Multi-view 3D Reconstruction for Dummies

Jianxiong Xiao

PRINCETON VISION GROUP

SFMedu Program with Code

Download from:

http://mit.edu/jxiao/Public/software/SFMedu/

What

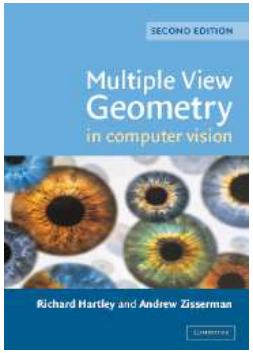
What will you learn in the following one hour?

After this Lecture

You should be able to

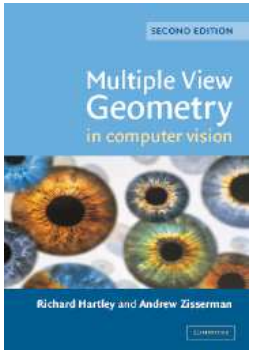
- Write your own structure from motion pipeline
- Implement a multiple view stereo system
- Know well about pinhole camera model
- Establish foundation to learn more multiple view geometry theories
- Know how to estimate the parameters for a model using linear system of equations
- Know how to solve non-linear least square problem in practice

How



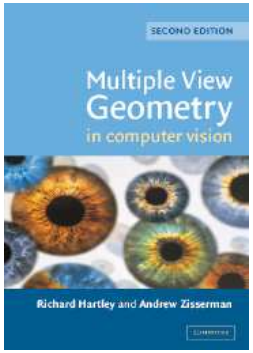
Reading: the MVG bible

How



Reading: the MVG bible
(need ~2 years)

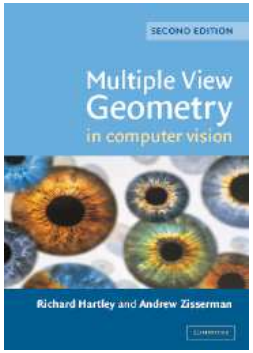
How



Reading: the MVG bible
(need ~2 years)

Coding

How

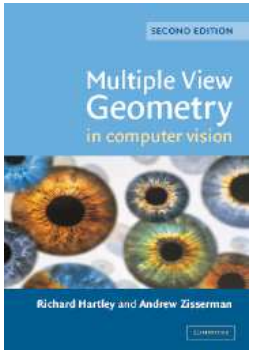


Reading: the MVG bible
(need ~2 years)

Coding

Crying: Not Working At All

How

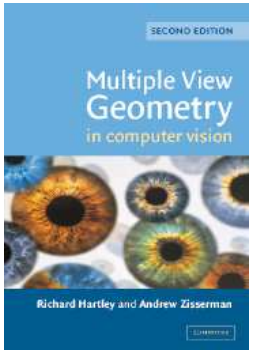


Reading: the MVG bible
(need ~2 years)

Coding

Crying: Not Working At All

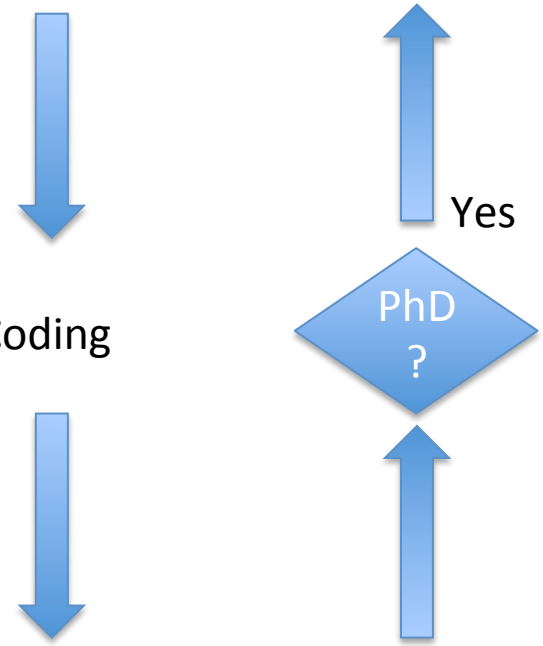
How



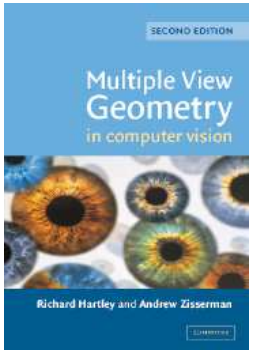
Reading: the MVG bible
(need ~2 years)

Coding

Crying: Not Working At All



How



Reading: the MVG bible
(need ~2 years)

5-6
years

Yes

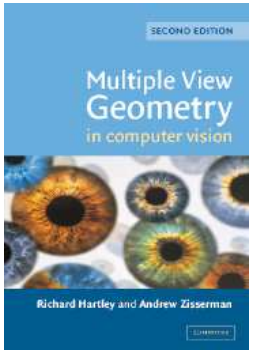
Coding

PhD
?

Crying: Not Working At All

How: a complete new way of learning

Standard Way



Reading: the MVG bible
(need ~2 years)

Coding

Crying: Not Working At All

5-6 years

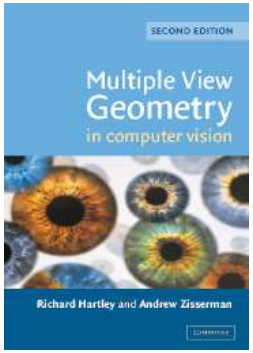
PhD ?

Yes

Our Way

How: a complete new way of learning

Standard Way



Reading: the MVG bible
(need ~2 years)

5-6 years

Yes

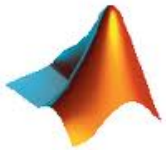
Coding

PhD ?

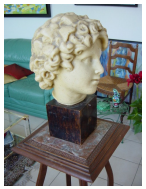
Crying: Not Working At All

Our Way

1 hour



Run a program



See pictures

Listen stories

$$x = PX$$

Play with math

Dance or sing

The Basics: My Life Story

My Life Story

My Life Story

Mr. Xiao

My Life Story

Xiao

My Life Story

X

I am a 3D Point

$\mathbf{X}$

I am a 3D Point

$\mathbf{X}$

I am a 3D Point

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

I am small

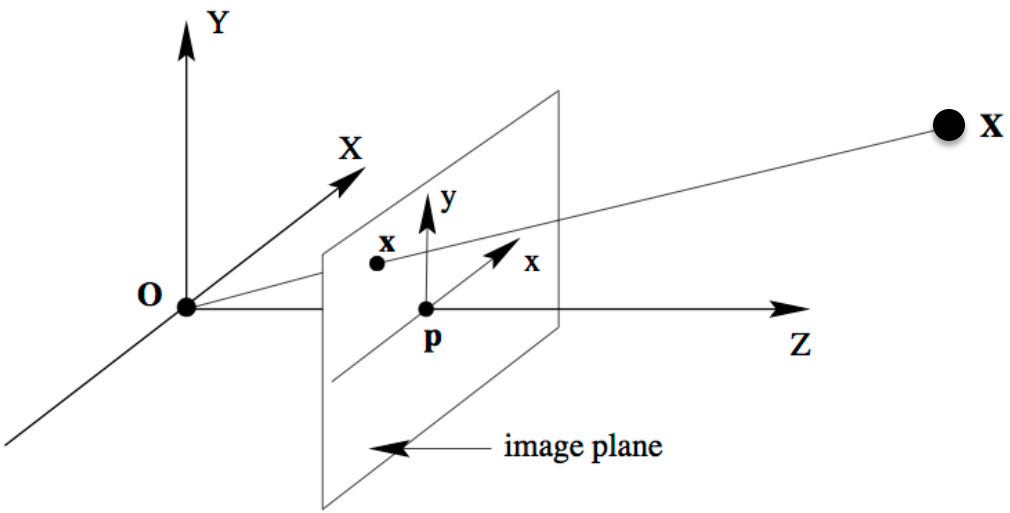
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

I am sexy

People loves to take picture of me.

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

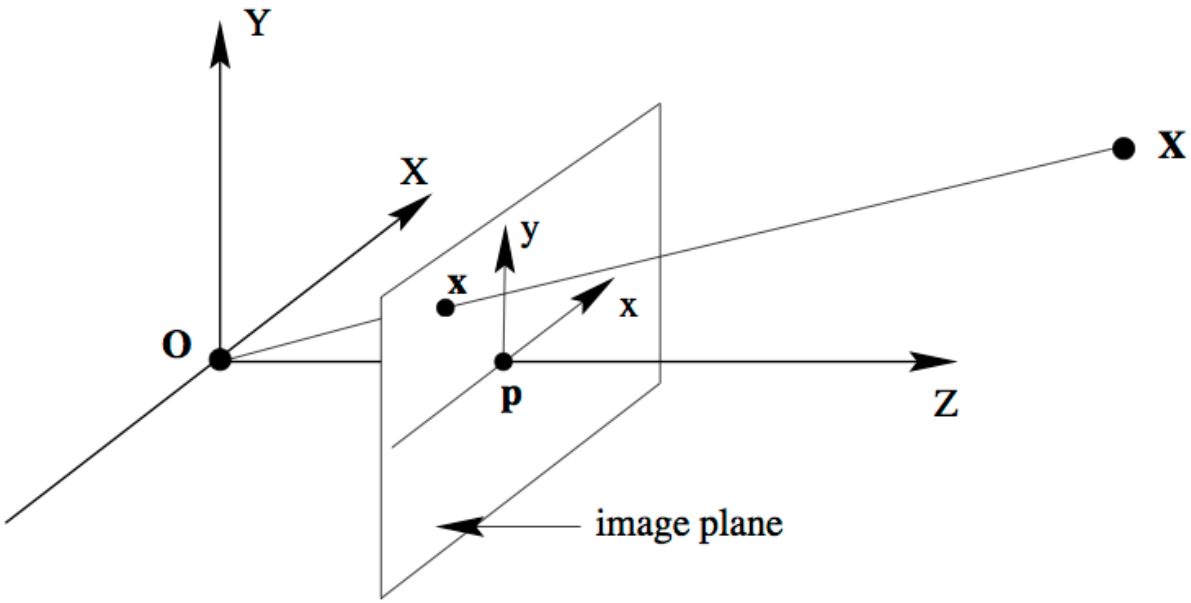
When they take a picture:



$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

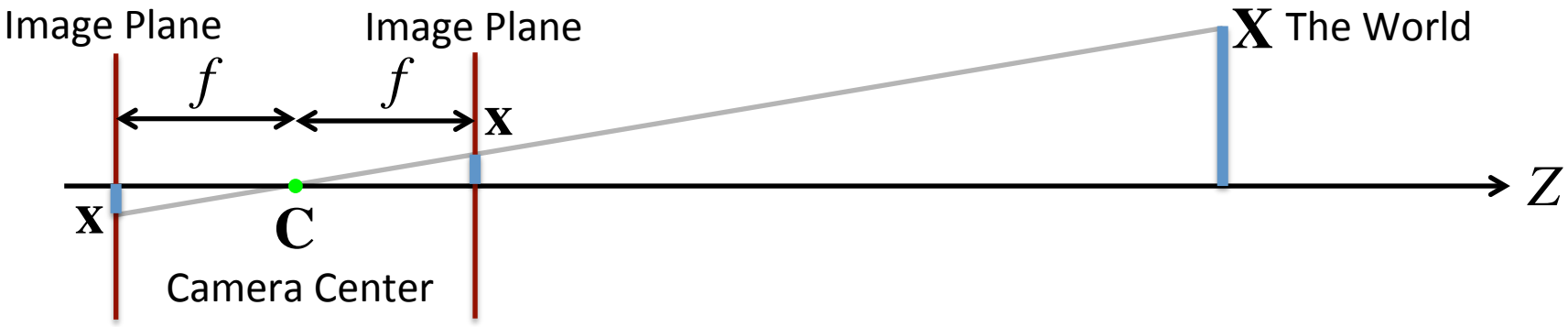
When they take a picture:

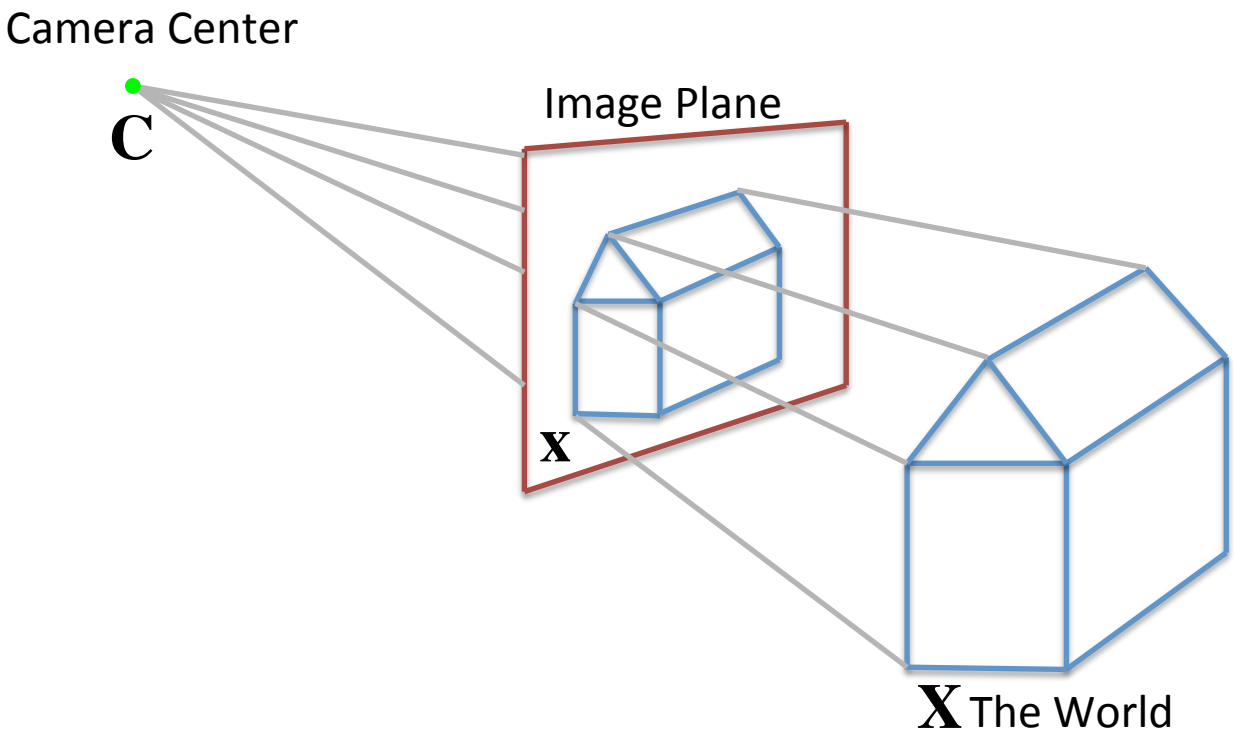


$$\lambda \begin{bmatrix} x \\ y \\ f \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \Leftrightarrow \begin{bmatrix} x \\ y \\ f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

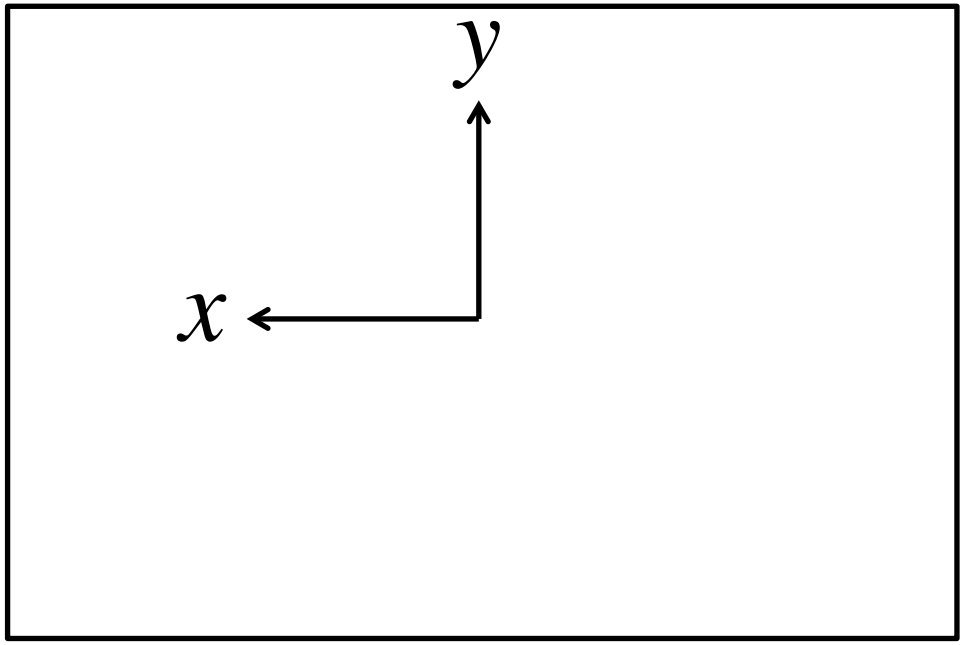
Up to a scale

Why the image plane is in front?



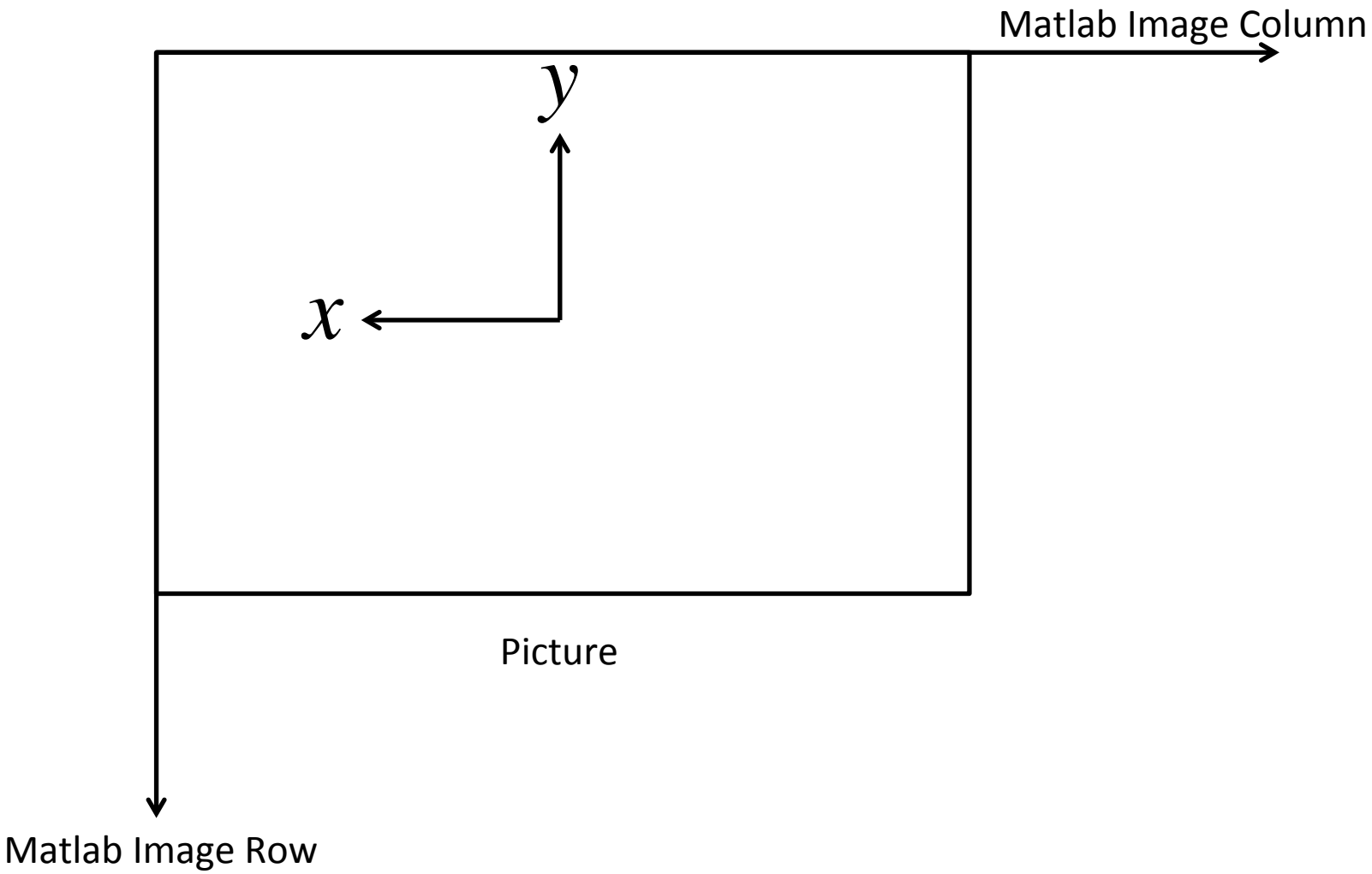


When they take a picture:

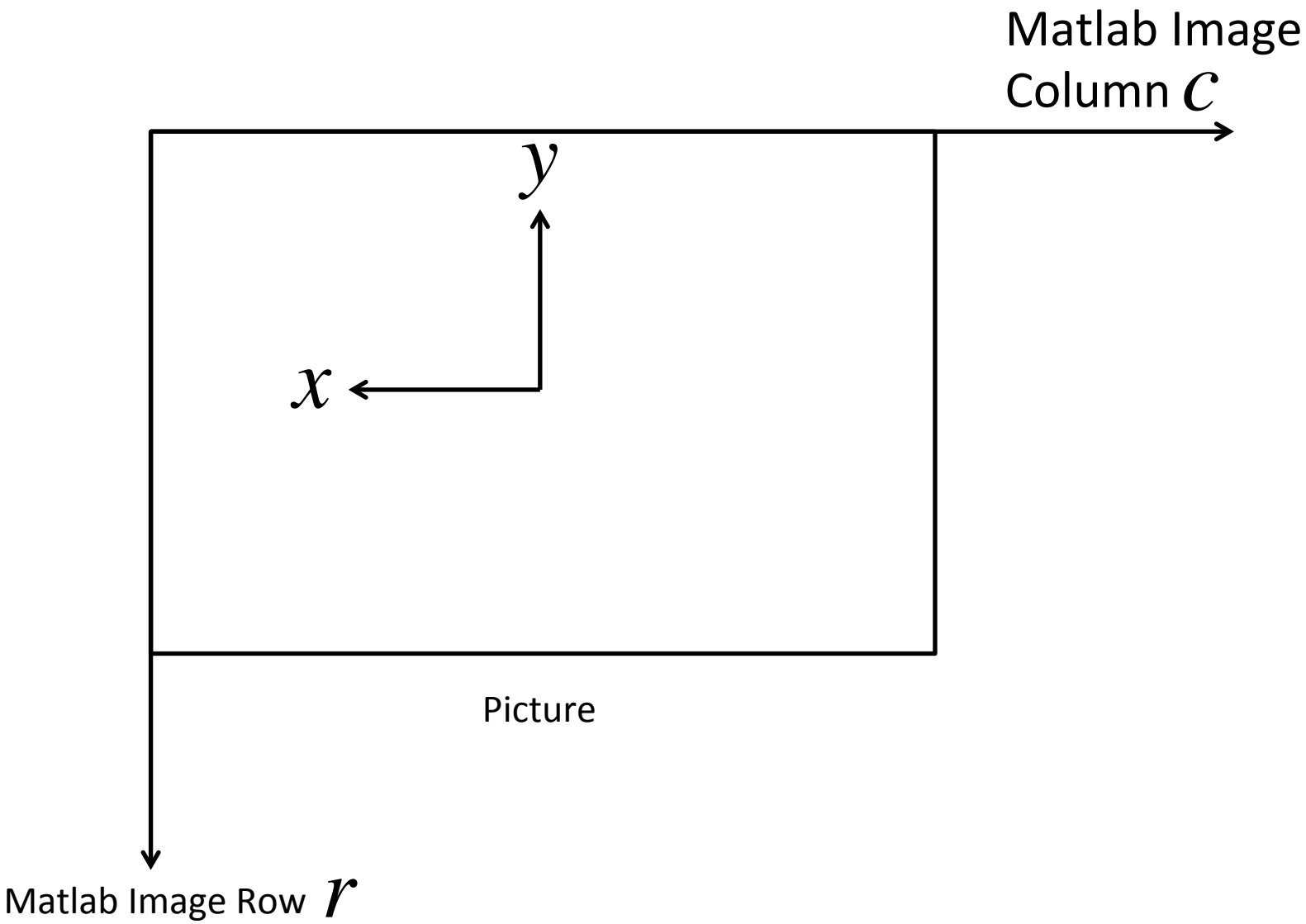


Picture

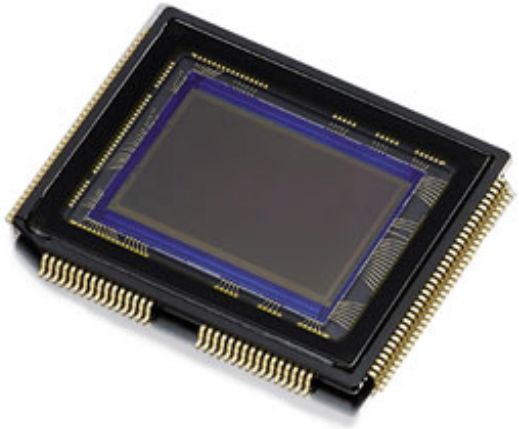
When they take a picture:

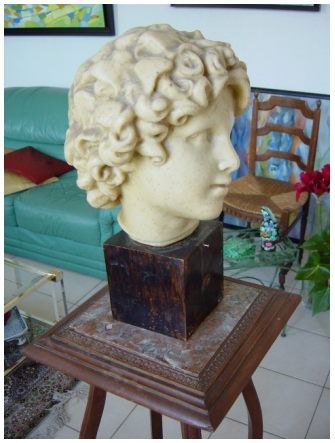


When they take a picture:



When they take a picture:





We let f to take care of this as well.
Unit of f is pixel/world unit.

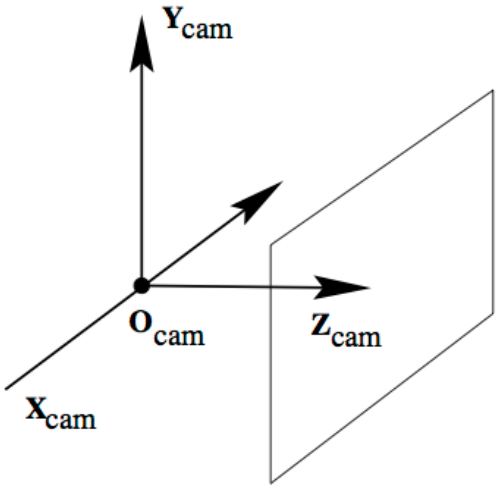
World Unit: e.g. Meters

Image Unit: Pixels

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

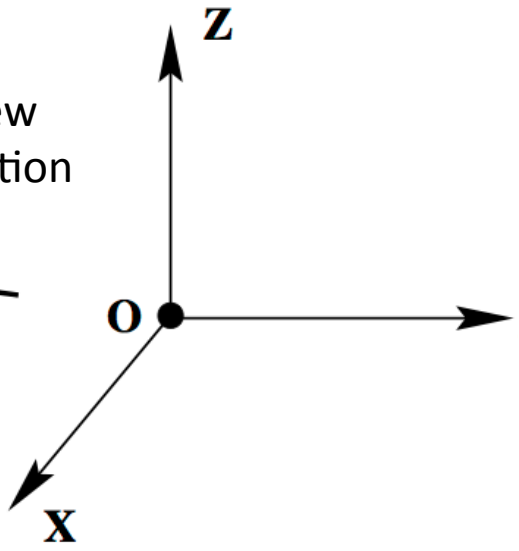
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

I live in a real world



Camera Coordinate System

Model-view
Transformation
R, t



World Coordinate System

$$\begin{pmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \\ 1 \end{pmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

World Coor. → Camera Coor.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}$$

$$\mathbf{x} = \mathbf{PX}$$

Camera Parameter
Camera Projection Matrix

$$\mathbf{P} = \mathbf{K}\left[\mathbf{R}|\mathbf{t}\right]$$

Intrinsic Extrinsic

I am sexy

- When people take a picture of me:

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R}\middle|\mathbf{t}\right]\mathbf{X}$$

I am very sexy

- When people take two pictures with same camera setting:

$$\mathbf{x}_1 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}$$

I am very very sexy

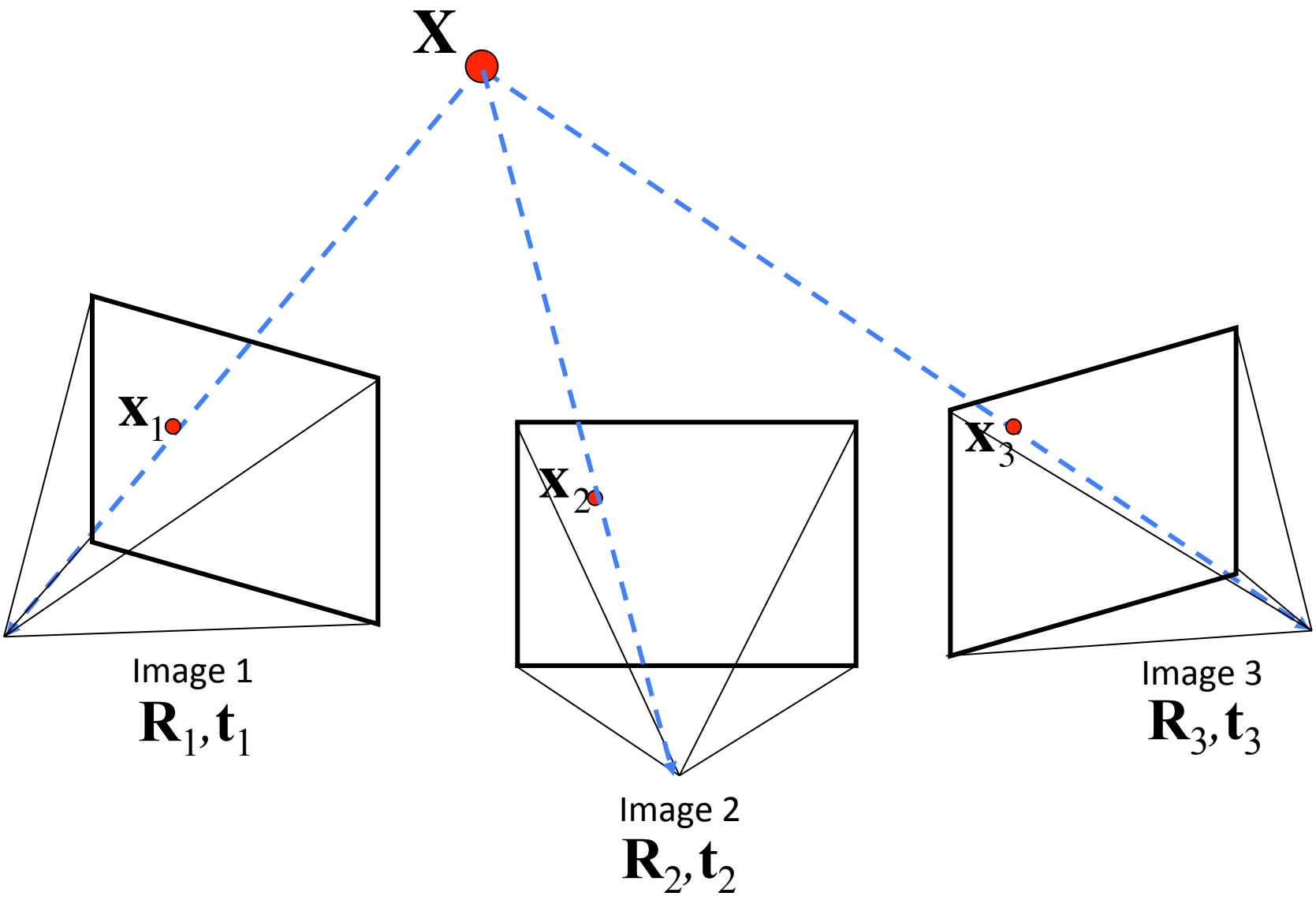
- When people take three pictures with same camera setting:

$$\mathbf{x}_1 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}$$

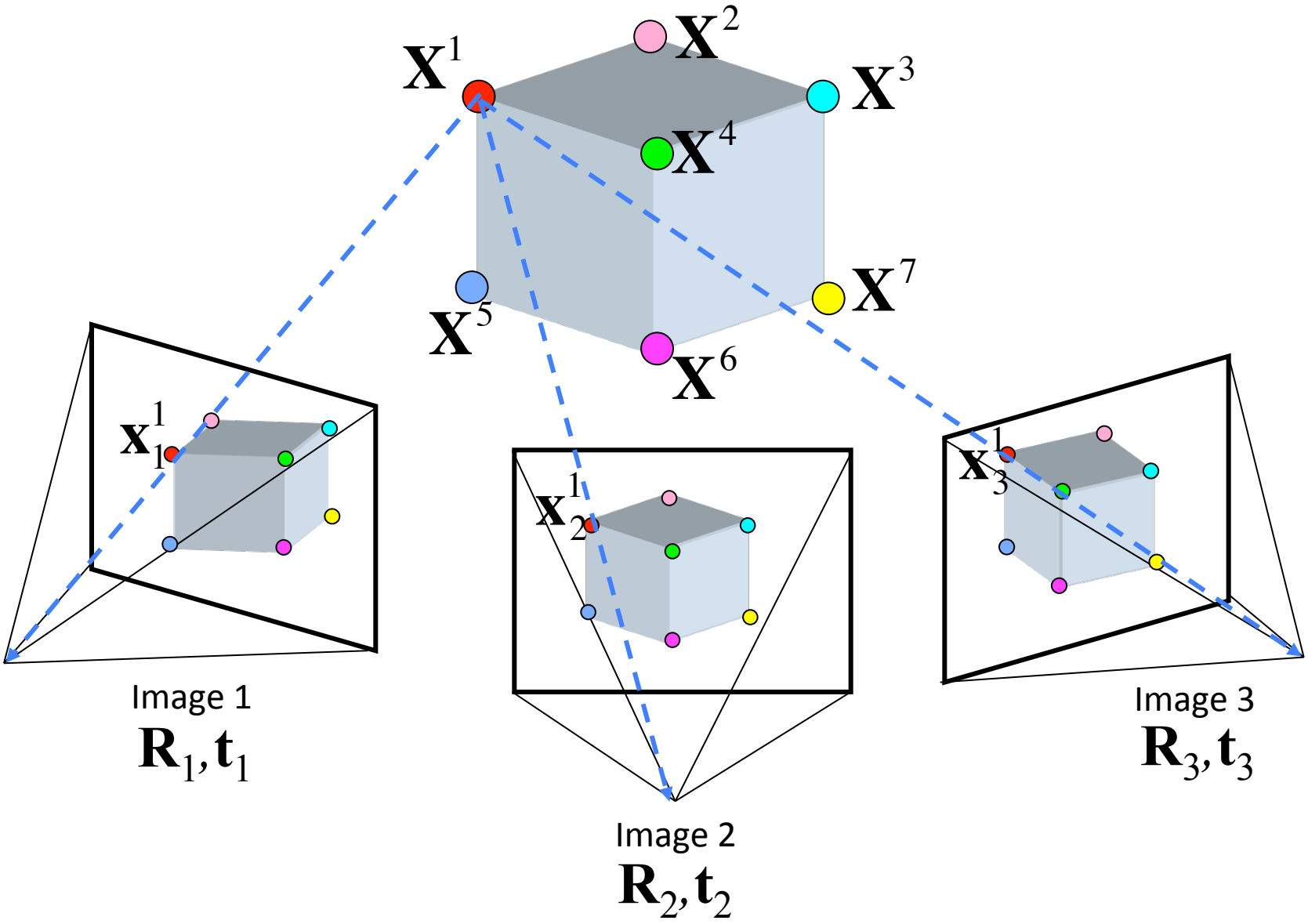
$$\mathbf{x}_3 = \mathbf{K}\left[\mathbf{R}_3 \middle| \mathbf{t}_3\right]\mathbf{X}$$

I am very very sexy



I join “America's Next Top Model”

I join “America's Next Top Model”

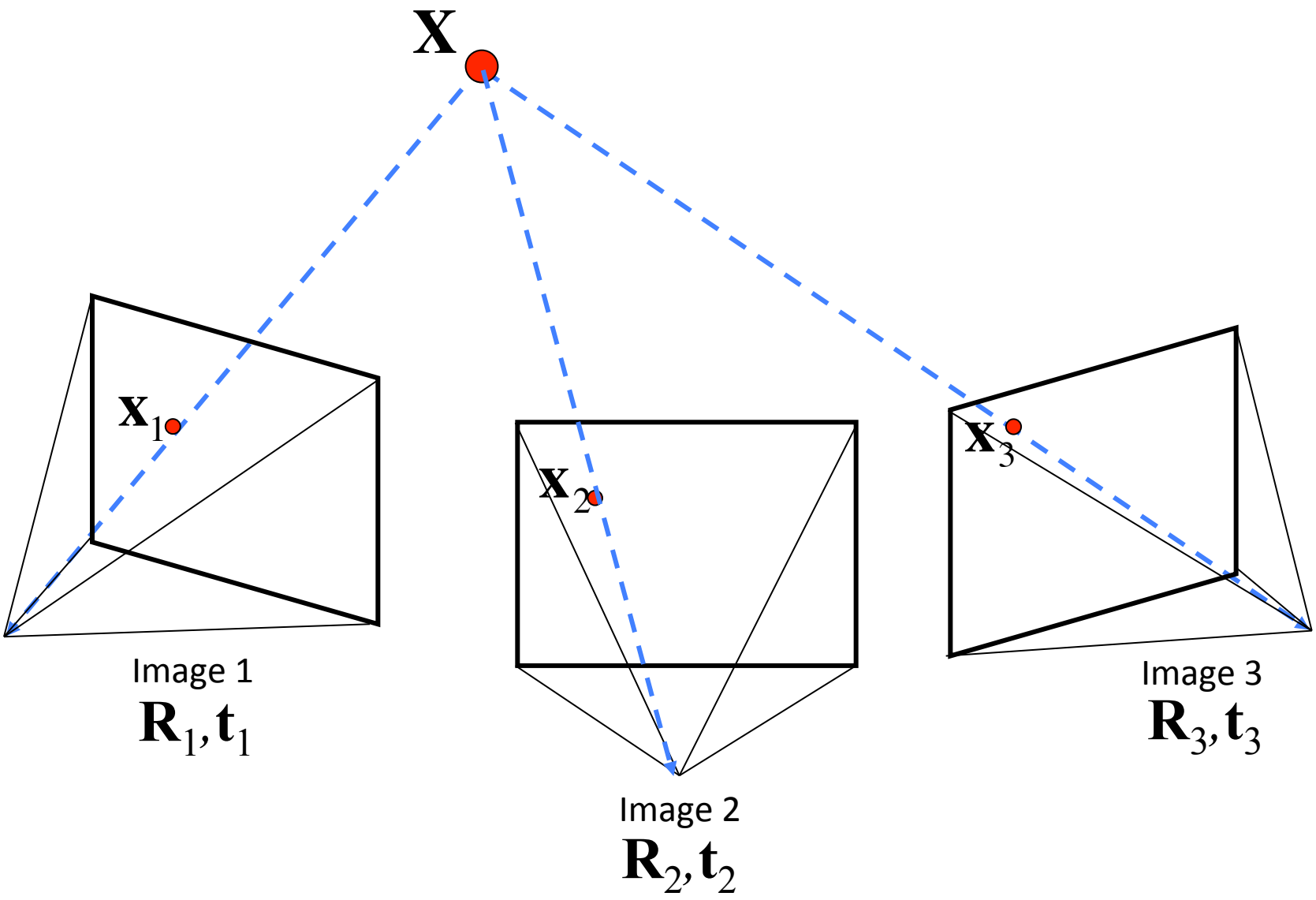


I join “America's Next Top Model”

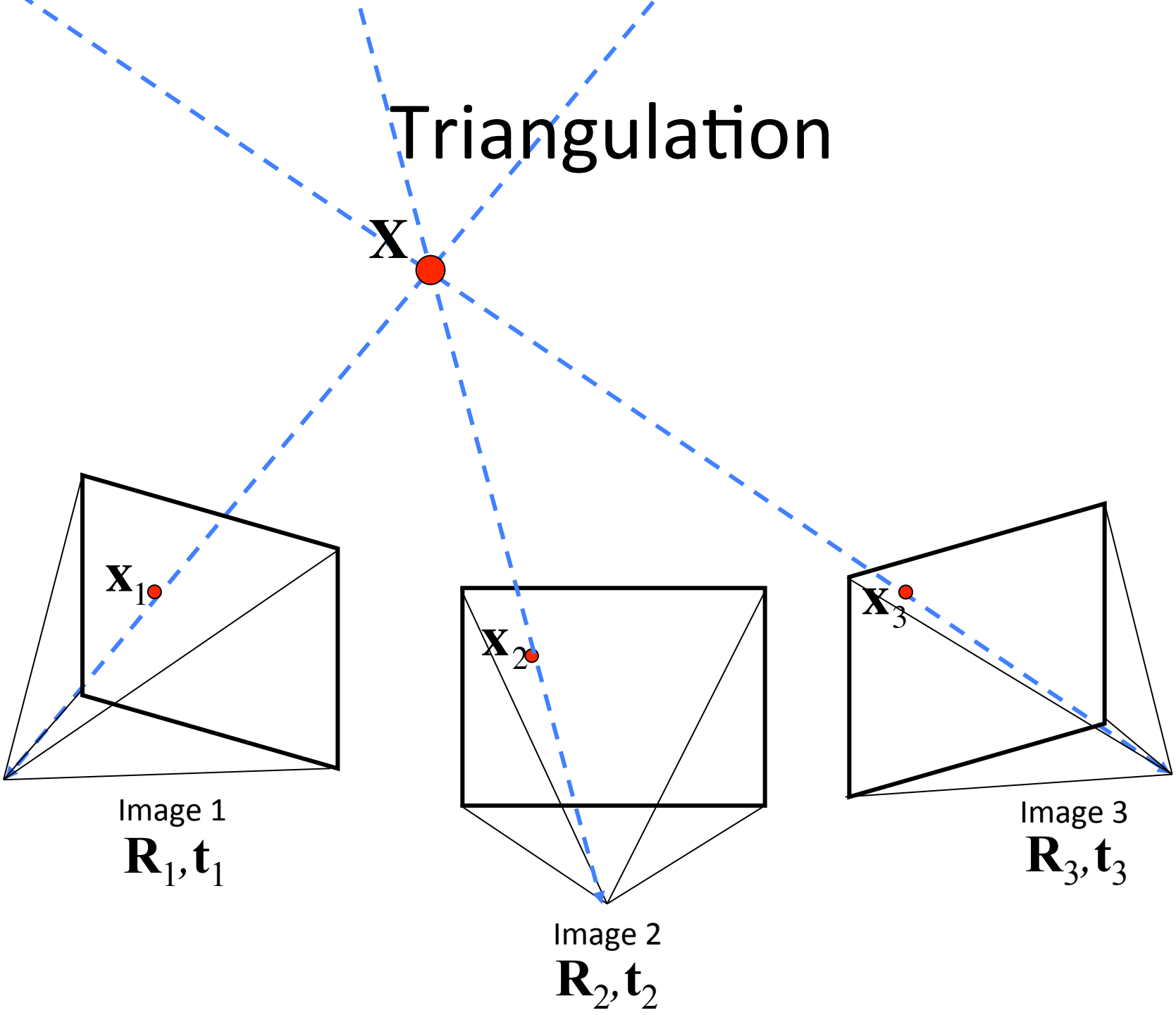
	Point 1	Point 2	Point 3
Image 1	$\mathbf{x}_1^1 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}^1$	$\mathbf{x}_1^2 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}^2$	
Image 2	$\mathbf{x}_2^1 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^1$	$\mathbf{x}_2^2 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^2$	$\mathbf{x}_2^3 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^3$
Image 3	$\mathbf{x}_3^1 = \mathbf{K}\left[\mathbf{R}_3 \middle| \mathbf{t}_3\right]\mathbf{X}^1$		$\mathbf{x}_3^3 = \mathbf{K}\left[\mathbf{R}_3 \middle| \mathbf{t}_3\right]\mathbf{X}^3$

Same Camera Same Setting = Same $\mathbf{K}$

Triangulation



Triangulation



Other Intrinsic Camera Parameters

- Principle point offset
  - especially when images are cropped (Internet)
- Skew

$$\mathbf{K} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Rightarrow \quad \mathbf{K} = \begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Radial distortion (due to optics of the lens)

$$r^2 = \|\mathbf{x}\|^2 = x^2 + y^2$$

$$\mathbf{x}' = (1 + k_1 r^2 + k_2 r^4)\mathbf{x}$$

before

after

Other Camera Models

- Fisheye
- Mirror
- Panorama
- Tilt-Shift Lens
- Biological Eyes

http://www.popgadget.net/2006/07/fisheye_camera.php

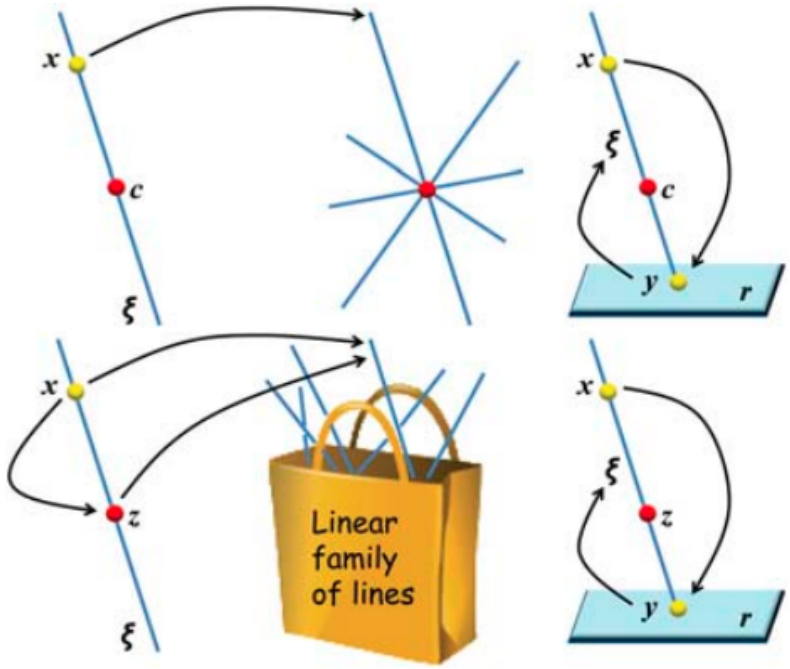
http://www.0-360.com/

http://sun360.csail.mit.edu

Canon TS-E 24mm f/3.5L II

What is a camera?

