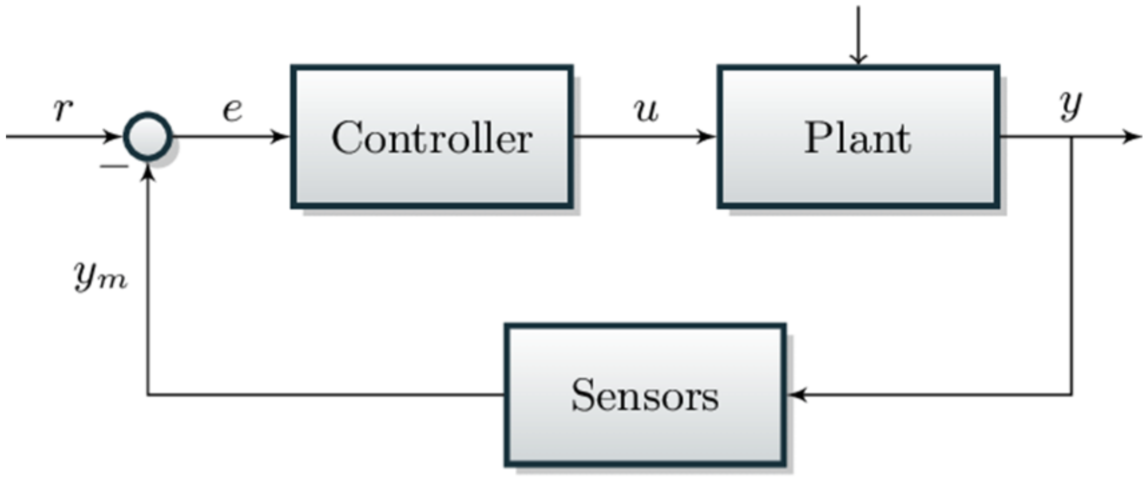
L6 Control

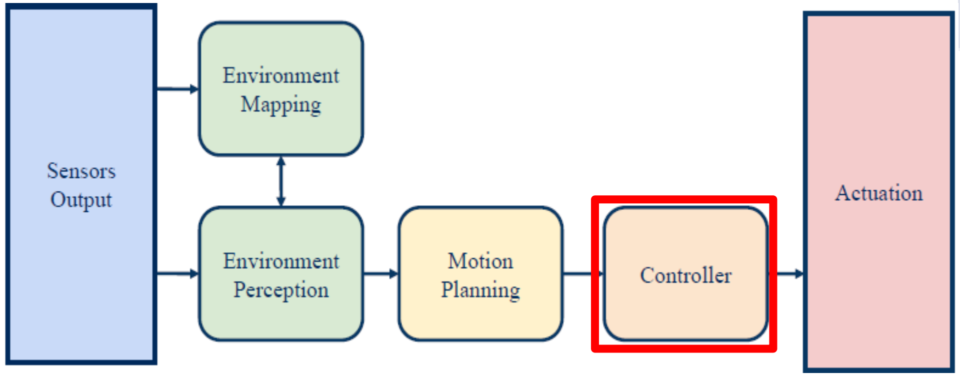


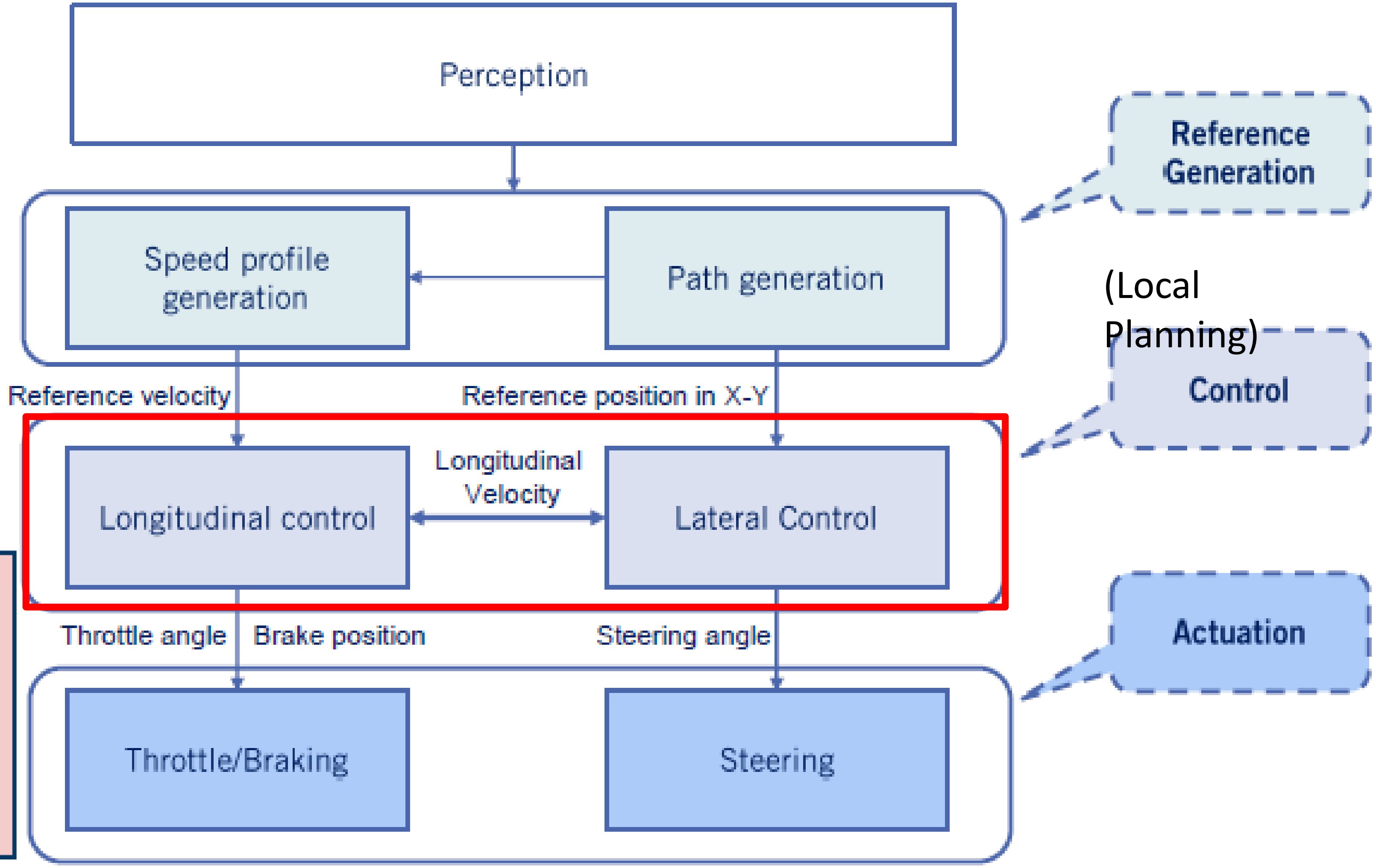
Zonghua Gu, Umeå University

Nov. 2023

Control in the AD Pipeline

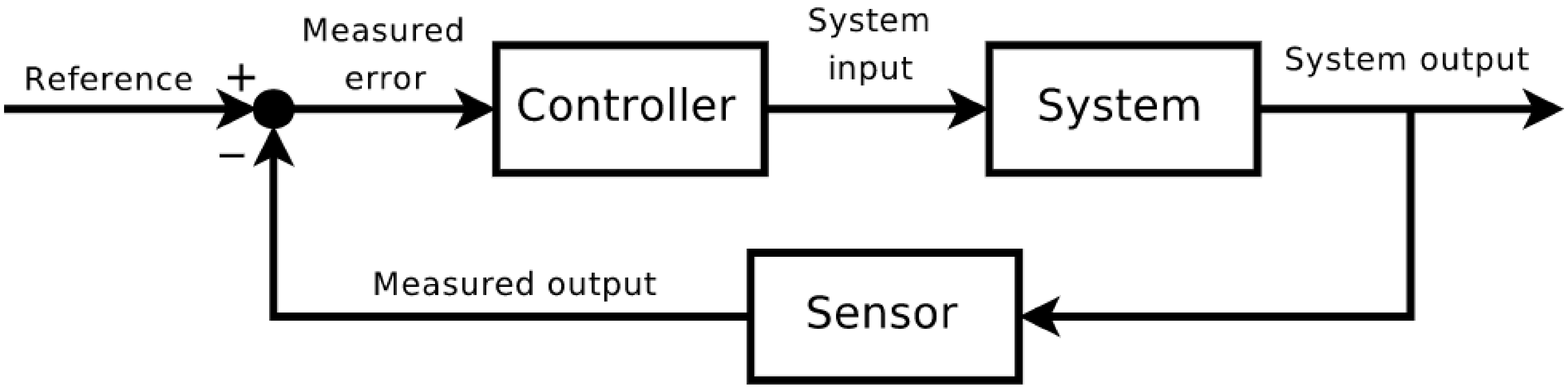
- The controller outputs control commands (brake, acceleration, steering) to follow the trajectory output from motion planning





Feedback Control Problem

- Given a system and a reference signal, find a control law such that the closed loop system is stable and follows the reference signal
- The most common control algorithms in automotive systems are Proportional–Integral–Derivative (PID) and Model Predictive Control (MPC)

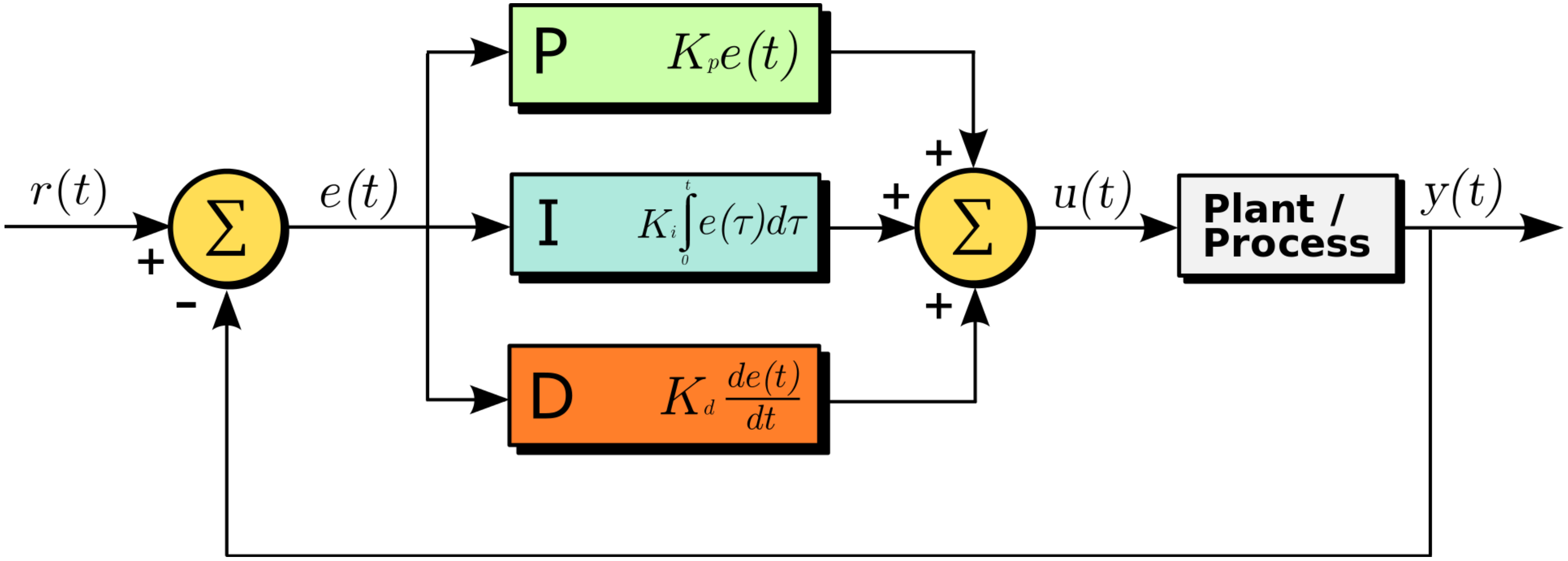


Outline

- PID Control
- MPC Control
- Kinematic bicycle model
- Twiddle() for PID control tuning

