Lecture 13
Sorting Algorithm

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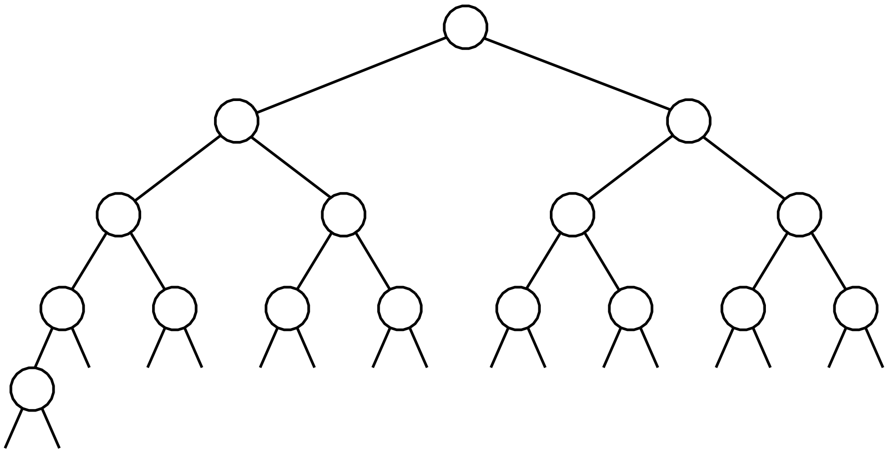
Lecture Goals

- We introduce *binary heap* for priority queue data abstract, which leads to an efficient sorting algorithm known as ***heapsort***.
- We introduce and implement the *randomized* ***quicksort*** algorithm and analyze its performance. We also consider randomized quickselect, a quicksort variant which finds the kth smallest item in linear time. Finally, consider 3-way quicksort, a variant of quicksort that works especially well in the presence of duplicate keys.
- We study the ***mergesort*** algorithm and show that it guarantees to sort any array of n items with at most nlg(n) compares. We also consider a nonrecursive, bottom-up version.

Heapsort: Binary Heap

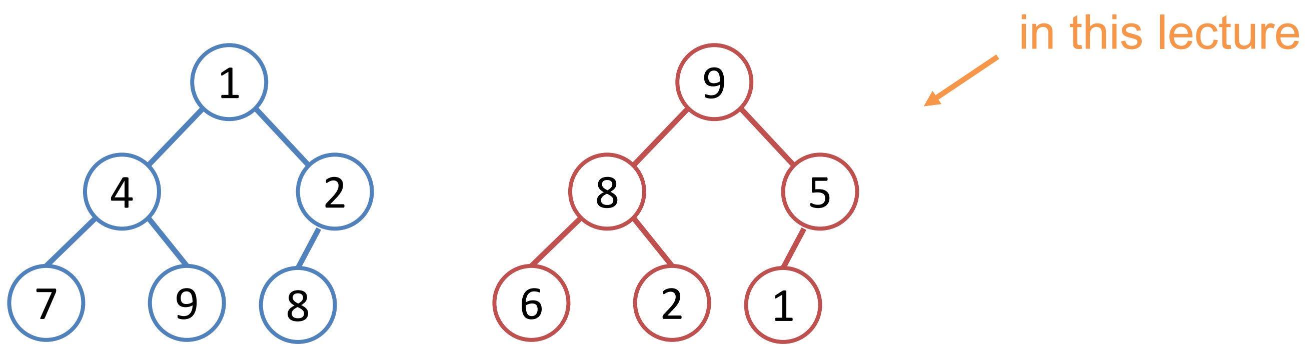
- In a **heap** the highest (or lowest) priority element is always stored at the root, hence the name "heap". A heap is useful data structure when you need to remove the object with the highest (or lowest) priority. A common use of a heap is to implement a **priority queue** and **heapsort**.
- A **binary heap** is a complete binary tree which is an efficient data structure satisfies the heap ordering property.
- In a *complete tree*, every level (except possibly the last) is completely filled; the last level is filled from left to right.



complete binary tree with n = 16 nodes (height = 4)

Heapsort: Binary Heap

- The heap ordering can be one of two types:
- The **min-heap property**: the value of each node is greater than or equal to the value of its parent, with the minimum-value element at the root.
- The **max-heap property**: the value of each node is less than or equal to the value of its parent, with the maximum-value element at the root.

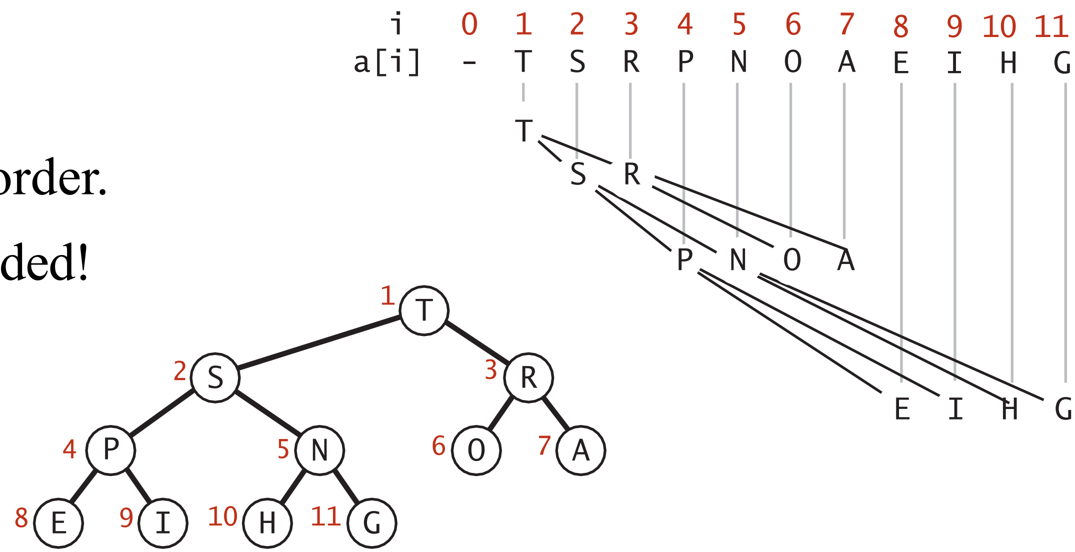


- A heap is not a sorted structure and can be regarded as partially ordered. As you see from the picture, there is no particular relationship among nodes on any given level, even among the siblings.
- Since a heap is a complete binary tree, it has a smallest possible height - a heap with N nodes always has ***O(log N)*** height.

Binary Heap: Array Representation

Array representation.

- Indices start at 1.
- Take nodes in level order.
- No explicit links needed!



Heap representations

Proposition. Largest key is a[1], which is root of binary tree.

Proposition. Can use array indices to move through tree.

