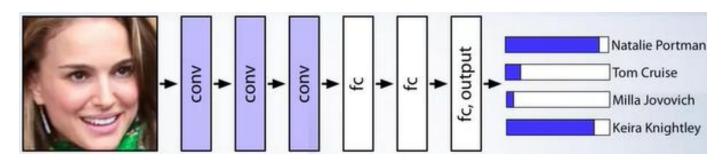
L3.1 CNN for Computer Vision

Zonghua Gu 2022



Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN Case Studies

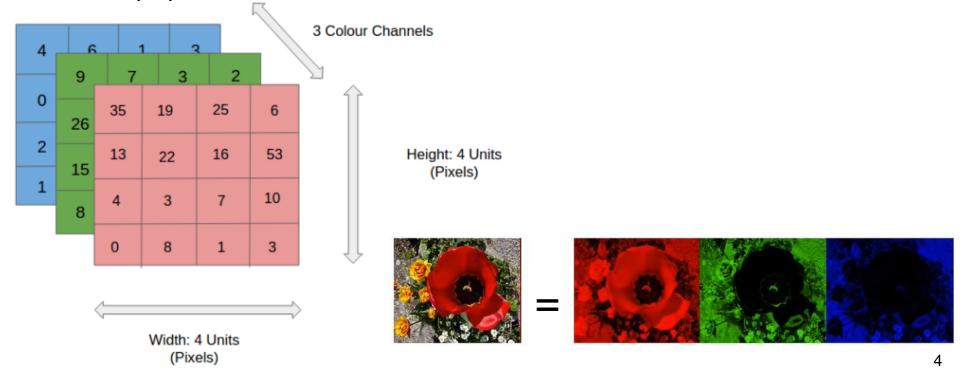
Classic Computer Vision

- Most "classic" (non-ML) CV algorithms are implemented in the OpenCV library, including
 - Core Operations:
 - basic operations on image like pixel editing, geometric transformations...
 - Image Processing
 - Thresholding, smoothing, edge detection, Hough Line Transform...
 - Feature Detection and Description
 - HOG, SIFT, SURF, BRIEF, ORB...
 - Video analysis
 - Object tracking w. optical flow
 - Camera Calibration and 3D Reconstruction
- They are simple, fast and reliable (e.g., for lane detection), and are often used in place of or in conjunction w. complex ML/DL algorithms, which may sometimes be unreliable and unpredictable.



Input Image Encoding

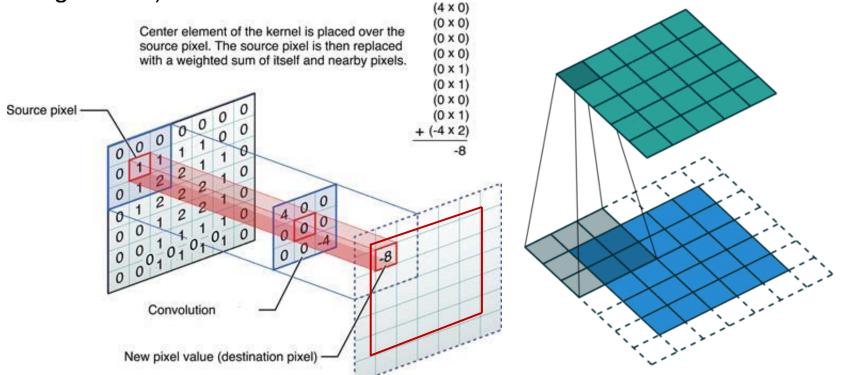
- A size $N \times N$ color image has volume $N \times N \times 3$, w. $N \times N$ pixels and 3 color components (Red, Green, and Blue, RGB) for each pixel
- A size $N \times N$ greyscale image has volume $N \times N \times 1$
- Color depth, or bit depth, is number of bits used for each color component of a single pixel
 - Typical value is 8, so pixel value has range [0, 255]
 - Larger depth is possible, e.g., true color (24-bit) is used in computer and phone displays for human eyes, but 8-bit is typically enough for CV



Filters/Kernels in Computer Vision

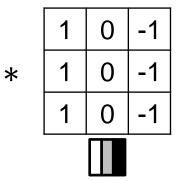
- Convolution operation: we slide each filter (also called kernel) across the width
 and height of the input volume, and compute dot products between the entries
 of the filter and the input. As we slide the filter over the width and height of the
 input volume, we will produce a 2D activation map (also called feature map) that
 gives the responses of that filter at every spatial position.
 - Dot product: elementwise multiplication of a filter w. corresponding input values, then summing them to generate one output value

Used to extract features for downstream tasks (classification or regression)

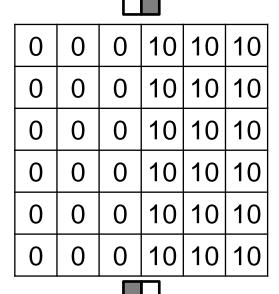


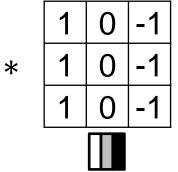
A Filter for Vertical Edge Detection

| 10 | 10 | 10 | 0 | 0 | 0 |
|----|----|----|---|---|---|
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |
| 10 | 10 | 10 | 0 | 0 | 0 |



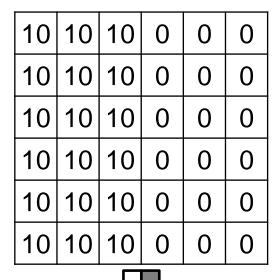
| 0 | 30 | 30 | 0 | | |
|---|----|----|---|--|--|
| 0 | 30 | 30 | 0 | | |
| 0 | 30 | 30 | 0 | | |
| 0 | 30 | 30 | 0 | | |
| | | | | | |

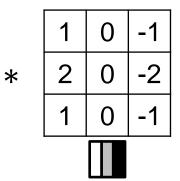




| 0 | -30 | -30 | 0 |
|---|-----|-----|---|
| 0 | -30 | -30 | 0 |
| 0 | -30 | -30 | 0 |
| 0 | -30 | -30 | 0 |
| | | | |

Sobel Filter for Vertical Edge Detection





| 0 | 40 | 40 | 0 |
|---|----|----|---|
| 0 | 40 | 40 | 0 |
| 0 | 40 | 40 | 0 |
| 0 | 40 | 40 | 0 |

| 0 | 0 | 0 | 10 | 10 | 10 |
|---|---|---|----|----|----|
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |
| 0 | 0 | 0 | 10 | 10 | 10 |

| | ~ | 0 | \ |
|---|---|---|----------|
| : | 2 | 0 | -2 |
| | 1 | 0 | -1 |
| | | | |

| 0 | -40 | -40 | 0 |
|---|-----|-----|---|
| 0 | -40 | -40 | 0 |
| 0 | -40 | -40 | 0 |
| 0 | -40 | -40 | 0 |
| | | | |

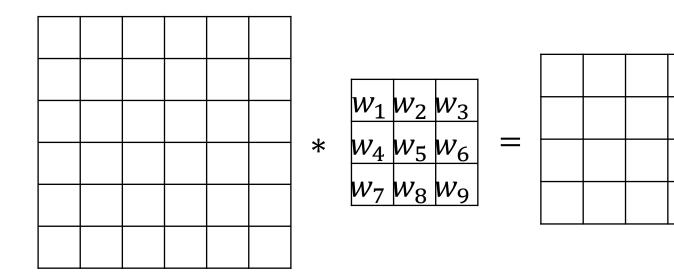
Common Filters in CV

| Operation | Kernel ω | Image result g(x,y) | 7 | |
|----------------|---|---------------------|--|--|
| Identity | $ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} $ | | Box blur (normalized) | $\frac{1}{9} \left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right]$ |
| | $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ | | Gaussian blur 3 × 3 (approximation) | $\frac{1}{16} \left[\begin{array}{ccc} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{array} \right]$ |
| Edge detection | $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ | | Gaussian blur 5 × 5 (approximation) | $\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$ |
| | $\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$ | | Unsharp masking 5 × 5 Based on Gaussian blur | 4 16 24 16 4 |
| Sharpen | $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ | | with amount as 1 and threshold as 0 (with no image mask) | |

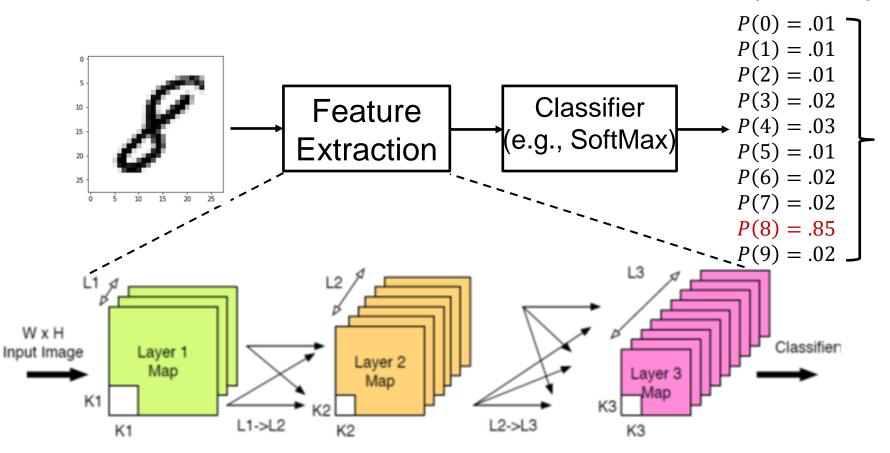
These filters were designed, or "hand-crafted", by CV researchers. They
extract features used by downstream tasks such as classification, image
segmentation, etc.

Machine Learning Meets CV

- Instead of hand-crafted filters in classic CV, why not learn custom filters from data by supervised learning?
 - For easy tasks like edge detection, learning may recover filters similar to hand-crafted ones.
 - For difficult tasks like cat vs. dog classification, learning is essential to achieving good results



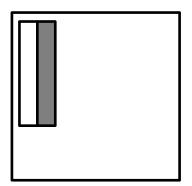
Convolutional Neural Networks (CNN)

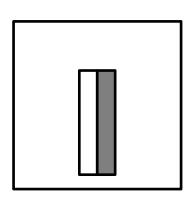


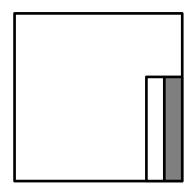
 A CNN (also called ConvNet) is a sequence of Convolutional (CONV) Layers, Pooling (POOL) Layers and non-linear activation functions for feature extraction, followed by one or more Fully-Connected (FC) Layers for classification based on the extracted features

Receptive Field and Parameter Sharing

- Each neuron in a CONV layer has local, sparse connectivity to a small patch of the input volume w. size of the filter, called its Receptive Field
 - Each neuron covers a limited, narrow "field-of-view"
 - In contrast, each neuron in a FC layer has RF that covers the entire input volume
- Parameter sharing: all neurons in the same CONV layer share the same filter params w, b
 - It helps to reduce the number of params significantly compared to fully-connected networks
 - It gives translation invariance, e.g., an edge can be detected regardless of its location in the image

