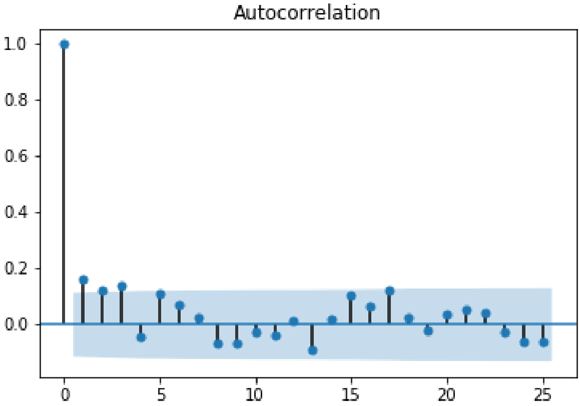
HMM vs headache

April 30, 2022

Short description of the dataset and the problem

The dataset contains a binary sequence of size 296. Each element has value 1 if in that they I had some headache and 0 if I had not.

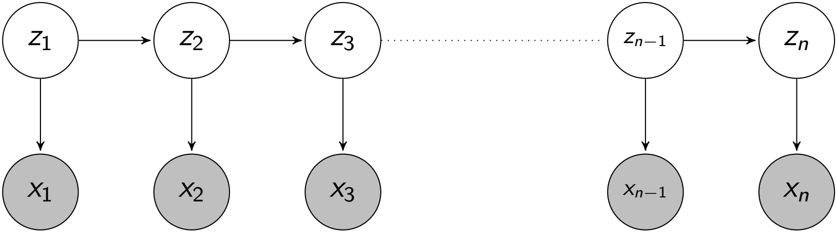
Autocorrelation



First three lags are statistical significant, so it makes sense to model the data as a time series. We decide to use short memory HMM models.

General model stucture

Z_t Markov process with memory parametrized by θ
$X_t|Z_t = z_t \sim Ber(g(z_t))$ where $g : A \subseteq \mathbb{R} \to [0,1]$ eventually paramatrized by ϕ
Z_t and g characterize the model



Model 1

Wiener Process

Z_t is a Wiener process if:

- $Z_t \sim N(0, t)$
- $Cov(Z_t, Z_s) = min\{t, s\}$

Our first model consists of:

- Z_t Wiener process
- $g(z) = e^{-\phi z^2}$

We can prove that given $S = \{i | x_i = 1\}$

$$p(x) = \sqrt{\frac{1}{1+\phi \sum_{i \in 1 \dots N \setminus S} i}} - \sum_{D \subset S} \sqrt{\frac{1}{1+\phi \sum_{i \in 1 \dots N \setminus D} i}}$$

Howewer the cardinality of the power set is $2^{|S|}$ so it is computationally unfeasible even if we have an analytical form. For this reason it is better to use approximation methods like SVI.

Model 1

For SVI we chosen $q_\theta(x)$ as a $Beta(\alpha, \beta)$. The optimal ϕ is 2.69.