MA4413 Autumn 2008 paper

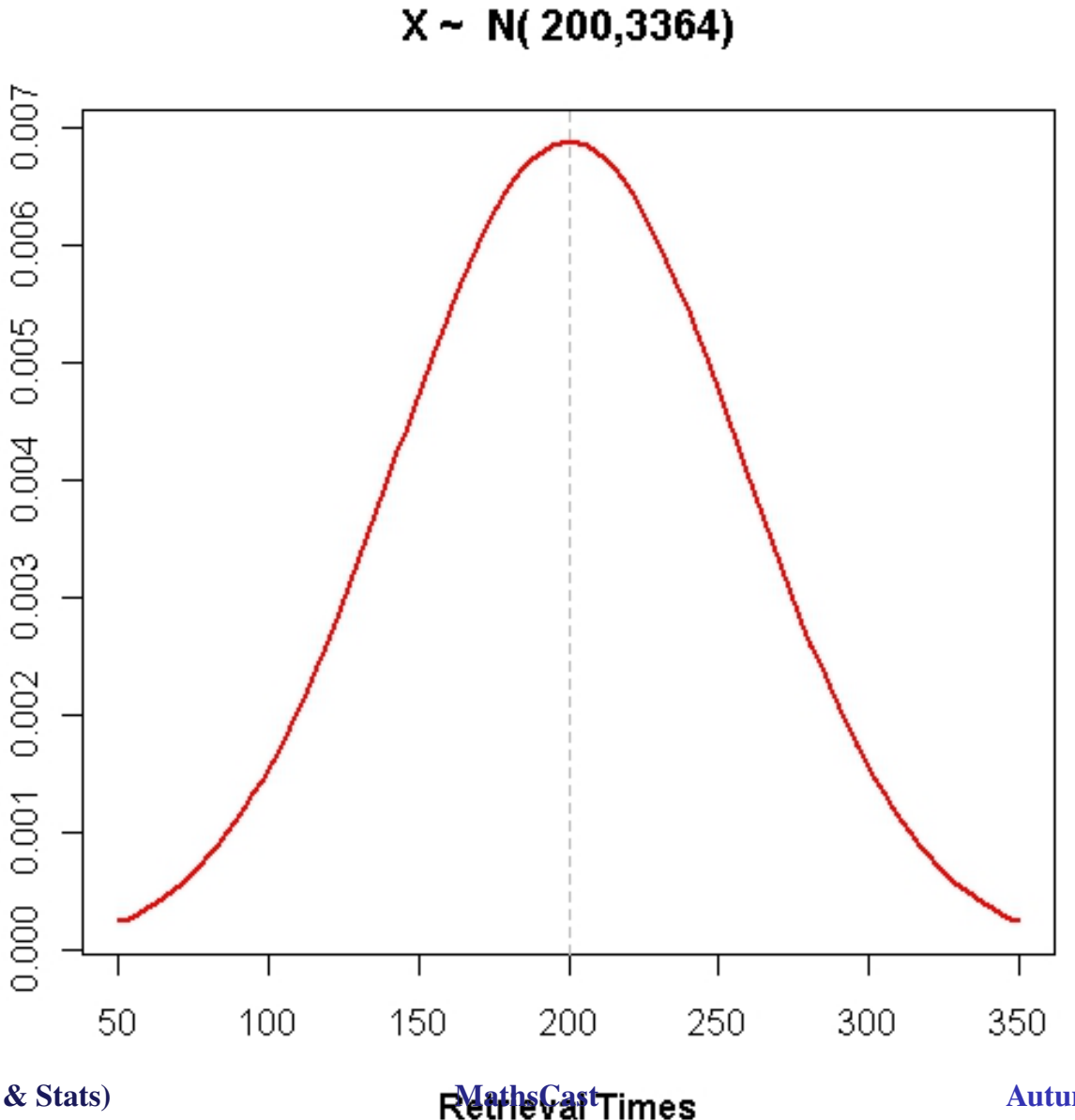
A model of an on-line computer system gives a mean times to retrieve a record from a direct access storage system device of 200 milliseconds, with a standard deviation of 58 milliseconds. If it can assumed that the retrieval times are normally distributed:

(i) What proportion of retrieval times will be greater than 75 milliseconds?

