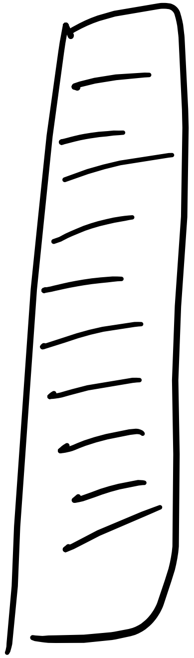
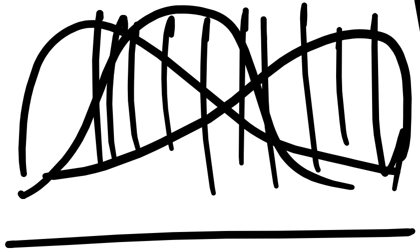
Central Limit Theorem:

Variable:

plot





Samples → S_1 → Mean (S_1)

S_2 → Mean (S_2)

S_2

. . .

S_n

(sample mean)

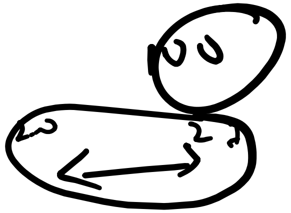
size > 25
Enough samples

9/10

plot the averages:

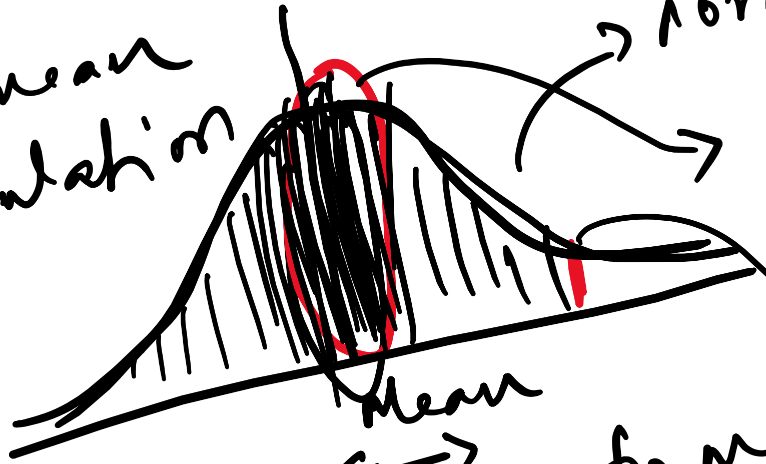
→ normal

2x



Sample mean → pop. mean

Variation → population



Mean

closer to mean

statistic

⇕

pop.

M

Sampling:- Taking a few samples out of population

Sampling error: -> Different results

-> sampling error / variation due to sampling

- 'Always'

Population

↑

Static - sample

Next month -> \$4.5

(\$4.2 - \$4.7)

→ Estimate the population parameter

↳ Good practice to give in a confidence Interval

Confidence Interval : Range ⟷ μ (lie)

What affects the width of C.I ?

1. The variation within the population ✓
2. The sample size ✓

① Variation ⇒ ↓

sample ⇒ ↓

$\boxed{C.I}$ → narrow

② Sample size → CLT simulation.

1000 = pop

N = 100

Heights

3.5

5.8

6.5

S1

S2

S3

S4

S5

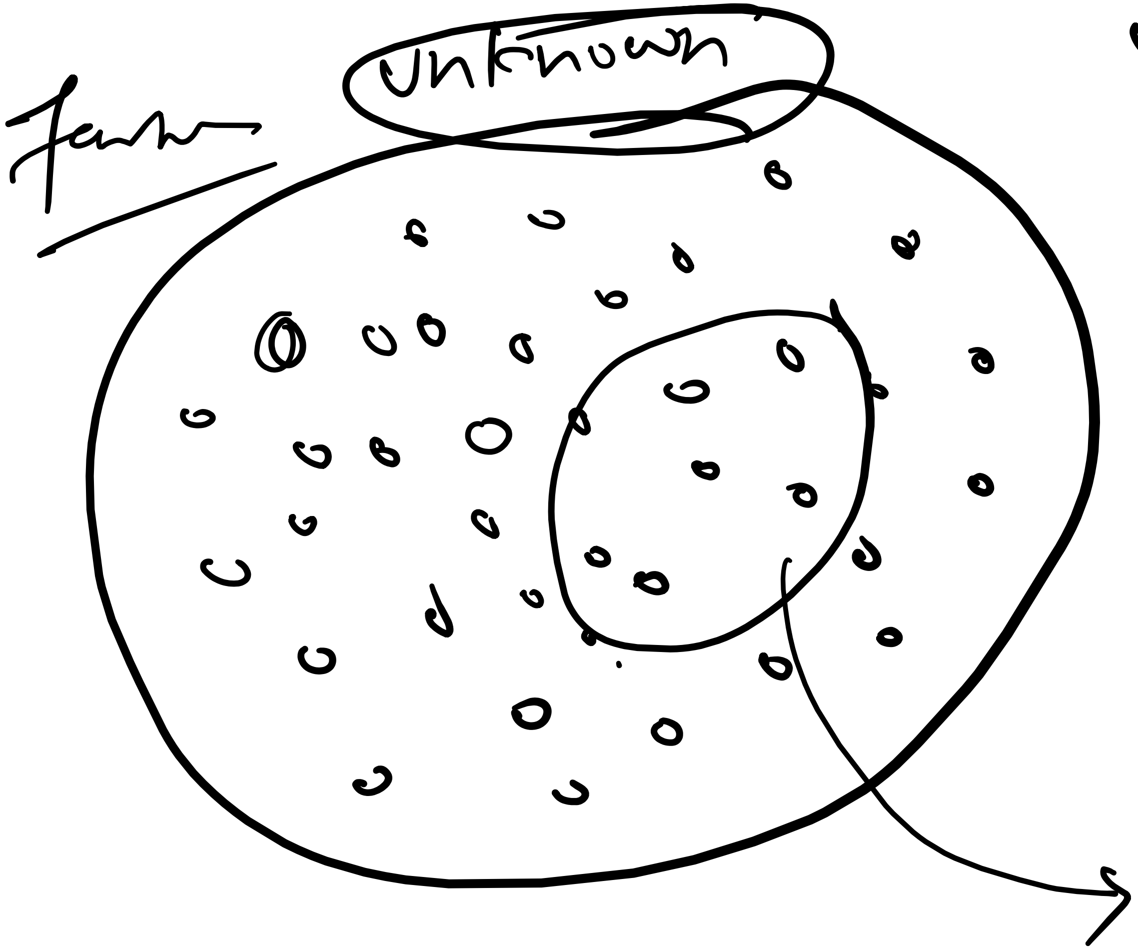
↑ sample

similar

3.5

7.5

Average out



'Size of an apple'

sample variation can be an indicator of my pop var?

variation

Weight of samples

Avg? estimate

from CLT:

C.I

Confidence Interval of a mean = $\boxed{\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)}$

$\bar{x} \rightarrow$ sample mean

$t \rightarrow$ t-value

$s \rightarrow$ Std of sample

$n \rightarrow$ sample size

Level of confidence

sample size number

$n = 15$; $\boxed{\bar{x} = 149.3}$; $\boxed{\sigma = 4.758}$

$\Rightarrow \left(s/\sqrt{n} \right) = 4.758/\sqrt{15} = 1.22$

(std error)

90% → level

df = sample size - 1 = 14

Margin of error = 2.145 * 1.22 ⇒ 2.62

spread

'means to be'

$\bar{x} + 2.62$ & $\bar{x} - 2.62$

Range $\Rightarrow$ $\boxed{146.67}$ to $\boxed{151.9}$

95% confident that the population parameter (μ) lies b/w above range, Misconception:-

(Apple eg) (150 — M — 170)

- The C.I is NOT the interval that will hold the weight of 95% of apples from the population