PID Control

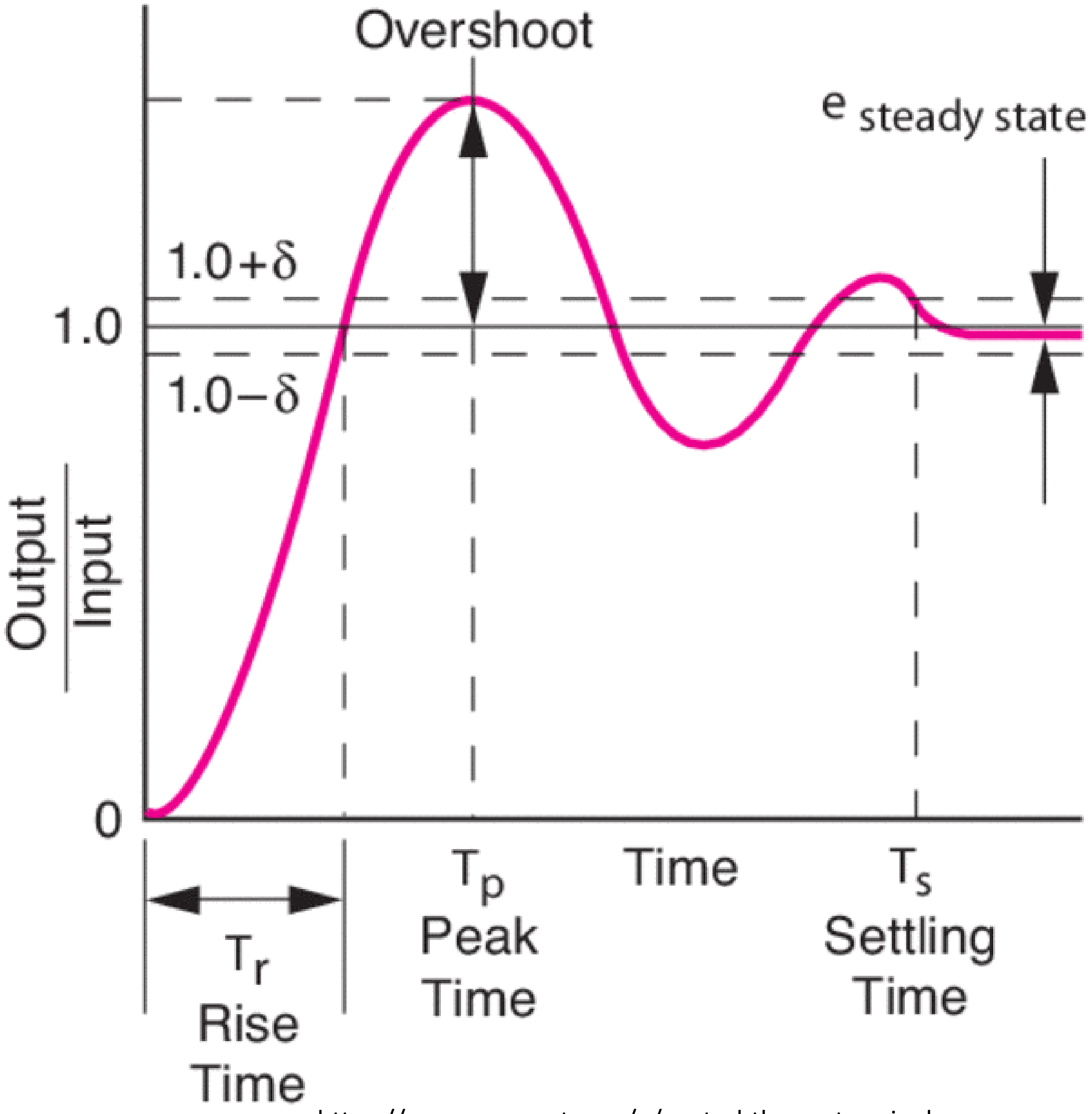
- Tracking Error: $e(t) = r(t) - y(t)$
- Control input: $u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \dot{e}(t)$
- Ref: Controlling Self Driving Cars
  - https://www.youtube.com/watch?v=4Y7zG48uHRo



https://en.wikipedia.org/wiki/PID_controller

Step Response Performance Metrics

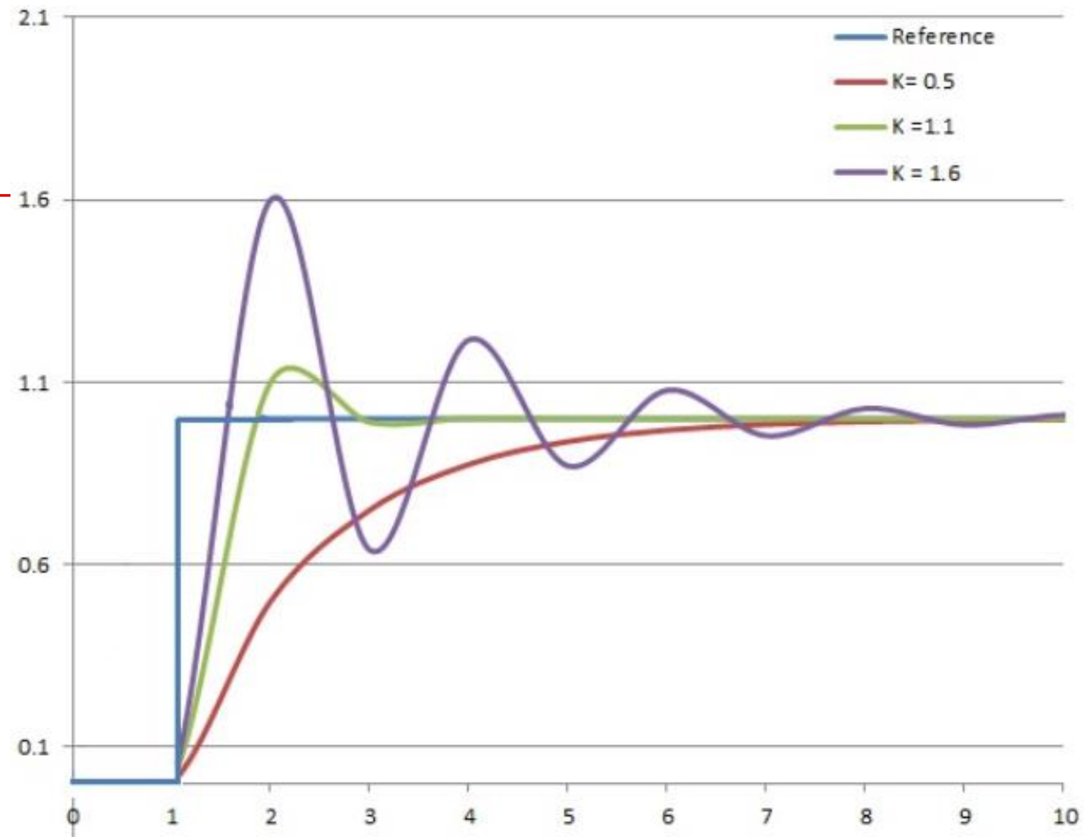
- The control input (reference input) jumps from 0 to a reference value (e.g., 1.0) at time 0, and the controller aims to track it closely
  - Rise time: the time it takes the transient response to move to $1.0 - \delta$ of the steady state response
  - Maximum overshoot: the amount (or percentage) by which the maximum value of the transient response exceeds the steady state value
  - Peak time: the time at which the maximum overshoot occurs
  - Settling time: the time after which the output is within a specified band around the steady state value $[1.0 - \delta, 1.0 + \delta]$

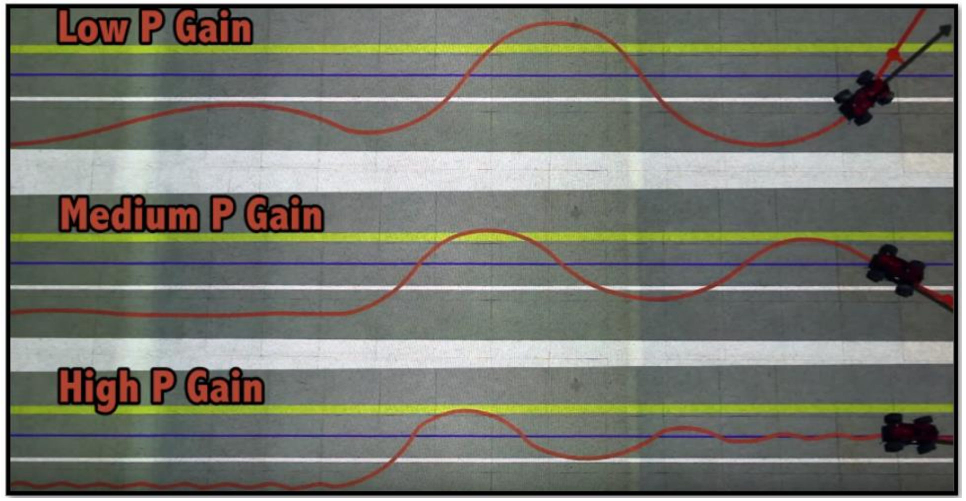


https://www.newport.com/n/control-theory-terminology

Proportional Term K_p

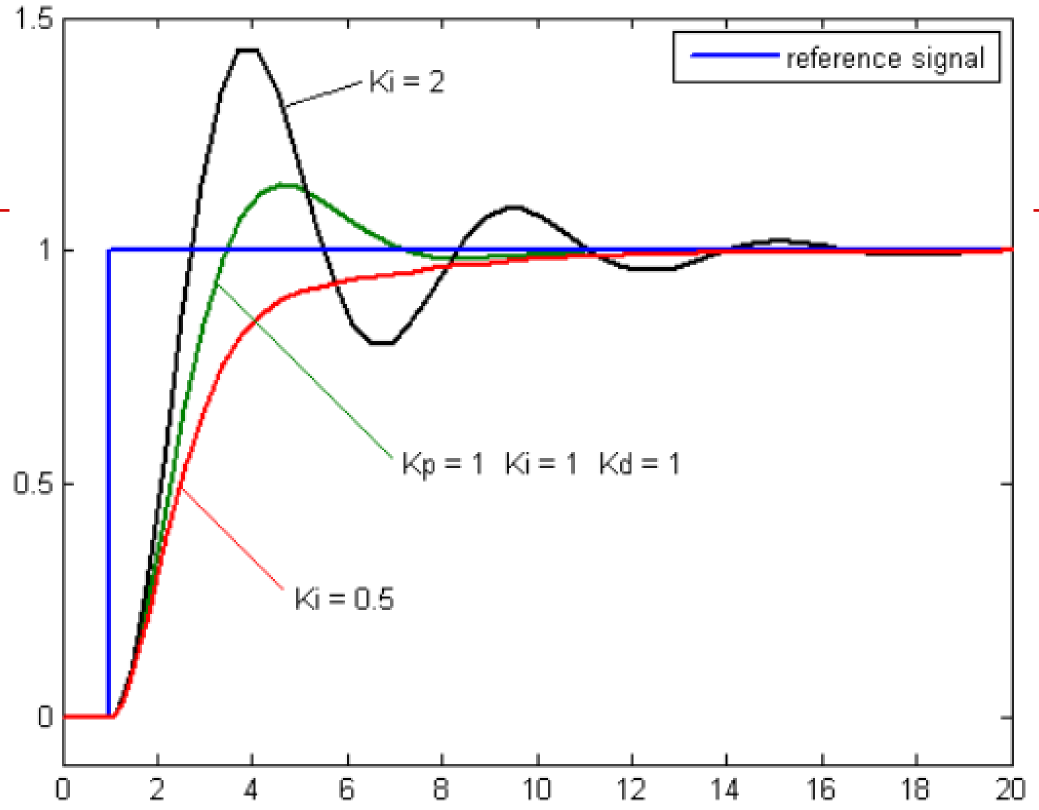
- $u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \dot{e}(t)$
- $u(t)$ proportional to error $e(t)$ with factor K_p
- Increasing K_p leads to:
  - Faster response
  - Bigger overshoot, oscillations
    - System may become unstable
  - Smaller but non-zero steady state error

