

These are probably  
filled with mistakes  
So if you think  
something's wrong  
let me know

Old Midterm 4)

[1 2 3 4 5 6]

[3 0 2 4 6]

48

$$EV = \text{avg. \# draws}$$

$$\text{avg} = \frac{0 + 0 + 0 + 2 + 4 + 6}{6}$$

$$= 2$$

$$EV = 48 \cdot 2 = 96$$

$$SE = \sqrt{\# \text{ draws}} \cdot SD$$

$$\sqrt{\frac{3(0-2)^2 + (2-2)^2 + (4-2)^2 + (6-2)^2}{6}}$$

$$= sd$$

$$SE = \sqrt{98} \cdot sd$$

$$SU = \frac{\text{value} - EV}{SE}$$

$$= \frac{104 - 96}{SE}$$

$p_{\text{norm}}(x)$  gives probability less than  $x$

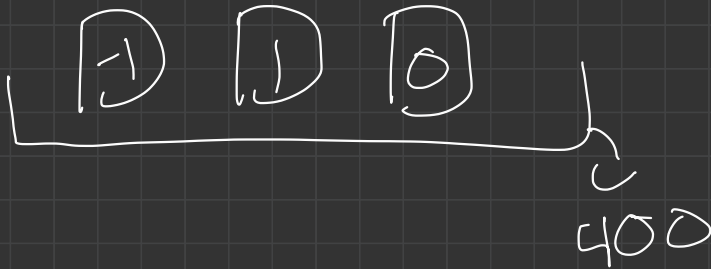
$\Rightarrow 1 - p_{\text{norm}}(x)$  gives more than  $x$

1 - pnorm(SU).

old midterm #5).

<u>Prob</u>	<u>dice shows</u>
$\frac{1}{3}$	1 or 2
$\frac{1}{3}$	5 or 6
$\frac{1}{3}$	3 or 4

<u>payout</u>
-1
1
0



$$\begin{aligned} EV &= 400 \cdot \text{avg} \\ &= 0 \end{aligned}$$

$$SE = \sqrt{400} \cdot sd$$

$$sd = \sqrt{\frac{(-1-0)^2 + (1-0)^2 + (0)^2}{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow SE = \sqrt{400} \cdot \frac{\sqrt{2}}{\sqrt{3}}$$

b) Check for normal approx.

1) at least 25 draws ✓

2)  $EV \pm 2 \cdot SE$  is  
a possible value. ✓

$$SE \approx 6$$

$$\Rightarrow -32 \text{ to } 32$$

$$SU = \frac{10 - EV}{SE} \quad \begin{array}{l} EV = 0 \\ SE \approx 16.33 \end{array}$$

$$1 - \text{pnorm}(SU).$$

c) Chance that I win more than 120 of the 400 games.

$$\underbrace{2(0) \quad 1}_{400} \quad \text{avg} = \frac{1}{3}$$

$$EV = \frac{400}{3}$$

$$SE = \sqrt{400 \cdot \frac{\left(0 - \frac{1}{3}\right)^2 + \left(0 - \frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}{3}}$$

$$SU = \frac{120 - EV}{SE}$$

1 -  $\text{pnorm}(SU)$ .

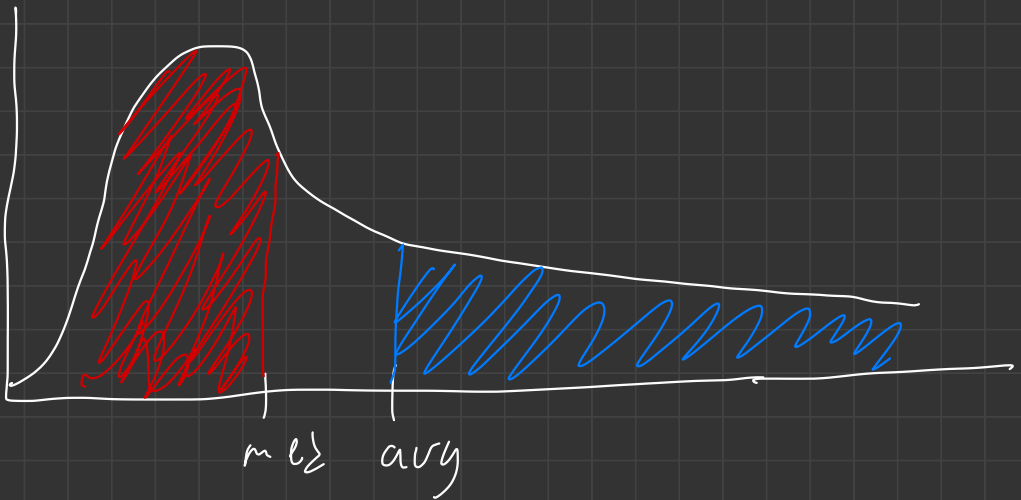
Ch 5: Rev 10.