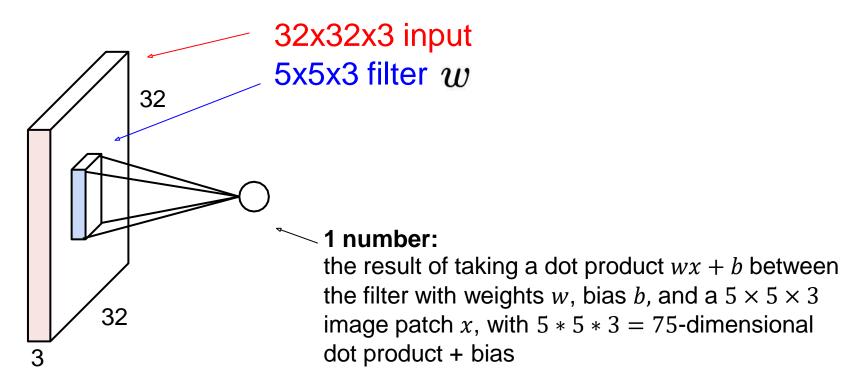


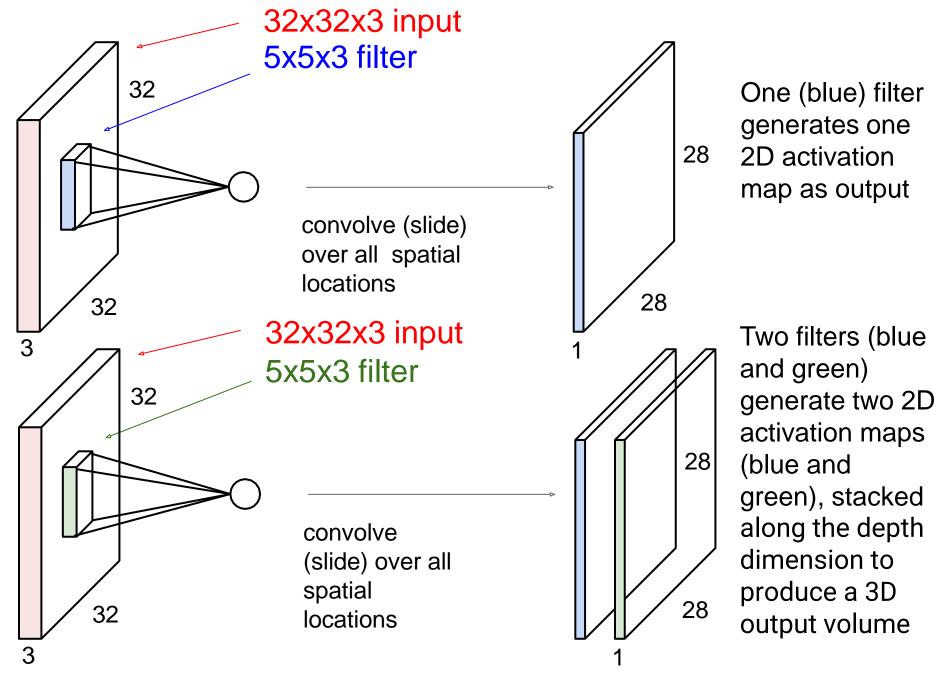




Convolution Operation

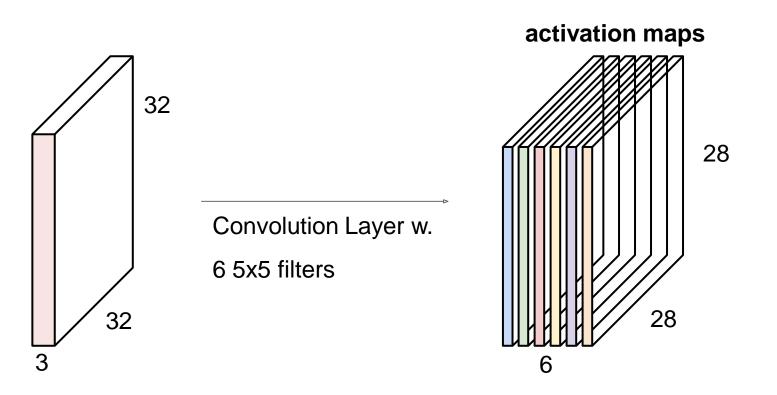
- Slide the filter over the image spatially, computing dot products $w^Tx + b$ to generate an activation map as output
- The input may be an input RGB image w. 3 channels, hence depth=3, or intermediate activation maps generated by hidden layers of a CNN. We use the terms "input volume" and "output volume" to emphasize they may be 3D tensors



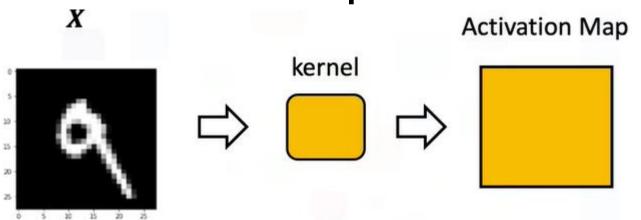


Stacked Activation Maps

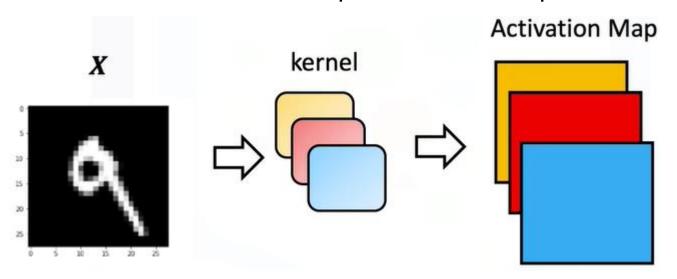
- If we have 6.5×5 filters, we'll get 6 different activation maps (feature maps), each computed by convolution of one filter with the input
 - For each 5x5 patch of the input, there are 6 different neurons looking at it, each extracting different features
- We stack these up to get an output volume (a new "image") of size 28 × 28 × 6, an intermediate representation to be passed to subsequent layers



Activation Maps Illustration



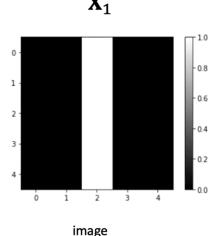
1 filter/kernel, 1 output activation map

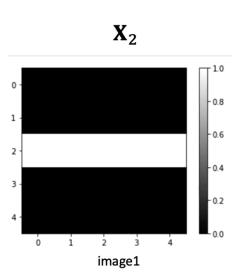


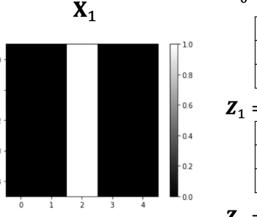
3 filters/kernels, 3 output activation maps

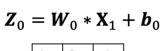
Concrete Example: 3 Filters

- 3 filters W_0 , W_1 , W_2 , each extracting different features. $(W_i * X_i)$ denotes convolution of filter W_i w. input X_i) (bias terms are assumed to be 0 here)
- Upper left: filter W_0 extracts vertical line features Z_0 from input image X_{1} (the other 2) filters do not extract any meaningful features)
- Lower left: filter W_1 extracts horizontal line features Z_1 from input image X_2 (the other 2 filters do not extract any meaningful features)

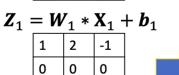


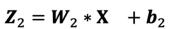






| 1 | 0 | -1 | |
|---|---|----|--|
| 1 | 0 | -2 | |
| 1 | 0 | -1 | |



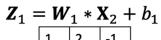


-2

| 1 | 1 | 1 | |
|---|---|---|--|
| 1 | 1 | 1 | |
| 1 | 1 | 1 | |

$$\boldsymbol{Z}_0 = \boldsymbol{W}_0 * \boldsymbol{X}_2 + \boldsymbol{b}_0$$

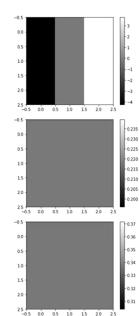
| 1 | 0 | -1 | |
|---|---|----|--|
| 1 | 0 | -2 | |
| 1 | 0 | -1 | |

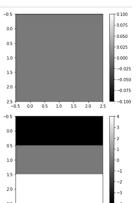


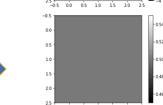
| | Τ. | | |
|----|----|----|--|
| 1 | 2 | -1 | |
| 0 | 0 | 0 | |
| -1 | -2 | -1 | |

$$\mathbf{Z}_2 = \mathbf{W}_2 * \mathbf{X}_2 + \mathbf{b}_2$$

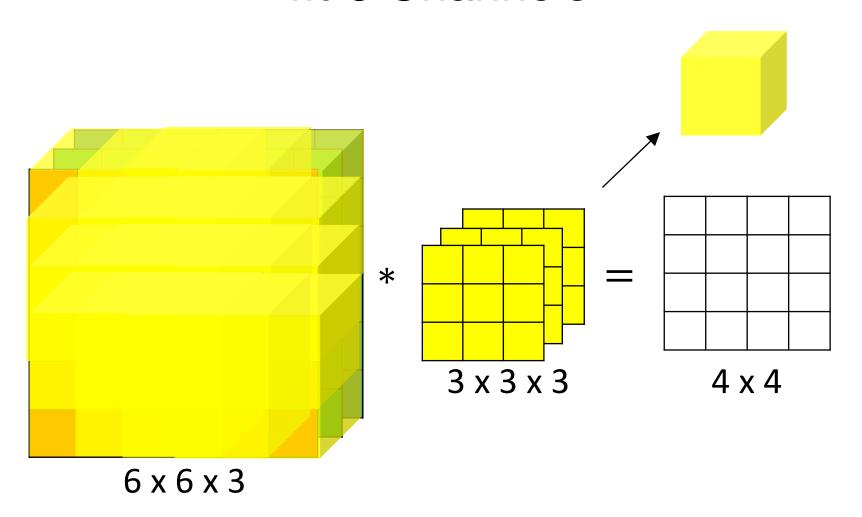
| 1 | 1 | 1 | |
|---|---|---|--|
| 1 | 1 | 1 | |
| 1 | 1 | 1 | |



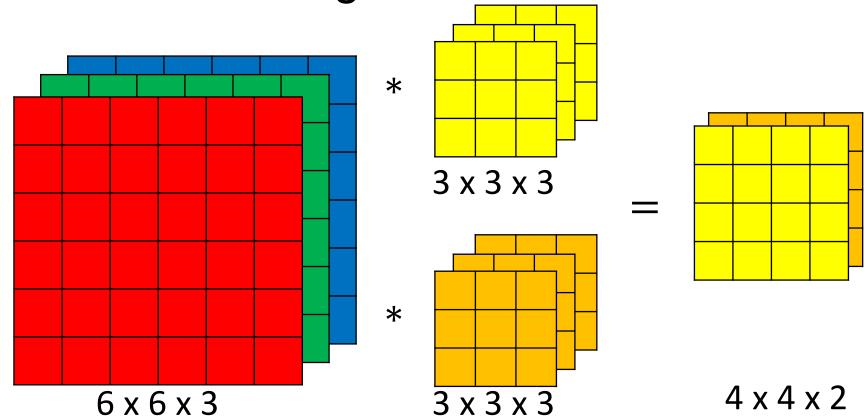




Convolution of a Filter on RGB Image w. 3 Channels



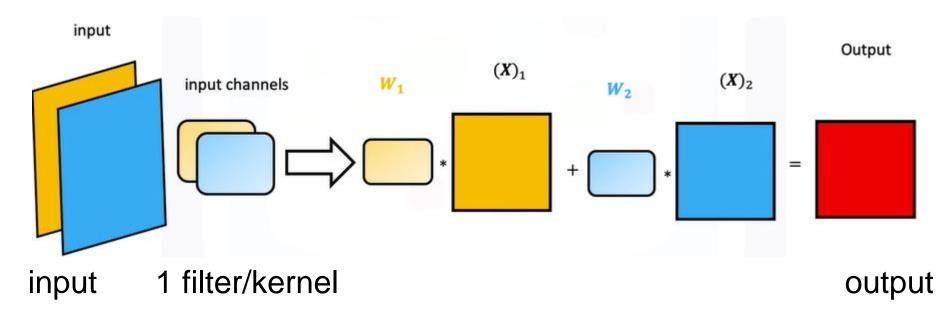
Convolution of 2 Filters on RGB Image w. 3 Channels



- 6x6 input feature map w. 3 channels; 2 3x3 filters with depth 3; 4x4 output feature map w. 2 channels
- # channels of input feature map == # depth of each filter (3)
- # channels of output feature map == # filters (2)

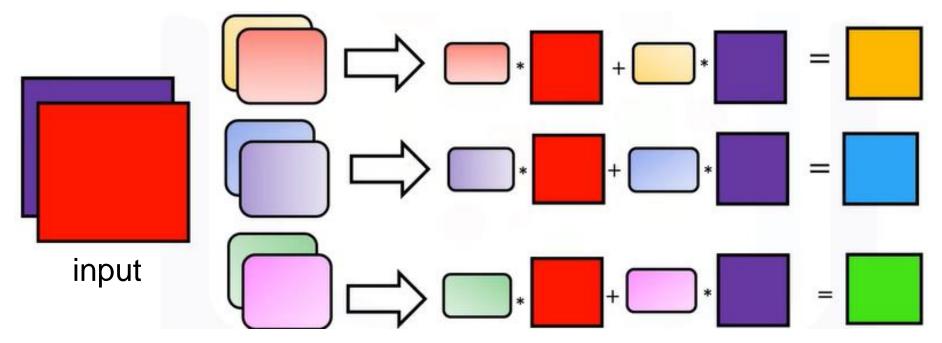
Convolution Example 1

- conv=nn.Conv2d(in_channels=2, out_channels=1, kernel_size=3)
 - Pytorch code for a CONV layer with an input image with 2 channels (in_channels=2), 1 3 × 3 filter (with depth 2), 1 output activation maps (out_channels=1).
 - (The biases are assumed to be 0)