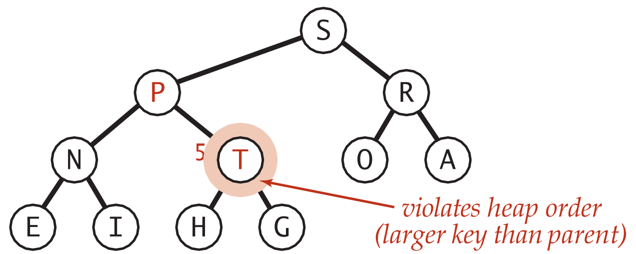
- Parent of node at k is at k/2.
- Children of node at k are at 2k and 2k+1.

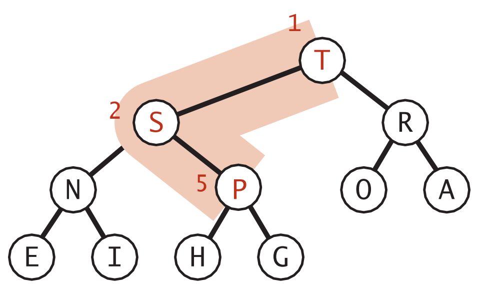
Binary Heap Operations: Promotion

- Scenario. A key becomes larger than its parent's key.
- To eliminate the violation:
- Exchange key in child with key in parent.
- Repeat until heap order restored.

```
private void swim(int k)
{
   while (k > 1 && less(k/2, k))
   {
      exch(k, k/2);
      k = k/2;
   }
}
```

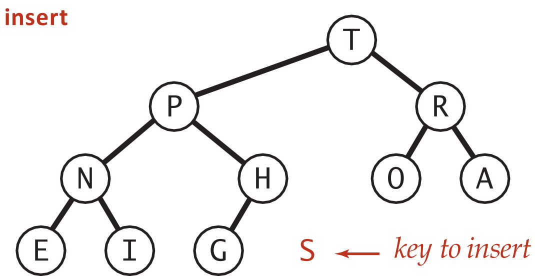
parent of node at k is at k/2

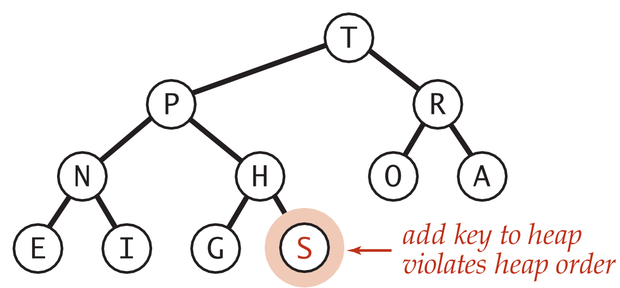


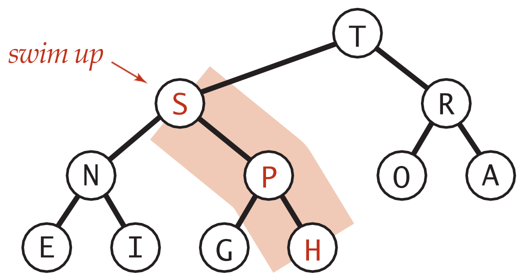


Binary Heap Operations: Insert

- Insert. Add node at end, then swim it up.
- Cost. At most 1 + lg n compares.







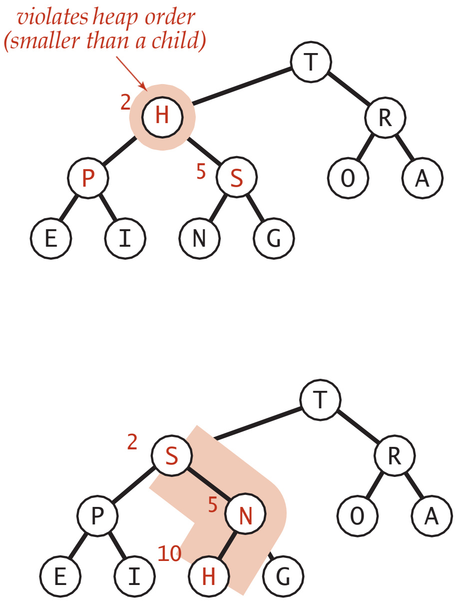

```
public void insert(Key x)
{
   pq[++n] = x;
   swim(n);
}
```

Binary Heap Operations: Demotion

- Scenario. A key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
- Exchange key in parent with key in larger child.
- Repeat until heap order restored.

```
private void sink(int k)
{
   while (2*k <= n)
   {
      int j = 2*k;
      if (j < n && less(j, j+1)) j++;
      if (!less(k, j)) break;
      exch(k, j);
      k = j;
   }
}
```

children of node at k are 2*k and 2*k+1

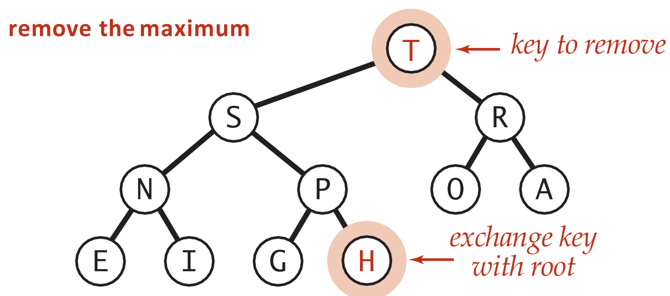


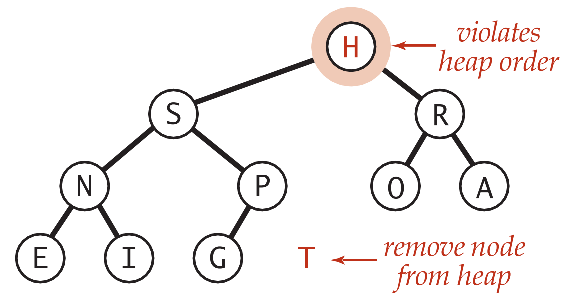
Top-down reheapify (sink)

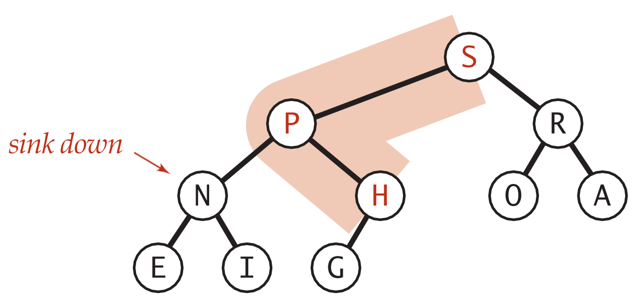
Binary Heap Operations: DeleteMax

- Delete max. Exchange root with node at end, then sink it down.
- Cost. At most 2 lg(n) compares.

DeleteRandom?





