Random Variables

A pair of dice is thrown. Let X denote the minimum of the two numbers which occur. Find the distributions and expected value of X.

Random Variables

A fair coin is tossed four times. Let X denote the longest string of heads. Find the distribution and expectation of X.

Random Variables

A fair coin is tossed until a head or five tails occurs. Find the expected number E of tosses of the coin.

Random Variables

A coin is weighted so that $P(H) = 0.75$ and $P(T) = 0.25$
The coin is tossed three times. Let X denote the number of heads that appear.

- (a) Find the distribution f of X.
- (b) Find the expectation $E(X)$.

Graphical Procedures for Statistics

- Bar-plots
- Histograms
- Boxplots

Histograms

- Consider an experiment in which each student in a class of 60 rolls a die 100 times.
- Each score is recorded, and a total score is calculated.
- As the expected value of rolled die is 3.5, the expected total is 350 for each student.
- At the end of the experiment the students reported their totals.
- The totals were put into ascending order, and tabulated as follows (next slide).

Outcomes of die-throw experiment

307	321	324	328	329	330	334	335	336	337
337	337	338	339	339	342	343	343	344	344
346	346	347	348	348	348	350	351	352	352
353	353	353	354	354	356	356	357	357	358
358	360	360	361	362	363	365	365	369	369
370	370	374	378	381	384	385	386	392	398

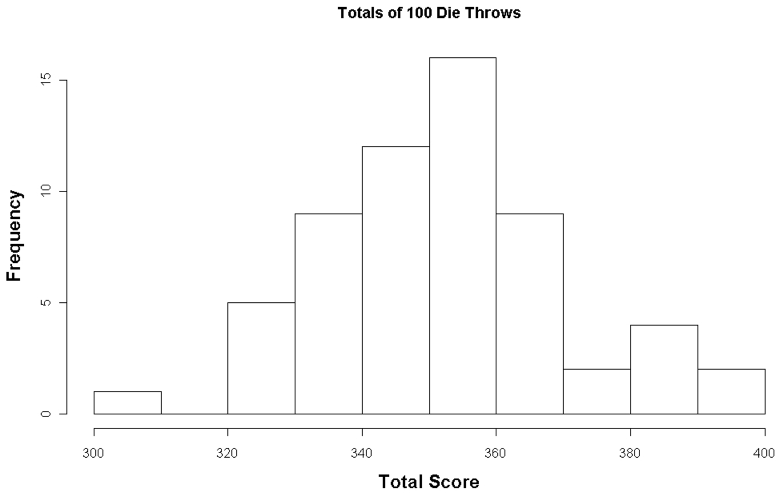
- What proportion of outcomes are less than or equal to 330? (Answer: 10%)
- What proportion of outcomes are greater than or equal to 370? (Answer: 16.66%)

What is a Histograms

TEXT HERE

Histograms

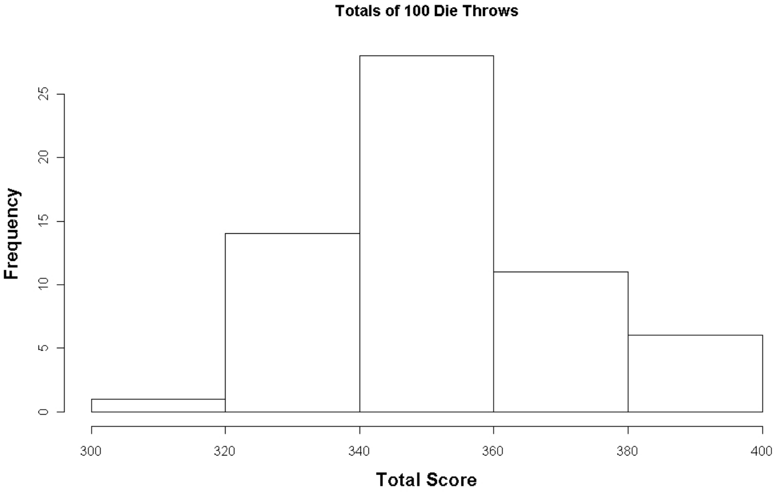
For the die-throw experiment;



Constructing Histograms

- Compute an appropriate number of class intervals.
- As a rule of thumb, the number of class intervals is usually approximately the square root of the number of observations.
- As there are 60 observations, we would normally use 7 or 8 class intervals.
- To save time, we will just use 5 class intervals.

Histograms

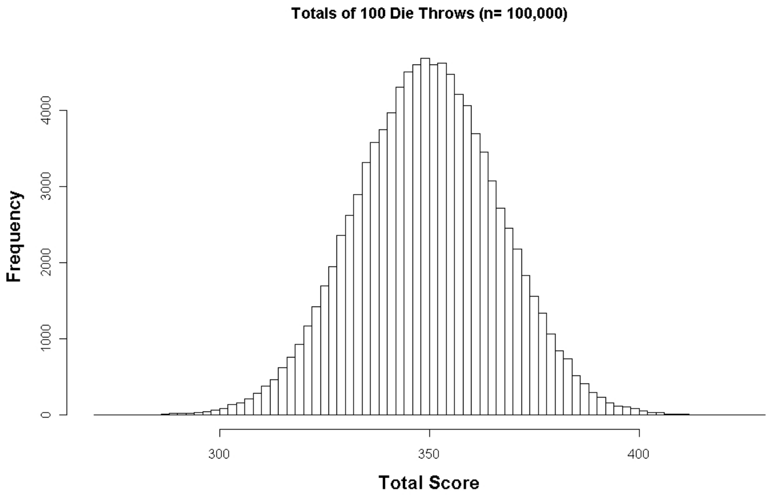


Totals of 100 Die Throws
Frequency
0
5
10
15
20
25
300
320
340
360
380
400
Total Score

Histograms

- Suppose that the experiment of throwing a die 100 times and recording the total was repeated 100,000 times.
- (If implemented on a computer, we would call this a simulation study)
- The histogram of data (with a class interval width of 2) is shown on the next slide.
- How should the shape of the histogram be described?
- “Bell-shaped” would be a suitable description.

Histograms



Totals of 100 Die Throws (n= 100,000)
Frequency
0
1000
2000
3000
4000
300
350
400
Total Score

Simulation Study

A couple of remarks about the simulation study, some of which will be relevant later on.

- Approximately 68.7% of the values in the simulation study are between 332 and 367.
- Approximately 95% of the values are between 316 and 383.
- 2.5% of the values output are less than 316.
- 2.5% of the values study output are greater than 383.
- 175 values are greater than or equal to 400, whereas 198 values are less than or equal to 300.
- Results such as these are unusual, but they are not impossible.

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The coin is tossed three times. Let X denote the number of heads that appear.

- (a) Find the distribution f of X.
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- Now consider an experiment with only two outcomes. Independent repeated trials of such an experiment are called Bernoulli trials, named after the Swiss mathematician Jacob Bernoulli (16541705).
- The term ***independent trials*** means that the outcome of any trial does not depend on the previous outcomes (such as tossing a coin).
- We will call one of the outcomes the “success” and the other outcome the “failure”.

- Let p denote the probability of success in a Bernoulli trial, and so $q = 1 - p$ is the probability of failure. A binomial experiment consists of a fixed number of Bernoulli trials.
- A binomial experiment with n trials and probability p of success will be denoted by

$$B(n,p)$$