 | 0.48 | 0.73 |
| US 10Y | 0.04 | -0.18 | -0.24 | -0.30 | 0.04 | -0.20 | 1.00 | 0.14 | 0.57 | 0.45 | -0.19 |
| UK 10Y | 0.42 | 0.42 | 0.43 | 0.34 | 0.41 | 0.54 | 0.14 | 1.00 | 0.19 | 0.44 | 0.42 |
| JP 10Y | 0.24 | -0.11 | -0.13 | -0.25 | -0.04 | -0.19 | 0.57 | 0.19 | 1.00 | 0.27 | -0.13 |
| US IG | 0.13 | 0.13 | 0.20 | 0.42 | 0.53 | 0.48 | 0.45 | 0.44 | 0.27 | 1.00 | 0.69 |
| US HY | 0.10 | 0.26 | 0.42 | 0.73 | 0.62 | 0.73 | -0.19 | 0.42 | -0.13 | 0.69 | 1.00 |

**2011-2017:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.00 | 0.16 | -0.25 | -0.17 | -0.06 | 0.34 | 0.35 | 0.37 | 0.18 | -0.04 |
| Oil | 0.00 | 1.00 | 0.34 | 0.39 | 0.16 | 0.26 | -0.32 | -0.03 | -0.05 | 0.07 | 0.31 |
| Copper | 0.16 | 0.34 | 1.00 | 0.37 | 0.17 | 0.17 | -0.27 | -0.01 | -0.14 | 0.07 | 0.19 |
| SP500 | -0.25 | 0.39 | 0.37 | 1.00 | 0.59 | 0.59 | -0.41 | 0.09 | -0.25 | 0.22 | 0.52 |
| Nikkei | -0.17 | 0.16 | 0.17 | 0.59 | 1.00 | 0.62 | -0.26 | 0.29 | -0.19 | 0.26 | 0.53 |
| FTSE | -0.06 | 0.26 | 0.17 | 0.59 | 0.62 | 1.00 | -0.34 | 0.50 | -0.18 | 0.36 | 0.78 |
| US 10Y | 0.34 | -0.32 | -0.27 | -0.41 | -0.26 | -0.34 | 1.00 | 0.13 | 0.56 | 0.51 | -0.10 |
| UK 10Y | 0.35 | -0.03 | -0.01 | 0.09 | 0.29 | 0.50 | 0.13 | 1.00 | 0.13 | 0.33 | 0.38 |
| JP 10Y | 0.37 | -0.05 | -0.14 | -0.25 | -0.19 | -0.18 | 0.56 | 0.13 | 1.00 | 0.40 | 0.04 |
| US IG | 0.18 | 0.07 | 0.07 | 0.22 | 0.26 | 0.36 | 0.51 | 0.33 | 0.40 | 1.00 | 0.68 |
| US HY | -0.04 | 0.31 | 0.19 | 0.52 | 0.53 | 0.78 | -0.10 | 0.38 | 0.04 | 0.68 | 1.00 |

**2018-2024:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | -0.02 | 0.35 | 0.23 | 0.14 | 0.11 | 0.28 | 0.40 | 0.34 | 0.36 | 0.21 |
| Oil | -0.02 | 1.00 | 0.41 | 0.37 | 0.15 | 0.27 | -0.28 | -0.04 | -0.22 | 0.03 | 0.25 |
| Copper | 0.35 | 0.41 | 1.00 | 0.47 | 0.20 | 0.40 | -0.21 | 0.19 | 0.01 | 0.12 | 0.24 |
| SP500 | 0.23 | 0.37 | 0.47 | 1.00 | 0.51 | 0.54 | -0.08 | 0.34 | 0.09 | 0.52 | 0.62 |
| Nikkei | 0.14 | 0.15 | 0.20 | 0.51 | 1.00 | 0.74 | 0.20 | 0.61 | 0.35 | 0.62 | 0.78 |
| FTSE | 0.11 | 0.27 | 0.40 | 0.54 | 0.74 | 1.00 | 0.03 | 0.61 | 0.19 | 0.53 | 0.76 |
| US 10Y | 0.28 | -0.28 | -0.21 | -0.08 | 0.20 | 0.03 | 1.00 | 0.54 | 0.74 | 0.62 | 0.23 |
| UK 10Y | 0.40 | -0.04 | 0.19 | 0.34 | 0.61 | 0.61 | 0.54 | 1.00 | 0.64 | 0.67 | 0.61 |
| JP 10Y | 0.34 | -0.22 | 0.01 | 0.09 | 0.35 | 0.19 | 0.74 | 0.64 | 1.00 | 0.67 | 0.39 |
| US IG | 0.36 | 0.03 | 0.12 | 0.52 | 0.62 | 0.53 | 0.62 | 0.67 | 0.67 | 1.00 | 0.79 |
| US HY | 0.21 | 0.25 | 0.24 | 0.62 | 0.78 | 0.76 | 0.23 | 0.61 | 0.39 | 0.79 | 1.00 |

