Modeling a Crossover Network with WDF

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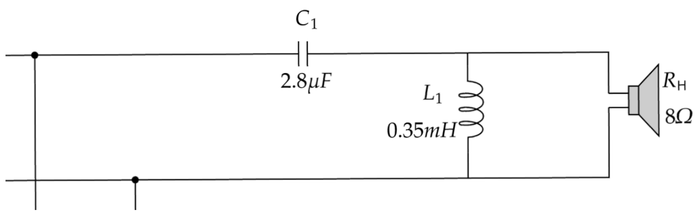
1 introduction

In this Homework we modeled a simple crossover circuit using Wave Digital Filters, that allows us to compute the output signals in a straightforward way without explicitly calculate any transfer Function.

2 Circuit and WDF Schematics

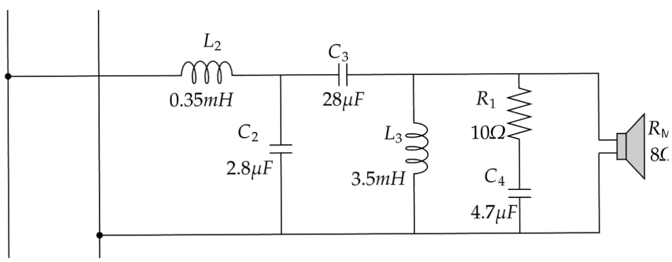
As advised in the Homework presentation we chose to split the circuit in 3 stages with same input. Note : In the WDF blocks Sn stands for 'Serial' junction while Pn stands for 'Parallel' Junction while Cn Ln and Rn are the circuit components

High Band Circuit



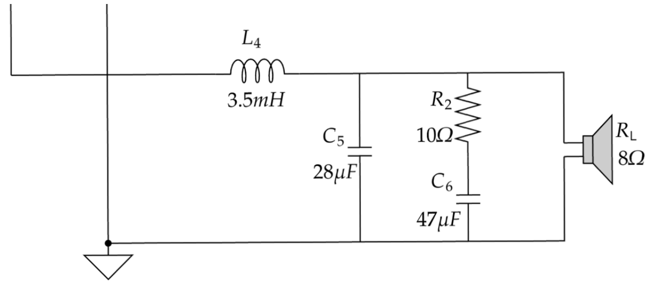

```
        -->IN
        a1    b1
       +---------+        +---------+
    a2 |       * | a3=b4  |         |  a5
C1     |   S1    |        |   P2    |      L2
   *b2 |        *| b3=a4  |         | *b5
       +---------+        +---------+
                            a6    *b6
                               Rh -->OUT
```

Mid Band Circuit




```
         --> IN                             C3                                                         R1
        b1      a1                      a8     *b8                                                 a17    *b17
       +----------+      +----------+     +----------+      +----------+      +----------+      +----------+
    a2 |          | a3=b4 |          |a6=b7 |          |a9=b10 |          |a12=b13|          |a15=b16|          | a18
L2     |    S1    |       |    P2    |      |    S3    |       |    P4    |       |    P5    |       |    S6    |       C4
   *b2 |         *| b3=a4 |         *|b6=a7 |         *|b9=a10 |         *|b12=a13|         *|b15=a16|          | *b18
       +----------+      +----------+     +----------+      +----------+      +----------+      +----------+
                           a5     *b5                        a11    *b11       a14  *b14
                              C2                                L3                Rm -->OUT
```

Low Band Circuit




```
         -->IN                                 R1-->OUT
         a1     b1                          a8    *b8
        +---------+       +---------+      +---------+       +---------+
    a2  |       * | a3=b4 |         |a6=b7 |         |a9=b10 |         |  a12
L4      |   S1    |       |   P2    |      |   P3    |       |   S4    |       R2
   *b2  |        *| b3=a4 |        *|b6=a7 |        *|b9=a10 |         | *b12
        +---------+       +---------+      +---------+*      +---------+
                          a5    *b5                          a11   *b11
                              C5                                 C6
```

3 WDF Implementation

As shown in the previous diagram we implemented several 3 ports junctions and modeled the circuit by their interconnections. To compute the Scattering matrices for each ports we used the following formulas

```
    I = eye(3);
    one = ones(3, 1);
    Z_Sh1 = [Z_h(1); Z_h(2); Z_h(3)];  % One of which follows adaptation condition
( SERIAL JUNCTION )
    K_Sh1 = 2 / sum(Z_Sh1);
    S_h1 = I - K_Sh1 * Z_Sh1 * one';

( PARALLEL JUNCTION )
    K_Ph1 = 2 / sum(Z_Ph1);
    P_h2 = K_Ph2 * G_Ph2 * one' - I;
```

