

Disperazione

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Theorem 1. *Let $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ differentiable such that $Vol(\phi(B)) = Vol(B) \forall B \subseteq \mathbb{R}^3$. Then $|Det(J_\phi)| = 1$.*

Proof. $Vol(\phi(B)) = \int_{\phi(B)} dx = \int_B |Det(J_\phi)| dx$

Also $Vol(B) = \int_B dx$

So we get $\int_B dx = \int_B |Det(J_\phi)| dx \forall B$, which means $|Det(J_\phi)| = 1 \forall x$. \square

Theorem 2. *Let $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ and C^1 such that $Vol(\phi(B)) = Vol(B) \forall B \subseteq \mathbb{R}^3$. Then $\phi(x) = Ax + b$ with $det(A) = 1$.*