**11 Assets, 4-year periods**

**1997-2000:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.00 | 0.12 | -0.13 | 0.18 | 0.05 | -0.07 | 0.05 | 0.32 | -0.08 | -0.14 |
| Oil | 0.00 | 1.00 | 0.40 | 0.04 | 0.22 | -0.27 | 0.04 | -0.01 | 0.18 | 0.06 | 0.11 |
| Copper | 0.12 | 0.40 | 1.00 | 0.22 | 0.18 | -0.15 | -0.09 | 0.03 | 0.13 | -0.13 | 0.11 |
| SP500 | -0.13 | 0.04 | 0.22 | 1.00 | 0.27 | 0.42 | -0.20 | 0.09 | -0.22 | 0.05 | 0.68 |
| Nikkei | 0.18 | 0.22 | 0.18 | 0.27 | 1.00 | 0.36 | -0.15 | 0.06 | 0.46 | 0.00 | 0.28 |
| FTSE | 0.05 | -0.27 | -0.15 | 0.42 | 0.36 | 1.00 | -0.16 | 0.07 | 0.05 | -0.01 | 0.24 |
| US 10Y | -0.07 | 0.04 | -0.09 | -0.20 | -0.15 | -0.16 | 1.00 | 0.52 | 0.24 | 0.85 | 0.02 |
| UK 10Y | 0.05 | -0.01 | 0.03 | 0.09 | 0.06 | 0.07 | 0.52 | 1.00 | 0.42 | 0.34 | -0.07 |
| JP 10Y | 0.32 | 0.18 | 0.13 | -0.22 | 0.46 | 0.05 | 0.24 | 0.42 | 1.00 | 0.02 | -0.38 |
| US IG | -0.08 | 0.06 | -0.13 | 0.05 | 0.00 | -0.01 | 0.85 | 0.34 | 0.02 | 1.00 | 0.40 |
| US HY | -0.14 | 0.11 | 0.11 | 0.68 | 0.28 | 0.24 | 0.02 | -0.07 | -0.38 | 0.40 | 1.00 |

**2001-2004:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|-------|------|-----|--------|-------|--------|------|--------|--------|--------|-------|-------|
| Gold | 1.00 | 0.29 | 0.43 | -0.02 | 0.23 | -0.15 | 0.36 | 0.46 | 0.34 | 0.34 | -0.22 |
| Oil | 0.29 | 1.00 | 0.20 | -0.09 | 0.17 | -0.20 | 0.04 | 0.04 | 0.27 | -0.12 | -0.07 |
| Copper | 0.43 | 0.20 | 1.00 | 0.34 | 0.39 | 0.25 | -0.26 | 0.07 | 0.15 | -0.30 | 0.04 |
| SP500 | -0.02 | -0.09 | 0.34 | 1.00 | 0.48 | 0.62 | -0.31 | -0.12 | 0.13 | -0.13 | 0.68 |
| Nikkei | 0.23 | 0.17 | 0.39 | 0.48 | 1.00 | 0.46 | -0.24 | -0.08 | 0.25 | -0.08 | 0.32 |
| FTSE | -0.15 | -0.20 | 0.25 | 0.62 | 0.46 | 1.00 | -0.34 | -0.05 | 0.10 | -0.18 | 0.38 |
| US 10Y | 0.36 | 0.04 | -0.26 | -0.31 | -0.24 | -0.34 | 1.00 | 0.54 | 0.28 | 0.78 | -0.11 |
| UK 10Y | 0.46 | 0.04 | 0.07 | -0.12 | -0.08 | -0.05 | 0.54 | 1.00 | 0.47 | 0.48 | -0.11 |
| JP 10Y | 0.34 | 0.27 | 0.15 | 0.13 | 0.25 | 0.10 | 0.28 | 0.47 | 1.00 | 0.34 | 0.15 |
| US IG | 0.34 | -0.12 | -0.30 | -0.13 | -0.08 | -0.18 | 0.78 | 0.48 | 0.34 | 1.00 | 0.23 |
| US HY | -0.22 | -0.07 | 0.04 | 0.68 | 0.32 | 0.38 | -0.11 | -0.11 | 0.15 | 0.23 | 1.00 |