Avg income 32 000

SD 26 000

Want the percentage of incomes in range 32-150 th.

Is it closer to 40, 50 or 60.



$\Rightarrow 40\%$

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Old mid term 7:

1. Normal, symmetric

$\Rightarrow \text{mean} = \text{median}$ .



$$f_{\text{norm}}(0.84) = \frac{67 - \text{avg}}{SD}$$

$$f_{\text{norm}}(0.87) = \frac{61 - \text{avg}}{SD}$$

[ 1 1 3 ]

Q4 Quiz  
5

$$1, 1 \rightarrow 1$$

$$3, 3 \rightarrow 9$$

$$1, 3 \rightarrow -4$$

$$P\left(\begin{matrix} 1, 3 \\ \text{or} \\ 3, 1 \end{matrix}\right) \rightarrow \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3}$$

$k$	$P(X=k)$
1	$\frac{2}{3} \cdot \frac{2}{3}$
9	$\frac{1}{3} \cdot \frac{1}{3}$
-4	$2 \cdot \frac{2}{3} \cdot \frac{1}{3}$

$$E(X) = \sum_k k \cdot P(X=k)$$

$$= 1 \cdot P(X=1) + 9 \cdot P(X=9) + (-4) \cdot P(X=-4)$$

$$= \frac{4}{9} + 9 \cdot \frac{1}{9} - 4 \cdot \frac{4}{9}$$

$$= -\frac{12}{9} + 1 = \frac{-3}{9} = -\frac{1}{3}$$

$$SE(X) = \sqrt{E((X - E(X))^2)}$$

$$= \sqrt{\sum_k (k - E(X))^2 P(X=k)}$$

$$= \sqrt{\left(1 - \left(-\frac{1}{3}\right)\right)^2 \cdot \frac{4}{9} + \left(9 - \left(-\frac{1}{3}\right)\right)^2 \cdot \frac{1}{9} + \left(-4 - \left(-\frac{1}{3}\right)\right)^2 \cdot \frac{4}{9}}$$

## old midterm Q2

a)  $P(\text{exactly 2 fours})$

$$= \binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

b)  $1 - P(\text{all rolls are 2 or more})$

$$1 - \binom{4}{4} \left(\frac{5}{6}\right)^4$$

c)  $P(\text{exactly 2 ones or exactly 2 sixes})$

$$\begin{aligned}
 &= P(\text{exactly 2 ones}) \\
 &\quad + P(\text{exactly 2 sixes}) \\
 &\quad - P(\text{exactly 2 ones and exactly 2 sixes})
 \end{aligned}$$

NOT mutually exclusive

$$\begin{array}{cc}
 \begin{array}{c} 1 \\ \hline \left(\frac{1}{6} \cdot \frac{1}{6}\right) \\ \left(\frac{1}{6}\right)^2 \end{array} & 
 \begin{array}{c} 1 \\ \hline \left(\frac{1}{6} \cdot \frac{1}{6}\right) \\ \left(\frac{1}{6}\right)^2 \end{array}
 \end{array}$$

# of ways to choose 2  
spots out of 4 spaces is

$$\binom{4}{2} \text{ So answer is}$$

$$\binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^2$$

## Section 112

$$\left[ 37 \quad \boxed{1} \quad \boxed{2} \quad \boxed{3} \quad \boxed{30} \right]$$

2018 Q4 30

a) In 30 draws.