df

100

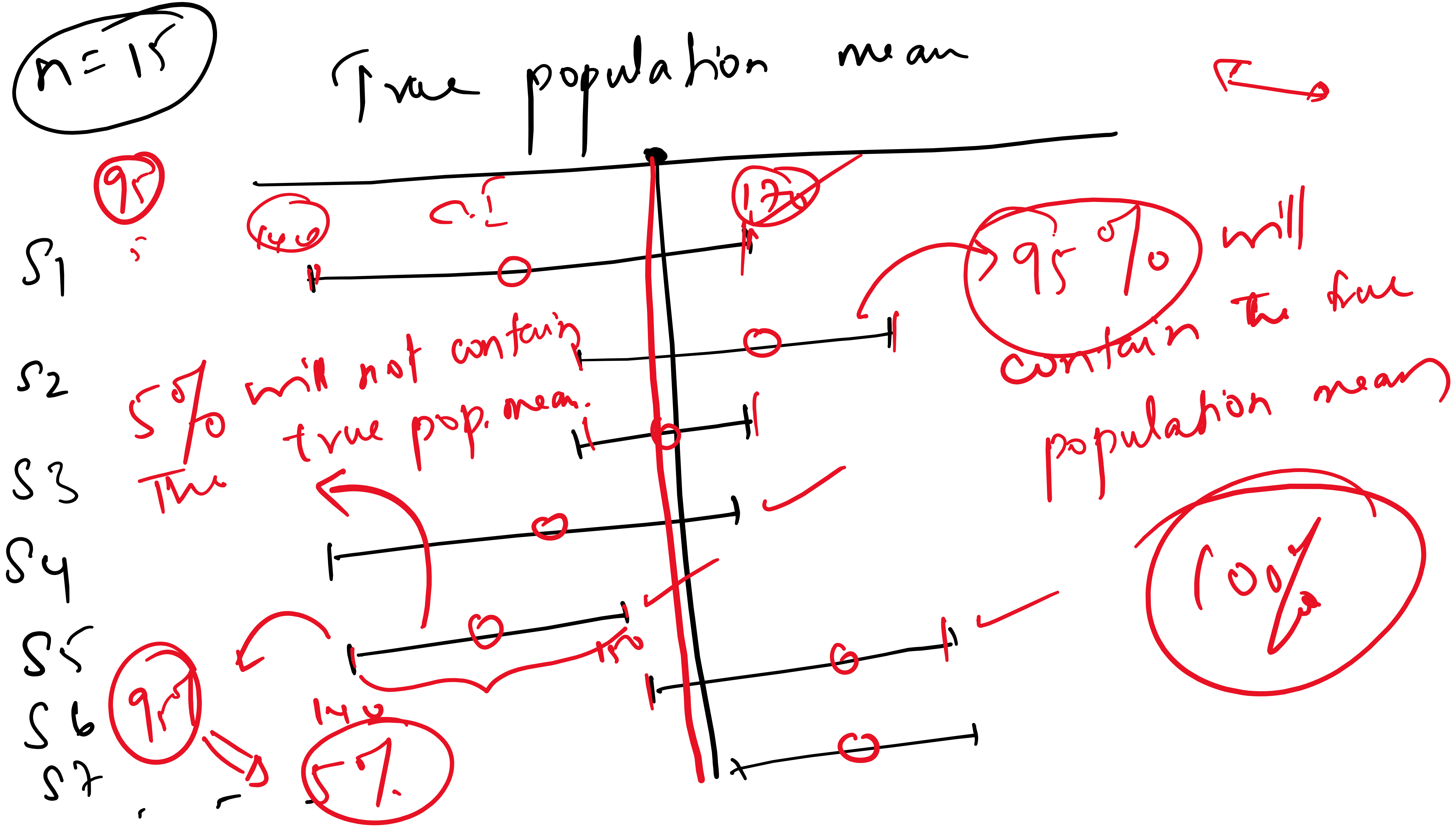
95

n = 15

Eg S1, S2, S3, S4 . . . . Sn

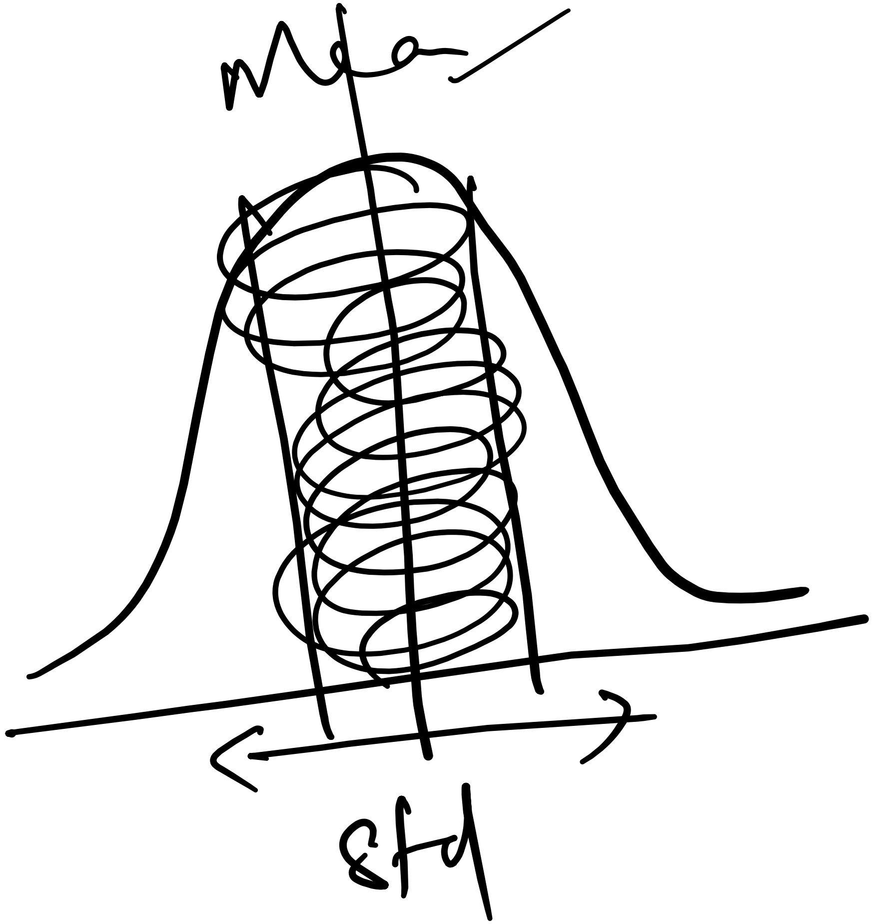
95% C.I

true population mean

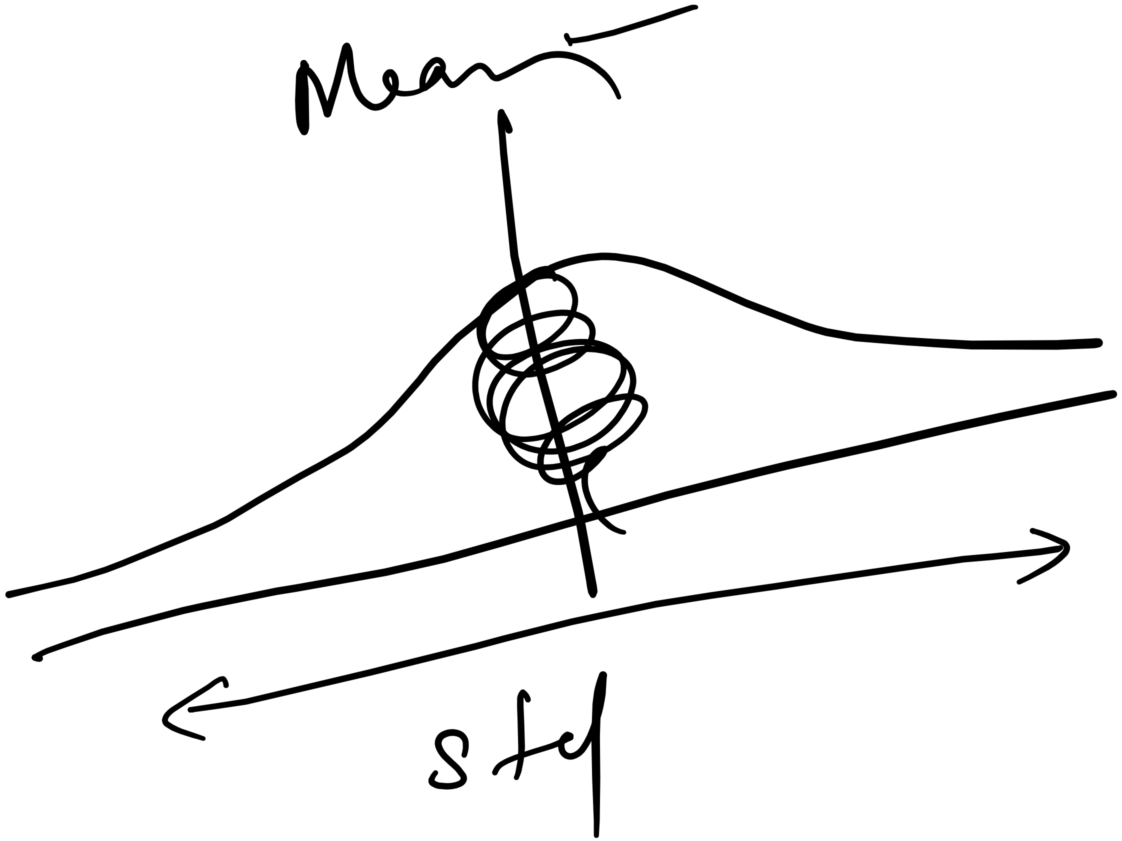

n=15
True population mean
95
S1
S2
S3
S4
S5
S6
S7
140
C.I
120
95% will contain the true population mean
5% will not contain the true pop. mean.
100%
150
95
140
5%

Normal Distribution:

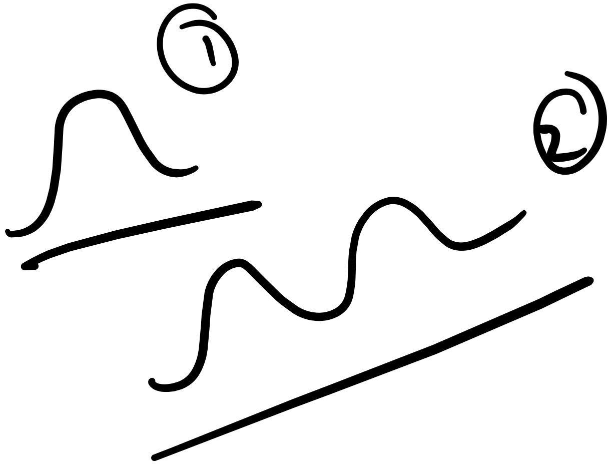
Parameter		Statistics
A number that describes the data from a population.	Mean	A number that describes the data from a sample
μ / σ	Std	$\bar{x}$ / s