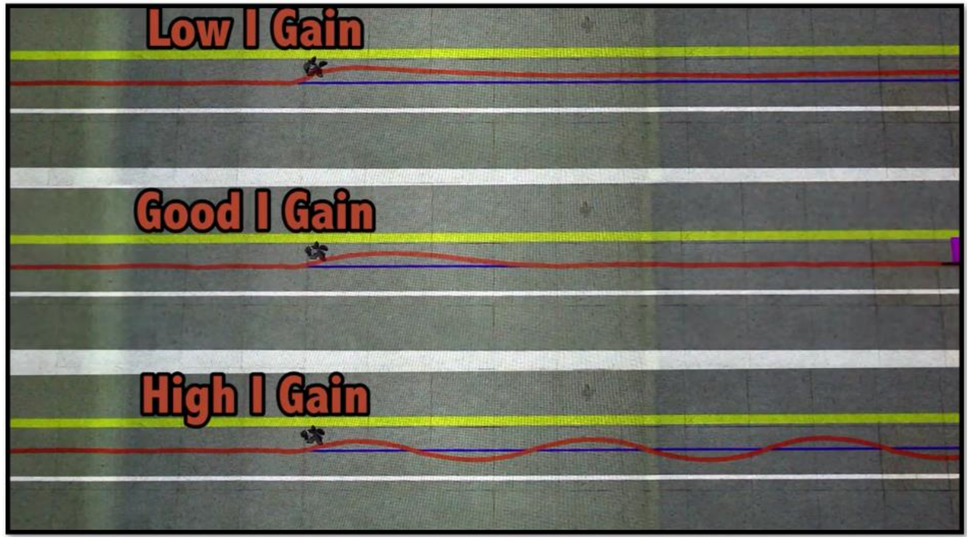




Integral Term K_i

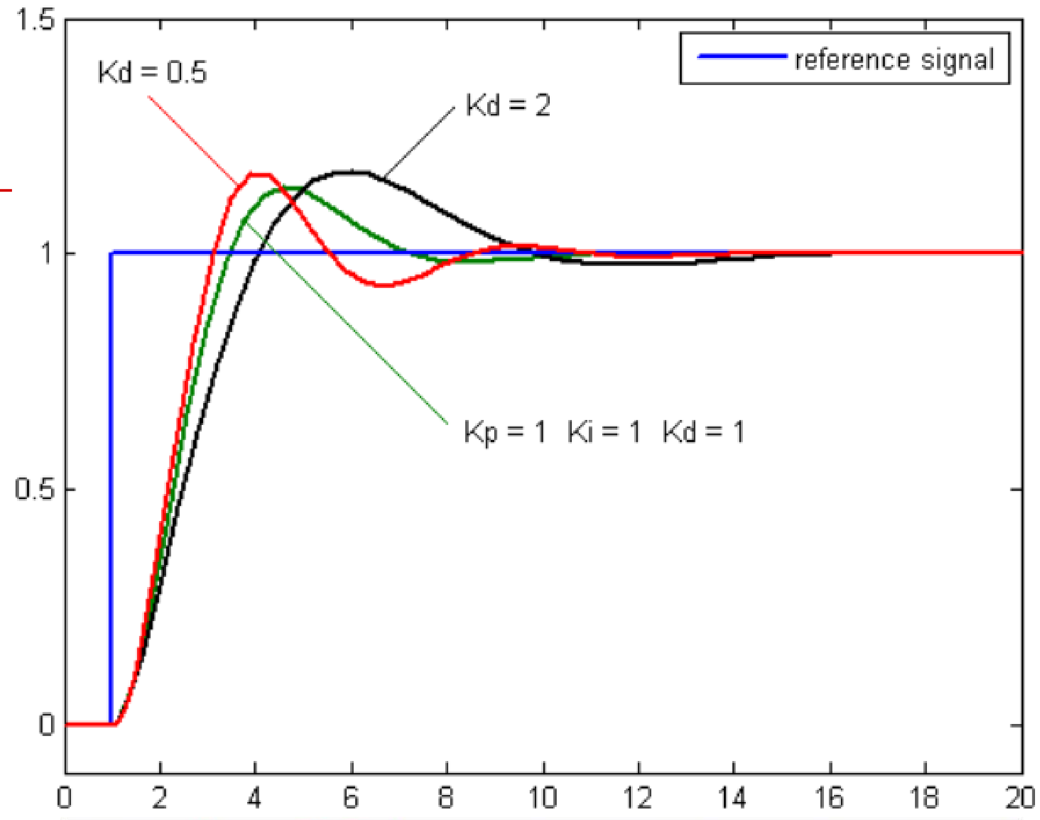
- $u(t) = K_p e(t) + {\color{red}K_i} \int_0^t e(\tau)d\tau + K_d \dot{e}(t)$
- K_i takes into account history of tracking error, and eliminates steady state error
- Increasing K_i leads to:
  - Increased overshoot
  - More robust to disturbances

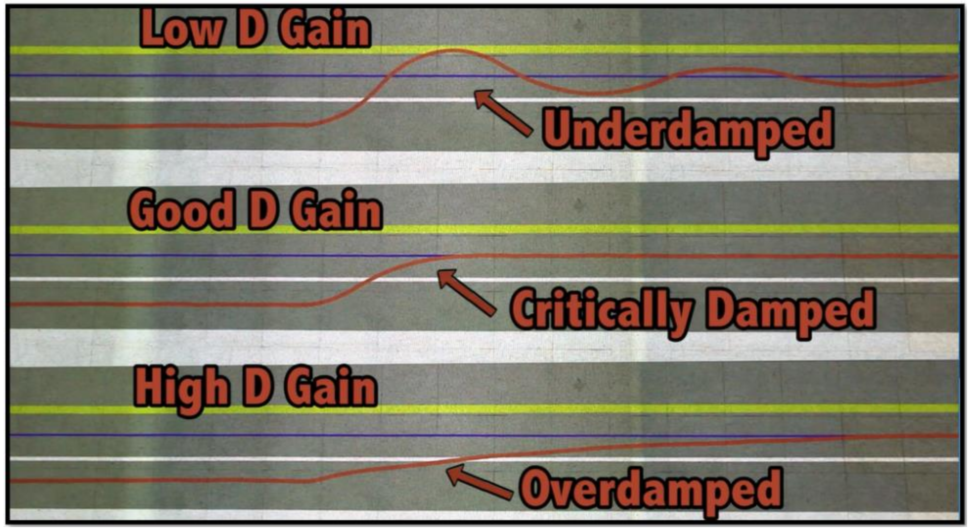




Derivative Term K_d

- $u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + {\color{red}K_d}\dot{e}(t)$
- $u(t)$ proportional to error derivative $\dot{e}(t)$ by factor K_d
- Increasing K_d leads to:
  - Reduced overshoot
  - Faster response
  - Little effect on steady state
  - More sensitive to measurement noise

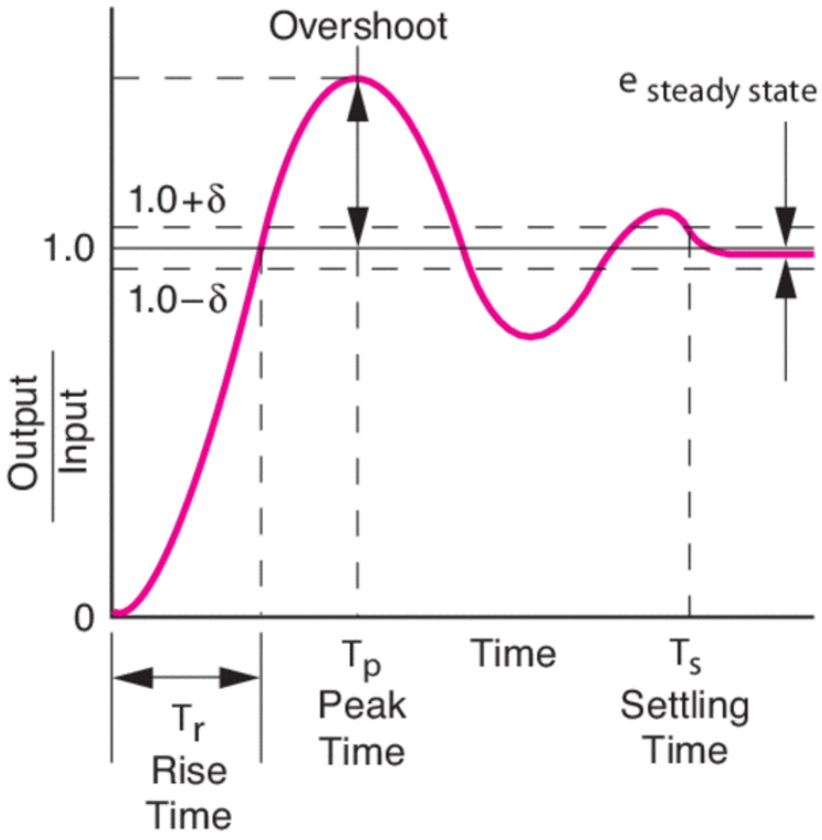




Effects of PID Gains

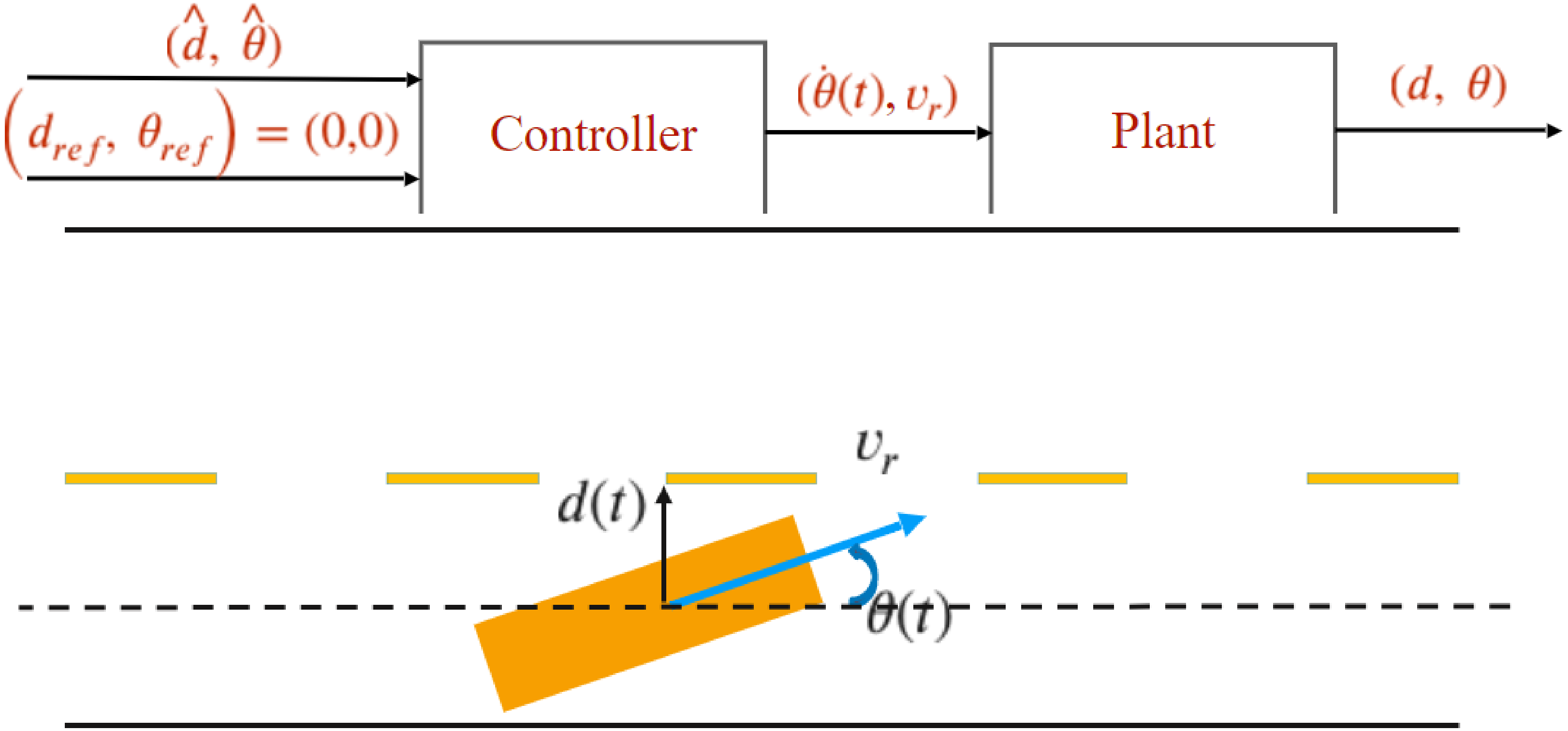
- $u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \dot{e}(t)$

