Math 327
Chapter 9

More about Odds and Odds Ratios

Intuition about Odds

- Given the probability of an outcome, p, the odds of that outcome is $\frac{p}{1-p}$

- Things we can say about the odds value
  - Odds is a multiplicative factor between the probability of a positive outcome and the probability of a negative outcome: $p = (Odds) \times (1 - p)$
  - Example: An odds value of 2 means the probability of a positive outcome is twice as big as (two times) the probability of a negative outcome
  - Example: An odds value of 1 means the probability of a positive outcome is equal to the probability of a negative outcome - the probabilities are both 50%
  - Example: An odds value of 0.1 means that the probability of a positive outcome is 1/10 as big as the probability of a negative outcome, i.e., the negative outcome is 10 times more probable than the positive outcome

What does "X times more likely" mean?

- When you hear the phrase "5 times more likely," what does that mean to you?
  - First, more likely than what?
  - Example: The chance of rain today is 5 times more likely than the chance of rain yesterday
    - Suppose the chance (probability) of rain yesterday was 10%. Then the chance of rain today is 50% (5 times more likely)
    - What if the chance (probability) of rain yesterday was 30%. What does "5 times more likely mean now?
- What does "10% more" mean?
  - Example: Yesterday, the chance of rain was 30%. Today it is 40%, which is 10% more
  - Example: Yesterday, the chance of rain was 30%. Today it is 33%, which is 10% more
    - $\frac{33-30}{30} \times 100\% = \frac{3}{30} \times 100\% = 10\%$
  - When working with percent values, always ask, "percent of what?"

Intuition about Odds Ratio

- Odds Ratio $= \dfrac{Odds\ of\ a\ positive\ outcome\ if\ X = x_0 + 1}{Odds\ of\ a\ positive\ outcome\ if\ X = x_0}$

  - Special case if X=0 or 1: Odds Ratio $= \dfrac{Odds\ of\ a\ positive\ outcome\ if\ X = 1}{Odds\ of\ a\ positive\ outcome\ if\ X = 0}$

- Things we can say about the Odds Ratio
  - An odds ratio is a ratio of two odds values
  - Those two odds values are usually (always in this class) both the odds of a positive outcome but under different conditions, such as a different value of a predictor variable
    - For comparison, a single odds value is about the positive outcome vs. the negative outcome under the same conditions
  - Specifically, the odds ratio for a predictor variable in logistic regression is the multiplicative factor between two odds values that corresponds to a one-unit increase in the predictor variable, X
  - So the predicted odds of a positive outcome for a specific value of X is multiplied by the odds ratio $(OR = e^{slope})$ to get the predicted odds of a positive outcome for X+1
    - $Predicted\ Odds\ for\ X + 1 = OR \times Predicted\ Odds\ for\ X$
  - For example, if the slope in a simple logistic regression (one predictor variable) is 0.6931 (0.6931=ln(2)), then a one-unit increase in the predictor variable corresponds the predicted odds of a positive outcome being two times larger $(odds\ ratio = e^{0.6931} = 2)$.