Convolution Example 2

- conv4=nn.Conv2d(in_channels=2, out_channels=3, kernel_size=3)
 - Pytorch code for a CONV layer with an input image with 2 channels (in_channels=2), 3 3 x 3 filters (with depth 2), 3 output activation maps (out_channels=3)
 - (The biases are assumed to be 0)



3 filters/kernels

Convolution Example 2: Filters and Input Image

conv4.state_dict()['weight'][0][0]]



| | 0 | 0 | 0 |
|---|---|-----|---|
| | 0 | 0.5 | 0 |
| I | 0 | 0 | 0 |

conv4.state_dict()['weight'][0][1]

 $W_{0,1}$

| 0 | 0 | 0 |
|---|-----|---|
| 0 | 0.5 | 0 |
| 0 | 0 | 0 |

Image4[1,0,:,:]

Image4[1,1,:,:]

conv4.state_dict()['weight'][2][0]]

 $W_{2,0}$

| 1 | 0 | -1 |
|---|---|----|
| 1 | 0 | -2 |
| 1 | 0 | -1 |

conv4.state_dict()['weight'][2][1]

 $W_{2,1}$

| 1 | 2 | -1 |
|----|----|----|
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Channel 1

| 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

conv4.state_dict()['weight'][1][0]]

$$W_{1,0}$$

| 0 | 0 | 0 |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 0 | 0 |

conv4.state_dict()['weight'][1][1]

 $W_{1,1}$

| 0 | 0 | 0 |
|---|----|---|
| 0 | -1 | 0 |
| 0 | 0 | 0 |

Channel 2

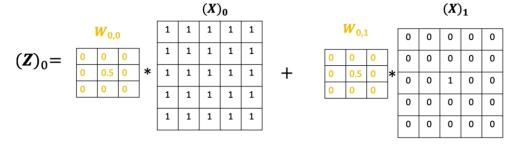
| 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

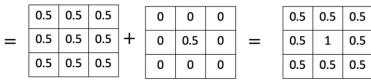
 3.3×3 filters

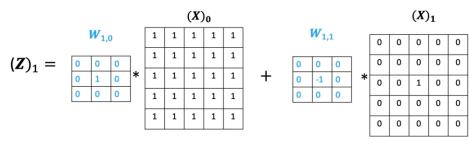
input image with 2 channels

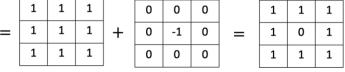
Convolution Example 2: Output

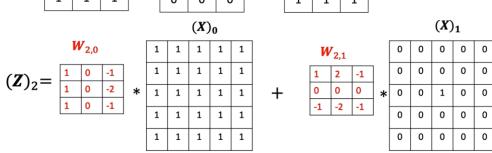
- Each of the 3 filters convolved with the input image generates an output activation map.
- The output volume consists of 3 3 × 3 activation maps, with volume 3 × 3 × 3









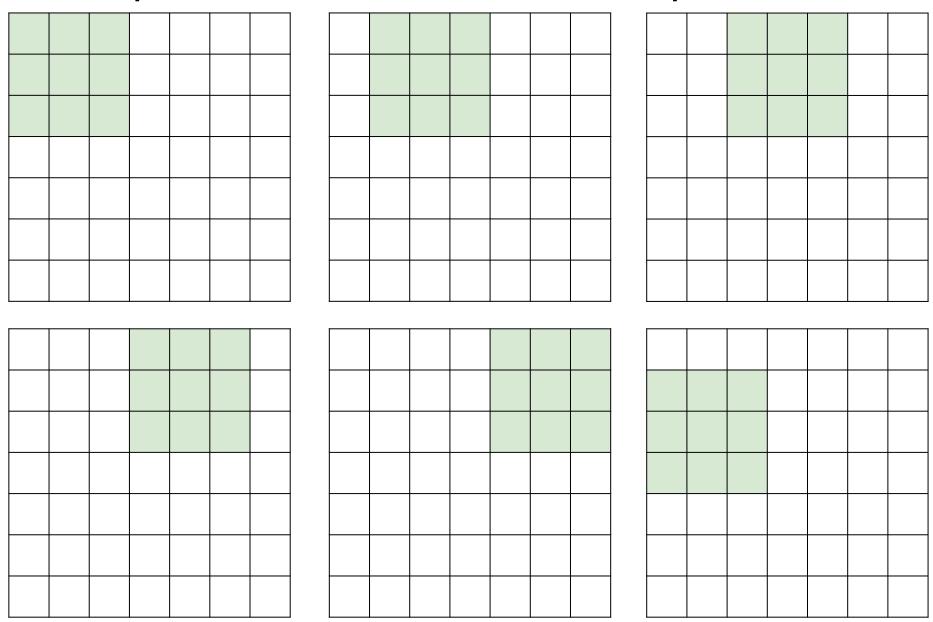


$$= \begin{array}{c|cccc} -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \end{array}$$

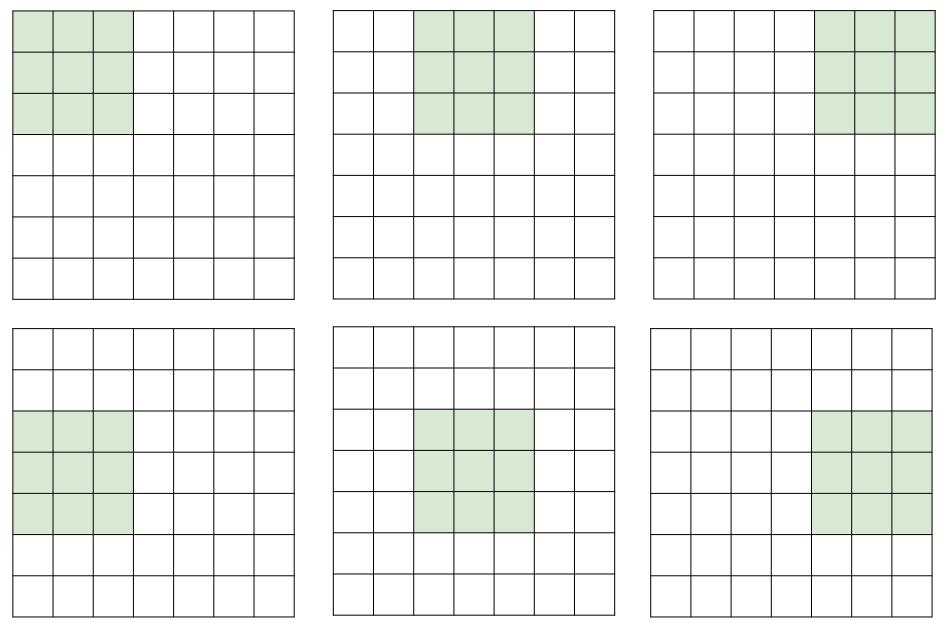
Filters and Activation Maps Example

filters Activations: one filter => example 5x5 filters one activation map (32 total) Activations:

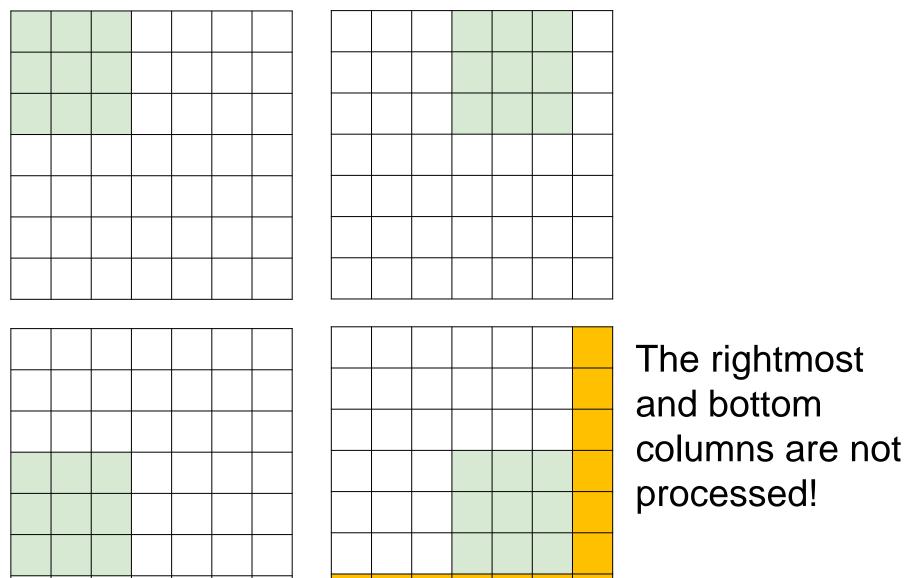
7x7 input, 3x3 filter, stride=1 \Rightarrow output: 5x5 filter



7x7 input, 3x3 filter, stride= $2 \Rightarrow$ output: 3x3 filter

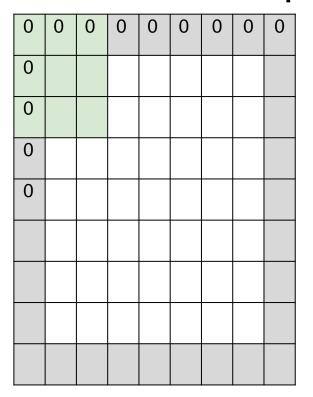


7x7 input, 3x3 filter, stride=3 \Rightarrow output: ???

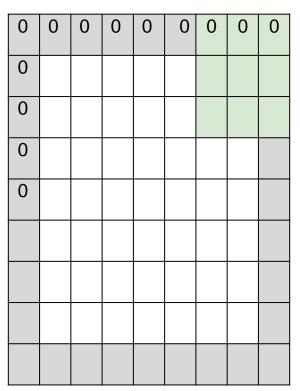


Solution: Add Padding

7x7 input, 3x3 filter, stride=3, zero padding
 w. 1 ⇒ output: 3x3 filter



| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| 0 | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |



Computation of CONV Layer Sizes

Summary. To summarize, the Conv Layer:

• Accepts a volume of size $W_1 imes H_1 imes D_1$

Requires four hyperparameters:

- Number of filters K,
- \circ their spatial extent F,
- the stride S,
- \circ the amount of zero padding P.

Common settings:

K = (powers of 2, e.g. 32, 64, 128, 512)

- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 1, S = 1, P = 0
- Produces a volume of size $W_2 imes H_2 imes D_2$ where:
 - $W_2 = (W_1 F + 2P)/S + 1$
 - $H_2 = (H_1 F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
 - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.
- If input has square shape, then we denote $N_1 = W_1 = H_1$; a filter is assumed to have square shape
- Each filter has the same depth D_1 as its input volume, and the number of filters K equals the depth D_2 of its output volume
- In practice, it is common to have stride S=1, filter size $F\times F$, and zero-pad $P=\frac{1}{2}(F-1)$. Then output activation map has same spatial size as input. This is called "same padding"
 - $W_2 = \frac{1}{5}(W_1 + 2P F) + 1 = \frac{1}{1}(W_1 + F 1 F) + 1 = W_1$; similarly, $H_2 = H_1$
 - e.g., $F < 3 \Rightarrow P = 0$; $F = 3 \Rightarrow P = 1$; $F = 5 \Rightarrow P = 2$

CONV Example 1: No Pad

- Input volume: $5 \times 5 \times 1$ ($W_1 = H_1 = N_1 = 32$, $D_1 = 1$)(e.g., a greyscale image)
- A $3 \times 3 \times 1$ filter $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (K = 1, F = 3) w. stride S = 1, no pad
- Output activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P F) + 1 = \frac{1}{1}(5 3) + 1 = 3$
 - Depth: $D_2 = K = 1$
- Output volume: 3 × 3 × 1
- Even though the fig shows sequential computation, convolution operations are inherently parallel, hence suitable for efficient implementation on parallel hardware, e.g., GPU, FPGA...

| 1 _{×1} | 1 _{×0} | 1 _{×1} | 0 | 0 |
|------------------------|------------------------|------------------------|---|---|
| 0,0 | 1, | 1,0 | 1 | 0 |
| 0 _{×1} | O _{×0} | 1 _{×1} | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |

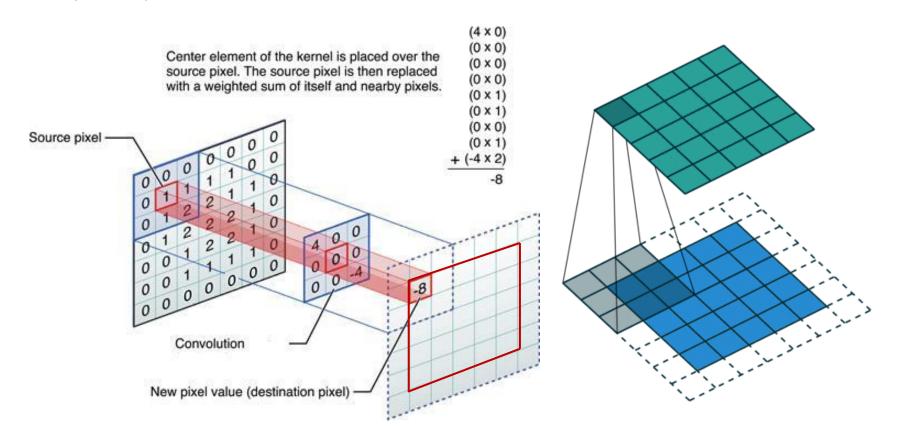
| 4 | |
|---|--|
| | |
| | |

Image

Convolved Feature

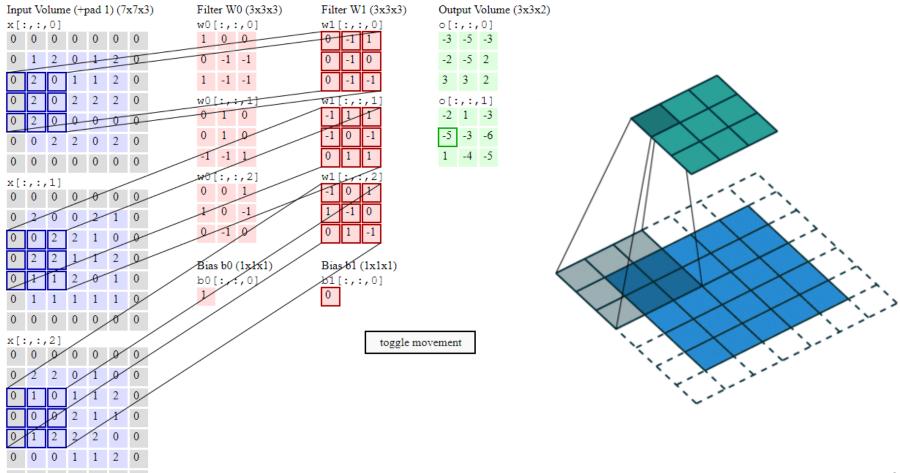
CONV Example 2: Same Padding

- Input volume: 5 × 5 × 1
- A $3 \times 3 \times 1$ filter (K = 1, F = 3) w. stride S = 1, pad P = 1
- Output volume: $5 \times 5 \times 1$ (since $\frac{1}{1}(5 + 2 3) + 1 = 5$)
- Output activation map has the same spatial dimension as input (5×5)



CONV Example 3: Stride S = 2

- Input volume: $5 \times 5 \times 3$
- $2 3 \times 3 \times 3$ filters (K = 2, F = 3) w. stride S = 2, pad P = 1
- Output volumes: $2 \ 3 \times 3 \times 1$ (since $\frac{1}{2}(5 + 2 * 1 3) + 1 = 3$)
 - Animation: https://cs231n.github.io/convolutional-networks/

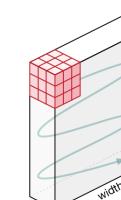


CONV Example 4: Input Depth $D_1 = 3$

| 0 | 0 | 0 | 0 | 0 | 0 | • |
|---|-----|-----|-----|-----|-----|------------|
| 0 | 156 | 155 | 156 | 158 | 158 | |
| 0 | 153 | 154 | 157 | 159 | 159 | |
| 0 | 149 | 151 | 155 | 158 | 159 | (i |
| 0 | 146 | 146 | 149 | 153 | 158 | |
| 0 | 145 | 143 | 143 | 148 | 158 | · |
| | | · | | | | |

| 0 | 0 | 0 | 0 | 0 | 0 | |
|---|-----|-----|-----|-----|-----|-----|
| 0 | 167 | 166 | 167 | 169 | 169 | |
| 0 | 164 | 165 | 168 | 170 | 170 | |
| 0 | 160 | 162 | 166 | 169 | 170 | |
| 0 | 156 | 156 | 159 | 163 | 168 | |
| 0 | 155 | 153 | 153 | 158 | 168 | ٠ |
| | | · | | | | ٠., |

| 0 | 0 | 0 | 0 | 0 | 0 | |
|---|-----|-----|-----|-----|-----|-----|
| 0 | 163 | 162 | 163 | 165 | 165 | |
| 0 | 160 | 161 | 164 | 166 | 166 | ::: |
| 0 | 156 | 158 | 162 | 165 | 166 | |
| 0 | 155 | 155 | 158 | 162 | 167 | |
| 0 | 154 | 152 | 152 | 157 | 167 | |
| | | | | | | |



Input Channel #1 (Red)

| -1 | -1 | 1 |
|----|----|----|
| 0 | 1 | -1 |
| 0 | 1 | 1 |

Kernel Channel #1



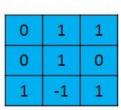
308

| 1 | 0 | 0 |
|---|----|----|
| 1 | -1 | -1 |
| 1 | 0 | -1 |

Kernel Channel #2



-498

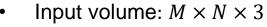


Kernel Channel #3



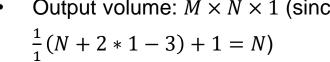
164 +1 = -25

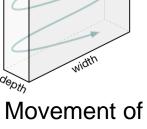




• A
$$3 \times 3 \times 3$$
 filter $(K = 1, F = 3)$ w. stride $S = 1$, pad $P = 1$

Output volume: $M \times N \times 1$ (since $\frac{1}{1}(M + 2 * 1 - 3) + 1 = M$,





height

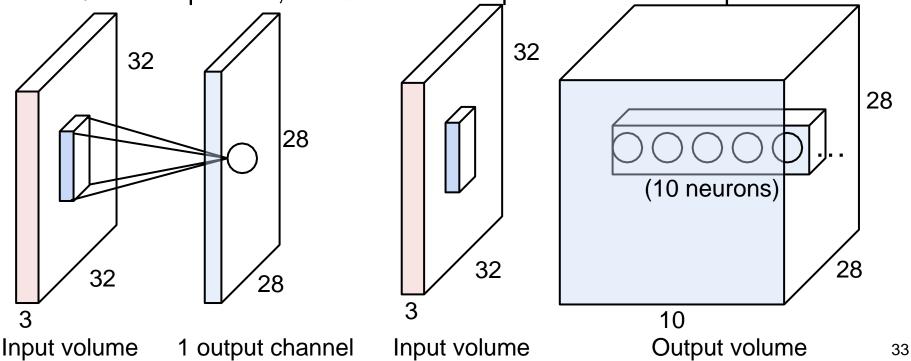
the filter

Output



CONV Example 5: Multiple Filters K = 10

- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$)
- $10.5 \times 5 \times 3$ filters (K = 10, F = 5) w. stride S = 1, no pad (P = 0)
- Each output activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P F) + 1 = \frac{1}{1}(32 5) + 1 = 28$
 - Depth: $D_2 = K = 10$
- Output volume: $28 \times 28 \times 10$
- No. params (weights and biases) in this layer: each filter has 5 * 5 * 3 + 1 = 76 params, so 10 filters add up to 76 * 10 = 760 params



CONV Example 6: Pad P=2

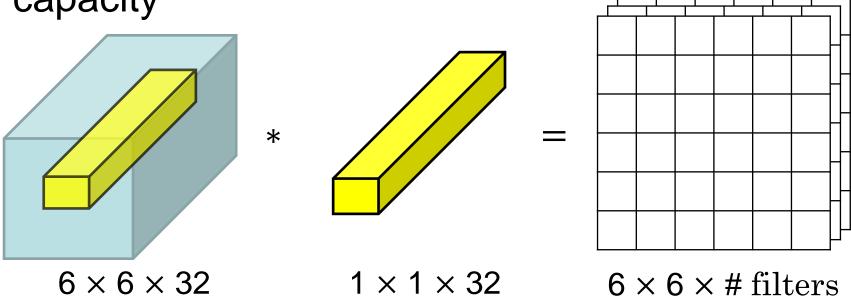
- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32$, $D_1 = 3$)
- $10.5 \times 5 \times 3$ filters (K = 10, F = 5) w. stride S = 1, pad P = 2
- Each activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P F) + 1 = \frac{1}{1}(32 + 2 * 2 5) + 1 = 32$
 - Depth: $D_2 = K = 10$
- Output volume: $32 \times 32 \times 10$
- No. params: each filter has 5 * 5 * 3 + 1 = 76 params, so 10 filters add up to 76 * 10 = 760 params

Pointwise Convolution with 1×1 Filter

• A 1×1 filter performs "mixing" of the input channels, then applies a non-linear activation function

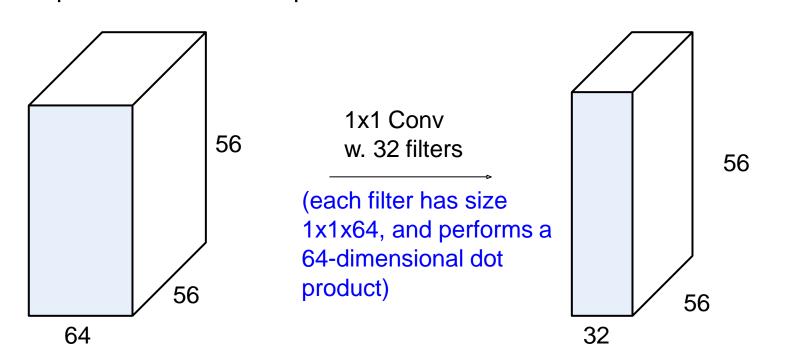
 Can be used to reduce the number of channels (volume depth); the non-linear activation function also helps increase model



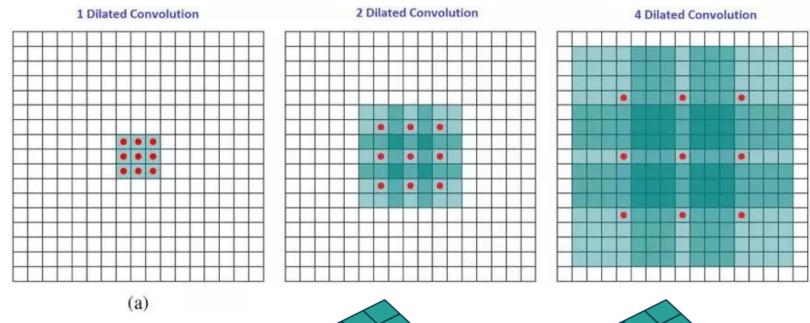


1 × 1 Filter Example

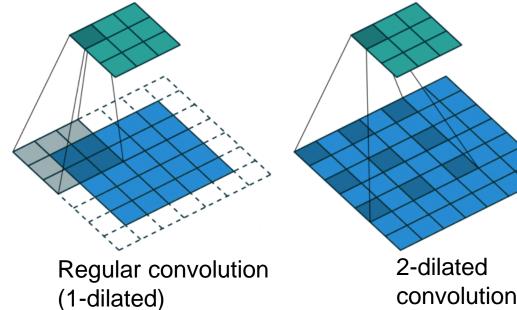
- Input volume: $56 \times 56 \times 64$ ($W_1 = H_1 = N_1 = 56, D_1 = 64$)
- 32 1 × 1 × 64 filters (K = 32, F = 1) w. stride S = 1, no pad
- Each activation map:
 - Spatial size: $W_2 = H_2 = N_2 = \frac{1}{S}(N_1 + 2P F) + 1 = \frac{1}{1}(56 1) + 1 = 56$
 - Depth: $D_2 = K = 32$
- Output volume: $56 \times 56 \times 32$
- No. params: each filter has 1 * 1 * 64 + 1 = 65 params, so 32 filters add up to 65 * 32 = 2080 params