size ↑



Characteristics of N.D:-



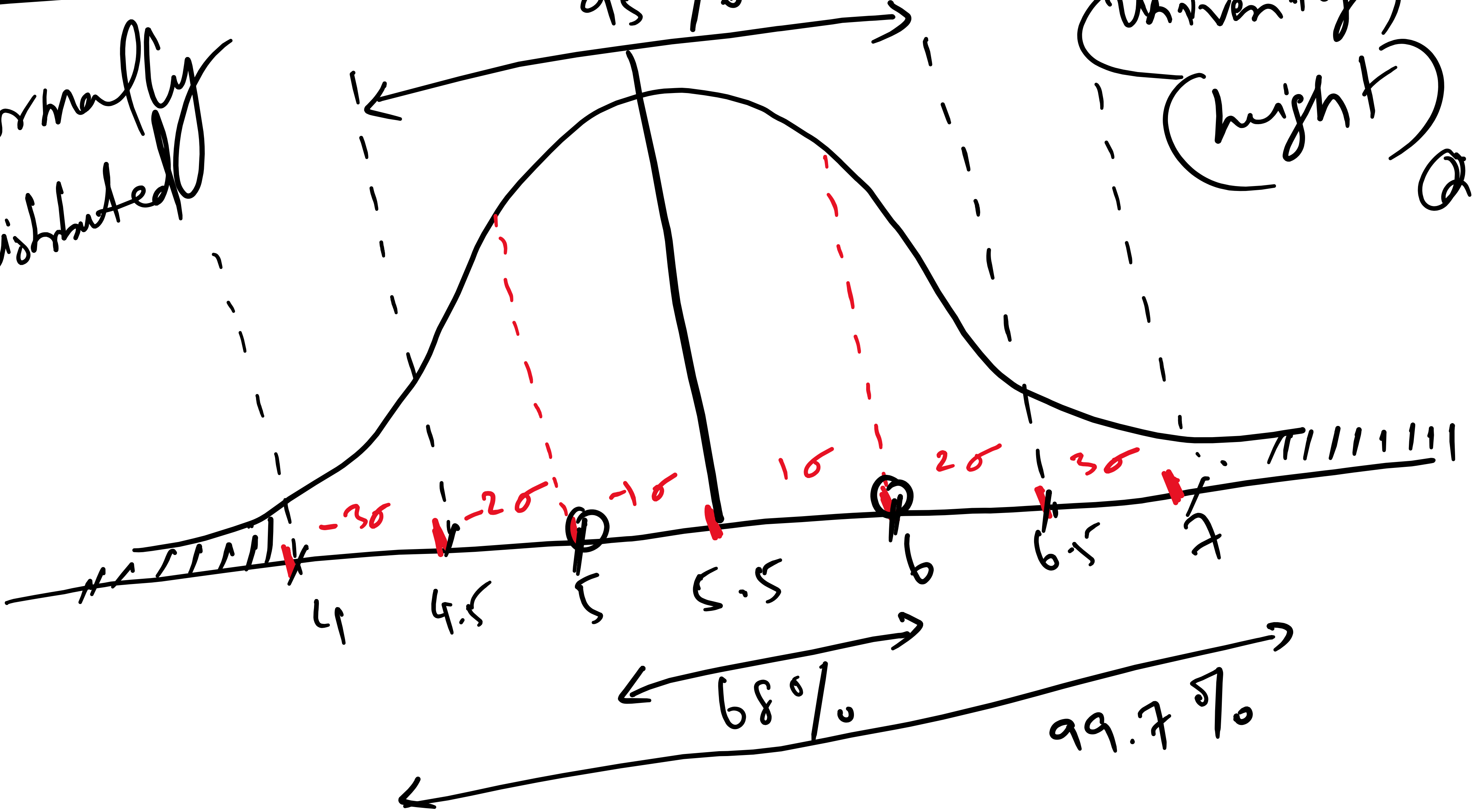
- It's a unimodal
- Symmetric about its mean
- The μ & σ normal distribution

68 - 95 - 99.7 Rule :- $\mu = 5.5$ feet ; $\sigma = 0.5$ feet

Normally distributed

University height

Q.3

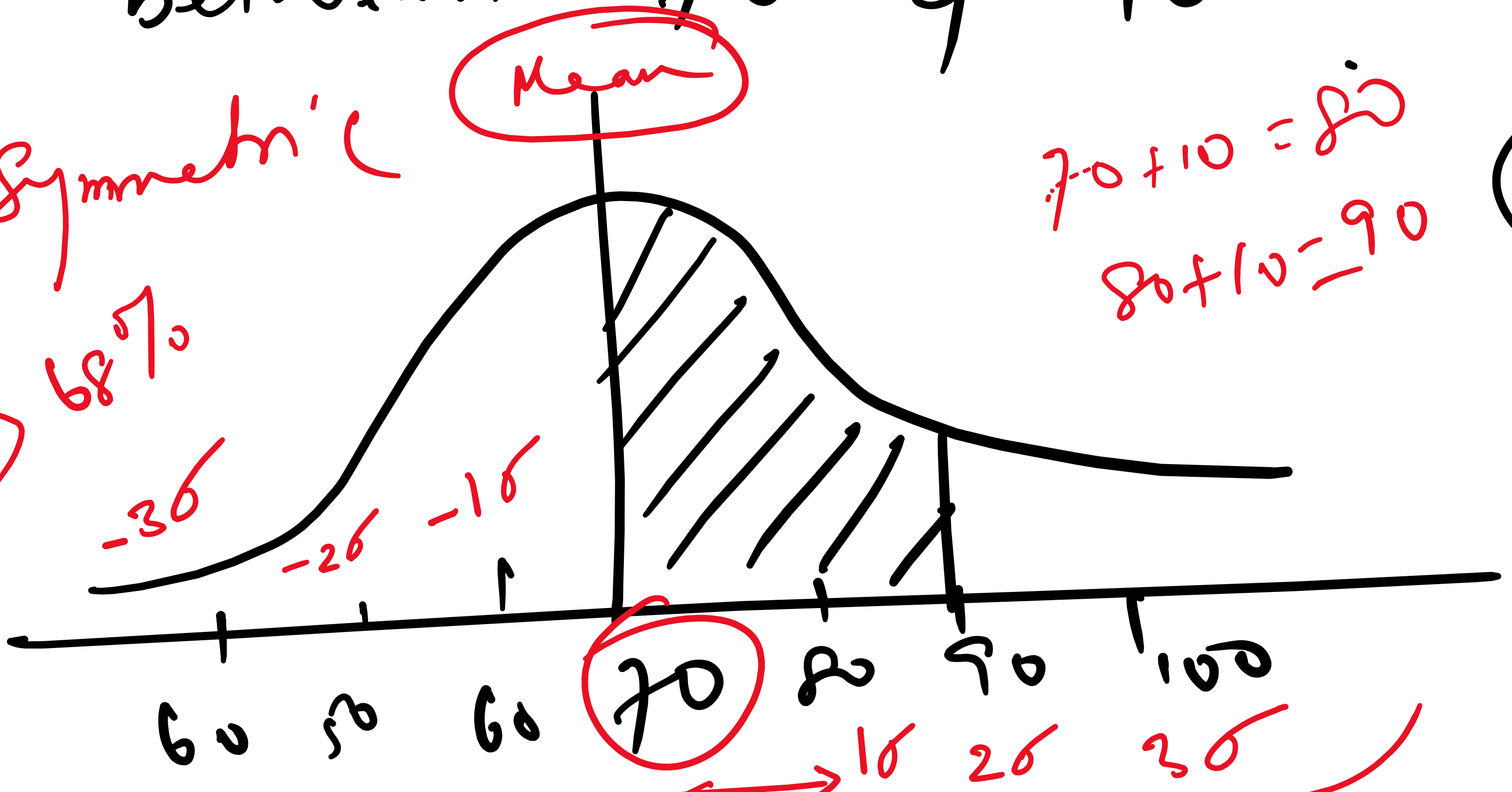


① The normal distribution has a $\boxed{\sigma = 10}$; $\boxed{\mu = 70}$; approximately what area is contained between 70 & 90?

