Introduction to Statistics and Probability

Probability : Contingency Tables

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Probability

- The symbol P is used to designate the probability of an event. Thus P(A) denotes the probability that event A will occur in a single observation or experiment.
- The smallest value that a probability statement can have is 0 (indicating the event is impossible) and the largest value it can have is 1 (indicating the event is certain to occur).
- Thus, in general: $0 \leq P(A) \leq 1$

Mutually exclusive events

- In a given observation or experiment, an event must either occur or not occur.
- Therefore, the sum of the probability of occurrence plus the probability of nonoccurrence always equals 1.
- Thus, where A' indicates the nonoccurrence of event A, we have $P(A) + P(A^c) = 1$

Mutually exclusive events

- Two or more events are mutually exclusive, or disjoint, if they cannot occur together. That is, the occurrence of one event automatically precludes the occurrence of the other event (or events).
- For instance, suppose we consider the two possible events "ace" and "king" with respect to a card being drawn from a deck of playing cards.
- These two events are mutually exclusive, because any given card cannot be both an ace and a king.
- Two or more events are nonexclusive when it is possible for them to occur together.

- Note that this definition does not indicate that such events must necessarily always occur jointly. For instance, suppose we consider the two possible events "ace" and "spade".
- These events are not mutually exclusive, because a given card can be both an ace and a spade; however, it does not follow that every ace is a spade or every spade is an ace.

General rule of addition

- For events that are not mutually exclusive, the probability of the joint occurrence of the two events is subtracted from the sum of the simple probabilities of the two events. We can represent the probability of joint occurrence by P(A and B).
- In the language of set theory this is called the intersection of A and B and the probability is designated by P(A and B). Thus, the rule of addition for events that are not mutually exclusive is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example

When drawing a card from a deck of playing cards, the events "ace" and "spade" are not mutually exclusive. The probability of drawing an ace (A) or spade (S) (or both) in a single draw is

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(S) - P(A \text{ and } B) && (1)\\
&= 4/52 + 13/52 - 1/52 && (2)\\
&= 16/52 && (3)\\
&= \mathbf{4/13} && (4)
\end{aligned}
$$

- Two events are independent when the occurrence or nonoccurrence of one event has no effect on the probability of occurrence of the other event.
- Two events are dependent when the occurrence or nonoccurrence of one event does affect the probability of occurrence of the other event.

- Historically, three different conceptual approaches have been developed for defining probability and for determining probability values: the classical, relative frequency, and subjective approaches.
- If N(A) possible elementary outcomes are favorable to event A, N(S) possible outcomes are included in the sample space, and all the elementary outcomes are equally likely and mutually exclusive, then the probability that event A will occur is

$$P(A) = \frac{N(A)}{N(S)}$$

Examples

When a fair dice is thrown, what are the possible outcomes? There are 6 possible outcomes.
The dice can role any number between one and six. Each outcome is equally likely. The probability of each outcome is 1/6.

In a well-shuffled deck of cards which contains 4 aces and 48 other cards, the probability of an ace (A) being obtained on a single draw is;

$$P(A) = N(A)/N(S) = 4/52 = 1/13$$

Bayes theorem

In its simplest algebraic form, Bayes theorem is concerned with determining the conditional probability of event A given that event B has occurred.
The general form of Bayes theorem is

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Probability Rules

- The probability of an event which cannot occur is 0.
- The probability of any event which is not in the sample space is zero.
- The probability of an event which must occur is 1.
- The probability of the sample space is 1.

Probability Rules

The Complement Rule

- The probability of an event not occurring is one minus the probability of it occurring.

$$P(E^C) = 1 - P(E)$$

Probability: Addition Rule for Any Two Events

- For any two events A and B, the probability of A or B is the sum of the probability of A and the probability of B minus the probability of both A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- We subtract the probability of $A \cap B$ to prevent it getting counted twice.
- *($A \cup B$ and $A \cap B$ denotes "A or B" and "A and B" respectively)*

Probability: Addition Rule for Any Two Events

- If events A and B are **mutually exclusive**, then the probability of A or B is the sum of the probability of A and the probability of B:

$$P(A \cup B) = P(A) + P(B)$$

- If A and B are mutually exclusive, then the probability of both A and B is zero.