Dilated Convolution

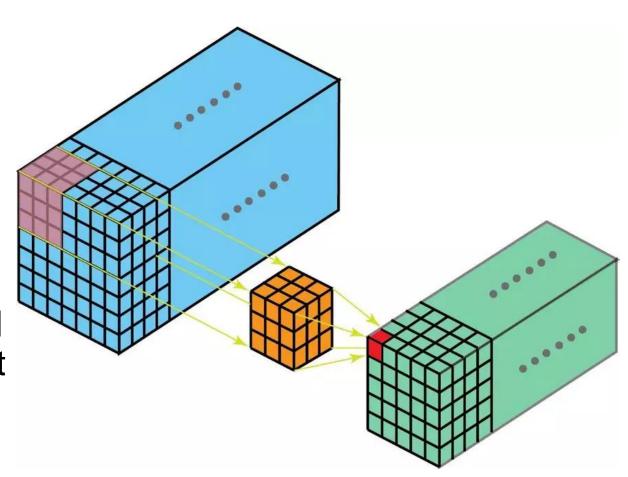


 Insert 0s between input elements to increase receptive field size without increasing # params



3D Convolution

- 3D filter slides along all 3 axes (width, height, depth). Very computation intensive
- Useful for 3D images such as medical CT/MRI images, or Point Clouds from Lidar



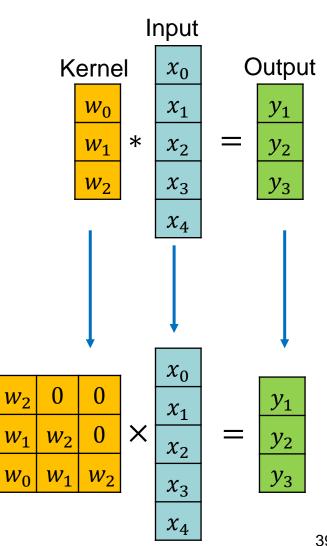
Converting Convolution to Matrix Multiplication: 1D CONV Example

 W_1

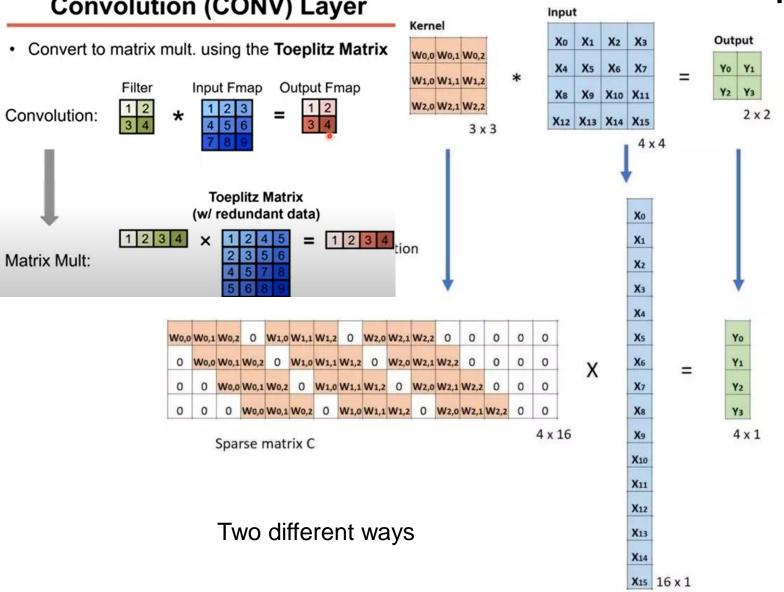
 W_0

0

 Since parallel hardware (GPU, FPGA...) can handle matrix multiplication efficiently, this conversion increases computation efficiency at the expense of increased memory size for storing the weights (the biases are not shown in fig) 0



Converting Convolution to Matrix Multiplication: 2D CONV Example

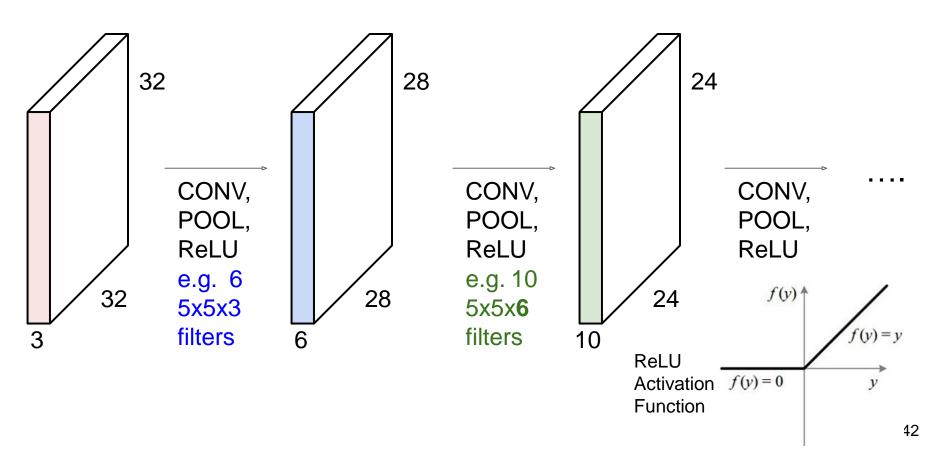


Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN Case Studies

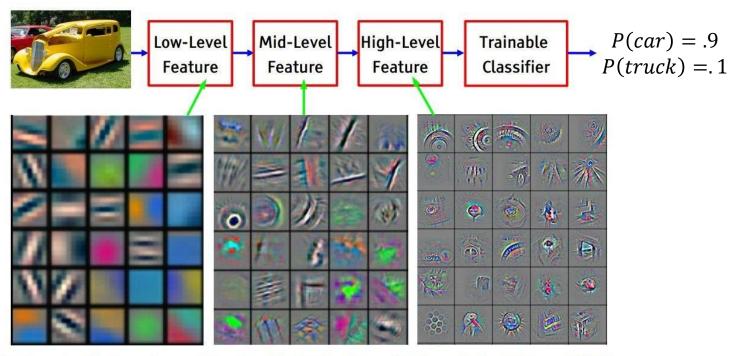
Typical CNN Architecture

- Multiple layers, each consisting of CONV, POOL and non-linear activation functions (e.g., ReLU), are stacked into a deep network
 - Many variants possible, e.g., multiple CONV layers can be stacked without POOL and activation functions in-between



Feature Hierarchy

 Multiple hidden layers extract a hierarchy of increasingly-abstract features layer-by-layer, until the last layer produces a classification result



Pooling (Sub-Sampling) Layer

- Accepts a volume of size $W_1 imes H_1 imes D_1$
- Requires two hyperparameters:
 - \circ their spatial extent F,
 - the stride S,
- Produces a volume of size $W_2 imes H_2 imes D_2$ where:

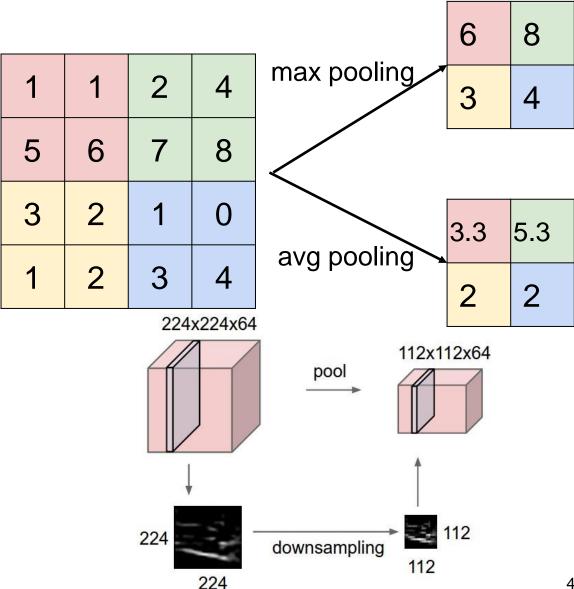
$$W_2 = (W_1 - F)/S + 1$$

$$\bullet \ H_2 = (H_1 - F)/S + 1$$

- $\circ D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- For Pooling layers, it is not common to pad the input using zero-padding.
- A pooling filter has depth 1, and operates over each activation map independently, hence the input volume and output volume have the same depth $D_1=D_2$
 - In contrast, a CONV filter has the same depth D₁ as its input volume, and the number
 of filters K equals the depth D₂ of its output volume
 - Common settings: F = 2, S = 2, or F = 3, S = 2
- Example: pooling w. a 2×2 filter w. stride S = 2, no pad
- Output volume: $\frac{W_1}{2} \times \frac{H_1}{2} \times D_1$ (since $\frac{1}{2}(W_1 2) + 1 = \frac{W_1}{2}$, $\frac{1}{2}(H_1 2) + 1 = \frac{H_1}{2}$)

Max Pooling w. Examples

- Max pooling: take the max element among the F * F elements in each F × F patch of each input activation map to reduce its dimension (F = 2, S = 2 in upper right fig)
- Alternative: average pooling is less commonly used
- Pooling is also called subsampling or downsampling



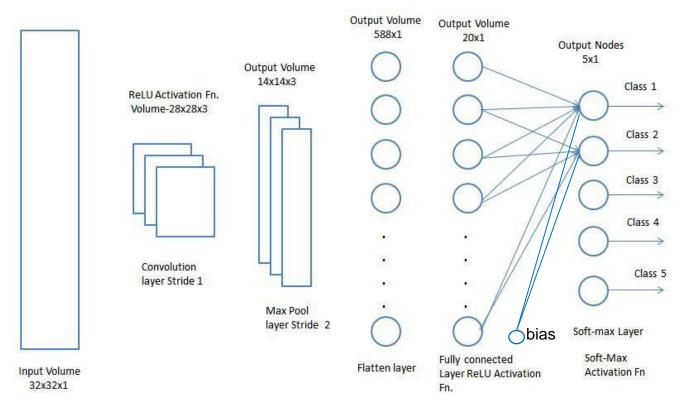
Overlapping Pooling

- Input volume: $N \times N \times D_1$
- A 3×3 pooling filter w. stride S = 1, no pad
- Output volume: $(N-2) \times (N-2) \times D_1(\text{since } \frac{1}{1}(N-3) + 1 = N-2)$
 - In practice, it is more common to have F=3, S=2 for overlapping pooling

| 1 | 3 | 2 | 1 | 3 | | | | |
|---|---|---|---|---|--|---|---|---|
| 2 | 9 | | 1 | 5 | max pool w. 3x3 filter and stride 1 | 9 | 9 | 5 |
| 1 | | | 3 | 2 | | 9 | 9 | 5 |
| 8 | 3 | 5 | | 0 | | 8 | 6 | 9 |
| 5 | 6 | 1 | 2 | 9 | ' | | 1 | 1 |

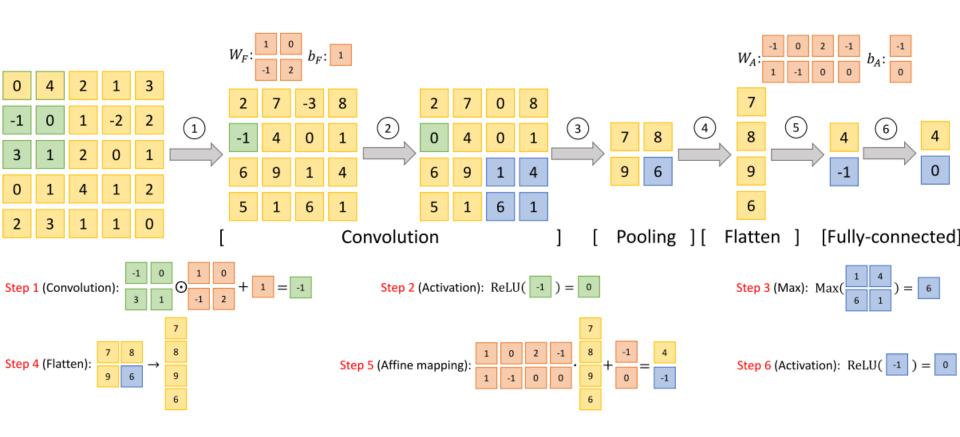
FC Layer

- Contains neurons that connect to the entire input volume w. no weight sharing
 - No. params for FC layer of size N_{out} connected to input layer of size N_{in} is $(N_{in} + 1) * N_{out}$



