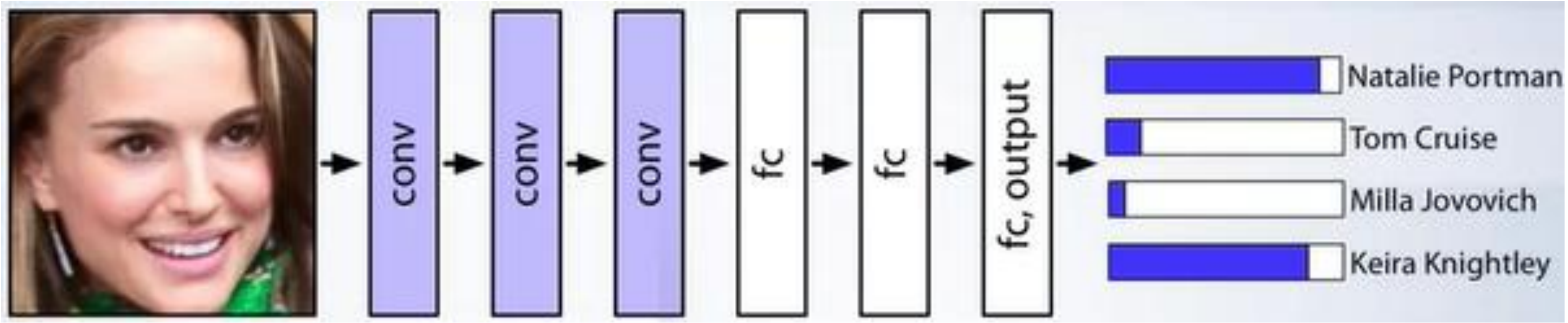
L3.1 Convolutional Neural Networks



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Acknowledgement: some contents taken from UC Berkeley CS231n https://cs231n.github.io
Coursera MOOC on CNN: https://www.coursera.org/learn/convolutional-neural-networks
Hung-yi Lee: https://speech.ee.ntu.edu.tw/~hylee/ml/2021-spring.html

Outline

- Convolution layers
- Pooling and Fully-Connected layers
- Well-known CNN architectures

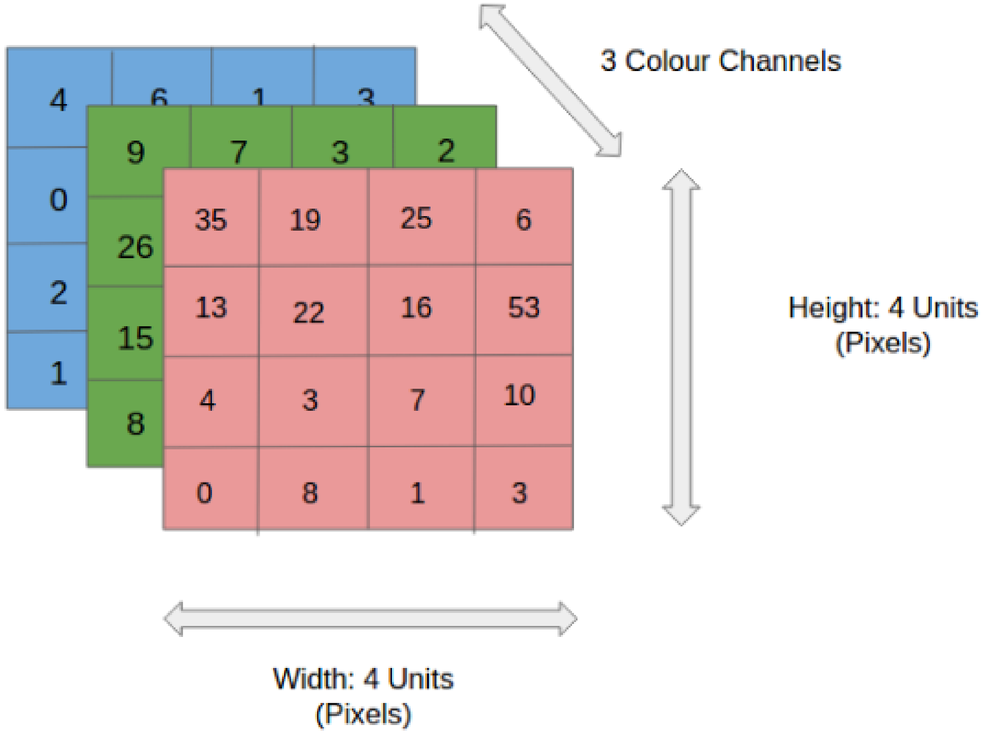
Classic Computer Vision

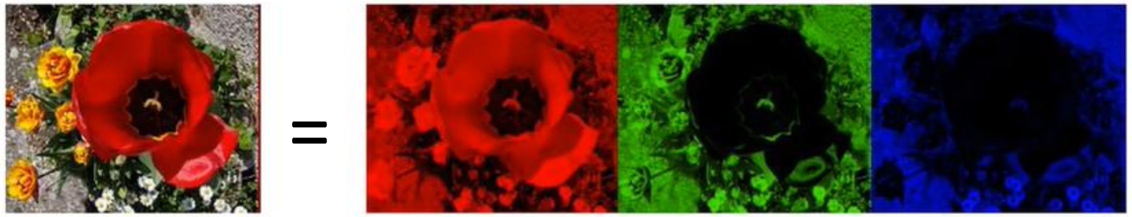
- Most “classic” (non-ML) CV algorithms are implemented in the OpenCV library, including
  - Core Operations:
    - basic operations on image like pixel editing, geometric transformations...
  - Image Processing
    - Thresholding, smoothing, edge detection, Hough Line Transform…
  - Feature Detection and Description
    - HOG, SIFT, SURF, BRIEF, ORB…
  - Video analysis
    - Object tracking w. optical flow
  - Camera Calibration and 3D Reconstruction
- They are simple, fast and reliable (e.g., for lane detection), and are often used in place of or in conjunction w. complex ML/DL algorithms, which may sometimes be unreliable and unpredictable

OpenCV

Input Image Encoding

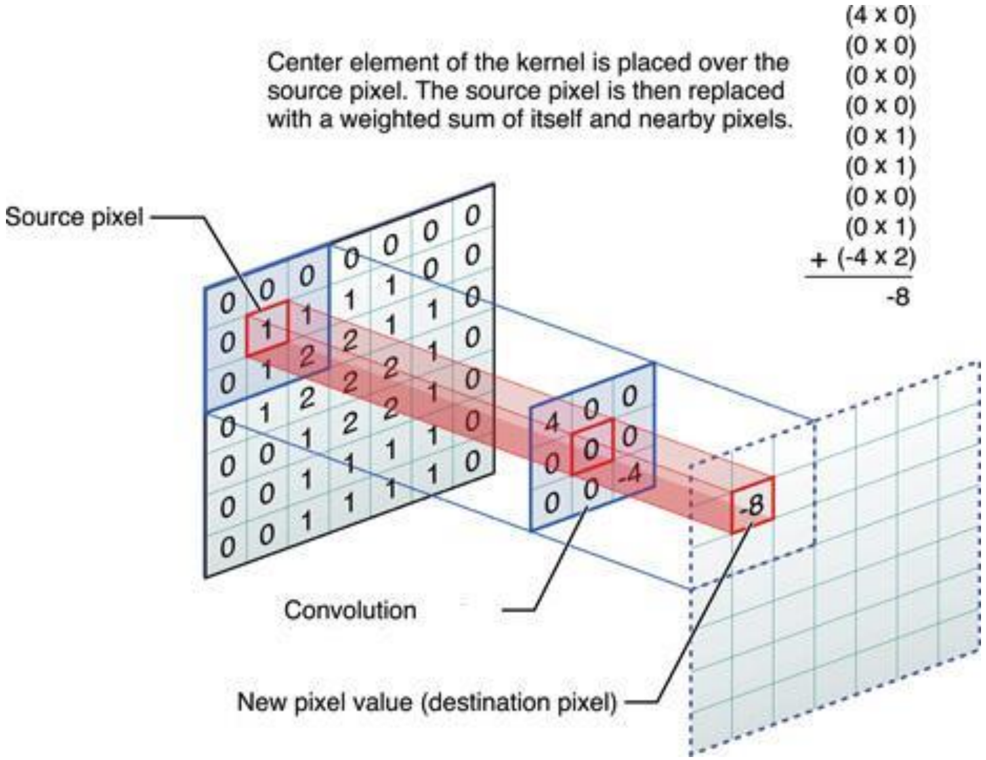
- A size $N \times N$ color image has volume $N \times N \times 3$, w. $N \times N$ pixels and 3 color components (Red, Green, and Blue, RGB) for each pixel
- A size $N \times N$ greyscale image has volume $N \times N \times 1$
- Color depth, or bit depth, is number of bits used for each color component of a single pixel
  - Typical value is 8, so pixel value has range $[0, 255]$
  - Larger depth is possible, e.g., true color (24-bit) is used in computer and phone displays for human eyes, but 8-bit is typically enough for CV tasks