%

Symmetric

-1 to $+1 \Rightarrow 68\%$



$70 + 10 = 80$

$80 + 10 = 90$

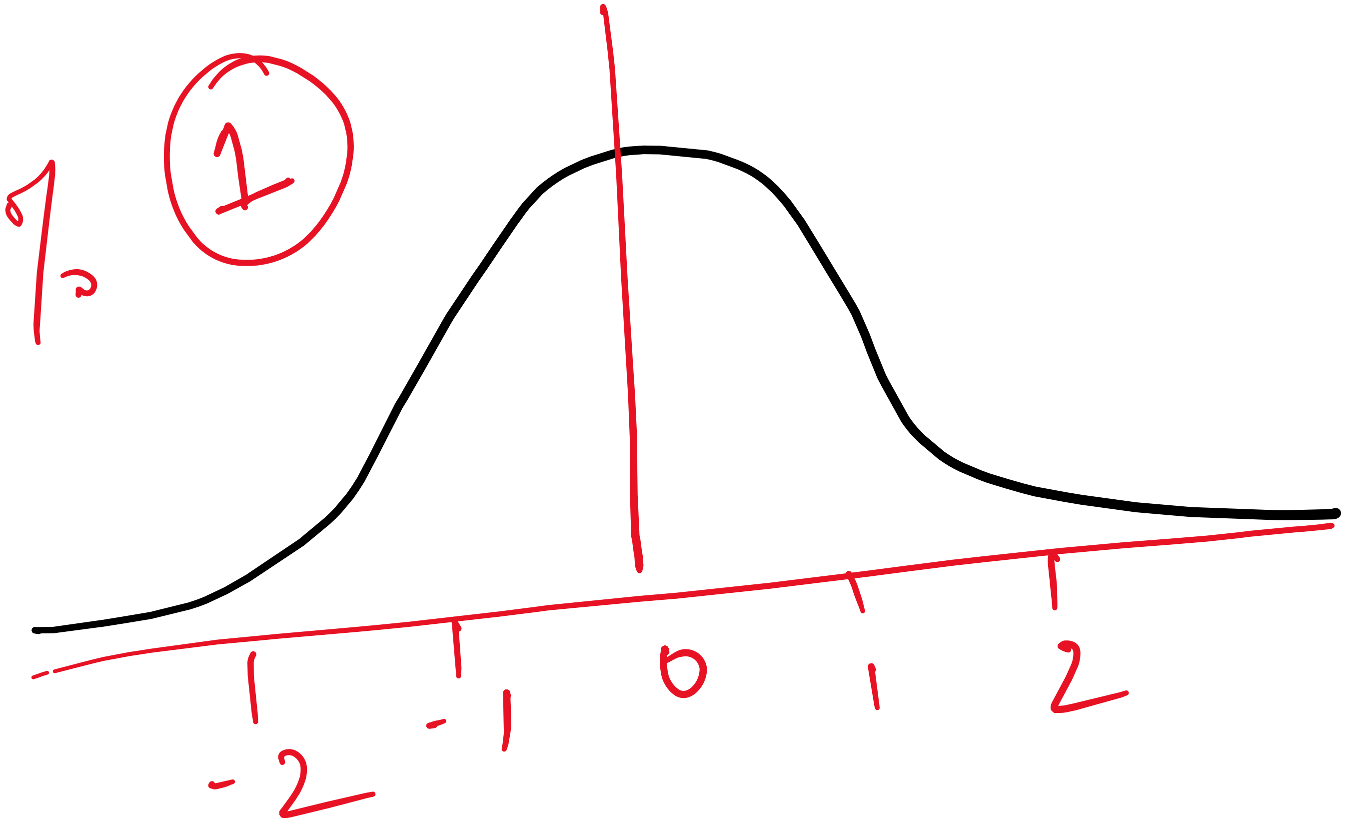
$\sigma = 10$

47.5%

81.5 % ①

$\mu = 0$

$\sigma = 1$

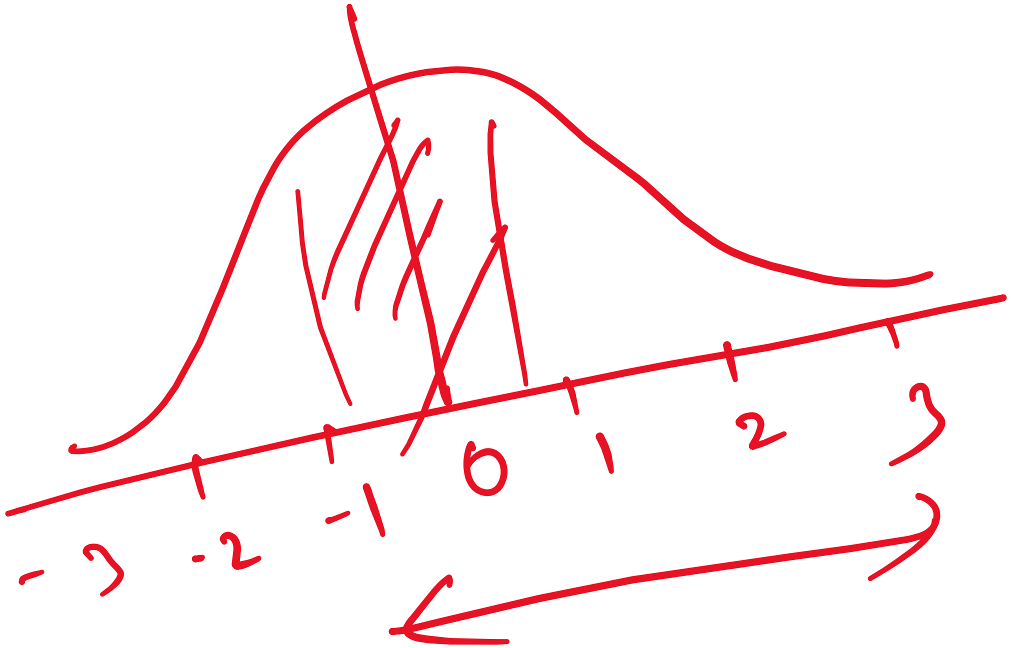


② Find the area b/w $-2 + 1$?

Z-score & standardization:- (ML preprocessing)

Standard normal distribution?

$\mu = 0$; $\sigma = 1$



z-score → -2

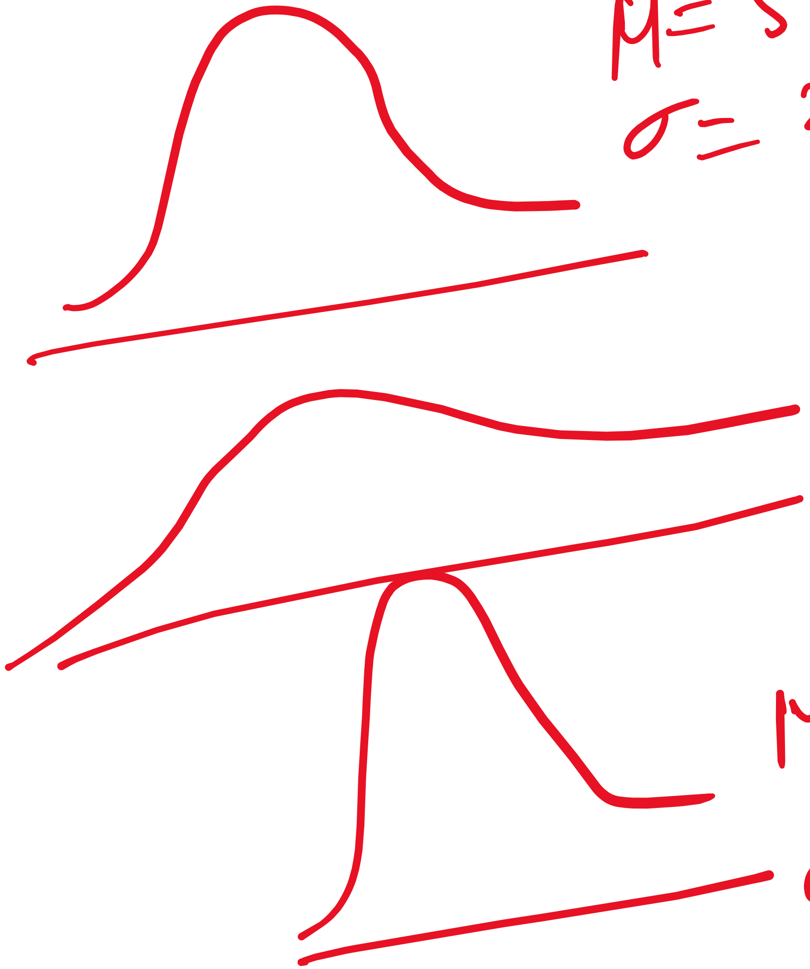
2 std

- Each number on the x-axis corresponds to 'Z-score'

- Z-score tells us how many 'σ' an observation is away from the mean

→ Calculate the area of distribution with the help of a Z-table

Normal distribution



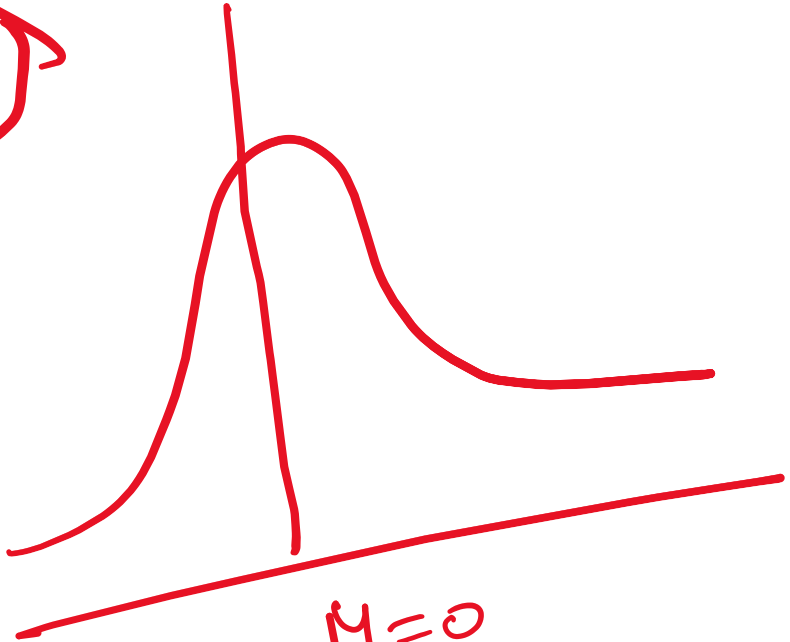
$\mu = 5$
$\sigma = 2$

$\mu = 16$
$\sigma = 3$

$\mu = 15$
$\sigma = 1$

Standardization

Std normal dist



$\mu = 0$
$\sigma = 1$

F Y

Formula: $\dfrac{x_i - \mu}{\sigma}$

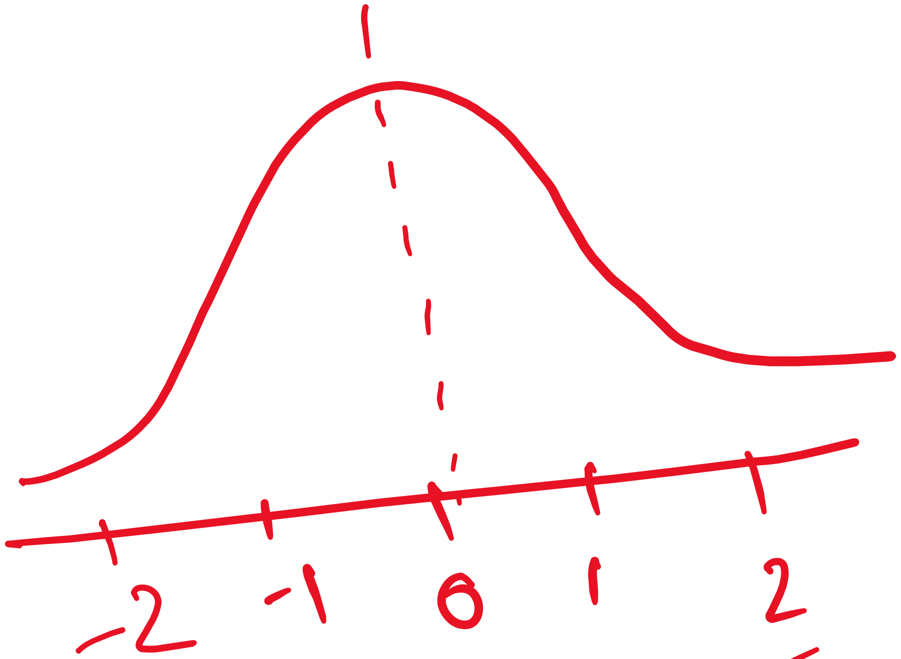
$x_i \rightarrow$ observation

$\mu \rightarrow$ population mean

$\sigma \rightarrow$ " std

Eg: 1 : Class x_i; scores of marks of students follow a normal distribution $\mu = 60$; $\sigma = 10$

$$\boxed{Z = \frac{x_i - \mu}{\sigma}}$$



$$\frac{40-60}{10} = \frac{-20}{10} = -2$$

$$\frac{50-60}{10} = -1$$

Std nml dist

$\mu = 0$ $\sigma = 1$

Q: What population of students scored less than 49 on the exam?