Definition: A camera is a two-parameter linear family of lines—that is, a degenerate regulus (rank-3 family), or a non-degenerate linear congruence (rank-4 family).



What is a Camera? J Ponce, 2009

White-board Exercise

$$\mathbf{K} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I brought a normal compact digital camera.

I took a picture at resolution 640x480 pixels.

What is the possible value of the focal length f ?

Focal Length
5.0 (W) - 40.0 (T) mm (35mm film equivalent: 28 (W) - 224 (T) mm)

Digital Zoom
4x

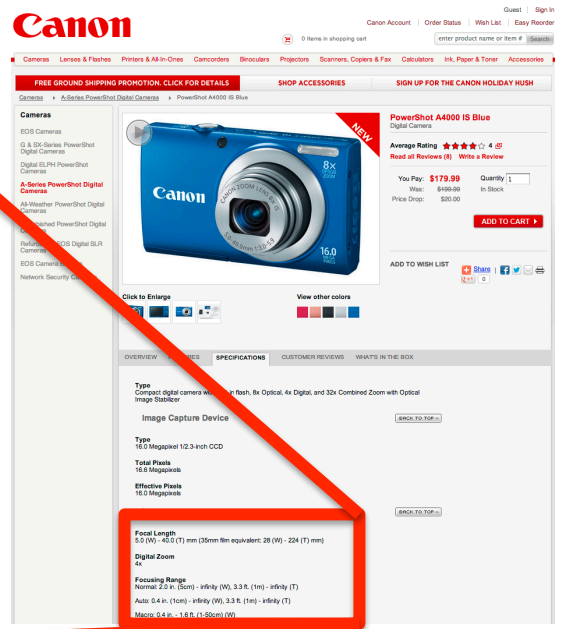
full-frame 35mm = width 36 mm height 24 mm
http://en.wikipedia.org/wiki/Angle_of_view

Focusing Range
Normal: 2.0 in. (5cm) - infinity (W), 3.3 ft. (1m) - infinity (T)

Auto: 0.4 in. (1cm) - infinity (W), 3.3 ft. (1m) - infinity (T)

Macro: 0.4 in. - 1.6 ft. (1-50cm) (W)

Autofocus System
TTL Autofocus



http://goo.gl/ENaw4

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Steps

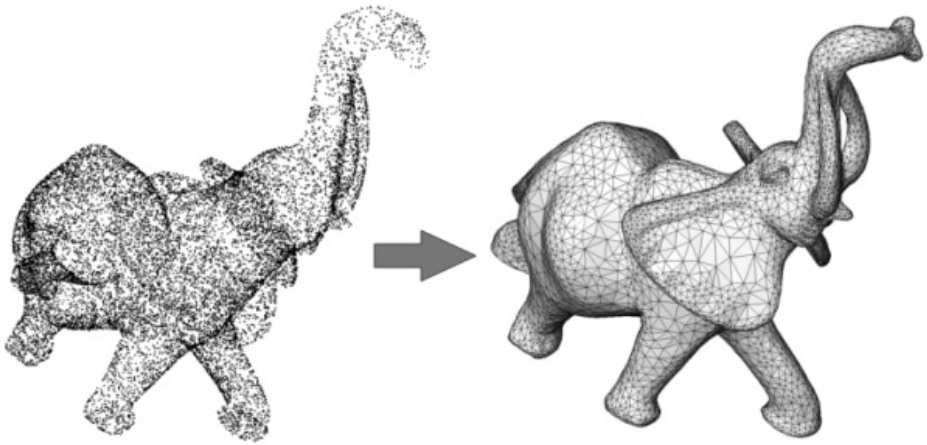
	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

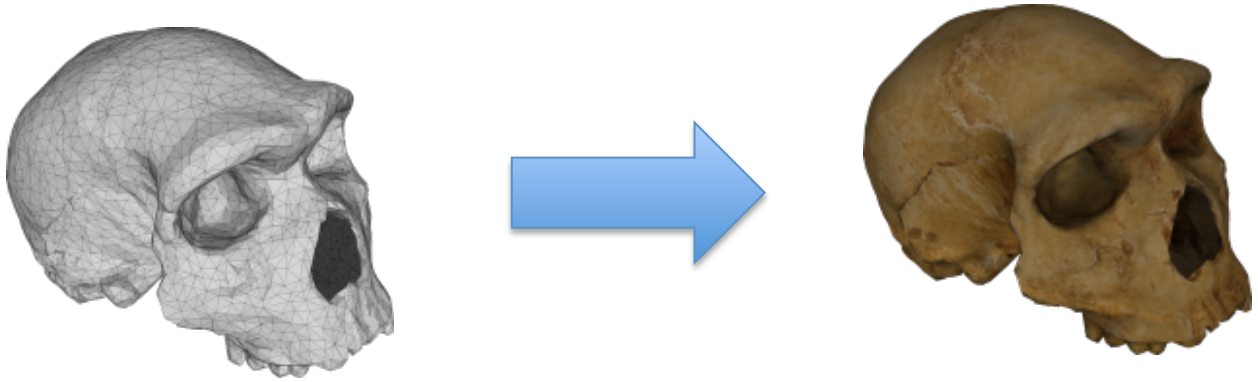
Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling



Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling



Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Image-based Modeling

Input Images

Image-based Street-side City Modeling. Xiao et al, 2009. http://people.csail.mit.edu/jxiao/CityRec/

Image-based Modeling

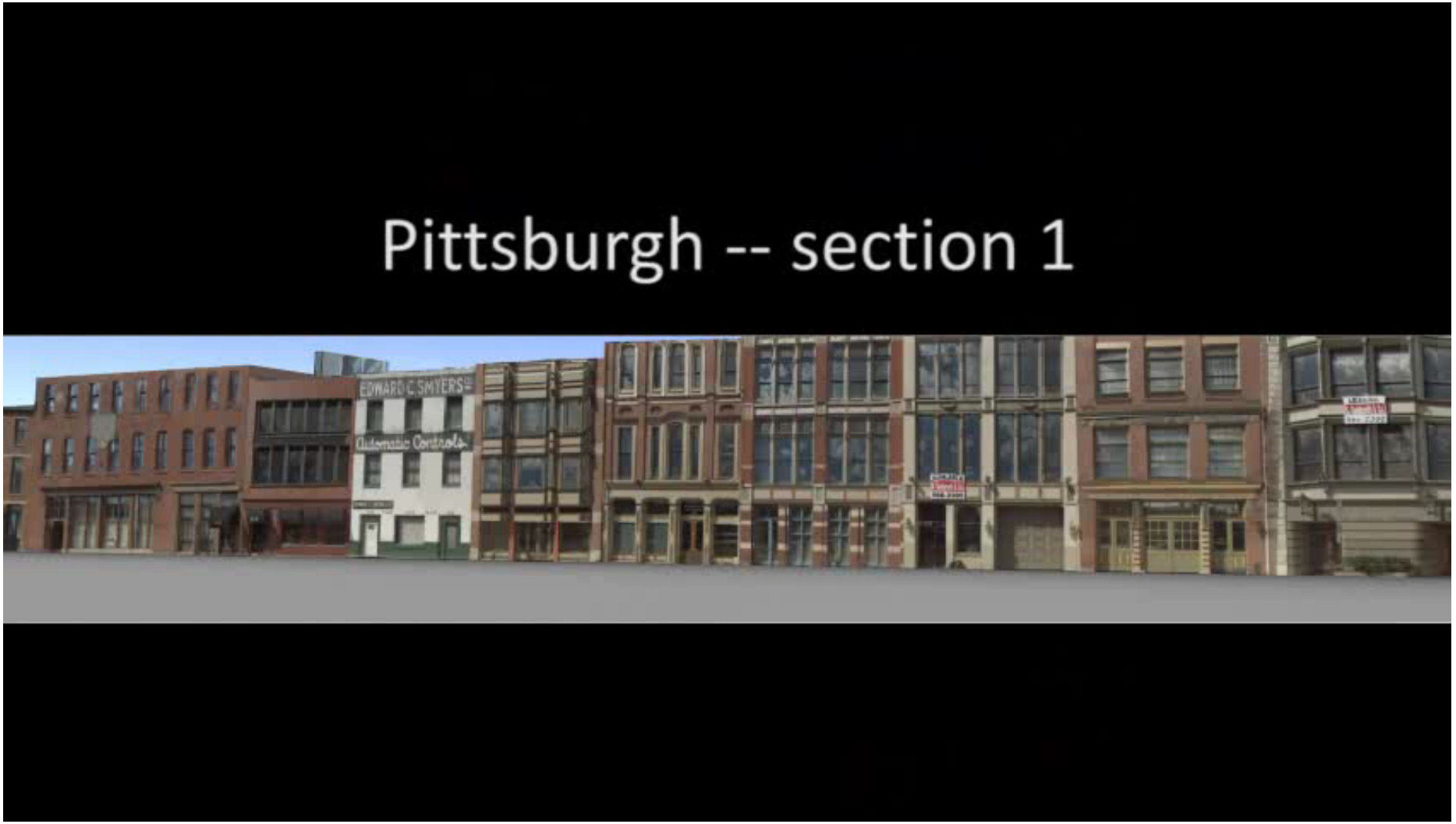


Image-based Street-side City Modeling. Xiao et al, 2009. http://people.csail.mit.edu/jxiao/CityRec/

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

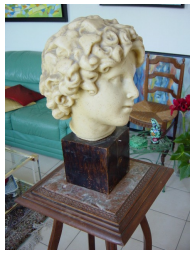
Steps

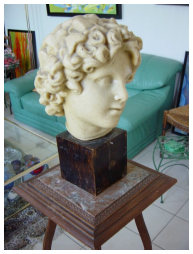
	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

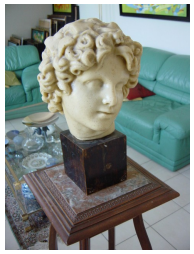
Structure From Motion

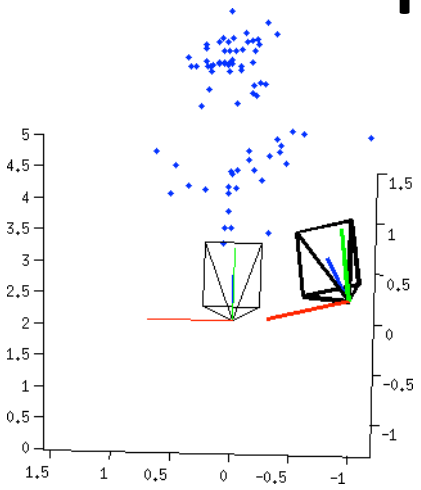
- Structure = 3D Point Cloud of the Scene
- Motion = Camera Location and Orientation
- SFM = Get the Point Cloud from Moving Cameras
- Structure and Motion: Joint Problems to Solve

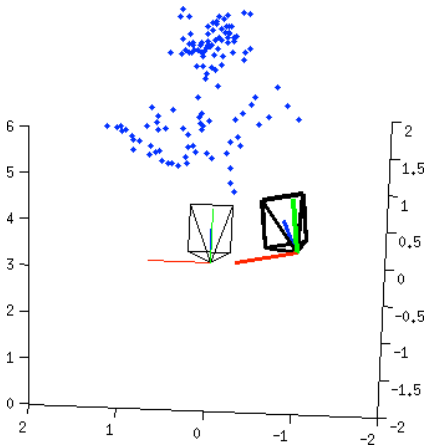
Pipeline

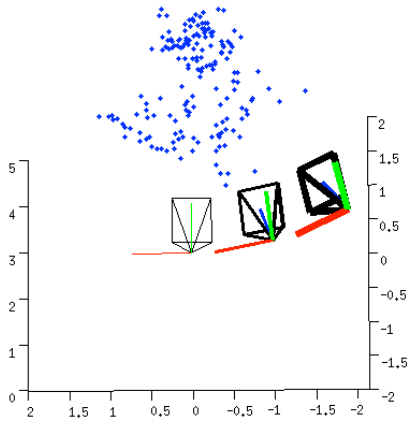


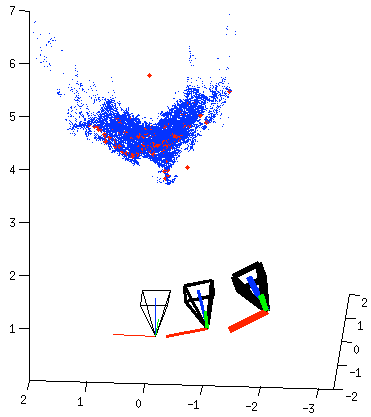








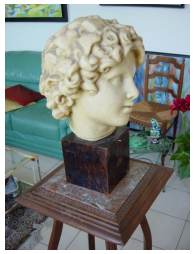


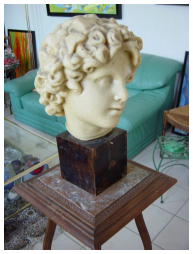


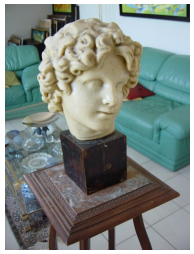
Structure from Motion (SFM)

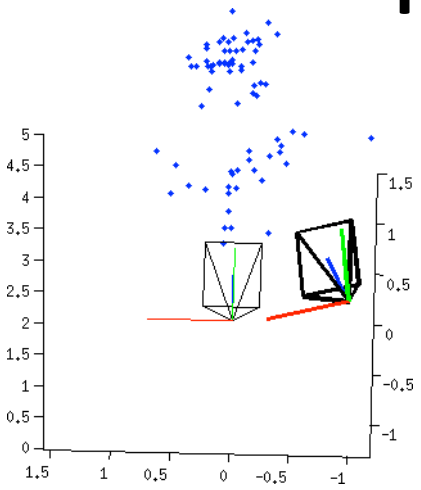
Multi-view Stereo (MVS)

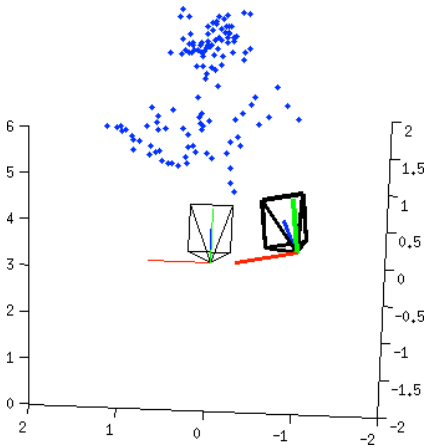
Pipeline

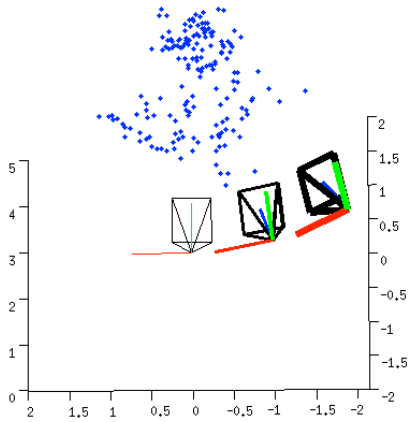


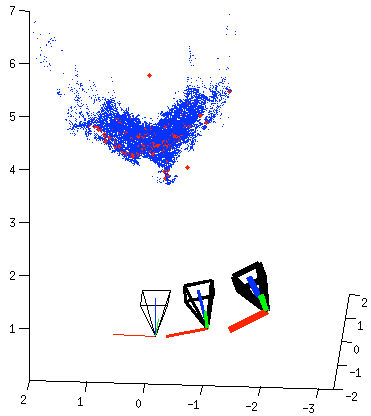








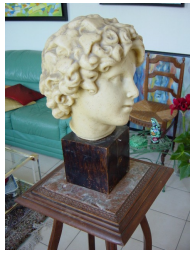


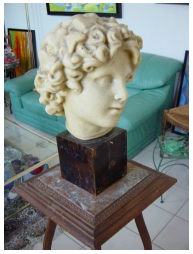


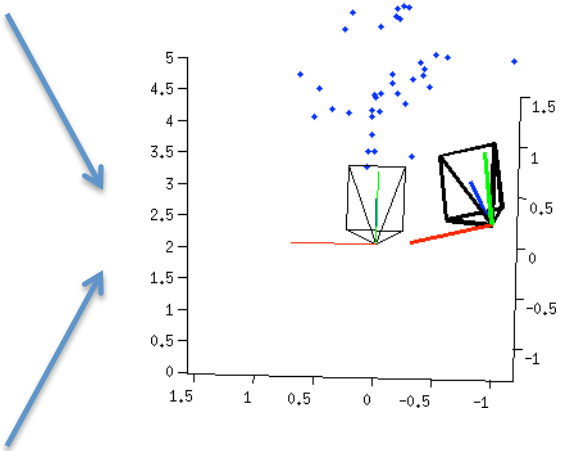
Structure from Motion (SFM)

Multi-view Stereo (MVS)

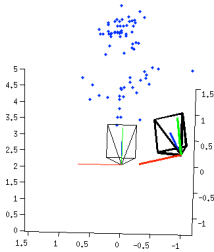
Two-view Reconstruction



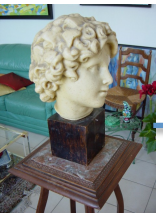
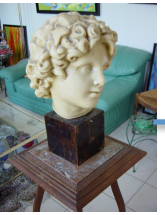




Two-view Reconstruction



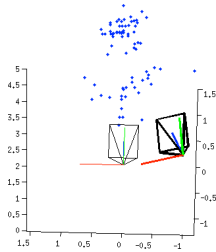
Two-view Reconstruction

keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation →

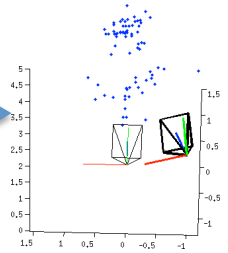


Keypoints Detection

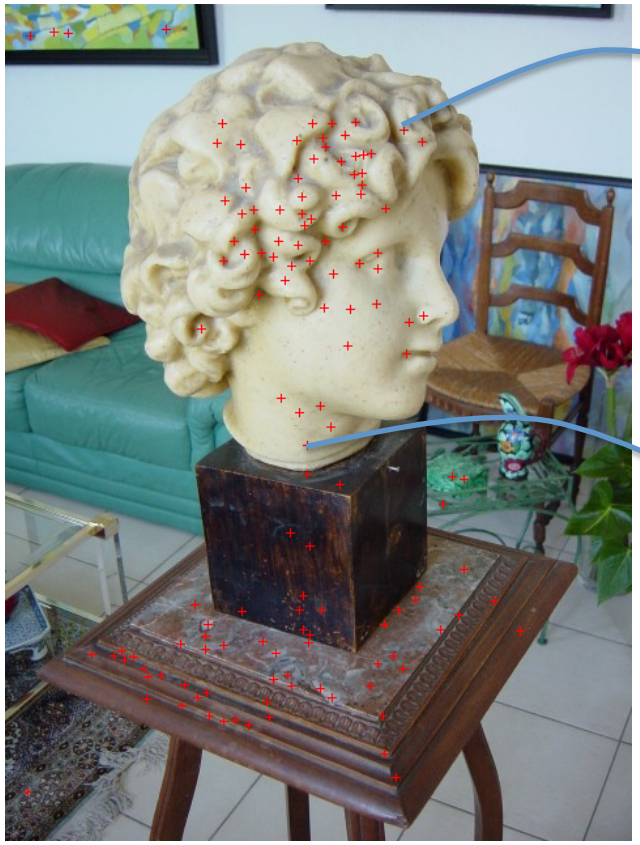
keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation



Descriptor for each point



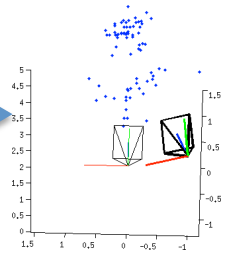
SIFT descriptor

SIFT descriptor

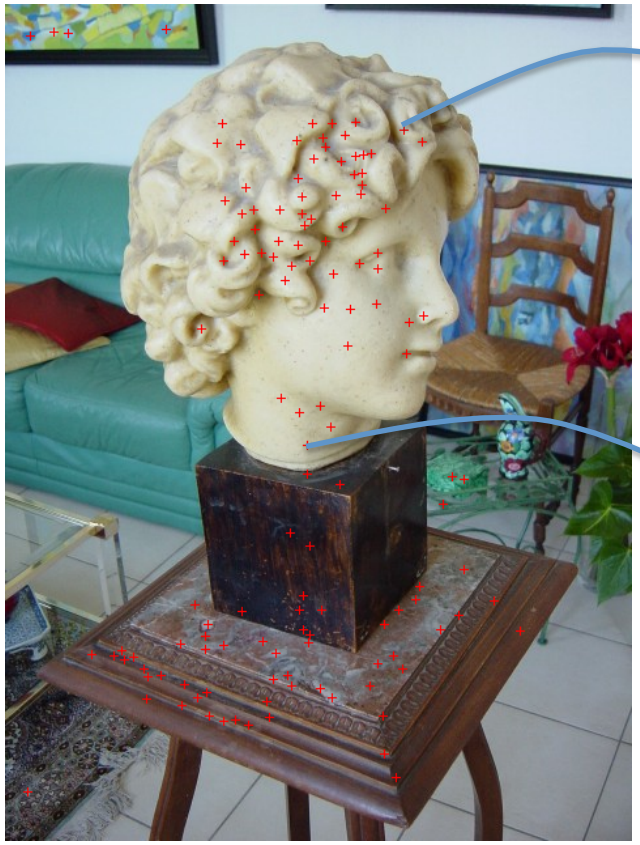
keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation



Same for the other images

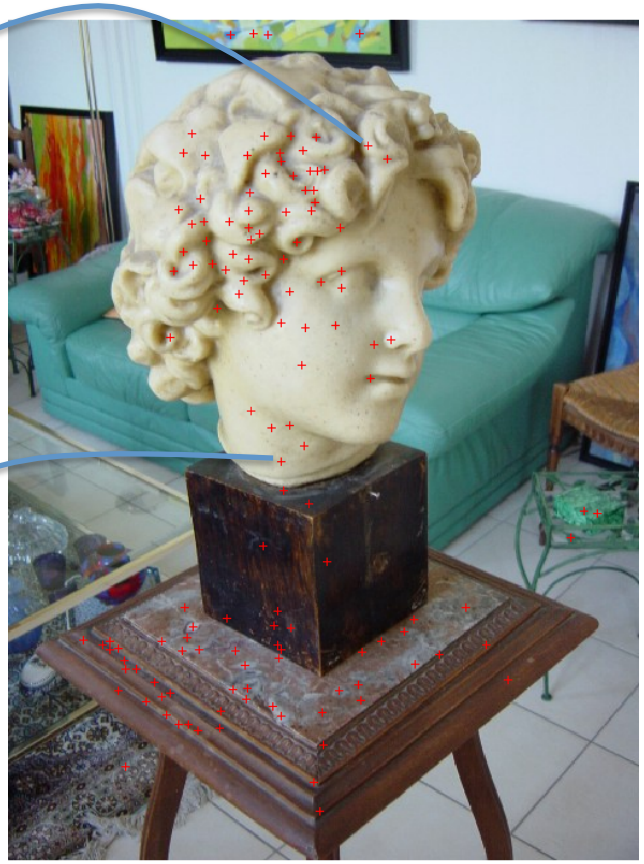


SIFT descriptor

SIFT descriptor

SIFT descriptor

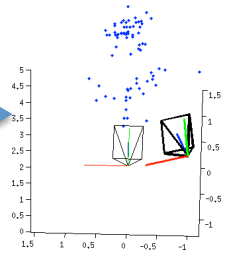
SIFT descriptor



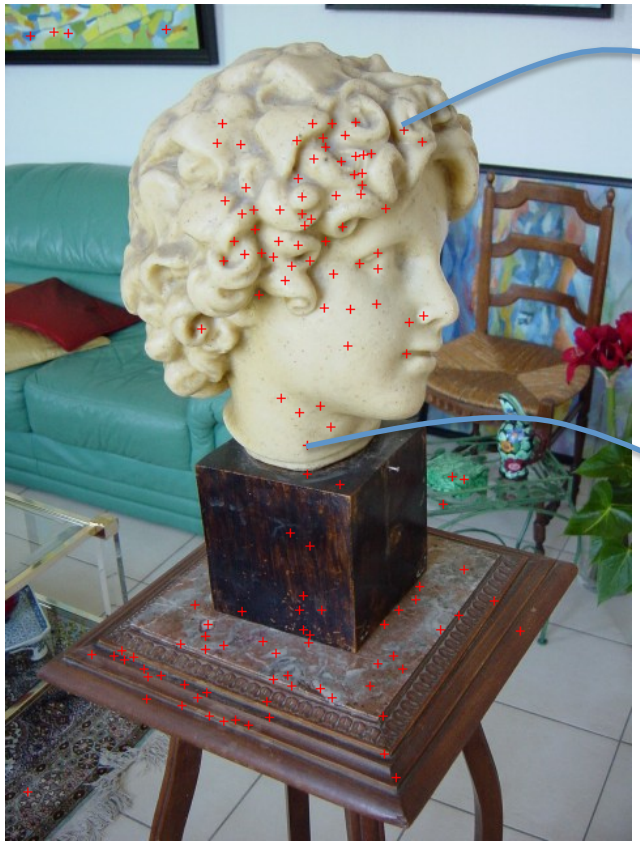
keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation



Point Match for correspondences

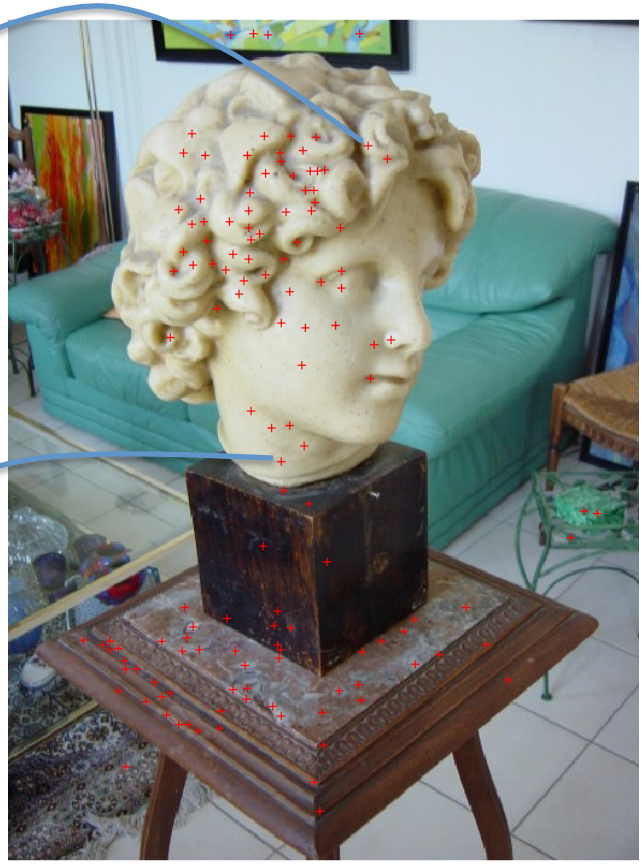


SIFT descriptor

SIFT descriptor

SIFT descriptor

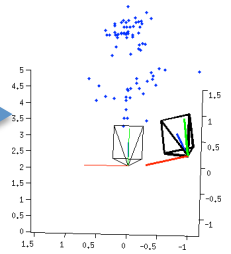
SIFT descriptor



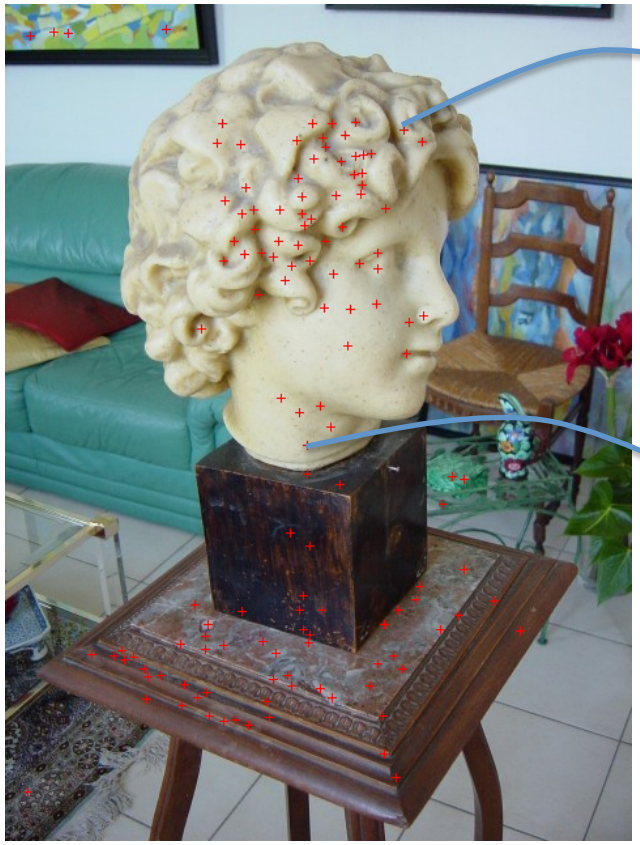
keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation



Point Match for correspondences

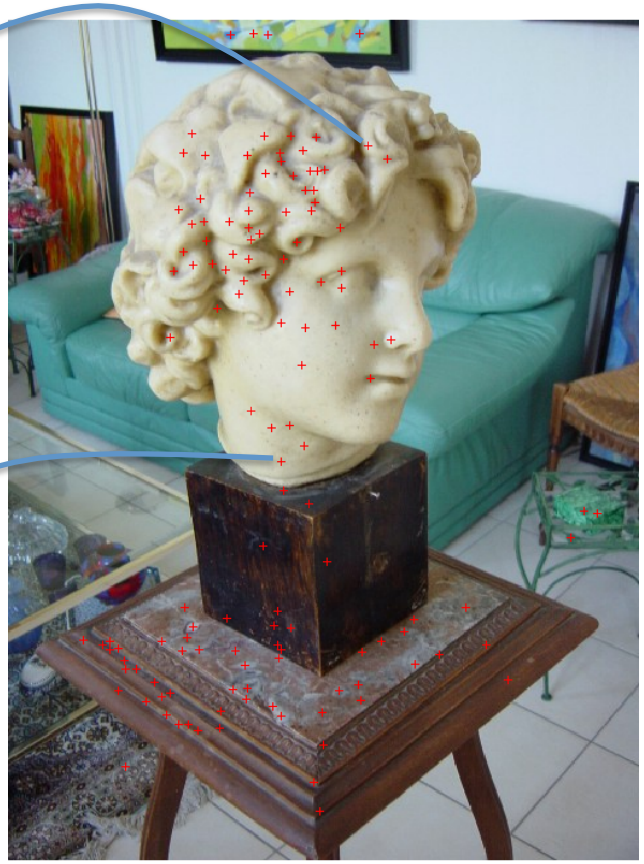


SIFT descriptor

SIFT descriptor

SIFT descriptor

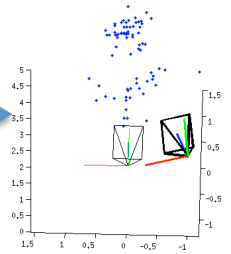
SIFT descriptor



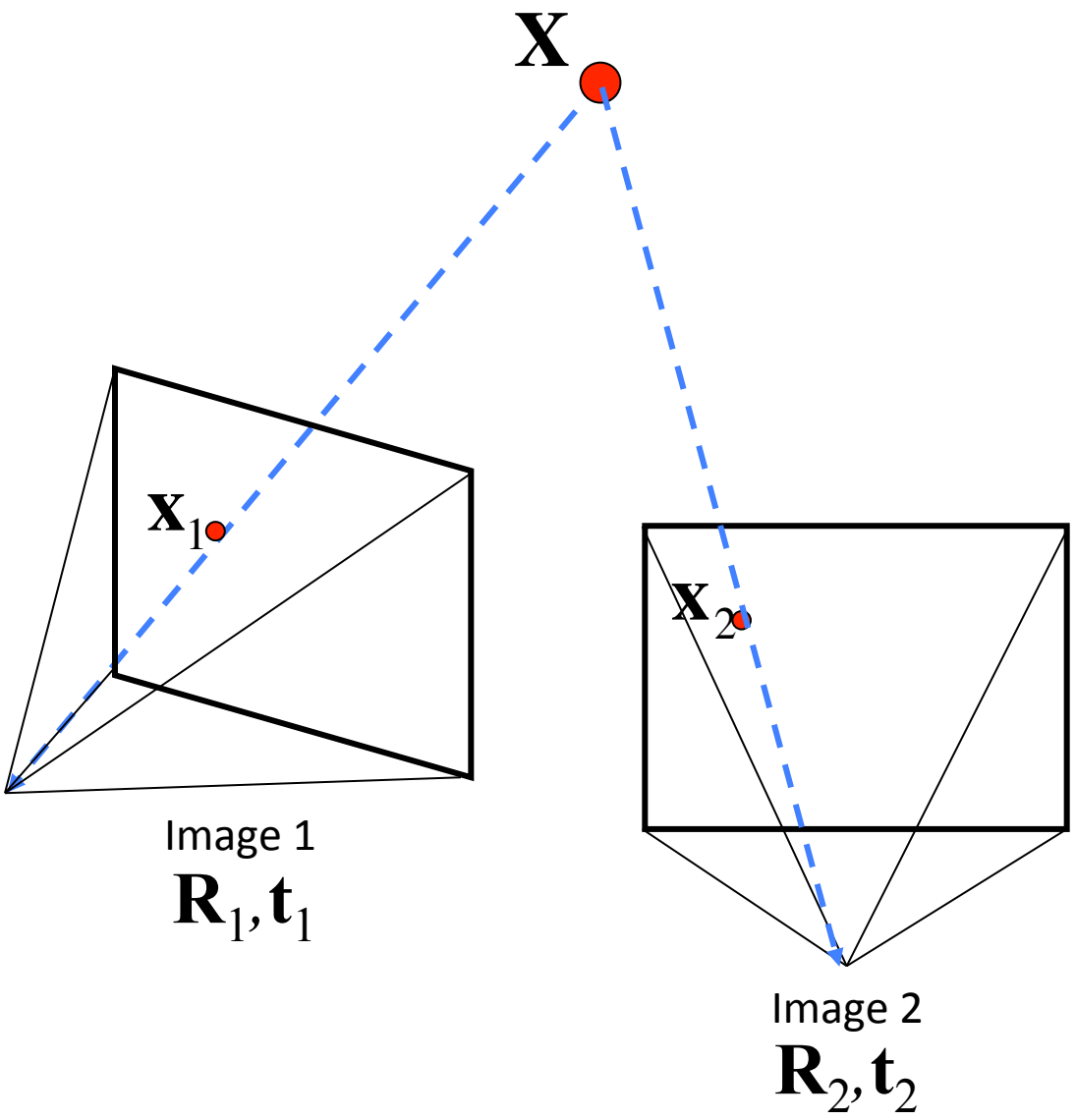
keypoints

keypoints

match → fundamental matrix → essential matrix → [R|t] → triangulation



Fundamental Matrix



$$\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$$

$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

Gangnam Style

Gangnam Style

Fundamental Matrix Song

Fundamental Matrix Song

After this song, there will be a quiz.
Listen carefully!

Exercise

What is the difference between Fundamental Matrix and Homography?

(Both of them are to explain 2D point to 2D point correspondences.)

