Minimized and Optimized Structural Information and Complexity Measurement of Network

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

Challenges

1. There is no such definition of optimal coding tree of structural information similar to that of a distribution $P = (p_1, p_2, \cdots, p_n)$.

2. There is no definition of minimized and optimal structural information.

3. There is no simple and explicit algorithm to estimate the structural information defined by Li and Pan [1]. i.e. How to estimate Two-Dimensional Structural Information and K-Dimensional Structural Information directly rather than traversing all the partitions of underlying graph.

4. And there is no algorithm to estimate the optimal structural information explicitly and directly rather than estimate all the k-Dimensional Structural Information where $1 \leq k \leq n$.

II. Preliminaries

In this section, we first present some definitions and notations of graph, then give the definition of k-core and degeneracy, and the algorithm to extract k-core decomposition. At last, basic theory of structural information is given.

A. Notations of Graph

B. Degeneracy and k-core Decomposition

Let G be a graph and G' be a subgraph of G induced by a set of vertices S. Then G' is called a k-core C_k of G, if it a maximal subgraph of G, and each vertex's degree $d_{G'}(v) \geq k$. Each k-core subgraph of G is unique, and not necessarily connected.

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C. Structural information

All the definitions about structural information are suggested by Li and Pan [1],

Definition 1: (Structural Information of a Graph by a Partitioning Tree): For a partitioning tree $\mathcal{T}$ of an undirected and connected graph $G = (V, E)$, thus the structural information of G by $\mathcal{T}$ is:

1) For each node $\alpha \in \mathcal{T}/\lambda$, where λ is the root of $\mathcal{T}$, then define the structural information of α is:

$$H^{\mathcal{T}}(G;\alpha) = -\frac{g_\alpha}{2m} log_2 \frac{V_\alpha}{V_{\alpha^-}} \tag{1}$$

where g_α is the number of edges from nodes in T_α to nodes outside T_α, V_α is the volume of set T_α.

2) The structural information of G is:

$$\mathcal{H}^{\mathcal{T}}(G) = \sum_{\alpha \in \mathcal{T}/\lambda} H^{\mathcal{T}}(G;\alpha) \tag{2}$$

Definition 2: (K-Dimensional Structural Information): Let $G = (V, E)$ is an undirect and connected graph, then K-dimensional structural information of G is defined as:

$$\mathcal{H}^{K}(G) = \min_{\mathcal{T}} \{\mathcal{H}^{\mathcal{T}}(G)\} \tag{3}$$

where $\mathcal{T}$ ranges over all of the partitioning trees of G of height K.

III. Optimal Structural Information

A. core-based Graph Partition

Definition 3: (Unconnected based Graph Partition): Let $G = (V, E)$ be a undirect and unconnected graph, and there are t connected sub-graphs $G_1, G_2, \cdots, G_t$ in G. Then the unconnected based graph Partition can be defined as

$$P(G) = \{G_1, G_2, \cdots, G_t\} \tag{4}$$

For example, in Fig. ...

Definition 4: (Core-based Graph Partition): Let $G = (V, E)$ be a undirect and connected graph. Then the core based graph partition can be defined as

$$P^{core}(G) = \{P(C_{\delta^*}), P(C_{\delta^*} - C_{\delta^*-1}), \cdots, P(C_2 - C_1)\} \tag{5}$$

where δ^* is degeneracy of G, $C_{\delta^*}, C_{\delta^*-1}, \cdots, C_2, C_1$ are the δ^*-core, $(\delta^* - 1)$-core, ..., 2-core, 1-core sub-graphs of G, respectively. And $P(\cdot)$ is a partition of graph $\cdot$ by the unconnected sub-graph(s) by following Def. 3.

For example, in Fig. ...

Theorem 1: Let $G = (V, E)$ be a undirect and connected graph, and $P^{core}(G)$ is the core based graph partition of G. Then $P^{core}(G)$ is the optimal partition of G by the minimized 2-dimensional structural entropy of G.