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady State Error
Increase K_p	Decrease	Increase	Small increase	Decrease
Increase K_i	Small decrease	Increase	Increase	Large decrease
Increase K_d	Small decrease	Decrease	Decrease	Minor change



Example: Vehicle Lateral Control

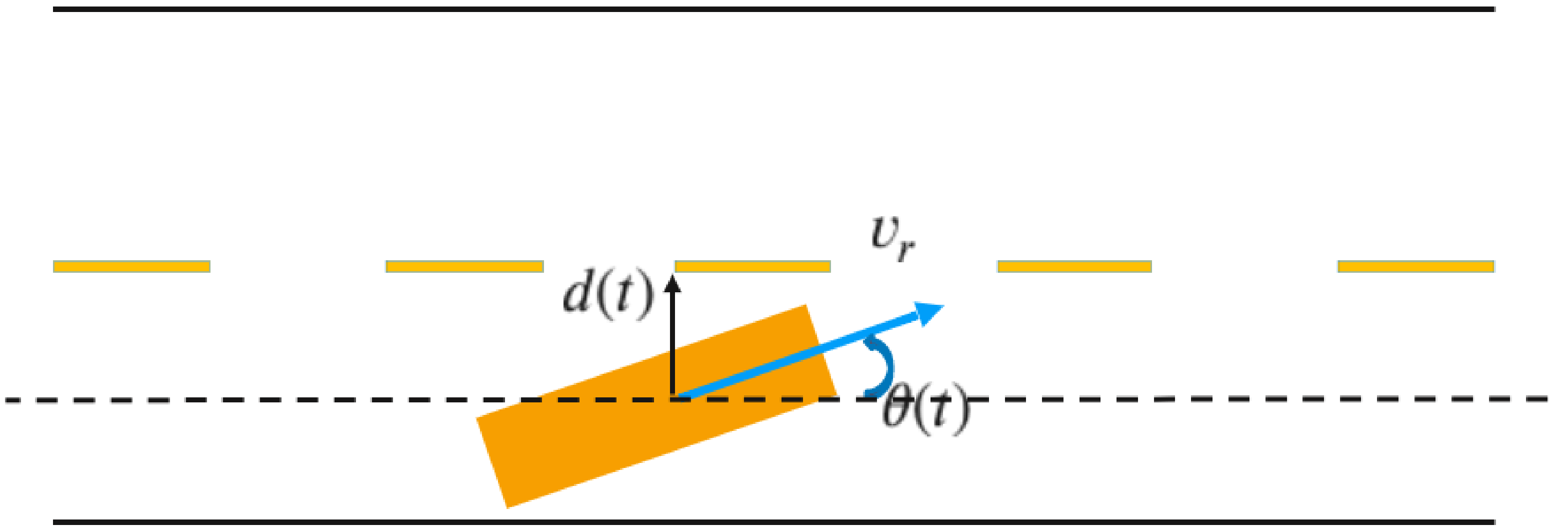
- State (d, θ)
  - d: distance to center of lane; θ: heading angle.
- Reference trajectory: $\left(d_{ref}, \theta_{ref}\right) = (0,0)$
  - The vehicle travels straight ahead at center of lane.
- Vehicle has constant speed v_r, so the only control input is $u(t) = \dot{\theta}(t)$, the angular velocity.
- Assume perfect sensor state estimation: $(\hat{d}, \hat{\theta}) = (d, \theta)$
  - Ref. Control Algorithms for Autonomous Vehicles https://www.icloud.com/keynote/035Qivaw_FXYD70xCbD5R9lDA#control_AV



System and P Control Modeling

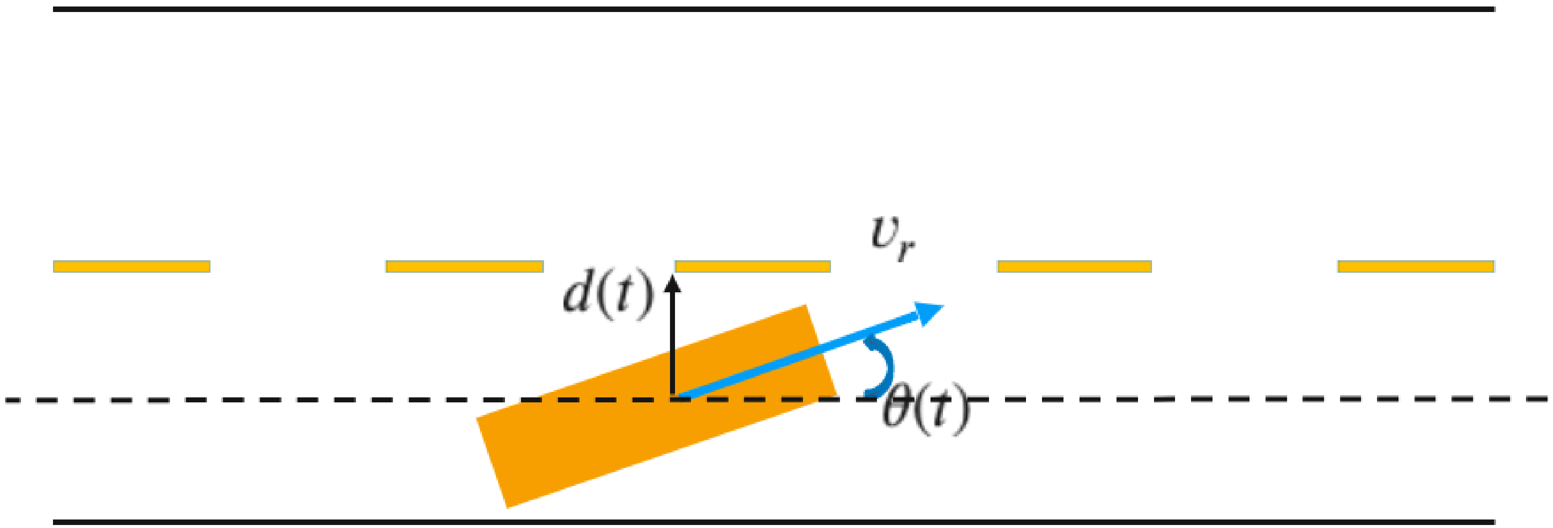
- Linear system dynamics:
  - $\begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & v_r \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$
  - $\dot{d}(t) = v_r \sin\theta(t) \approx v_r\theta(t)$ assuming $\theta(t)$ is small (linearized kinematic model).
- P Controller:
  - $u(t) = \begin{bmatrix} K_d & K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$
- Closed-loop dynamics is obtained by plugging $u(t)$ into system dynamics:
  - $\begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & v_r \\ K_d & K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$

- Ref trajectory: $\begin{bmatrix} d_{ref} \\ \theta_{ref} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- Error signal:
  - $\begin{bmatrix} e_d(t) \\ e_\theta(t) \end{bmatrix} = \begin{bmatrix} d_{ref} \\ \theta_{ref} \end{bmatrix} - \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix} = -\begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$
- Error dynamics:
  - $\begin{bmatrix} \dot{e_d}(t) \\ \dot{e_\theta}(t) \end{bmatrix} = -\begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -v_r \\ -K_d & -K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$
- Control objective: design K_d, K_θ to drive error $\begin{bmatrix} e_d(t) \\ e_\theta(t) \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$



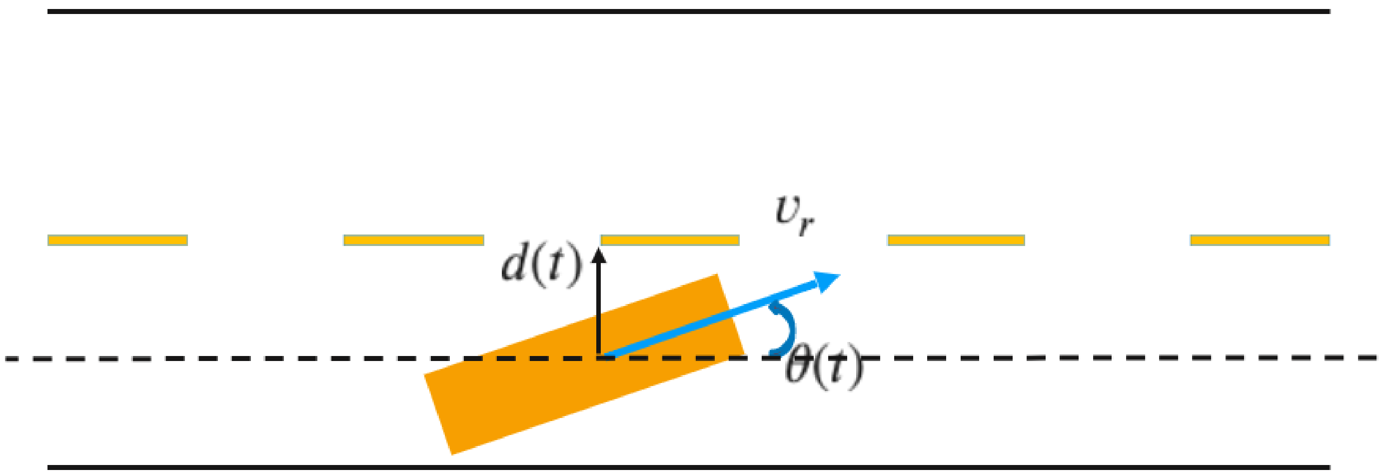
P Control Design with Pole Placement (not covered in this course)

- Closed-loop poles:
  - $0 = \det(\lambda I - \begin{bmatrix} 0 & -v_r \\ -K_d & -K_\theta \end{bmatrix}) = \lambda^2 + K_\theta \lambda - v_r K_d$
  - Solution $\lambda_{1,2} = -\frac{K_\theta}{2} \pm \frac{1}{2}\sqrt{K_\theta^2 + 4v_r k_d}$
- Critically-damped dynamics (repeated real roots):
  - $\sqrt{K_\theta^2 + 4v_r K_d} = 0 \Rightarrow K_d = -\frac{K_\theta^2}{4v_r}$
  - K_θ chosen empirically



P Control Design Cont'd

- Linear systems dynamics relies on the small angle approximation $\sin\theta(t) \approx \theta(t)$, assuming $|\theta(t)| < \theta_{th}, \forall d(0)$
- Closed-loop dynamics $\dot{\theta}(t) = K_d d(t) + K_\theta \theta(t)$
  - Set $\dot{\theta}(t) = -\frac{K_\theta^2}{4v_r}\text{sat}(d(t), d_{th}) + K_\theta\theta(t)$,
  - where: $\text{sat}(d(t), d_{th}) = \begin{cases} -d_{th} & \text{if } d(t) < -d_{th} \\ d(t) & \text{if } d(t) \in [-d_{th}, d_{th}] \\ d_{th} & \text{if } d(t) > d_{th} \end{cases}$
- Empirical param settings: $\theta_{th} = \frac{\pi}{6}$, $d_{th} = \left|\frac{K_\theta\theta_{th}}{K_d}\right|$