Introduction to Probability

Combining Probabilities

Events rarely occur in isolation. Usually we are interested in a combination or compound of events; for example

- The probability that two sections of a factory will be understaffed on the same day
- The probability of having a car accident today, given that you have had a car accident in the last five years.

We will look at two laws of probability for combining events

- The Addition Law
- The multiplication Law

Introduction to Probability

Conditional Probability The conditional probability of an event is the probability that an event A occurs given that another event B has already occurred. This type of probability is calculated by restricting the sample space that were working with to only the set B.
The formula for conditional probability can be rewritten using some basic algebra. Instead of the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Introduction to Probability

Probability trees The setting out of solutions to problems requiring the manipulation of the probabilities of mutually exclusive and independent events can sometimes be helped by the use of probability tree diagrams. These have useful applications in decision theory.

The best choice of probability tree structure often depends upon the question and the natural order in which events like A and B above occur.

Introduction to Probability

Histograms A histogram is constructed from a frequency table. The intervals are shown on the X-axis and the number of scores in each interval is represented by the height of a rectangle located above the interval. A histogram of the response times from the dataset Target RT is shown below.

Introduction to Probability

Cumulative Distribution Function

The cumulative distribution function (c.d.f.) of a discrete random variable X is the function F(t) which tells you the probability that X is less than or equal to t. So if X has p.d.f. P(X = x), we have:

$$F(t) = P(X \leq 1)$$

In other words, for each value that X can be which is less than or equal to t, work out the probability that X is that value and add up all such results.

Introduction to Probability

Example

In the above example where the die is thrown repeatedly, lets work out $P(X \leq t)$ for some values of t.

P(X $\leq$ 1) is the probability that the number of throws until we get a 6 is less than or equal to 1. So it is either 0 or 1.

- P(X = 0) = 0
- $P(X = 1) = 1/6$.
- Hence $P(X \leq 1) = 1/6$

Similarly, $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
= 0 + 1/6 + 5/36 = 11/36

Conditional Probability

Suppose B is an event in a sample space S with $P(B) > 0$. The probability that an event A occurs once Bhas occurred or, specifically, the conditional probability of A given B (written $P(A|B)$), is defined as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- This can be expressed as a multiplication theorem

$$P(A \cap B) = P(A|B) \times P(B)$$

- The symbol $|$ is a vertical line and does not imply division.
- Also $P(A|B)$ is not the same as $P(B|A)$.

Remark: The Prosecutor's Fallacy , with reference to the O.J. Simpson trial.

Independent Events

Events A and B in a probability space S are said to be independent if the occurrence of one of them does not influence the occurrence of the other.

More specifically, B is independent of A if $P(B)$ is the same as $P(B|A)$. Now substituting $P(B)$ for $P(B|A)$ in the multiplication theorem from the previous slide yields.

$$P(A \cap B) = P(A) \times P(B)$$

We formally use the above equation as our definition of independence.

Mutually Exclusive Events

- Two events are mutually exclusive (or disjoint) if it is impossible for them to occur together.
- Formally, two events A and B are mutually exclusive if and only if $A \cap B = \varnothing$

Consider our die example

- Event A = 'observe an odd number' = $\{1,3,5\}$
- Event B = 'observe an even number' = $\{2,4,6\}$
- $A \cap B = \varnothing$ (i.e. the empty set), so A and B are mutually exclusive.

Addition Rule

The addition rule is a result used to determine the probability that event A or event B occurs or both occur. The result is often written as follows, using set notation:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- $P(A)$ = probability that event A occurs.
- $P(B)$ = probability that event B occurs.
- $P(A \cup B)$ = probability that either event A or event B occurs, or both occur.
- $P(A \cap B)$ = probability that event A and event B both occur.

Remark: $P(A \cap B)$ is subtracted to prevent the relevant outcomes being counted twice.

Addition Rule (Continued)

For mutually exclusive events, that is events which cannot occur together: $P(A \cap B) = 0$. The addition rule therefore reduces to

$$P(A \cup B) = P(A) + P(B)$$

Addition Rule: Worked Example

Suppose we wish to find the probability of drawing either a Queen or a Heart in a single draw from a pack of 52 playing cards. We define the events Q = 'draw a queen' and H = 'draw a heart'.