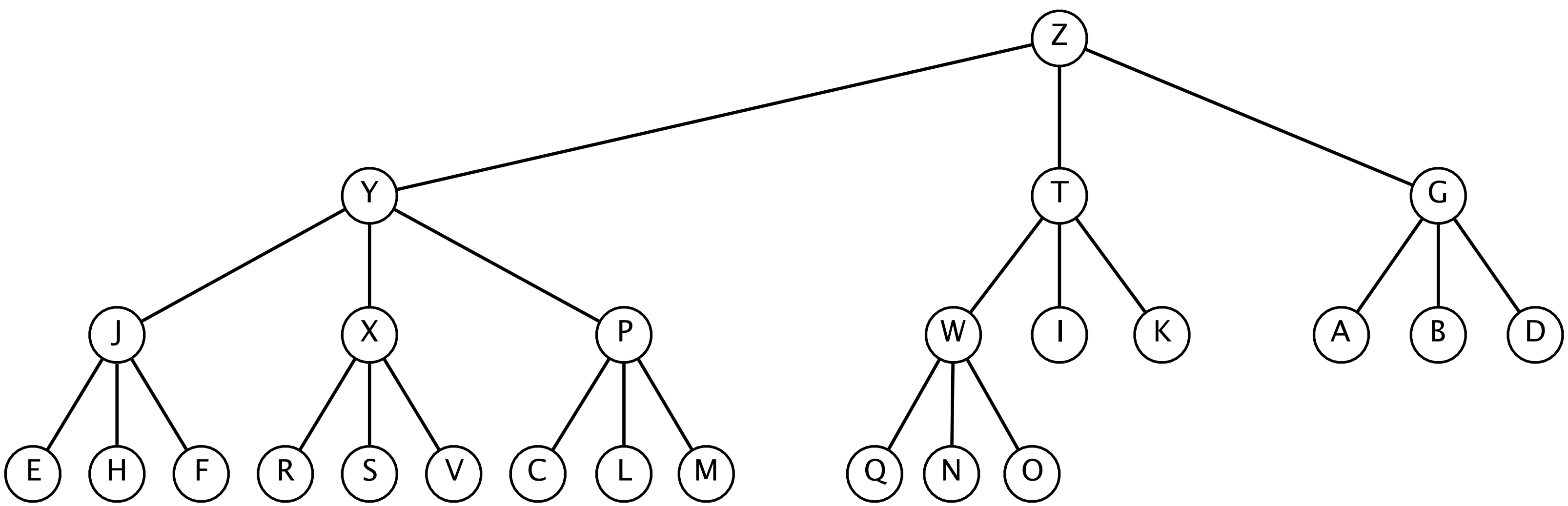



```
public Key delMax()
{
   Key max = pq[1];
   exch(1, n--);
   sink(1);
   pq[n+1] = null;
   return max;
}
```

prevent loitering

Binary Heap: Practical improvements

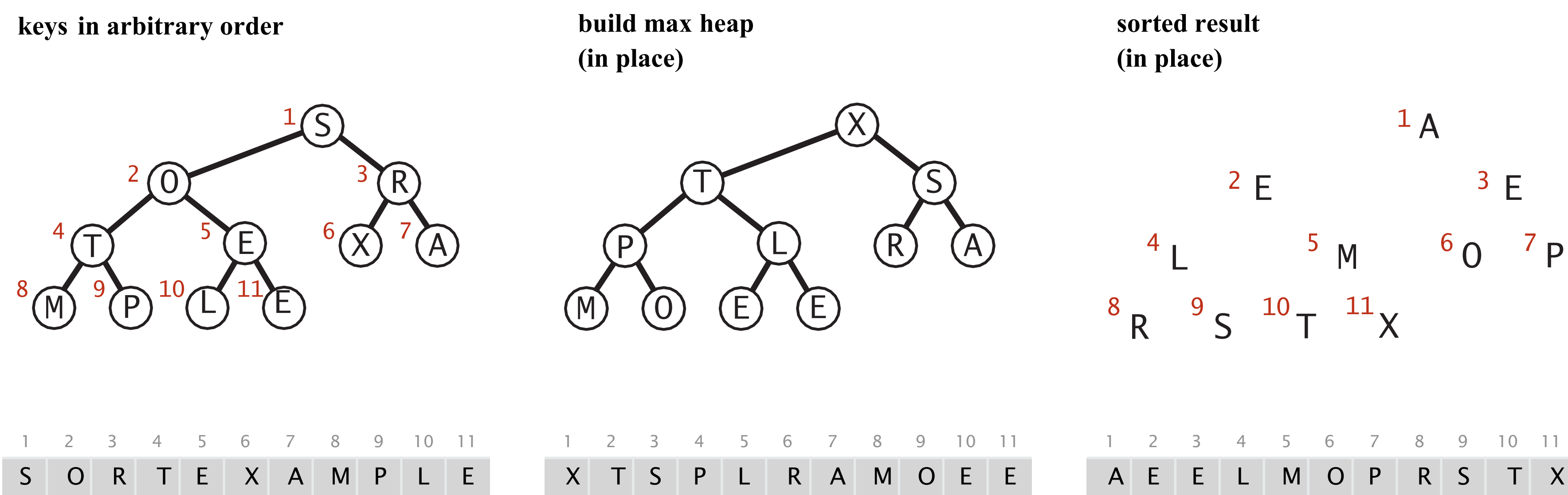
- Multiway heaps. Complete d-way tree.
- Parent's key no smaller than its children's keys.
- Fact. Height of complete d-way tree on n nodes is $\sim \log_d n$.