(ii) What proportion of retrieval times will be between 150 and 250 milliseconds?

(iii) What is the retrieval time below which 10% of retrieval times will be?

Normal Distribution



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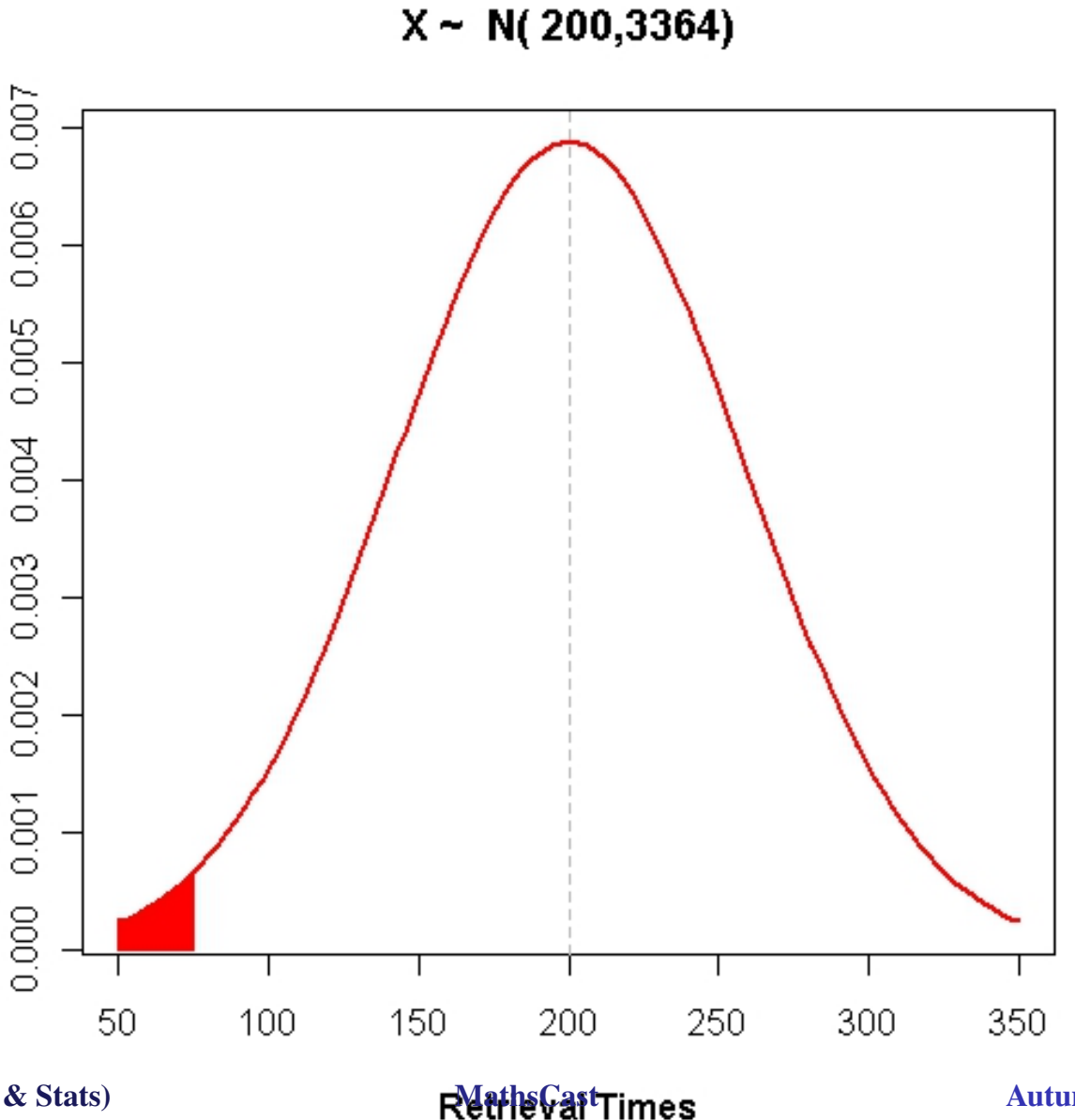
What proportion of retrieval times will be greater than 75 milliseconds?