Std

-3.49

$.09$

$p(x < 49)$?

$$\boxed{Z = \frac{x_i - 60}{10}} \Rightarrow \frac{49 - 60}{10} = -1.1$$

$p(z < -1.1) \Rightarrow$ Look up in the z table

$= 0.1357$

→ How much area is associated with the z-score → calculated.

(found std.)

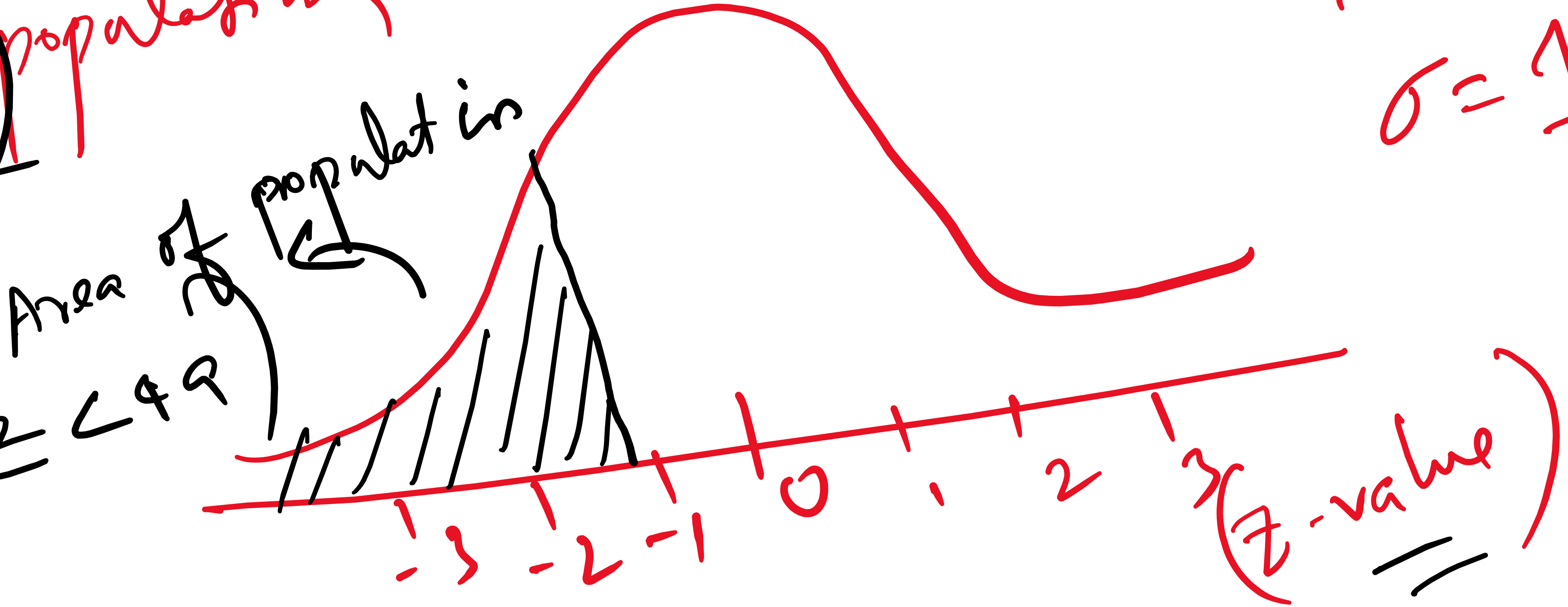
→ z value of -1.1 is '0.13' population

$\mu = 0$

$\sigma = 1$

0.13

$P(Z < a)$



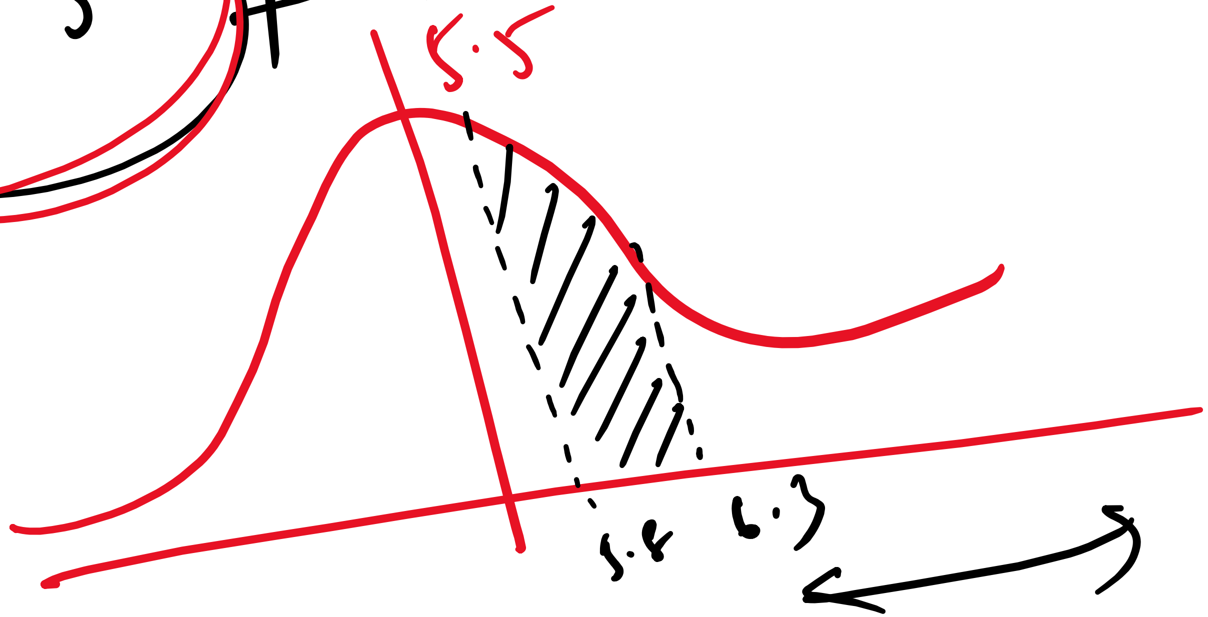
Eg: 2 Heights of students

$\mu = 5.5$ feet ; $\sigma = 0.5$ feet

Q: What proportion of students b/w 5.81 and 6.3 feet tall

① Standardization

② Z value.