3-way heap

Heapsort Algorithm

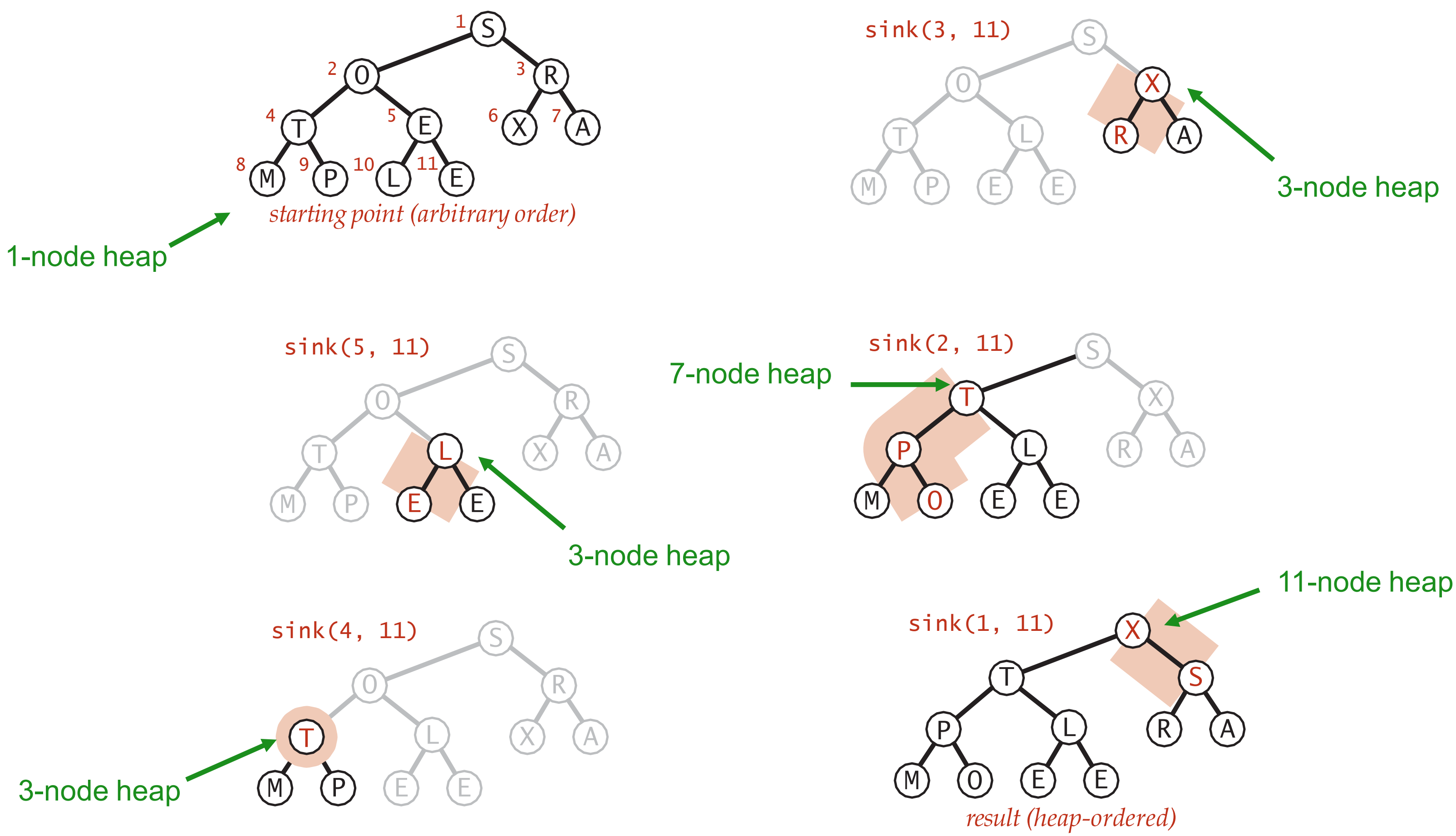
- Basic plan for in-place sort.
- View input array as a complete binary tree.
- Heap construction: build a max-heap with all n keys.
- Sortdown: repeatedly remove the maximum key.



Heapsort: Heap Construction

First pass. Build heap using bottom-up method.

```
for (int k = n/2; k >= 1; k--)
   sink(a, k, n);
```

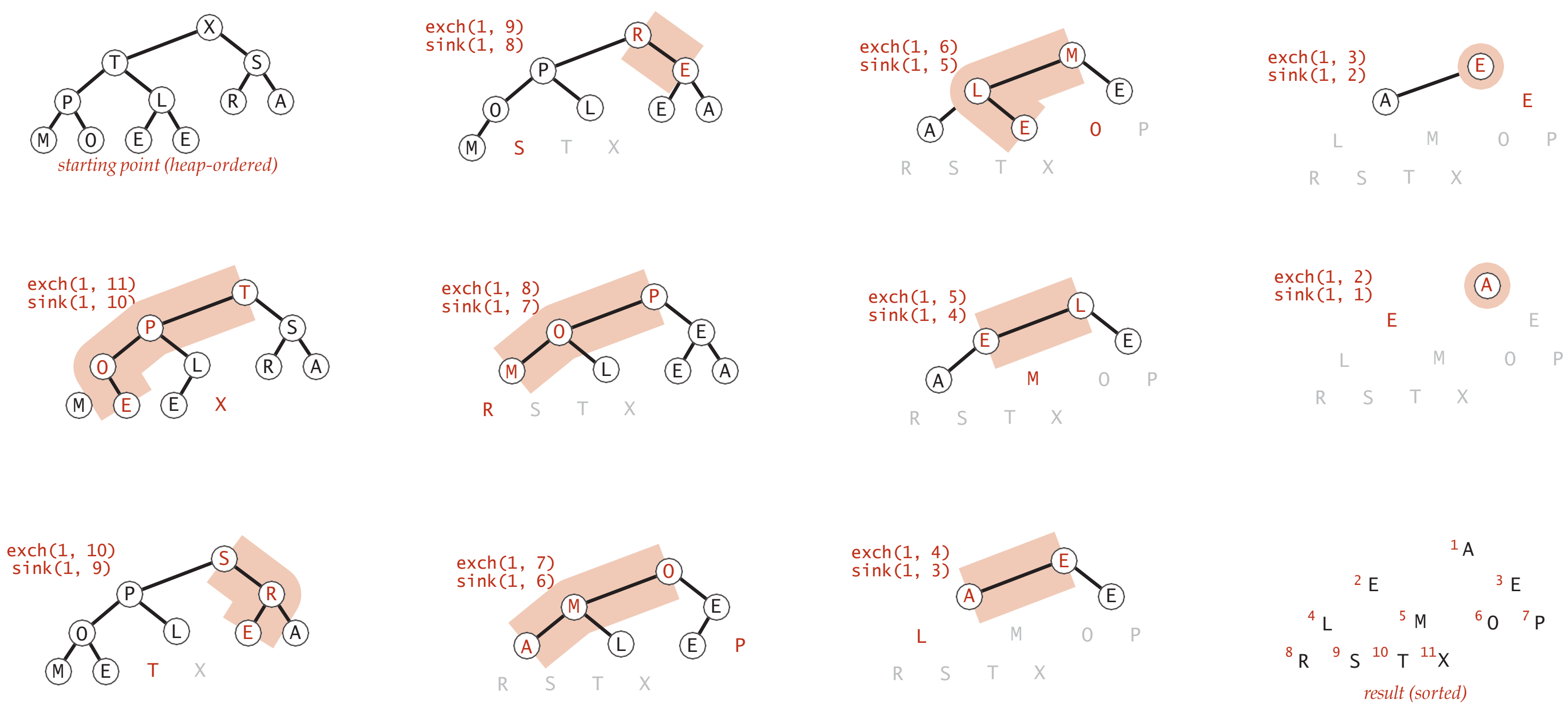


Heapsort: Sortdown

Second pass.

