Machine Learning
Assignment # 1
Universität Bern

Due date: 9/10/2019

Late submissions will incur a penalty. Submit your answers in ILIAS (as a pdf or as a picture of your written notes if the handwriting is very clear). Submission instructions will be provided via email. For any clarification about the problem set ask the teaching assistant. You are not allowed to work with others.

Linear algebra review [Total 100 points]

Solve each of the following problems and show all the steps of your working.

1. $S = \{v_1, \ldots, v_n\}$ be an orthogonal set of non-zero vectors in R^n. **[10 points]**
   Prove that the vectors in S are linearly independent.

2. Given a square matrix $A \in \mathbb{R}^{n\times n}$ and a vector $x \in \mathbb{R}^n$ show that $x^T Ax = x^T(\frac{1}{2}A + \frac{1}{2}A^T)x$ **[15 points]**

3. Show that if $(A + B)^{-1} = A^{-1} + B^{-1}$ then $AB^{-1}A = BA^{-1}B$ **[15 points]**

4. Use the definition of trace to show that $tr(A + B) = trA + trB$, where $A, B \in \mathbb{R}^{n\times n}$. **[15 points]**

5. Show that if (λ_i, x_i) are the i-th eigenvalue and i-th eigenvector of a non-singular and symmetric matrix $A \in \mathbb{R}^{n\times n}$, then $(\frac{1}{\lambda_i}, x_i)$ are the i-th eigenvalue and i-th eigenvector of A^{-1}. **[15 points]**
   Hint: use the eigendecomposition of A.

6. Show that rank$(A) \leq \min\{m, n\}$, where $A \in \mathbb{R}^{m\times n}$. **[10 points]**

7. In each of the following cases, state whether the real matrix A is guaranteed to be singular or not. Justify your answer in each case.
   (a) $A \in \mathbb{R}^{(n+1)\times n}$ is a full rank matrix. **[4 points]**
   (b) $|A| = 0$. **[4 points]**
   (c) A is an orthogonal matrix. **[4 points]**
   (d) A has no eigenvalue equal to zero. **[4 points]**
   (e) A is a symmetric matrix with non-negative eigenvalues. **[4 points]**