Estimating Fundamental Matrix

- Given a correspondence

$$\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$$

- The basic incidence relation is

$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0 \quad \Longleftrightarrow \quad \begin{bmatrix} x_1x_2, x_1y_2, x_1, y_1x_2, y_1y_2, y_1, x_2, y_2, 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

Need 8 points

Estimating Fundamental Matrix

$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$ for 8 point correspondences:

$$\mathbf{x}_1^1 \leftrightarrow \mathbf{x}_2^1, \mathbf{x}_1^2 \leftrightarrow \mathbf{x}_2^2, \mathbf{x}_1^3 \leftrightarrow \mathbf{x}_2^3, \mathbf{x}_1^4 \leftrightarrow \mathbf{x}_2^4, \mathbf{x}_1^5 \leftrightarrow \mathbf{x}_2^5, \mathbf{x}_1^6 \leftrightarrow \mathbf{x}_2^6, \mathbf{x}_1^7 \leftrightarrow \mathbf{x}_2^7, \mathbf{x}_1^8 \leftrightarrow \mathbf{x}_2^8$$

$$\begin{bmatrix} x_1^1x_2^1 & x_1^1y_2^1 & x_1^1 & y_1^1x_2^1 & y_1^1y_2^1 & y_1^1 & x_2^1 & y_2^1 & 1 \\ x_1^2x_2^2 & x_1^2y_2^2 & x_1^2 & y_1^2x_2^2 & y_1^2y_2^2 & y_1^2 & x_2^2 & y_2^2 & 1 \\ x_1^3x_2^3 & x_1^3y_2^3 & x_1^3 & y_1^3x_2^3 & y_1^3y_2^3 & y_1^3 & x_2^3 & y_2^3 & 1 \\ x_1^4x_2^4 & x_1^4y_2^4 & x_1^4 & y_1^4x_2^4 & y_1^4y_2^4 & y_1^4 & x_2^4 & y_2^4 & 1 \\ x_1^5x_2^5 & x_1^5y_2^5 & x_1^5 & y_1^5x_2^5 & y_1^5y_2^5 & y_1^5 & x_2^5 & y_2^5 & 1 \\ x_1^6x_2^6 & x_1^6y_2^6 & x_1^6 & y_1^6x_2^6 & y_1^6y_2^6 & y_1^6 & x_2^6 & y_2^6 & 1 \\ x_1^7x_2^7 & x_1^7y_2^7 & x_1^7 & y_1^7x_2^7 & y_1^7y_2^7 & y_1^7 & x_2^7 & y_2^7 & 1 \\ x_1^8x_2^8 & x_1^8y_2^8 & x_1^8 & y_1^8x_2^8 & y_1^8y_2^8 & y_1^8 & x_2^8 & y_2^8 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0 \quad \Rightarrow \quad \mathbf{Ax} = \mathbf{b}$$

$$\mathbf{Af} = \mathbf{0}$$

Direct Linear Transformation (DLT)

Algebraic Error vs. Geometric Error

- Algebraic Error

$$\min \|\mathbf{Af}\|$$

- Geometric Error (better) Unit: pixel

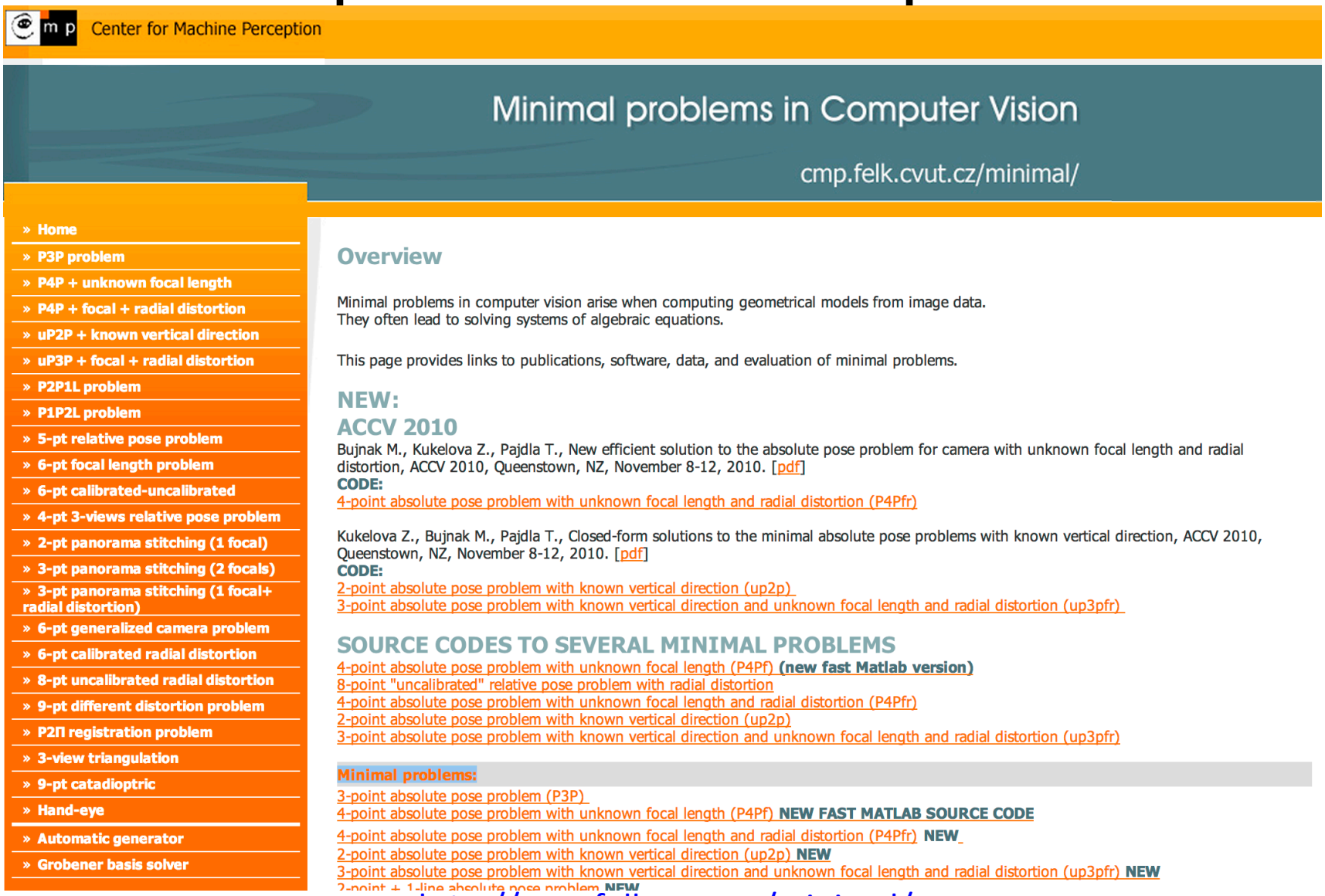
$$\min \sum_j d\left(\mathbf{x}_1^j, \mathbf{F}\mathbf{x}_2^j\right)^2 + d\left(\mathbf{x}_2^j, \mathbf{F}^T\mathbf{x}_1^j\right)^2$$

Solved by (non-linear) least square solver (e.g. Ceres)

RANSAC to Estimate Fundamental Matrix

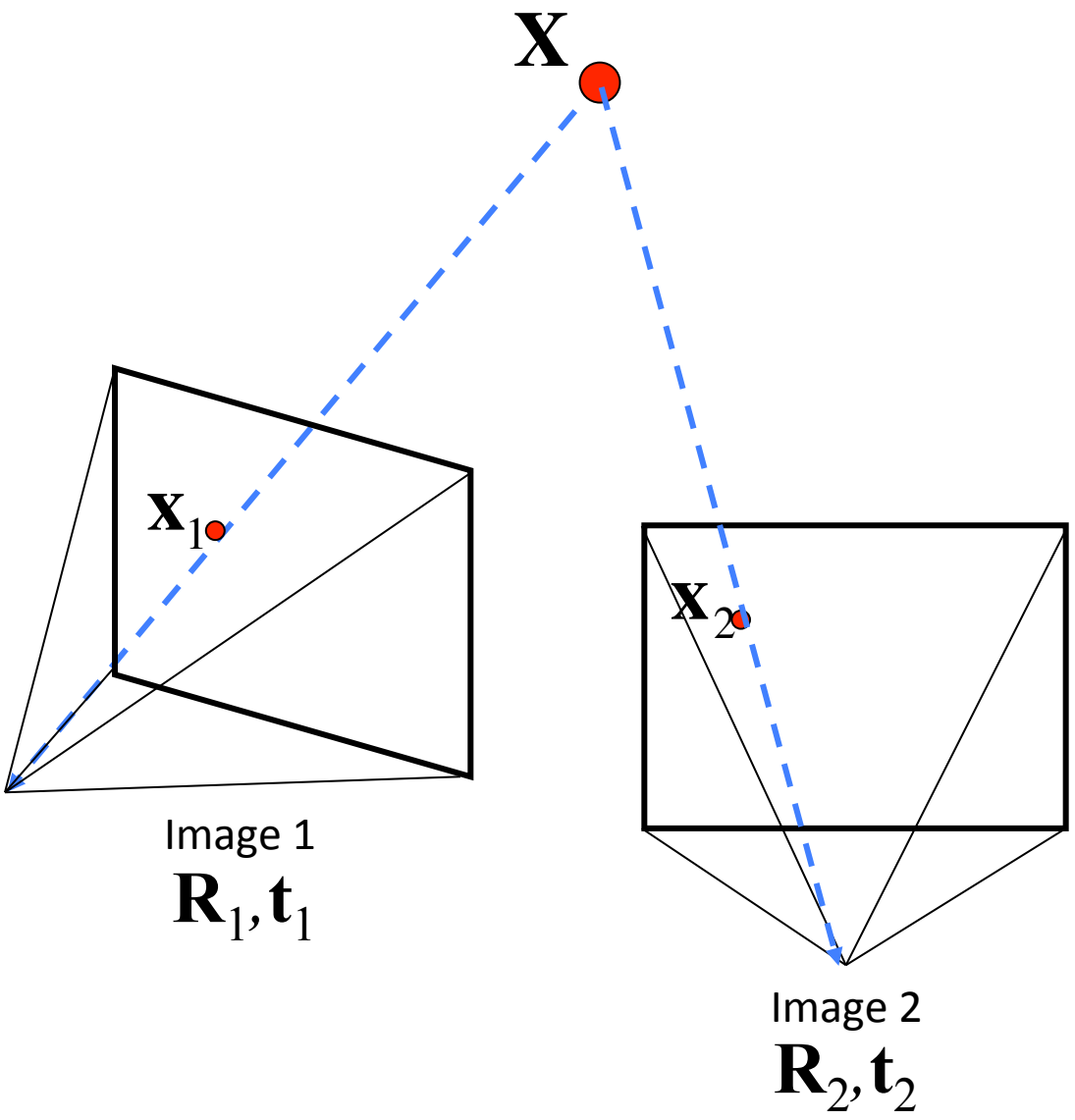
- For many times
  - Pick 8 points
  - Compute a solution for $\mathbf{F}$ using these 8 points
  - Count number of inliers
- Pick the one with the largest number of inliers

Minimal problems in Computer Vision



http://cmp.felk.cvut.cz/minimal/

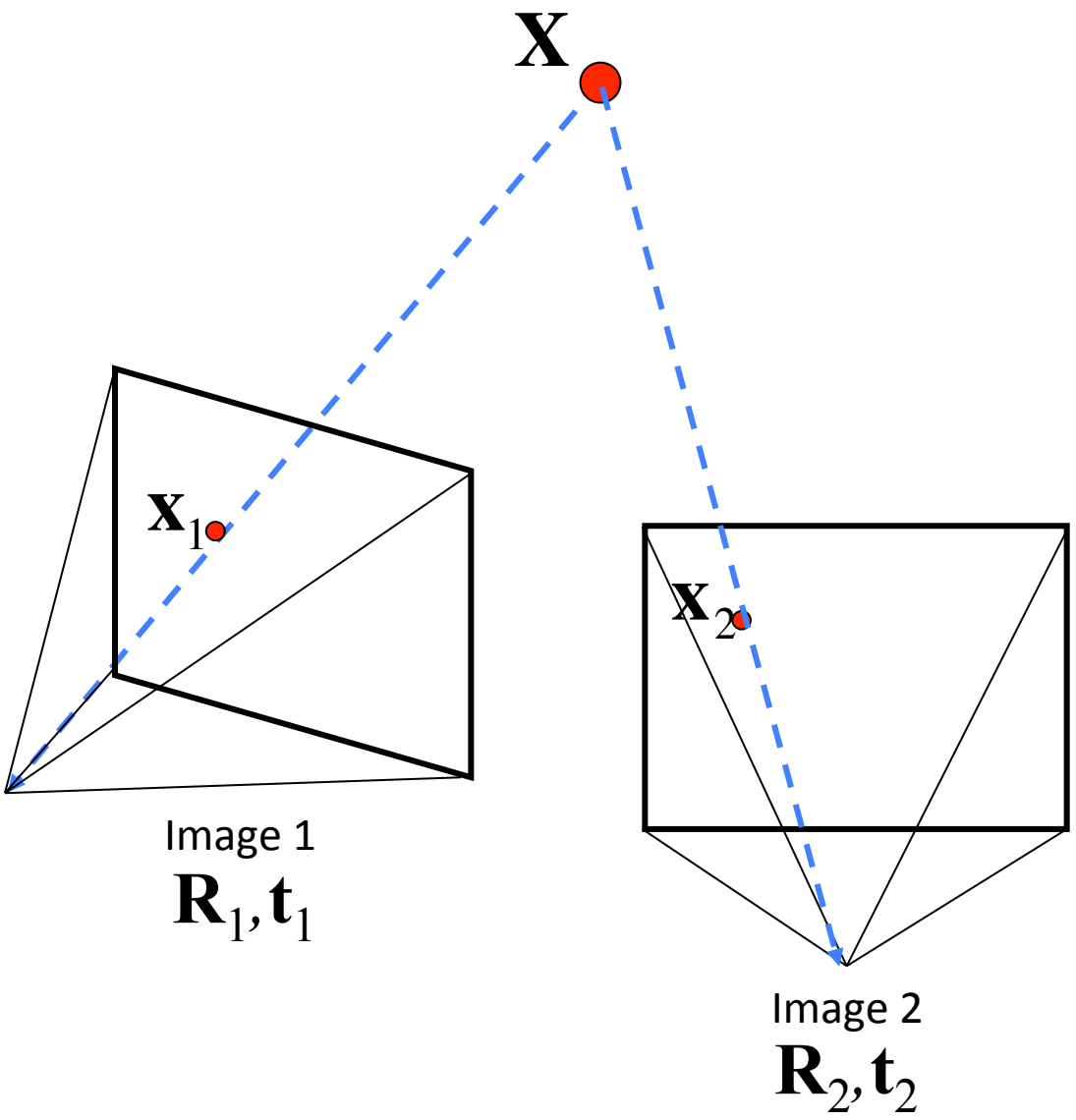
Fundamental Matrix → Essential Matrix



$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

$$\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2$$

Essential Matrix → $[\mathbf{R}|\mathbf{t}]$



$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

$$\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2$$

Essential Matrix → $\left[\mathbf{R}|\mathbf{t}\right]$

Result 9.19. For a given essential matrix

$$\mathbf{E} = \mathbf{U}\mathrm{diag}\left(1,1,0\right)\mathbf{V}^T,$$

and the first camera matrix $\mathbf{P}_1 = \left[\mathbf{I}|\mathbf{0}\right]$, there are four possible choices for the second camera matrix $\mathbf{P}_2$:

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}\mathbf{V}^T|+\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}\mathbf{V}^T|-\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}^T\mathbf{V}^T|+\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}^T\mathbf{V}^T|-\mathbf{u}_3\right]$$

$$\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Page 259 of the bible (Multiple View Geometry, 2nd Ed)

Four Possible Solutions

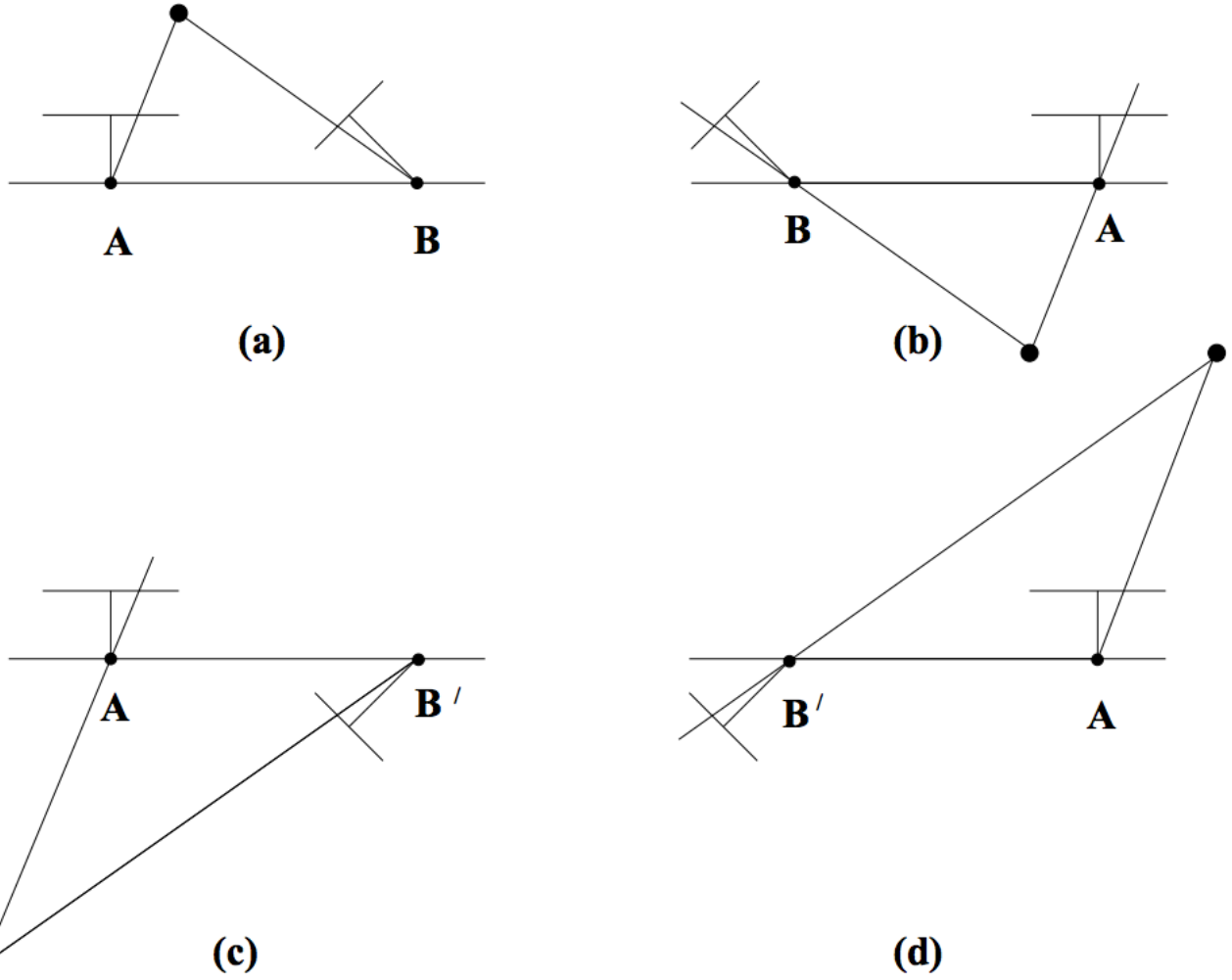
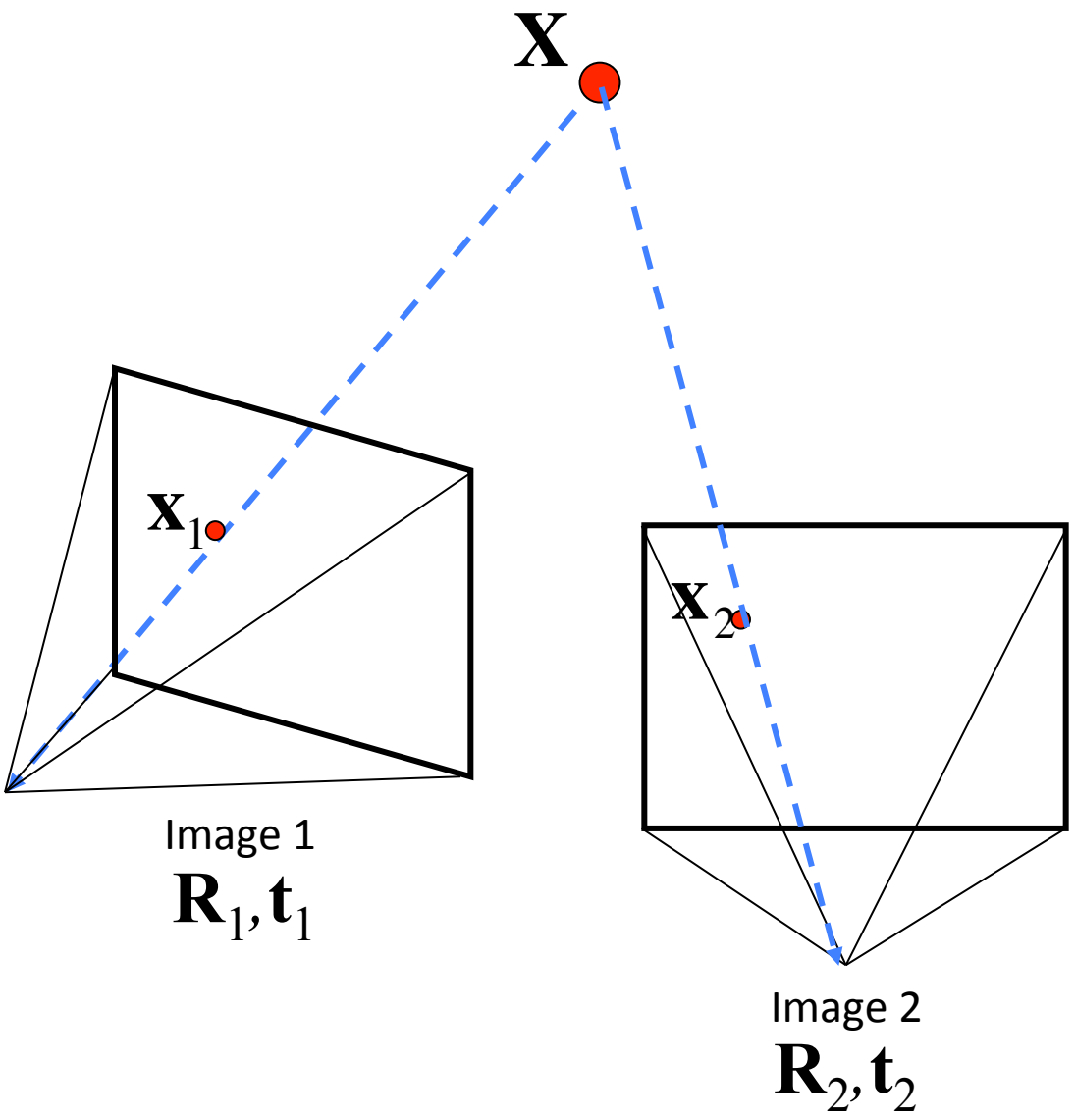


Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates* 180° *about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

Page 260 of the bible (Multiple View Geometry, 2nd Ed)

Triangulation



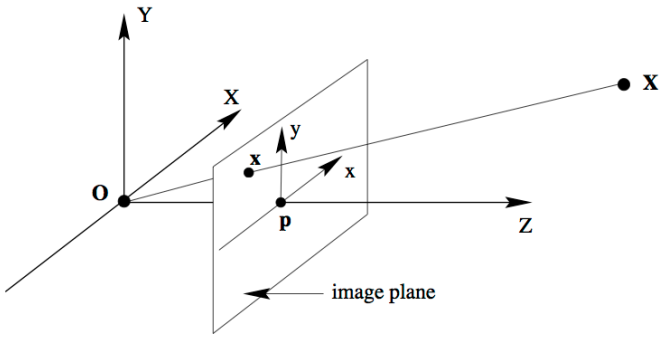
In front of the camera?

- Camera Extrinsic $[\mathbf{R}|\mathbf{t}]$

$$\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} = \mathbf{R}\begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} + \mathbf{t} \quad \Leftrightarrow \quad \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} = \mathbf{R}^{-1}\left(\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} - \mathbf{t}\right) = \mathbf{R}^{T}\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} - \mathbf{R}^{T}\mathbf{t}$$

- Camera Center

$$\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \mathbf{C} = \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} = \mathbf{R}^{T}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \mathbf{R}^{T}\mathbf{t} = -\mathbf{R}^{T}\mathbf{t}$$



- View Direction

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left(\mathbf{R}^{T}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \mathbf{R}^{T}\mathbf{t}\right) - (\mathbf{C}) = \left(\mathbf{R}(3,:)^{T} - \mathbf{R}^{T}\mathbf{t}\right) - \left(-\mathbf{R}^{T}\mathbf{t}\right) = \mathbf{R}(3,:)^{T}$$

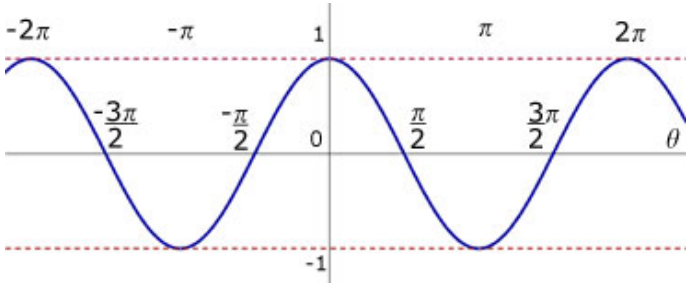
Camera Coordinate System

World Coordinate System

In front of the camera?

- A point $\mathbf{X}$
- Direction from camera center to point $\mathbf{X} - \mathbf{C}$
- Angle Between Two Vectors

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \|\mathbf{B}\| \cos\theta$$



- Angle Between $\mathbf{X} - \mathbf{C}$ and View Direction
- Just need to test

$$(\mathbf{X} - \mathbf{C}) \cdot \mathbf{R}(3,:)^T > 0 ?$$

Pick the Solution

With maximal number of points in front of both cameras.

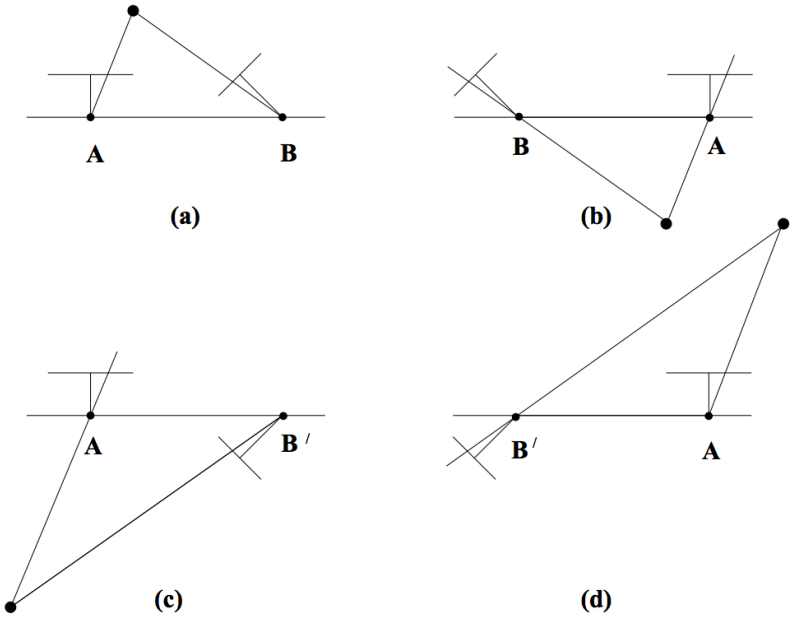
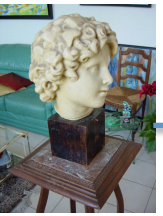
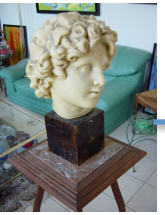


Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates* 180° *about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

Page 260 of the bible (Multiple View Geometry, 2nd Ed)

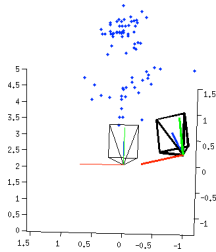
Two-view Reconstruction

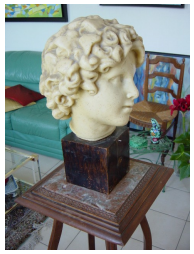
keypoints

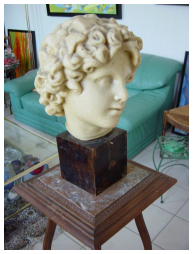
keypoints

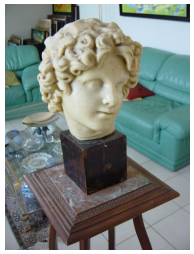
match → fundamental matrix → essential matrix → [R|t] → triangulation →

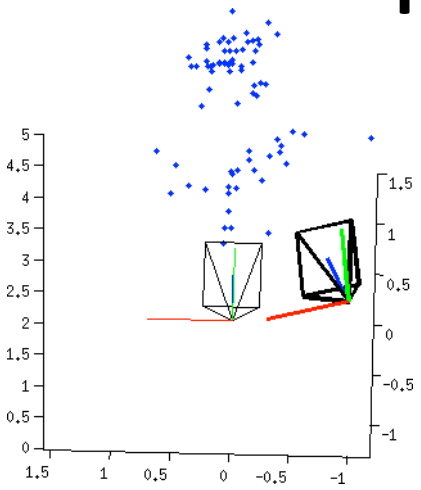


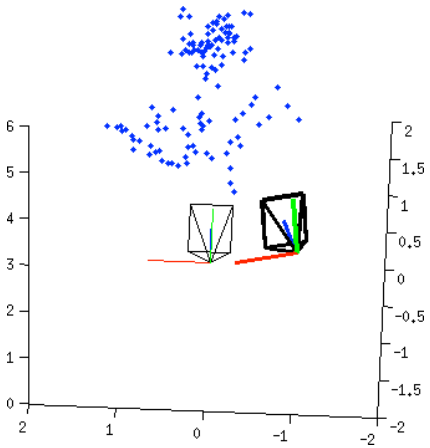
Pipeline

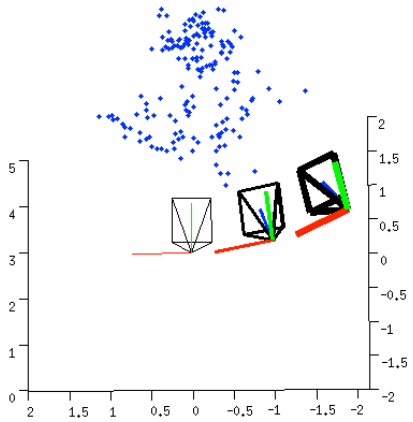


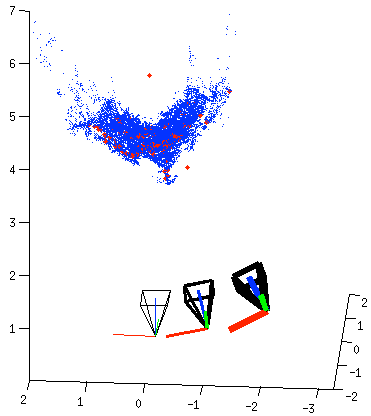








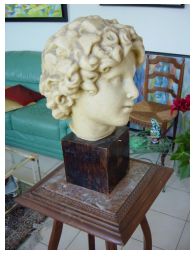


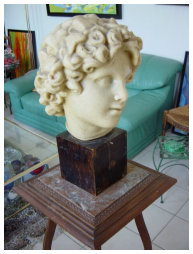


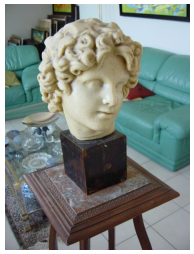
Structure from Motion (SFM)

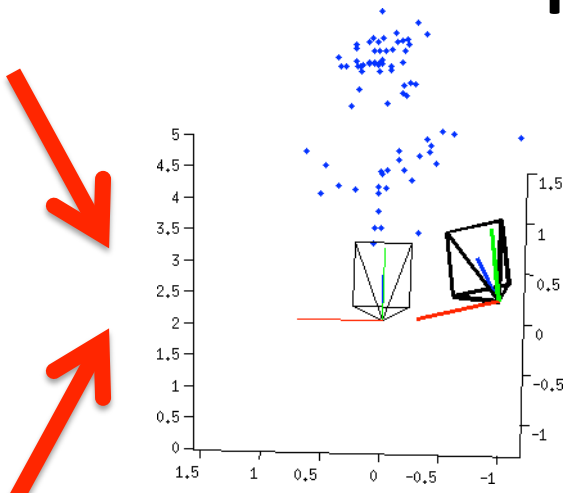
Multi-view Stereo (MVS)

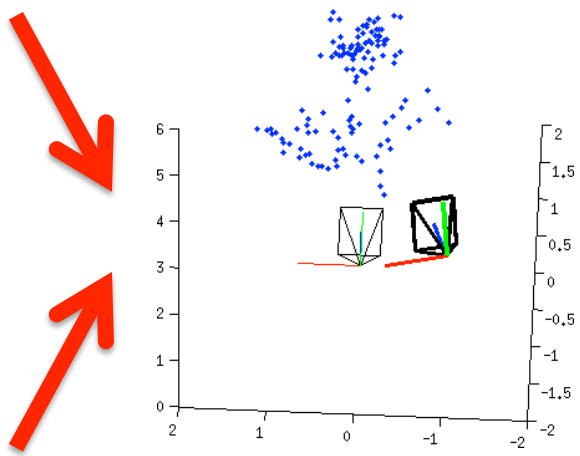
Pipeline

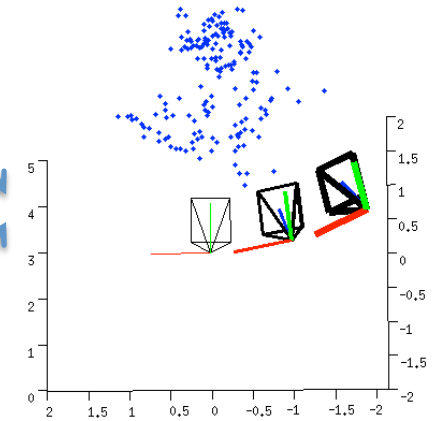


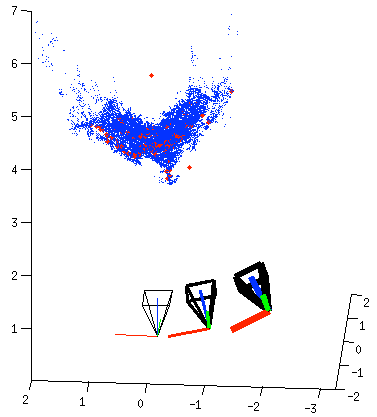








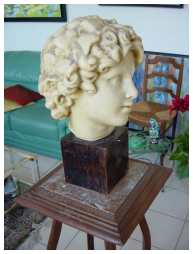


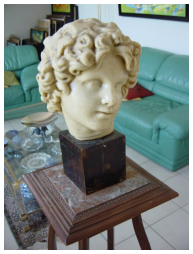


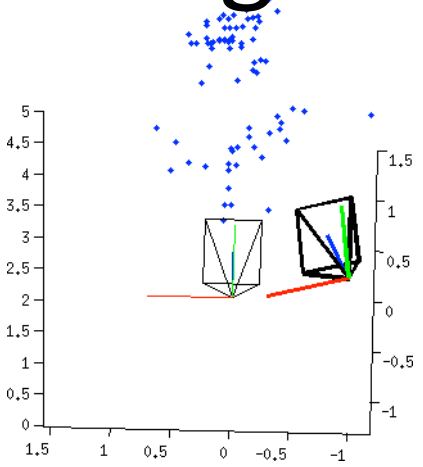
Taught

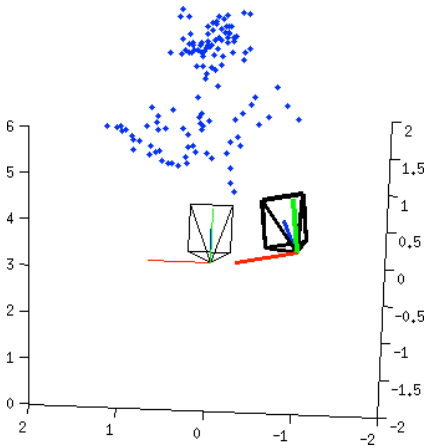
Next

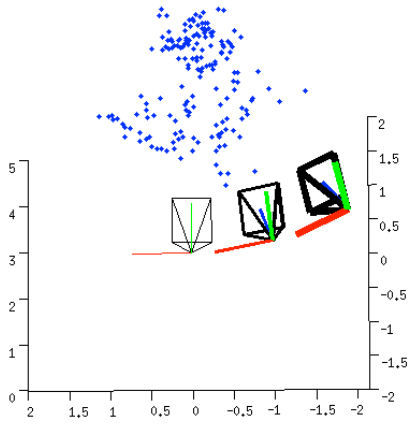
Merge Two Point Cloud



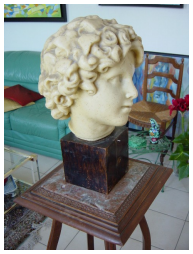


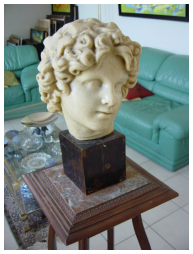


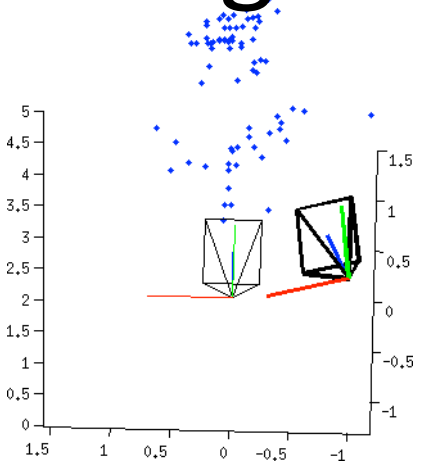


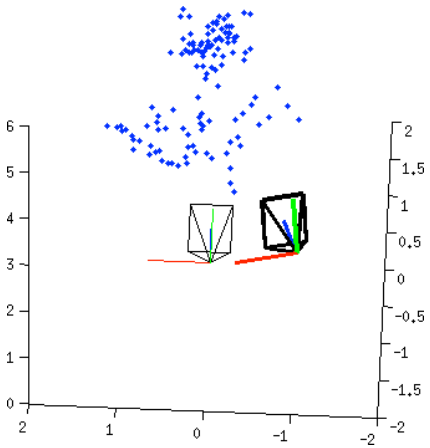


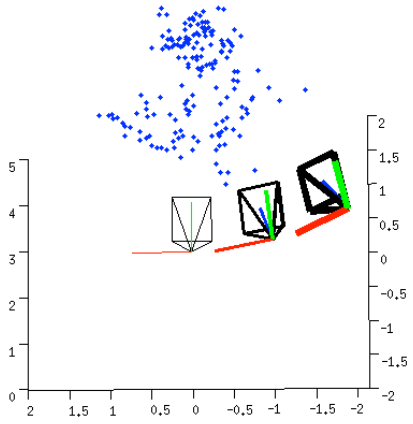
Merge Two Point Cloud









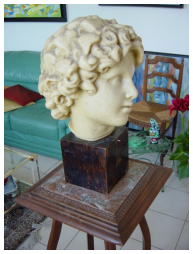


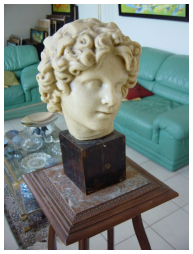
There can be only one $\left[\mathbf{R}_2 | \mathbf{t}_2\right]$

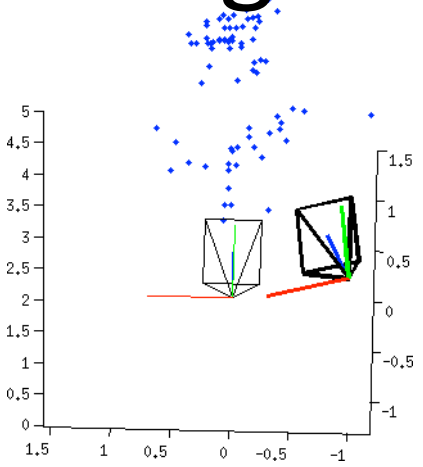
Merge Two Point Cloud

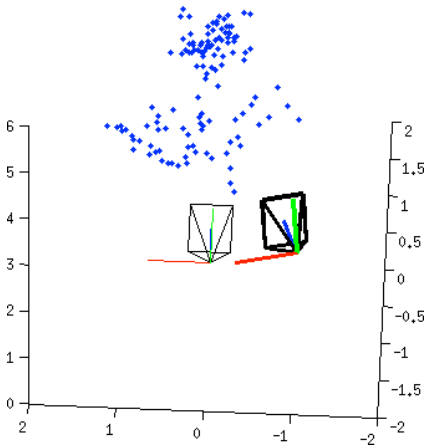
- From the 1st and 2nd images, we have $\left[\mathbf{R}_1|\mathbf{t}_1\right]$ and $\left[\mathbf{R}_2|\mathbf{t}_2\right]$
- From the 2nd and 3rd images, we have $\left[\mathbf{R}_2|\mathbf{t}_2\right]$ and $\left[\mathbf{R}_3|\mathbf{t}_3\right]$
- **Exercise**: How to transform the coordinate system of the second point cloud to align with the first point cloud so that there is only one $\left[\mathbf{R}_2|\mathbf{t}_2\right]$?