The signal is propagated through the ports by Forward and Backward Scan in witch at each step the corresponding port reflected wave $bn[k]$ is calculated with a matrix multiplication and propagated to the next port.

```
( FORWARD SCAN  P_h2 -> S_h1 )
    b_h(4:6) = P_h2 * a_h(4:6);
    a_h(3) = b_h(4);
```

The Input is introduced on local root scattering

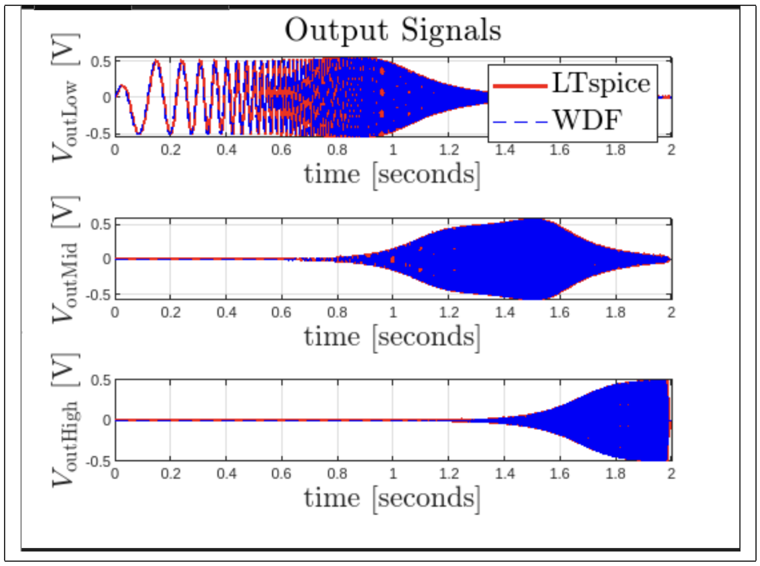
```
    a_h(1) = 2 * Vin(ii) - b_h(1);
```

The Output is calculated at the end by conversion from Wave domain to Kirchoff Domain

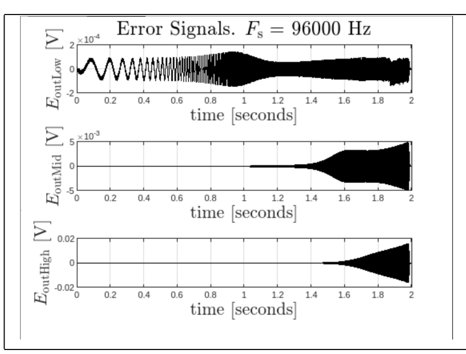
```
    VoutHigh(ii) = -(a_h(6) + b_h(6))/2;
```

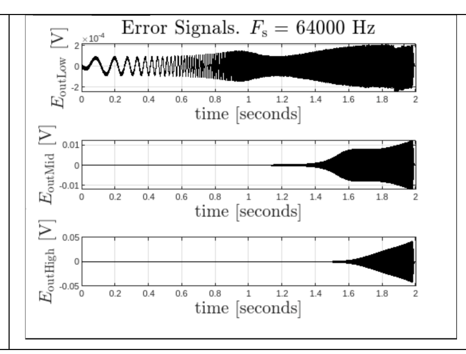
4 Results

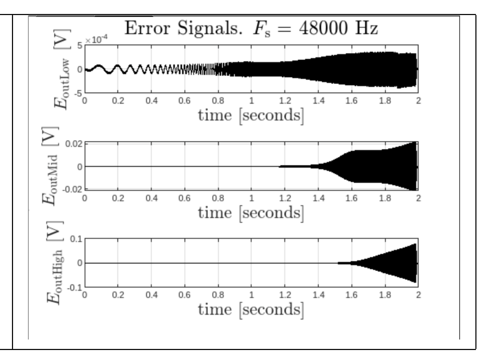
Now we show the output plots of an incoming Sinusoidal Frequency Swiped Signal to test if the crossover network is faithfully splitting in frequency sub-bands. The WDF out model is shown in 'red' color while the expected behaviour (ground-truth) is shown in 'blue'



We now provide the errors plots for 3 different sampling frequencies







5 Questions and Answers

5.1 Answer 1

In general (for each sampling frequencies) from the plots we can see how the global trend of the error signal across the frequencies is very different for each sub-band circuit.

in First we notice how the overall magnitude of the error is smaller for the lower frequency splitted Bands and higher for the highest frequency Band.