CNN Toy Example

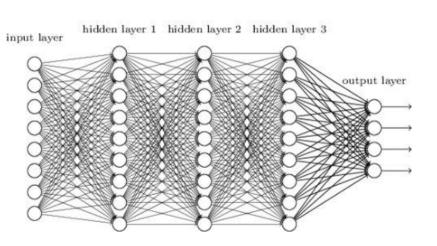
A CNN with 1 CONV layer and 1 FC layer



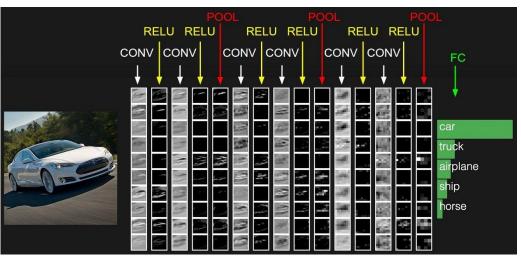
No. Params in Each Layer

| | CONV | POOL | FC |
|-------------|-----------------------------|-------------------------------|--------------------------|
| | F $\times K$ | $F \downarrow \frac{F}{\max}$ | $N_{ m in}$ $N_{ m out}$ |
| Input size | $W_1 \times H_1 \times D_1$ | $W_1 \times H_1 \times D_1$ | N_{in} |
| Output Size | $W_2 \times H_2 \times K$ | $W_2 \times H_2 \times K$ | N_{out} |
| No. params | $(F * F * D_1 + 1) * K$ | 0 | $(N_{in}+1)*N_{out}$ |

Fully-Connected NN vs. CNN



- In a FCNN, all layers are Fully-Connected
- Cannot alter input image size
- No translation invariance
- No. params can grow very large, prune to overfitting

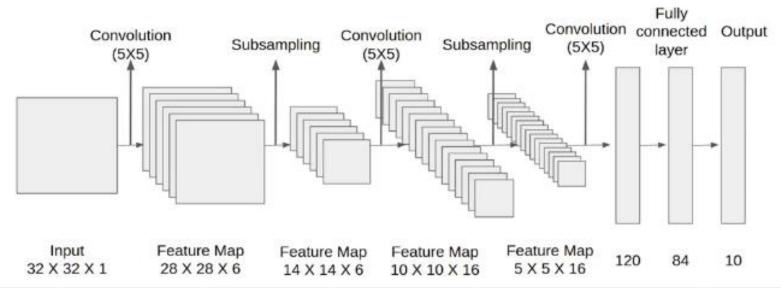


- In a CNN, only the last few (typically <=3) layer(s) are FC
- CONV layers can handle images of arbitrary size
- Translation invariance
- Fewer params than MLP

Outline

- CNN Convolution layers
- Pooling and Fully-Connected layers
- CNN Case Studies

LeNet-5



| Layer | Input $W_1 \times H_1 \times D_1$ | No. Filters | Filter $K \times K \times D/S$ | Output $W_2 \times H_2 \times D_2$ | No. params |
|---------|-----------------------------------|----------------|--------------------------------|------------------------------------|------------|
| C1:CONV | $32 \times 32 \times 1$ | 6 | $5 \times 5 \times 1$ | $28 \times 28 \times 6$ | 156 |
| S2:POOL | $28 \times 28 \times 6$ | 6 | $2 \times 2 \times 1/2$ | $14 \times 14 \times 6$ | 0 |
| C3:CONV | $14 \times 14 \times 6$ | 16 | $5 \times 5 \times 6$ | $10 \times 10 \times 16$ | 2416 |
| S4:POOL | $10 \times 10 \times 16$ | 16 | $2 \times 2 \times 1/2$ | $5 \times 5 \times 16$ | 0 |
| C5:CONV | $5 \times 5 \times 16$ | 120 | $5 \times 5 \times 16$ | $1 \times 1 \times 120$ | 48120 |
| F6 | FC | - | _ | 84 | 10164 |
| Output | FC | | | 10 | 850 |

υ**2**

LeNet-5 Details

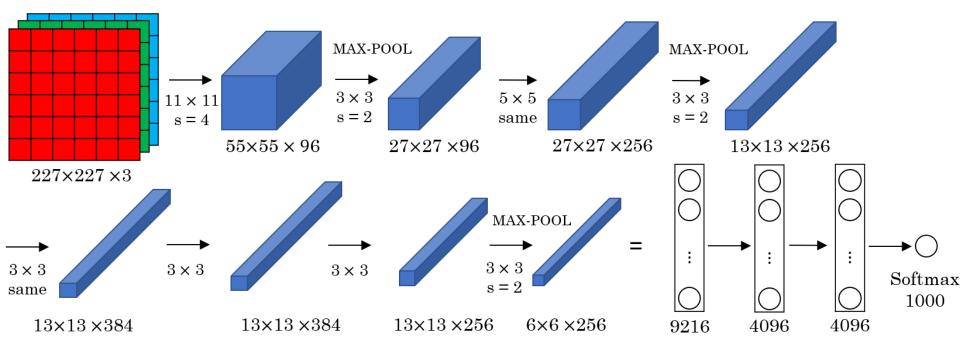
- Input image: $32 \times 32 \times 1$ (grey-scale images of hand-written digits w. size 32×32 pixels)
- Conv filters $5 \times 5 \times 1$ w. stride 1; Pooling filters 2×2 w. stride 2
- Conv layer C1 maps from input volume $32 \times 32 \times 1$ to 6 feature maps w. volume $28 \times 28 \times 6$ (since $\frac{1}{1}(32-5)+1=28$). No params: (5*5*1+1)*6=156
- Pooling layer S2 maps from input volume $28 \times 28 \times 6$ to 6 feature maps w. volume $14 \times 14 \times 6$ (since $\frac{1}{2}(28-2)+1=14$).
- Conv layer C3 maps from input volume $14 \times 14 \times 6$ to 16 feature maps w. volume $10 \times 10 \times 16$ (since $\frac{1}{1}(14-5)+1=10$). No params: (5*5*6+1)*16=2416
- Pooling layer S4 maps from input volume $10 \times 10 \times 16$ to 16 feature maps w. volume $5 \times 5 \times 16$ (since $\frac{1}{2}(10-2)+1=5$)
- Conv layer C5 maps from input volume $5 \times 5 \times 16$ to 120 feature maps w. volume $1 \times 1 \times 120$ (since $\frac{1}{1}(5-5)+1=1$). No params: (5*5*16+1)*120=48120
 - You can also view it as an equivalent Fully-Connected layer that maps from the flattened input of size 400×1 (5 * 5 * 16 = 400) to output of size 120×1 . For details, refer to L4.2 "Turning FC layer into CONV Layers"
- FC layer F6 maps from input of size 120×1 to output of size 84×1 . No params: (120 + 1) * 84 = 10164
- Output layer (SoftMax) maps from input of size 84×1 to output of size 10. No params: (84 + 1) * 10 = 850

AlexNet [Krizhevsky et al. 2012]

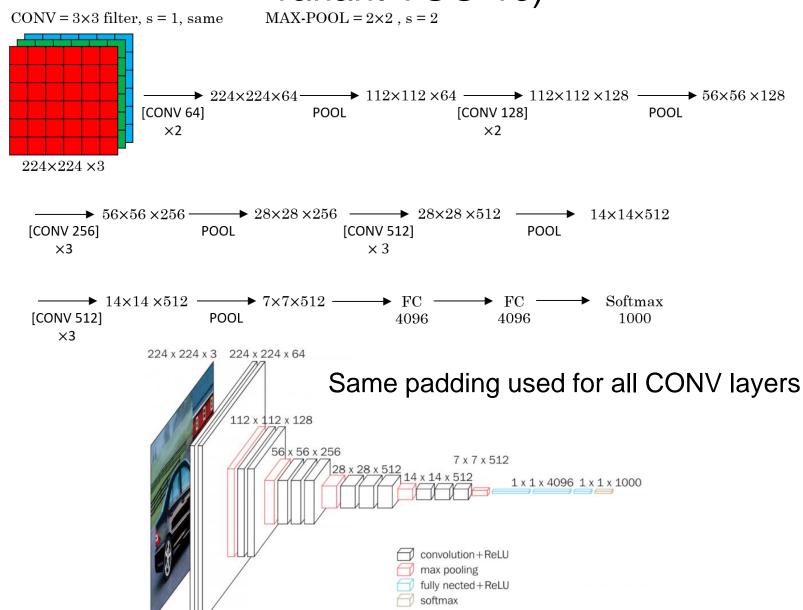
- Input image: 227 × 227 × 3
- 1st layer (CONV1): 96 11 × 11 filters w. stride S = 4, w. ReLU activation function
- Output volume: $55 \times 55 \times 96$ (since $\frac{1}{4}(227 11) + 1 = 55$).
- 2nd layer (POOL1): 3×3 filters w. stride S = 2 (overlapping)
- Output volume: $27 \times 27 \times 96$ (since $\frac{1}{2}(55 3) + 1 = 27$)

•

- Total No. params: 60M
- Introduced ReLU activation function



VGGNet [Simonyan 2014] (the best performing variant VGG-16)



VGG-16 Details

- VGG-16 has 16 weight layers, not including POOL layers w. 0 weight
- Input image: 224 × 224 × 3
- 1st and 2nd CONV layers: 64.3×3 filters w. stride S = 1, pad P = 1
 - Output volume: $224 \times 224 \times 64$ (since $\frac{1}{1}(224 + 2 * 1 3) + 1 = 224$)
- 3rd POOL layer: 2×2 filters w. stride S = 2
 - Output volume: $112 \times 112 \times 64$ (since $\frac{1}{2}(224 2) + 1 = 112$)
- 4th and 5th CONV layers: 128.3×3 filters w. stride S = 1, pad P=1
- Output volume: $112 \times 112 \times 128$ (since $\frac{1}{1}(112 + 2 * 1 3) + 1 = 112$)
 6th POOL layer: 2×2 filters w. stride S = 2
- Output volume: $56 \times 56 \times 128$ (since $\frac{1}{2}(112 2) + 1 = 56$)
- Total No. params: 60M
- ImageNet top 5 error: 7.3%

Stacked 3 × 3 CONV Layers

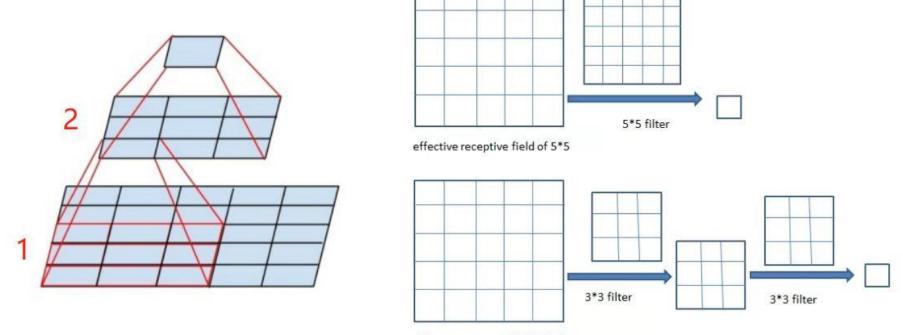
2 stacked 3×3 CONV layers w. pad P = 1 have the same effective receptive field as a 5×5 CONV layer; 3 stacked 3×3 CONV layers w. pad P = 1 have RF of 7×7 ; L stacked 3×3 CONV layers w. pad P = 1 have RF of 1 + 2L

• Benefits:

Fewer params. Suppose all volumes have the same depth D, then a 7×7 CONV layer has $(7*7*D+1)*D \approx 49D^2$ params, while three stacked 3x3 CONV layers have only $(3*3*D+1)*D*3) \approx 27D^2$ params

Two layers of non-linear activation functions increases CNN depth. hence larger model

capacity



effective receptive field of 5*5

VGGNet No. Params

```
INPUT: [224x224x3] memory: 224*224*3=150K weights: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M weights: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M weights: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K weights: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M weights: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M weights: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K weights: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K weights: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K weights: (3*3*256)*256=589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K weights: (3*3*256)*256=589,824
POOL2: [28x28x256] memory: 28*28*256=200K weights: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K weights: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K weights: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K weights: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K weights: 0
CONV3-512: [14\times14\times512] memory: 14*14*512=100K weights: (3*3*512)*512=2,359,296
CONV3-512: [14x14x512] memory: 14*14*512=100K weights: (3*3*512)*512 = 2,359,296
CONV3-512: [14\times14\times512] memory: 14*14*512=100K weights: (3*3*512)*512=2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K weights: 0
FC: [1x1x4096] memory: 4096 weights: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 weights: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 weights: 4096*1000 = 4,096,000
TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters
```