- $P(Q)$ probability that a random selected card is a Queen
- $P(H)$ probability that a randomly selected card is a Heart.
- $P(Q \cap H)$ probability that a randomly selected card is the Queen of Hearts.
- $P(Q \cup H)$ probability that a randomly selected card is a Queen or a Heart.

Solution

- Since there are 4 Queens in the pack and 13 Hearts, so the $P(Q) = 4/52$ and $P(H) = 13/52$ respectively.
- The probability of selecting the Queen of Hearts is $P(Q \cap H) = 1/52$.
- We use the addition rule to find $P(Q \cup H)$:

$$P(Q \cup H) = (4/52) + (13/52) - (1/52) = 16/52$$

- So, the probability of drawing either a queen or a heart is $16/52 (= 4/13)$.

Multiplication Rule

The multiplication rule is a result used to determine the probability that two events, A and B, both occur. The multiplication rule follows from the definition of conditional probability.

The result is often written as follows, using set notation:

$$P(A|B) \times P(B) = P(B|A) \times P(A) \qquad (= P(A \cap B))$$

Recall that for independent events, that is events which have no influence on one another, the rule simplifies to:

$$P(A \cap B) = P(A) \times P(B)$$

Multiplication Rule

From the first year intake example, check that

$$P(E|F) \times P(F) = P(F|E) \times P(E)$$

- $P(E|F) \times P(F) = 0.58 \times 0.38 = 0.22$
- $P(F|E) \times P(E) = 0.55 \times 0.40 = 0.22$

Law of Total Probability

The law of total probability is a fundamental rule relating marginal probabilities to conditional probabilities. The result is often written as follows, using set notation:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

where $P(A \cap B^c)$ is probability that event A occurs and B does not.

Using the multiplication rule, this can be expressed as

$$P(A) = P(A|B) \times P(B) + P(A|B^c) \times P(B^c)$$

Law of Total Probability

From the first year intake example , check that

$$P(E) = P(E \cap M) + P(E \cap F)$$

with $P(E) = 0.40$, $P(E \cap M) = 0.18$ and $P(E \cap F) = 0.22$

$$0.40 = 0.18 + 0.22$$

Remark: M and F are complement events.

Bayes' Theorem

Bayes' Theorem is a result that allows new information to be used to update the conditional probability of an event.

Recall the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Using the multiplication rule, gives Bayes' Theorem in its simplest form:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Probability: Worked Example

An electronics assembly subcontractor receives resistors from two suppliers: Deltatech provides 70% of the subcontractors's resistors while another company, Echelon, supplies the remainder.
1% of the resistors provided by Deltatech fail the quality control test, while 2% of the resistors from Echelon also fail the quality control test.

1. What is the probability that a resistor will fail the quality control test?
2. What is the probability that a resistor that fails the quality control test was supplied by Echelon?

Probability: Worked Example

Firstly, let's assign names to each event.

- D : a randomly chosen resistor comes from Deltatech.
- E : a randomly chosen resistor comes from Echelon.
- F : a randomly chosen resistor fails the quality control test.
- P : a randomly chosen resistor passes the quality control test.

We are given (or can deduce) the following probabilities:

- $P(D) = 0.70$,
- $P(E) = 0.30$.

Probability: Worked Example

We are given two more important pieces of information:

- The probability that a randomly chosen resistor fails the quality control test, given that it comes from Deltatech: $P(F|D) = 0.01$.
- The probability that a randomly chosen resistor fails the quality control test, given that it comes from Echelon: $P(F|E) = 0.02$.

Probability: Worked Example

The first question asks us to compute the probability that a randomly chosen resistor fails the quality control test. i.e. $P(F)$.