- Remove the maximum, one at a time.
- Leave in array, instead of nulling out.

```
while (n > 1)
{
   exch(a, 1, n--);
   sink(a, 1, n);
}
```



Heapsort: Java Implementation

```
public class Heap
{
   public static void sort(Comparable[] a)
   {
      int n = a.length;
      for (int k = n/2; k >= 1; k--)
           sink(a,  k,  n);
      while (n >   1)
      {
            exch(a,  1,  n);
            sink(a,  1,  --n);
      }
   }

   private static void sink(Comparable[] a, int k, int n)
   {  /* as before */  }

   private static boolean less(Comparable[] a, int i, int j)
   {  /* as before */  }

   private static void exch(Object[] a, int i, int j)
   {  /* as before */  }
}
```

but make static (and pass arguments)

but convert from 1-based indexing to 0-base indexing

O(nlogn)

Heapsort: Trace

sink(K, N)

a[i]

N	k	0	1	2	3	4	5	6	7	8	9	10	11
initial	*values*		S	O	R	T	E	X	A	M	P	L	E
11	5		S	O	R	T	L	X	A	M	P	E	E
11	4		S	O	R	T	L	X	A	M	P	E	E
11	3		S	O	X	T	L	R	A	M	P	E	E
11	2		S	T	X	P	L	R	A	M	O	E	E
11	1		X	T	S	P	L	R	A	M	O	E	E
heap-ordered			X	T	S	P	L	R	A	M	O	E	E
10	1		T	P	S	O	L	R	A	M	E	E	X
9	1		S	P	R	O	L	E	A	M	E	T	X
8	1		R	P	E	O	L	E	A	M	S	T	X
7	1		P	O	E	M	L	E	A	R	S	T	X
6	1		O	M	E	A	L	E	P	R	S	T	X
5	1		M	L	E	A	E	O	P	R	S	T	X
4	1		L	E	E	A	M	O	P	R	S	T	X
3	1		E	A	E	L	M	O	P	R	S	T	X
2	1		E	A	E	L	M	O	P	R	S	T	X
1	1		A	E	E	L	M	O	P	R	S	T	X
sorted	*result*		A	E	E	L	M	O	P	R	S	T	X

3-node heap

7-node heap

11-node heap

red: exchanged

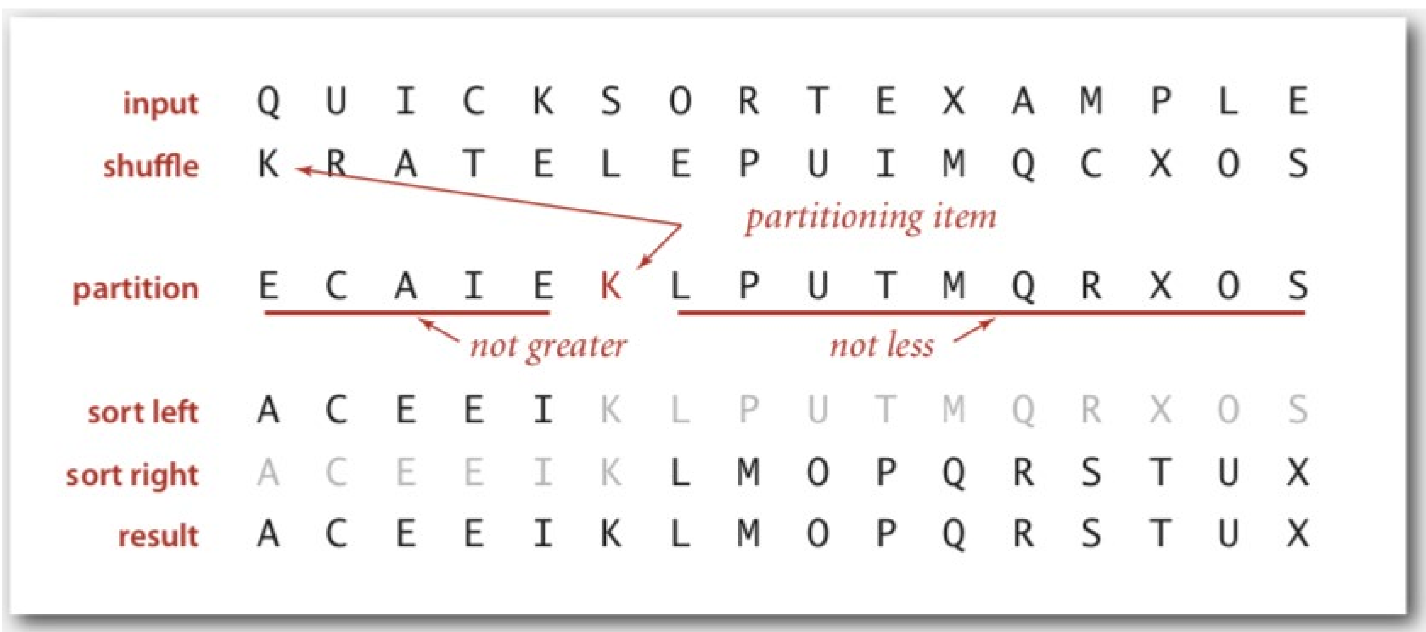
black: compared

Heapsort trace (array contents just after each sink)

Quicksort

- Basic plan.

1. Shuffle the array.
2. Partition so that, for some j
   - entry a[j] is in place
   - no larger entry to the left of j
   - no smaller entry to the right of j
3. Sort each piece recursively.