**2005-2008:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|-------|------|-----|--------|-------|--------|------|--------|--------|--------|-------|-------|
| Gold | 1.00 | 0.44 | 0.48 | 0.12 | 0.16 | 0.19 | 0.09 | 0.51 | 0.36 | 0.15 | 0.08 |
| Oil | 0.44 | 1.00 | 0.33 | -0.05 | 0.07 | 0.21 | 0.06 | 0.31 | 0.21 | -0.02 | -0.24 |
| Copper | 0.48 | 0.33 | 1.00 | 0.19 | -0.02 | 0.31 | -0.20 | 0.26 | 0.08 | -0.12 | 0.06 |
| SP500 | 0.12 | -0.05 | 0.19 | 1.00 | 0.47 | 0.56 | -0.24 | -0.01 | -0.40 | 0.02 | 0.56 |
| Nikkei | 0.16 | 0.07 | -0.02 | 0.47 | 1.00 | 0.55 | -0.16 | 0.12 | -0.20 | 0.03 | 0.38 |
| FTSE | 0.19 | 0.21 | 0.31 | 0.56 | 0.55 | 1.00 | -0.32 | 0.50 | -0.20 | -0.03 | 0.55 |
| US 10Y | 0.09 | 0.06 | -0.20 | -0.24 | -0.16 | -0.32 | 1.00 | 0.31 | 0.70 | 0.90 | -0.12 |
| UK 10Y | 0.51 | 0.31 | 0.26 | -0.01 | 0.12 | 0.50 | 0.31 | 1.00 | 0.58 | 0.38 | 0.16 |
| JP 10Y | 0.36 | 0.21 | 0.08 | -0.40 | -0.20 | -0.20 | 0.70 | 0.58 | 1.00 | 0.57 | -0.22 |
| US IG | 0.15 | -0.02 | -0.12 | 0.02 | 0.03 | -0.03 | 0.90 | 0.38 | 0.57 | 1.00 | 0.23 |
| US HY | 0.08 | -0.24 | 0.06 | 0.56 | 0.38 | 0.55 | -0.12 | 0.16 | -0.22 | 0.23 | 1.00 |

**2009-2012:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | -0.12 | -0.10 | -0.33 | -0.52 | -0.38 | 0.15 | 0.02 | 0.01 | -0.27 | -0.46 |
| Oil | -0.12 | 1.00 | 0.48 | 0.36 | 0.32 | 0.19 | -0.09 | 0.21 | 0.04 | 0.26 | 0.31 |
| Copper | -0.10 | 0.48 | 1.00 | 0.56 | 0.46 | 0.29 | -0.24 | 0.03 | -0.02 | 0.35 | 0.50 |
| SP500 | -0.33 | 0.36 | 0.56 | 1.00 | 0.62 | 0.60 | -0.41 | 0.06 | 0.07 | 0.48 | 0.66 |
| Nikkei | -0.52 | 0.32 | 0.46 | 0.62 | 1.00 | 0.70 | -0.18 | 0.23 | 0.14 | 0.50 | 0.63 |
| FTSE | -0.38 | 0.19 | 0.29 | 0.60 | 0.70 | 1.00 | -0.38 | 0.46 | 0.09 | 0.51 | 0.74 |
| US 10Y | 0.15 | -0.09 | -0.24 | -0.41 | -0.18 | -0.38 | 1.00 | 0.10 | 0.53 | 0.04 | -0.45 |
| UK 10Y | 0.02 | 0.21 | 0.03 | 0.06 | 0.23 | 0.46 | 0.10 | 1.00 | 0.18 | 0.43 | 0.26 |
| JP 10Y | 0.01 | 0.04 | -0.02 | 0.07 | 0.14 | 0.09 | 0.53 | 0.18 | 1.00 | 0.48 | 0.11 |
| US IG | -0.27 | 0.26 | 0.35 | 0.48 | 0.50 | 0.51 | 0.04 | 0.43 | 0.48 | 1.00 | 0.74 |
| US HY | -0.46 | 0.31 | 0.50 | 0.66 | 0.63 | 0.74 | -0.45 | 0.26 | 0.11 | 0.74 | 1.00 |