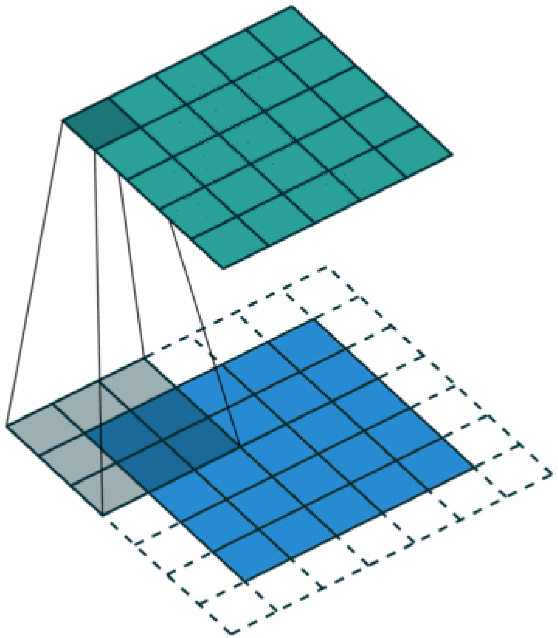




Filters/Kernels in Computer Vision

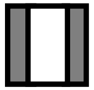
- Convolution operation: we slide each filter (also called kernel) across the width and height of the input volume, and compute dot products between the entries of the filter and the input. As the filter slides over the width and height of the input volume, a 2D feature map (also called activation map) is produced that gives the responses of that filter at every spatial position.
  - Dot product: elementwise multiplication of a filter w. corresponding input values, then summing them to generate one output value
- Filters extract features used by downstream tasks such as classification, image segmentation, etc.



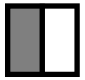


A Filter for Vertical Edge Detection

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

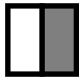


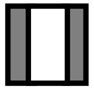
$$\begin{bmatrix} 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \end{bmatrix}$$



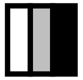
Sobel Filter for Vertical Edge Detection

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 40 & 40 & 0 \\ 0 & 40 & 40 & 0 \\ 0 & 40 & 40 & 0 \\ 0 & 40 & 40 & 0 \end{bmatrix}$$





$$\begin{bmatrix} 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -40 & -40 & 0 \\ 0 & -40 & -40 & 0 \\ 0 & -40 & -40 & 0 \\ 0 & -40 & -40 & 0 \end{bmatrix}$$

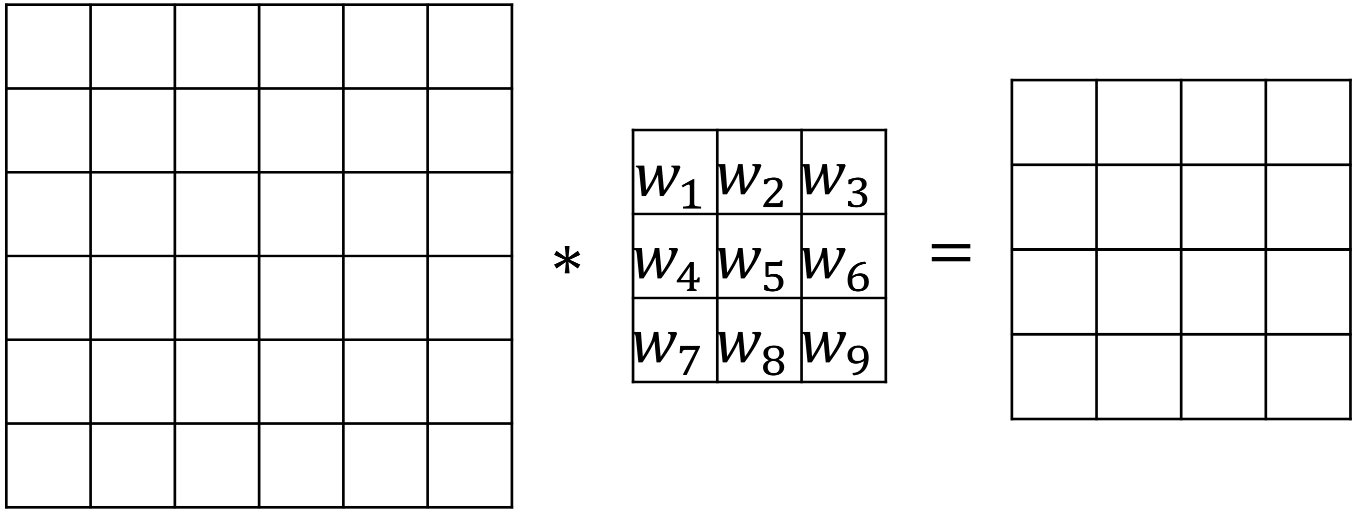

Common Filters in CV

- These filters were designed, or “hand-crafted”, by human experts

Operation	Kernel ω	Image result g(x,y)
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Box blur (normalized)	$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	
Gaussian blur 3 × 3 (approximation)	$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	
Gaussian blur 5 × 5 (approximation)	$\frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$	
Unsharp masking 5 × 5 Based on Gaussian blur with amount as 1 and threshold as 0 (with no image mask)	$\frac{-1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & -476 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$	

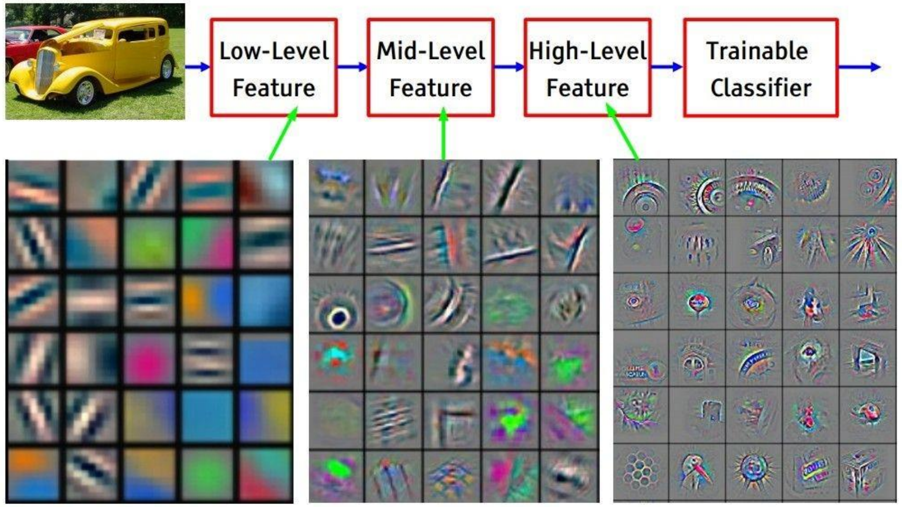
Machine Learning Meets CV

- Instead of hand-crafted filters in classic CV, why not learn custom filters from data by supervised learning?
  - For easy tasks like edge detection, learning may recover filters similar to hand-crafted ones.
  - For difficult tasks like cat vs. dog classification, learning is essential to achieving good results