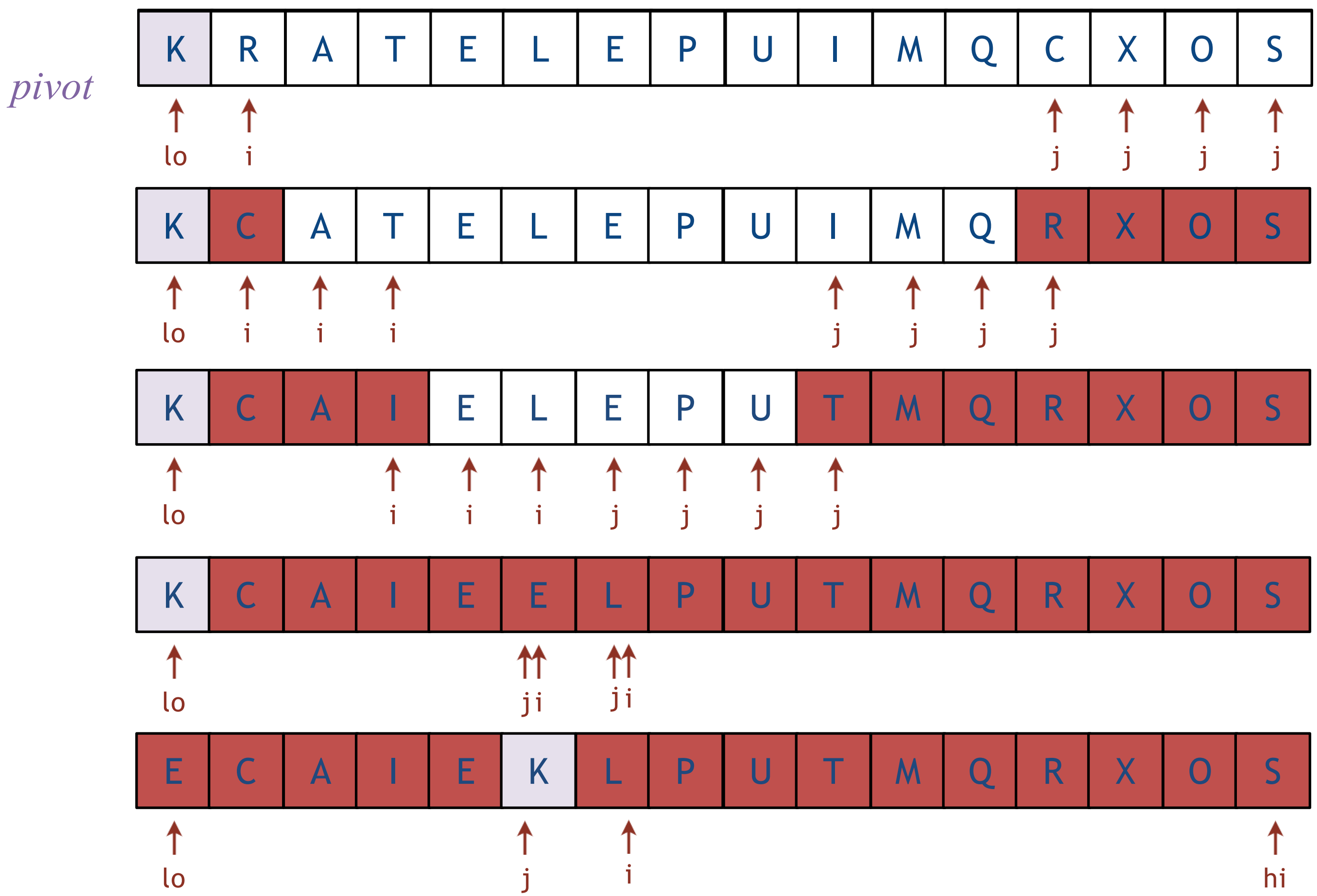
Partition Operation

Repeat until i and j pointers cross.

- Scan i from left to right so long as (a[i] < a[lo]).
- Scan j from right to left so long as (a[j] > a[lo]).
- Exchange a[i] with a[j].

When pointers cross.

- Exchange a[lo] with a[j].



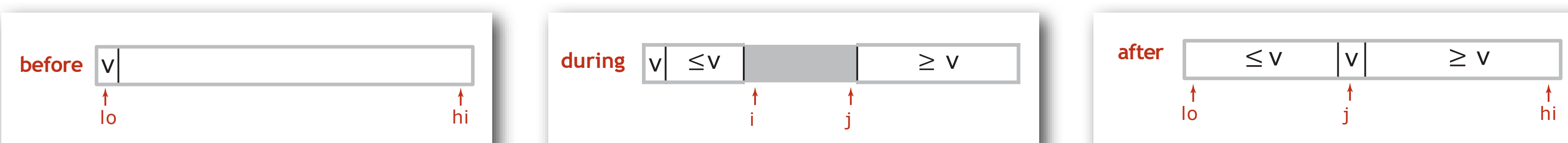
Partition Operation: Java Implementation

```
private static int partition(Comparable[] a, int lo, int hi)
{
    int i = lo, j = hi+1;
    while (true)
    {
        while (less(a[++i], a[lo]))                 find item on left to swap
        if (i == hi) break;

        while (less(a[lo], a[--j]))                 find item on right to swap
        if (j == lo) break;

        if (i >= j) break;                          check if pointers cross
        exch(a, i, j);                              swap
    }

    exch(a, lo, j);                                 swap with partitioning item
    return j;                                       return index of item now known to be in place
}
```



Quicksort: Java Implementation

```
public class Quick
{
  private static int    partition(Comparable[]  a,  int   lo,  int   hi)
     {/* see previous slide /     } *

    public static void sort(Comparable[] a)
    {
        StdRandom.shuffle(a);
        sort(a, 0, a.length - 1);
    }

    private static void sort(Comparable[] a, int lo, int hi)
    {
        if (hi <= lo) return;
        int j = partition(a, lo, hi);
        sort(a, lo, j-1);
        sort(a, j+1, hi);
   }
}
```

shuffle needed for performance guarantee (stay tuned)

Quicksort: Trace

	lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
initial values				Q	U	I	C	K	S	O	R	T	E	X	A	M	P	L	E
random shuffle				K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
	0	5	15	E	C	A	I	E	K	L	P	U	T	M	Q	R	X	O	S
	0	3	4	E	C	A	E	I	K	L	P	U	T	M	Q	R	X	O	S
	0	2	2	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	0	0	1	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	1		1	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	4		4	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
	6	6	15	A	C	E	E	I	K	L	P	U	T	M	Q	R	X	O	S
no partition for subarrays of size 1	7	9	15	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	7	7	8	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	8		8	A	C	E	E	I	K	L	M	O	P	T	Q	R	X	U	S
	10	13	15	A	C	E	E	I	K	L	M	O	P	S	Q	R	T	U	X
	10	12	12	A	C	E	E	I	K	L	M	O	P	R	Q	S	T	U	X
	10	11	11	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	10		10	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	14	14	15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
	15		15	A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X
result				A	C	E	E	I	K	L	M	O	P	Q	R	S	T	U	X

Quicksort trace (array contents after each partition)

Quicksort: Best-case Analysis

Best case. Number of compares is ~ N lg N.

			a[ ]														
lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
initial values			H	A	C	B	F	E	G	D	L	I	K	J	N	M	O
random shuffle			H	A	C	B	F	E	G	D	L	I	K	J	N	M	O
0	7	14	D	A	C	B	F	E	G	H	L	I	K	J	N	M	O
0	3	6	B	A	C	D	F	E	G	H	L	I	K	J	N	M	O
0	1	2	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
0		0	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
2		2	A	B	C	D	F	E	G	H	L	I	K	J	N	M	O
4	5	6	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
4		4	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
6		6	A	B	C	D	E	F	G	H	L	I	K	J	N	M	O
8	11	14	A	B	C	D	E	F	G	H	J	I	K	L	N	M	O
8	9	10	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
8		8	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
10		10	A	B	C	D	E	F	G	H	I	J	K	L	N	M	O
12	13	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
12		12	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
14		14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

Quicksort: Worst-case Analysis

Worst case. Number of compares is $\sim \frac{1}{2} N^2$.

lo	j	hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
initial values			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
random shuffle			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	0	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	1	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
2	2	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3	3	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
4	4	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	5	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
6	6	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
7	7	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
8	8	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
9	9	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
10	10	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
11	11	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
12	12	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
13	13	14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
14		14	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
			A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

Quicksort: Practical Improvements

Insertion sort small subarrays.

- Even quicksort has too much overhead for tiny subarrays.
- Cutoff to insertion sort for ≈ 10 items.
- Note: could delay insertion sort until one pass at end.

```
private static void sort(Comparable[] a, int lo, int hi)
{
    if (hi <= lo + CUTOFF - 1)
    {
        Insertion.sort(a, lo, hi);
        return;
    }
    int j = partition(a, lo, hi);
    sort(a, lo, j-1);
    sort(a, j+1, hi);
}
```

Quicksort: Summary of Performance Characteristics

Worst case. Number of compares is quadratic.

