What do I need on the next test? How to estimate marginal grade improvement?

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Introduction

(Practical example with actual scores and what not)

Students always ask their professors: "What do I need to make on the next test in order to get a B?". I developed an application in R to help answer these questions. Even though I already derived the formula (see below) to do it, I am trying a second methodology (using partial derivatives) and it does not seem to work, which explains this present question.

1- Preliminaries

Grade categories

Courses have several grade categories (e.g. tests, homework assignments, quizzes, ...). Let the n grade categories of a given course be X_i with $i \in \{1, 2, ..., n\}$.

Scores in each grade category

Each grade category usually has several assessments throughout the semester/quarter. Let X_{ij} denote the j^{th} assessment score in the category X_i with $j \in \{1, 2, ..., n_i\}$. For example, if X_1 refers to exams, then X_{13} is the score obtained in the third exam. Note that X_{ij} is a score out of 100. n_i is the total number of assessments in X_i.

Grade category weights

The grade categories in a course are often weighed differently. Let w_i represent the weight of grade category X_i with $i \in \{1, 2, ..., n\}$. It is important to note that w_i is in decimal form and not in percentage. For instance, if grade category X_1 accounts for 50% of the total grade, then $w_1 = 0.5$.

Grade category averages

$\bar{X_1}$ represent the average score of all n_1 assessments within the grade category X_1. If there were 3 assessments in the category X_1 and the scores were: $X_{11} = 75$, $X_{12} = 80$ and $X_{13} = 85$, then $\bar{X_1} = 80$. The general formula to compute $\bar{X_i}$ is:

$$\bar{X_i} = \frac{\sum_{k=1}^{n_i} X_{ik}}{n_i}$$

Total grade

Computing a student's total grade consists in:

- computing the weighted average of each grade category
- adjusting the sum of weighted averages by the sum of weights

The second step is important especially if the student wants to know their total grade before a specific grade category has been assessed. For example, let's consider a course with 3 grade categories: exams (50%), quizzes (20%) and homework (30%). If only exams and quizzes have been given (and no homework), then the sum of weighted averages of exams and quizzes must be adjusted by 0.7 (i.e. 0.5 + 0.2 = 0.7). Let G be the total grade, it is computed as:

$$G = \frac{\sum_{i=1}^{n} w_i \bar{X}_i}{\sum_{i=1}^{n} w_i}$$

2- Computing the answer

My first methodology

Let G^* be a student's desired total grade. The student wants to know what they should make in the next assessment of X_1 in order to reach G^*. The student's performance in the next assessment of X_1 is $X_{1(n_1+1)}$. Essentially, the student is hoping to bring up $\bar{X}_1$ high enough so that the total grade equal G^*. Let's denote the desired/target grade category average as $\bar{X_1}^*$

Given the above, G^* can be expressed as:

$$G^* = \frac{\sum_{i=1}^{n} w_i \bar{X}_i}{\sum_{i=1}^{n} w_i}$$

$$G^* = \frac{w_1 \bar{X_1}^* + \sum_{i=2}^{n} w_i \bar{X}_i}{\sum_{i=1}^{n} w_i}$$

Solving for $\bar{X_1}^*$, we obtain:

$$\bar{X_1}^* = \frac{\sum_{i=1}^{n} w_i \; G^* - \sum_{i=2}^{n} w_i \bar{X}_i}{w_1} \text{(equation 1)}$$

At this point, we need to expand $\bar{X_1}^*$:

$$\bar{X_1}^* = \frac{\sum_{i=1}^{n_1+1} X_{1i}}{n_1 + 1}$$

which can be rewritten as:

$$\bar{X_1}^* = \frac{\sum_{i=1}^{n_1} X_{1i} + X_{1(n_1+1)}}{n_1 + 1}$$

The last set of steps is to set *(equation 1)* to the expression for $\bar{X_1}^*$ and then solve for $X_{1(n_1+1)}$. The final result is:

$$X_{1(n_1+1)} = \frac{n_1 + 1}{w_1}(\sum_{i=1}^{n} w_i \; G^* - \sum_{i=2}^{n} w_i \bar{X}_i) - \sum_{i=1}^{n_1} X_{1i}$$