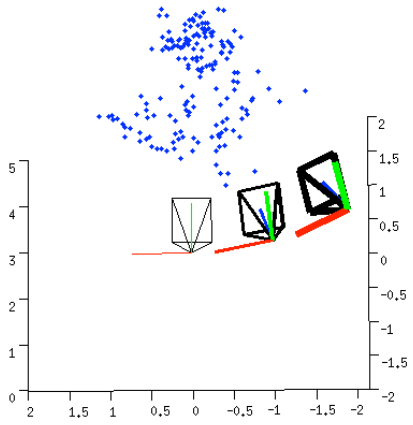
Merge Two Point Cloud



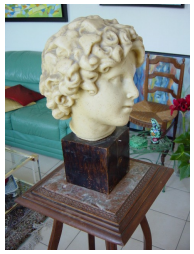


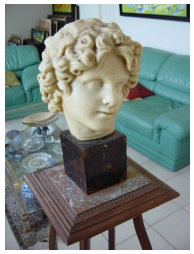


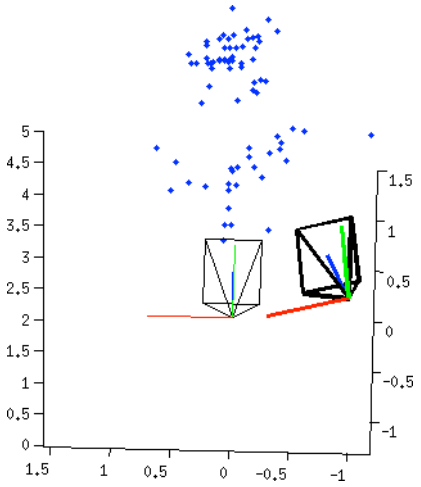


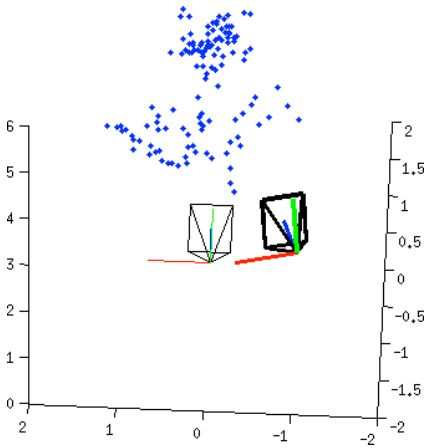


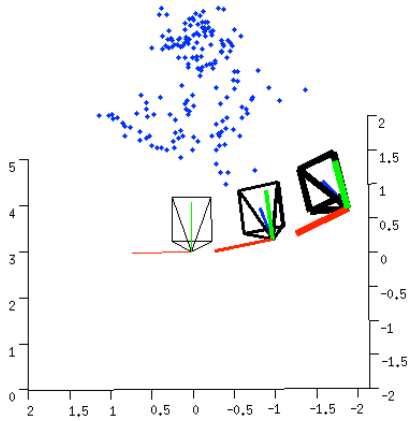
Oops

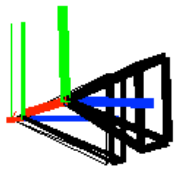






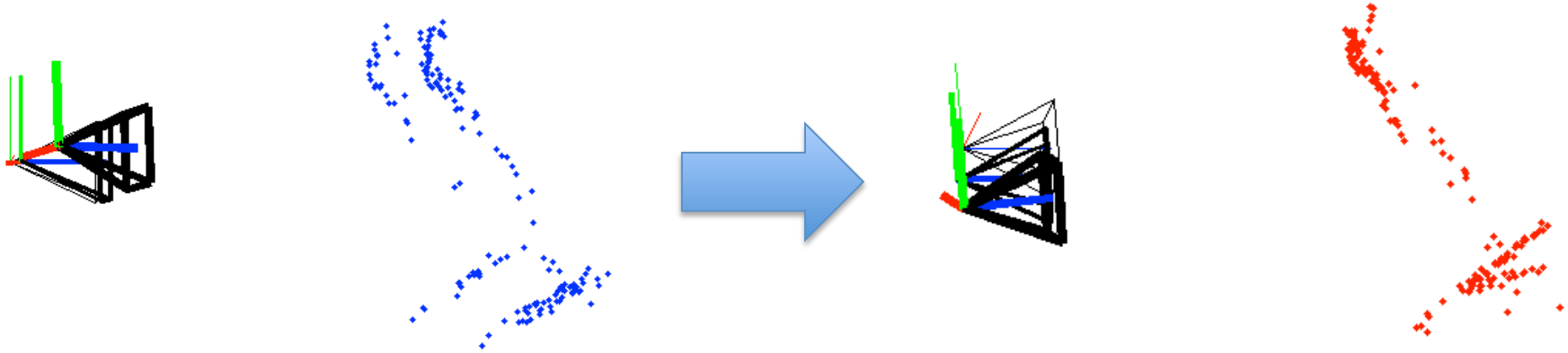




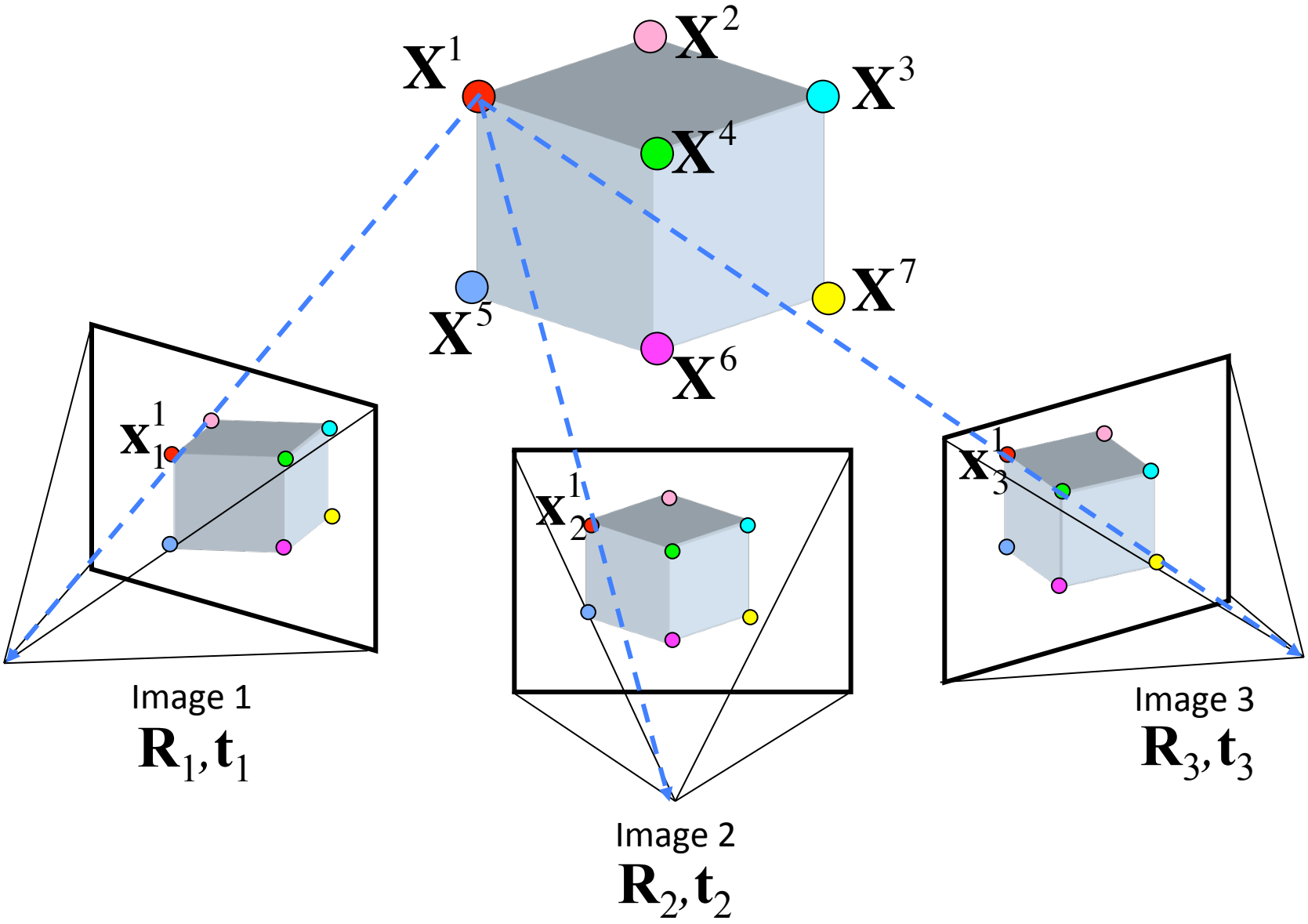


See From a Different Angle

Bundle Adjustment



“America's Next Top Model”



“America's Next Top Model”

	Point 1	Point 2	Point 3
Image 1	$\mathbf{x}_1^1 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}^1$	$\mathbf{x}_1^2 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}^2$	
Image 2	$\mathbf{x}_2^1 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^1$	$\mathbf{x}_2^2 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^2$	$\mathbf{x}_2^3 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}^3$
Image 3	$\mathbf{x}_3^1 = \mathbf{K}\left[\mathbf{R}_3 \middle| \mathbf{t}_3\right]\mathbf{X}^1$		$\mathbf{x}_3^3 = \mathbf{K}\left[\mathbf{R}_3 \middle| \mathbf{t}_3\right]\mathbf{X}^3$

Rethinking the SFM problem

- Input: Observed 2D image position

$$\begin{array}{ccc} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{array}$$

- Output:

Unknown Camera Parameters (with some guess)

$$\left[\mathbf{R}_1|\mathbf{t}_1\right],\left[\mathbf{R}_2|\mathbf{t}_2\right],\left[\mathbf{R}_3|\mathbf{t}_3\right]$$

Unknown Point 3D coordinate (with some guess)

$$\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\ldots$$

Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let

Re-projection

$$\begin{array}{lll} \mathbf{x}_1^1=\mathbf{K}[\mathbf{R}_1|\mathbf{t}_1]\mathbf{X}^1 & \mathbf{x}_1^2=\mathbf{K}[\mathbf{R}_1|\mathbf{t}_1]\mathbf{X}^2 & \\ \mathbf{x}_2^1=\mathbf{K}[\mathbf{R}_2|\mathbf{t}_2]\mathbf{X}^1 & \mathbf{x}_2^2=\mathbf{K}[\mathbf{R}_2|\mathbf{t}_2]\mathbf{X}^2 & \mathbf{x}_2^3=\mathbf{K}[\mathbf{R}_2|\mathbf{t}_2]\mathbf{X}^3 \\ \mathbf{x}_3^1=\mathbf{K}[\mathbf{R}_3|\mathbf{t}_3]\mathbf{X}^1 & & \mathbf{x}_3^3=\mathbf{K}[\mathbf{R}_3|\mathbf{t}_3]\mathbf{X}^3 \end{array}$$

=

Observation

$$\begin{array}{lll} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{array}$$

Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let the Re-projection close to the Observation, i.e. to minimize the reprojection error

$$\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^j - \mathbf{K}\left[\mathbf{R}_i|\mathbf{t}_i\right]\mathbf{X}^j \right)^2$$

Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let the Re-projection close to the Observation, i.e. to minimize the reprojection error

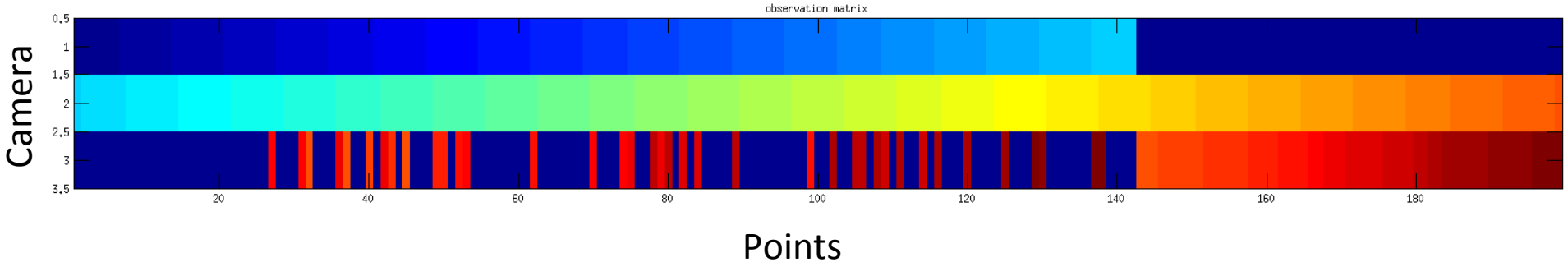
$$\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^j - \mathbf{K}\left[\mathbf{R}_i|\mathbf{t}_i\right]\mathbf{X}^j \right)^2$$

Question: What is the unit of this objective function?

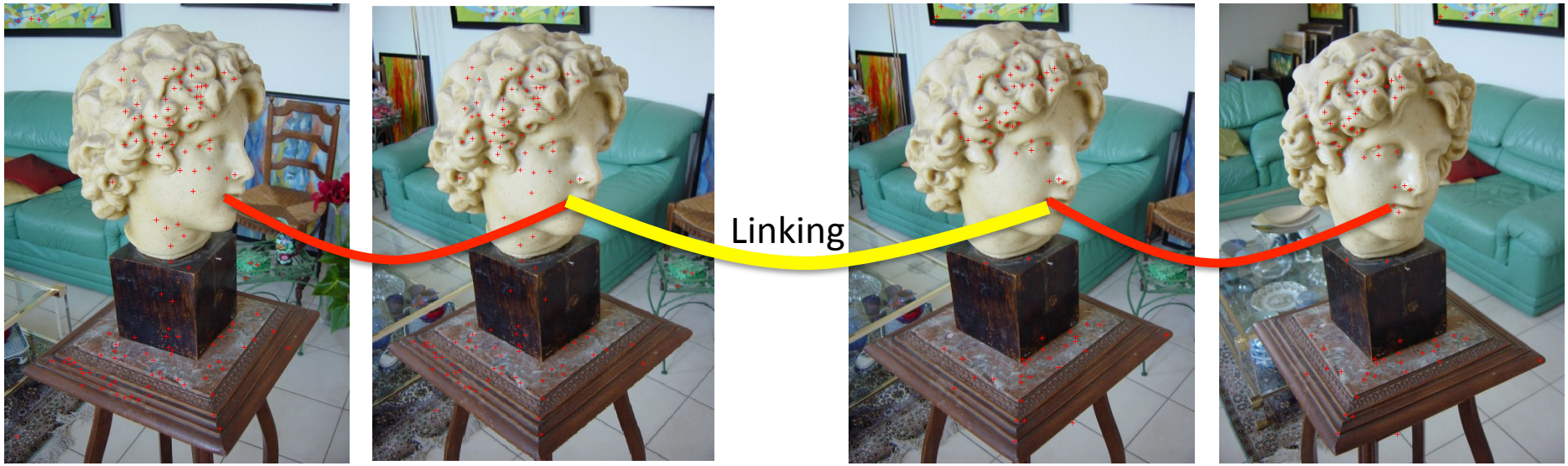
Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let the Re-projection close to the Observation, i.e. to minimize the reprojection error

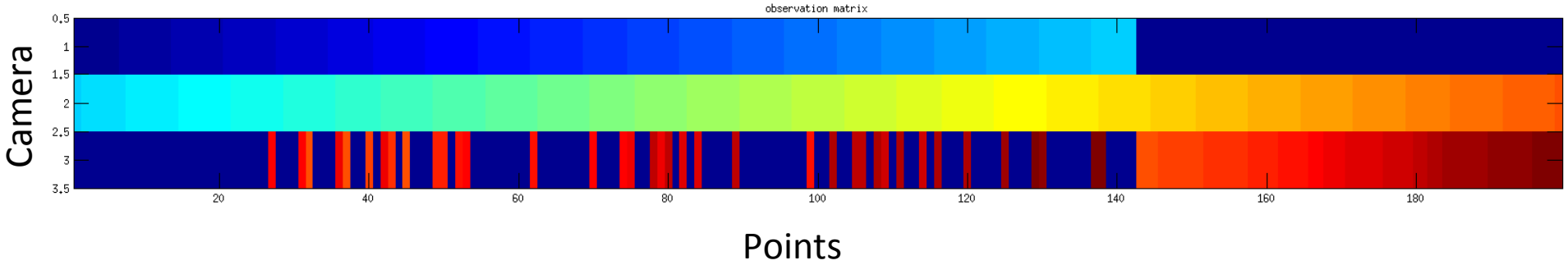
$$\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^j - \mathbf{K}\left[\mathbf{R}_i|\mathbf{t}_i\right]\mathbf{X}^j \right)^2$$



Linking



SIFT Matching **SIFT Matching**



Solving This Optimization Problem

- Theory:

  The Levenberg–Marquardt algorithm

  http://en.wikipedia.org/wiki/Levenberg-Marquardt_algorithm

- Practice:

  The Ceres-Solver from Google

  http://code.google.com/p/ceres-solver/

Ceres-solver: A Nonlinear Least Squares Minimizer

Toy problem to solve $\min\left(10 - x\right)^2$

```
class SimpleCostFunction
  : public ceres::SizedCostFunction<1 /* number of residuals */,
                                    1 /* size of first parameter */> {
 public:
  virtual ~SimpleCostFunction() {}
  virtual bool Evaluate(double const* const* parameters,
                        double* residuals,
                        double** jacobians) const {
    const double x = parameters[0][0];
    residuals[0] = 10 - x;  // f(x) = 10 - x
    // Compute the Jacobian if asked for.
    if (jacobians != NULL && jacobians[0] != NULL) {
      jacobians[0][0] = -1;
    }
    return true;
  }
};
```

Ceres-solver: A Nonlinear Least Squares Minimizer

Toy problem to solve $\min\left(10 - x\right)^2$

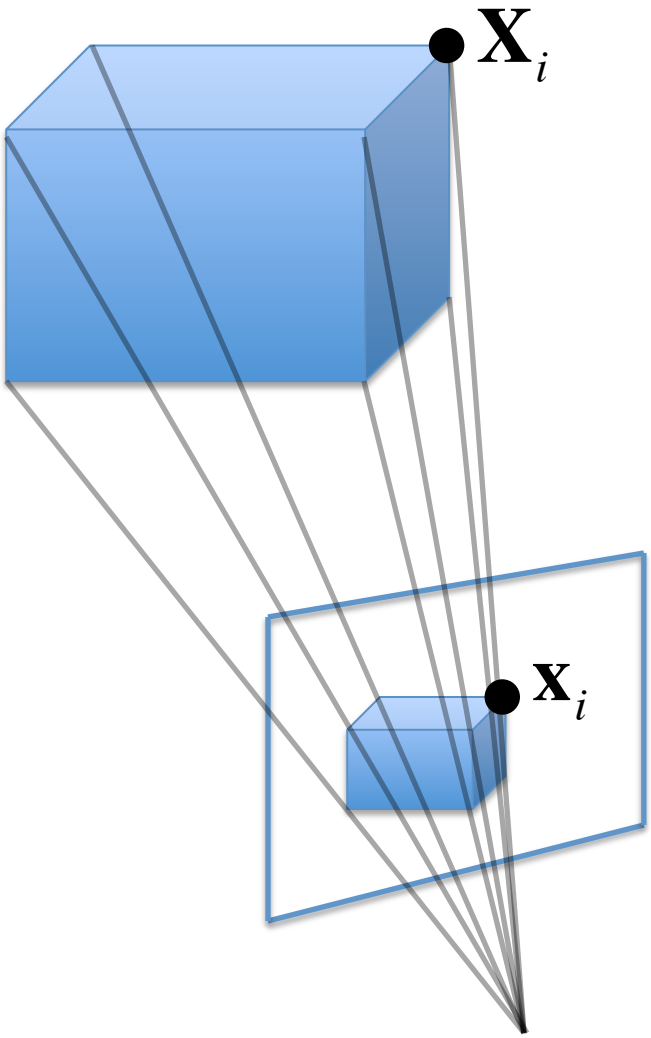
```
int main(int argc, char** argv) {
  double x = 5.0;
  ceres::Problem problem;

  // The problem object takes ownership of the newly allocated
  // SimpleCostFunction and uses it to optimize the value of x.
  problem.AddResidualBlock(new SimpleCostFunction, NULL, &x);

  // Run the solver!
  Solver::Options options;
  options.max_num_iterations = 10;
  options.linear_solver_type = ceres::DENSE_QR;
  options.minimizer_progress_to_stdout = true;
  Solver::Summary summary;
  Solve(options, &problem, &summary);
  std::cout << summary.BriefReport() << "\n";
  std::cout << "x : 5.0 -> " << x << "\n";
  return 0;
}
```

Ceres-solver: A Nonlinear Least Squares Minimizer

Toy problem to solve $\min\left(10-x\right)^2$

```
0: f: 1.250000e+01 d: 0.00e+00 g: 5.00e+00 h: 0.00e+00 rho: 0.00e+00 mu: 1.00e-04 li:  0
1: f: 1.249750e-07 d: 1.25e+01 g: 5.00e-04 h: 5.00e+00 rho: 1.00e+00 mu: 3.33e-05 li:  1
2: f: 1.388518e-16 d: 1.25e-07 g: 1.67e-08 h: 5.00e-04 rho: 1.00e+00 mu: 1.11e-05 li:  1
Ceres Solver Report: Iterations: 2, Initial cost: 1.250000e+01,  \
Final cost: 1.388518e-16, Termination: PARAMETER_TOLERANCE.
x : 5 -> 10
```

Non-linear Least Square Cuboid Reconstruction



Observation:

Pixel location of the 7 (or 6) corners

Parameters to be estimated:

Cuboid: Height h x Width w x 1

Camera: Intrinsic K and Extrinsic R, t

$$\min \sum_i \left\| \mathbf{x}_i - \mathbf{K}\left(\mathbf{R}\mathbf{X}_i + \mathbf{t}\right) \right\|^2$$

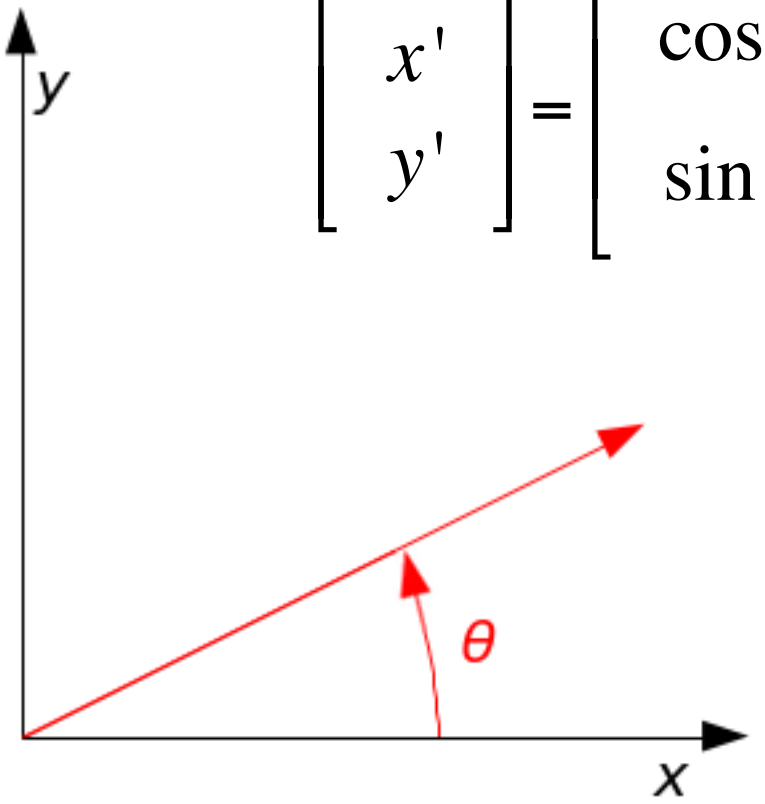
$$\mathbf{X} = \begin{bmatrix} h & h & h & h & -h & -h & -h & -h \\ w & w & -w & -w & w & w & -w & -w \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

http://mit.edu/jxiao/Public/software/fitCuboid/

Parameterizing Rotation Matrix

- 2D Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$\mathbf{R}^T = \mathbf{R}^{-1}, \det \mathbf{R} = 1$$

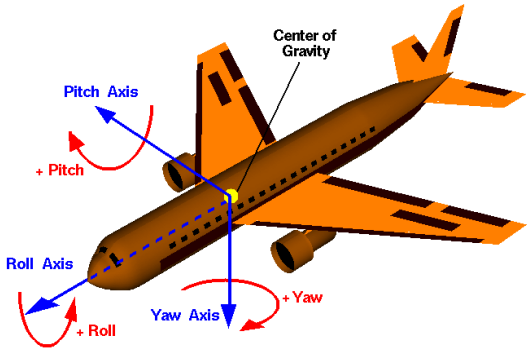
3D Rotation

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

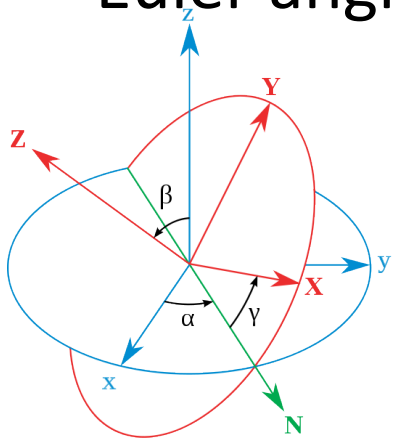
$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Yaw, pitch and roll are α, β, γ



$$\mathbf{R}_x(\gamma)\mathbf{R}_y(\beta)\mathbf{R}_z(\alpha)$$

Euler angles are α, β, γ

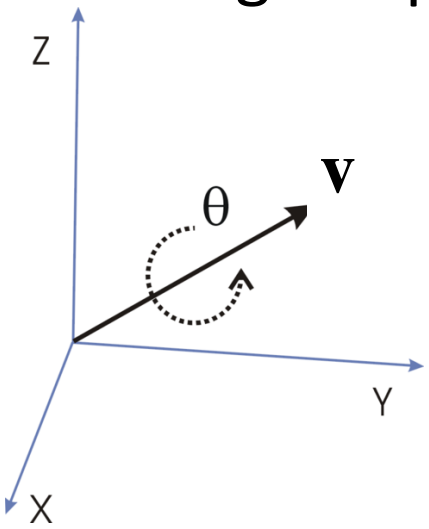


$$\mathbf{R}_z(\gamma)\mathbf{R}_x(\beta)\mathbf{R}_y(\alpha)$$

http://en.wikipedia.org/wiki/Rotation_matrix

3D Rotation

Axis-angle representation



Quaternions

$$\mathbf{q} = \left(\mathbf{v} \sin\left(\frac{\theta}{2}\right), \cos\left(\frac{\theta}{2}\right) \right)^T$$

Avoid Gimbal Lock!

Recommendation!

Triplet Representation $\mathbf{v}\theta$ (3 dof) ← Not over-parameterized

Rodrigues' rotation formula

$$\mathbf{k}_{rot} = \mathbf{k}\cos\theta + (\mathbf{v} \times \mathbf{k})\sin\theta + \mathbf{v}(\mathbf{v} \cdot \mathbf{k})(1 - \cos\theta)$$

http://en.wikipedia.org/wiki/Axis-angle_representation
http://en.wikipedia.org/wiki/Quaternions_and_spatial_rotation
http://en.wikipedia.org/wiki/Rodrigues%27_rotation_formula

Matlab Symbolic Operation

How to derive complicated equations (after passing math classes)?

```
syms  h, w, K, alpha, beta, gamma, tx, ty, tz
X = [h; w; f];
Y = [...] * X;
Z = K* Y;
x = Z(1:2,:) ./ Z([3,3],:);
ccode(x,'file','x.cpp'); % ←matlab will generate C code for x
```

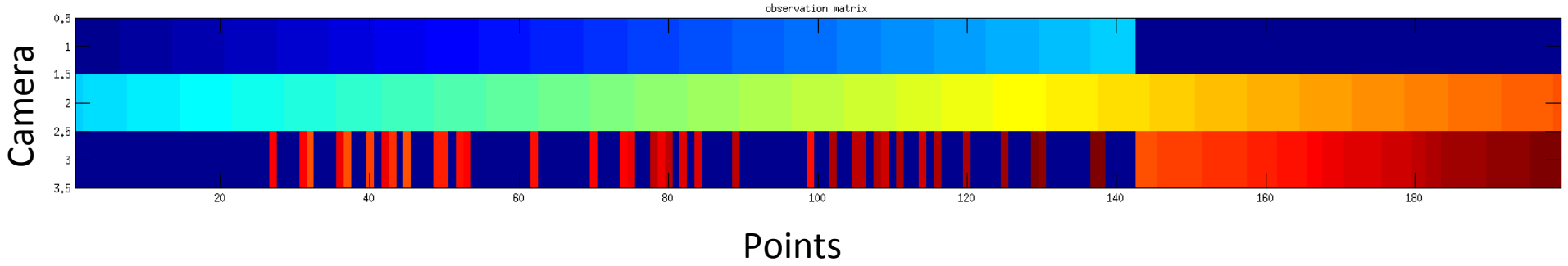
http://www.mathworks.com/help/symbolic/sym.html

http://mit.edu/jxiao/Public/software/fitCuboid/fitCuboid/derive4cuboid.m

Ceres for Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let the Re-projection close to the Observation, i.e. to minimize the reprojection error

$$\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^j - \mathbf{K}\left[\mathbf{R}_i|\mathbf{t}_i\right]\mathbf{X}^j \right)^2$$



ba2D.cc

```
struct AlignmentError2D {
  AlignmentError2D(double* observed_in): observed(observed_in) {}
  template <typename T>
  bool operator()(const T* const camera_extrinsic,
                  const T* const point,
                  T* residuals) const {
    T p[3];
    // camera_extrinsic[0,1,2] are the angle-axis rotation.
    ceres::AngleAxisRotatePoint(camera_extrinsic, point, p);
    // camera_extrinsic[3,4,5] are the translation.
    p[0] += camera_extrinsic[3];
    p[1] += camera_extrinsic[4];
    p[2] += camera_extrinsic[5];

    // let p[2] ~= 0
    if (T(0.0)<=p[2]){
        if(p[2]<EPS){
            p[2] = EPS;
        }
    }else{
        if (p[2]>-EPS){
            p[2] = -EPS;
        }
    }
    // project it
    p[0] = T(fx) * p[0] / p[2] + T(px);
    p[1] = T(fy) * p[1] / p[2] + T(py);
    // reprojection error
    residuals[0] = p[0] - T(observed[0]);
    residuals[1] = p[1] - T(observed[1]);
    return true;
  }
  double* observed;
};
```

$\mathbf{RX}$

$\mathbf{RX} + \mathbf{t} = \left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}$

To avoid divide by 0

$\mathbf{x} = \mathbf{K}\left[\mathbf{R}|\mathbf{t}\right]\mathbf{X}$

$\mathbf{x} - \tilde{\mathbf{x}}$

Initialization Matters

- Input: Observed 2D image position

$$\begin{matrix} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{matrix}$$

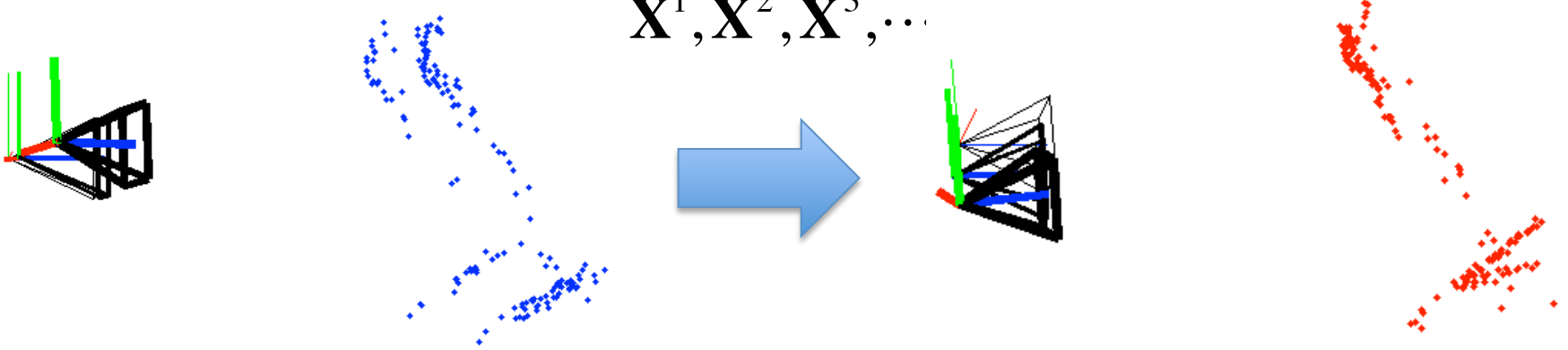
- Output:

Unknown Camera Parameters (with some guess)

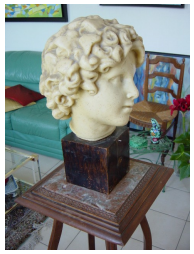
$$\left[\mathbf{R}_1|\mathbf{t}_1\right],\left[\mathbf{R}_2|\mathbf{t}_2\right],\left[\mathbf{R}_3|\mathbf{t}_3\right]$$

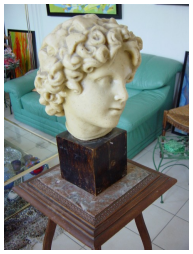
Unknown Point 3D coordinate (with some guess)

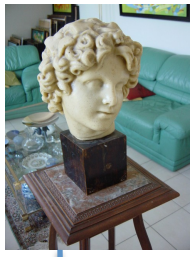
$$\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$$

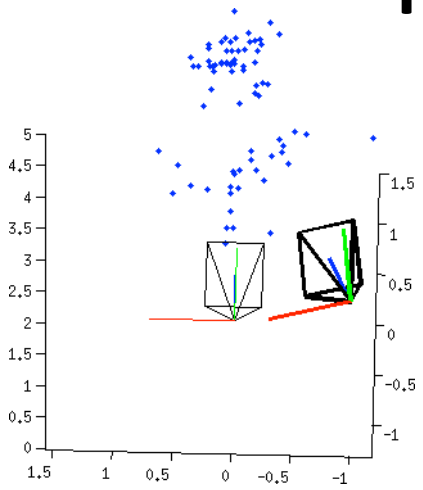


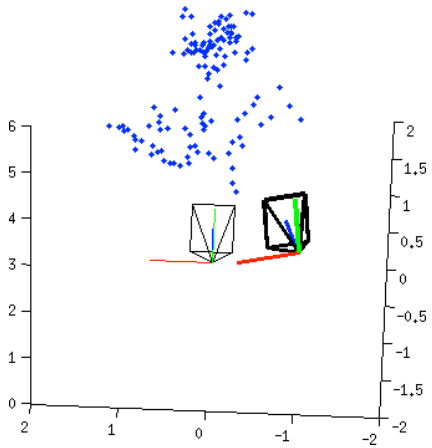
Pipeline

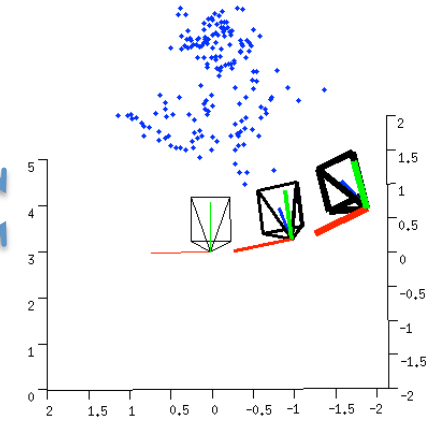


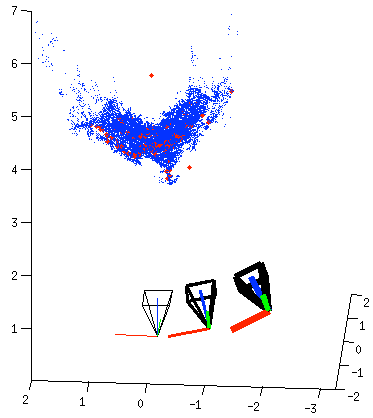








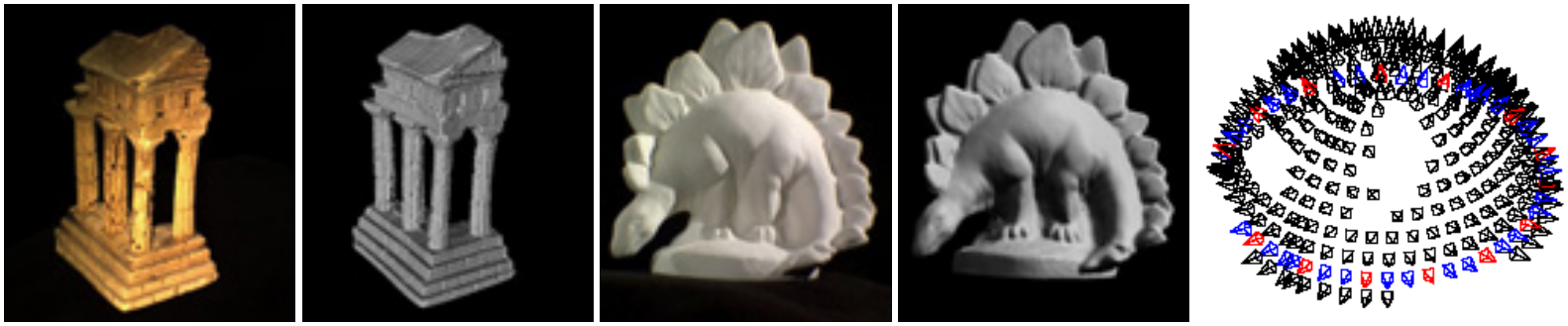




Taught

Next

Multiple View Stereo



State-of-the-art:

PMVS: http://grail.cs.washington.edu/software/pmvs/

Accurate, Dense, and Robust Multi-View Stereopsis, Y Furukawa and J Ponce, 2007.

Benchmark:

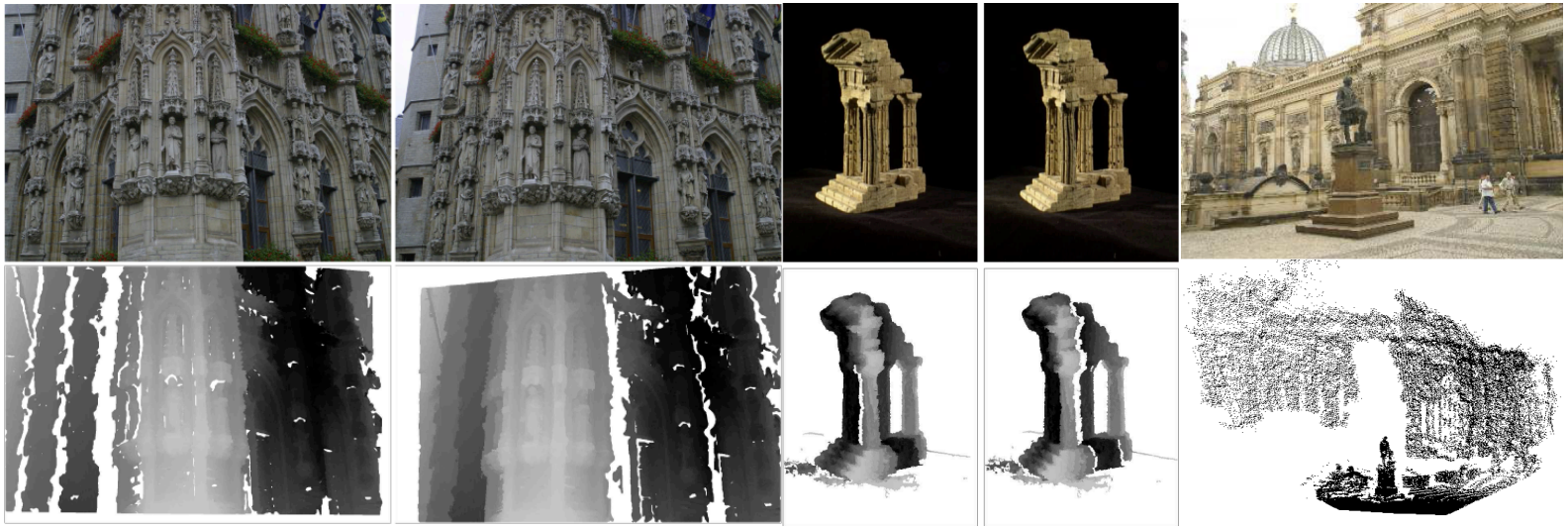
http://vision.middlebury.edu/mview/

A Comparison and Evaluation of Multi-View Stereo Reconstruction Algorithms. SM Seitz, B Curless, J Diebel, D Scharstein, R Szeliski. 2006.

Baseline:

Multi-view stereo revisited. M Goesele, B Curless, SM Seitz. 2006.

Key idea: Matching Propagation



[1] Learning Two-view Stereo Matching, J Xiao, J Chen, DY Yeung, and L Quan, 2008.

[2] Accurate, Dense, and Robust Multi-View Stereopsis, Y Furukawa and J Ponce, 2007.

[3] Multi-view stereo revisited. M Goesele, B Curless, SM Seitz. 2006.