Algorithm 1 Partitioning G by the core based graph partition

Input: An undirect and connected graph $G = (V, E)$, with n vertexes and m edges.
Output: Core based graph partition $P^{core}(G)$.

```
Let C_1, C_2, ..., C_{δ*} be the 1-core, 2-core, ..., δ*-core of G.
P^core(G) = P(C_{δ*})
for each i ∈ [1, δ* − 1] do
    P^core(G) = P^core(G) ∪ P(C_i − C_{i+1})
end for
return P^core(G)
```

B. 2-dimensional and Optimal Structural Information

Definition 5: (Two-dimensional Structural Information): Let $G = (V, E)$ be a undirect and connected graph, and $P^{core}(G) = \{X_1, X_2, \cdots, X_L\}$ be the core based graph partition of G. Then the two-dimensional structural information of G can be defined as

$$\mathcal{H}^2(G) = \sum_{j=1}^{L} \frac{V_j}{2m} \cdot H(\frac{d_1^j}{V_j}, \cdots, \frac{d_{n_j}^j}{V_j}) - \sum_{j=1}^{L} \frac{g_j}{2m} log_2 \frac{V_j}{2m} \tag{6}$$

Def. 5 is a variant of Def. 9 in [1]. However, by Def. 5, we can estimate the two-dimensional structural information directly with an extremely lower time complexity. Here we suggest an algorithm to estimate two-dimensional structural information of G as Alg. 2.

Definition 6: (Optimal Structural Information): Let $G = (V, E)$ is an undirect and connected graph, then optimal structural information of G is defined as:

$$\mathcal{H}^{Optimal}(G) = \min_{K \in [1,n]} \{\mathcal{H}^K(G)\} \tag{7}$$

Definition 7: (Normalized Optimal Structural Information): Let $G = (V, E)$ is an undirect and connected graph, then normalized optimal structural information of G is defined as:

$$\mathcal{H}^{Normalized}(G) = \frac{\mathcal{H}^{Optimal}(G)}{\mathcal{H}^1(G)} \tag{8}$$

Algorithm 2 Estimating 2-dimensional structural information of graph

Input: An undirect and connected graph $G = (V, E)$, with n vertices and m edges.
Output: 2-dimensional structural information $\mathcal{H}^2(G)$.

```
P^core(G) = {X_1, X_2, ..., X_L}
H^2(G) = 0
for each j ∈ [1, L] do
    n_j = |X_j|
    H(X_j) = − Σ_{i=1}^{n_j} (d_i^(j) / V_j) log_2 (d_i^(j) / V_j)
    H^2(G) += (V_j / 2m) H(X_j) − (g_j / 2m) log_2 (V_j / 2m)
end for
return H^2(G)
```

C. Characterization

IV. Estimation Algorithm of Optimal Structural Information

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
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- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

A. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{9}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(9)", not "Eq. (9)" or "equation (9)", except at the beginning of a sentence: "Equation (9) is . . ."

B. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the

order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

BibTeX does not work by magic. It doesn't get the bibliographic data from thin air but from .bib files. If you use BibTeX to produce a bibliography you must send the .bib files.

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C. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).
- Do not use the word "essentially" to mean "approximately" or "effectively".
- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
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- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

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The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

E. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

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a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Fig. 1. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization $\{$A[m(1)]$\}$", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

Please number citations consecutively within brackets [**?**]. The sentence punctuation follows the bracket [**?**]. Refer simply to the reference number, as in [**?**]—do not use "Ref. [**?**]" or "reference [**?**]" except at the beginning of a sentence: "Reference [**?**] was the first ..."

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [**?**]. Papers that have been accepted for publication should be cited as "in press" [**?**]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [**?**].

REFERENCES

[1] A. Li and Y. Pan, "Structural information and dynamical complexity of networks," *IEEE Transactions on Information Theory*, vol. 62, no. 6, pp. 3290–3339, June 2016.