**2013-2016:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.02 | 0.15 | -0.23 | -0.10 | 0.05 | 0.23 | 0.32 | 0.45 | 0.24 | 0.19 |
| Oil | 0.02 | 1.00 | 0.52 | 0.24 | 0.15 | 0.33 | -0.46 | 0.03 | -0.14 | -0.12 | 0.25 |
| Copper | 0.15 | 0.52 | 1.00 | 0.12 | -0.07 | 0.09 | -0.15 | -0.17 | -0.13 | 0.08 | 0.12 |
| SP500 | -0.23 | 0.24 | 0.12 | 1.00 | 0.46 | 0.48 | -0.33 | -0.11 | -0.56 | -0.08 | 0.27 |
| Nikkei | -0.10 | 0.15 | -0.07 | 0.46 | 1.00 | 0.51 | -0.03 | 0.31 | -0.24 | 0.12 | 0.39 |
| FTSE | 0.05 | 0.33 | 0.09 | 0.48 | 0.51 | 1.00 | -0.13 | 0.46 | -0.15 | 0.23 | 0.72 |
| US 10Y | 0.23 | -0.46 | -0.15 | -0.33 | -0.03 | -0.13 | 1.00 | 0.19 | 0.37 | 0.81 | 0.25 |
| UK 10Y | 0.32 | 0.03 | -0.17 | -0.11 | 0.31 | 0.46 | 0.19 | 1.00 | 0.22 | 0.25 | 0.40 |
| JP 10Y | 0.45 | -0.14 | -0.13 | -0.56 | -0.24 | -0.15 | 0.37 | 0.22 | 1.00 | 0.24 | 0.01 |
| US IG | 0.24 | -0.12 | 0.08 | -0.08 | 0.12 | 0.23 | 0.81 | 0.25 | 0.24 | 1.00 | 0.66 |
| US HY | 0.19 | 0.25 | 0.12 | 0.27 | 0.39 | 0.72 | 0.25 | 0.40 | 0.01 | 0.66 | 1.00 |

**2017-2020:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | -0.07 | 0.18 | -0.14 | -0.04 | 0.02 | 0.17 | 0.40 | 0.33 | 0.22 | 0.29 |
| Oil | -0.07 | 1.00 | 0.31 | 0.56 | 0.29 | 0.35 | -0.45 | -0.07 | -0.27 | -0.20 | 0.32 |
| Copper | 0.18 | 0.31 | 1.00 | 0.31 | 0.15 | 0.20 | -0.30 | -0.04 | -0.17 | -0.27 | 0.04 |
| SP500 | -0.14 | 0.56 | 0.31 | 1.00 | 0.60 | 0.56 | -0.49 | 0.12 | -0.47 | -0.09 | 0.50 |
| Nikkei | -0.04 | 0.29 | 0.15 | 0.60 | 1.00 | 0.66 | -0.38 | 0.35 | -0.32 | 0.00 | 0.59 |
| FTSE | 0.02 | 0.35 | 0.20 | 0.56 | 0.66 | 1.00 | -0.39 | 0.49 | -0.41 | 0.09 | 0.65 |
| US 10Y | 0.17 | -0.45 | -0.30 | -0.49 | -0.38 | -0.39 | 1.00 | -0.08 | 0.57 | 0.78 | -0.05 |
| UK 10Y | 0.40 | -0.07 | -0.04 | 0.12 | 0.35 | 0.49 | -0.08 | 1.00 | 0.03 | 0.17 | 0.35 |
| JP 10Y | 0.33 | -0.27 | -0.17 | -0.47 | -0.32 | -0.41 | 0.57 | 0.03 | 1.00 | 0.39 | -0.14 |
| US IG | 0.22 | -0.20 | -0.27 | -0.09 | 0.00 | 0.09 | 0.78 | 0.17 | 0.39 | 1.00 | 0.47 |
| US HY | 0.29 | 0.32 | 0.04 | 0.50 | 0.59 | 0.65 | -0.05 | 0.35 | -0.14 | 0.47 | 1.00 |