For instance let's examine the case with Fs = 96kHz In the Low-Band the error stays around the 10^-4 order and keeps a similar trend over the frequencies. For the Mid-Band signal the error actually grows to higher orders (10^-3) at high frequencies, that's also the case for the High-Band error that stays in the (10^-2) range.

In each case the nature of the error is oscillating, with same frequency and phase of the input, that suggests the model has less accuracy in the high level input range, most likely due to the absence of nonlinearities of the circuit components (that have been modeled as ideal and linear).

Moreover for high frequency we saw we have more error, we can figure out the reason we think to the discrete-time to continous-time domain transformation through the so called 'Trapezoidal Rule' that is at the base of WDF theory. In particular the phenomena of of frequency warping is involved that originates from the tangent based relation in the continuous to discrete frequency mapping in the formula

$$\omega_c = \frac{2}{T_s} \tan(\tilde{\omega}\frac{T_s}{2}) \tag{1}$$

So we expect to have warping distortion effects at high frequencies in digital domain that produces deviations from the Analog case.

5.2 Answer 2

Increasing the sampling frequency has a positive impact on the accuracy of the results in the three bands. This improvement can be observed by considering equation (1), where increasing the sampling frequency (Fs = 1/Ts) results in a smaller Ts. That reduces the warping effects and expands the range in which ω_c and $\tilde{\omega}$ are more similar.

5.3 Answer 3

If a single diode is added in parallel with the tweeter resistor RH in the reference circuit, the computability of the WD structure is affected. The addition of the diode introduces two elements that cannot be adapted, namely the ideal generator and the diode itself. Consequently, the WDF (Wave Digital Filter) structure would have two roots. In order to solve this modified circuit, iterative methods would be required since the two elements cannot be combined into a single root.

In the scenario where the ideal voltage source is replaced with a resistive voltage source, the WDF implementation is further influenced. By using a resistive generator that can be adapted, we can solve the circuit by considering the diode (which remains as the sole element that cannot be adapted) as the root, rather than the generator. This alteration allows for a different approach in solving the WDF implementation with both the diode and the resistive voltage source.

5.4 Answer 4

We have map continuos Laplace variables in Z domain using the backward Euler method:

$$s = j\omega \leftrightarrow \frac{1 - z^{-1}}{T_s} \tag{2}$$

The Inductor characteristic in the Laplace Domain

$$V(s) = sLI(s) \tag{3}$$

We substitute s (notice z^-1 is a delay block)

$$V[k] = \frac{L}{T_s}I[k] - \frac{L}{T_s}I[k-1] \tag{4}$$

We now want to figure out the adapted scattering relation, we first consider the voltage current relation

$$v[k] = R_e[k]i[k] - V_e \tag{5}$$

in our case we have in the constants

$$R_e[k] = L/T_s and V_e[k] = L/T_S i[k-1] \tag{6}$$

by substituting equation 5 in kirchoff to wave conversion relation :

$$v[k] = \frac{a[k] + b[k]}{2}, i[k] = \frac{a[k] - b[k]}{2Z[k]} \tag{7}$$

we obtain the scattering relation

$$b[k] = a[k]\frac{R_e - Z[k]}{R_e + Z[k]} - V_e\frac{2Z[k]}{R_e[k] + Z[k]} \tag{8}$$

that can be adapted if the WDF component free parameter is

$$Z[k] = R_e[k] = L/T_s \tag{9}$$

that eliminates the relation from $b[k]$ and $a[k]$, the reflected wave will be

$$b[k] = V_e[k] = R_e[k]i[k-1] = Z[k]\frac{a[k-1] - b[k-1]}{2Z[k-1] = \frac{a[k-1]-b[k-1]}{2}} \tag{10}$$

Inductor Equation 1	**Reflected Wave when adapted**	**Adaptation condition**
$V(s) = L\frac{di(t)}{dt}$	$b[k] = \frac{a[k-1]-b[k-1]}{2}$	$Z[k] = \frac{L}{T_s}$