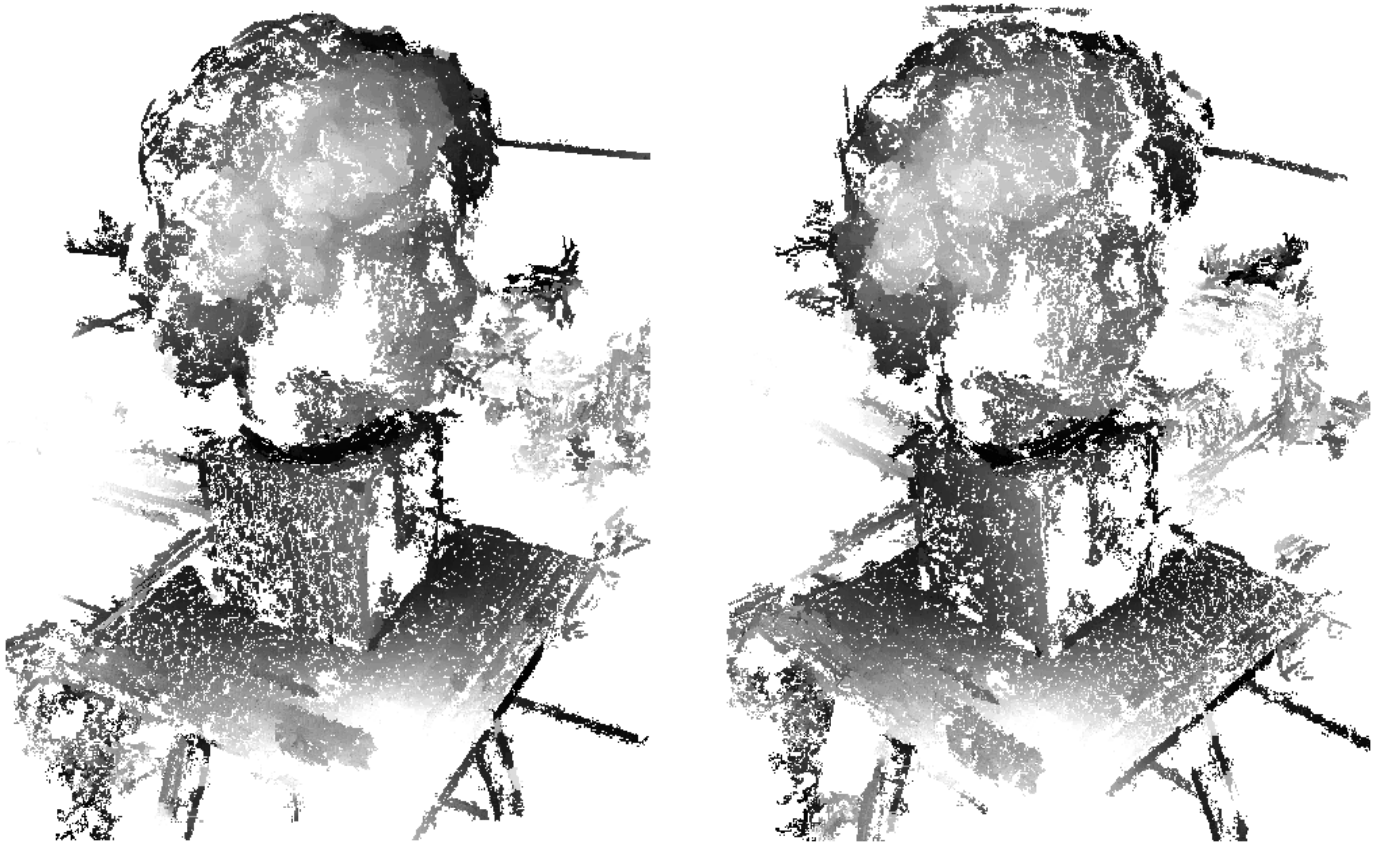
[4] Robust Dense Matching Using Local and Global Geometric Constraints, M Lhuillier & L Quan, 2000.

In another context:

[5] PatchMatch: A Randomized Correspondence Algorithm for Structural Image Editing, C Barnes, E Shechtman, A Finkelstein, and DB Goldman, 2009.

Simplest Matching Propagation

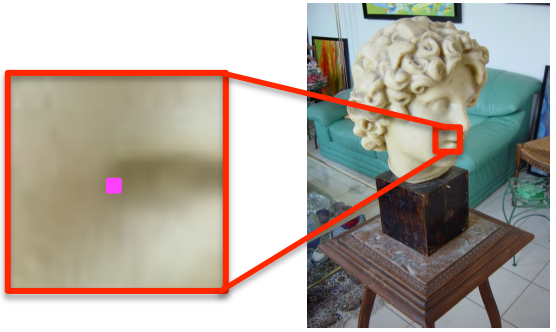
We are going to learn a very simple algorithm that is implemented in the SFMedu program.



Descriptor: ZNCC (Zero-mean Normalized Cross-Correlation)

- Invariant to linear radiometric changes
- More conservative than others such as sum of absolute or square differences in uniform regions
- More tolerant in textured areas where noise might be important

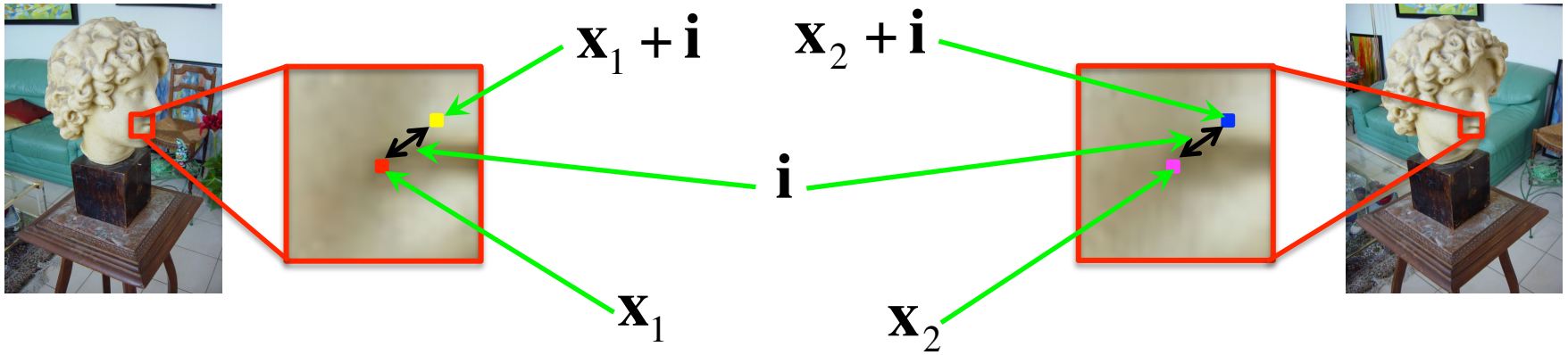
$$ZNCC(\mathbf{x}_1,\mathbf{x}_2)=\frac{\sum_{\mathbf{i}}\left(I(\mathbf{x}_1+\mathbf{i})-\bar{I}(\mathbf{x}_1)\right)\left(I(\mathbf{x}_2+\mathbf{i})-\bar{I}(\mathbf{x}_2)\right)}{\sqrt{\sum_{\mathbf{i}}\left(I(\mathbf{x}_1+\mathbf{i})-\bar{I}(\mathbf{x}_1)\right)^2\sum_{\mathbf{i}}\left(I(\mathbf{x}_2+\mathbf{i})-\bar{I}(\mathbf{x}_2)\right)^2}}$$



Descriptor: ZNCC (Zero-mean Normalized Cross-Correlation)

- Invariant to linear radiometric changes
- More conservative than others such as sum of absolute or square differences in uniform regions
- More tolerant in textured areas where noise might be important

$$ZNCC(\mathbf{x}_1,\mathbf{x}_2)=\frac{\sum_{\mathbf{i}}\left(I(\mathbf{x}_1+\mathbf{i})-\bar{I}(\mathbf{x}_1)\right)\left(I(\mathbf{x}_2+\mathbf{i})-\bar{I}(\mathbf{x}_2)\right)}{\sqrt{\sum_{\mathbf{i}}\left(I(\mathbf{x}_1+\mathbf{i})-\bar{I}(\mathbf{x}_1)\right)^2\sum_{\mathbf{i}}\left(I(\mathbf{x}_2+\mathbf{i})-\bar{I}(\mathbf{x}_2)\right)^2}}$$



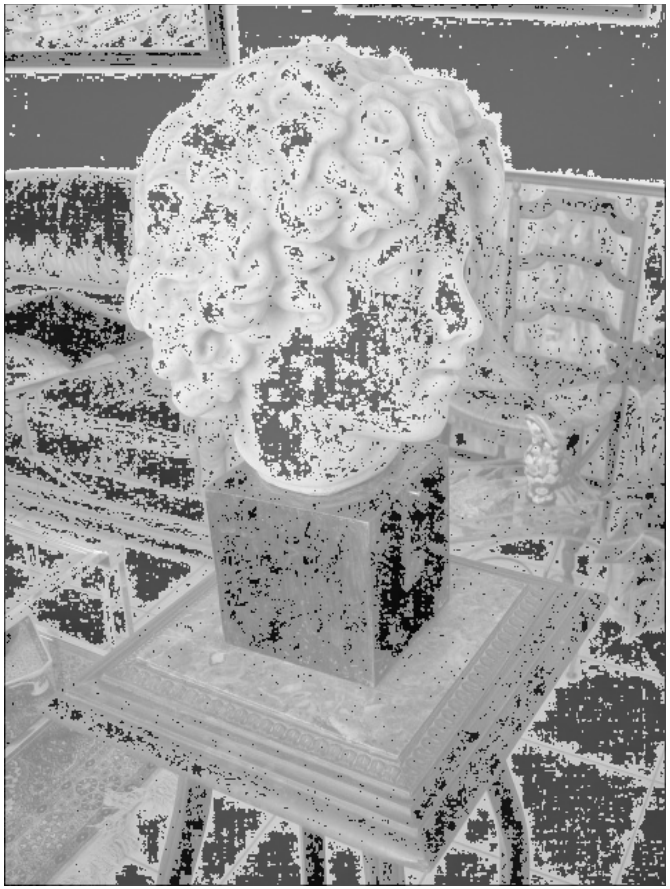
Seed for propagation

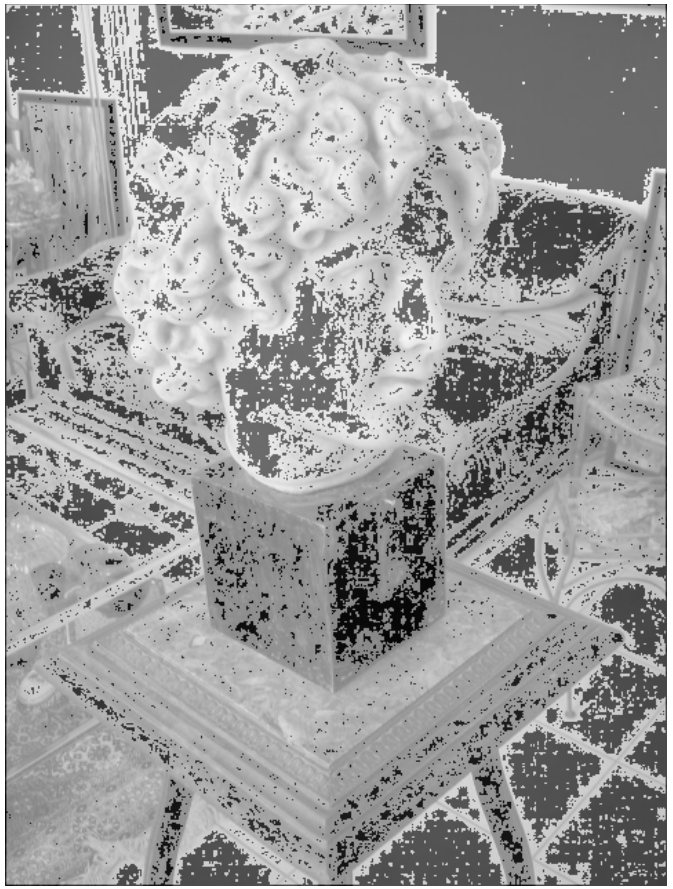
Matching Propagation (propagate.m)

- Maintain a priority queue Q
- Initialize: Put all seeds into Q with their ZNCC values as scores
- For each iteration:
  - Pop the match with best ZNCC score from Q
  - Add new potential matches in their immediate spatial neighborhood into Q
- Safety: handle uniqueness, and propagate only on matchable area

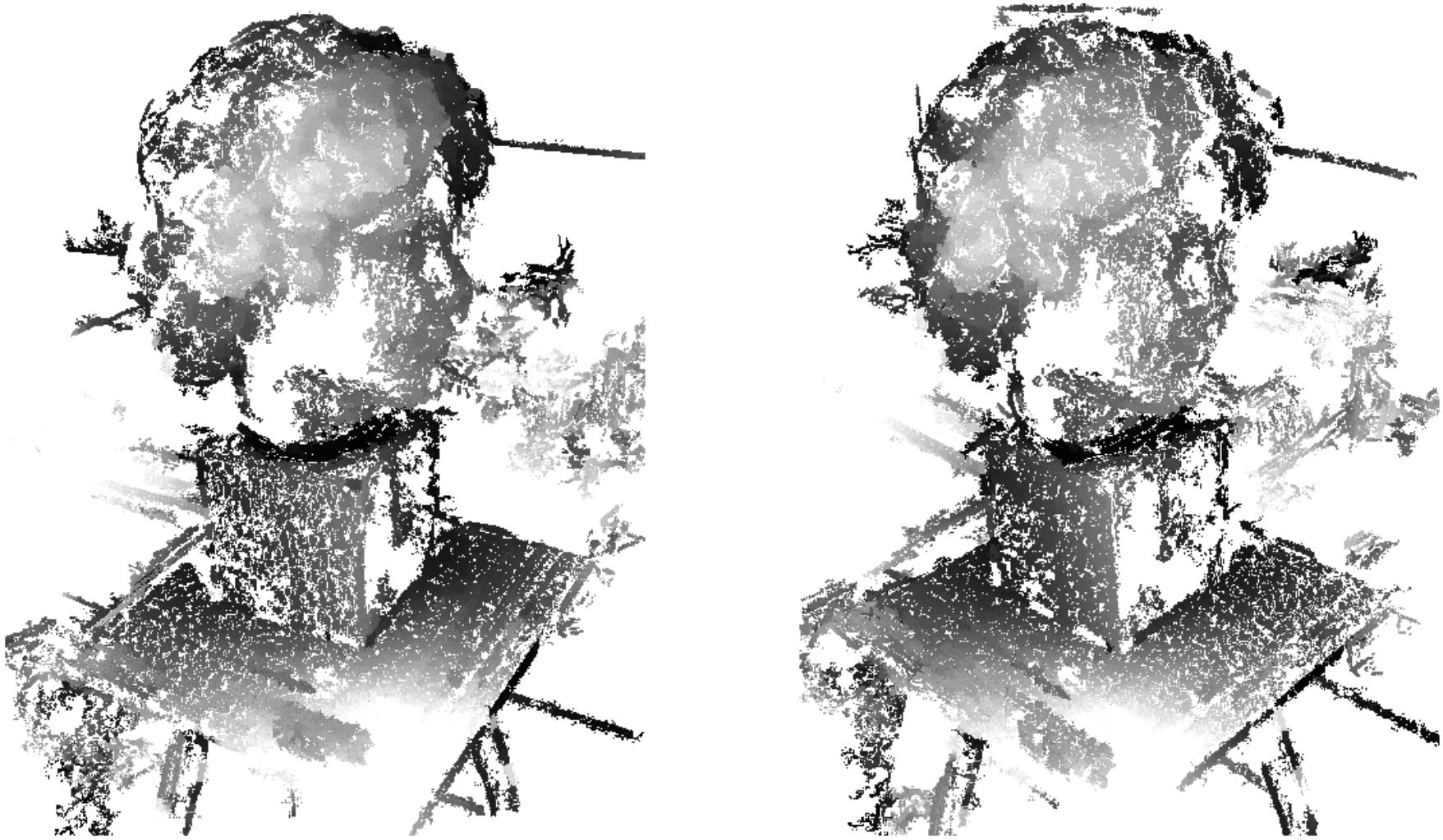
Matchable Area

the area with maximal gradience > threshold

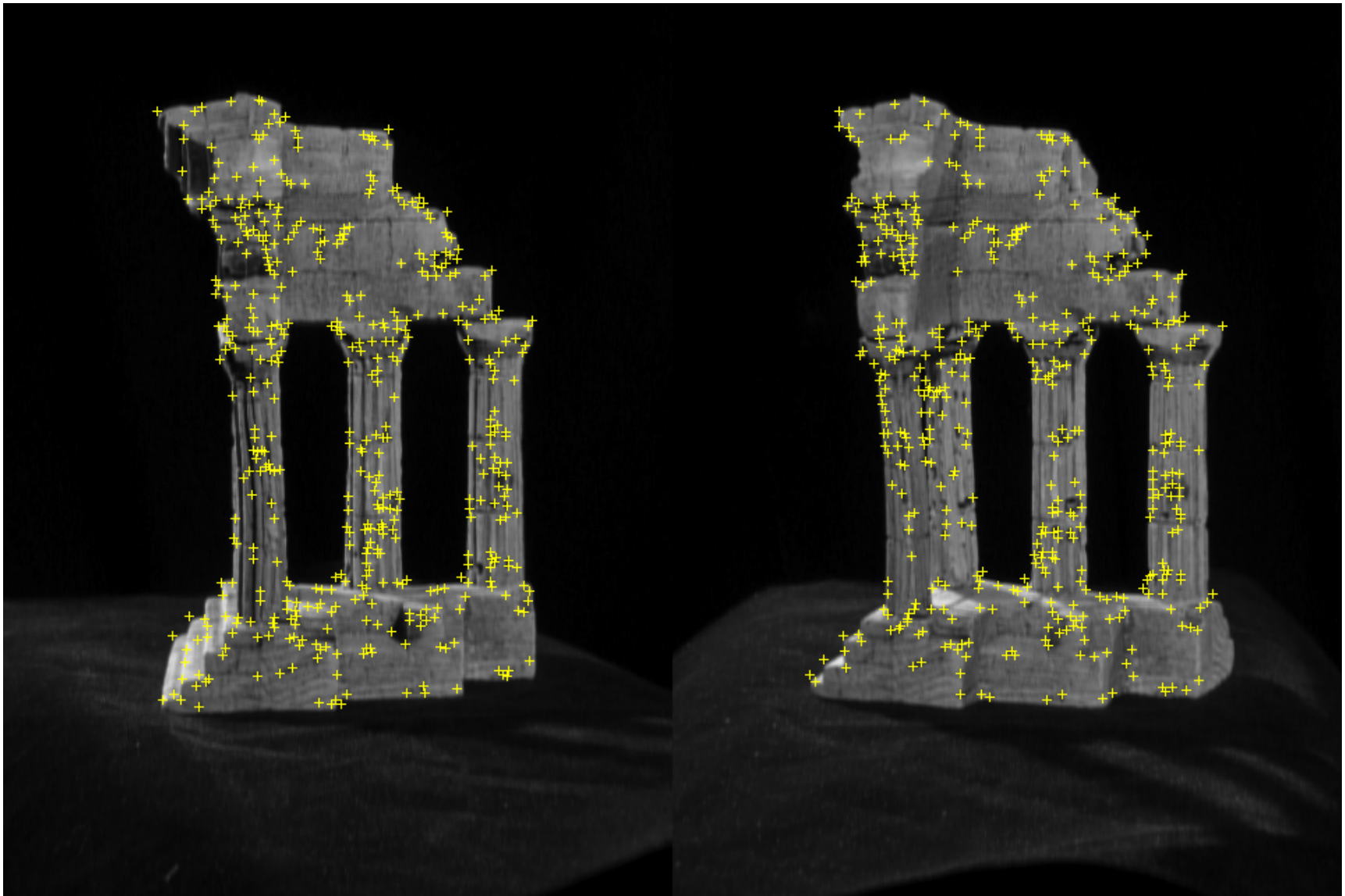




Result (denseMath/run.m)



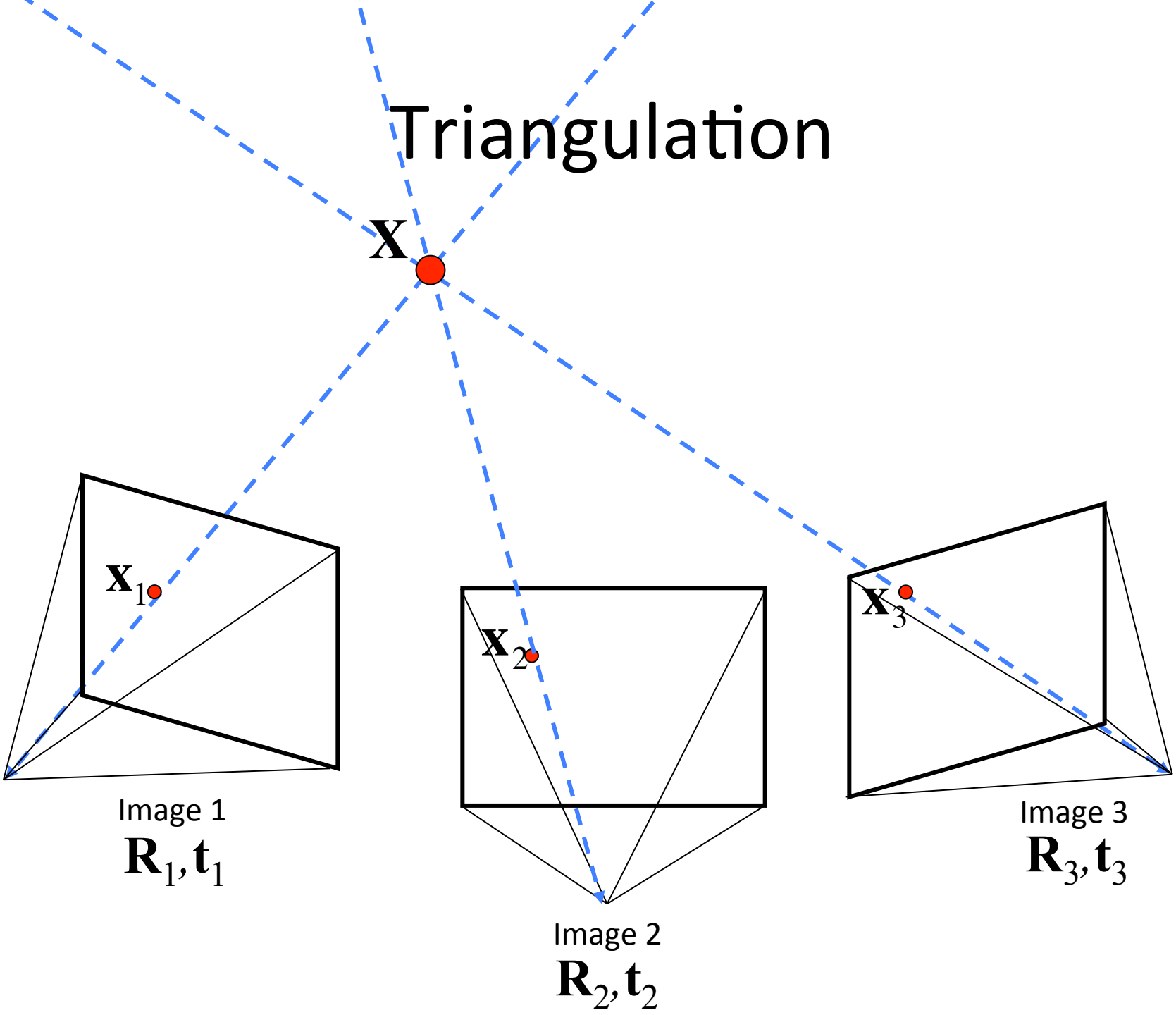
Another Example



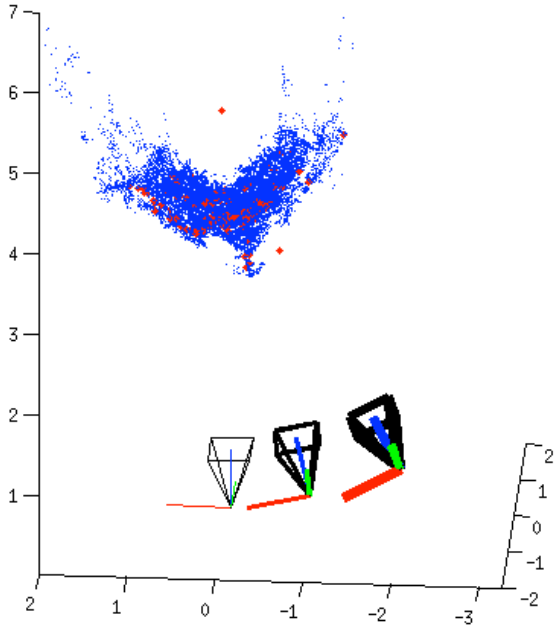
Another Example

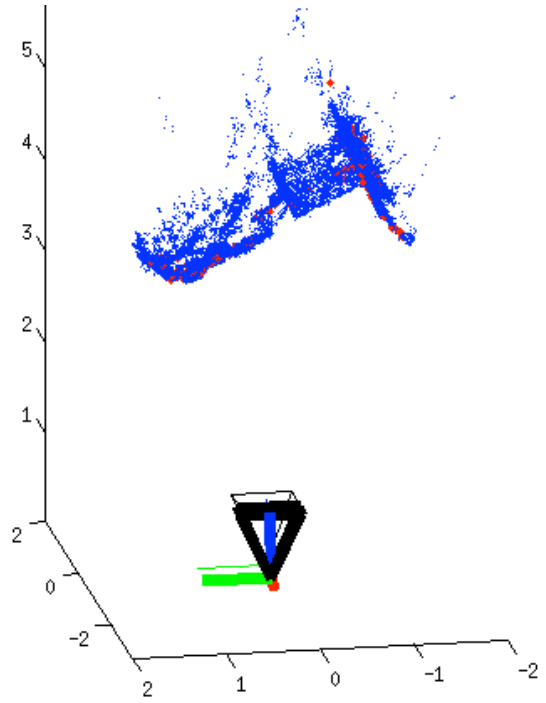
Another Example

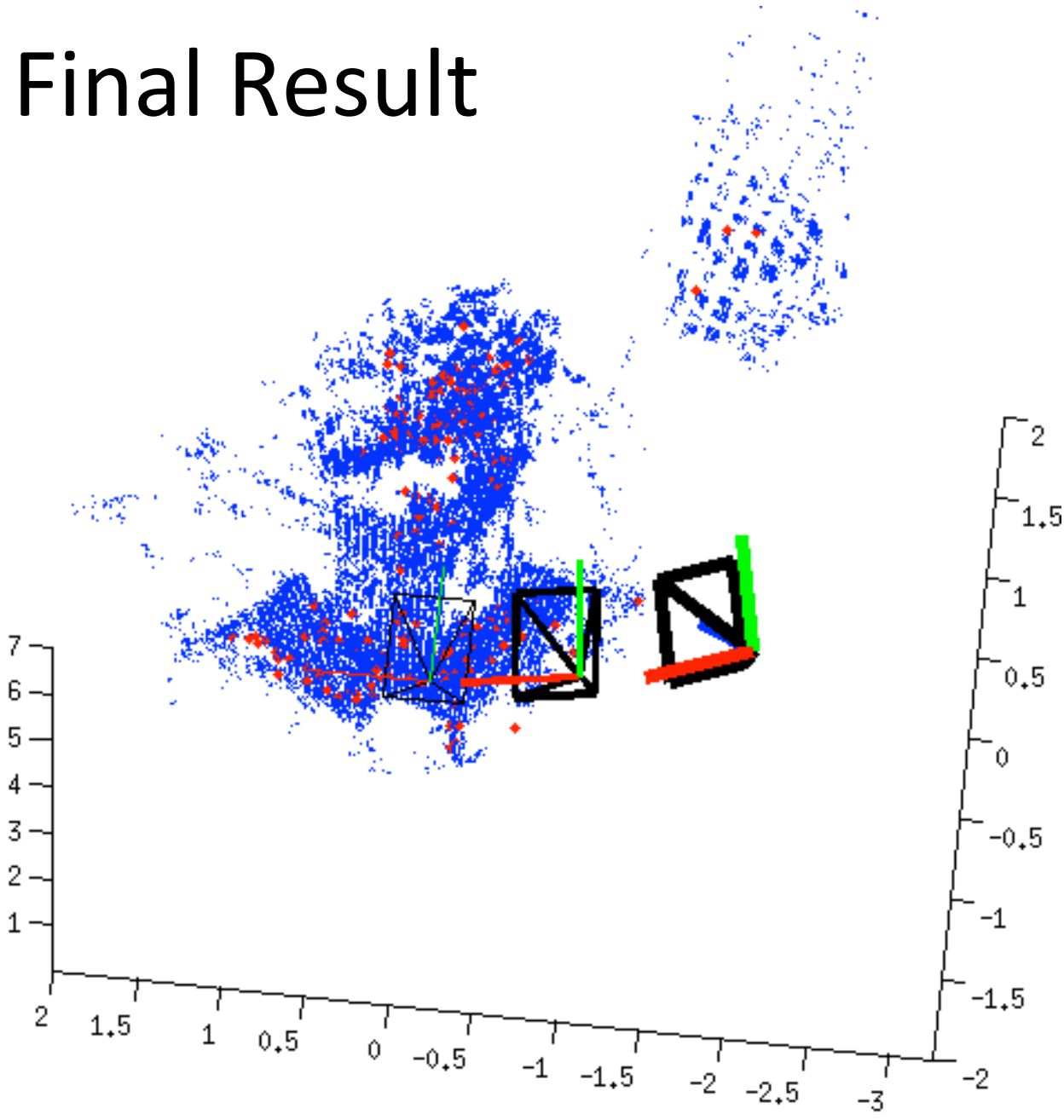
Triangulation



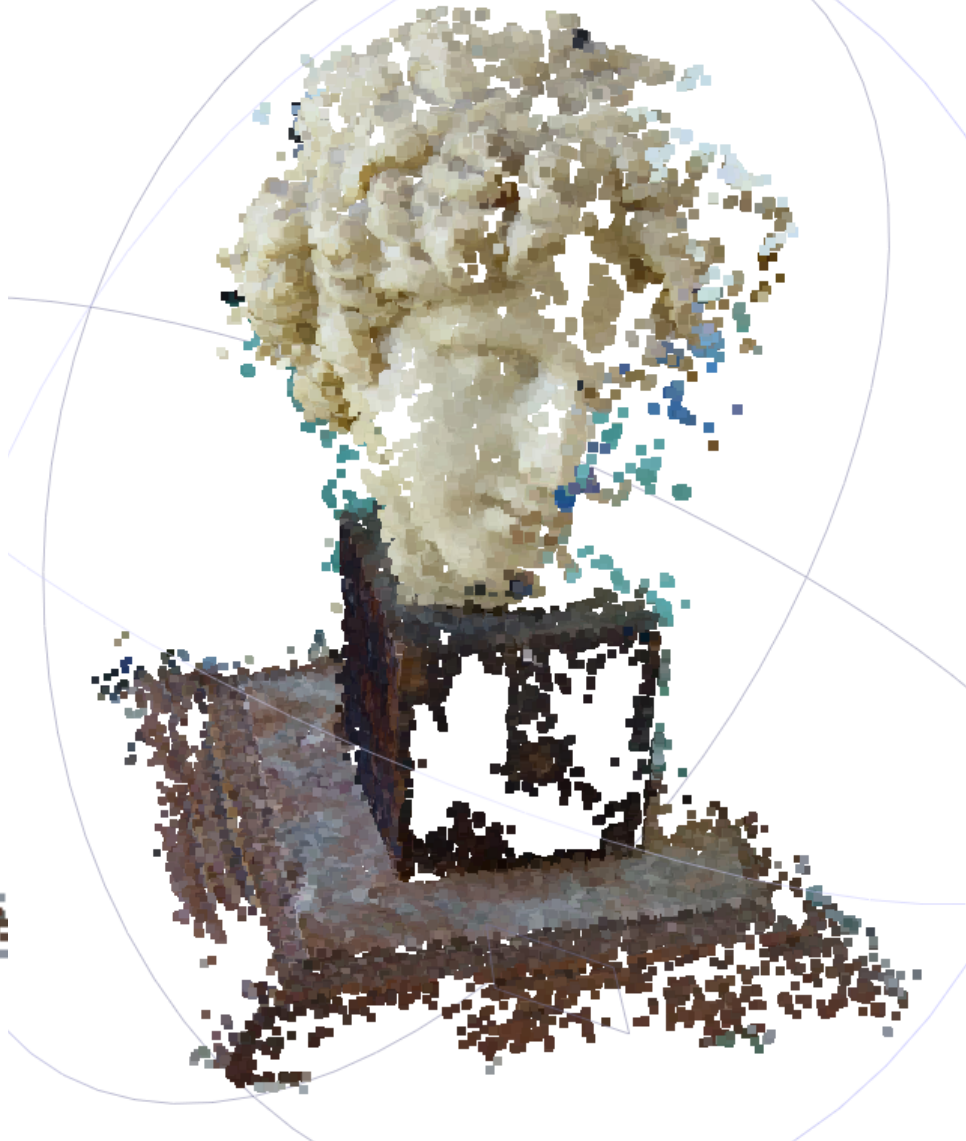
Final Result



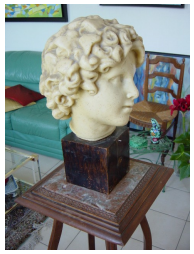


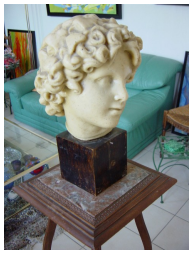


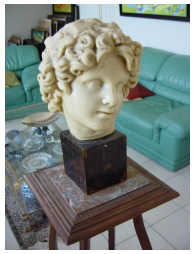
Colorize the Point Cloud

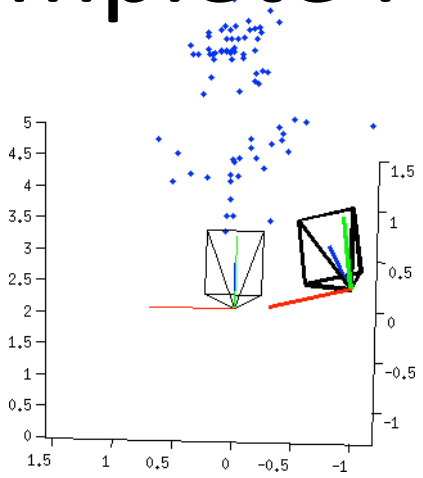


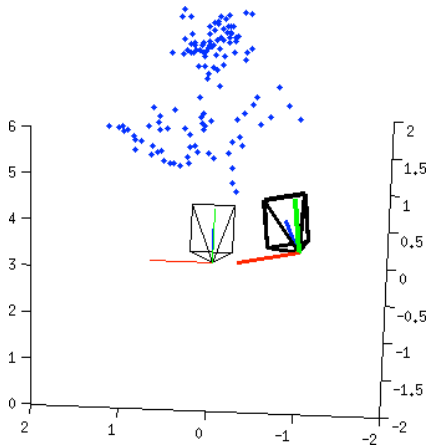
Complete Pipeline in SFMedu

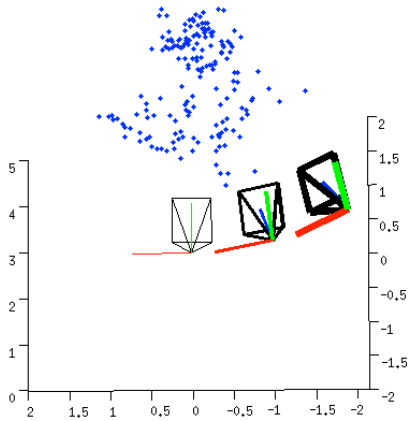


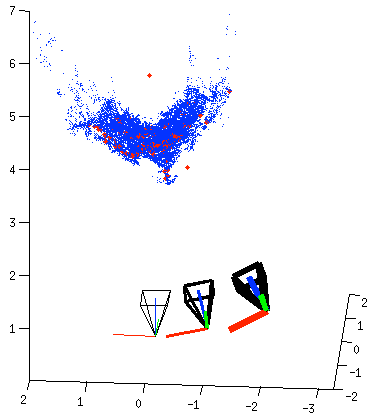












Structure from Motion (SFM)

Multi-view Stereo (MVS)

Wait: How to get the focal length?

- Auto-calibration

  Self-Calibration and Metric Reconstruction in spite of Varying and Unknown Internal Camera Parameters, M Pollefeys, R Koch and L Van Gool, 1998.
  http://mit.edu/jxiao/Public/software/autocalibrate/autocalibration_lin.m

- Grid Search to look for the solution with minimal reprojection error

  for f=min_f:max_f

  do everything, then obtain reprojection error after bundle adjustment

- Optimize for this value in bundle adjustment
- Camera Calibration (with checkerboard)

  http://www.vision.caltech.edu/bouguetj/calib_doc/

- EXIF of JPEG file recorded from digital camera

  Read the code of Bundler to understand how to convert EXIF into focal length value
  http://phototour.cs.washington.edu/bundler/

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Steps

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

Real World Applications

- Streetview Reconstruction and Recognition

  http://people.csail.mit.edu/jxiao/StreetSeg/

  http://people.csail.mit.edu/jxiao/CityRec/

- Photo Tourism http://phototour.cs.washington.edu/
- Microsoft Photosynth http://photosynth.net/
- 2d3, boujor (Matchmovers) and movies

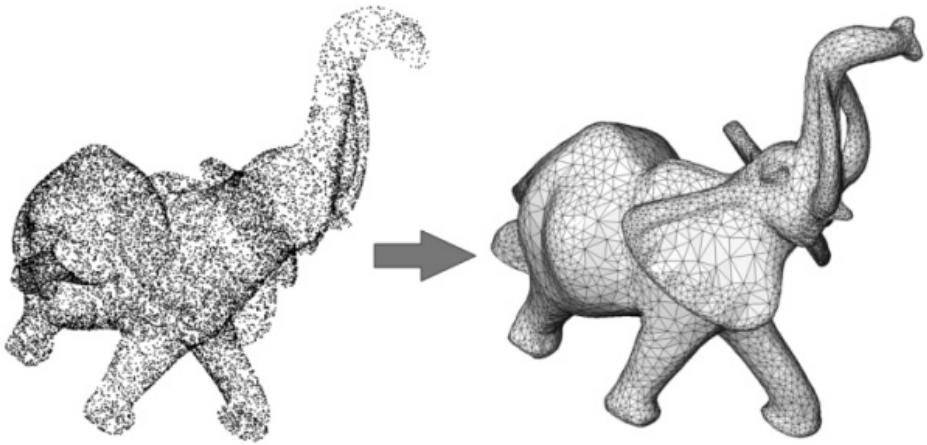
  http://www.2d3.com/ http://www.vicon.com/boujou/

- Robotics: SLAM http://openslam.org/

  Simultaneous Localization And Mapping

Steps

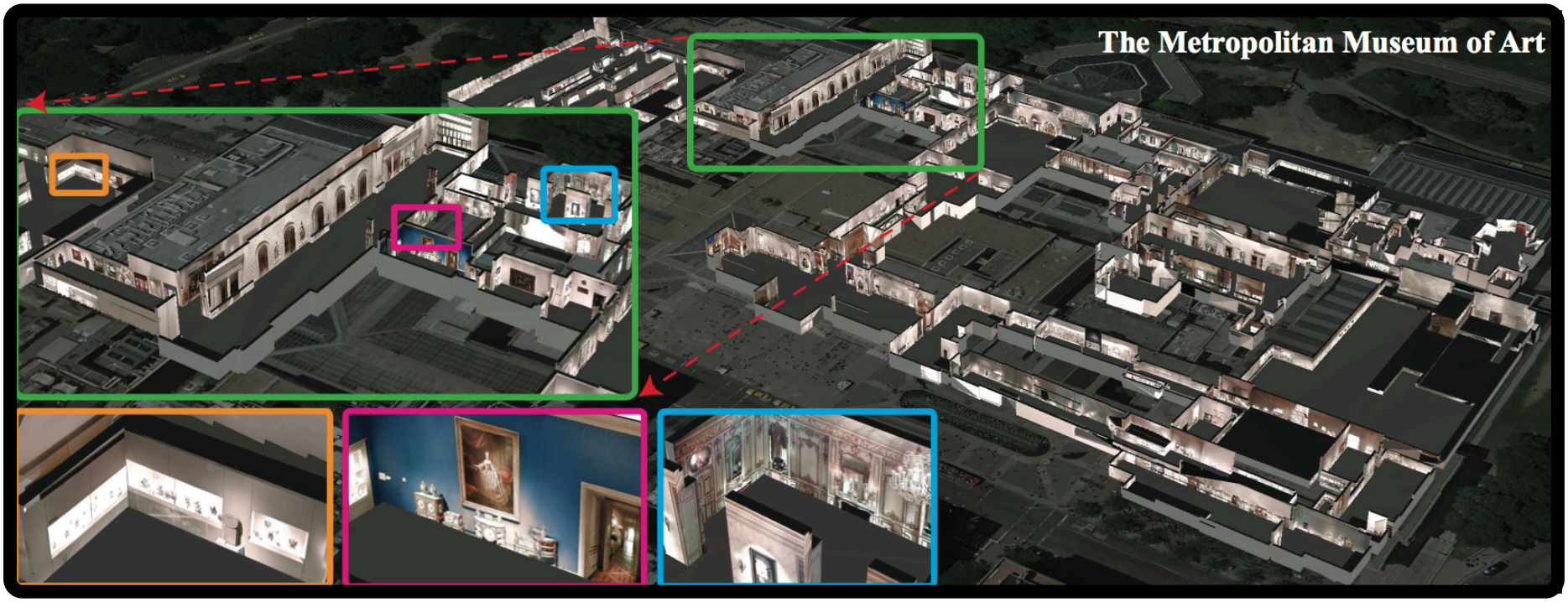
	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling



Point Cloud → 3D Mesh Model

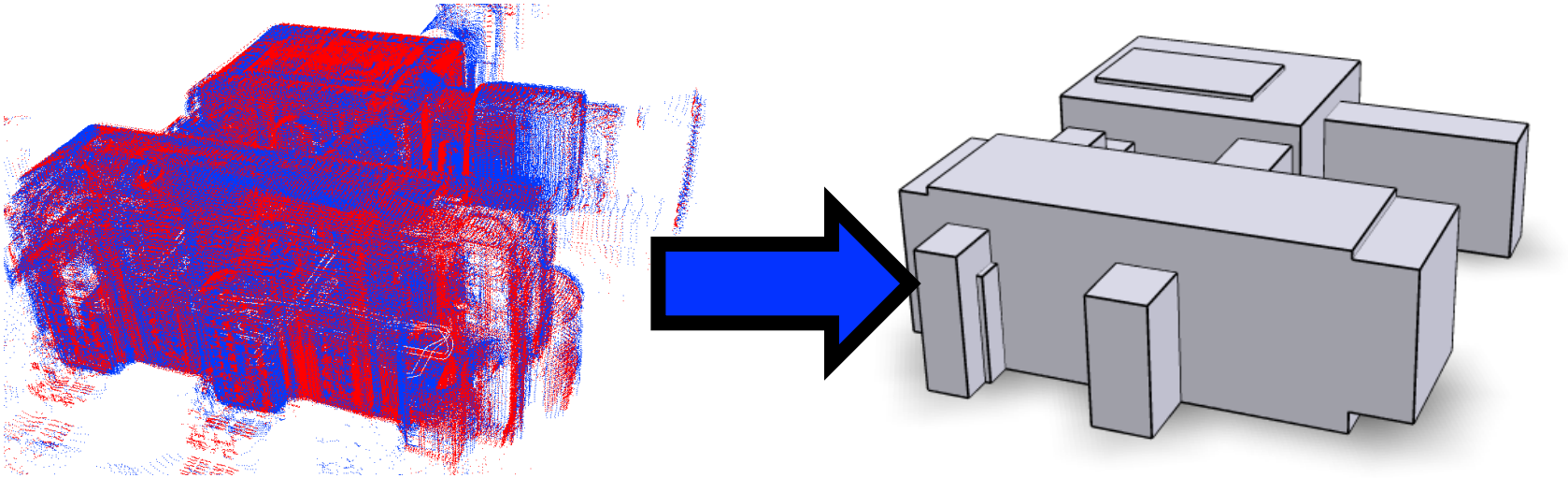
- Surface Reconstruction
  - Marching Cubes: http://en.wikipedia.org/wiki/Marching_cubes
  - Poisson Surface Reconstruction http://research.microsoft.com/en-us/um/people/hoppe/proj/poissonrecon/
- Model Fitting
  - RANSAC & J-linkage http://www.diegm.uniud.it/fusiello/demo/jlk/
  - InverseCSG
  - GlobFit http://code.google.com/p/globfit/
  - Face Model Fitting http://research.microsoft.com/apps/pubs/default.aspx?id=73211

InverseCSG Algorithm

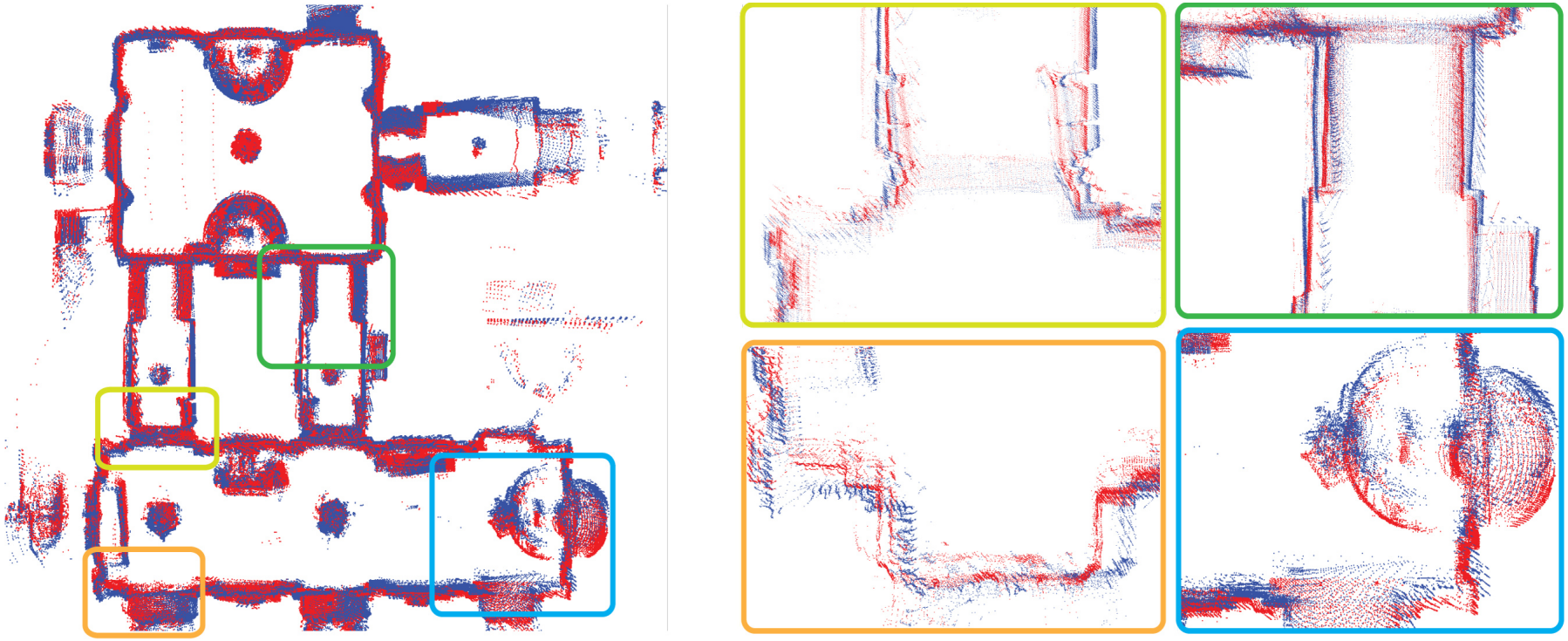


Reconstructing the World's Museums, J. Xiao and Y. Furukawa, 2012.