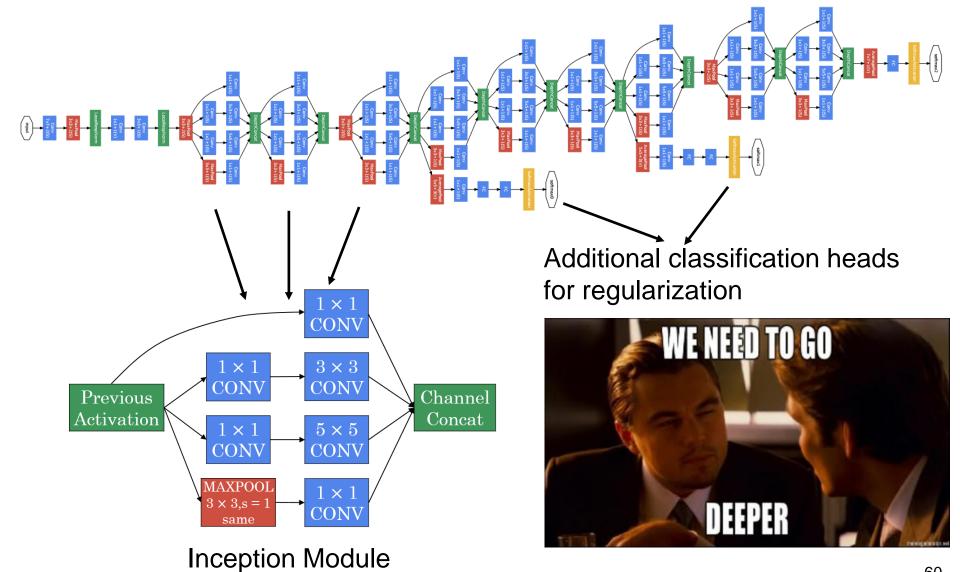
- Memory refers to memory size of activation maps
- For ease of calculation, only the No. weights are counted, not the biases

VGGNet Variants

Best performing variant VGG-16

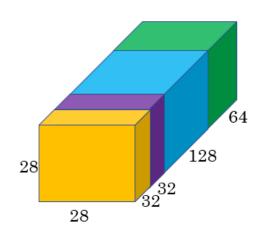
| | 4 | ConvNet C | onfiguration | | | |
|-------------------------------------|---|------------------------|-------------------------------------|---|--|--|
| A | A-LRN | В | C D | | E | |
| 11 weight layers | 11 weight layers | 13 weight layers | 16 weight layers | 16 weight layers | 19 weight layers | |
| | i | nput $(224 \times 2$ | 24 RGB image |) | | |
| conv3-64 conv3-64 LRN | | conv3-64 conv3-64 | conv3-64 conv3-64 | conv3-64 conv3-64 | conv3-64 conv3-64 | |
| * | 2 | max | pool | C2514500000000000000000000000000000000000 | 1- | |
| conv3-128 | conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 | conv3-128 conv3-128 | |
| | | max | pool | | | |
| conv3-256 conv3-256 conv3-256 | | conv3-256 conv3-256 | conv3-256 conv3-256 conv1-256 | conv3-256 conv3-256 conv3-256 | 6 conv3-256 | |
| | | max | pool | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 conv1-512 | conv3-512 conv3-512 conv3-512 | conv3-512 conv3-512 conv3-512 conv3-512 | |
| | 4 | max | pool | | | |
| conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 | conv3-512 conv3-512 conv1-512 | conv3-512 conv3-512 conv3-512 | conv3-512 conv3-512 conv3-512 conv3-512 | |
| | | max | pool | | | |
| | | | 4096 | | | |
| | | | 4096 | | | |
| | | | 1000 | | | |
| | | soft | -max | | | |

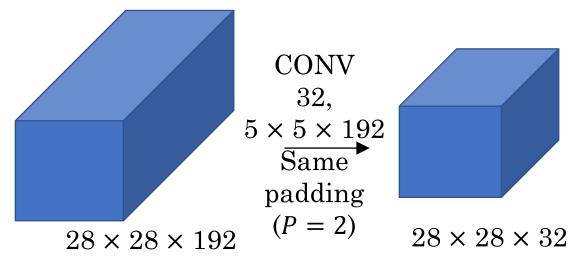
GoogLeNet [Szegedy et al., 2014]



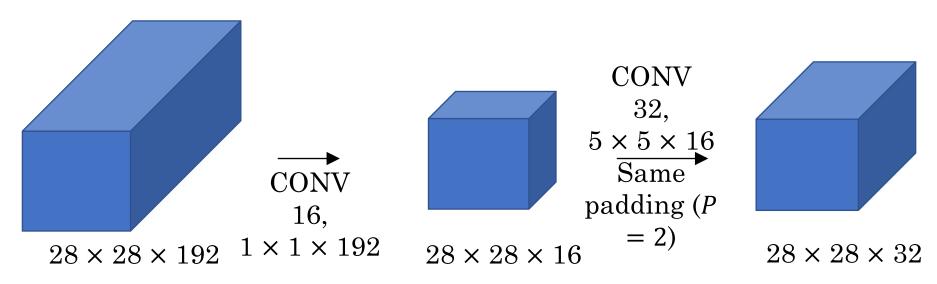
Inception Module

- Can't make up your mind about filter size? Have them all in the Inception Module!
 - But this increases computation load
- Additional 1 × 1 CONV layers serve as bottleneck to reduce number of parameters and computation load
- Can't make up your mind about filter size? Have them all in the Inception Module!
 - But this increases computation load
- Additional 1×1 CONV layers serve as bottleneck to reduce number of parameters and computation load





Without the bottleneck layer: No. params: 5 * 5 * 192 * 32 = 153600;
 No. multiplications: 5 * 5 * 192 * 32 * 28 * 28 = 120M



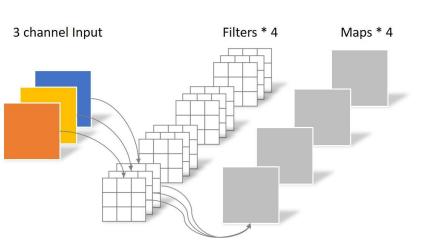
With the bottleneck layer: No. params: 1 * 1 * 192 * 16 + 5 * 5 * 16 * 32 = 15872; No. multiplications: 1 * 1 * 192 * 16 * 28 * 28 + 5 * 5 * 16 * 32 * 28 * 28 = 12.4M

GoogLeNet Size

- Compared to AlexNet:
 - 12x less params (only 5M, due to no FC layers), 2x
 more compute (due to more CONV layers)

| type | patch size/ stride | output size | depth | #1×1 | #3×3 reduce | #3×3 | #5×5 reduce | #5×5 | pool proj | params | ops |
|----------------|-----------------------|----------------|-------|------|----------------|------|----------------|------|--------------|--------|------|
| convolution | 7×7/2 | 112×112×64 | 1 | | | | | | | 2.7K | 34M |
| max pool | 3×3/2 | 56×56×64 | 0 | | | | | | | | |
| convolution | 3×3/1 | 56×56×192 | 2 | | 64 | 192 | | | | 112K | 360M |
| max pool | 3×3/2 | 28×28×192 | 0 | | | | | | | | |
| inception (3a) | | 28×28×256 | 2 | 64 | 96 | 128 | 16 | 32 | 32 | 159K | 128M |
| inception (3b) | | 28×28×480 | 2 | 128 | 128 | 192 | 32 | 96 | 64 | 380K | 304M |
| max pool | 3×3/2 | 14×14×480 | 0 | | | | | | | | |
| inception (4a) | | 14×14×512 | 2 | 192 | 96 | 208 | 16 | 48 | 64 | 364K | 73M |
| inception (4b) | | 14×14×512 | 2 | 160 | 112 | 224 | 24 | 64 | 64 | 437K | 88M |
| inception (4c) | | 14×14×512 | 2 | 128 | 128 | 256 | 24 | 64 | 64 | 463K | 100M |
| inception (4d) | | 14×14×528 | 2 | 112 | 144 | 288 | 32 | 64 | 64 | 580K | 119M |
| inception (4e) | | 14×14×832 | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 840K | 170M |
| max pool | 3×3/2 | 7×7×832 | 0 | | | | • | | | | 1-0 |
| inception (5a) | | 7×7×832 | 2 | 256 | 160 | 320 | 32 | 128 | 128 | 1072K | 54M |
| inception (5b) | | 7×7×1024 | 2 | 384 | 192 | 384 | 48 | 128 | 128 | 1388K | 71M |
| avg pool | 7×7/1 | 1×1×1024 | 0 | | | 4 | | | | | |
| dropout (40%) | | 1×1×1024 | 0 | | | | | | | | 6 |
| linear | | 1×1×1000 | 1 | | V. | 8 | | | | 1000K | 1M |
| softmax | | 1×1×1000 | 0 | | | | | | | | |

Xception [Chollet 2017] MobileNets [Howard et al. 2017]: Depthwise Separable Convolution



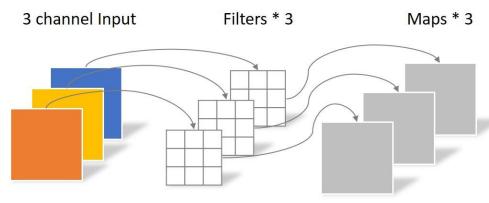
Each filter is convolved with all input channels

Regular Convolution

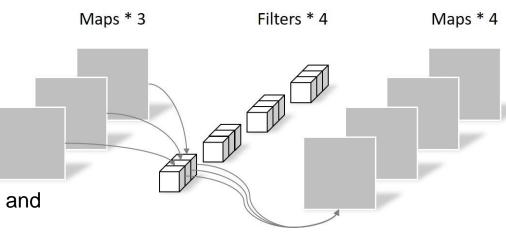
The intermediate feature maps serve as bottleneck to reduce number of parameters and computation load

(Optional) Depthwise Separable Convolution - A FASTER CONVOLUTION!

https://www.youtube.com/watch?v=T7o3xvJLuHk



Each filter is convolved with one input channel

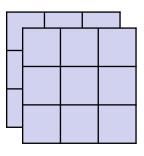


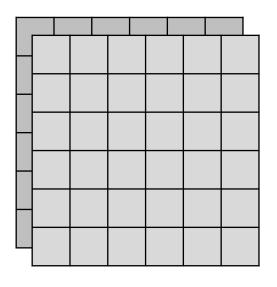
Followed by pointwise convolution

Depthwise Separable Convolution:

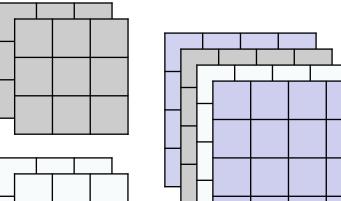
Example: Regular Convolution

Input feature map





2 channels

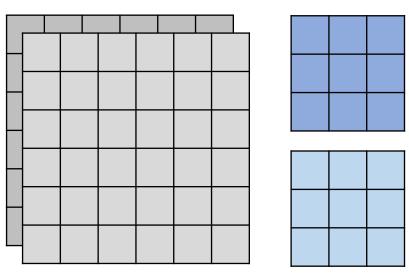


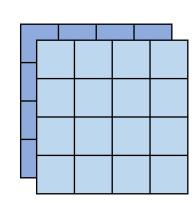
No. params: 3 * 3 * 2 * 4 = 72(Four $3 \times 3 \times 2$ filters) (not counting biases)

No. MULs: 3 * 3 * 2 * 4 * 4 * 4 = 1152 (3 * 3 * 2 MULs to compute each output element; 4 * 4 * 4 output elements)

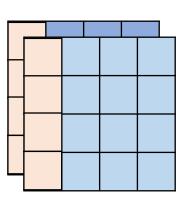
Example: Depthwise Separable Convolution