All resistors come from either Deltatech or Echelon. So, using the ***law of total probability***, we can express $P(F)$ as follows:

$$P(F) = P(F \cap D) + P(F \cap E)$$

Probability: Worked Example

Using the ***multiplication rule*** i.e. $P(A \cap B) = P(A|B) \times P(B)$, we can re-express the formula as follows

$$P(F) = P(F|D) \times P(D) + P(F|E) \times P(E)$$

We have all the necessary probabilities to solve this.

$$P(F) = 0.01 \times 0.70 + 0.02 \times 0.30 = 0.007 + 0.006 = 0.013$$

Probability: Worked Example

- The second question asks us to compute probability that a resistor that fails the quality control test was supplied by Echelon.
- In other words; of the resistors that did fail the quality test only, what is the probability that a randomly selected resistor was supplied by Echelon?
- We can express this mathematically as $P(E|F)$.
- We can use ***Bayes' theorem*** to compute the answer.

Probability: Worked Example

Recall Bayes' theorem

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$$P(E|F) = \frac{P(F|E) \times P(E)}{P(F)} = \frac{0.02 \times 0.30}{0.013} = 0.46$$

More on probability

For this lecture and the next.

1. Contingency Tables
2. Conditional Probability: Worked Examples
3. Joint Probability Tables
4. The Multiplication Rule
5. Law of Total Probability
6. Bayes' Theorem
7. Exam standard Probability Question
8. Random Variables

This Presentation

1. Contingency Tables
2. Conditional Probability: Worked Examples
3. Joint Probability Tables
4. The Multiplication Rule
5. Law of Total Probability

What is a contingency table?

A contingency table is essentially a display format used to analyse and record the relationship between two or more categorical variables. It is the categorical equivalent of the scatterplot used to analyse the relationship between two continuous variables.

Contingency Tables

Suppose there are 100 students in a first year college intake.

- 44 are male and are studying computer science,
- 18 are male and studying engineering
- 16 are female and studying computer science,
- 22 are female and studying engineering.

Contingency Tables

We assign the names M, F, C and E to these events that a student, randomly selected from this group is:

- **M** Male
- **F** Female
- **C** Studying Computer Science
- **E** Studying Engineering

Contingency Tables

- The most effective way to handle this data is to draw up a table. We call this a ***contingency table***.
- A contingency table is a table in which all possible outcomes for one variable are listed as row headings and all possible outcomes for a second variable are listed as column headings.
- The value entered in each cell of the table is the frequency of each joint occurrence.

Contingency Tables

For the Student Intake example

	C	E	Total
M	44	18	62
F	16	22	38
Total	60	40	100

Contingency Tables

It is now easy to deduce the probabilities of the respective events, by looking at the totals for each row and column.

We call these probabilities the ***marginal probabilities***.

- P(C) = 60/100 = 0.60
- P(E) = 40/100 = 0.40
- P(M) = 62/100 = 0.62
- P(F) = 38/100 = 0.38

Marginal Probabilities

- In the context of joint probability tables, a ***marginal probability*** is so named because it is a marginal total of a row or a column.
- Whereas the probability values in the cells of the table are probabilities of joint occurrence, the marginal probabilities are the simple (i.e. unconditional) probabilities of particular events.

Remark:
The information we were originally given can also be expressed as:

- $P(C \cap M) = 44/100 = 0.44$
- $P(C \cap F) = 16/100 = 0.16$
- $P(E \cap M) = 18/100 = 0.18$
- $P(E \cap F) = 22/100 = 0.22$

We can call these probabilities the ***joint probabilities***.

Joint Probability Tables

- A ***joint probability table*** is similar to a contingency table, but for that the value entered in each cell of the table is the probability of each joint occurrence.
- Often, the probabilities in such a table are based on observed frequencies of occurrence for the various joint events.

Joint Probability Tables

	C	E	Total
M	0.44	0.18	0.62
F	0.16	0.22	0.38
Total	0.60	0.40	1.00

Conditional Probabilities : Example 1

Recall the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- $P(A|B)$: probability of A **given** B,
- $P(A \cup B)$: joint probability of A and B,
- $P(B)$: probability of B.

Using the conditional probability formula, compute the following:

(1) $P(C|M)$: Probability that a student is a computer science student, given that he is male.

(2) $P(E|M)$: Probability that a student studies engineering, given that he is male.

(3) $P(F|E)$: Probability that a student is female, given that she studies engineering.