Simplifying Assumptions

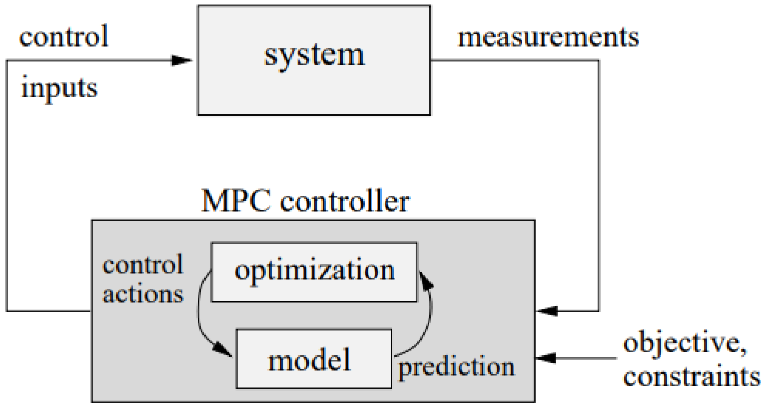
- Did not consider:
  - Estimation uncertainty due to measurement noise
    - We assume perfect state estimation $(\hat{d}, \hat{\theta}) = (d, \theta)$
  - Estimation latency:
    - Time from measurements (d, θ) to state estimation $(\hat{d}, \hat{\theta})$ availability to the controller
  - Constraints (e.g., actuator limits)
    - Need to impose a maximum curvature radius to simulate a real car.
  - Discrete time (multi-rate), non-uniform sampling:
    - Our controller is continuous time, but the actual implementation runs in discrete time.
    - Sampling rate of the estimator (slower) may be different than that of actuation (faster), or may be variable

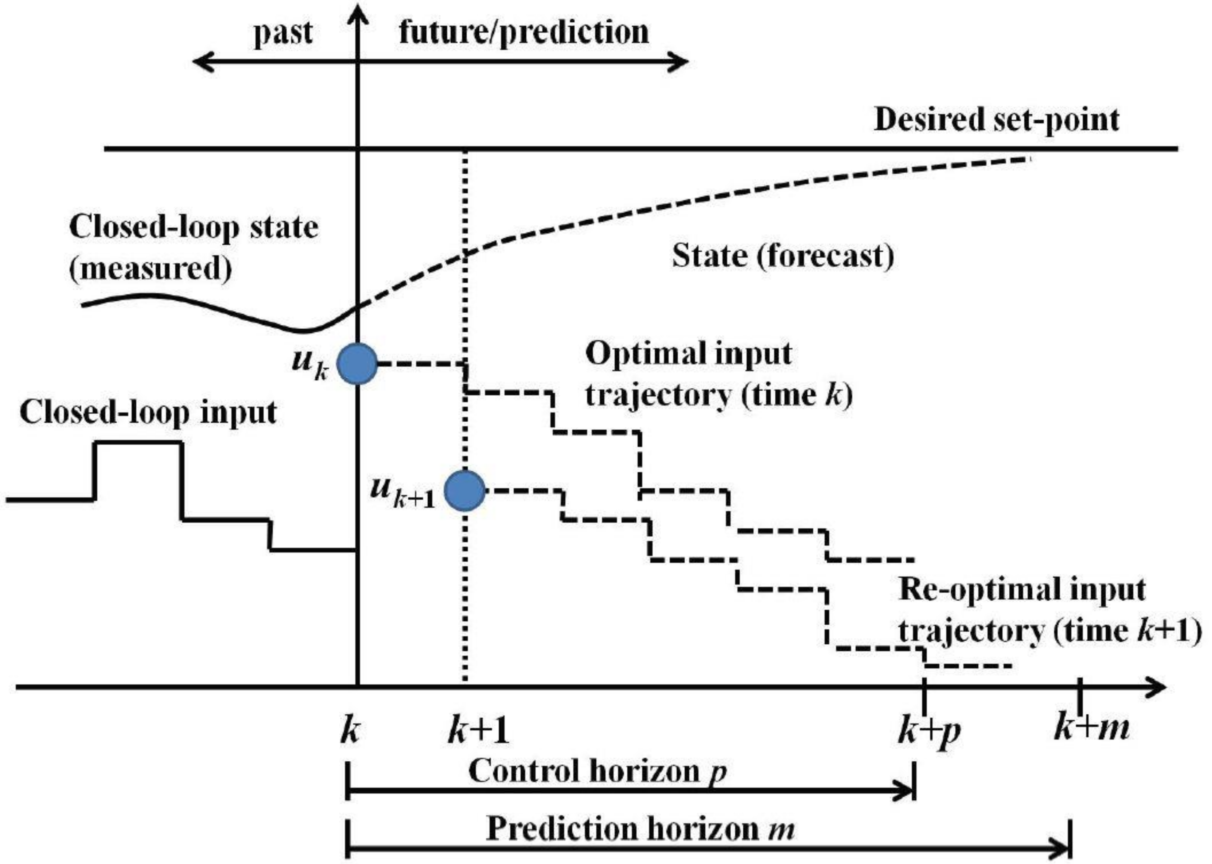
Outline

- PID Control
- MPC Control
- Kinematic bicycle model
- Twiddle() for PID control tuning

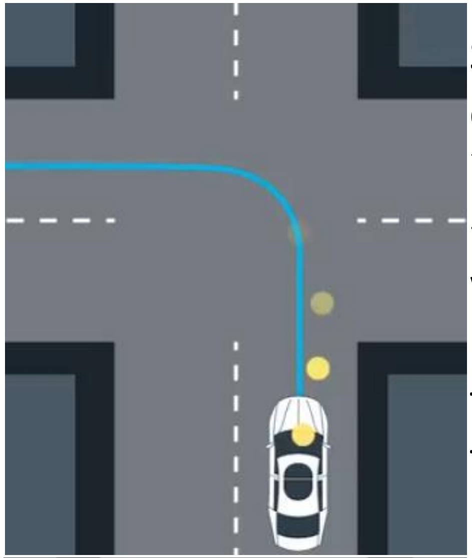
MPC (Model-Predictive Control)

- Also called Receding Horizon Control
- Choose prediction horizon m and control horizon p
- At each time step k:
  - Set initial state to predicted state $x[k]$
  - Solve a constrained optimization problem over lookahead window $[k, k+m]$, to get a sequence of control inputs u, while in the time interval $[k-1, k]$
  - Apply 1^{st} control command $u[k]$ at time step k
- Control horizon p and prediction horizon m may be different, but often the same
  - denoted as lookahead horizon T in the next slides (current time is denoted as t)

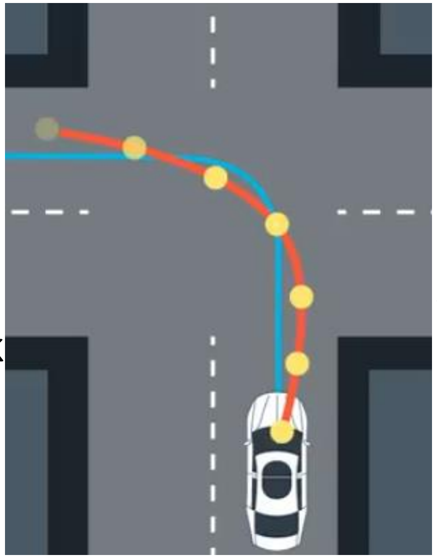




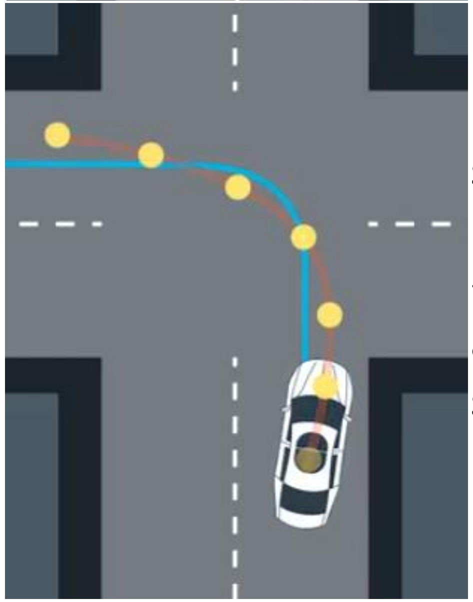
MPC Example



Solve for opt control inputs $\mathrm{U} = \{\mathrm{u_t}, \dots, \mathrm{u_{t+T-1}}\}$ within lookahead horizon T to track the blue desired traj



Execute control input u_t at 1st timestep



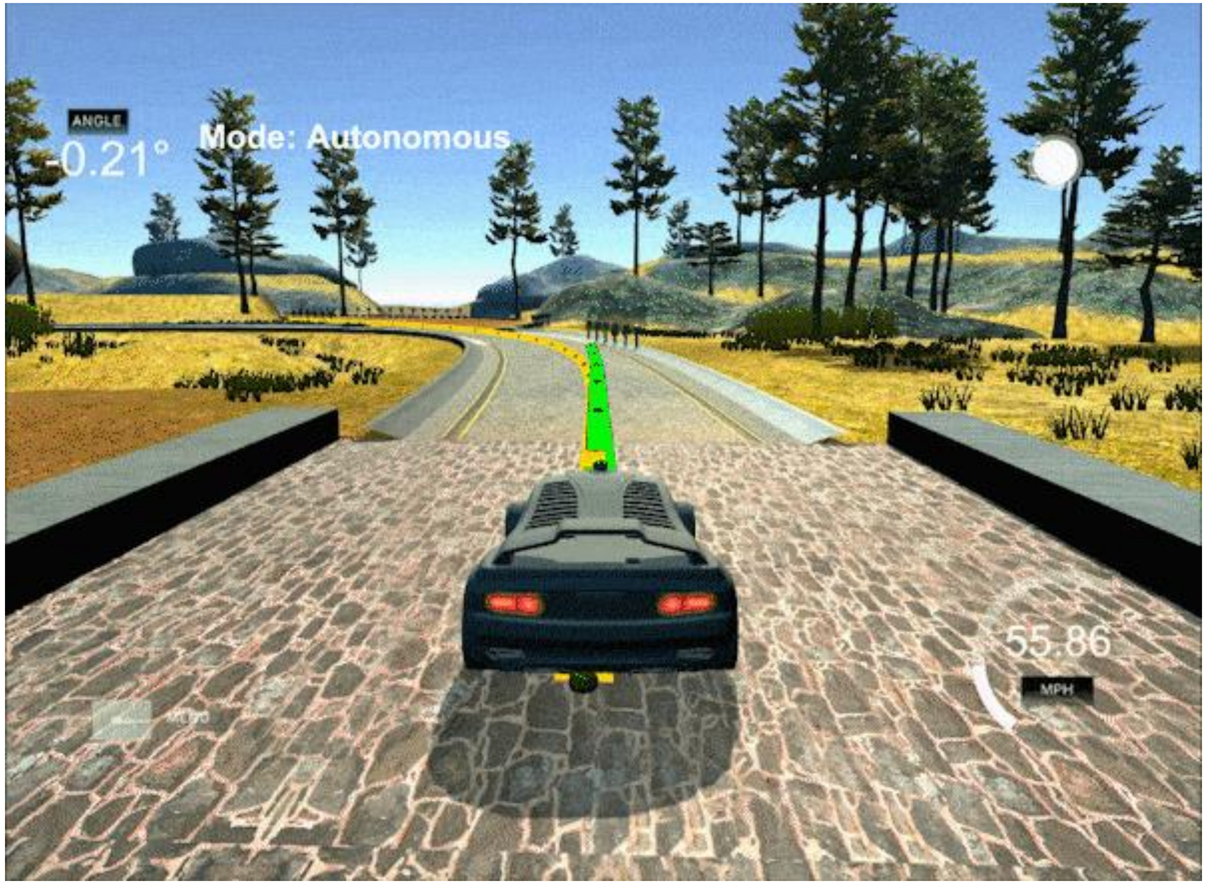
Replan a new set of control inputs $\mathrm{U} = \{\mathrm{u_t}, \dots, \mathrm{u_{t+T-1}}\}$ at next time step t