$$P(\text{sum draws} \leq 30)$$

$$\text{avg} = \frac{37 + 2 + 3 + 30}{40}$$

$$= 2$$

$$SD = \sqrt{\frac{37 \cdot (1-2)^2 + 0^2 + (3-2)^2 + (30-2)^2}{40}}$$

$$= \sqrt{\frac{37 + 1 + 36^2}{40}}$$

$$= 5.9$$

$$EV = \# \text{ draws} - 2 = 60$$

$$SE = \sqrt{\# \text{ draws} \cdot SD} = \sqrt{30 \cdot 5.9} \\ \approx 32.3$$

$$EV - 2SE < 0.$$

$\Rightarrow$  normal approx bad

$$P(\text{sum 30 draws} \leq 30)$$

$$= P(\text{exactly 30 in 30 draws})$$

$$= P(\text{drawing 30 } \boxed{1}\text{'s})$$

out of 30 draws.

$$= \left(\frac{37}{40}\right)^{30}$$

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sd in R uses

$$\sqrt{\frac{1}{n-1} \sum (x - \text{avg})^2}$$

we use

$$\sqrt{\frac{1}{n} \sum (x - \text{avg})^2}$$



$z$  is a vector in  $\mathbb{R}$

$$\text{ours} = \text{sd}(z) \cdot \frac{\text{sqrt}(n-1)}{\text{sqrt}(n)}$$

$n = \text{length of vector } z.$

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$p$  (at least heart or at least  
overspace!)

$$= p(\text{at least heart}) + p(\text{at least 1 space})$$

$$- p(\text{at least 1 heart and at least 1 space})$$

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$$P(\text{at least 1 heart}) = P(hs \text{ or } sh \text{ or } hh)$$

$$= 2 \frac{13}{52} \cdot \frac{13}{51} + \frac{13}{52} \cdot \frac{12}{51}$$

$$= P(\text{at least 1 spade})$$

$$P(\text{at least 1 heart and at least 1 spade})$$

$$= P(hs \text{ or } sh)$$

$$= 2 \frac{13}{52} \cdot \frac{13}{51}$$

## Quiz 4 choose question

From a group of 7 men and 6 women five people are to be selected to form a committee with at least 3 men.

Choose (binomial coefficient)

$$\begin{matrix} 3 \text{ men} & 4 \text{ men} & \text{or} & 5 \text{ men} \end{matrix}$$
$$\binom{7}{3} \cdot \binom{6}{2} + \binom{7}{4} \cdot \binom{6}{1}$$

$$+ \binom{7}{5}$$

$$\begin{aligned} \text{Choose } (7, 3) &= \binom{7}{3} \\ &= \frac{7!}{(7-3)! \cdot 3!} \end{aligned}$$

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ChS A2 (Section 12)

13, 9, 11, 7, 10

$$SD = \frac{\text{value-avg} \leftarrow EV}{SD \leftarrow SE}$$

$$\frac{13 + 9 + 11 + 7 + 10}{5} = 10.$$

$$SD = \sqrt{\frac{(13-10)^2 + (9-10)^2 + (11-10)^2 + (7-10)^2 + (10-10)^2}{5}}$$

$$= \sqrt{\frac{9 + 1 + 1 + 9 + 0}{5}}$$

$$= \sqrt{\frac{20}{5}} = 2.$$

$$\text{values} - \text{avg} = (3, -1, 1, -3, 0)$$

$$\frac{\text{values only}}{52} = \left( \frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{3}{2}, 0 \right)$$

Gambler's Roulette

$P(X=R)$	$R$	↙ 1 draw.
$\frac{2}{38}$	17	
$\frac{36}{38}$	-1	

$$\left[ 2 \boxed{17} \quad 36 \boxed{-1} \right] \rightarrow 29$$

$$\left[ 1 \boxed{17} \quad 18 \boxed{-1} \right] \rightarrow 25$$

If  $S_n = \text{sum of } n$   
ind. random  $X$   
variables

$$E(S_n) = n E(X)$$

$$SE(S_n) = \sqrt{n} \cdot SE(X)$$

$$EV = \# \text{ draws} \cdot \text{avg}$$

$$SE = \sqrt{\# \text{ draws}} \cdot SD$$