0.945 to 0.7257

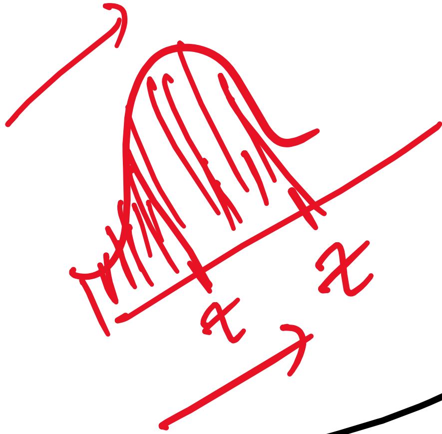
z-value

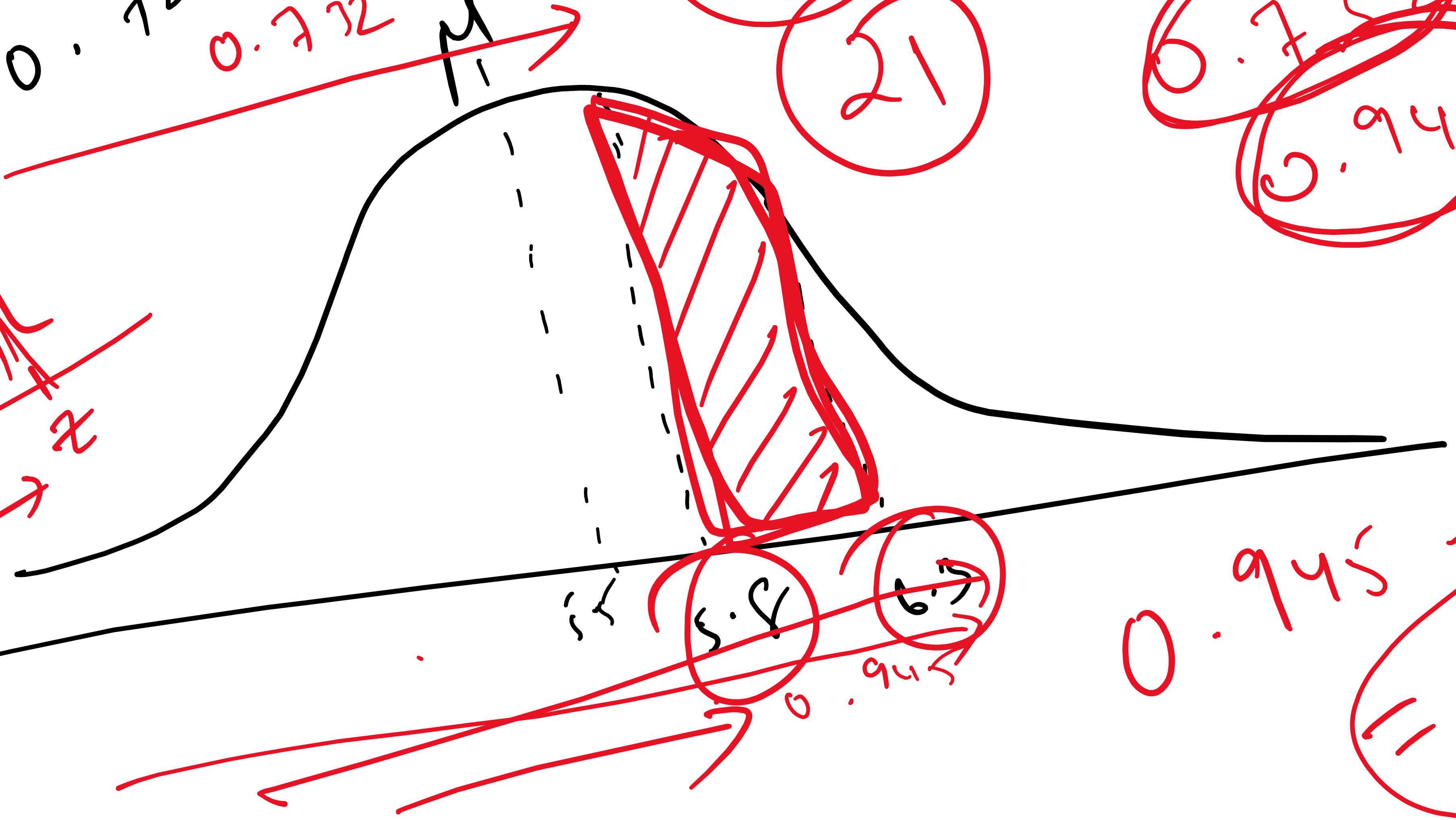
100

21

0.62 to 1.6

0.732





0.732

0.945

low

high

0.945 − 0.732

= 0.213

X = Age 25, 35, 45

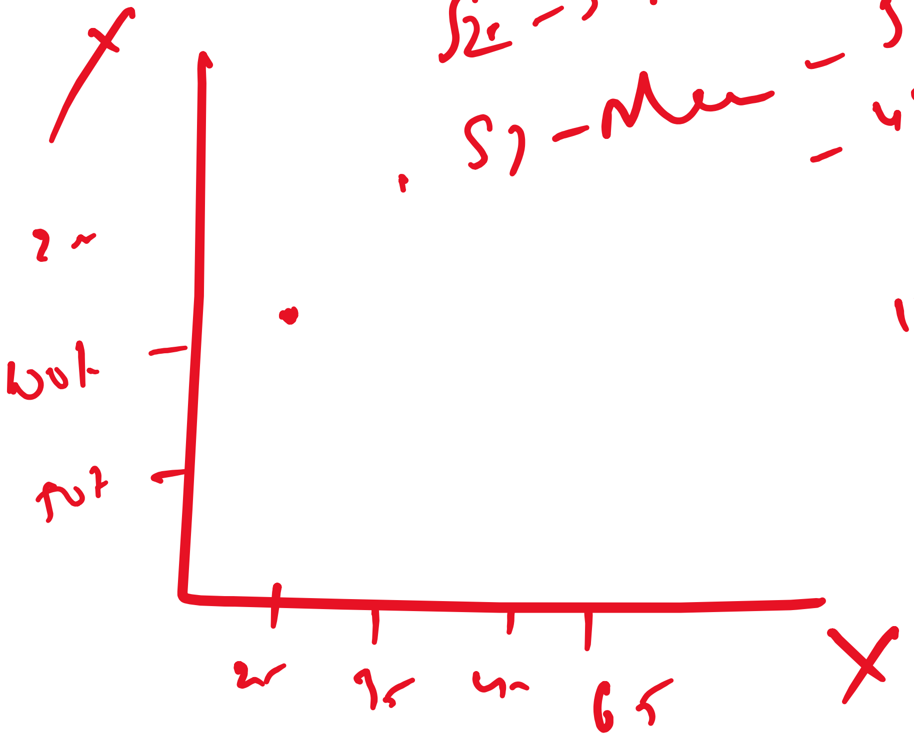
Y = Salary 1000, 2000 ...

S1 — S Mean — 50

S2 -> Mean — 52

S3 - Mean — 55

49



Density / Distribution

Hypothesis Testing :- (Assumption)

Covid

1. Person 1 — 3

2. Person 2 — 4

$n = 100$

Med 1

Cure	Not
97	3

S1

Med 1

X

α

Med 2

Cure	Not
3	97

S2

Med 2

1 / 2

65 35

60 40

55 45

Sklearn

H_2O.ai

fast.ai

$n = 200$

P value

Med 1

C	N
73	125

37% S3

X_Med 2

C	N
59	131

S4 31%

Eg:2 'Snickers' – 200 g ; 70 g of nuts

$\overset{V_1}{D} = \overset{V_2}{D} \times$ Ha

T $V_1 \neq V_2$

$V_1 < V_2$

$V_1 > V_2$

$\frac{200}{100}$

~~$H_0 \Rightarrow 70\,g$~~

$H_a < 70\,g$

H_1

α > α

α $\mu = 68.7\,g$

'p' value = 0.18

18%

S1	S2	S3	S4
70.1	69.2	69	71.1

69.9 1m

69.4 70

68.2 70.1

69.4 72

68.9

Hypothesis Testing: $H_0 \rightarrow$ Null ; $H_a \rightarrow$ Alternate

- Frame a hypothesis - (about a population)
- Set a significance level (α)
- Sampling
- Calculate 'p' value
- Decision (Accept / Reject the H_0)

Z-test :-

Scenario :- A complaint that The students are malnutritioned

Age = 10 ; n = 25 ; $\mu = 32$ kg ; $\sigma = 9$ kg   statistical data chart

Sampling :- $\boxed{\bar{x} = 29.5 \text{ kg}}$

A 32 vs. B

$\neq$ 29.5 / 31.

Proof

① $H_0 \Rightarrow \mu = 32$ (No significant diff)

$H_a \Rightarrow \mu < 32$ (Significant differen)

$\neq$, $<$, $>$

(2) $\boxed{\alpha = 0.05}$ 0.025   0.10

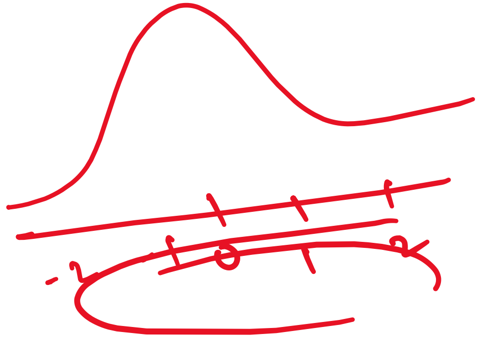
95%   97.5   90%

(3) Sampling $\mu \le 32$

(4) Calculate the Z value

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{29.5 - 32}{9/\sqrt{25}} = \frac{-2.5}{9/5} = \frac{-2.5}{1.8} = -1.39$$



$\bar{x}$ → sample mean

μ → pop. mean

σ → " std

n → sample size

$\boxed{Z = -1.39}$

1 → −1.3

② .09 → Y

→ find the p-value (from the Z table)

$\boxed{\alpha = 0.05}$

$P(Z = -1.39) = 0.0823$

$\boxed{P = 0.0823} > 0.05$

$\alpha = 0.02$ Reject

'If p value is low; Null must go'

Accept

$p(z) < 0.05$;

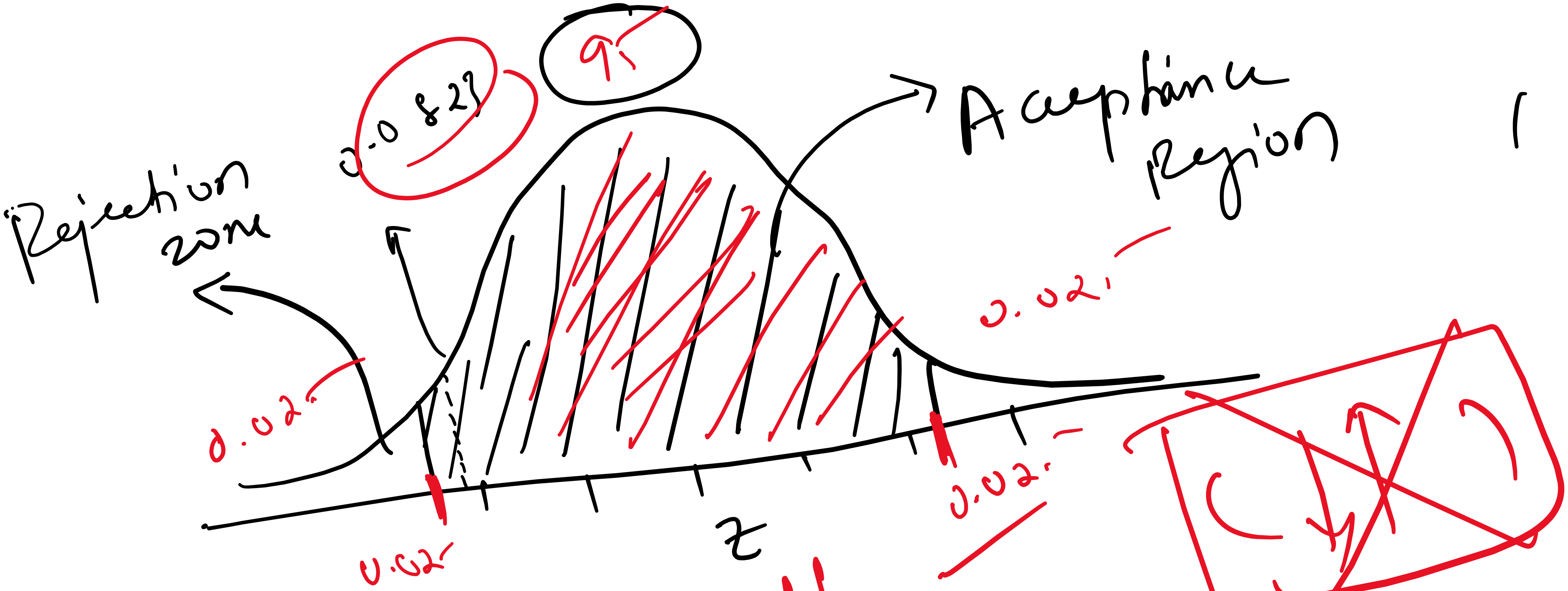
$p(z) > 0.05$;

