

Table S1: Computational time (in seconds) to run different Bayesian variable selection methods for 100 MCMC iterations, as a function of sample size and the number of covariates. Compared here are Bayes Ridge (BRR), Bayes Lasso (BL), Bayes LMM (BLMM), and Bayes C $\pi$ . BAKR is assessed under both its full model specification ( $V = 1$ ), as well as under its empirical factor representation ( $V \approx 0.9$  and  $0.8$ , respectively). Sample sizes considered were  $n = 100, 500, 1000$ , and  $2000$ . The number of covariates considered were  $p = 1 \times 10^3, 1 \times 10^4, 5 \times 10^4$ , and  $1 \times 10^5$ , respectively. Note that only cases in which  $p > n$  were timed. Results are based on 100 different simulated datasets. Standard errors across these replicates for each model are given the parentheses.

Samples	Covariates	Other Methods				BAKR Models		
		BRR	BL	BLMM	BC $\pi$	$V = 1$	$V \approx 0.9$	$V \approx 0.8$
$n = 100$	$1 \times 10^3$	0.09 (0.04)	0.20 (0.09)	0.14 (0.02)	0.12 (0.09)	0.03 (0.00)	0.02 (0.00)	<b>0.01 (0.00)</b>
	$1 \times 10^4$	0.58 (0.09)	0.97 (0.12)	0.60 (0.11)	0.62 (0.18)	0.03 (0.00)	0.02 (0.00)	<b>0.01 (0.00)</b>
	$5 \times 10^4$	2.44 (0.20)	4.46 (0.12)	2.72 (0.21)	2.85 (0.17)	0.03 (0.00)	0.02 (0.00)	<b>0.01 (0.00)</b>
	$1 \times 10^5$	4.76 (0.19)	8.64 (0.21)	4.81 (0.20)	5.48 (0.14)	0.03 (0.00)	0.02 (0.00)	<b>0.01 (0.00)</b>
$n = 500$	$1 \times 10^3$	0.39 (0.18)	0.36 (0.17)	2.29 (0.58)	<b>0.30 (0.15)</b>	2.64 (0.01)	1.61 (0.00)	0.93 (0.00)
	$1 \times 10^4$	1.91 (0.27)	2.36 (0.23)	3.57 (0.43)	2.08 (0.23)	2.66 (0.02)	1.79 (0.01)	<b>1.13 (0.00)</b>
	$5 \times 10^4$	8.21 (0.51)	10.61 (0.49)	9.91 (0.10)	8.94 (0.32)	2.67 (0.02)	1.83 (0.00)	<b>1.19 (0.00)</b>
	$1 \times 10^5$	15.89 (0.36)	19.71 (1.43)	18.80 (1.93)	17.81 (0.21)	2.65 (0.01)	1.84 (0.01)	<b>1.20 (0.00)</b>
$n = 1000$	$1 \times 10^3$	–	–	–	–	–	–	–
	$1 \times 10^4$	<b>3.57 (0.37)</b>	3.57 (0.48)	11.09 (0.47)	3.70 (0.53)	21.06 (0.13)	13.71 (0.07)	8.53 (0.02)
	$5 \times 10^4$	14.80 (0.53)	16.62 (0.72)	22.92 (0.76)	15.71 (0.11)	21.20 (0.32)	14.36 (0.17)	<b>9.17 (0.08)</b>
	$1 \times 10^5$	28.93 (0.41)	33.47 (0.57)	36.57 (0.46)	31.17 (0.39)	21.01 (0.10)	14.44 (0.10)	<b>9.30 (0.09)</b>
$n = 2000$	$1 \times 10^3$	–	–	–	–	–	–	–
	$1 \times 10^4$	<b>6.37 (0.63)</b>	6.91 (0.83)	51.42 (2.11)	6.56 (0.83)	164.40 (0.58)	104.97 (1.01)	63.98 (0.90)
	$5 \times 10^4$	<b>29.55 (0.69)</b>	32.29 (0.55)	75.37 (3.20)	29.98 (0.63)	163.57 (1.81)	109.40 (0.52)	70.17 (0.60)
	$1 \times 10^5$	<b>58.86 (0.30)</b>	63.63 (0.43)	101.69 (1.47)	61.16 (0.42)	163.53 (1.94)	111.45 (1.11)	72.09 (0.98)