Execute control input u_t at 1st timestep

…

MPC Illustration Con’t

- Yellow: reference trajectory from planner
- Green: trajectory from running control inputs $u[0, \ldots, T-1]$ computed by MPC based on system model for lookahead horizon T



https://medium.com/@david010/vehicle-mpc-controller-33ae813cf3be

Linear vs. Nonlinear MPC

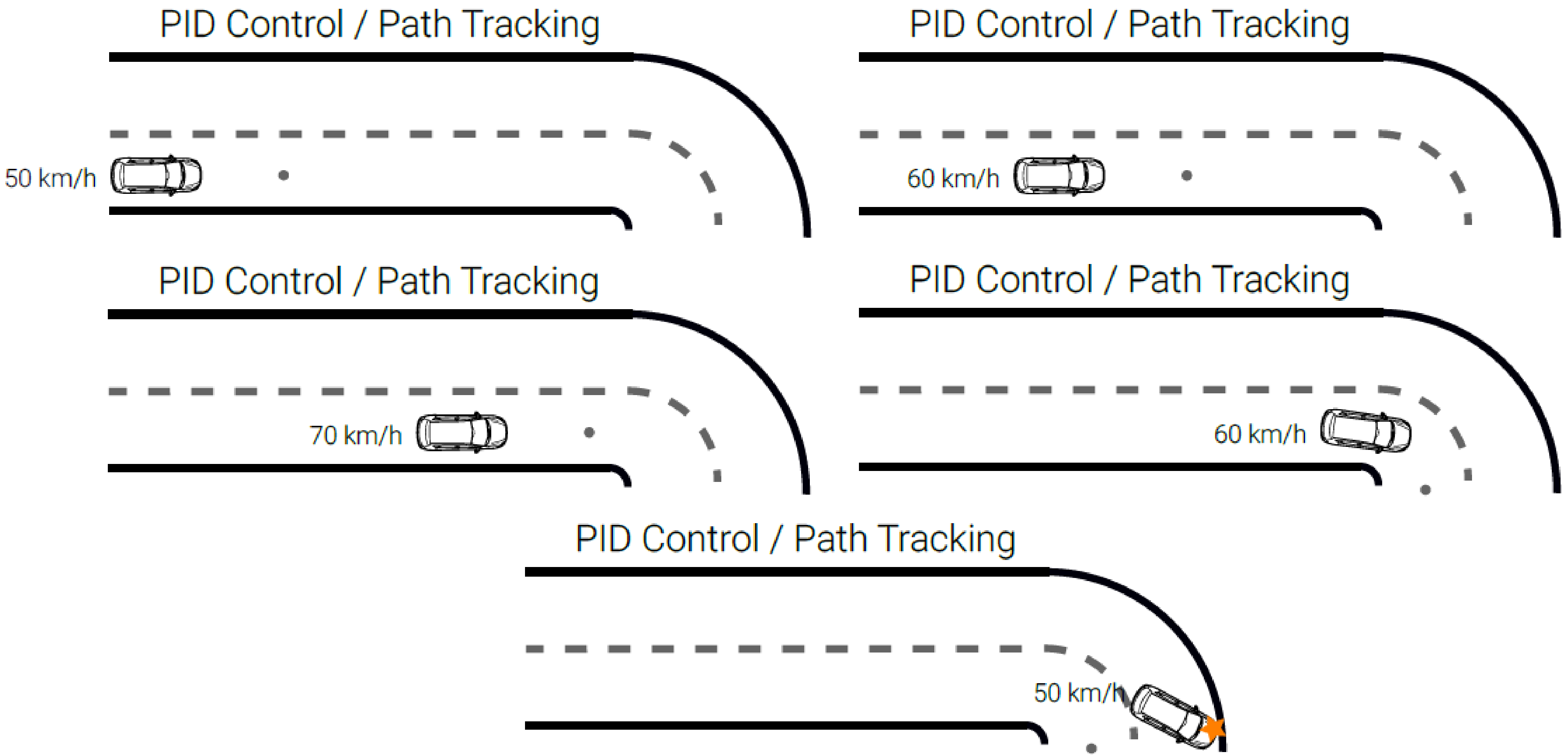
- Linear MPC (no constraints)
  - $\min_{\mathrm{U}=\{\mathrm{u_t},\ldots,\mathrm{u_{t+T-1}}\}} J(x(t),U) = x_{t+T}^T Q_f x_{t+T} + \sum_{j=t}^{t+T-1}(x_j^T Q x_j + u_j^T R u_j)$
  - s.t. for $t \le j \le t+T-1$
  - $x_{j+1} = Ax_j + Bu_j$
  - x_j is the difference between actual state and ref state, which should be minimized with the term $x_{t+T}^T Q_f x_{t+T}$
  - u_j is control input, which should be minimized with the term $u_j^T R u_j$ (to reduce control effort)
  - Relative magnitudes of Q and R encode relative importance of the two objectives
  - Can be solved analytically at each time step $u_t = -Kx_t$
- Nonlinear MPC (with constraints)
  - $\min_{\mathrm{U}=\{\mathrm{u_t},\ldots,\mathrm{u_{t+T-1}}\}} J(x(t),U) = \sum_{j=t}^{t+T} C(x_j, u_j)$
  - s.t. for $t \le j \le t+T-1$
  - $x_{j+1} = f(x_j, u_j)$
  - $x_{min} \le x_{j+1} \le x_{max}$
  - $u_{min} \le u_j \le u_{max}$
  - $g(x_j, u_j) \le 0$
  - $h(x_j, u_j) = 0$
  - Both objective $J(x(t), U)$ and system dynamics $f(x_j, u_j)$ may be nonlinear.
- (Note: in x_{t+T}^T, superscript T denotes "vector transpose"; subscript T denotes "lookahead horizon")

MPC Pros and Cons

- Pros
  - Predictive control with lookahead
    - PID control is like driving your car by looking in the rearview mirror
  - Handles constraints explicitly
    - PID control cannot handle constraints
  - Applicable to both linear and nonlinear systems
    - Pole placement for PID control design is applicable to linear systems only
    - Empirical PID param tuning is model-free and applicable to any system (blackbox)
- Cons
  - Requires accurate yet efficient system model (whitebox)
  - Optimizer computation may be expensive, esp. for non-linear MPC
    - Select lookahead horizon T to tradeoff between control performance and computation overhead; Larger $T \rightarrow$ better control performance but higher overhead

PID Control Example (No Look-Ahead)

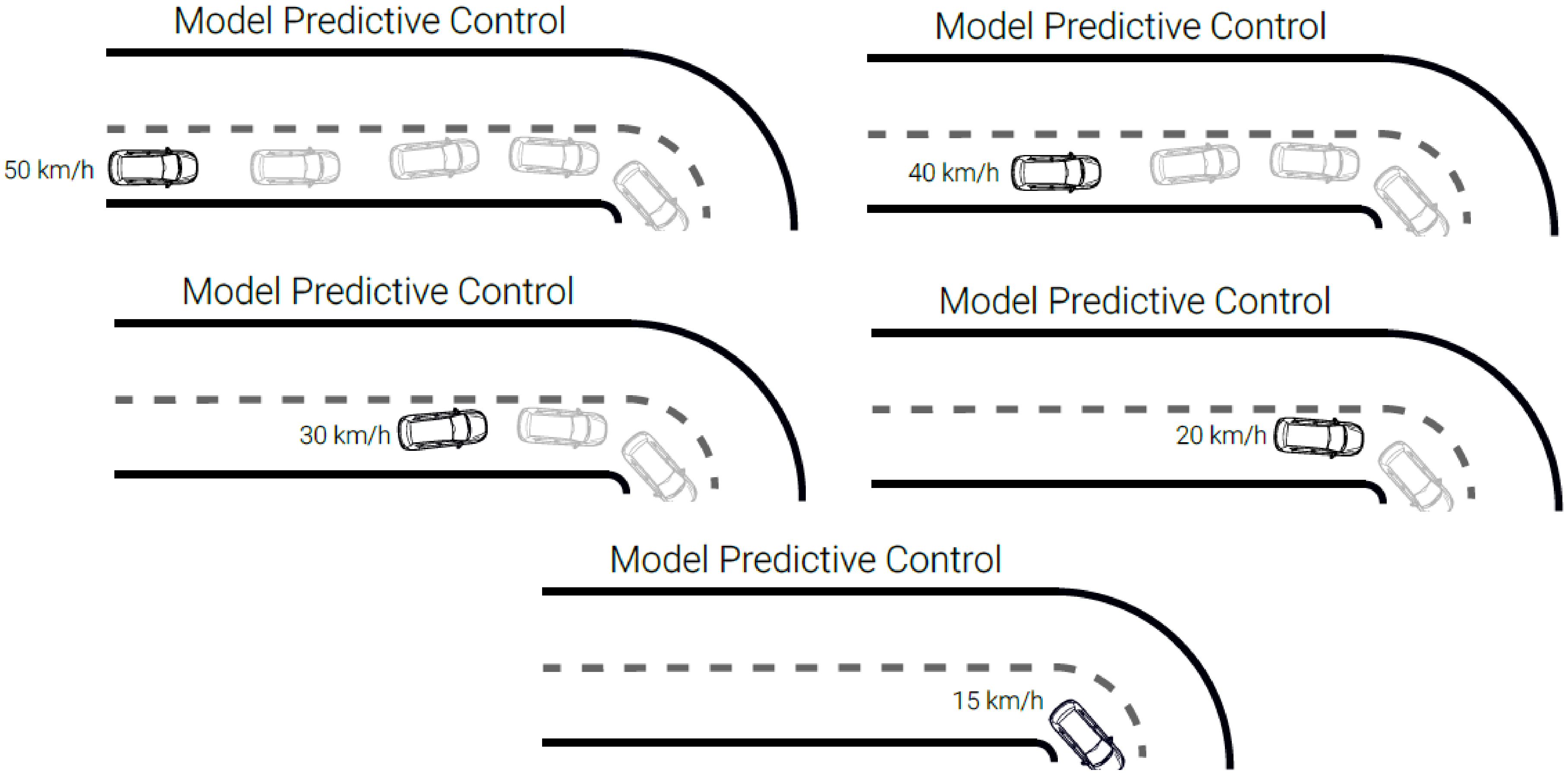
- Vehicle accelerates to track setpoint speed of 70 km/h
- Then decelerates upon encountering a sharp turn, but it is too late and cannot track the lane



https://uni-tuebingen.de/fakultaeten/mathematisch-naturwissenschaftliche-fakultaet/fachbereiche/informatik/lehrstuehle/autonomous-vision/lectures/self-driving-cars/

MPC Control Example (Look-Ahead)

- MPC computes control actions based on a lookahead window into the future
- Vehicle decelerates early in anticipation of the sharp turn



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MPC Quiz

- Which from the below statement about MPC are true?
  - 1) Horizon is a finite window of time
  - 2) Prediction horizon keeps being shifted at each time step
  - 3) Full optimization over the time horizon is performed at each iteration
  - 4) Only the first control action from the optimization is applied at time t
  - 5) All of the above
- ANS: 5