InverseCSG Algorithm



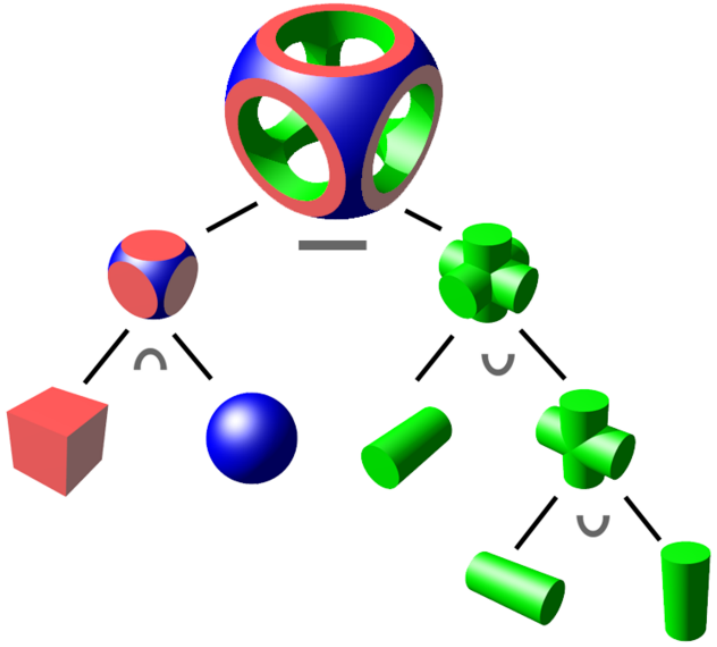
Noisy Points



top-down view of input points

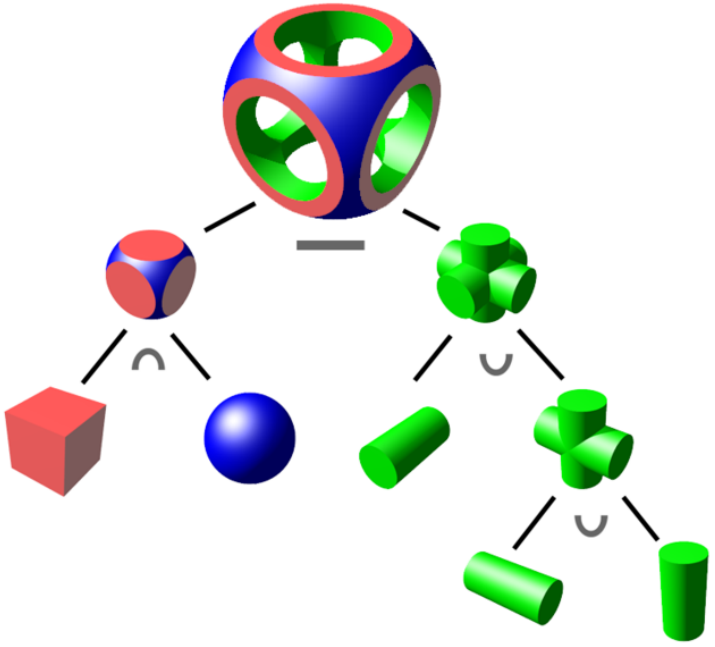
InverseCSG Algorithm

Constructive Solid Geometry (CSG)



InverseCSG Algorithm

Constructive Solid Geometry (CSG)

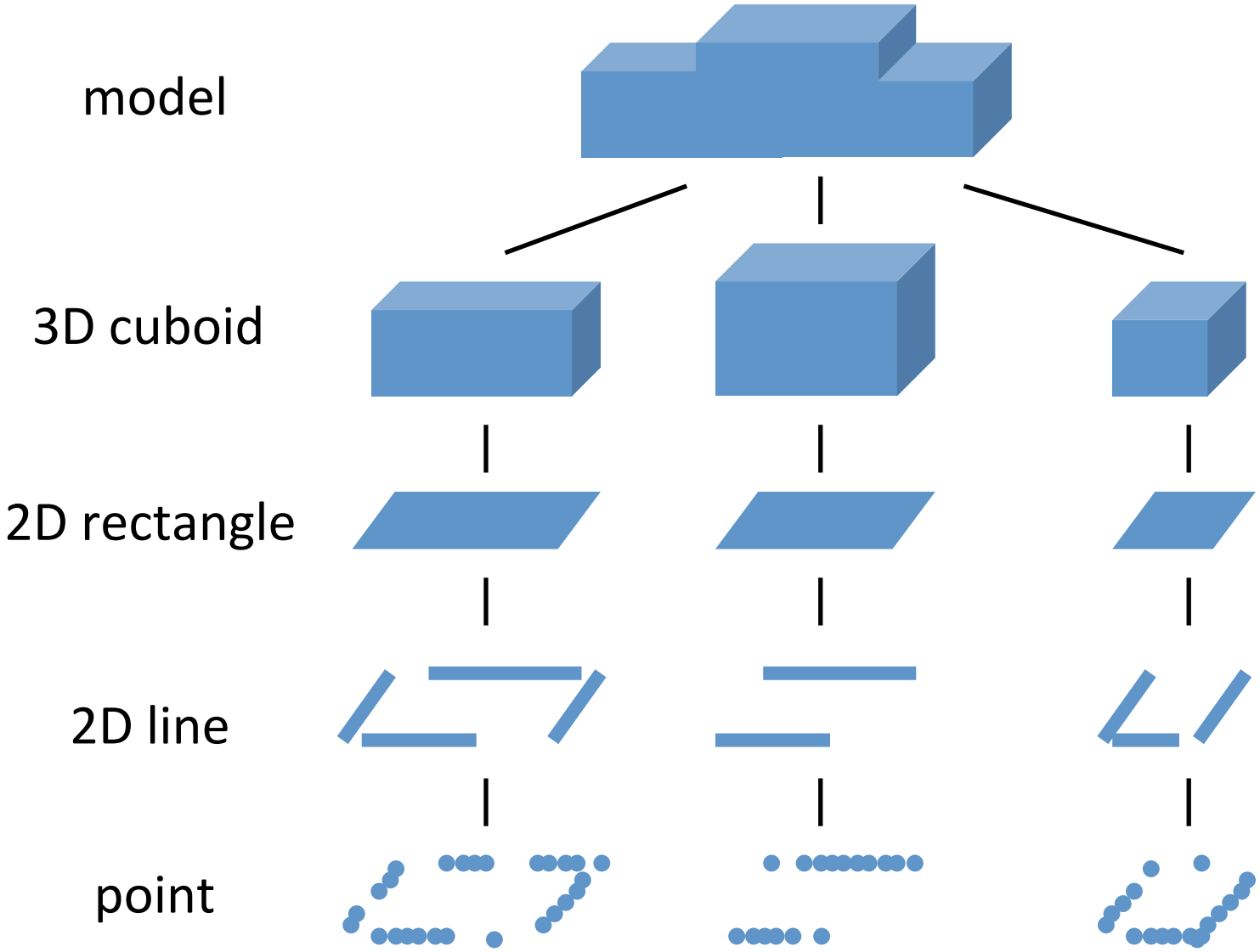


Cuboids as primitives

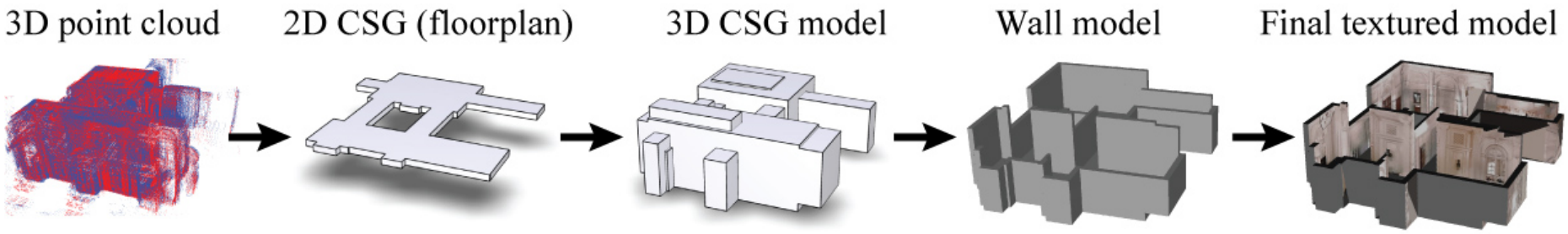
Xiao et al. NIPS 2012

Xiao et al. Siggraph Asia 2012a

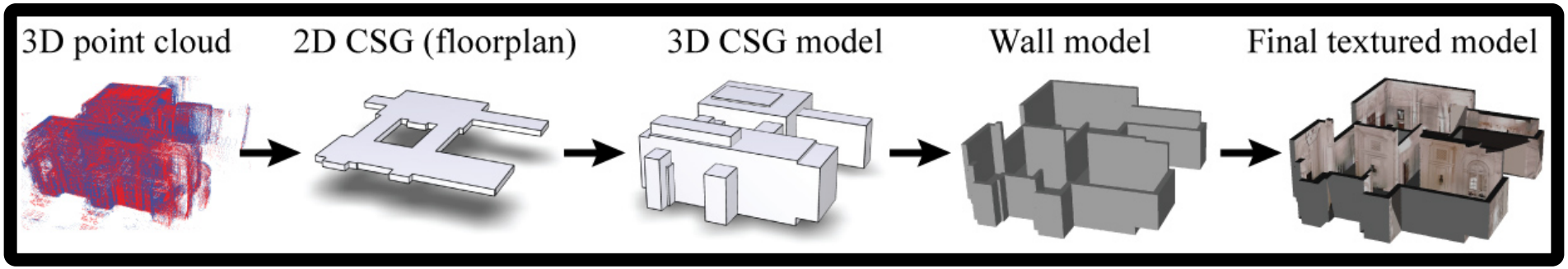
Bottom-up & Top-down



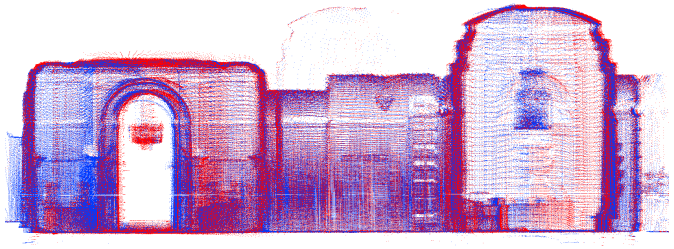
System Overview



System Overview

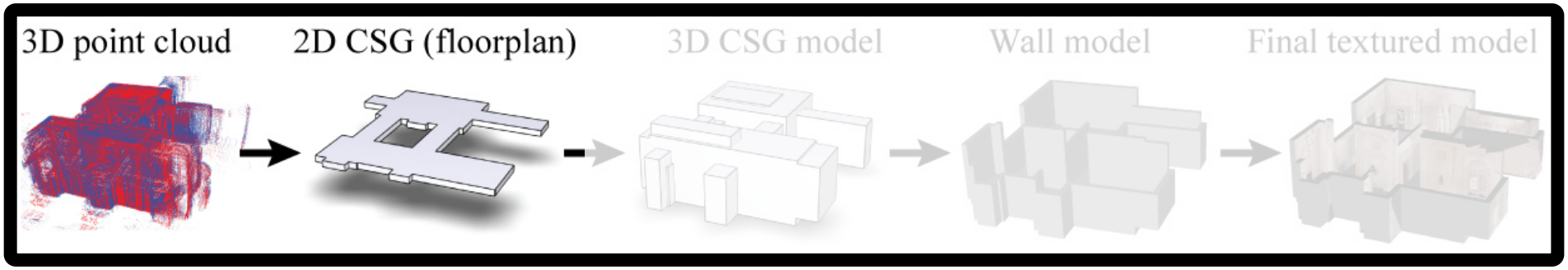


Cut into Slices

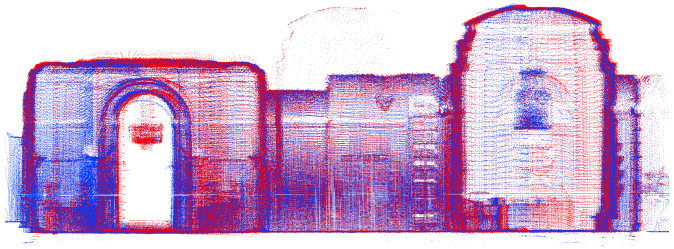


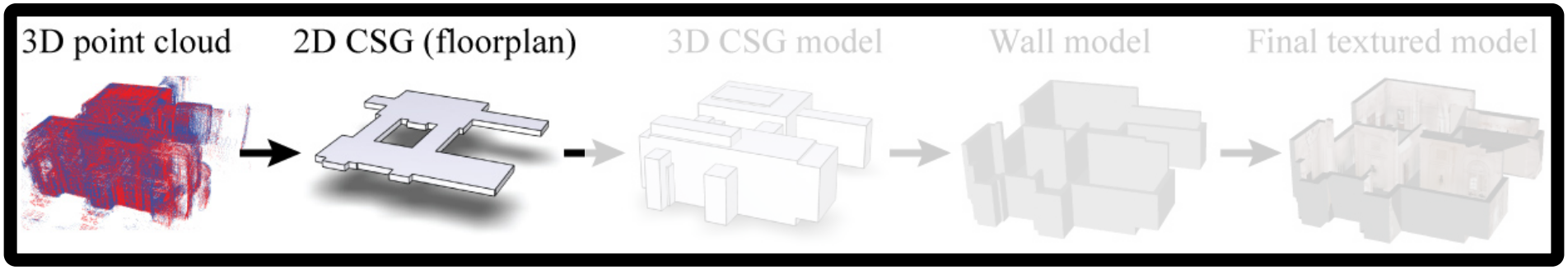
side view

gravity

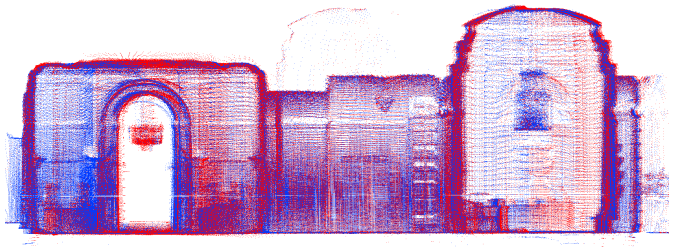


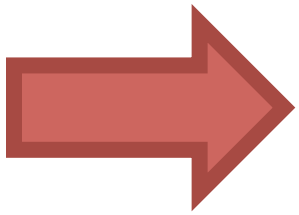
Cut into Slices

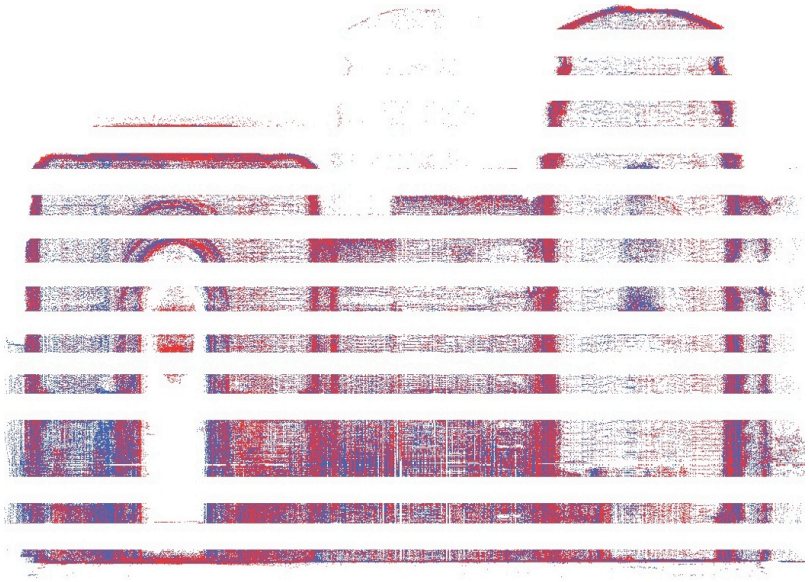


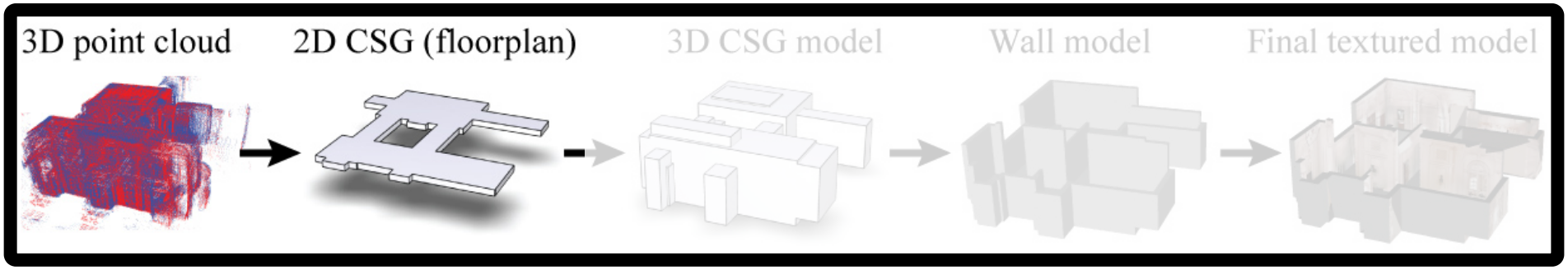


Cut into Slices





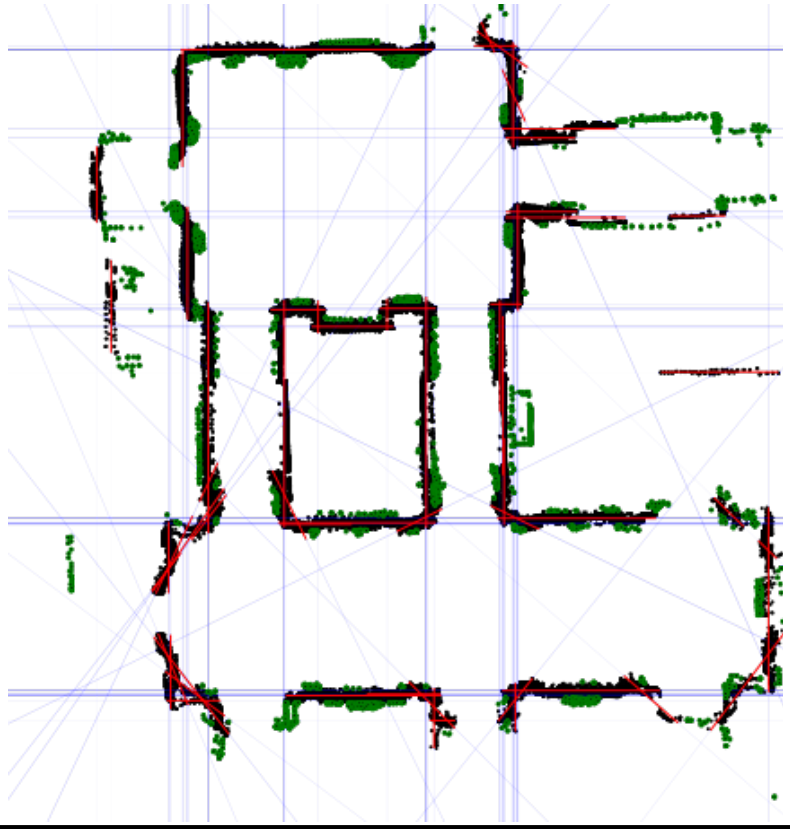


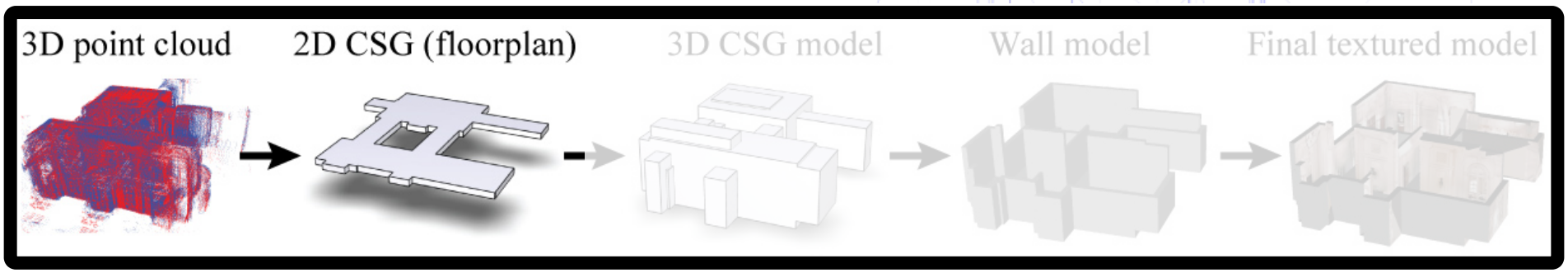


2D CSG Reconstruction

1. Generate primitives

2. Choose a subset

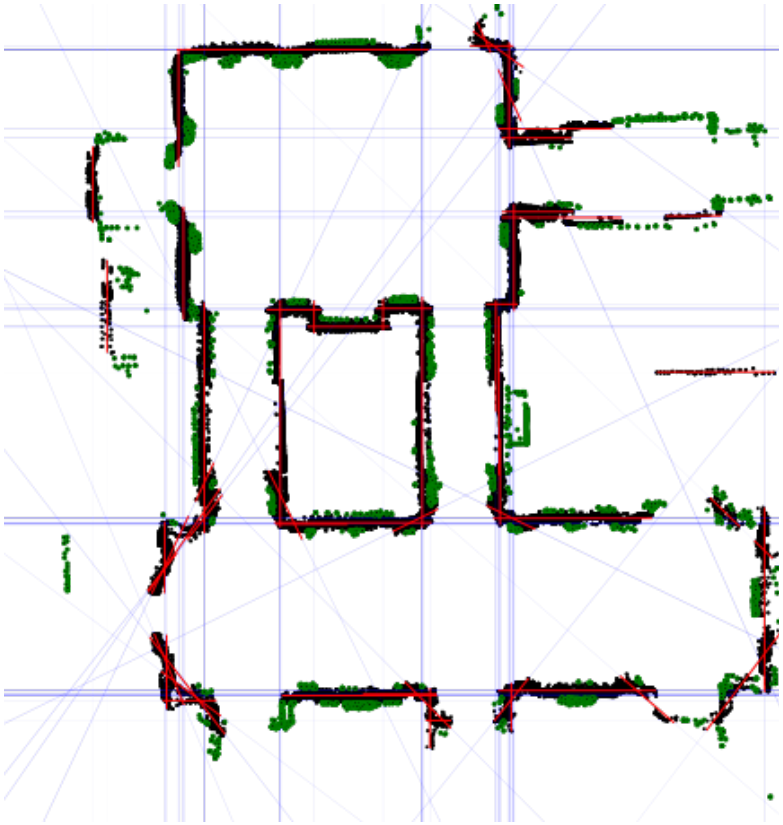




2D CSG Reconstruction

1. Generate primitives

point → line



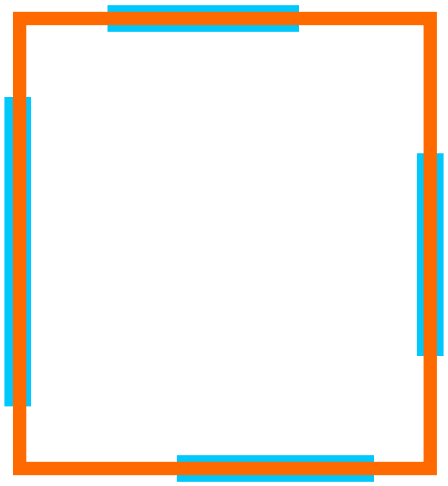
2D CSG Reconstruction

1. Generate primitives

point → line

line → rectangle

From 4 line segments



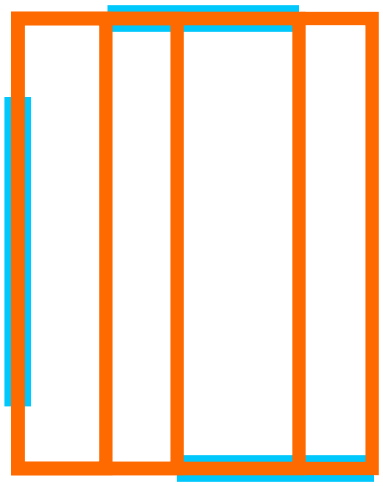
2D CSG Reconstruction

1. Generate primitives

point → line

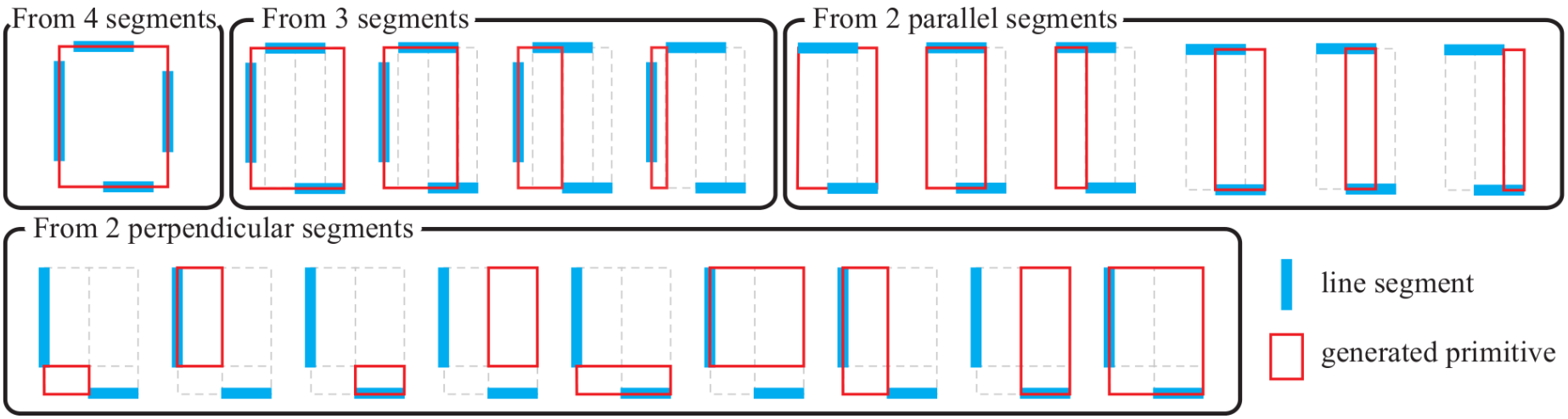
line → rectangle

From 3 line segments



2D CSG Reconstruction

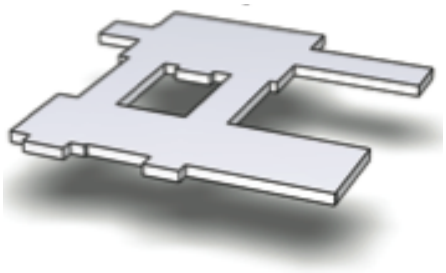
1. Generate primitives

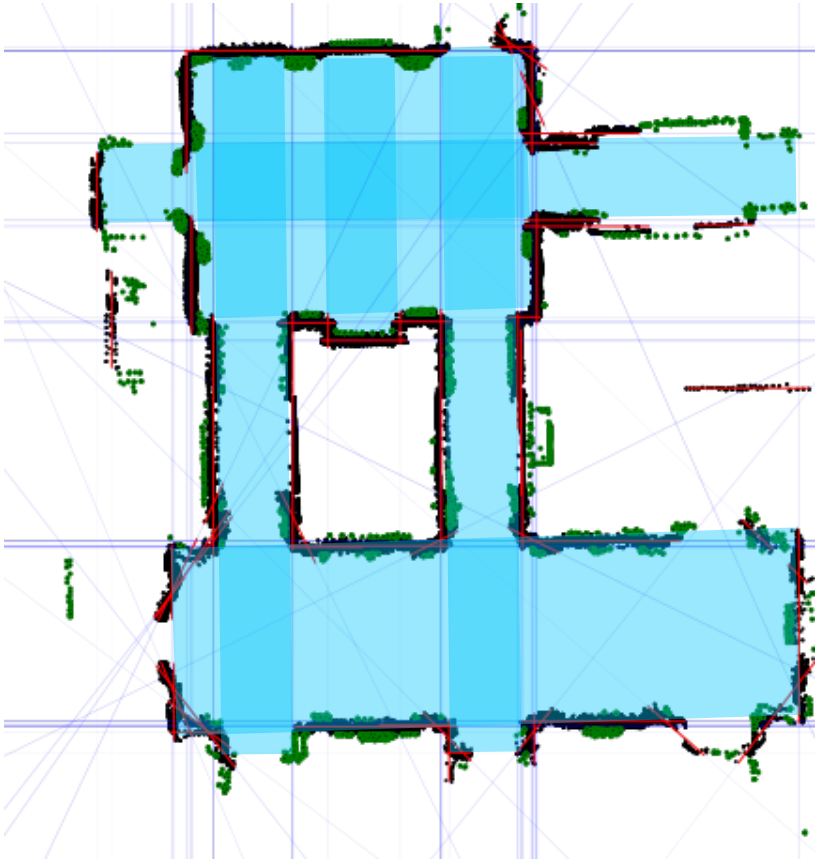


2D CSG Reconstruction

1. Generate primitives

2. Choose a subset



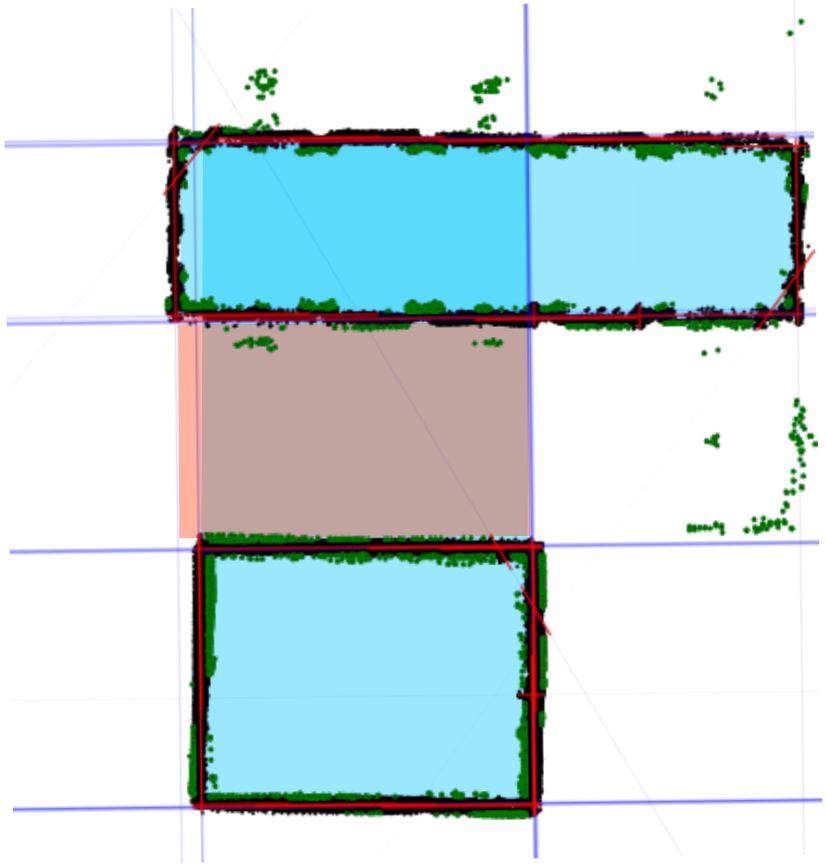


2D CSG Reconstruction

1. Generate primitives

2. Choose a subset

Repeat in each slice



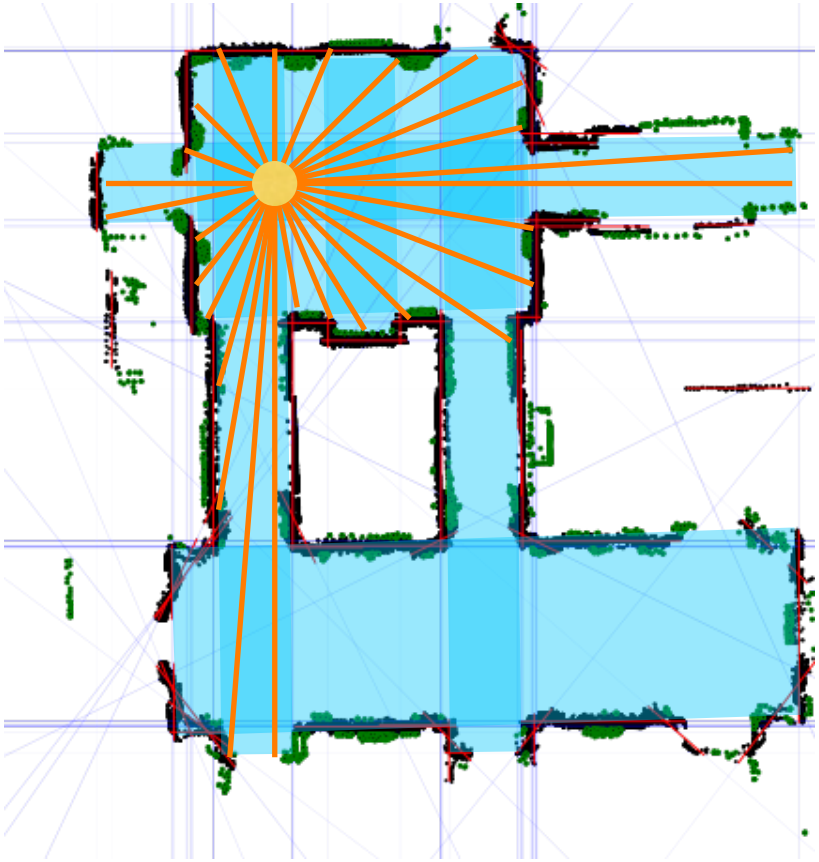
2D CSG Reconstruction

Explain the data

- Free space
- Laser points

Simple

- Regularization



Objective Function

$$E_1(T) = \frac{\{\text{Sum of free-space scores inside } T\}}{\{\text{Total sum in the domain without negative scores}\}}$$

$$E_2(T) = \frac{\{\#\text{ of points on the surface of } T\}}{\{\text{total \# of points}\}}$$

$$E_3(T) = \frac{\{\text{perimeter of } T \text{ near laser points (within 0.2 meters)}\}}{\{\text{total perimeter of } T\}}$$

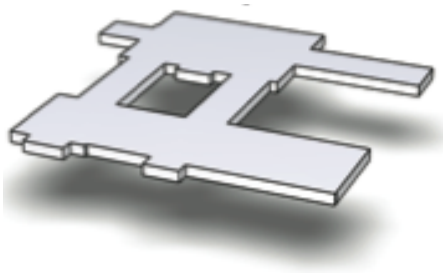
$$E(T) = w_1 E_1(T) + w_2 E_2(T) + w_3 E_3(T)$$