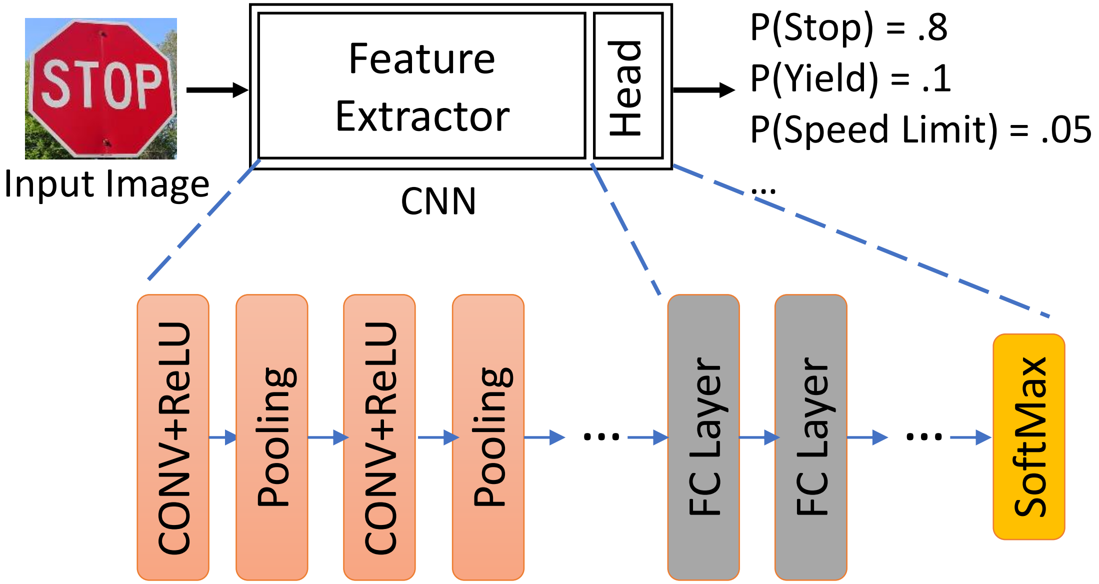


Convolutional Neural Network (CNN)

- Since the Deep Learning revolution that started a decade ago, Deep Neural Networks (DNNs) are widely deployed in many application domains
  - Multiple hidden layers of a DNN extract a hierarchy of increasingly-abstract features layer-by-layer, until the last layer produces a classification result
- A CNN, or ConvNet, consists of a sequence of Convolutional (CONV) Layers, Pooling (POOL) Layers and non-linear activation functions for feature extraction, followed by one or more Fully-Connected (FC) Layers for classification based on the extracted features

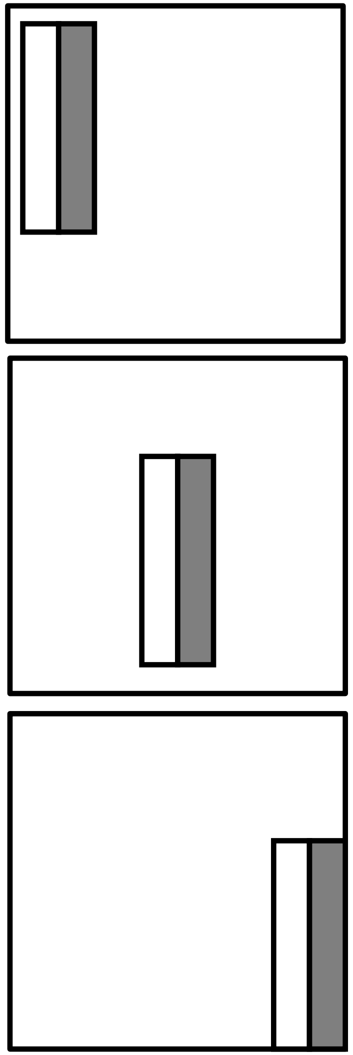


Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]



Receptive Field and Parameter Sharing

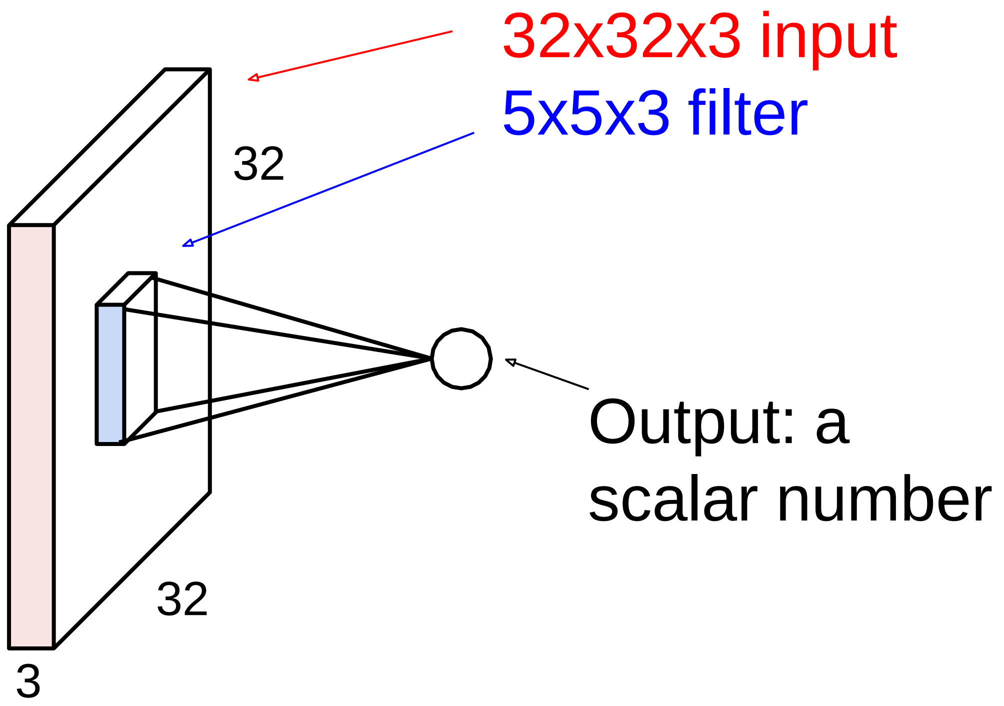
- Each neuron in a CONV layer has local, sparse connectivity to a small patch of the input volume w. size of the filter, called its Receptive Field (e.g., 3x3, 5x5, etc.)
  - Each neuron covers a limited, narrow “field-of-view”
  - In contrast, each neuron in a FC layer has RF that covers the entire input volume
- Parameter sharing: all neurons in the same CONV layer share the same filter params w, b
  - It helps to reduce the number of params significantly compared to fully-connected networks
  - It gives translation invariance, e.g., an edge can be detected regardless of its location in the image