(4) $P(E|F)$: Probability that a student studies engineering, given that she is female.

Part 1) Probability that a student is a computer science student, given that he is male.

$$P(C|M) = \frac{P(C \cap M)}{P(M)} = \frac{0.44}{0.62} = 0.71$$

Part 2) Probability that a student studies engineering, given that he is male.

$$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{0.18}{0.62} = 0.29$$

Part 3) Probability that a student is female, given that she studies engineering.

$$P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{0.22}{0.40} = 0.55$$

Part 4) Probability that a student studies engineering, given that she is female.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.22}{0.38} = 0.58$$

Remark: $P(E \cap F)$ is the same as $P(F \cap E)$.

Multiplication Rule

- This useful multiplication rule follows from the definition of conditional probability.
- First we algebraically re-arrange the conditional probability equation.

$$P(A \cap B) \; = \; P(A|B) \times P(B).$$

- Equivalently $P(A \cap B) \; = \; P(B|A) \times P(A)$.
- Therefore we can say:

$$P(A|B) \times P(B) \; = \; P(B|A) \times P(A).$$

Multiplication Rule

As an aside, for **independent events**, (events which have no influence on one another), the multiplication rule simplifies to:

$$P(A \cap B) = P(A) \times P(B)$$

Multiplication Rule

Going back to our example:

From the first year intake example, check that

$$P(E|F) \times P(F) = P(F|E) \times P(E)$$

- $P(E|F) \times P(F) = 0.58 \times 0.38 = 0.22$
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Law of Total Probability

- The law of total probability is a fundamental rule relating marginal probabilities to conditional probabilities.
- The result is often written as follows:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

- Here $P(A \cap B^c)$ is joint probability that event A occurs and B does not.

Law of Total Probability

Using the multiplication rule, this can be expressed as

$$P(A) = [P(A|B) \times P(B)] + [P(A|B^c) \times P(B^c)]$$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

Law of Total Probability

From the first year intake example , check that

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$$0.40 = 0.18 + 0.22$$

Remark: M and F are complement events.

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Probability Trees

- The setting out of solutions to problems requiring the manipulation of the probabilities of mutually exclusive and independent events can sometimes be helped by the use of probability tree diagrams. These have useful applications in decision theory.
- The best choice of probability tree structure often depends upon the question and the natural order in which events like A and B above occur.

Probability Trees: Example

- Two gamblers, A and B, are playing each other in a tournament to win a jackpot worth $\$6,000$.
- The first gambler to win 5 rounds, wins the tournament, and the jackpot outright.
- Each player has an equal chance of winning each round. Also, a tie is not possible.

Probability Trees: Example

- The tournament is suspended after the seventh round. At this point A has won 3 rounds, while B has won 4.
- They agree to finish then and divide up the jackpot, according to how likely an outright victory would have been for both.

How much money did A end up with?

Probability Trees: Example

- Consider that A needed to win two more rounds, while B only need to win one more.
- One could suppose that B was twice as likely as A to win the jackpot.
- That would mean that the shares of the jackpot would be $\$2,000$ for A and $\$4,000$ for B.

Probability Trees

Probability Trees: Example

- Consider that A needed to win two more rounds, while B only need to win one more.
- One could suppose that B was twice as likely as A to win the jackpot.
- That would mean that the shares of the jackpot would be $\$2,000$ for A and $\$4,000$ for B.
- WRONG!

Probability Trees: Example

- At the end of the seventh round, A had a 25% chance of winning the jackpot.
- A's share of the jackpot is the $\$1,500$.
- B had a 75% chance of winning, so gets $\$4,500$.

Sample Spaces

- A complete list of all possible outcomes of a random experiment is called **sample space** or possibility space and is denoted by $\mathscr{S}$.
- A sample space is a set or collection of outcome of a particular random experiment.
- For example, imagine a dart board. You are trying to find the probability of getting a bullseye. The dart board is the sample space. The probability of a dart hitting the dart board is 1.0.
- For another example, imagine rolling a six sided die. The sample space is $\{1,2,3,4,5,6\}$.