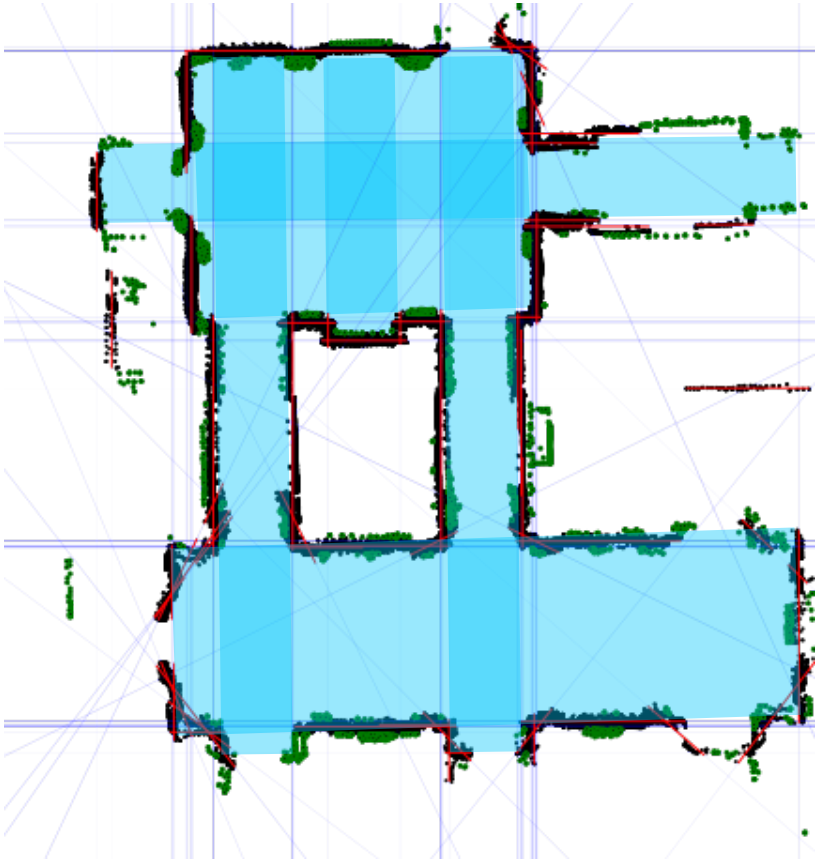
Free space Laser points Regularization

2D CSG Reconstruction

1. Generate primitives

2. Choose a subset

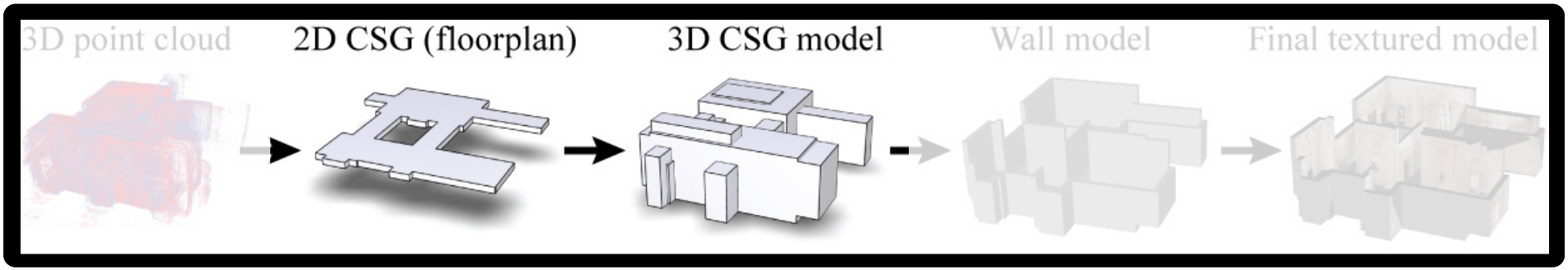




3D CSG Reconstruction

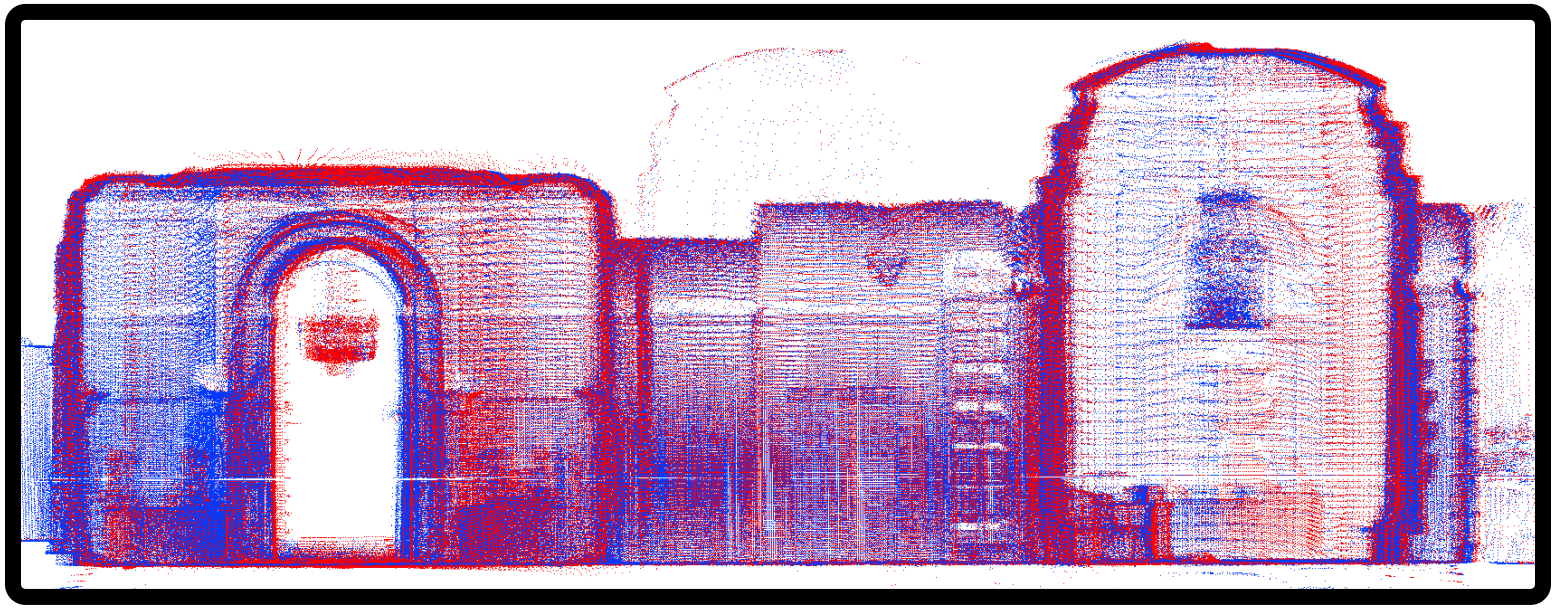
1. Generate primitives (cuboids)

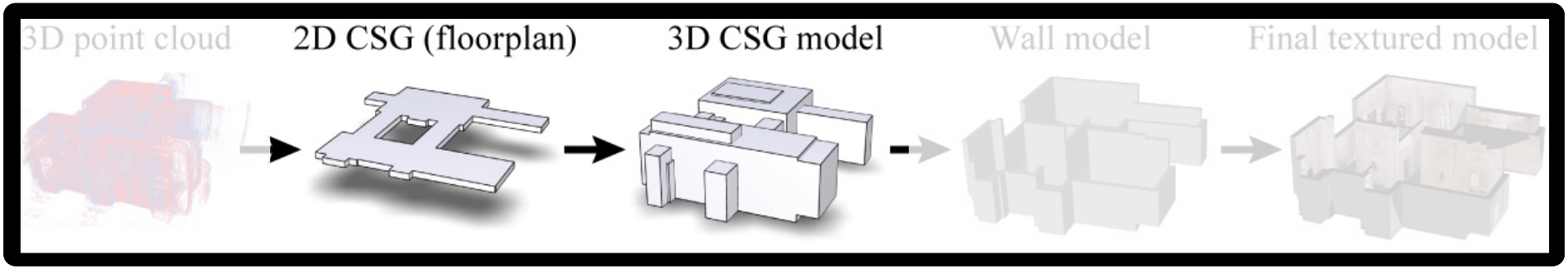
2. Choose a subset (out of primitive candidates)



3D CSG Reconstruction

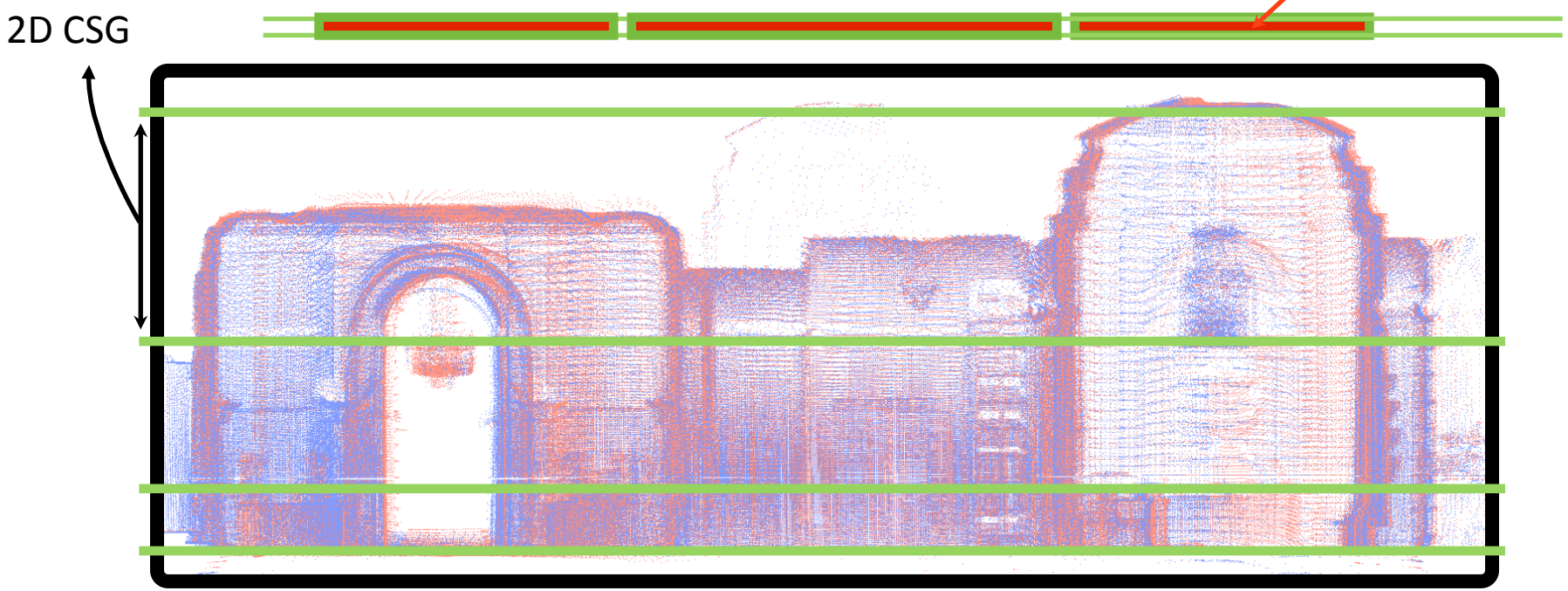
1. Generate primitives (cuboids)

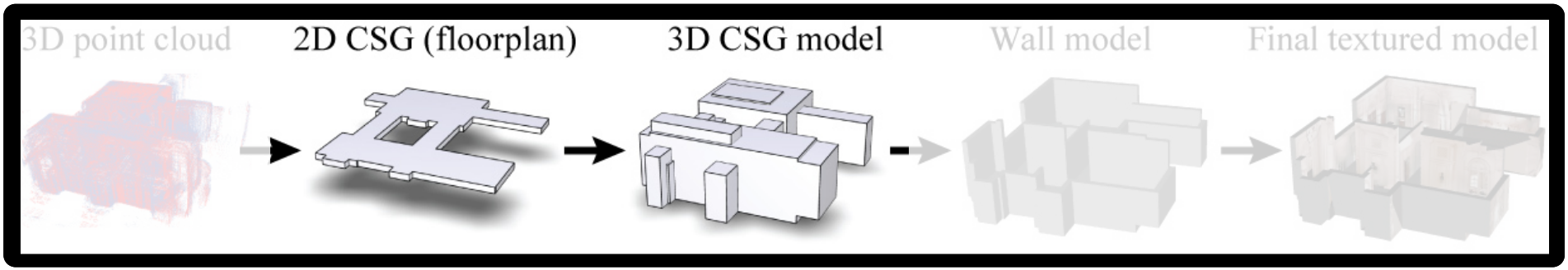




3D CSG Reconstruction

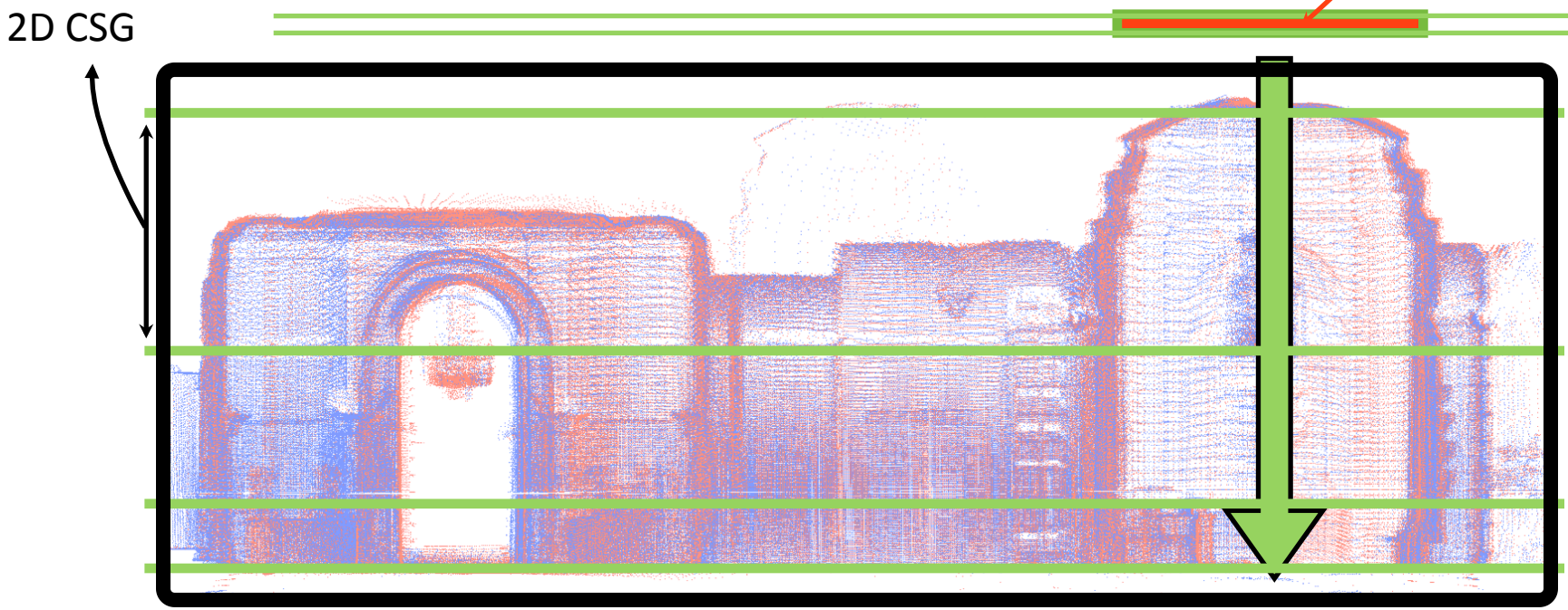
1. Generate primitives (cuboids)

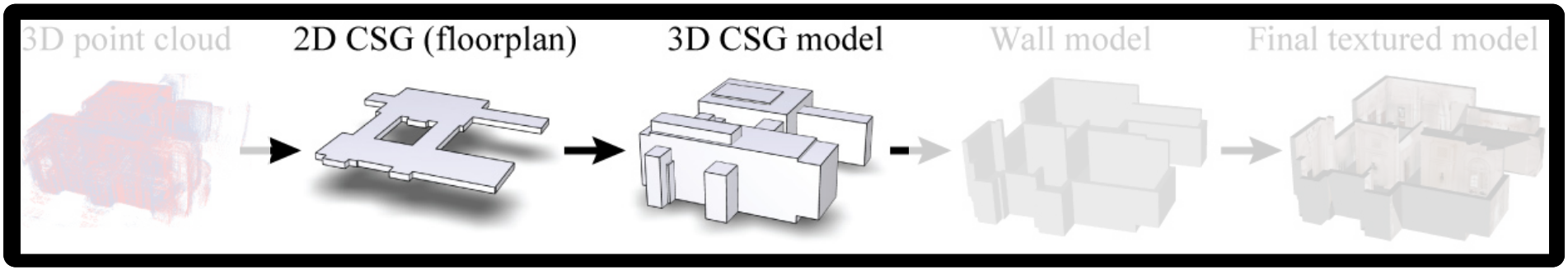




3D CSG Reconstruction

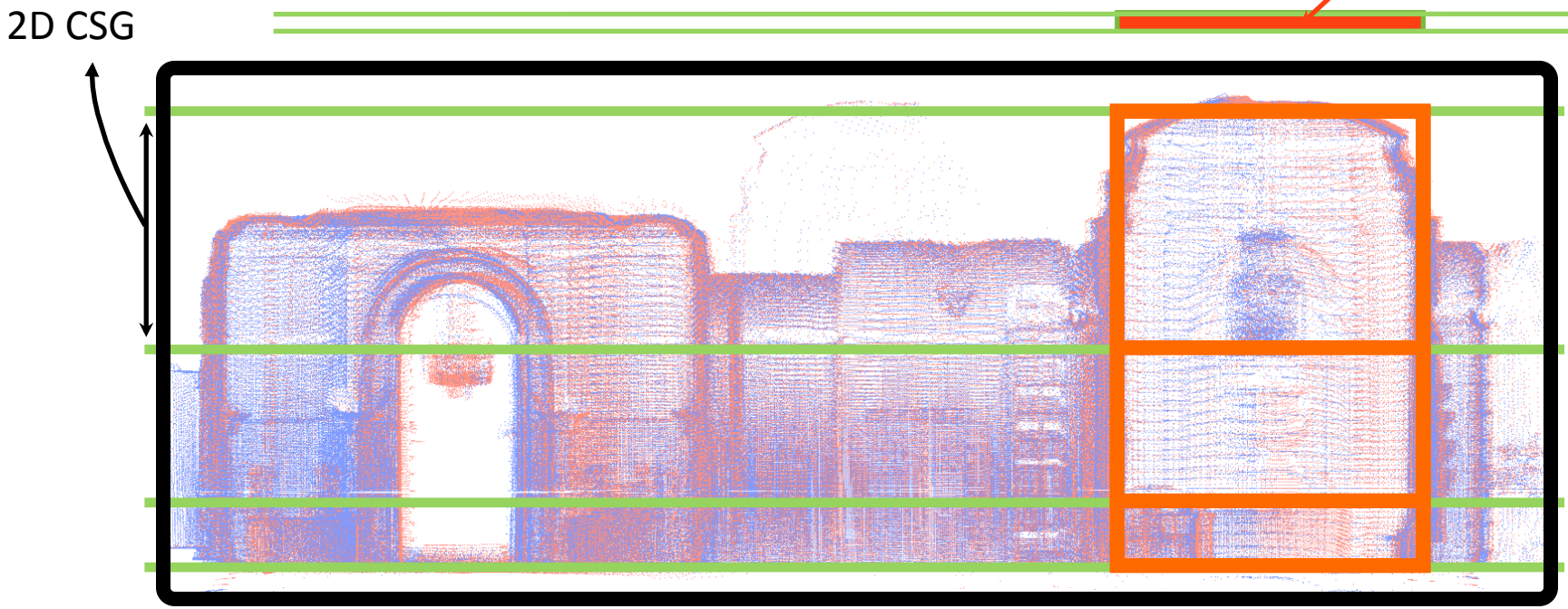
1. Generate primitives (cuboids)

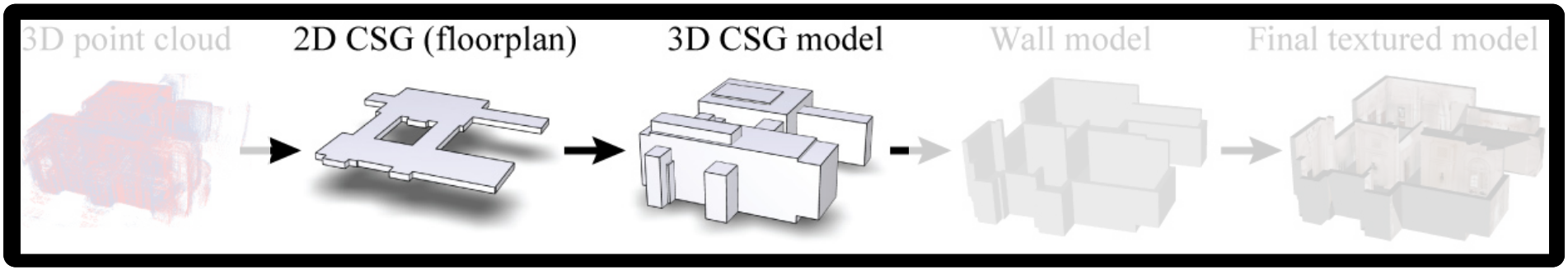




3D CSG Reconstruction

1. Generate primitives (cuboids)

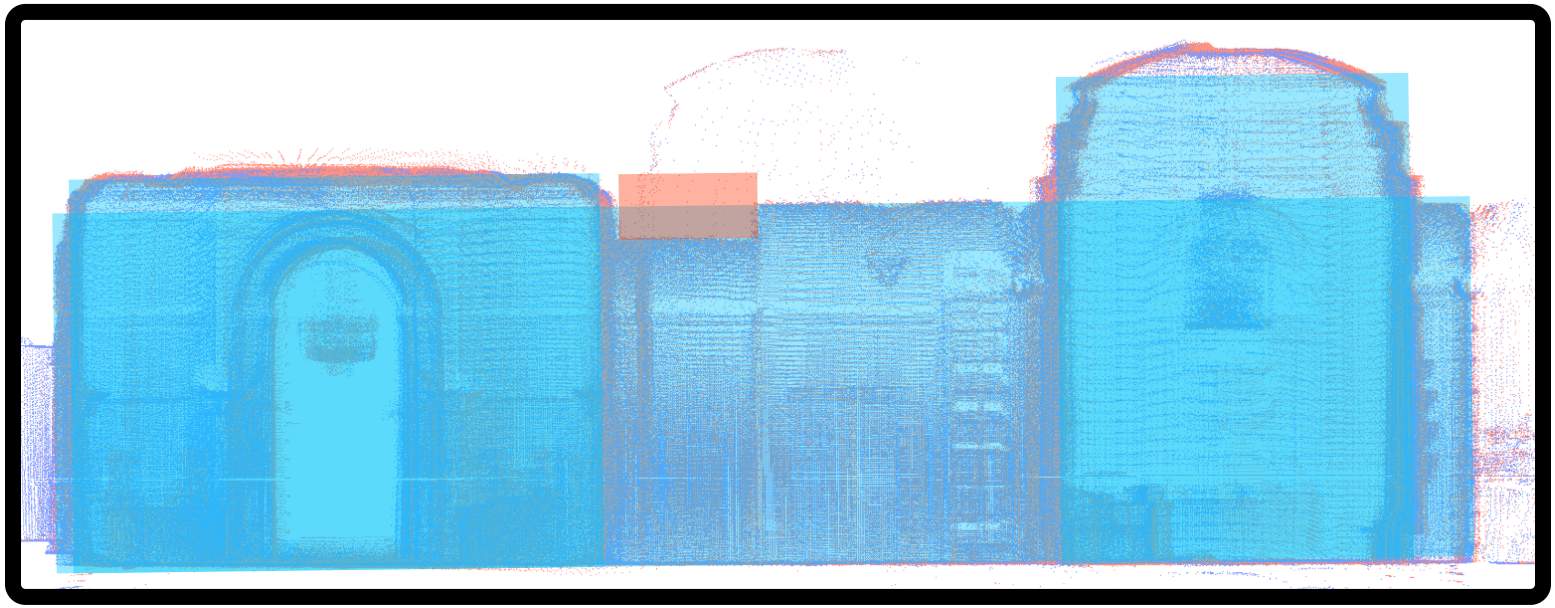


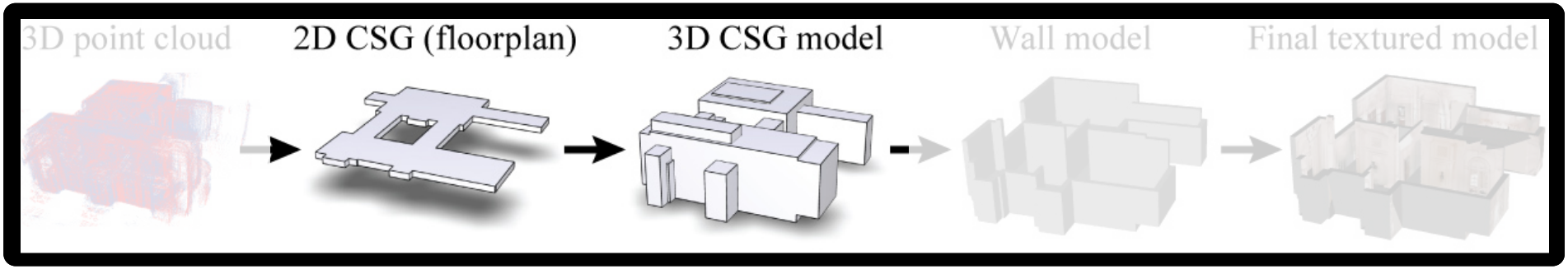


3D CSG Reconstruction

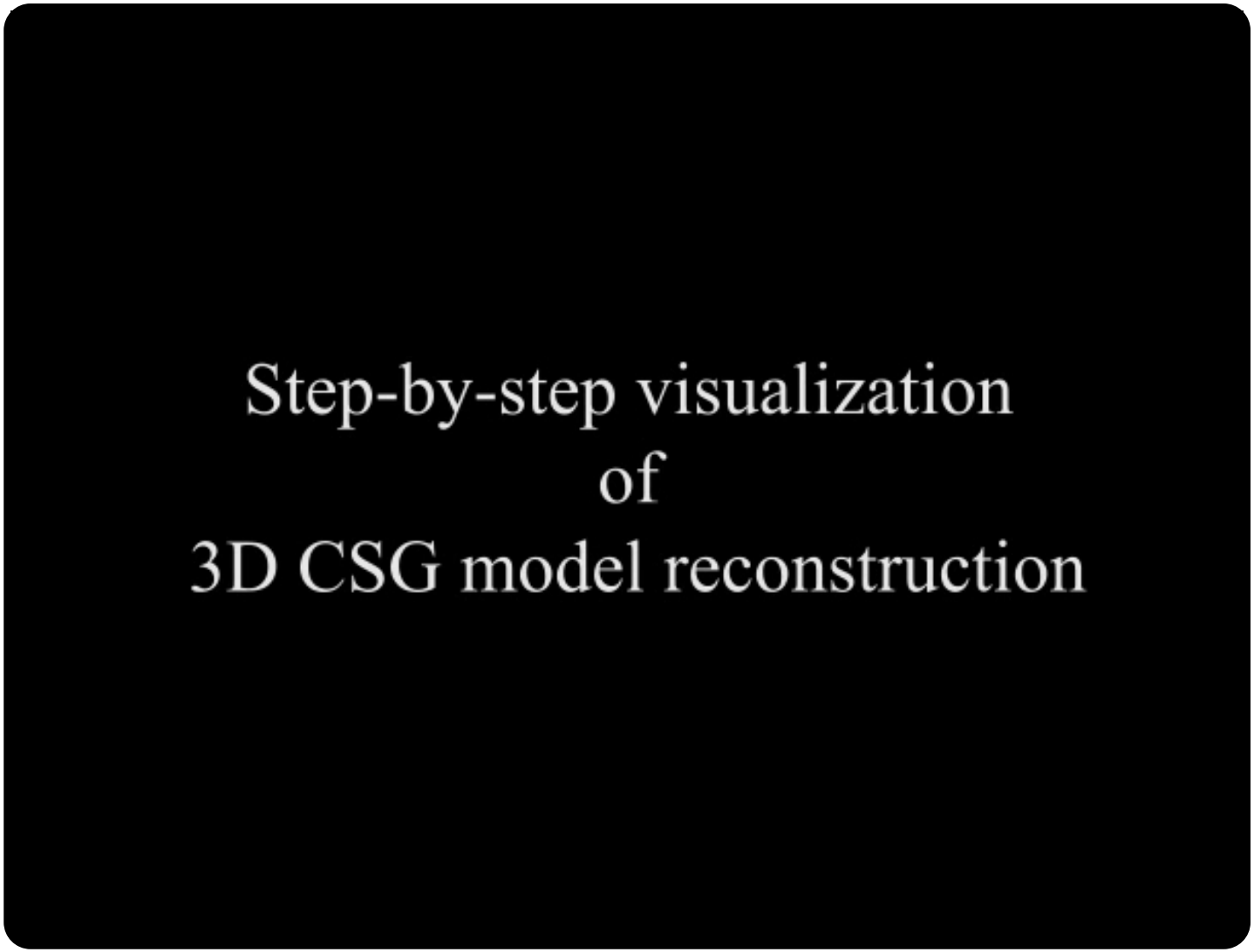
1. Generate primitives (cuboids)

2. Choose a subset





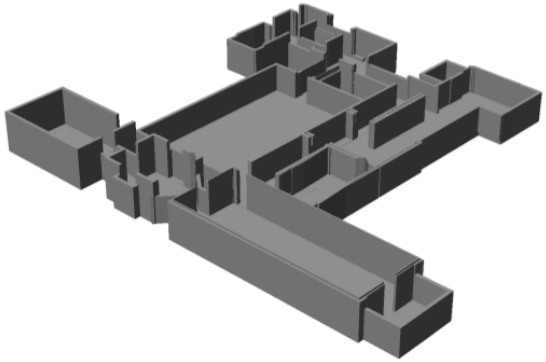
Algorithm on Run



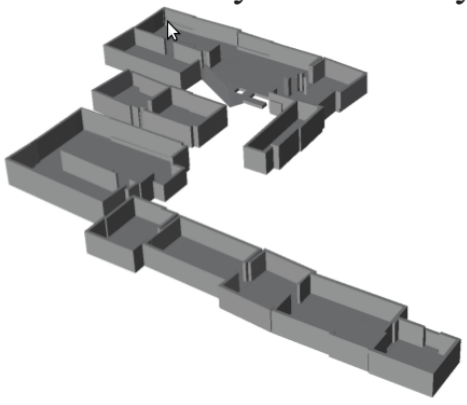
Result

Shaded mesh

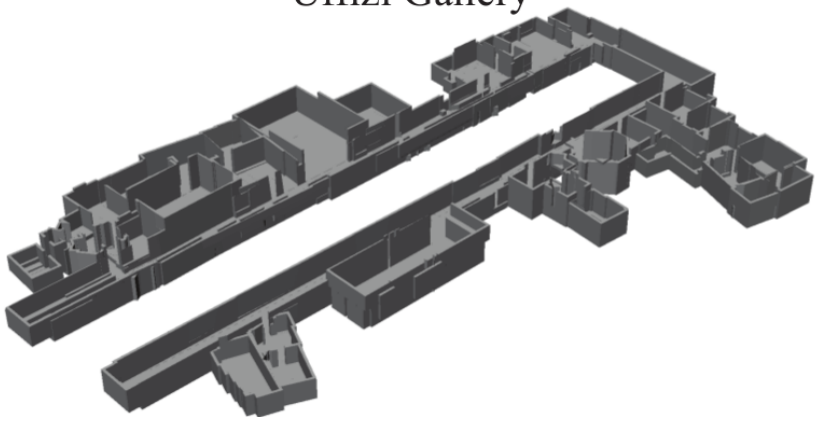
The Frick Collection



The State Tretyakov Gallery

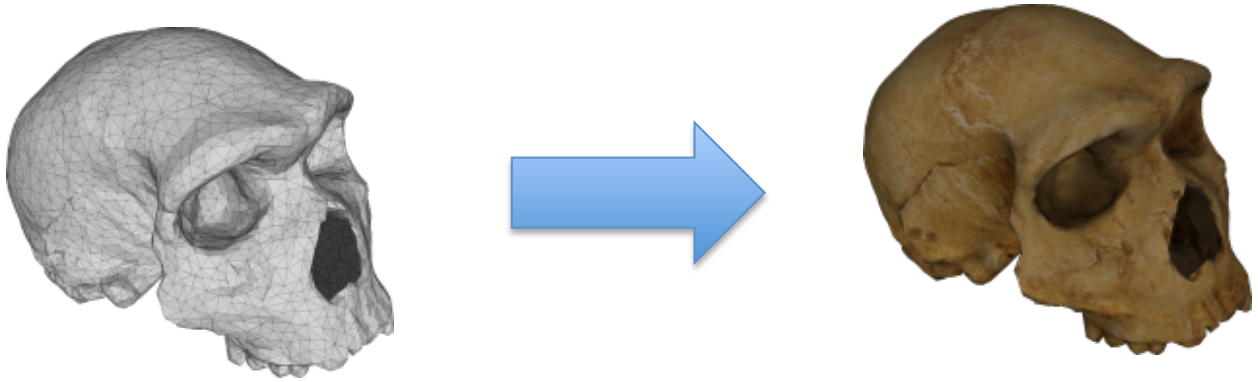


Uffizi Gallery



Steps

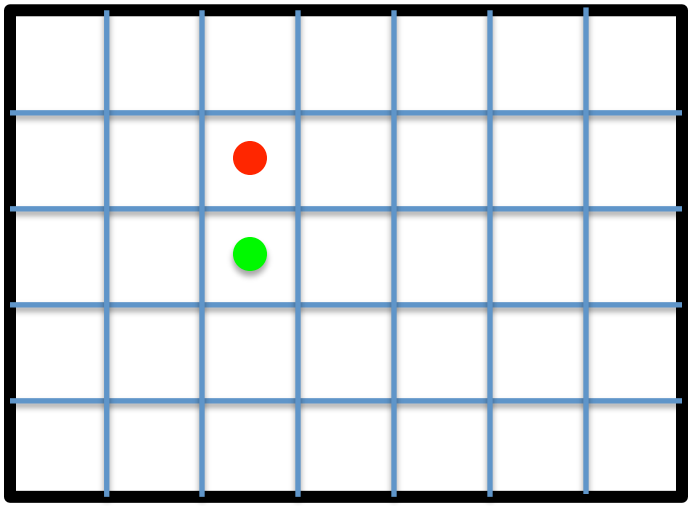
	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling



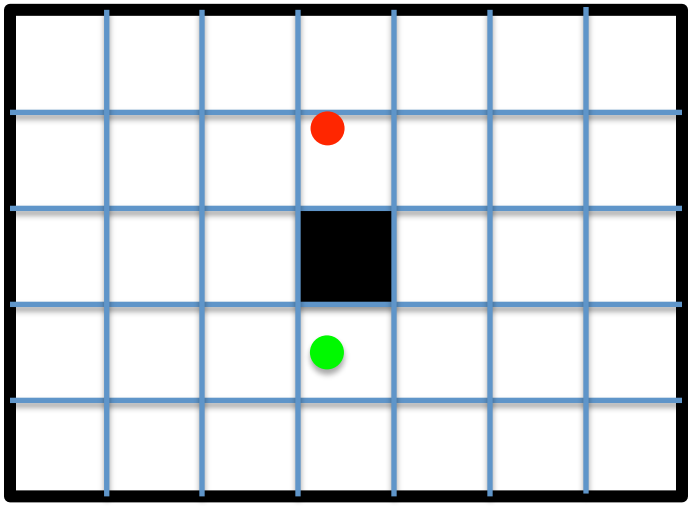
Texture Mapping

- Texture Stitching
- View-based Texture Mapping
- View Interpolation and Warping
- Interactive Visualization

