Statistics and Probability

Probability Functions

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Probability Functions

Suppose X is a random variable with probability density function

$$f_X(x) = a + bx^2$$

over the range $(0, 2)$, where a and b are constants. The mean value of X is 1.5.

1. Find a and b,
2. Find $F_X(x)$, the cumulative distribution function of X,
3. Find the variance of X.

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Question 1

- Compute the coefficients a and b.

Remarks:

Total area under the curve defined by the probability density function (between 0 and 2) must equal 1. (by definition.)

The expected value μ (or $E(X)$) of random variable X is 1.5.

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Remark 1

Total area under the curve defined by the probability density function (between 0 and 2) must equal 1. (by definition.)

$$\int_0^2 a + bx^2 \, dx = 1$$

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Remark 2

The expected value μ (or $E(X)$) of random variable X is 1.5.

$$\mu = \int_0^2 x f_X(x)\, dx = 1.5$$

$$\mu = \int_0^2 x(a + bx^2)\, dx = 1.5$$

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- $2a + \frac{8b}{3} = 1$
- $2a + 4b = 1.5$

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Question 2

- Find $F_X(x)$, the cumulative distribution function of X,

$$F_X(x) = \int_0^x f_X(x)\,dx.$$

$$F_X(x) = \int_0^x (a + bx^2)\,dx.$$

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$$F_X(x) = \int_0^x \frac{3}{8}x^2\, dx.$$

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Computing the Variance

Definitions:

$$\mathrm{Var}(X) = \sigma^2 = \int (x - \mu)^2 \, f_X(x) \, dx$$

$$\mathrm{Var}(X) = \sigma^2 = \int x^2 \, f_X(x) \, dx \; - \mu^2$$

For this example :

$$= \int_0^2 x^2 \, \left(a + bx^2\right) \, dx \; - (1.5)^2$$

$$= \int_0^2 (ax^2 + bx^4) \, dx \; - (1.5)^2$$