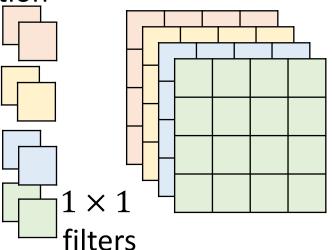
1. Depthwise Convolution





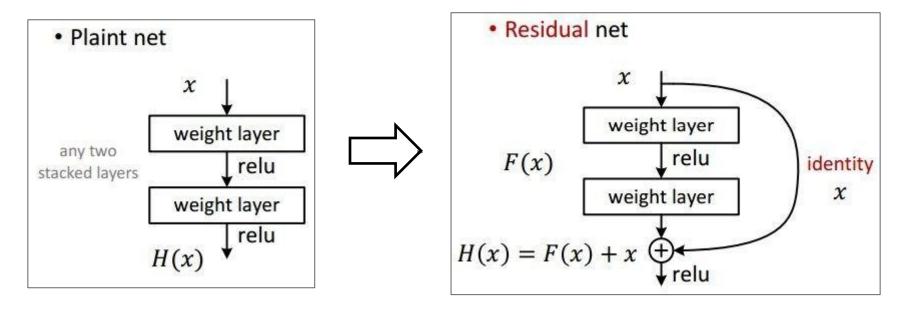
2. Pointwise Convolution





Residual Networks (ResNet) [He et al. 2015]

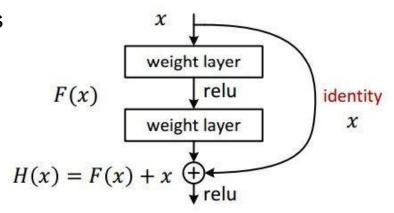
- Based on VGG-19, adding more layers and skip connections
- ImageNet top 5 error: 3.6%

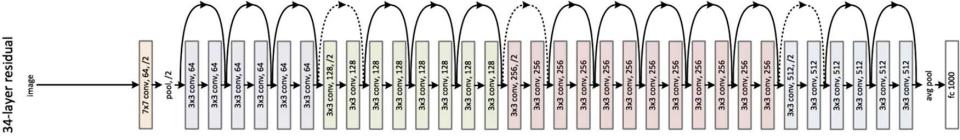


ResNet Skip Connection

- In a standard network, output from a given layer is F(x)
- In ResNet w. the identity skip (or short-cut) connection, output from a given layer is H(x) = F(x) + x
- Benefits:
 - Residual connections help in handling the vanishing gradient problem in very deep NNs
 - If identify mapping is close to optimal, then weights can be small to capture minor differences only, in other words, "unnecessary layers" can learn to be identity mapping. This allows stacking many layers (e.g., 152) without overfitting

Residual net



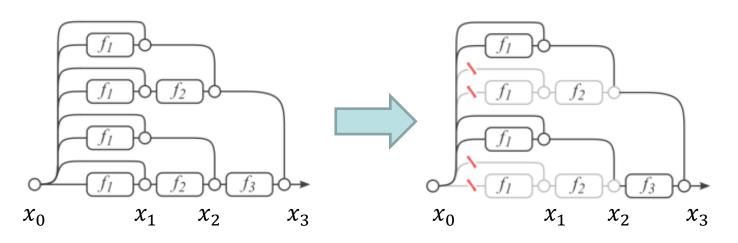


Consider a 3-layer Network

Standard NN:

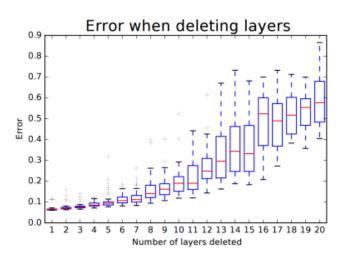
-
$$x_3 = f_3(f_2(f_1(x_0)))$$

- ResNet:
 - $x_1 = f_1(x_0) + x_0$
 - $x_2 = f_2(x_1) + x_1 = f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0$
 - $x_3 = f_3(x_2) + x_2 = f_3(f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0) + f_2(f_1(x_0) + x_0) + f_1(x_0) + x_0$
- Suppose $f_2(x_1)$ is a vector of very small values (layer 2 is "off"/skipped), then it looks like the input x_0 bypassed the second layer completely on its way to the output x_3



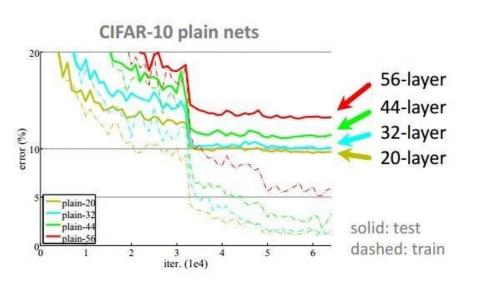
ResNet is an Ensemble of Models

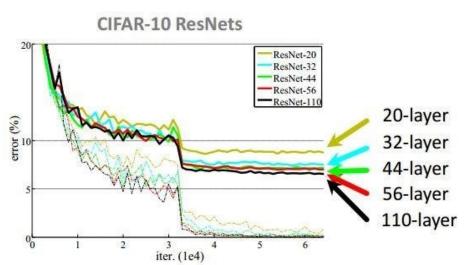
- Every input x_0 to ResNet may activate a unique path to the output. Total number of possible paths is 2^N , where N is the total number of layers in the network, since each layer may be either "on" or "off" for a given input x_0
 - Compare w. a standard network, where there is only one single path for any input corresponding to all layers being "on", and no layer is skipped
- Consequences:
 - Resilience to layer deletion: deleting 1-3 layers in a large ResNet introduces only around 6-7% error
 - Shortening of effective paths: w. 152layer ResNet, most paths are only 20-30 levels deep!



Deeper Nets have Better Performance

CIFAR-10 experiments





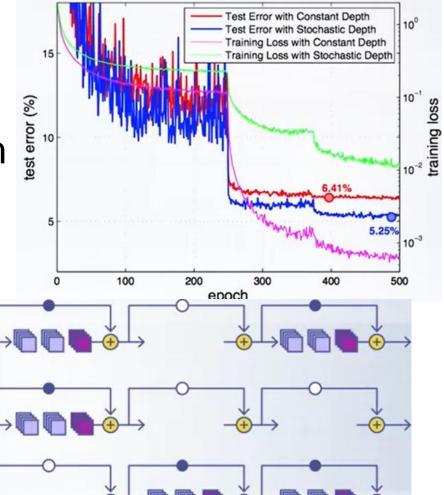
ResNet Training with Stochastic Depth

 For each minibatch of inputs, randomly skip some layers (replaced w. identity mapping)

 Reduced network depth during training; full depth during inference

MB2

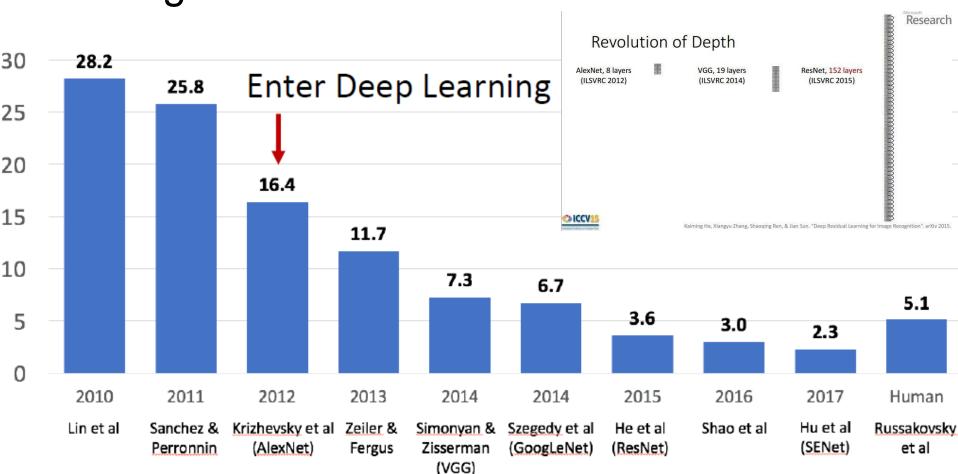
MB3



110-layer ResNet on CIFAR-10

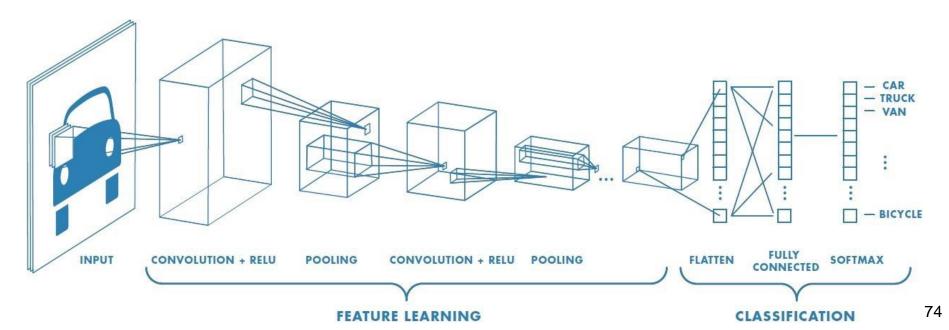
ImageNet Large Scale Visual Recognition Challenge

1,000 object classes, 1.4 M labeled images



CNN Layer Patterns

- A typical CNN architecture looks like: INPUT->[[CONV->RELU]*N->POOL?]*M->[FC->RELU]*K->FC
 - where * indicates repetition, and POOL? indicates an optional pooling layer. $N \ge 0$ (usually $N \le 3$), $M \ge 0$, $K \ge 0$ (and usually K < 3)
- Some common architectures:
 - INPUT->FC, implements a linear classifier. Here N = M = K = 0.
 - INPUT->CONV->RELU->FC
 - INPUT->[CONV->RELU->POOL] *2->FC->RELU->FC (fig below). There is a single CONV layer between every POOL layer.
 - INPUT->[CONV->RELU->CONV->RELU->POOL] *3->[FC->RELU] *2->FC There are two CONV layers stacked before every POOL layer, e.g., two stacked 3 × 3 CONV Layers. This is generally a good idea for larger and deeper networks, because multiple stacked CONV layers can develop more complex features of the input volume before the destructive pooling operation.



Layer Sizing Rules-of-Thumb

- The input layer (that contains the image) should be divisible by 2 many times. Common numbers include 32 (e.g. CIFAR-10), 64, 96 (e.g. STL-10), or 224 (e.g. ImageNet), 384, and 512.
- The CONV layers should use small filters (e.g. 3x3 or at most 5x5), stride S=1. The input volume should have "same padding", i.e., the conv layer does not alter the spatial size of the input. For any F, pad P=(F-1)/2 preserves the input size, e.g., when F=3, P=1; when F=5, P=2. This means the CONV layers only transform the input volume depth-wise, but do not perform downsampling. (c.f. CONV Example 3 and VGGNet).
- The POOL layers are in charge of downsampling the spatial dimensions of the input. The
 most common setting is to use max-pooling with 2x2 receptive fields (F=2), with stride of 2
 (S=2). A less common setting is to use F=3, S=2. It is uncommon to see receptive field
 sizes for max pooling that are larger than 3, because the pooling is then too lossy and
 aggressive.
- In some cases (especially in early layers), the memory size can build up very quickly with the rules of thumb presented above. For example, filtering a 224x224x3 image with three 3x3 CONV layers with 64 filters each and padding 1 would create 3 activation volumes, each with size 224x224x64. This amounts to a total of about 10 million activations, or 72MB of memory (per image, for both activations and gradients). Since GPUs are often bottlenecked by memory, it may be necessary to compromise. In practice, make the compromise at only the first CONV layer that is looking at the input image. For example, AlexNet uses filter size of 11x11 and stride of 4 in the first CONV layer.

Memory Size Considerations

- From the intermediate volume sizes:
 - These are the raw number of activations at every layer of the CNN, and also their gradients (of equal size). Usually, most of the activations are on the earlier CONV layers of a CNN. These are kept around because they are needed for backpropagation during training, but for inference, we can store only the current activations at the current layer and discarding the activations from previous layers.
- From the parameter sizes:
 - These are the weights and biases, and their gradients during backprop, and also a step cache if the optimization is using momentum, Adagrad, or RMSProp. Therefore, the memory to store the parameter vector alone usually should be multiplied by a factor of at least 3 or so.
- Each number may need 4 B storage space for floating point, 8 B for double, or 1 B or smaller for optimized fixed-point implementations.

Transfer Learning

- Instead of training your CNN from scratch, start from a pre-trained CNN (e.g., ResNet) and fine-tune it for your task
- First, replace SoftMax layer (classification head) with your own
- Next, train the CNN while keeping parameters frozen for
 - all CONV layers and only train the SoftMax layer
 - or part of the earlier CONV layers close to the input layer (since earlier layers extract lower-level features that are more likely to be common among different tasks)
 - or none of the layers
 - The decision depends on how much training data you have, and how similar your task is to that of the pre-trained CNN

