Overview of Current Part of Course

Probability Distributions (Question 2 for End Of Year Exam)

- Discrete Probability Distributions
  - Binomial Probability Distribution (Week 3)
  - Geometric Probability Distribution (Week 3)
  - Poisson Probability Distribution (Week 3/4)
- Continuous Probability Distributions
  - Exponential Probability Distribution (Week 4)
  - Uniform Probability Distribution (Week 4)
  - Normal Probability Distribution (Week 4/5)

Continuous Random variables

- Previously we have been studying discrete random variables, such as the Binomial and the Poisson random variables.
- Now we turn our attention to continuous random variables.
- Recall that a continuous random variable is one which takes an infinite number of possible values, rather than just a countable number of distinct values.
- Continuous random variables are usually measurements.
- Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

Exact Probabilities

Remarks: This is for continuous distributions only.

- The probability that a continuous random variable will take an exact value is infinitely small. We will usually treat it as if it was zero.
- When we write probabilities for continuous random variables in mathematical notation, we often retain the equality component (i.e. the "...or equal to..").
  For example, we would write expressions $P(X \leq 2)$ or $P(X \geq 5)$.
- Because the probability of an exact value is almost zero, these two expression are equivalent to $P(X < 2)$ or $P(X > 5)$.
- Also, the complement of $P(X \geq k)$ can be written as $P(X \leq k)$.

Functions and Definite integrals

- Integration is not part of the syllabus, and it is assumed that students are not familiar with how to compute definite integrals.
- However, it is useful to know what the purpose of definite integrals are, because we will be using the results derived from definite integrals.
- It is assumed that students are familiar with functions.

Functions

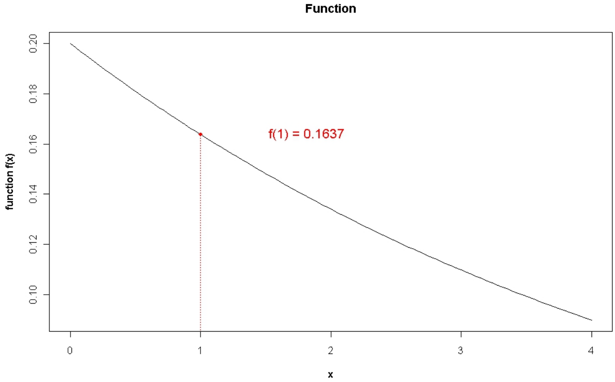
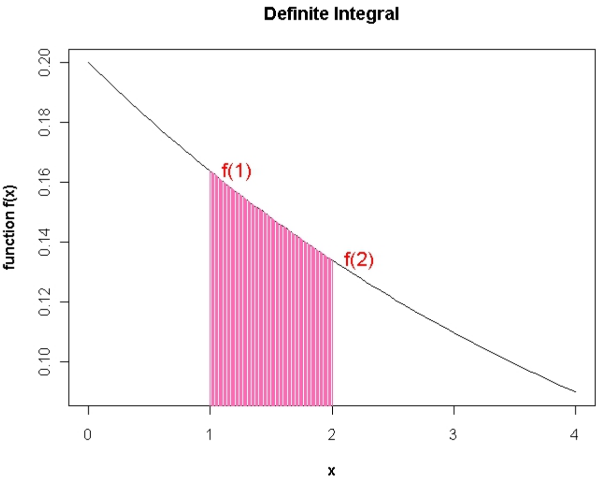


Figure:

Some function $f(x)$ evaluated at $x = 1$.

Definite Integral



Definite integral of function is area under curve between X=1 and X=2.

Definite Integral

- Definite integrals are used to compute the "***area under curves***".
- Definite integrals are defined by a lower and upper limit.
- The area under the curve between X=1 and X=2 is depicted in the previous slide.
- By computing the definite integral, we are able to determine a value for this area.
- Probability can be represented as an area under a curve.

Probability Density Function

- In probability theory, a ***probability density function*** (PDF) (or "density" for short) of a continuous random variable is a function that describes the relative likelihood for this random variable to occur at a given point.
- The PDF for a continuous random variable X is often denoted $f(x)$.
- The probability density function can be integrated to obtain the probability that the random variable takes a value in a given interval.
- The probability for the random variable to fall within a particular interval is given by the integral of this variable's density over the region.
- The probability density function is non-negative everywhere, and its integral over the entire space is equal to one.

Density Curves

- A plot of the PDF is referred to as a '***density curve***'.
- A density curve that is always on or above the horizontal axis and has total area underneath equal to one.
- Area under the curve in a range of values indicates the proportion of values in that range.
- Density curves come in a variety of shapes, but the normal distribution's bell-shaped densities are perhaps the most commonly encountered.
- Remember the density is only an approximation, but it simplifies analysis and is generally accurate enough for practical use.

The Cumulative Distribution Function

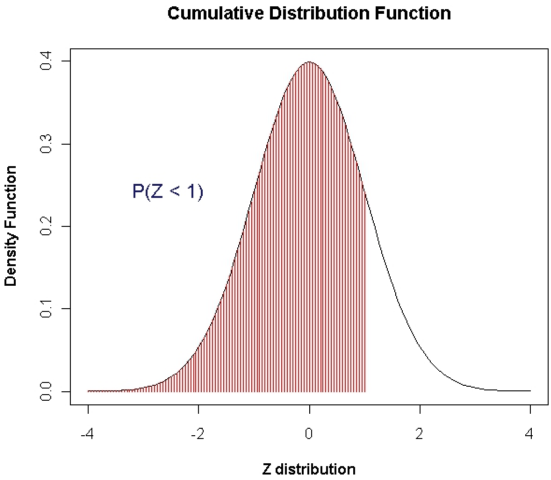
Recall:

- The ***cumulative distribution function*** (CDF), (or just distribution function), describes the probability that a continuous random variable X with a given probability distribution will be found at a value less than or equal to x.

$$F(x) = P(X \leq x)$$

- Intuitively, it is the "area so far" function of the probability distribution.

Cumulative Distribution Function



Cumulative Distribution Function $P(Z \leq 1)$
Here the random variable is called Z (we will see why later)

Continuous Random Variables

- Probability Density Function
- Cumulative Density Function

If X is a continuous random variable then we can say that the probability of obtaining a **precise** value x is infinitely small, i.e. close to zero.

$$P(X = x) \approx 0$$

Consequently, for continuous random variables (only), $P(X \leq x)$ and $P(X < x)$ can be used interchangeably.

$$P(X \leq x) \approx P(X < x)$$

Continuous Uniform Distribution

A random variable X is called a continuous uniform random variable over the interval (a,b) if it's probability density function is given by

$$f_X(x) = \frac{1}{b-a} \qquad \text{when } a \leq x \leq b$$

The corresponding cumulative density function is

$$F_x(x) = \frac{x-a}{b-a} \qquad \text{when } a \leq x \leq b$$

The mean of the continuous uniform distribution is

$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

The Memoryless property

The most interesting property of the exponential distribution is the ***memoryless*** property. By this , we mean that if the lifetime of a component is exponentially distributed, then an item which has been in use for some time is a good as a brand new item with regards to the likelihood of failure.
The exponential distribution is the only distribution that has this property.