

Generative Models

First line of research consist in studying **generative models** for shape optimization of complex geometries with a large number of parameters; the objective is to learn the shape of the geometry using a minor number of parameters, for example for modeling naval hulls, and creating new artificial geometries similar to real data, as creating new real geometries can be costly.

Optimal transport

In the second line **semidiscrete optimal transport** is used to obtain an optimal transport map to find some intermediate geometries with some regularity constraints. An application is design optimization, in which the quantity of material for similar models must be the same. The extension of semidiscrete optimal transport to three meshes will also be discussed.

2 - Optimal Transport

Given Ω a Borel set and two measures μ and ν on Ω such that $\mu(\Omega) = \nu(\Omega)$, c a convex function $T : \Omega \rightarrow \Omega$ such that

$$\begin{cases} \nu(X) = \mu(T^{-1}(X)) & \text{for any Borel (i.e. measurable) subset } X \text{ of } \Omega \\ \int_{\Omega} c(x, T(x)) d\mu & \text{is minimal} \end{cases}$$

is called the **optimal transport** map from μ to ν . We are interested in μ continuous and ν discrete. In this case we talk of Semi Discrete Optimal Transport.

We can approximate the optimal transport map with the following algorithm:

Data: Two tetrahedral meshes M and M' , and k the desired number of vertices in the result

Result: A tetrahedral mesh G with k vertices and a pair of points p_i^0 and p_i^1 attached to each vertex. Transport is parameterized by time $t \in [0, 1]$ with $p_i(t) = (1 - t)p_i^0 + tp_i^1$.

(1) Sample M' with a set Y of k points

(2) Compute the weight vector W that realizes the optimal transport between M and Y

(3) Construct $E = Del(Y)$ where Del is the Delaunay Triangulation.

(4) For each $i \in [1 \dots k]$, $(p_i)^0 \leftarrow \text{centroid}(\text{Pow}_W(y_i) \cap M)$, $(p_i)^1 \leftarrow y_i$

G will be the mesh defined by the topology of E with the pair of points $(p_i)^0, (p_i)^1$.

It can be proven that the linear map does not clash particles. This algorithm is implemented in the library **Geogram** created by Bruno Levy.

We modified the map to be

$$\phi_{MM'}(t) = Vol(M)^{\frac{1}{3}} \frac{tM' + (1-t)M}{Vol(tM' + (1-t)M)^{\frac{1}{3}}}$$

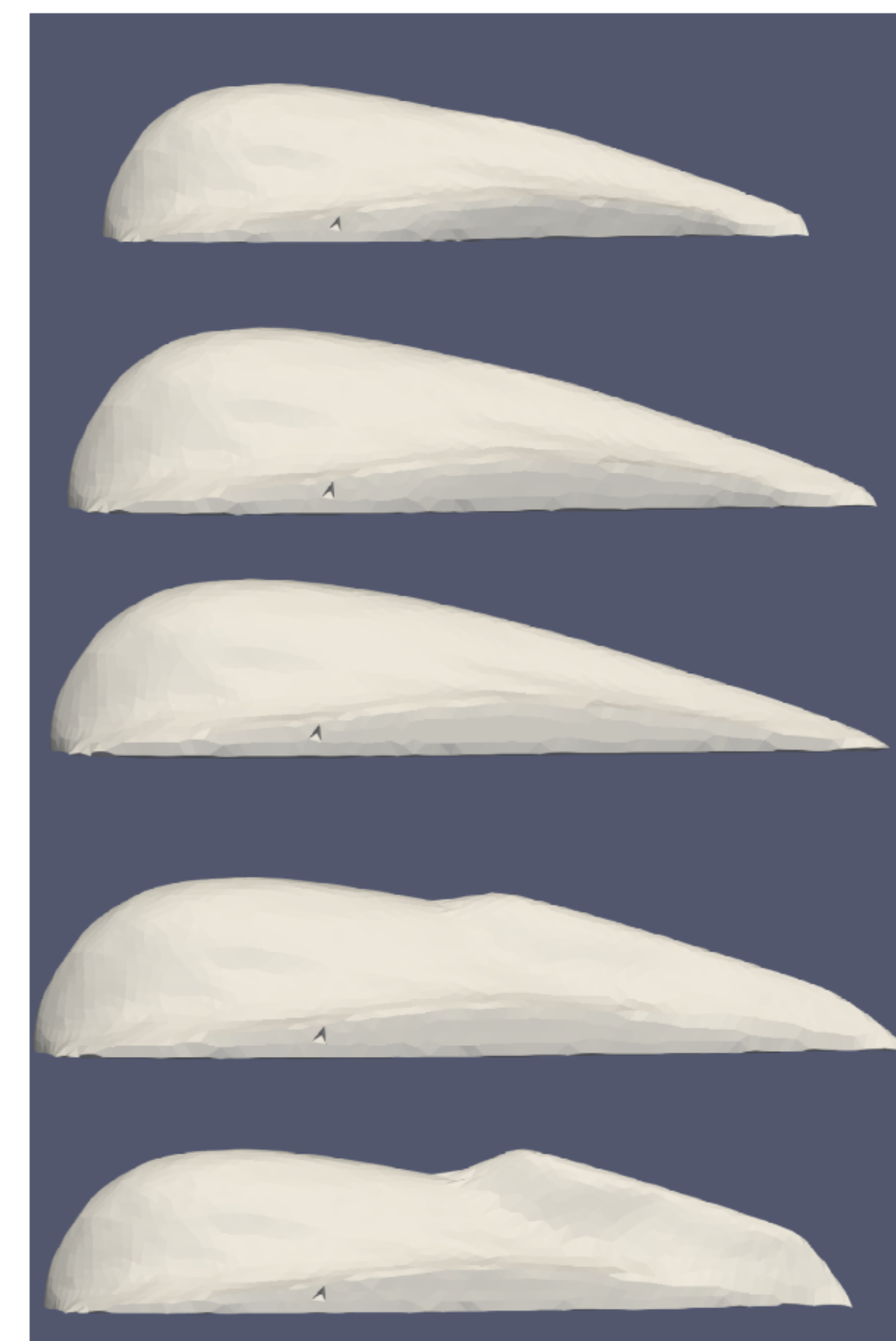
in order to preserve volume in intermediate times.