95%

95%

90%

$\alpha = 0.10$



$H_0 \Rightarrow$ Accept H_0

$H_a \Rightarrow$ Reject H_a

One tail vs two tail :

① $H_0 \Rightarrow \mu = 100g$ ✓ ×

② $H_a \Rightarrow \mu \neq 100g$

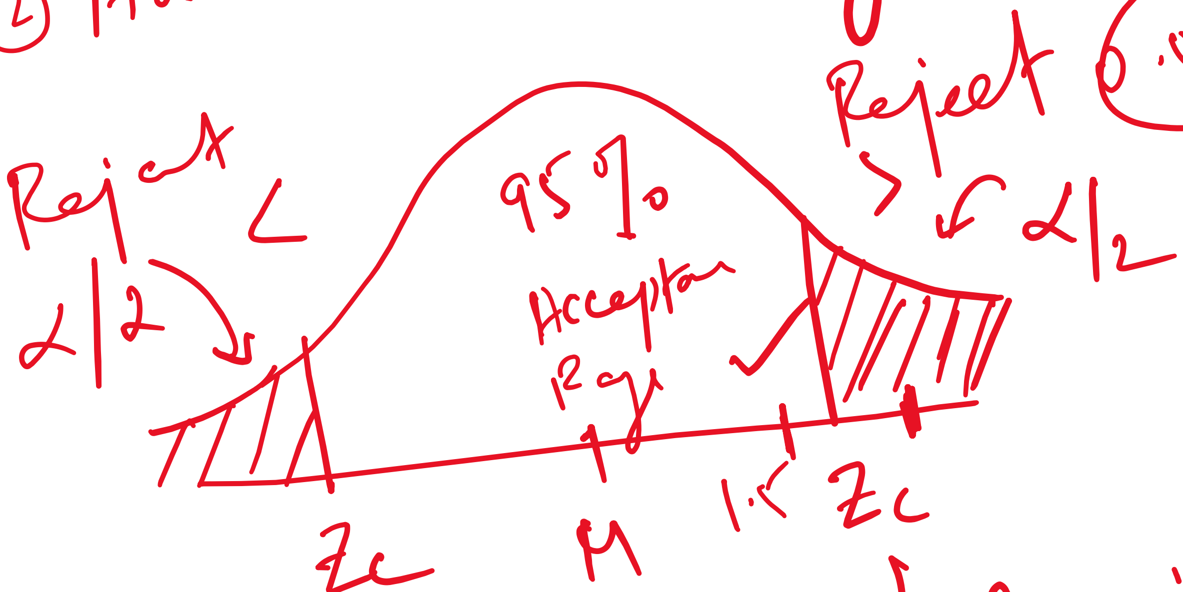
$\boxed{\alpha = 0.05}$

$0.025 < 0.05$ (α)

Sampling :