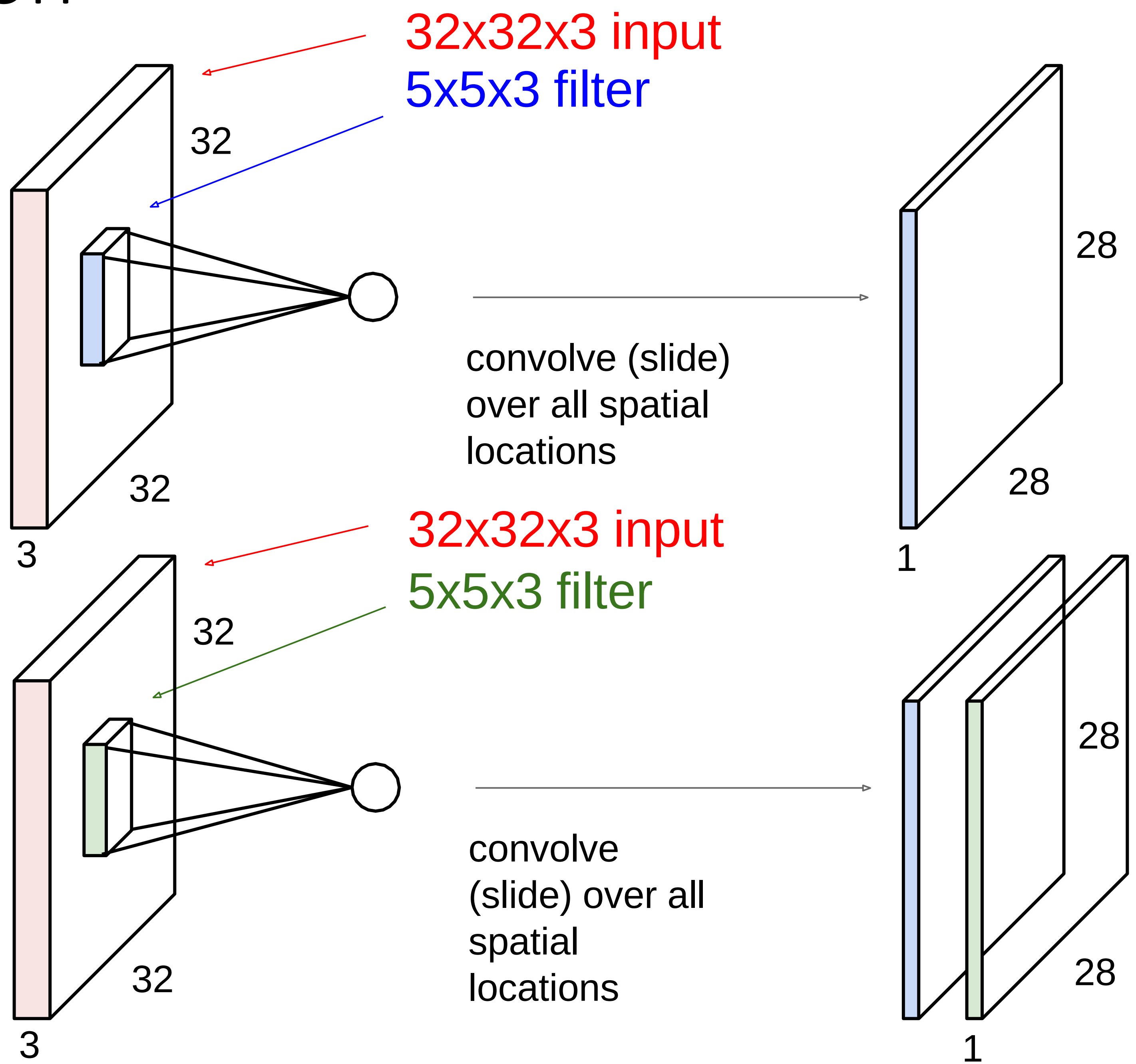
A filter sliding across different parts of the image

Convolution Operation

- A filter slides over the image spatially, computing dot products $\boldsymbol{w}^T \boldsymbol{x} + b$ to generate a feature map as output
- Input may be an input RGB image w. 3 channels, hence depth=3, or intermediate feature maps generated by hidden layers of a CNN. We use the terms "input volume" and "output volume" to emphasize they may be 3D tensors
- At each position, output is a scalar number, computed by taking dot product $\boldsymbol{w}^T \mathbf{x} + b$ between the $5 \times 5 \times 3$ filter with weights w, bias b, and a $5 \times 5 \times 3$ image patch x, with $5 * 5 * 3 = 75$ multiply operations and one addition of the bias



Convolution Operation

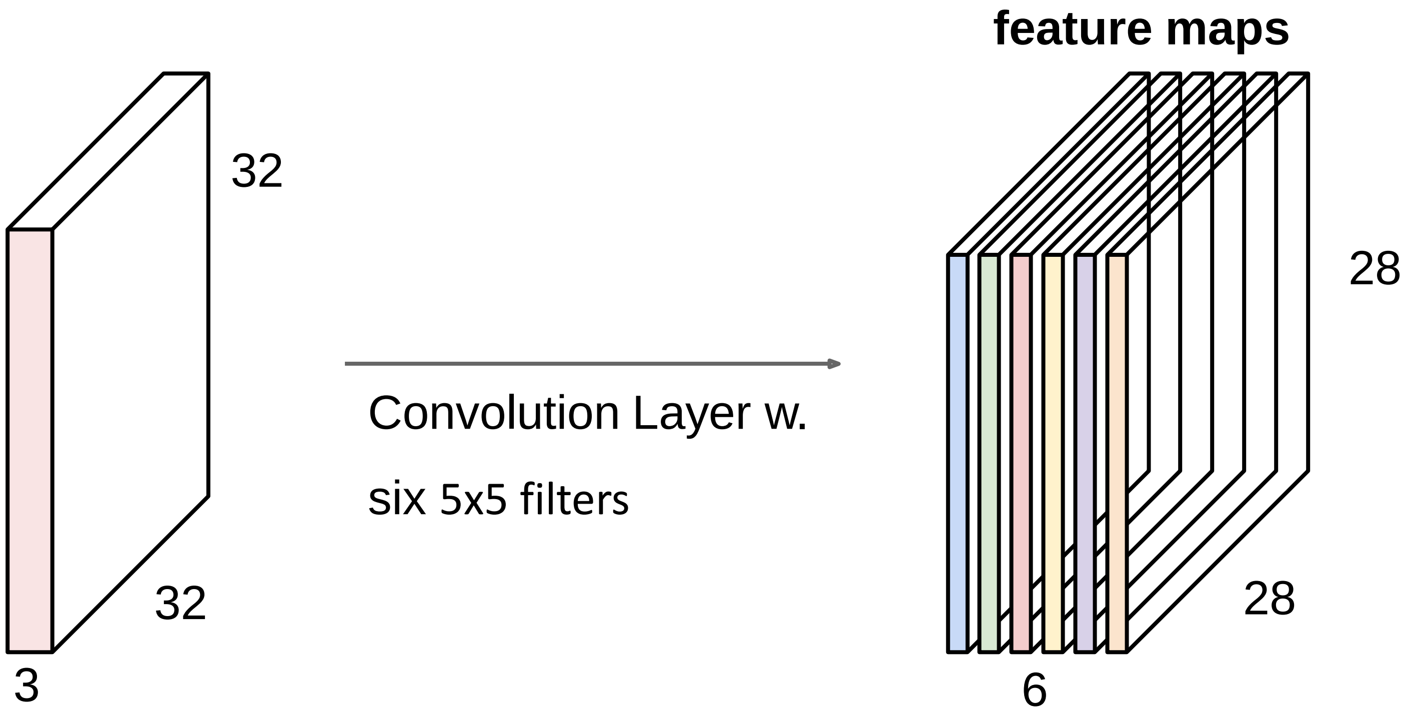


One (blue) filter generates one 2D feature map as output

Two filters (blue and green) generate two 2D feature maps (blue and green), stacked along the depth dimension to produce a 3D output volume