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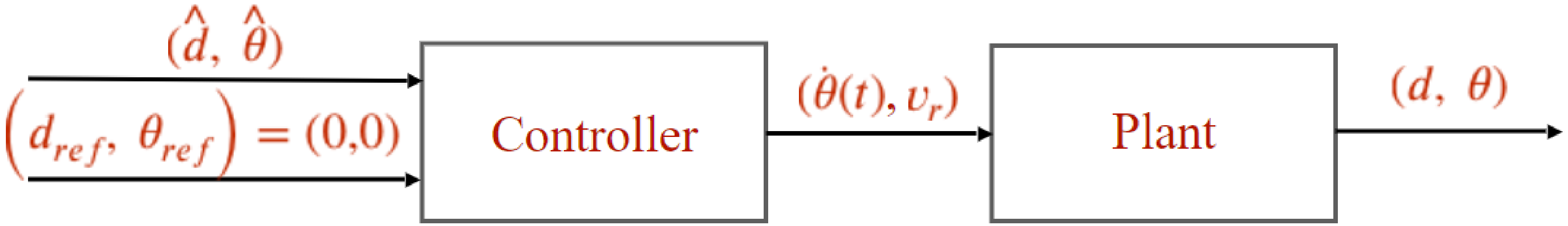
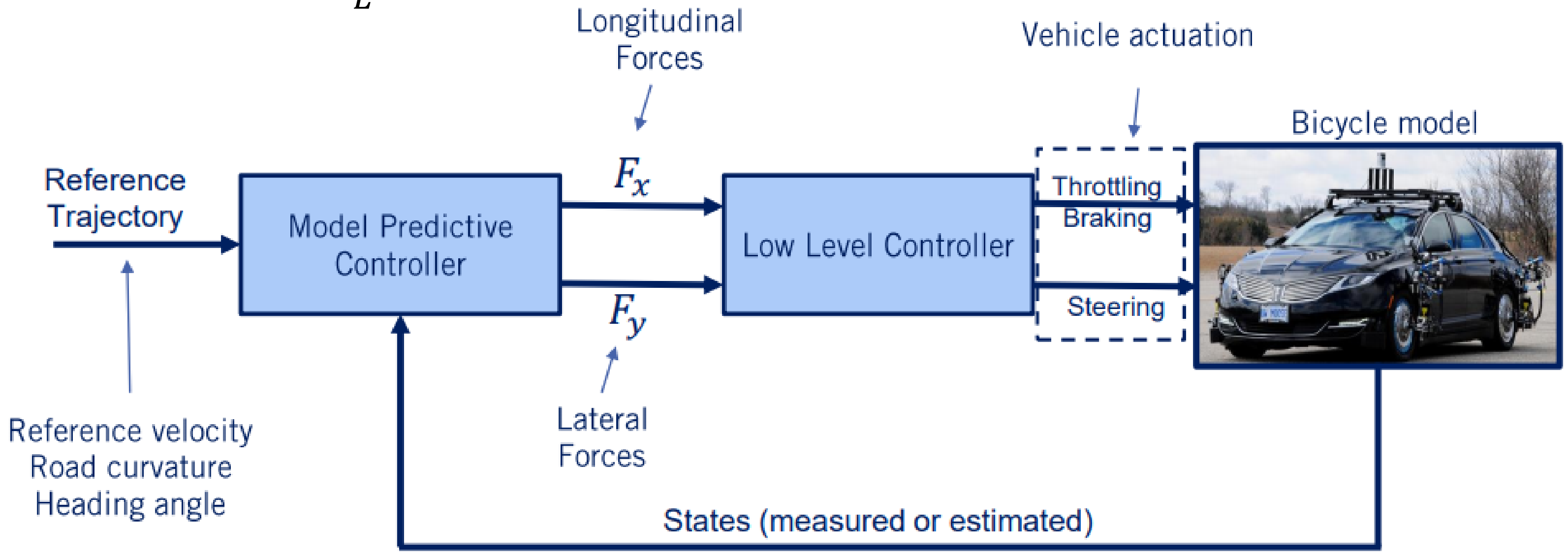
Kinematics vs. Dynamics

- Kinematics is study of motion without considering the forces that affect the motion. It deals with the geometric relationships that govern the system
  - A kinematic model: $\dot{x} = v, \dot{v} = \ddot{x} = a$
  - Uses position, velocity, acceleration (and/or further derivatives) as control input (e.g. the kinematic bicycle model)
- Dynamics is the study of motion taking into account the forces that affect it. It is described by the equations of motion.
  - A dynamic model: $F = M\ddot{x} + B\dot{x}$ (Newton's law with friction)
  - Uses force and torque as control input, and takes mass and inertia into consideration.
  - e.g., vehicle dynamics model (longitudinal and lateral)
- Consider two vehicles with the same geometry but different mass/weight turning a tight corner
  - They may have the same kinematic model, but different dynamic model due to different mass M.
  - Both may be controllable kinematically by control input a
    - The light vehicle may be controllable dynamically by control input F.
    - The heavy vehicle may not be controllable dynamically by control input F. Due to large M, F may exceed the max actuator limit.
- Orthogonal issue from control algorithm
  - Control algorithms, e.g., PID or MPC, may be either kinematic or dynamic control

Kinematic vs. Dynamic Control



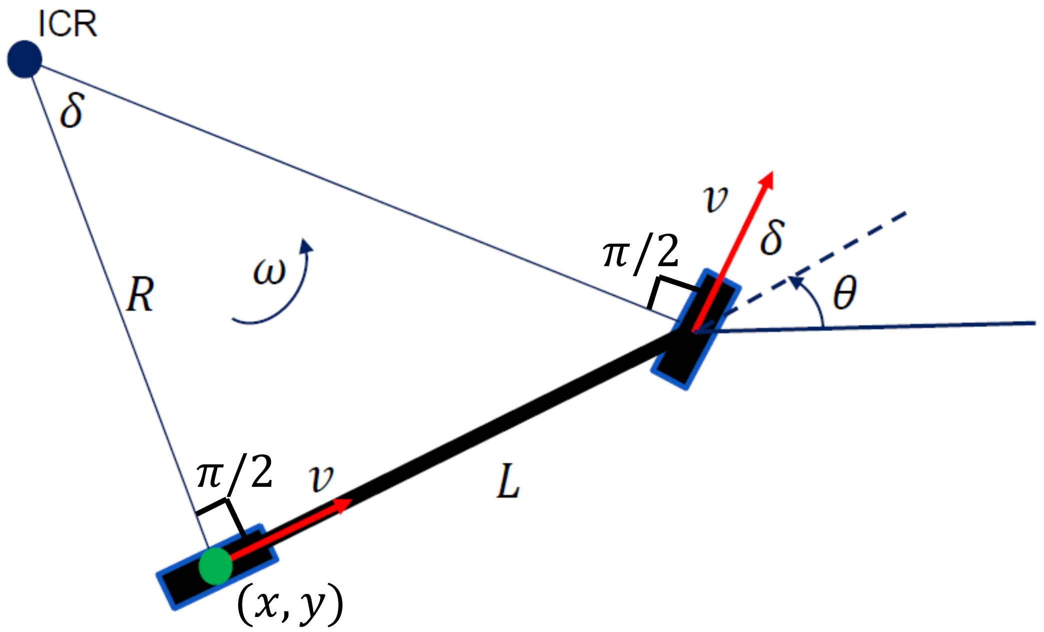
PID controller for kinematic bicycle model with heading angle rate $(\dot{\theta})$ and velocity $(v_r$, constant here) as control input. ($\dot{\theta}$ should be controlled indirectly by controlling steering angle δ, with $\dot{\theta} = \frac{v \tan \delta}{L}$.)



MPC for dynamic vehicle model (high-level controller) with forces (longitudinal and lateral) as control input.

Kinematic Bicycle Model

- Front wheel steering. Assuming here rear wheel as reference point (may also use front wheel or center of gravity).
- State vector: $[x \quad y \quad \theta]^T$: vehicle pose includes its position (x, y) and heading angle θ.
- Control inputs: $[\delta \quad v]^T$: steering angle δ and vehicle speed v (assumed to be constant).
- $\tan\delta = \frac{L}{R}$
  - L: vehicle length (distance between 2 wheels); R: rotation radius of Instantaneous Center of Rotation (ICR), equal to distance between ICR and rear wheel. Curvature $\kappa = \frac{1}{R}$.
  - Line from ICR to each wheel is perpendicular to it.
- $\dot{\theta} = \omega = \frac{v}{R} = \frac{v \tan\delta}{L}$
  - Angular velocity is speed v divided by rotation radius R
- Typically, angles δ, θ are based on counter-clockwise convention w.r.t reference direction.
  - In the fig, $\delta \approx \frac{\pi}{6}$ w.r.t ref direction v; $\theta \approx \frac{\pi}{6}$ w.r.t ref direction east (horizontal); $\dot{\theta} > 0$ for counter-clockwise rotation.



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{\tan\delta}{L} \end{bmatrix}$$

(Non-linear in δ;
Linear in v)

State Update Equation

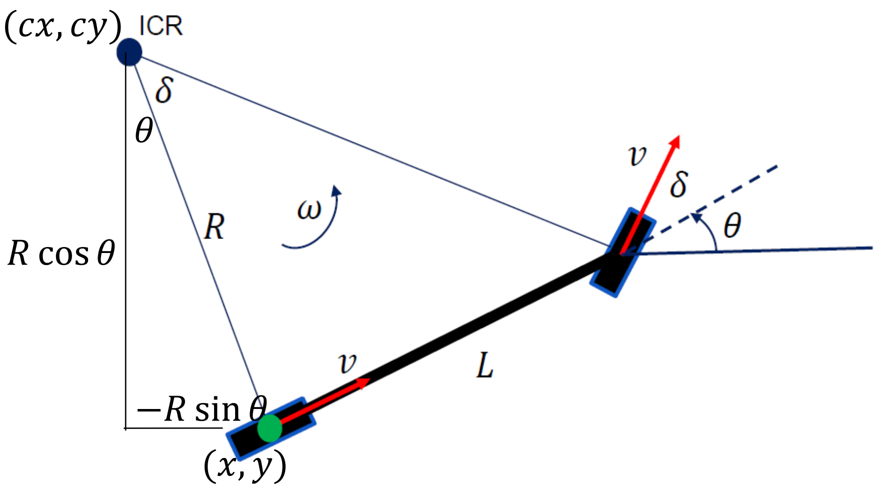
- Circular motion around ICR (accurate):
- $\begin{bmatrix} x(t+dt) \\ y(t+dt) \\ \theta(t+dt) \end{bmatrix} = \begin{bmatrix} x(t) - R\sin\theta + R\sin(\theta + \dot{\theta}dt) \\ y(t) + R\cos\theta - R\cos(\theta + \dot{\theta}dt) \\ \theta + \dot{\theta}dt \end{bmatrix}$
  - $R = \frac{L}{\tan\delta} = \frac{v}{\dot{\theta}}$
- Straight-line motion for small $\dot{\theta}dt$ (approximate):
- $\begin{bmatrix} x(t+dt) \\ y(t+dt) \\ \theta(t+dt) \end{bmatrix} = \begin{bmatrix} x(t) + vdt\cos\theta \\ y(t) + vdt\sin\theta \\ \theta + \dot{\theta}dt \end{bmatrix}$
  - $\delta \to 0, \dot{\theta}dt \to$

State Update in Python Code

- # apply noise
- steering2 = random.gauss(steering, self.steering_noise)
- distance2 = random.gauss(distance, self.distance_noise)
- # apply steering drift
- steering2 += self.steering_drift
- # noise and drift are all set to 0, so steering2 is steering angle δ, distance2 is distance traveled per time step vdt
- # Execute motion
- turn = np.tan(steering2) * distance2 / self.length ($\dot{\theta}dt = \frac{vdt\tan\delta}{L}$)
- **if abs(turn) < tolerance:** (with small $\dot{\theta}dt$)
- # approximate by straight line motion
- self.x += distance2 * np.cos(self.orientation) ($x += vdt\cos\theta$)
- self.y += distance2 * np.sin(self.orientation) ($y += vdt\sin\theta$)
- self.orientation = (self.orientation + turn) % (2.0 * np.pi) ($\theta = (\theta + \dot{\theta}dt)\%(2\pi)$)

$\%(2\pi)$ (modulo 2π) keeps angles to be within 2π; It can be omitted since it does not affect results of $\cos, \sin$ functions (assuming no numeric overflow).

