Random Variables

- A random variable is defined as a numerical event whose value is determined by a chance process.
- When probability values are assigned to all possible numerical values of a random variable X, either by a listing or by a mathematical function, the result is a ***probability distribution***.

Random Variables

- The sum of the probabilities for all the possible numerical outcomes must equal one.
- Individual probability values may be denoted by the symbol $f(x)$, which indicates that a mathematical function is involved, by $P(X = x)$, which recognizes that the random variable can have various specific values, or simply by P(x).

Random Variables

- The outcome of an experiment need not be a number, for example, the outcome when a coin is tossed can be ‘heads’ or ‘tails’.
- However, we often want to represent outcomes as numbers.
- A ***random variable*** is a function that associates a unique numerical value with every outcome of an experiment.
- The value of the random variable will vary from trial to trial as the experiment is repeated.

Random Variables

- Numeric values can be assigned to outcomes that are not usually considered numeric.
- For example, we could assign a ‘head’ a value of 0, and a ‘tail’ a value of 1, or vice versa.

Random Variables

There are two types of random variable - discrete and continuous. The distinction between both types will be important later on in the course.

Examples

- A coin is tossed ten times. The random variable X is the number of tails that are noted. X can only take the values $\{0, 1, ..., 10\}$, so X is a discrete random variable.
- A light bulb is burned until it burns out. The random variable Y is its lifetime in hours. Y can take any positive real value, so Y is a continuous random variable.

Discrete Random Variable

- A discrete random variable is one which may take on only a countable number of distinct values such as $\{0, 1, 2, 3, 4, ...\}$.
- Discrete random variables are usually (but not necessarily) counts.
- If a random variable can take only a finite number of distinct values, then it must be discrete.

Discrete Random Variable

- Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

Continuous Random Variable

- A continuous random variable is one which takes an infinite number of possible values.
- Continuous random variables are usually measurements.
- Examples include height, weight, the amount of sugar in an orange, the time required to run a computer simulation.

Discrete Random Variables

- For a discrete random variable observed values can occur only at isolated points along a scale of values. In other words, observed values must be integers.
- Consider a six sided die: the only possible observed values are 1, 2, 3, 4, 5 and 6.
- It is not possible to observe values that are real numbers, such as 2.091.
- *(Remark: it is possible for the average of a discrete random variable to be a real number.)*

Discrete Random Variables

- Therefore, it is possible that all numerical values for the variable can be listed in a table with accompanying probabilities.
- There are several standard probability distributions that can serve as models for a wide variety of discrete random variables involved in business applications.

Discrete probability distributions

The discrete probability distributions that described in this course are

- the binomial distribution,
- the geometric distribution,
- the hypergeometric distribution,
- the Poisson distributions.