**2021-2024:**

| Asset | Gold | Oil | Copper | SP500 | Nikkei | FTSE | US 10Y | UK 10Y | JP 10Y | US IG | US HY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gold | 1.00 | 0.09 | 0.56 | 0.33 | 0.21 | 0.21 | 0.32 | 0.37 | 0.33 | 0.34 | 0.09 |
| Oil | 0.09 | 1.00 | 0.28 | -0.14 | -0.23 | 0.12 | -0.23 | -0.06 | -0.36 | -0.22 | -0.20 |
| Copper | 0.56 | 0.28 | 1.00 | 0.39 | 0.01 | 0.29 | -0.07 | 0.14 | 0.05 | 0.06 | -0.03 |
| SP500 | 0.33 | -0.14 | 0.39 | 1.00 | 0.35 | 0.38 | 0.29 | 0.47 | 0.27 | 0.41 | 0.36 |
| Nikkei | 0.21 | -0.23 | 0.01 | 0.35 | 1.00 | 0.65 | 0.65 | 0.76 | 0.65 | 0.79 | 0.85 |
| FTSE | 0.21 | 0.12 | 0.29 | 0.38 | 0.65 | 1.00 | 0.52 | 0.80 | 0.53 | 0.67 | 0.77 |
| US 10Y | 0.32 | -0.23 | -0.07 | 0.29 | 0.65 | 0.52 | 1.00 | 0.79 | 0.85 | 0.93 | 0.68 |
| UK 10Y | 0.37 | -0.06 | 0.14 | 0.47 | 0.76 | 0.80 | 0.79 | 1.00 | 0.79 | 0.89 | 0.77 |
| JP 10Y | 0.33 | -0.36 | 0.05 | 0.27 | 0.65 | 0.53 | 0.85 | 0.79 | 1.00 | 0.86 | 0.64 |
| US IG | 0.34 | -0.22 | 0.06 | 0.41 | 0.79 | 0.67 | 0.93 | 0.89 | 0.86 | 1.00 | 0.81 |
| US HY | 0.09 | -0.20 | -0.03 | 0.36 | 0.85 | 0.77 | 0.68 | 0.77 | 0.64 | 0.81 | 1.00 |

## B.4 Macroeconomic Summary

### 5 Assets, 10-year periods

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1971-1980 | 3.6% | 7.2% |
| 1981-1990 | 3.5% | 4.7% |
| 1991-2000 | 3.4% | 2.7% |
| 2001-2010 | 1.7% | 2.5% |
| 2011-2020 | 2.4% | 1.8% |

### 5 Assets, 5-year periods

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1971-1975 | 3.4% | 6.2% |
| 1976-1980 | 4.7% | 7.8% |
| 1981-1985 | 3.1% | 6.0% |
| 1986-1990 | 3.7% | 3.6% |
| 1991-1995 | 2.5% | 3.2% |
| 1996-2000 | 4.4% | 2.3% |
| 2001-2005 | 2.3% | 2.3% |
| 2006-2010 | 0.6% | 2.4% |
| 2011-2015 | 2.1% | 2.1% |
| 2016-2020 | 2.5% | 1.9% |

### 9 Assets, 7-year periods

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1990-1996 | 2.5% | 3.5% |
| 1997-2003 | 3.4% | 2.3% |
| 2004-2010 | 1.6% | 2.6% |
| 2011-2017 | 2.2% | 1.6% |
| 2018-2024 | 2.5% | 3.7% |

**9 Assets, 5-year periods**

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1990-1994 | 2.0% | 3.9% |
| 1995-1999 | 3.8% | 2.4% |
| 2000-2004 | 2.4% | 2.5% |
| 2005-2009 | 2.1% | 3.3% |
| 2010-2014 | 2.2% | 2.1% |
| 2015-2019 | 2.5% | 1.5% |
| 2020-2024 | 2.3% | 4.5% |

**11 Assets, 7-year periods**

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1997-2003 | 3.4% | 2.3% |
| 2004-2010 | 1.6% | 2.6% |
| 2011-2017 | 2.2% | 1.6% |
| 2018-2024 | 2.5% | 3.7% |

**11 Assets, 4-year periods**

| Time Period | Average Annual Real GDP Growth | Average CPI Growth |
|---|---|---|
| 1997-2000 | 4.6% | 2.0% |
| 2001-2004 | 1.8% | 2.2% |
| 2005-2008 | 2.8% | 3.2% |
| 2009-2012 | 0.6% | 1.5% |
| 2013-2016 | 2.5% | 1.1% |
| 2017-2020 | 2.7% | 2.1% |
| 2021-2024 | 3.8% | 5.6% |