- Let X be the retrieval times, with $X \sim \mathrm{N}(200, 58^2)$.
- The first question asks us to find $P(X \geq 75)$.
- First compute the z score.

$$z_o = \frac{x_o - \mu}{\sigma} = \frac{75 - 200}{58} = -2.15$$

Normal Distribution



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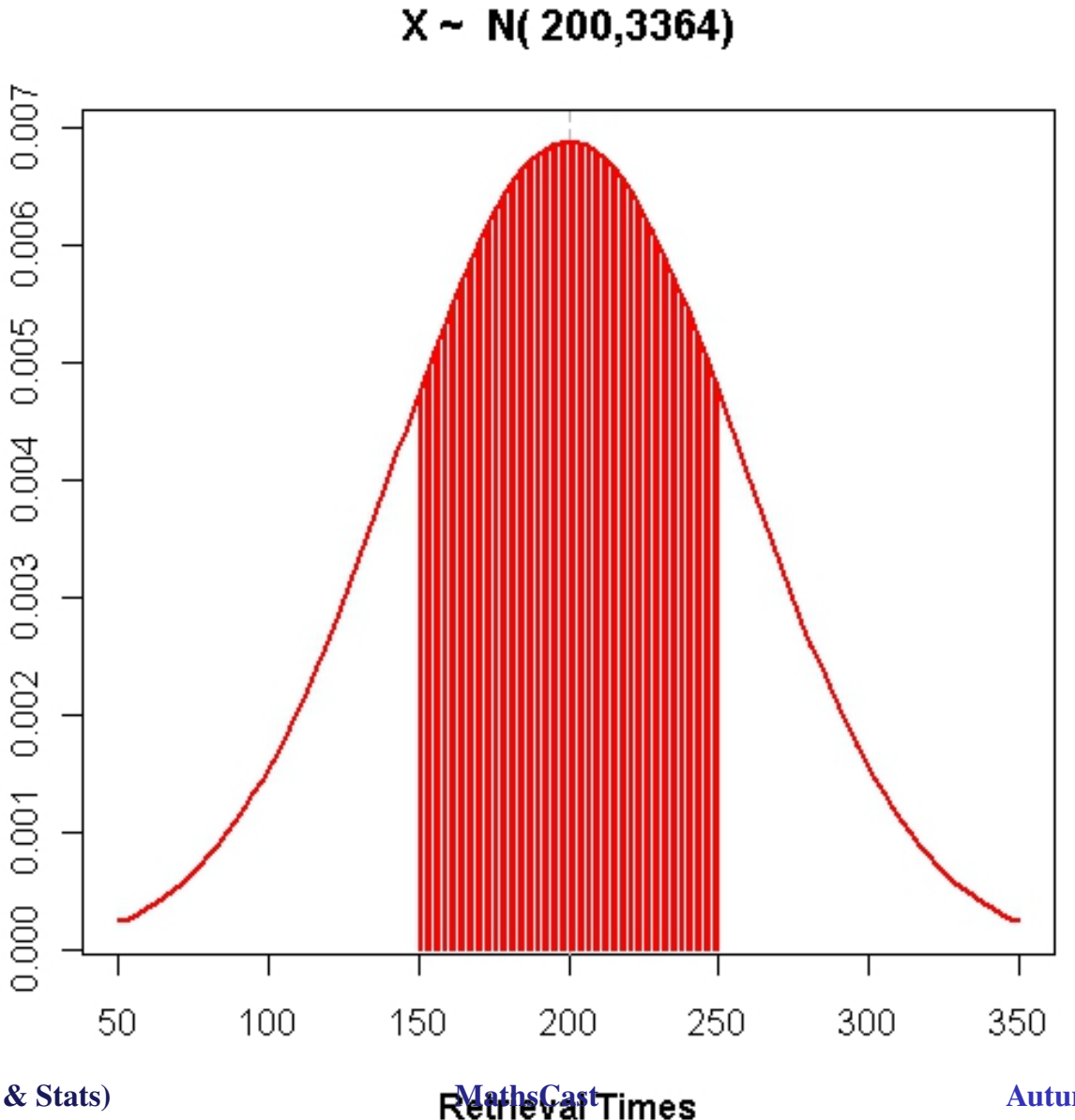
- We can say
$$P(X \geq 75) = P(Z \geq -2.15)$$
- Using symmetry rule and complement rule
$$P(Z \geq -2.15) = P(Z \leq 2.15) = 1 - P(Z \geq 2.15)$$
- From tables $P(Z \geq 2.15) = 0.0158$
- Therefore $P(Z \leq 2.15) = 0.9842$
- Furthermore $P(X \geq 75) = \mathbf{0.9842}$ [Answer].