- $N + (N - 1) + (N - 2) + \ldots + 1 \sim \frac{1}{2} N^2$.
- More likely that your computer is struck by lightning bolt.

Average case. Number of compares is $\sim 1.39 N \lg N$.

- 39% more compares than mergesort.
- But faster than mergesort in practice because of less data movement.

Random shuffle.

- Probabilistic guarantee against worst case.
- Basis for math model that can be validated with experiments.

Quickselect

Goal. Given an array of N items, find a k^{th} smallest item.

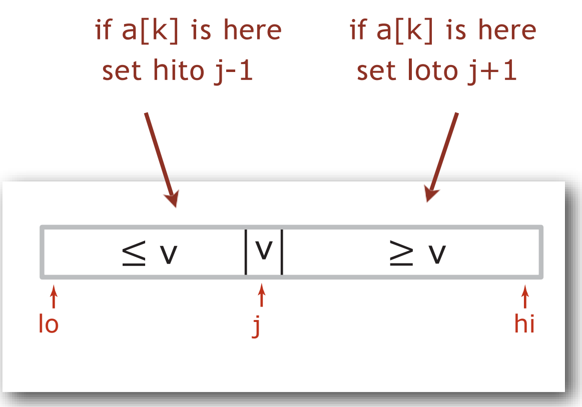
Ex. Min ($k = 0$), max ($k = N - 1$), median ($k = N/2$).

Partition array so that:

- Entry a[j] is in place.
- No larger entry to the left of j.
- No smaller entry to the right of j.

Repeat in one subarray, depending on j; finished when j equals k.

```
public static Comparable select(Comparable[] a, int k)
{
    StdRandom.shuffle(a);
    int lo = 0, hi = a.length - 1;
    while (hi > lo)
    {
        int j = partition(a, lo, hi);
        if (j < k) lo = j + 1;
        else if (j > k) hi = j - 1;
        else
            return a[k];
    }
    return a[k];
}
```



Mergesort Algorithm

Basic plan.

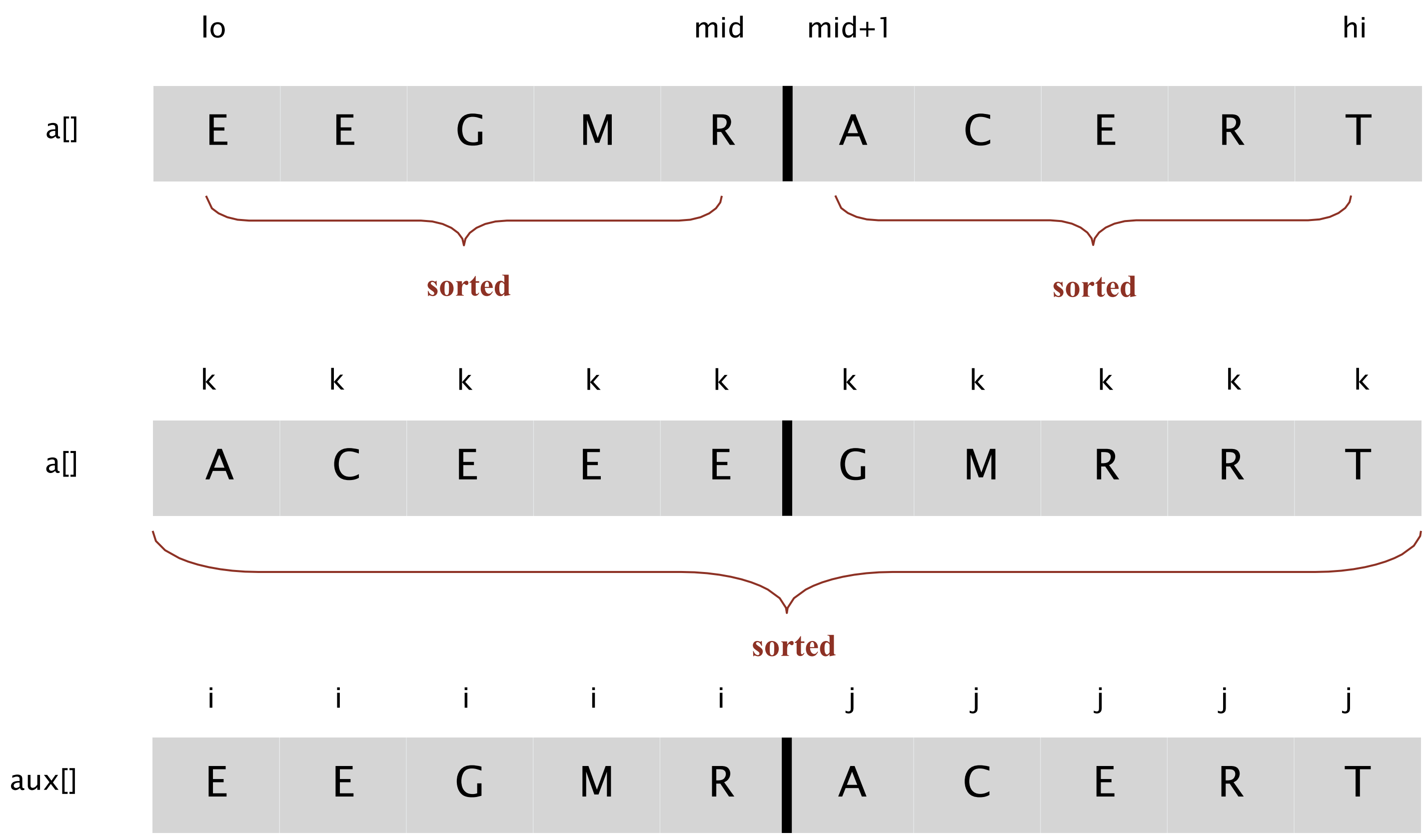
1. Divide array into two halves.
2. Recursively sort each half.
3. Merge two halves.

input	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
sort left half	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
sort right half	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge results	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Mergesort overview

Merge Operation

Goal. Given two sorted subarrays a[lo] to a[mid] and a[mid+1] to a[hi], replace with sorted subarray a[lo] to a[hi].



one subarray exhausted, take from other

Merge Operation: Java Implementation

```
private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid, int hi)
{
   assert isSorted(a, lo, mid);     // precondition: a[lo..mid]   sorted
   assert isSorted(a, mid+1, hi);   // precondition: a[mid+1..hi] sorted

   for (int k = lo; k <= hi; k++)
      aux[k] = a[k];

   int i = lo, j = mid+1;
   for (int k = lo; k <= hi; k++)
   {
      if      (i > mid)              a[k] = aux[j++];
      else if (j > hi)               a[k] = aux[i++];
      else if (less(aux[j], aux[i])) a[k] = aux[j++];
      else                           a[k] = aux[i++];
   }

   assert isSorted(a, lo, hi);      // postcondition: a[lo..hi] sorted
}
```

copy

merge

	lo			i	mid			j		hi
aux[]	A	G	L	O	R	H	I	M	S	T

						k				
a[]	A	G	H	I	L	M				

Can enable or disable at runtime.