Z value

P value



Critical region

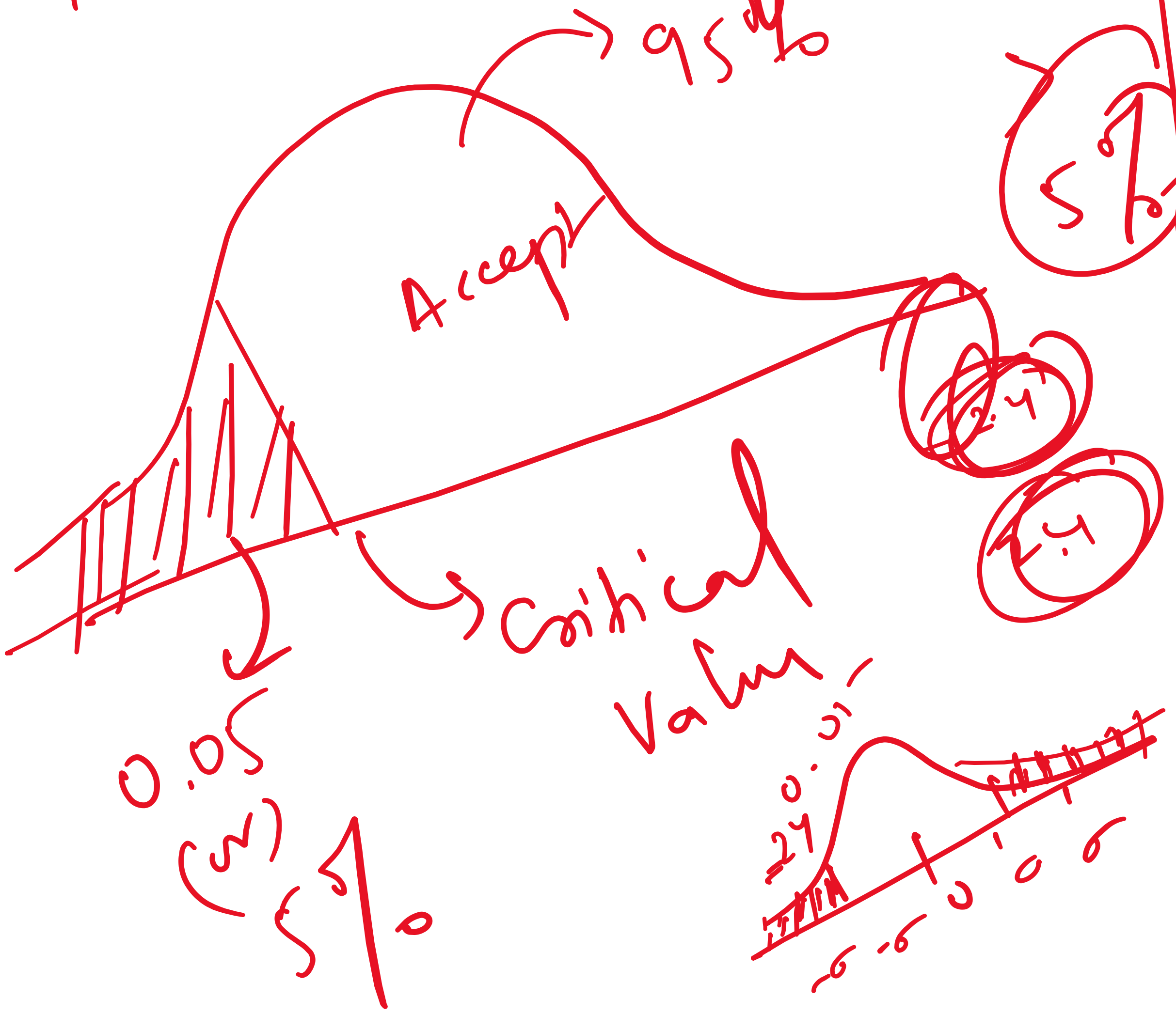
✓ $\boxed{-0.05 < Z < +0.5}$

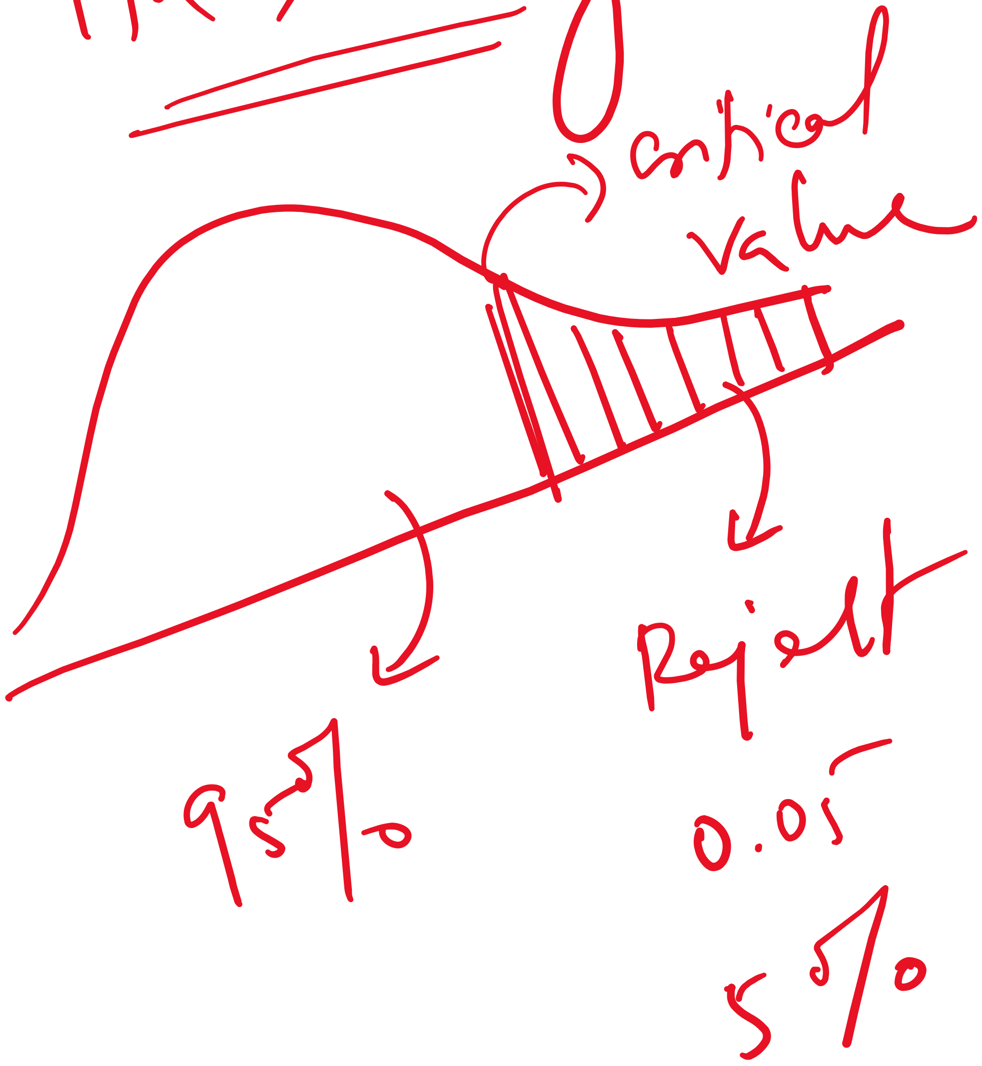
95%

Ha < 100g

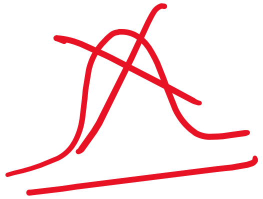
$\alpha = 0.05$

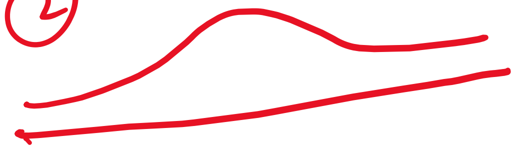
Ha > 100g





T- Distribution:

1 

2 

z vs t

- Works with normal distribution
- **Small** samples
- For large samples, t-dist = z-score (normal distribution)
- T-Dist ⇒ high standard deviation.

→ if $n > 30$; 't' similar to 'z'.

Parameter μ σ → take a sample → Calculate statistic

Param | Stats

For eg:- 'mean' ($\bar{x}$)

→ Add a 'margin of error' to our statistic ($\bar{x}$)

$\boxed{\frac{\bar{x} - \mu}{s/\sqrt{n}}}$

⇒ Statics $\pm$ $t^* \frac{s}{\sqrt{n}}$

σ → population s/d

s → sample std

$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$ $\boxed{\bar{x} \pm t^* \frac{s}{\sqrt{n}}}$ $\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$

→ 'σ' → 's' ; High variability

→ df ⇒ sample size − 1 ; $n \uparrow \Rightarrow$ t ⇒ 'z'

Constructing a C.I. (95%) for pop. mean :-

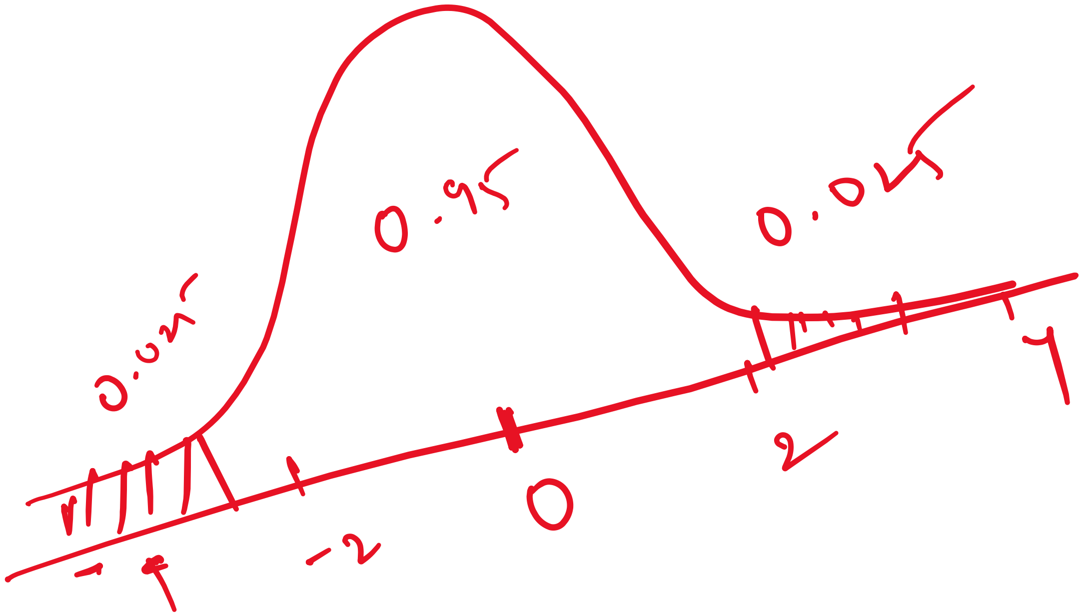
→ for 't' distribution ; 'σ' is unknown

If 'σ' is known

(Z)

$$\bar{x} \pm Z * \sigma/\sqrt{n}$$

$\boxed{\text{Regardless of 'n'}}$



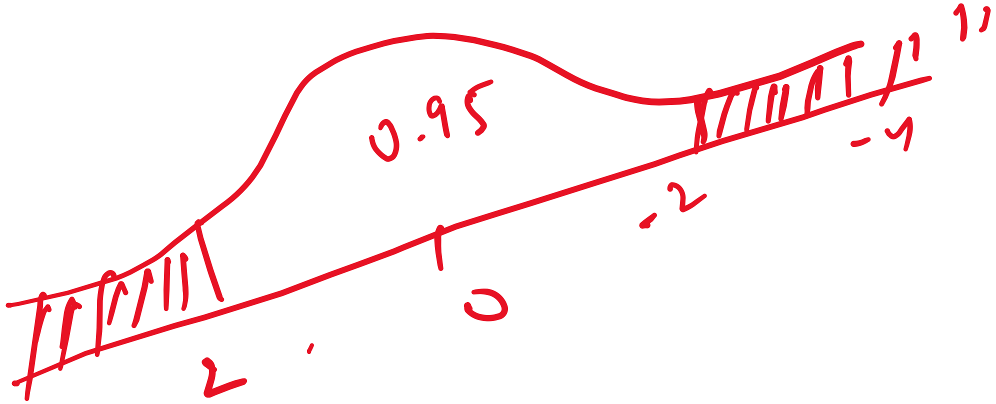
If 'σ' is Unknown

(t)

$$\bar{x} \pm ? * s/\sqrt{n}$$

$$\bar{x} \pm t * s/\sqrt{n}$$

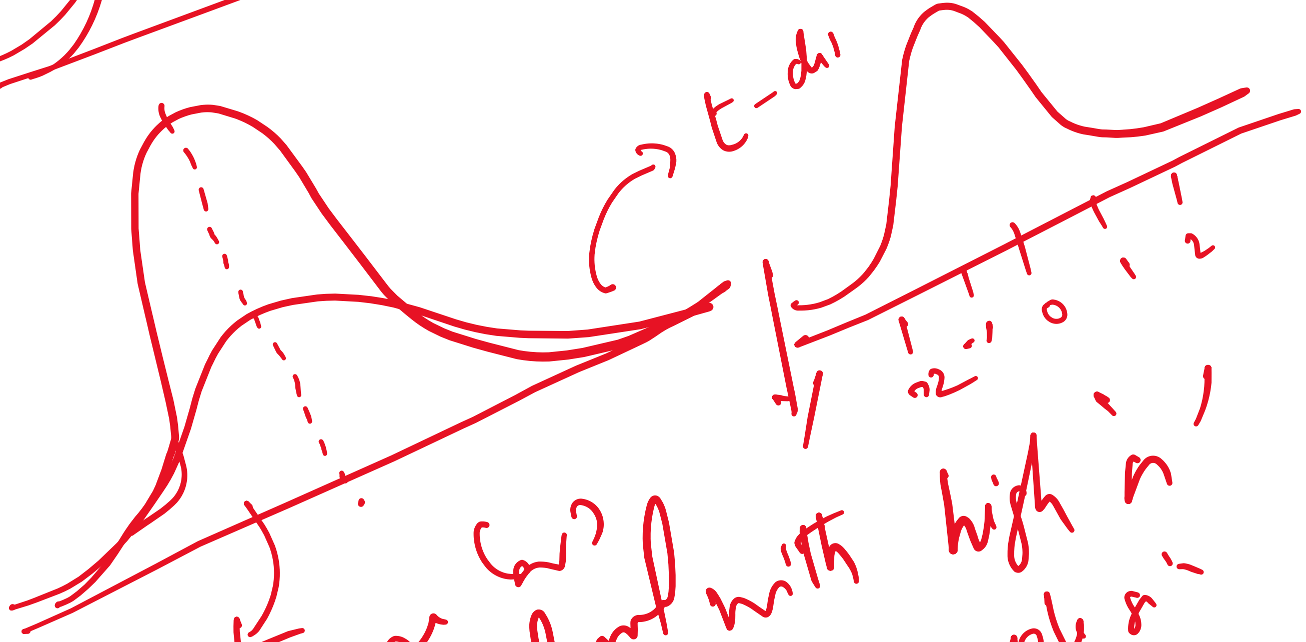
'less sample' if I have more 'n'



$\underline{t_{0.025}}$ for (97.5% C.I)

n	df	$t_{0.025}$
6	5	2.571
11	10	2.228
31	30	2.042
61	60	2.000
81	80	1.990
101	100	1.984
∞	∞	1.960

C.I	Z-score
90	1.65
95	1.96
99	2.5



z-score (or) standard dist with high 'n' sample size

Misconception:

'If $n > 30$; forget 't'; just use 'z' ✗

$df > 30$; t-value $\Rightarrow$ 2.042 ; $z = 1.96$

$df = 100$; $t \Rightarrow$ 1.98 ; $z = 1.96$ (2)

'Margin of error'

diff (margin error)

C.I

$\bar{x} \pm z * \frac{s}{\sqrt{n}}$

(or)

$\bar{x} \pm t * \frac{s}{\sqrt{n}}$

$\boxed{\text{'t' test not 'z'}}$