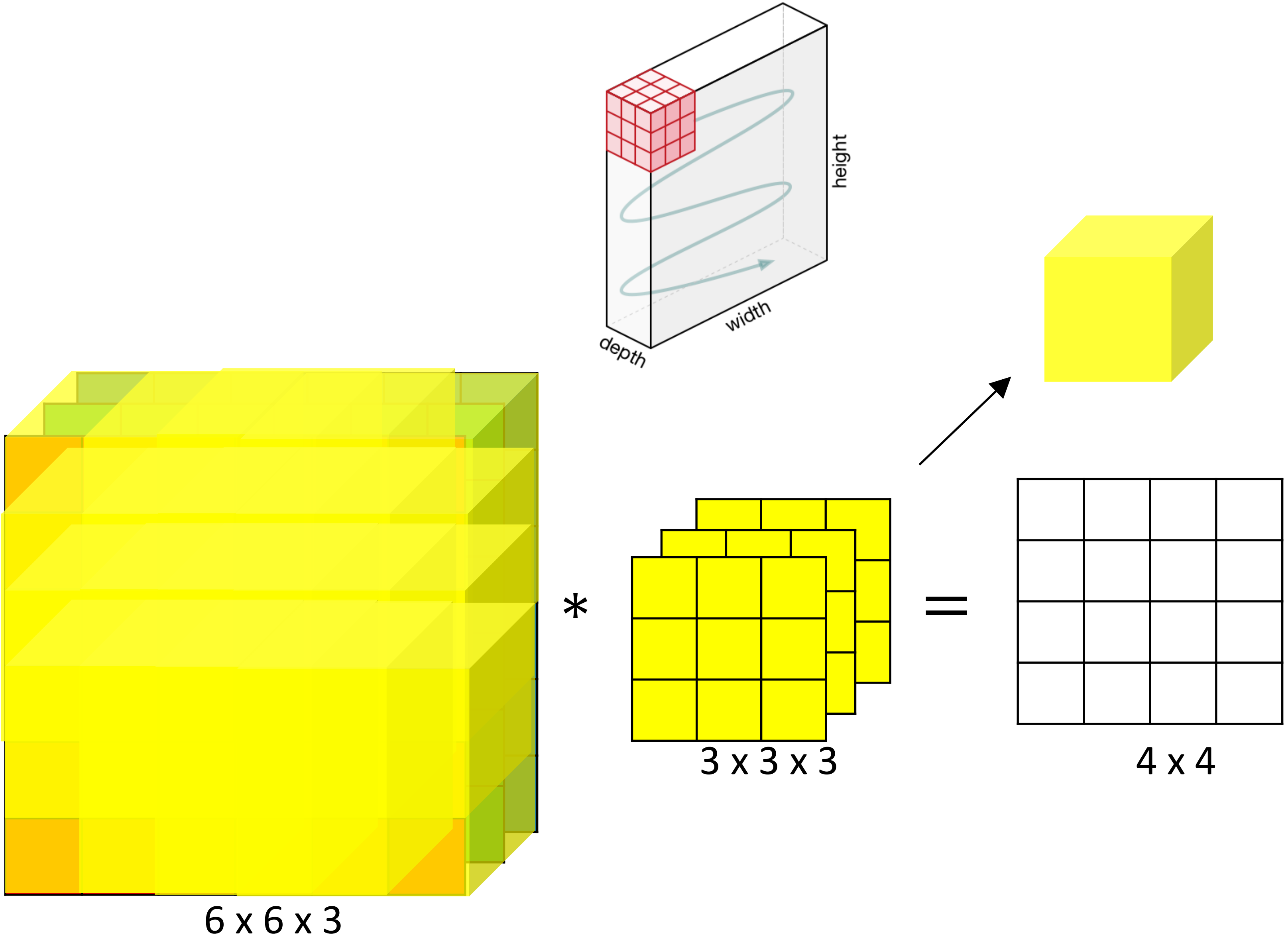
Stacked feature maps

- If we have six 5×5 filters, we'll get six different feature maps, each computed by convolution of one filter with the input
  - For each 5×5 patch of the input, there are 6 different neurons looking at it, each extracting different features
- We stack these up to get an output volume (a new "image") of size $28 \times 28 \times 6$, an intermediate representation to be passed to subsequent layers



Convolution of a Filter on an RGB Image w. 3 Channels

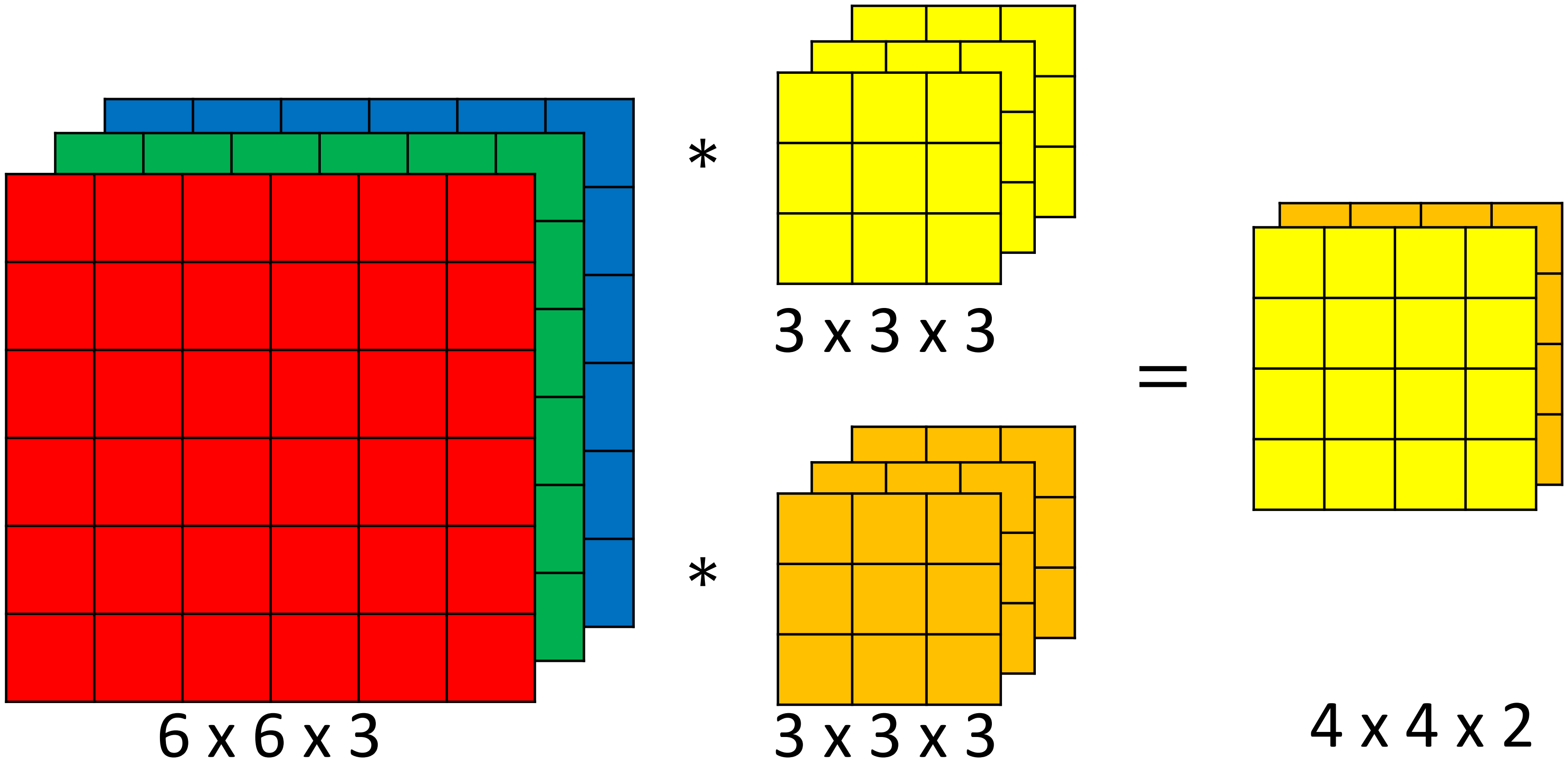
- 6x6 input feature map w. 3 channels; one 3x3 filter with depth 3; 4x4 output feature map w. one channel
- # channels of input feature map == # depth of each filter (3)
- # channels of output feature map == # filters (1)
- (Even though the fig shows sequential computation, convolution operations are inherently parallel, hence suitable for implementation on parallel hardware like GPUs)