Normal Distribution



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- What proportion of retrieval times will be between 150 and 250 milliseconds?
- Find $P(150 \leq X \leq 250)$
- Use the 'Too Low / Too High ' approach.
- Too low $P(X \leq 150)$
- Too high $P(X \geq 250)$
- Find the z-scores for each.

$$z_{150} = \frac{150 - 200}{58} = -0.86$$

$$z_{250} = \frac{250 - 200}{58} = 0.86$$

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- We can now say

$$1. P(X \leq 150) = P(Z \leq -0.86)$$

$$2. P(X \geq 250) = P(Z \geq 0.86)$$

- By symmetry rule, $P(Z \leq -0.86) = P(Z \geq 0.86)$

$$P(X \leq 150) = P(X \geq 250)$$

- Let's compute $P(X \geq 250)$. Using tables

$$P(X \geq 250) = P(Z \geq 0.86) = 0.1949$$

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- Too high: $P(X \geq 250) = 0.1949$
- Too low: $P(X \leq 150) = 0.1949$
- Probability of being inside interval:

$$P(150 \leq X \leq 250) = 1 - [P(X \leq 150) + P(X \geq 250)]$$

- $P(150 \leq X \leq 250) = 1 - [0.1949 + 0.1949] = \mathbf{0.6102}$

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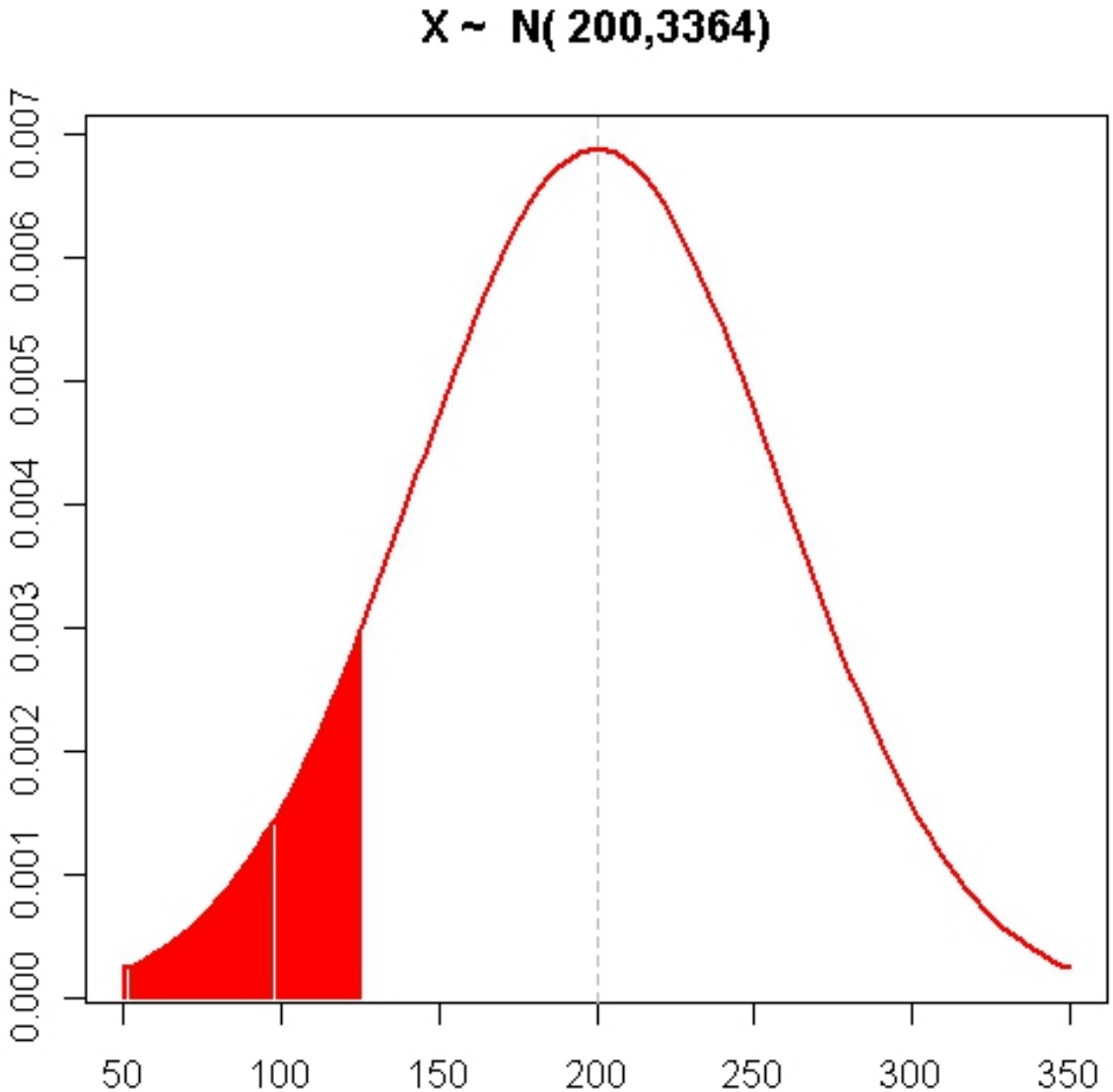
- What is the retrieval time below which 10% of retrieval times will be?
- Find A such that $P(X \leq A) = 0.10$.
- What z-score would correspond to A? Lets call it z_A.
- $P(Z \leq z_A) = 0.10$
- Remark: z_A could be negative.
- Using symmetry $P(Z \geq -z_A) = 0.10$
- Remark: $-z_A$ could be positive.

Normal Distribution



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- Use the Murdoch Barnes tables to get an approximate value for $-z_A$.
- The nearest value we can get is 1.28. ($P(Z \geq 1.28) = 0.1003$).
- If $-z_A = 1.28$, then $z_A = -1.28$
- We can now say

$$P(X \leq A) = P(Z \leq -1.28)$$

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- Necessarily A and Z_A are related by the standardization formula
- Recall that $\mu = 200$ and $\sigma = 58$.

$$-1.28 = \frac{A - 200}{58}$$

- Re-arranging (multiply both sides by 58)

$$-74.24 = A - 200$$

- Re-arranging again (Add 200 to both sides)

$$125.76 = A$$

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- Now we know the retrieval time below which 10% of retrieval times will be.
- $P(X \leq 125.76) = 0.10$ [Answer].