Sample Spaces

- The following list consists of sample spaces of examples of random experiments and their respective outcomes.
- The tossing of a coin, sample space is $\{Heads, Tails\}$.
- The roll of a die, sample space is 1, 2, 3, 4, 5, 6
- The selection of a numbered ball (1-50) in an urn, sample space is $\{1,2,3,4,5,....,50\}$

Sample Spaces

- Percentage of calls dropped due to errors over a particular time period, sample space is $\{2\%, 14\%, 23\%, ...\}$
- The time difference between two messages arriving at a message centre, sample space is 0, ..., infinity
- The time difference between two different voice calls over a particular network, sample space is 0, ..., infinity

Sample Spaces

Consider couplse that have two children. Treating the gender of the children as an ***ordered pair*** outcome of a random experiment, the sample space is

$$\boldsymbol{S} = \{(b,b),(b,g),(g,b),(g,g)\}.$$

Let us assume that each sample point is ***equiprobable***, with probability 0.25 for each sample point.

Sample Spaces

Find the probability that both children are girls if it is known that:

(a) at least one of the children is a girl,

(b) the older child is a girl.

Part a

Find the probability that both children are girls if it is known that at least one of the children is a girl.

$$\boldsymbol{S} = \{(b,b),(b,g),(g,b),(g,g)\}.$$

Part b

Find the probability that both children are girls if it is known that the older child is a girl.

$$\boldsymbol{S} = \{(b,b),(b,g),(g,b),(g,g)\}.$$

Random Variables

- A random variable is defined as a numerical event whose value is determined by a chance process.
- When probability values are assigned to all possible numerical values of a random variable X, either by a listing or by a mathematical function, the result is a ***probability distribution***.

Random Variables

- The sum of the probabilities for all the possible numerical outcomes must equal one.
- Individual probability values may be denoted by the symbol $f(x)$, which indicates that a mathematical function is involved, by $P(X = x)$, which recognizes that the random variable can have various specific values, or simply by P(x).

Random Variables

- The outcome of an experiment need not be a number, for example, the outcome when a coin is tossed can be 'heads' or 'tails'.
- However, we often want to represent outcomes as numbers.
- A ***random variable*** is a function that associates a unique numerical value with every outcome of an experiment.
- The value of the random variable will vary from trial to trial as the experiment is repeated.

Random Variables

- Numeric values can be assigned to outcomes that are not usually considered numeric.
- For example, we could assign a ‘head’ a value of 0, and a ‘tail’ a value of 1, or vice versa.

Random Variables

There are two types of random variable - discrete and continuous. The distinction between both types will be important later on in the course.

Examples

- A coin is tossed ten times. The random variable X is the number of tails that are noted. X can only take the values $\{0, 1, ..., 10\}$, so X is a discrete random variable.
- A light bulb is burned until it burns out. The random variable Y is its lifetime in hours. Y can take any positive real value, so Y is a continuous random variable.

Discrete Random Variable

- A discrete random variable is one which may take on only a countable number of distinct values such as $\{0, 1, 2, 3, 4, ...\}$.
- Discrete random variables are usually (but not necessarily) counts.
- If a random variable can take only a finite number of distinct values, then it must be discrete.

Discrete Random Variable

- Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

Continuous Random Variable

- A continuous random variable is one which takes an infinite number of possible values.
- Continuous random variables are usually measurements.
- Examples include height, weight, the amount of sugar in an orange, the time required to run a computer simulation.

Discrete Random Variables

- For a discrete random variable observed values can occur only at isolated points along a scale of values. In other words, observed values must be integers.
- Consider a six sided die: the only possible observed values are 1, 2, 3, 4, 5 and 6.
- It is not possible to observe values that are real numbers, such as 2.091.
- *(Remark: it is possible for the average of a discrete random variable to be a real number.)*

Discrete Random Variables

- Therefore, it is possible that all numerical values for the variable can be listed in a table with accompanying probabilities.
- There are several standard probability distributions that can serve as models for a wide variety of discrete random variables involved in business applications.

Discrete probability distributions

The discrete probability distributions that described in this course are

- the binomial distribution,
- the geometric distribution,
- the hypergeometric distribution,
- the Poisson distributions.