Important

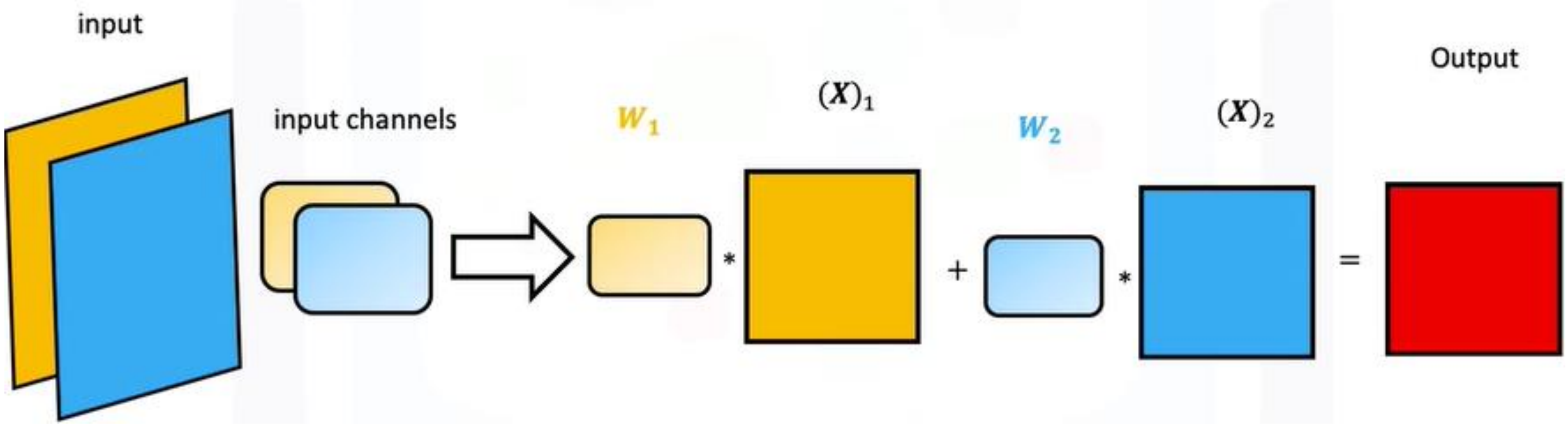
Convolution of 2 Filters on an RGB Image w. 3 Channels

- 6x6 input feature map w. 3 channels; two 3x3 filters with depth 3; 4x4 output feature map w. two channels
- # channels of input feature map == # depth of each filter (3)
- # channels of output feature map == # filters (2)



Convolution in PyTorch

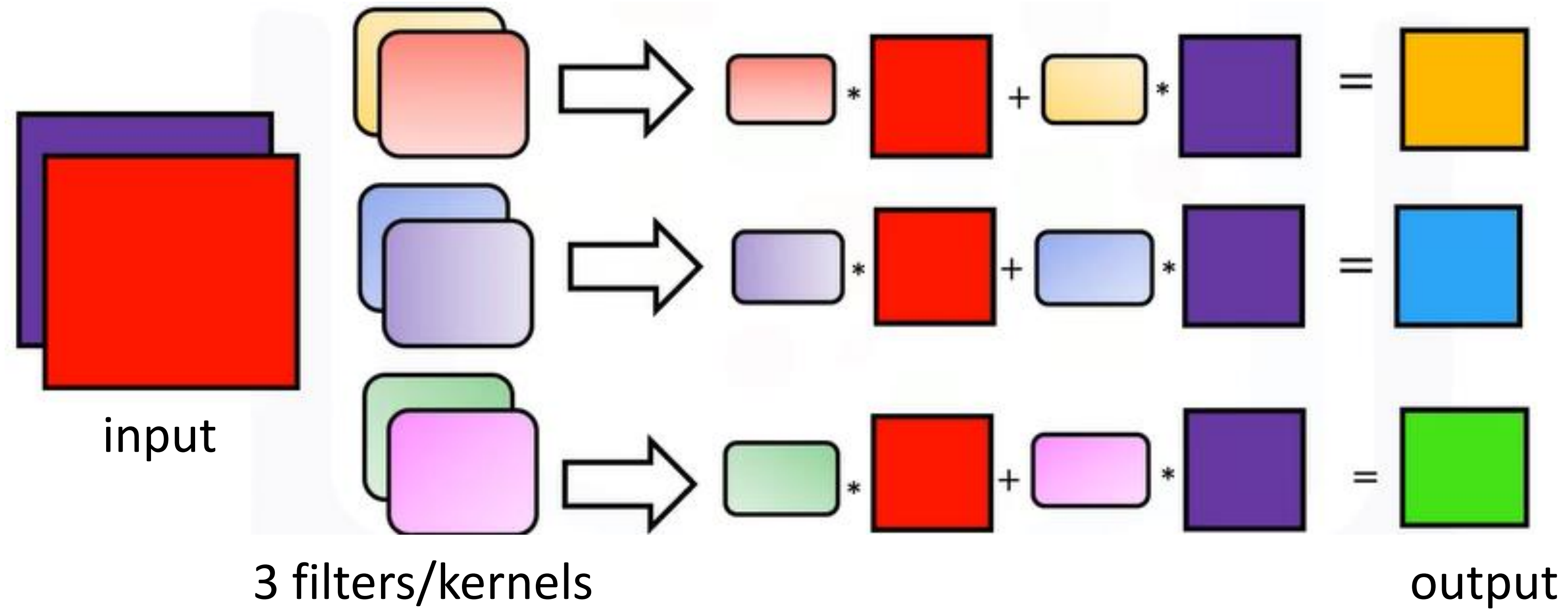
- `conv=nn.Conv2d(in_channels=2, out_channels=1, kernel_size=3)`
  - A CONV layer with an input image with 2 channels (in_channels=2), 1 3×3 filter (with depth 2), 1 output feature maps (out_channels=1).
  - (Bias terms are assumed to be 0)