Let M, M', M'' three meshes. Let us suppose we calculate the OTM from M to M' and from M' selecting M'' to be sampled in the same way in both transports, so the resulting mesh will be the same (because in the algorithm does not depend on the other). So with the same assumption as before we can define a new transport map

$$\phi_{MM'M^{(2)}}(t) = \begin{cases} Vol(M')^{\frac{1}{3}} \frac{2tM' + (1-2t)M}{Vol(2tM' + (1-2t)M)^{\frac{1}{3}}} & 0 \leq t \leq \frac{1}{2} \\ Vol(M')^{\frac{1}{3}} \frac{(2t-1)M'' + (2-2t)M'}{Vol((2t-1)M'' + (2-2t)M')^{\frac{1}{3}}} & \frac{1}{2} \leq t \leq 1 \end{cases}$$

that goes from M to M' and then from M' to M'' .

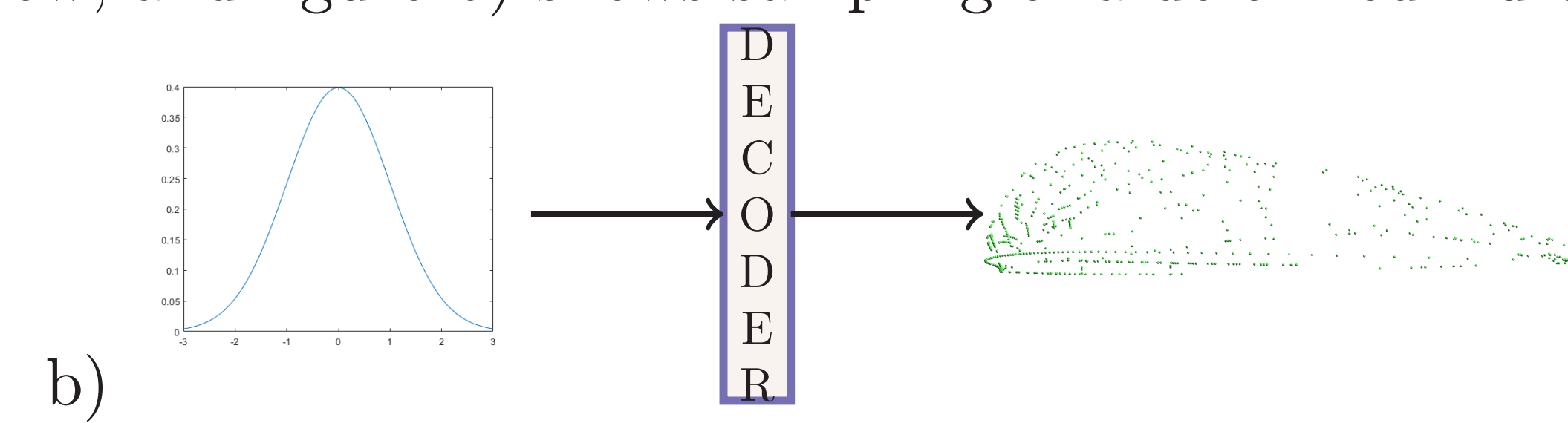
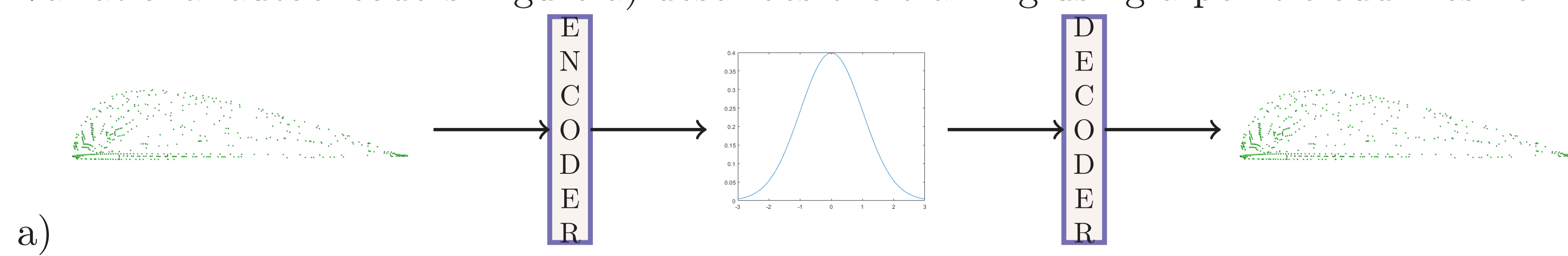
Here is shown an example of three mesh OTM of a Bulbous bow, the three meshes are images in odd position, while in even position we have the intermediate meshes:



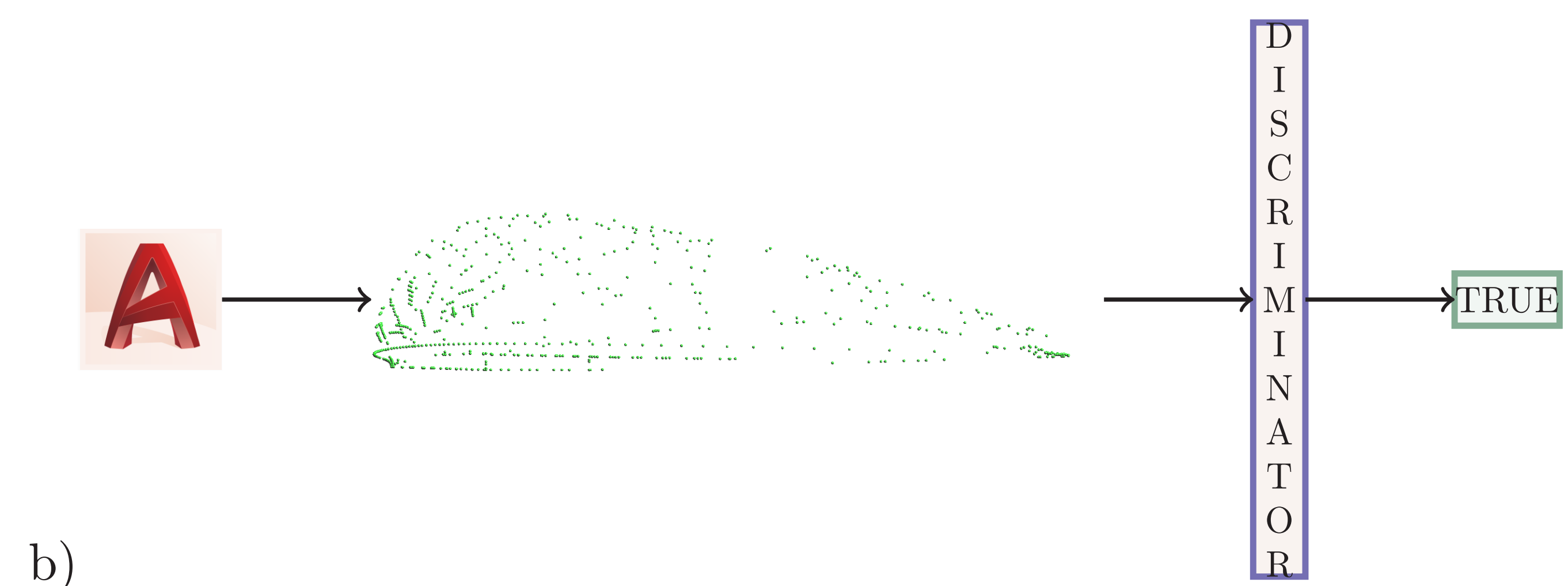
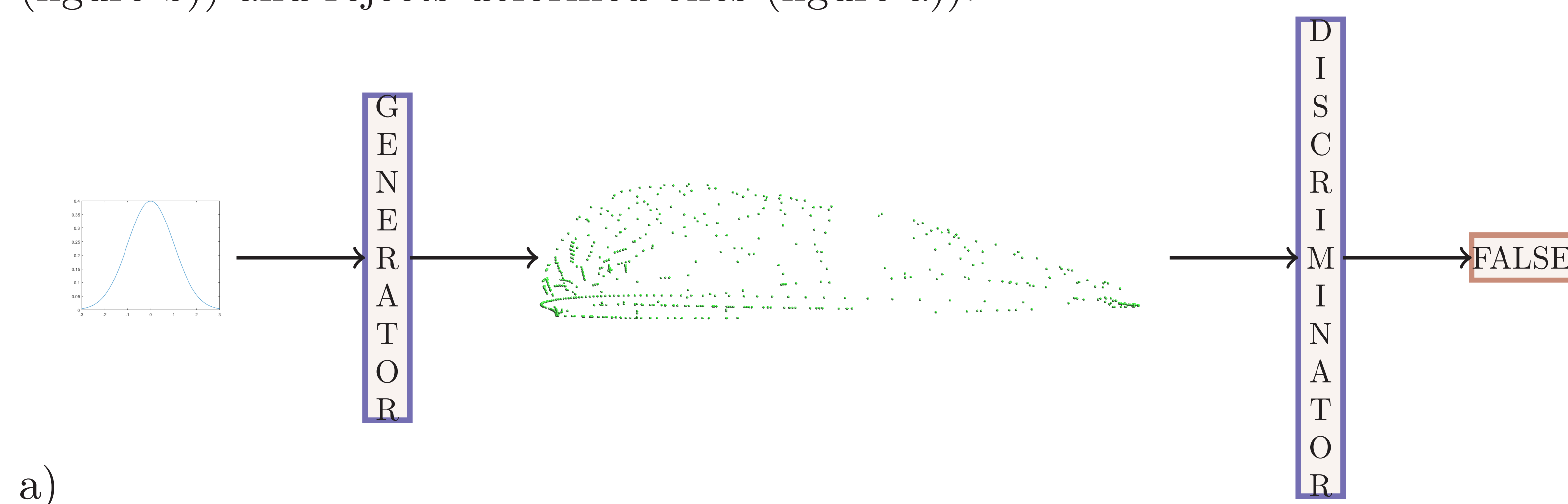
4 - Generative Models

Two main model classes:

- Variational autoencoders: figure a) describes the training using a point cloud mesh of Bulbous bow, and figure b) shows sampling of a deformed Bulbous



- Generative adversarial network: it is characterized by a generator that samples point cloud mesh of deformed Bulbous bow and by a discriminator that accepts real Bulbous (figure b)) and rejects deformed ones (figure a)).



5 - Preliminary results and future work

Summary of our results:

- We are able to do volume preserving continuous deformation between three bulbous bows meshes using semidiscrete optimal transport. Our next step is to generalize it for more than three meshes, which is nontrivial (as M and M' are nonsampled).
- We are able to sample bulbous bows meshes using Variational Autoencoders. However, the space Z is too much sparse, so we started studying Generative Adversarial Networks.

Bibliography and Software References

- Geogram, <http://alice.loria.fr/software/geogram/doc/html/index.html>
- A numerical algorithm for L_2 semi-discrete optimal transport in 3D, Bruno Levy, arXiv, 2014
- Variational Autoencoders for Deforming 3D Mesh Models, Qingyang Tan, arXiv, 2018