Image Warping: Forward Warping



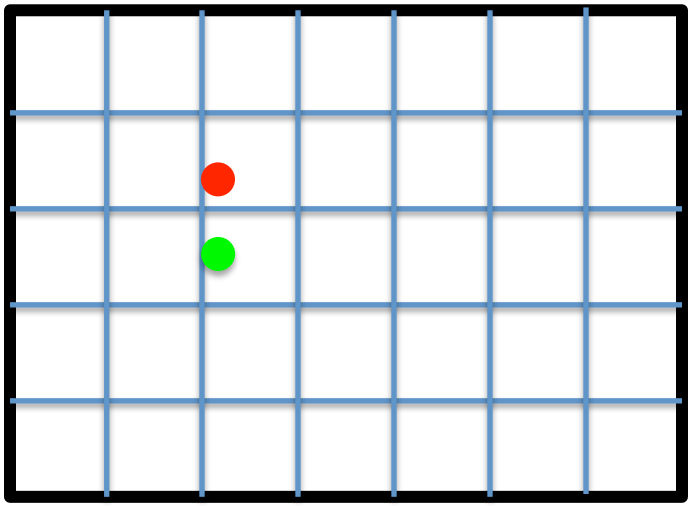
Source Image



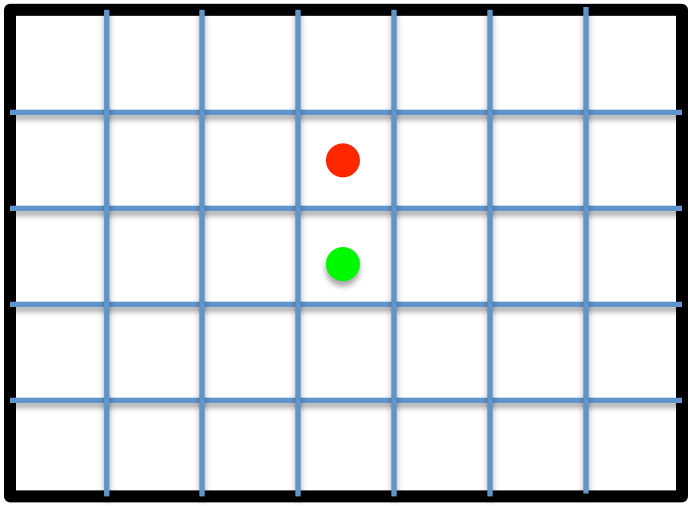
Target Image

Matlab interp2 example: http://www.cse.psu.edu/~rcollins/CSE486/lecture14.pdf

Image Warping: Backward Warping



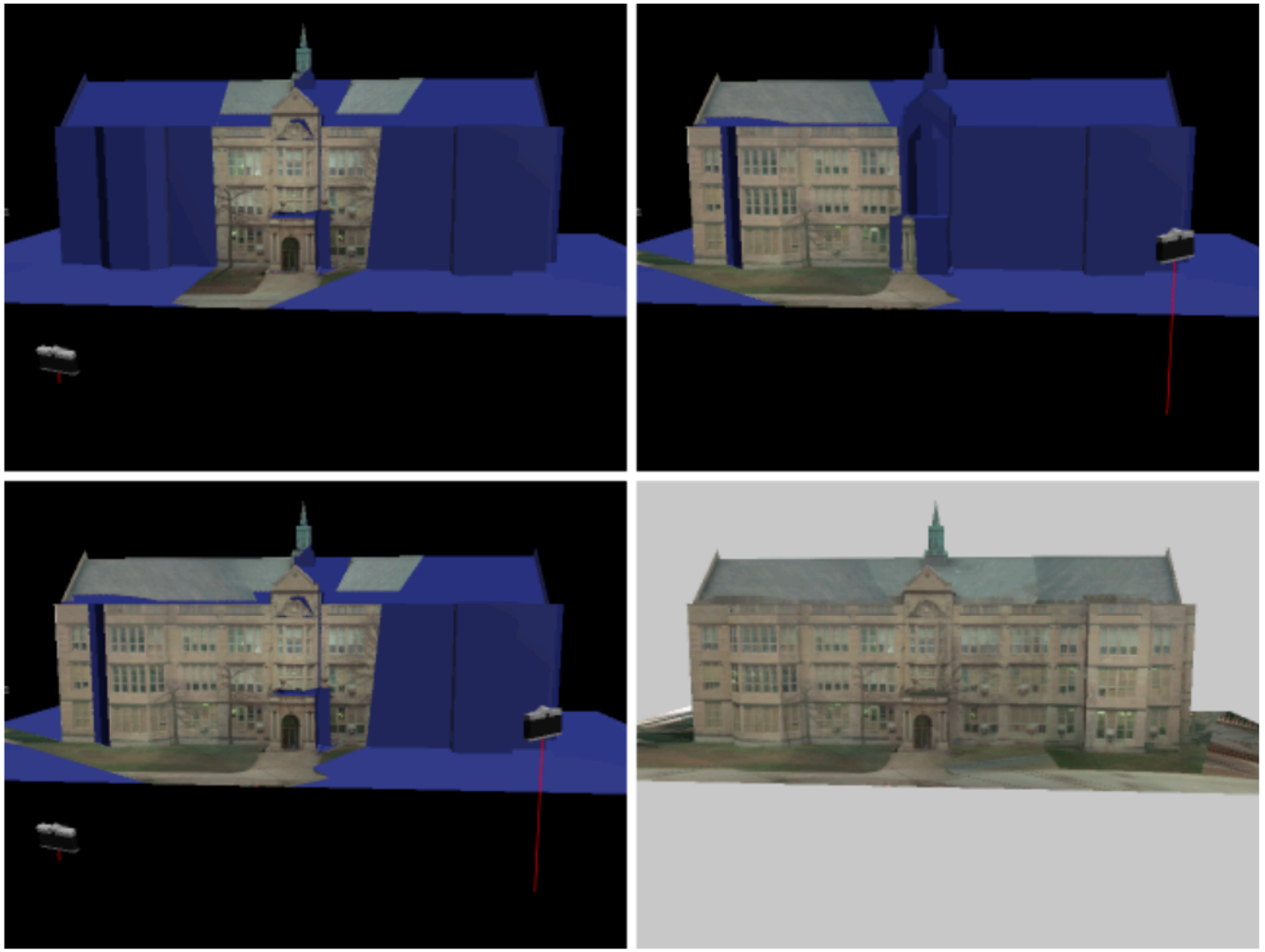
Source Image



Target Image

Matlab interp2 example: http://www.cse.psu.edu/~rcollins/CSE486/lecture14.pdf

Texture Stitching



The process of assembling projected images to form a composite rendering

Image from Paul Debevec's Thesis

Texture Stitching

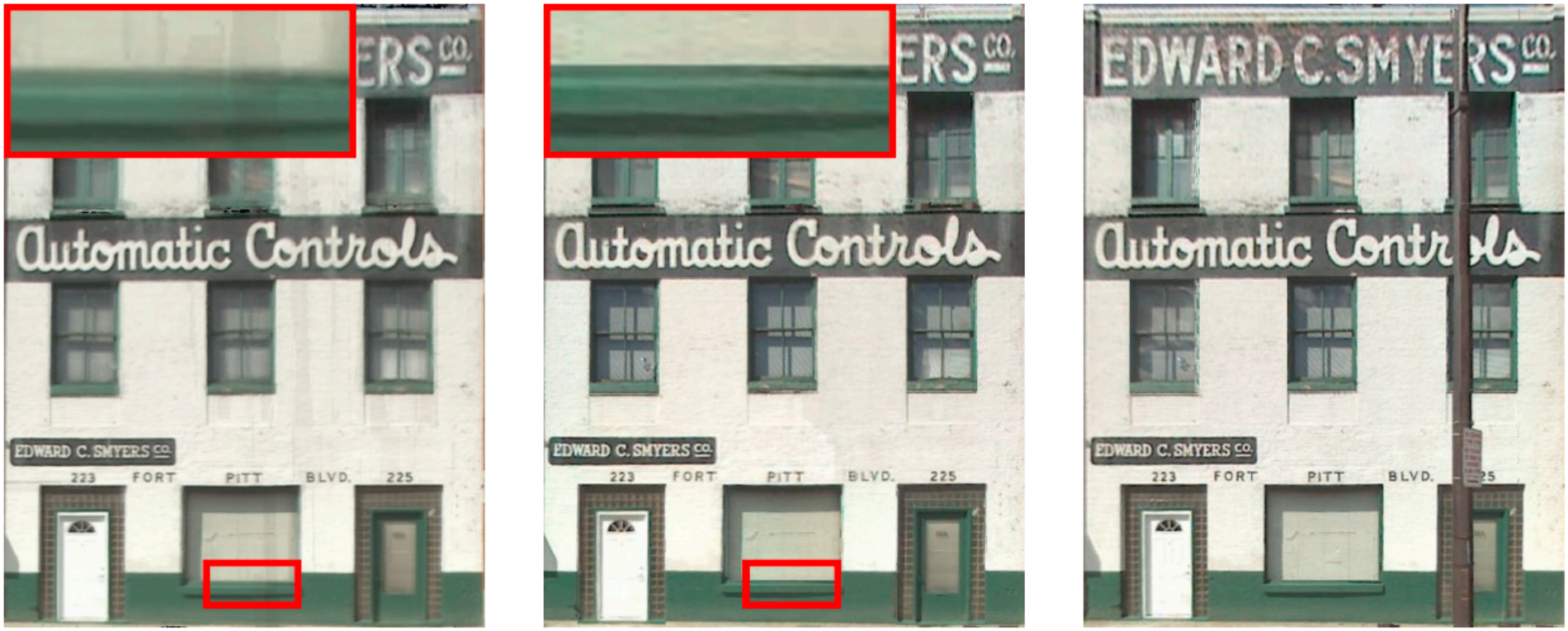
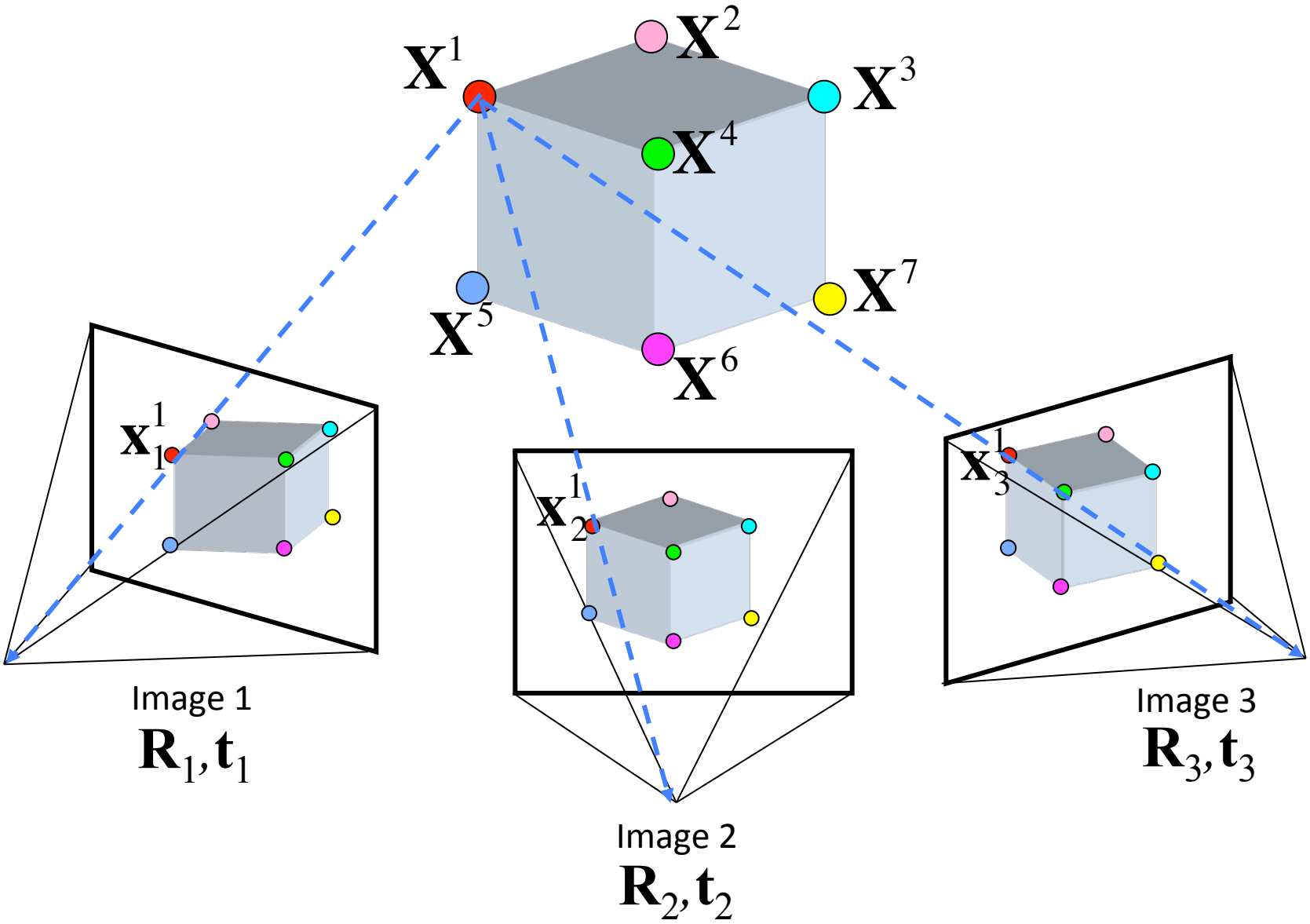


Image-based Street-side City Modeling. Xiao et al, 2009.

Texture Stitching



Texture Stitching

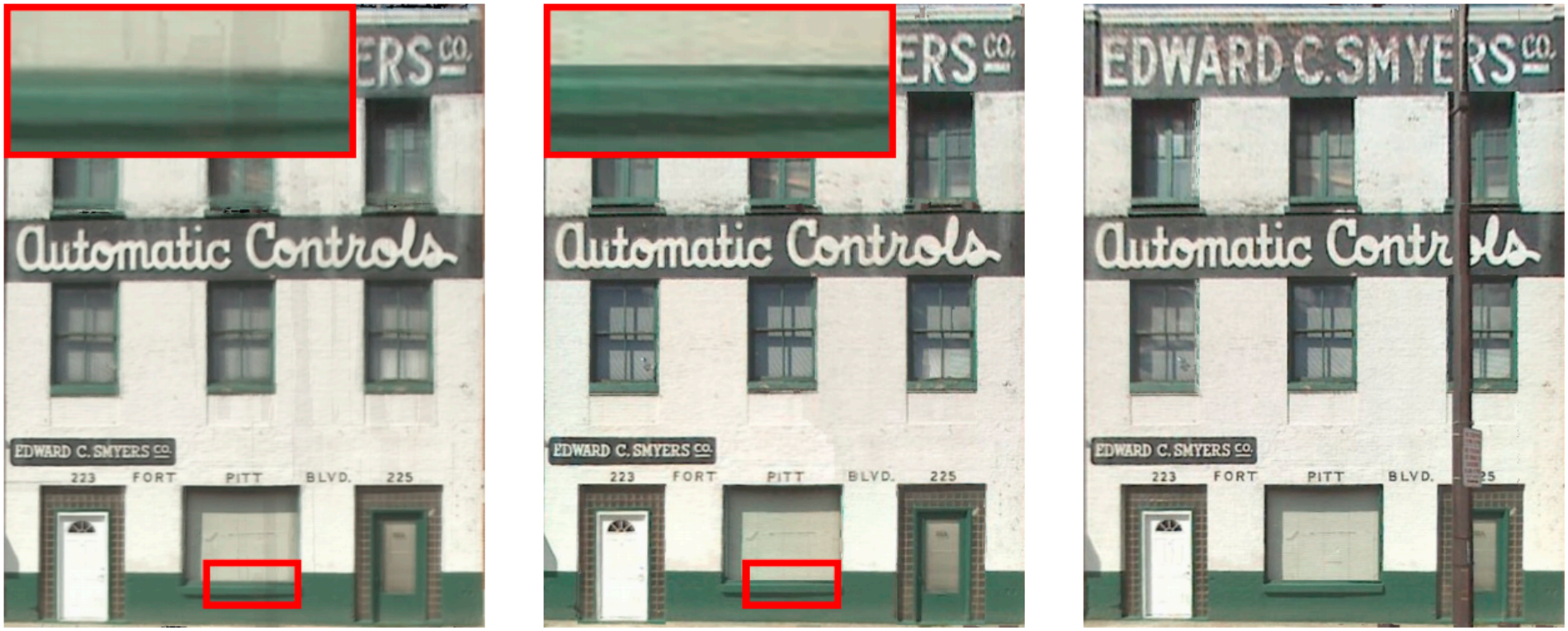
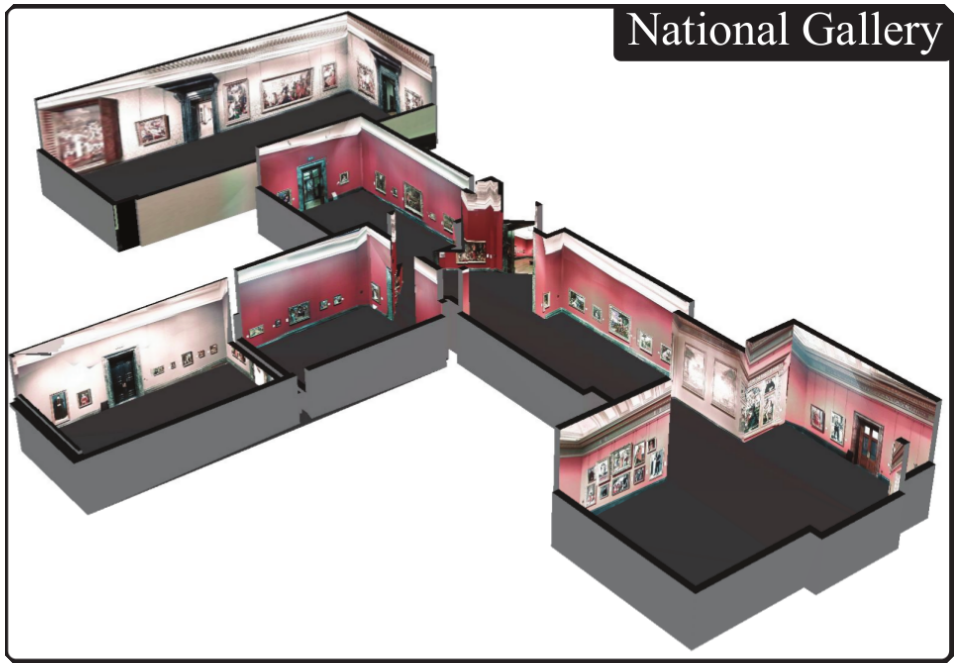


Image-based Street-side City Modeling. Xiao et al, 2009.


The Frick Collection

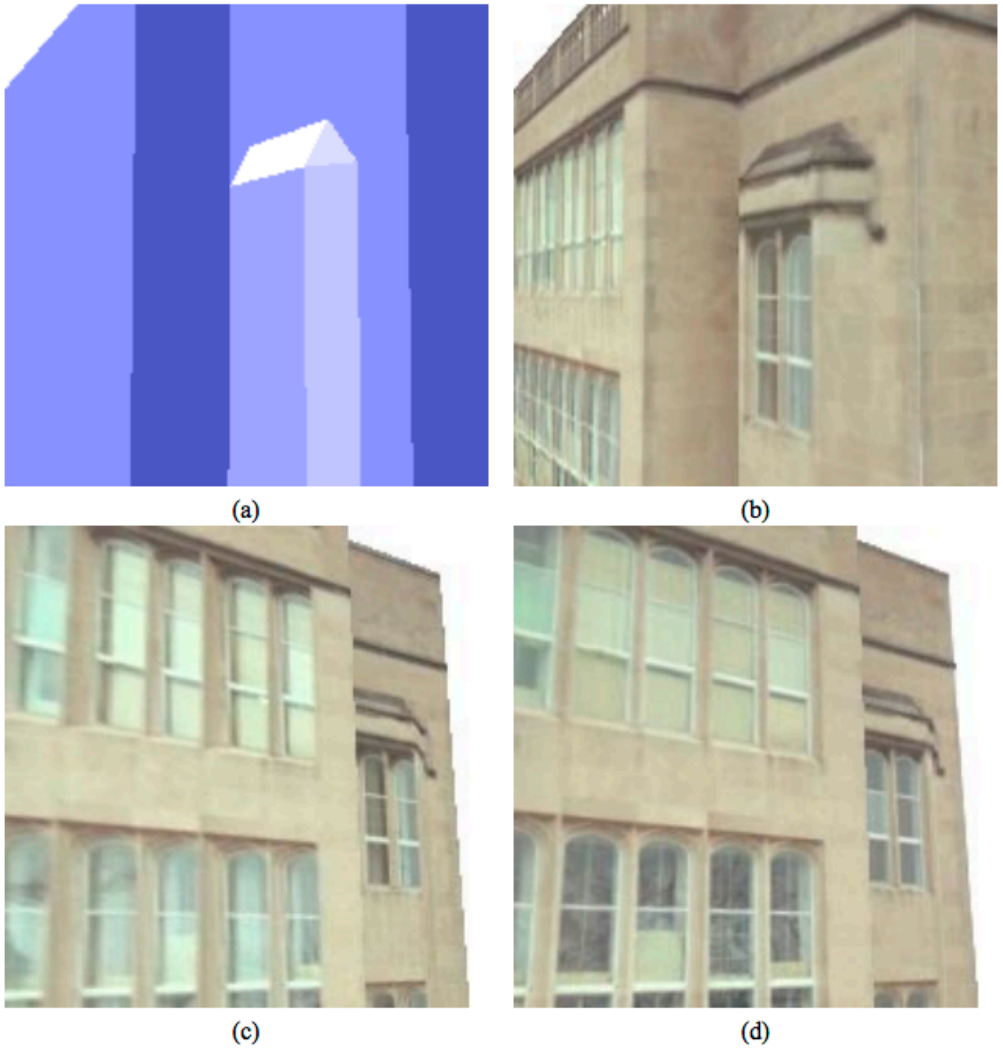

National Gallery


The State Tretyakov Gallery

The State Hermitage Museum

Uffizi Gallery

View-Dependent Texture Mapping



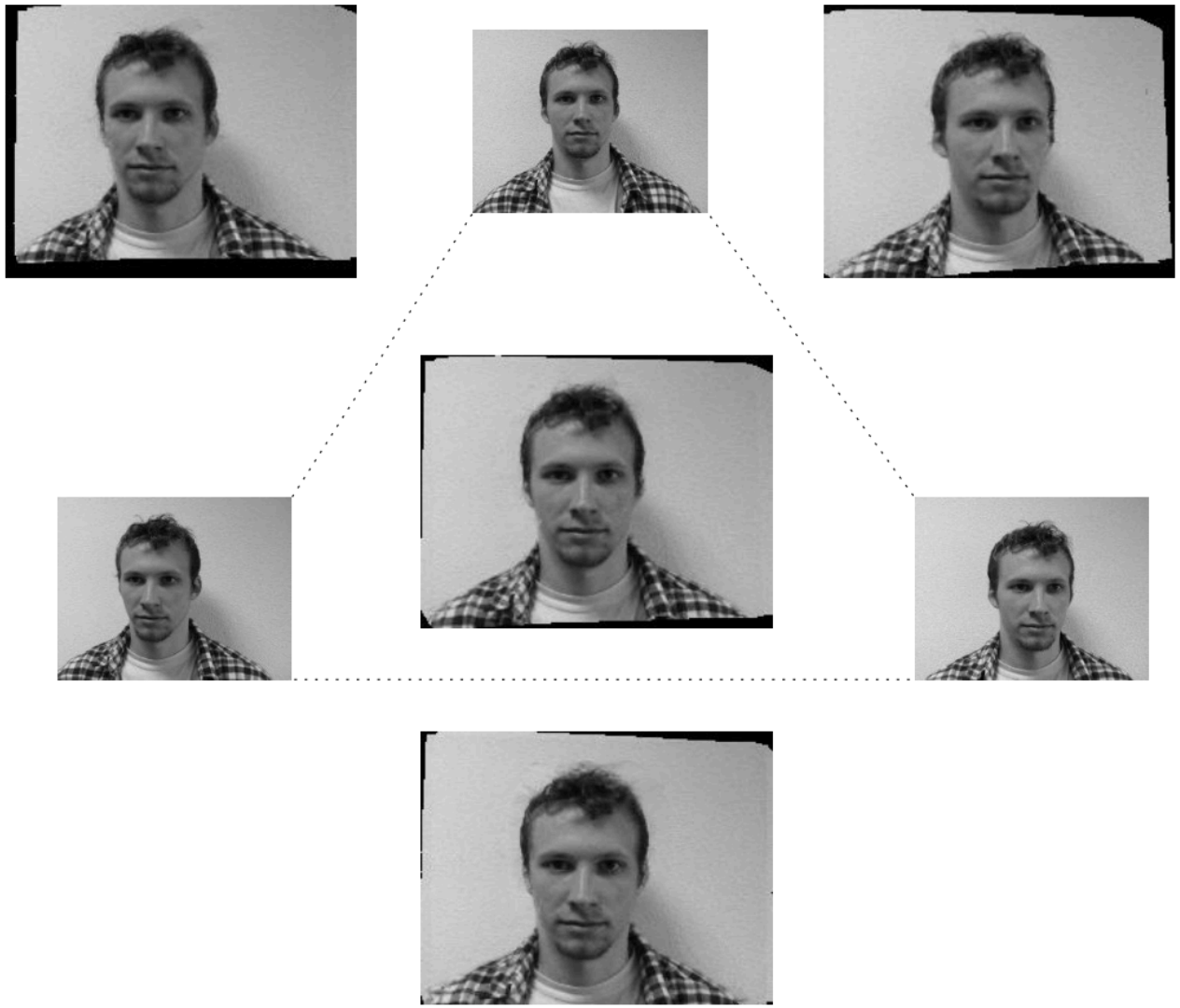
Modeling and Rendering Architecture from Photographs,
Paul Debevec, PhD Thesis, UC Berkeley, 1996

View Interpolation

Match Propagation from Image-based Modeling and Rendering
M Lhuillier and L Quan, 2012

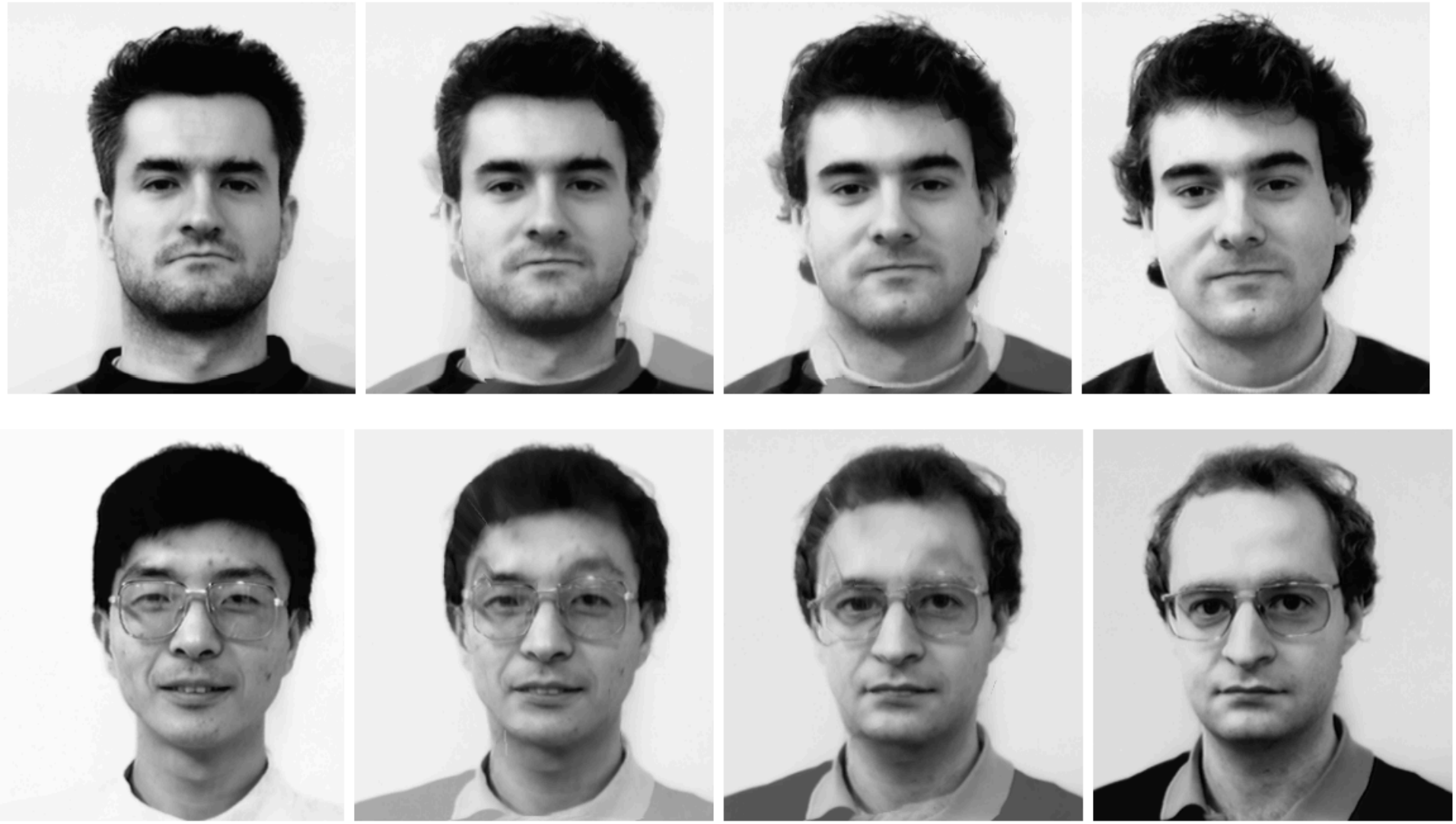
View Interpolation

View Interpolation



Match Propagation from Image-based Modeling and Rendering, M Lhuillier and L Quan, 2012

View Interpolation

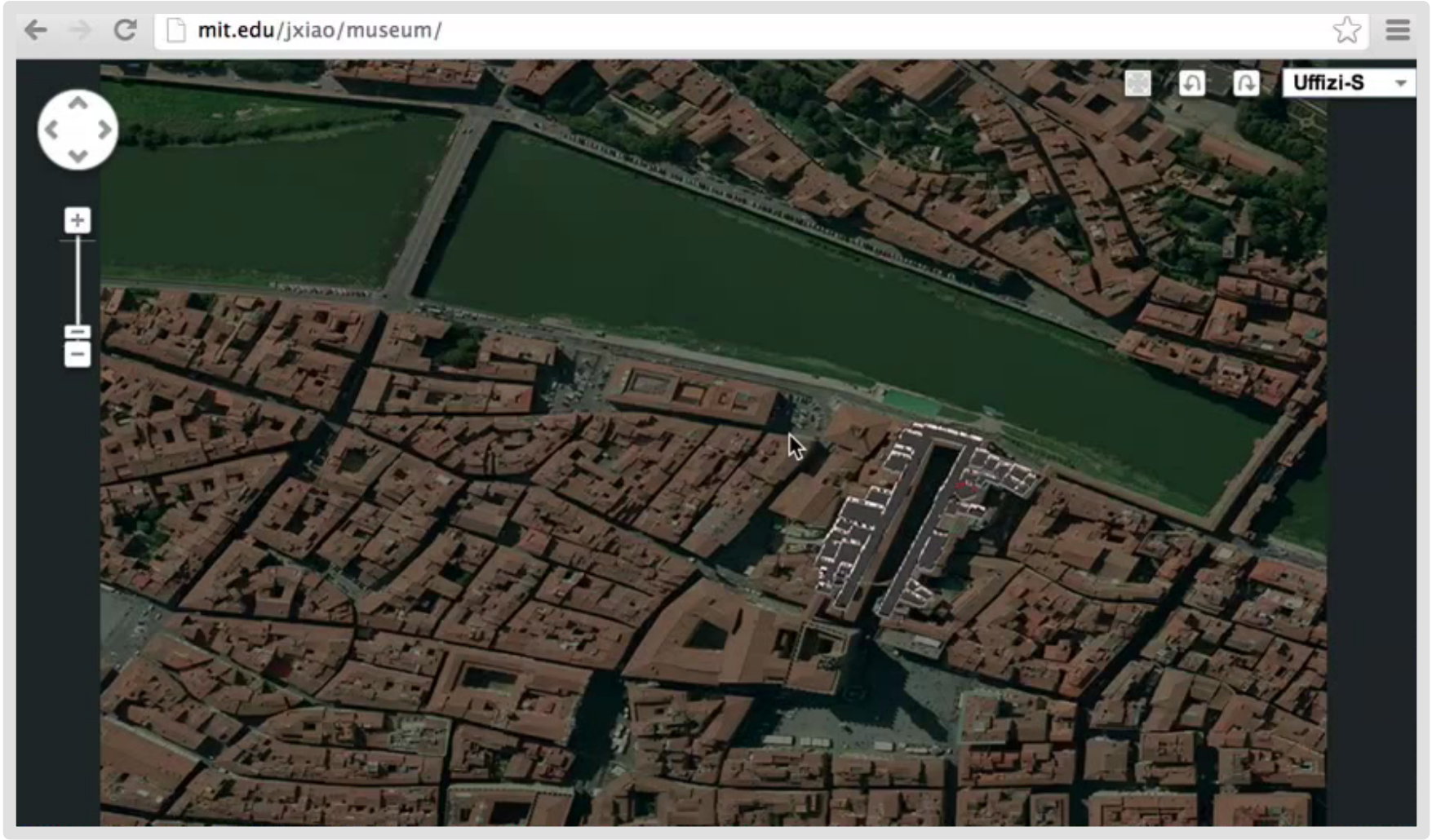


Match Propagation from Image-based Modeling and Rendering, M Lhuillier and L Quan, 2012

Interactive Visualization

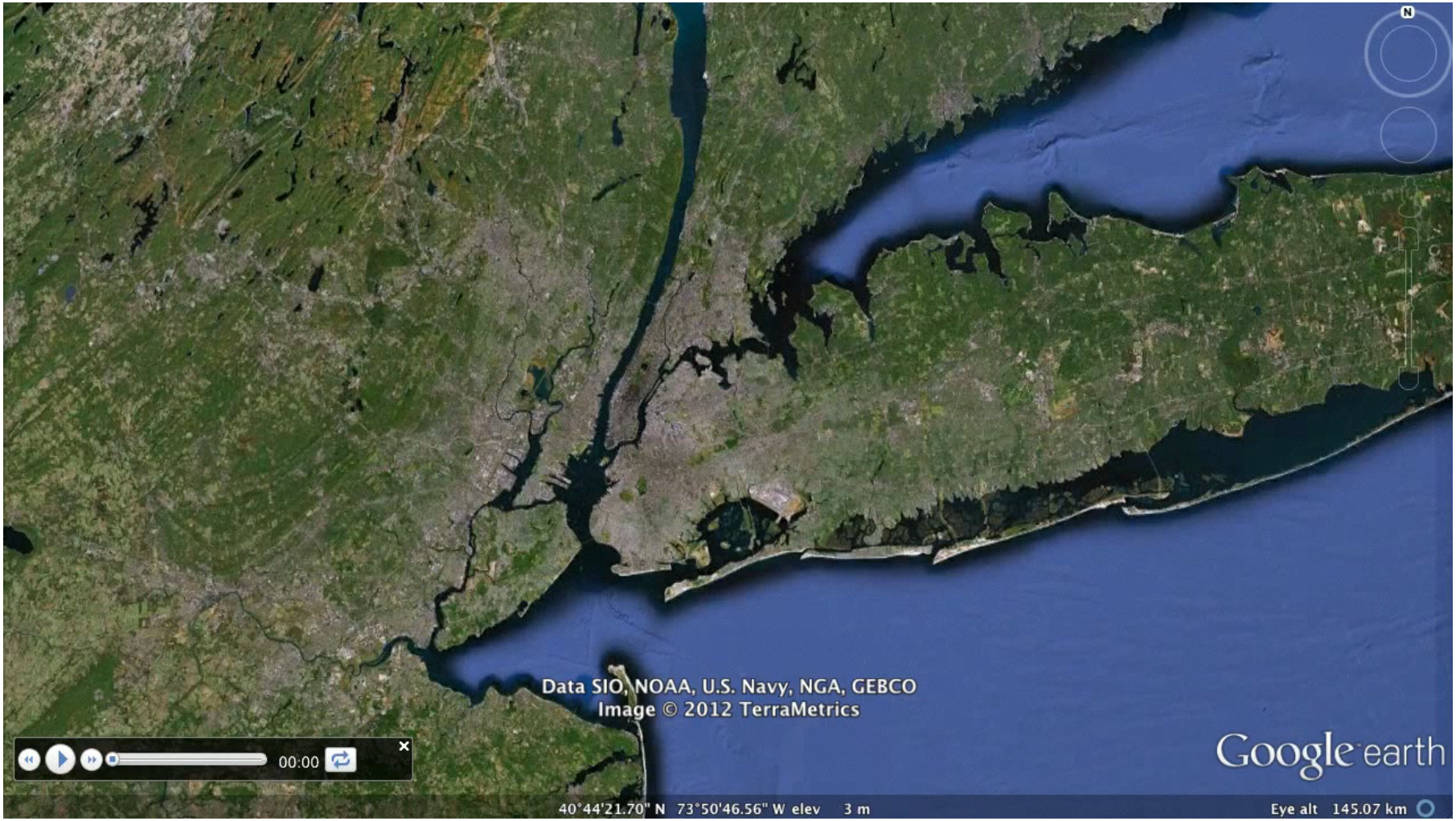
Reconstructing Building Interiors from Images, Y Furukawa, B Curless, SM Seitz, R Szeliski, 2009
Download the Viewer from: http://grail.cs.washington.edu/projects/interior/

Bird’s-eye View Indoor Maps



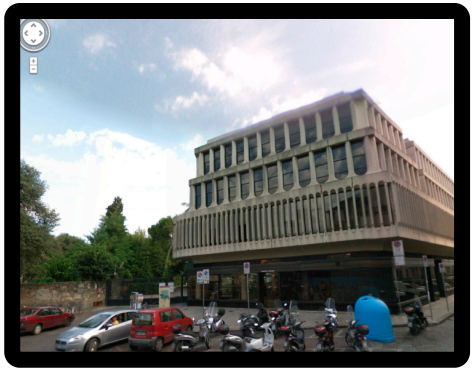
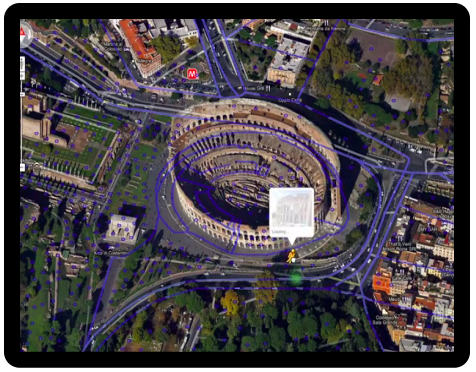
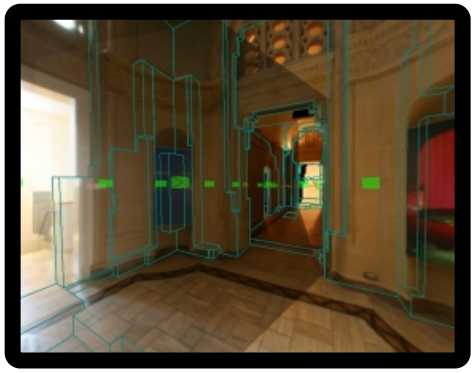
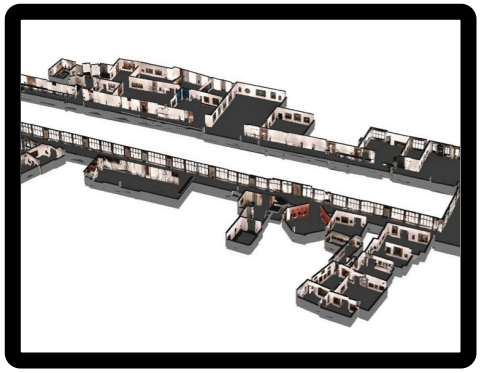
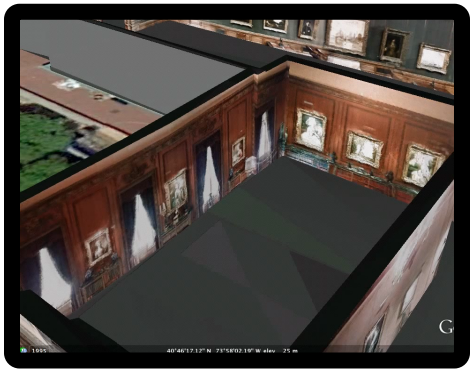
Reconstructing the World's Museums. J. Xiao and Y. Furukawa, 2012. http://mit.edu/jxiao/museum/

Aerial+Ground Visualization



Reconstructing the World's Museums. J. Xiao and Y. Furukawa, 2012. http://mit.edu/jxiao/museum/

Ground vs. Aerial → Ground + Aerial

	Ground	Aerial	Ground+Aerial
Outdoor	 Google Streetview	 Google/Bing/NASA …	 Google MapsGL
Indoor	 Furukawa et al.	 Xiao et al.	 Xiao et al.

Finish our Journey!

	Images → Points:	Structure from Motion
	Points → More points:	Multiple View Stereo
	Points → Meshes:	Model Fitting
+	Meshes → Models:	Texture Mapping
=	Images → Models:	Image-based Modeling

History of 3D Reconstruction (an 1-min over-simplified view)

50 years of 3D reconstruction

1960s Problem Definition

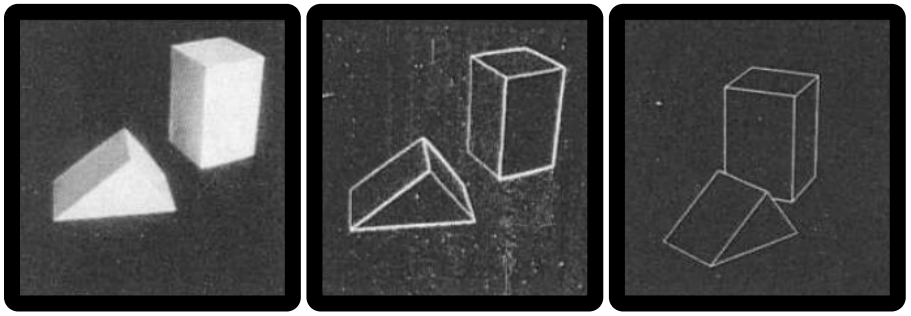
1970s Image Formulation

1980s

1990s Geometry

2000s Reconstruction

2010s Visualization



Input Gradient Output

Machine Perception of Three-Dimensional Solids, Roberts, PhD Thesis, MIT, 1963.

50 years of 3D reconstruction

1960s Problem Definition

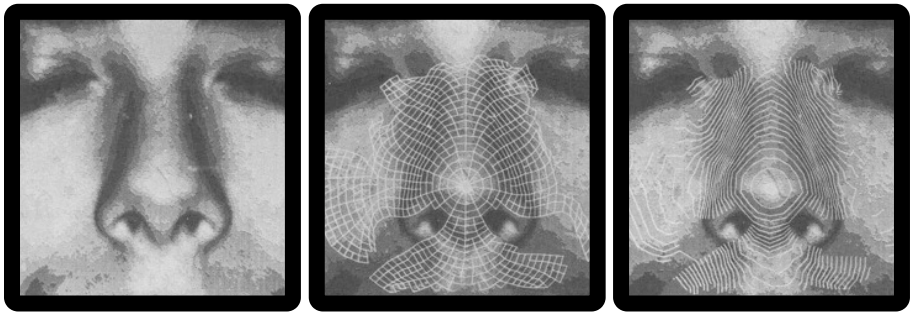
1970s Image Formulation

1980s

1990s Geometry

2000s Reconstruction

2010s Visualization



Shape from Shading,
Horn, MIT AI Memos 232, 1970.

50 years of 3D reconstruction

1960s Problem Definition

1970s Image Formulation

1980s

1990s Geometry

2000s Reconstruction

2010s Visualization

Essential Matrix

$$X'_\mu Q_{\mu\nu} X_\nu = 0 \quad (11)$$

Dividing equation (11) by $X'_3 X_3$ we arrive at the desired relationship between the image coordinates:

$$x'_\mu Q_{\mu\nu} x_\nu = 0 \quad (12)$$

A computer algorithm for reconstructing a scene from two projections, Longuet-Higgins, Nature, 1981.

50 years of 3D reconstruction

1960s Problem Definition

1970s Image Formulation

1980s

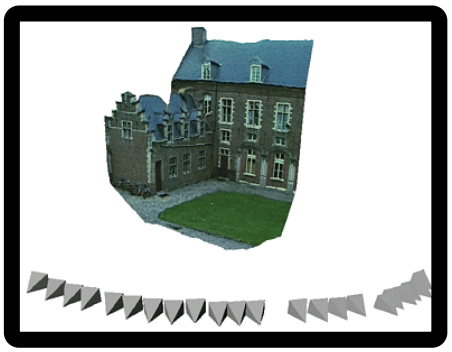
Geometry

1990s

2000s Reconstruction

2010s Visualization

Structure from Motion



Pollefeys et al

Multi-view Stereo

Furukawa & Ponce

3D Model Fitting

Xiao et al

50 years of 3D reconstruction

1960s Problem Definition

1970s Image Formulation

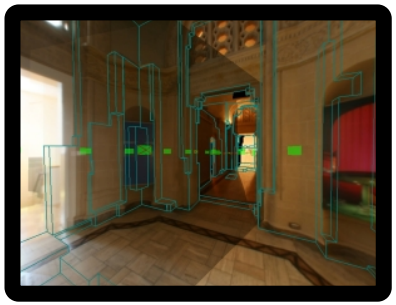
1980s

1990s Geometry

2000s Reconstruction

2010s Visualization

Snavely et al



Furukawa et al

Xiao et al

50 years of 3D reconstruction

1960s Problem Definition

1970s Image Formulation

1980s Geometry

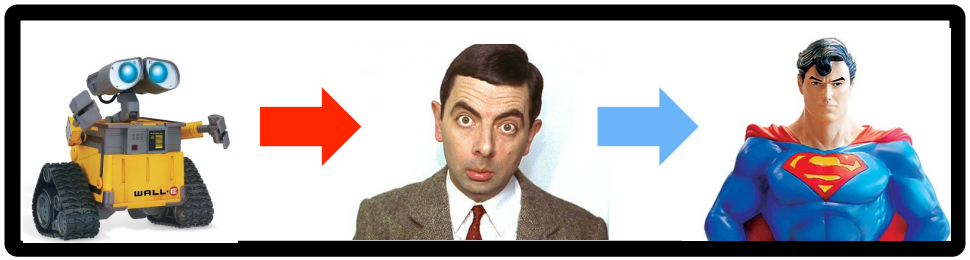
1990s

2000s Reconstruction

2010s Visualization

Goals of Computer Vision:

- Let machines see
- Let humans see better



Longer Summary of the History

Research Landmarks for 3D Reconstruction

Steve Seitz

"History of 3D Computer Vision"

NSF Frontiers in Computer Vision, 2011

http://www.youtube.com/watch?v=kyIzMr917Rc

http://homes.cs.washington.edu/~seitz/talks/3Dhistory.pdf

Homework: Play with SFMedu

- Run the SFMedu program
- Debug the code step by step in Matlab
- Link the code and the theory we cover in this lecture
- Try some other images
- Locate the unstable part of the algorithm

Final Project Ideas: Improve SFMedu

- 5-point algorithm
- Estimate focal length on the fly
- Beyond pairwise neighboring matching
- Tri-focal tensor
- Make it stable for different images input
- Make it work for a video input
- Make it work faster
- Check all the theories and fix all bugs ☺

Q & A