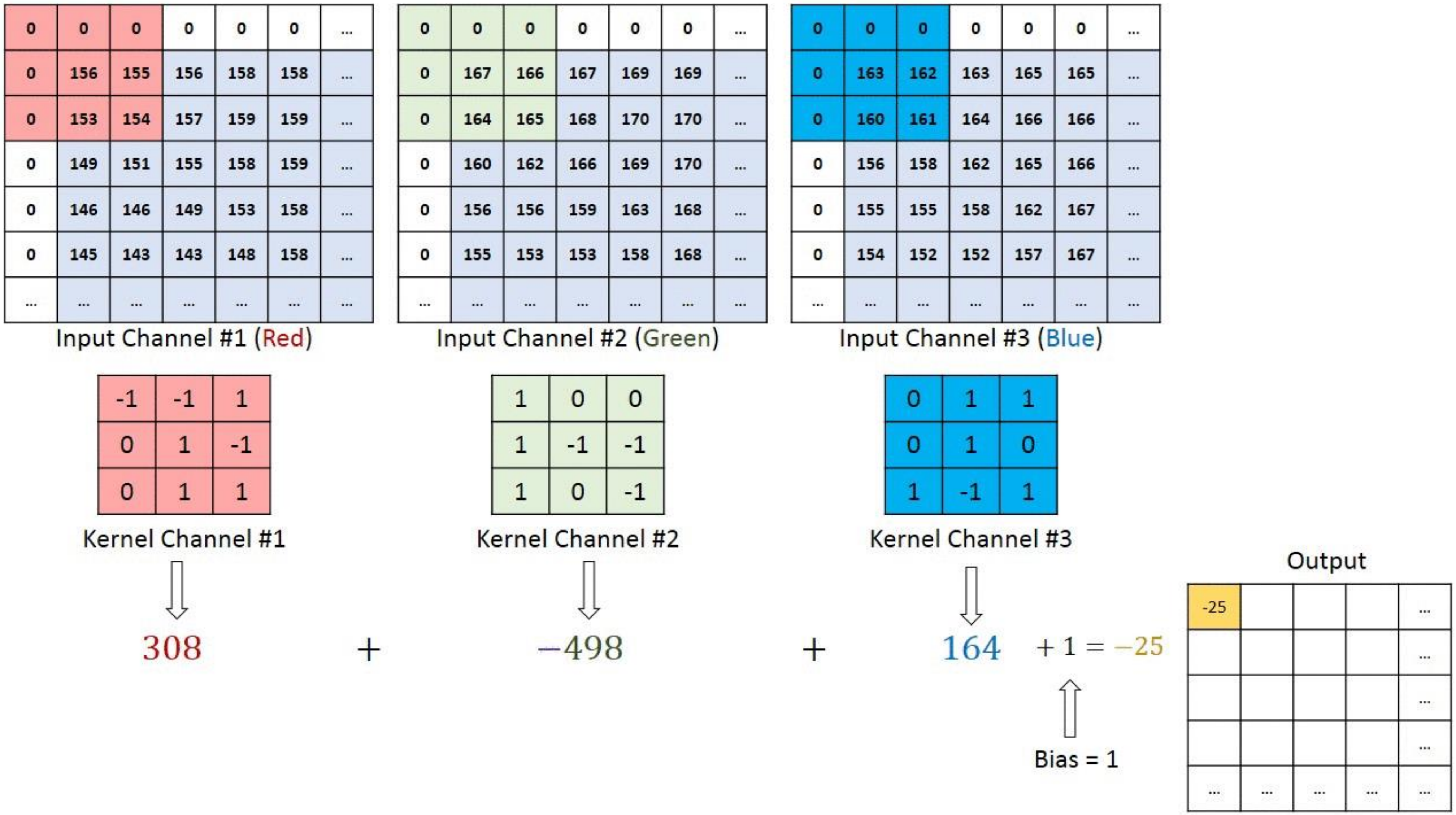
input 1 filter/kernel output

Convolution in PyTorch Cont'

- `conv4=nn.Conv2d(in_channels=2, out_channels=3, kernel_size=3)`
  - Pytorch code for a CONV layer with an input image with 2 channels (in_channels=2), 3 3×3 filters (with depth 2), 3 output feature maps (out_channels=3)
  - (Bias terms are assumed to be 0)



Convolution Example 1: Computing a Single Output Element



https://towardsdatascience.com/a-comprehensive-guide-to-convolutional-neural-networks-the-eli5-way-3bd2b1164a53

Convolution Example 2

Image4[1,0,:,:]

Channel 1

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

Image4[1,1,:,:]

Channel 2

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Input image with 2 channels

conv4.state_dict()['weight'][0][0]

$\boldsymbol{W}_{0,0}$

0	0	0
0	0.5	0
0	0	0

conv4.state_dict()['weight'][0][1]

$\boldsymbol{W}_{0,1}$

0	0	0
0	0.5	0
0	0	0

conv4.state_dict()['weight'][1][0]

$\boldsymbol{W}_{1,0}$

0	0	0
0	1	0
0	0	0

conv4.state_dict()['weight'][1][1]

$\boldsymbol{W}_{1,1}$

0	0	0
0	-1	0
0	0	0

conv4.state_dict()['weight'][2][0]

$\boldsymbol{W}_{2,0}$

1	0	-1
1	0	-2
1	0	-1

conv4.state_dict()['weight'][2][1]

$\boldsymbol{W}_{2,1}$

1	2	-1
0	0	0
-1	-2	-1

Three 3×3 filters

Convolution Example 2 Cont'

- Each of the three filters convolved with the input image generates an output feature map.
- The output volume consists of three 3×3 feature maps, with volume $3 \times 3 \times 3$

$$(\boldsymbol{Z})_0 = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{W}_{0,0}} * \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}}_{(\boldsymbol{X})_0} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{W}_{0,1}} * \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{(\boldsymbol{X})_1}$$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$(\boldsymbol{Z})_1 = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{W}_{1,0}} * \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}}_{(\boldsymbol{X})_0} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{W}_{1,1}} * \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{(\boldsymbol{X})_1}$$

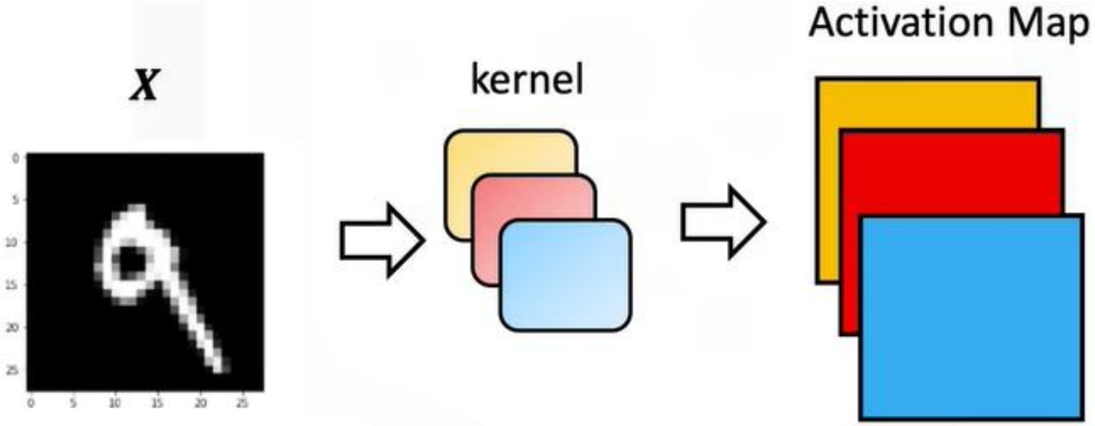
$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(\boldsymbol{Z})_2 = \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}}_{\boldsymbol{W}_{2,0}} * \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}}_{(\boldsymbol{X})_0} + \underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}}_{\boldsymbol{W}_{2,1}} * \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{(\boldsymbol{X})_1}$$