- **else:** (with large $\dot{\theta}dt$)
- # Circular motion around ICR
- radius = distance2 / turn ($R = \frac{v}{\dot{\theta}}$; can also use $R = \frac{L}{\tan\delta}$)
- # compute ICR's coordinates (cx, cy)
- cx = self.x - (np.sin(self.orientation) * radius) ($cx = x - R\sin\theta$)
- cy = self.y + (np.cos(self.orientation) * radius) ($cy = y + R\cos\theta$)
- self.orientation = (self.orientation + turn) % (2.0 * np.pi) ($\theta = (\theta + \dot{\theta}dt)\%(2\pi)$)
- # compute vehicle's x, y coordinate after rotation around ICR
- self.x = cx + (np.sin(self.orientation) * radius) ($x = cx + R\sin\theta$)
- self.y = cy - (np.cos(self.orientation) * radius) ($y = cy - R\cos\theta$)



Straight Line vs. Circular Motion

- Straight line motion (with steering angle $\delta = 0$) is a special case of circular motion with radius ∞.
  - $\delta \to 0 \Rightarrow R = \frac{L}{\tan \delta} \to \infty, \dot{\theta} = \frac{v}{R} \to 0$ (small steering angle δ leads to slow angular velocity $\dot{\theta}$.)
  - We use this special case to improve computational efficiency for small $\delta, \dot{\theta}$, and avoid division by 0.
- $x(t + dt) = x(t) - R \sin\theta + R \sin(\theta + \dot{\theta}dt) = x + R(\sin(\theta + \dot{\theta}dt) - \sin\theta) = x + \frac{v}{\dot{\theta}} * \cos\theta * \dot{\theta}dt = x + vdt \cos\theta$
  - $\sin(\theta + \dot{\theta}dt) - \sin\theta \approx \frac{d}{dt}\sin\theta * \dot{\theta}dt = \cos\theta * \dot{\theta}dt$, for small $\dot{\theta}dt$.
- $y(t + dt) = y(t) + R \cos\theta - R \cos(\theta + \dot{\theta}dt) = y - R(\cos(\theta + \dot{\theta}dt) - \cos\theta) = y + \frac{v}{\dot{\theta}} * \sin\theta * \dot{\theta}dt = y + vdt \sin\theta$
  - $\cos(\theta + \dot{\theta}dt) - \cos\theta \approx \frac{d}{dt}\cos\theta * \dot{\theta}dt = -\sin\theta * \dot{\theta}dt$, for small $\dot{\theta}dt$.
- Four special cases of straight line motion:
  - $\theta = 0 \Rightarrow x' = x + vdt, y' = y$ (east)
  - $\theta = \pi \Rightarrow x' = x - vdt, y' = y$ (west)
  - $\theta = \frac{\pi}{2} \Rightarrow x' = x, y' = y + vdt$ (north)
  - $\theta = \frac{3\pi}{2} \Rightarrow x' = x, y' = y - vdt$ (south)

PID Control Lab Setup

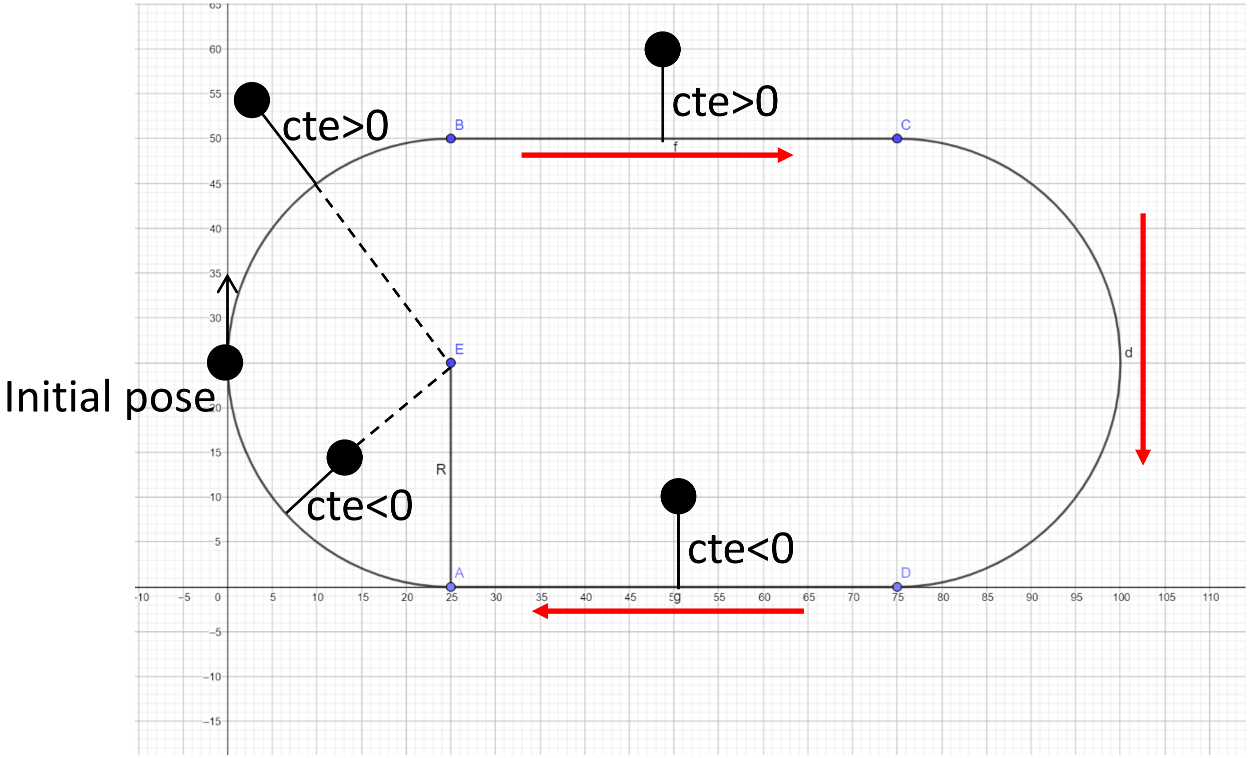
- Based on Udacity course Lesson AI for Robotics, Lesson 15: PID Control
  - https://classroom.udacity.com/courses/cs373/
- Lateral control of a car to run along a race track (either straight line or circular) with fixed speed.

Outline

- PID Control
- MPC Control
- Kinematic bicycle model
- Twiddle() for PID control tuning

Udacity: Racetrack Control

- Lesson 16: Problem Set 5, 4. Quiz: Racetrack control.
- Cross-track error (cte): lateral distance (of rear wheel) to desired trajectory, defined as:
  - if $x \in [radius, 3 * radius]$: deviation from the straight horizontal track (with the implicit assumption that the car will not deviate from the track for more than $radius$.)
  - Otherwise: deviation from each semi-circle track.
- For clockwise traversal, cte>0 if car is outside of the track region; cte<0 if inside.



run()

- Implements the PID controller for with PID gains
  - params[0] as K_p; params[1] as K_d; params[2] as K_i.
  - Returns actual trajectory as arrays of x_trajectory[], y_trajectory[]; and average error, defined as sum of squared cte
- For clockwise travel direction:
  - If cte>0, car is outside of track region; steering angle δ should decrease (turn right according to the counter-clockwise convention)
  - If cte<0, car is inside of track region; steering angle δ should increase (turn left)
  - Based on the above, K_p should be positive (this can be a sanity check for your final solution)

```python
def run(robot, params, n=100, speed=1.0):
    x_trajectory = []
    y_trajectory = []
    err = 0
    # TODO: your code here
    prev_cte = robot.y
    int_cte = 0
    for i in range(2 * n):
        cte = robot.y
        diff_cte = cte - prev_cte
        int_cte += cte
        prev_cte = cte
        steer = -params[0] * cte - params[1] * diff_cte - params[2] * int_cte
        robot.move(steer, speed)
        x_trajectory.append(robot.x)
        y_trajectory.append(robot.y)
        if i >= n:
            err += cte ** 2
    return x_trajectory, y_trajectory, err / n
```

This is for straight line track; need to call myrobot.cte() for circular track.

PID control law

For General dt

- The Python code assumes controller time step size $dt = 1$.
  - Also, v is constant. δ's range is unconstrained (steering wheel can be turned to arbitrary angle).
- For general dt, the following needs to be modified:
  - Call myrobot.move(steer, speed*dt), to match its definition move(self, steering, distance,…)
  - In the PID control law:
    - Differential of error $diffcte = \frac{cte-prevcte}{dt}$
    - Integral of error $intcte \mathrel{+}= cte * dt$

twiddle()

- twiddle: “twist, move, or fiddle with (something), typically in a purposeless or nervous way”

Adjust each $p[i]$ in turn.

Adjust $p[i]$ to $p[i] + dp[i]$ and run(). If error decreases, we adjusted in the right direction, keep the original adjustment $p[i] + dp[i]$, and increase step size to $1.1 * dp[i]$.

If error increases, we adjusted in the wrong direction, adjust in the opposite direction to $p[i] - 2 * dp[i]$. Run again.

If error decreases, we adjusted in the right direction, keep the original adjustment $p[i] + dp[i]$, and increase step size to $1.1 * dp[i]$.

If error increases, we adjusted in the wrong direction, reduce step size to $.9 * dp[i]$. We do not reverse direction here to avoid oscillation around current $p[i]$ with no progress.

```python
def twiddle(tol=0.2):
    # TODO: Add code here
    # Don't forget to call `make_robot` before you call `run`!
    p = [0.0, 0.0, 0.0]
    dp = [1.0, 1.0, 1.0]
    robot = make_robot()
    x_trajectory, y_trajectory, best_err = run(robot, p)

    it = 0
    while sum(dp) > tol:
        # print("Iteration {}, best error = {}".format(it, best_err))
        for i in range(len(p)):
            p[i] += dp[i]
            robot = make_robot()
            x_trajectory, y_trajectory, err = run(robot, p)

            if err < best_err:
                best_err = err
                dp[i] *= 1.1
            else:
                p[i] -= 2 * dp[i]
                robot = make_robot()
                x_trajectory, y_trajectory, err = run(robot, p)

                if err < best_err:
                    best_err = err
                    dp[i] *= 1.1
                else:
                    p[i] += dp[i]
                    dp[i] *= 0.9
        it += 1
    return p, best_err
```

Changing to P Control

- Setting $dparams[1] = dparams[2] = 0$ turns it into a P controller, since K_d, K_i are initialized to 0 and never changed. P_p is adjusted in twiddle()
- twiddle() is a local optimization algorithm, so perf is dependent on parameter initialization. Here are initialized to 0 for simplicity.
  - https://classroom.udacity.com/courses/cs373/lessons/91f48b5b-a063-41f9-ace6-5fb9e7508941/concepts/b740218e-b0eb-40dc-80af-343912305293

```
#### ENTER CODE BELOW THIS LINE

n_params = 3
dparams     = [1.0 for row in range(n_params)]
params      = [0.0 for row in range(n_params)]
dparams[2] = 0.0
dparams[1] = 0.0

best_error = run(params)
n = 0
while sum(dparams) > tol:
    for i in range(len(params)):
```

twiddle() vs. Pole Placement

- twiddle() is a model-free approach to tuning PID controller by Gradient descent. It adjusts each PID gain parameter systematically, gradually decreasing or increasing step of adjustment $dp[i]$ (the gradient) until convergence to minimum best_err.
- Pole placement for PID controller design is model-based, but requires a linear model
  - $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{\tan\delta}{L} \end{bmatrix} \approx \begin{bmatrix} v \\ v\theta \\ u \end{bmatrix}$
  - With simplifying assumptions:
    - θ is small
    - Simplified kinematic model: $u = \dot{\theta}$ as control input instead of δ