⇒ No cost in production code.

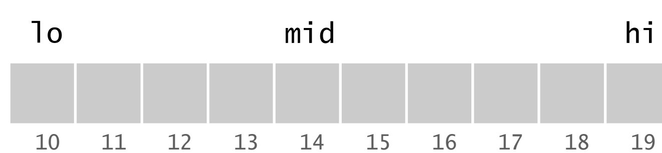
```
java -ea MyProgram  // enable assertions
java -da MyProgram  // disable assertions (default)
```

Mergesort: Java implementation

```
public class Merge
{
   private static void merge(...)
   {  /* as before */  }

   private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
   {
      if (hi <= lo) return;
      int mid = lo + (hi - lo) / 2;
      sort(a, aux, lo, mid);
      sort(a, aux, mid+1, hi);
      merge(a, aux, lo, mid, hi);
   }

   public static void sort(Comparable[] a)
   {
      aux = new Comparable[a.length];
      sort(a, aux, 0, a.length - 1);
   }
}
```



Mergesort: Trace

a[]

lo, hi	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 0, 0, 1)	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 2, 2, 3)	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 0, 1, 3)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 4, 5)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 6, 6, 7)	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 5, 7)	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E
merge(a, aux, 0, 3, 7)	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
merge(a, aux, 8, 8, 9)	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E
merge(a, aux, 10, 10, 11)	E	E	G	M	O	R	R	S	E	T	A	X	M	P	L	E
merge(a, aux, 8, 9, 11)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, aux, 12, 12, 13)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, aux, 14, 14, 15)	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L
merge(a, aux, 12, 13, 15)	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, aux, 8, 11, 15)	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge(a, aux, 0, 7, 15)	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

result after recursive call

Mergesort: Practical Improvement

Use insertion sort for small subarrays.

- Mergesort has too much overhead for tiny subarrays.
- Cutoff to insertion sort for ≈ 7 items.

```
private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
{
   if (hi <= lo + CUTOFF - 1)
   {
      Insertion.sort(a, lo, hi);
      return;
   }
   int mid = lo + (hi - lo) / 2;
   sort (a, aux, lo, mid);
   sort (a, aux, mid+1, hi);
   merge(a, aux, lo, mid, hi);
}
```

Mergesort: Practical Improvement

Stop if already sorted.

- Is biggest item in first half ≤ smallest item in second half?
- Helps for partially-ordered arrays.

A	B	C	D	E	F	G	H	I	J	M	N	O	P	Q	R	S	T	U	V
A	B	C	D	E	F	G	H	I	J	M	N	O	P	Q	R	S	T	U	V

```
private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
{
   if (hi <= lo) return;
   int mid = lo + (hi - lo) / 2;
   sort (a, aux, lo, mid);
   sort (a, aux, mid+1, hi);
   if (!less(a[mid+1], a[mid])) return;
   merge(a, aux, lo, mid, hi);
}
```

Bottom-up Mergesort

Basic plan.

1. Pass through array, merging subarrays of size 1.
2. Repeat for subarrays of size 2, 4, 8, 16, ...

Simple and non-recursive version of mergesort. but about 10% slower than recursive, top-down mergesort on typical systems

a[i]

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
sz = 1	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
`merge(a, aux, 0, 0, 1)`	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
`merge(a, aux, 2, 2, 3)`	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
`merge(a, aux, 4, 4, 5)`	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
`merge(a, aux, 6, 6, 7)`	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
`merge(a, aux, 8, 8, 9)`	E	M	G	R	E	S	O	R	E	T	X	A	M	P	L	E
`merge(a, aux, 10, 10, 11)`	E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
`merge(a, aux, 12, 12, 13)`	E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
`merge(a, aux, 14, 14, 15)`	E	M	G	R	E	S	O	R	E	T	A	X	M	P	E	L
sz = 2																
`merge(a, aux, 0, 1, 3)`	E	G	M	R	E	S	O	R	E	T	A	X	M	P	E	L
`merge(a, aux, 4, 5, 7)`	E	G	M	R	E	O	R	S	E	T	A	X	M	P	E	L
`merge(a, aux, 8, 9, 11)`	E	G	M	R	E	O	R	S	A	E	T	X	M	P	E	L
`merge(a, aux, 12, 13, 15)`	E	G	M	R	E	O	R	S	A	E	T	X	E	L	M	P
sz = 4																
`merge(a, aux, 0, 3, 7)`	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
`merge(a, aux, 8, 11, 15)`	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
sz = 8																
`merge(a, aux, 0, 7, 15)`	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Stability of Sorting Algorithms

A stable sort preserves the relative order of items with equal keys.

i	min	0	1	2
0	2	B_1	B_2	A
1	1	A	B_2	B_1
2	2	A	B_2	B_1
		A	B_2	B_1

selectsort is not stable

i	j	0	1	2	3
		B_1	C_1	C_2	A_1
1	3	B_1	C_1	C_2	A_1
1	3	B_1	A_1	C_2	C_1
0	1	A_1	B_1	C_2	C_1

quickcksort is not stable

i	j	0	1	2	3	4
0	0	B_1	A_1	A_2	A_3	B_2
1	0	A_1	B_1	A_2	A_3	B_2
2	1	A_1	A_2	B_1	A_3	B_2
3	2	A_1	A_2	A_3	B_1	B_2
4	4	A_1	A_2	A_3	B_1	B_2
		A_1	A_2	A_3	B_1	B_2

insertsort is stable

0	1	2	3	4
A_1	A_2	A_3	B	D

5	6	7	8	9	10
A_4	A_5	C	E	F	G

mergesort is stable

0	1	2	3
B	A	C_1	C_2
B	C_2	C_1	A
C_1	C_2	B	A
C_2	A	B	C_1
B	A	C_2	C_1
A	B	C_2	C_1

heapsort is not stable

Summary

	inplace?	stable?	best	average	worst	remarks
selection	✔		$\frac{1}{2} n^2$	$\frac{1}{2} n^2$	$\frac{1}{2} n^2$	n exchanges
insertion	✔	✔	n	$\frac{1}{4} n^2$	$\frac{1}{2} n^2$	use for small n
merge		✔	$\frac{1}{2} n \lg n$	$n \lg n$	$n \lg n$	n log n guarantee; stable
quick	✔		$n \lg n$	$2 n \ln n$	$\frac{1}{2} n^2$	n log n probabilistic guarantee; fastest in practice
heap	✔		$3 n$	$2 n \lg n$	$2 n \lg n$	n log n guarantee; in-place
?	✔	✔	n	$n \lg n$	$n \lg n$	holy sorting grail

Additional Resources