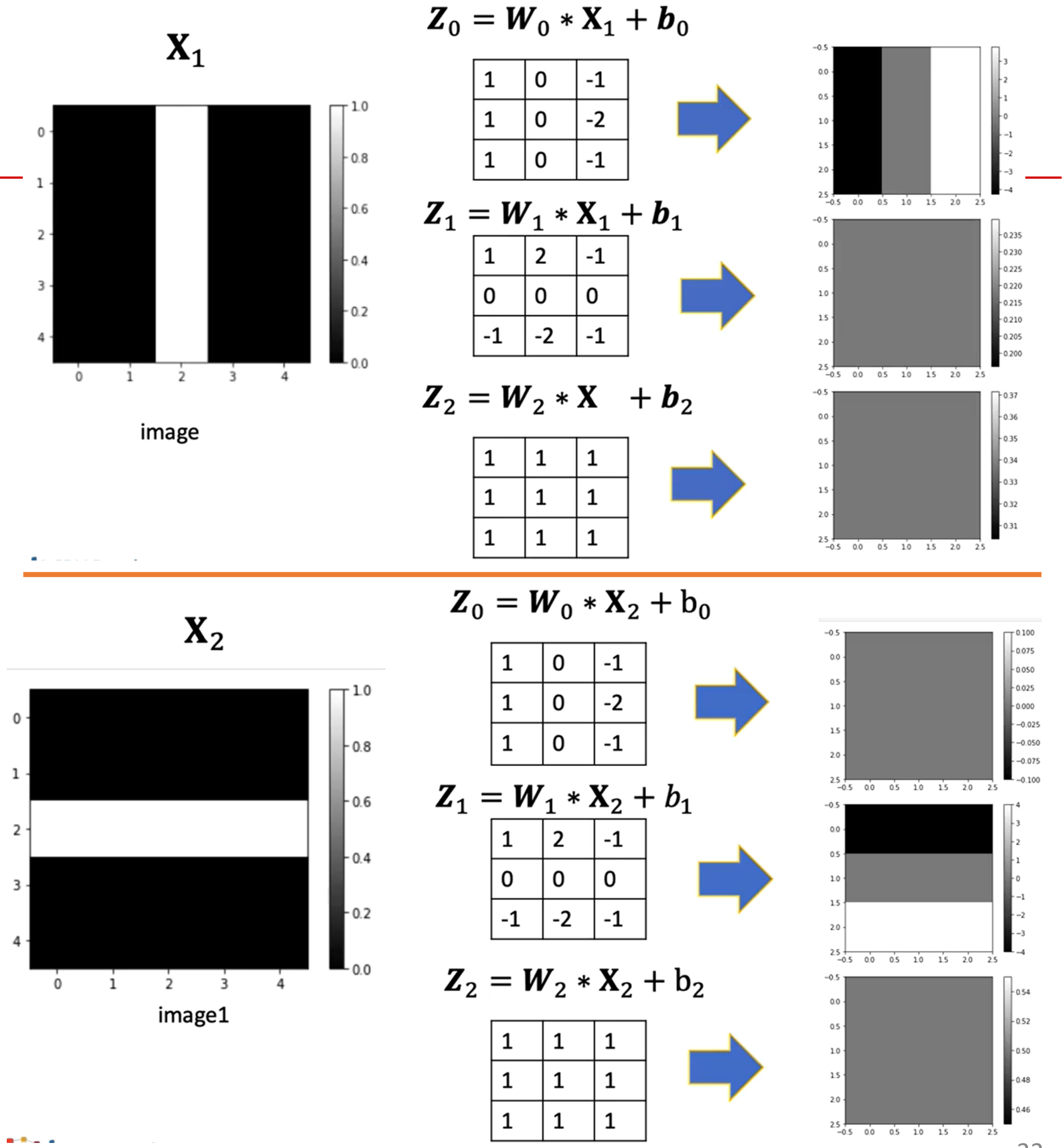
$$= \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Convolution Example 3

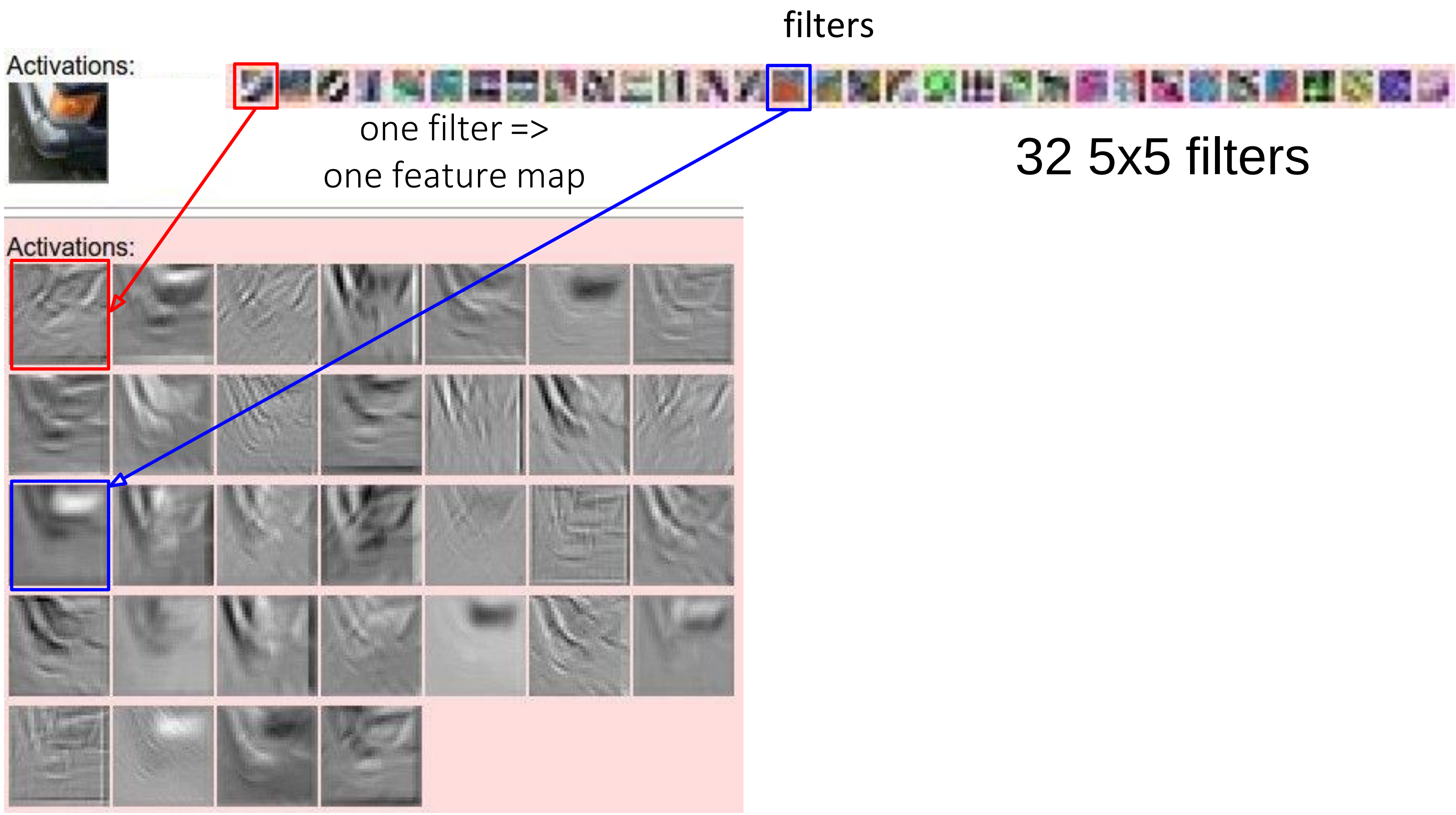
- 3 filters $\boldsymbol{W_0}, \boldsymbol{W_1}, \boldsymbol{W_2}$, each extracting different features. ($\boldsymbol{W_i} * \boldsymbol{X_j}$ denotes convolution of filter $\boldsymbol{W_i}$ w. input $\boldsymbol{X_j}$) (bias terms are assumed to be 0 here)
- Upper left: filter $\boldsymbol{W_0}$ extracts vertical line features $\boldsymbol{Z_0}$ from input image $\boldsymbol{X_1}$ (the other 2 filters do not extract any meaningful features)
- Lower left: filter $\boldsymbol{W_1}$ extracts horizontal line features $\boldsymbol{Z_1}$ from input image $\boldsymbol{X_2}$ (the other 2 filters do not extract any meaningful features)



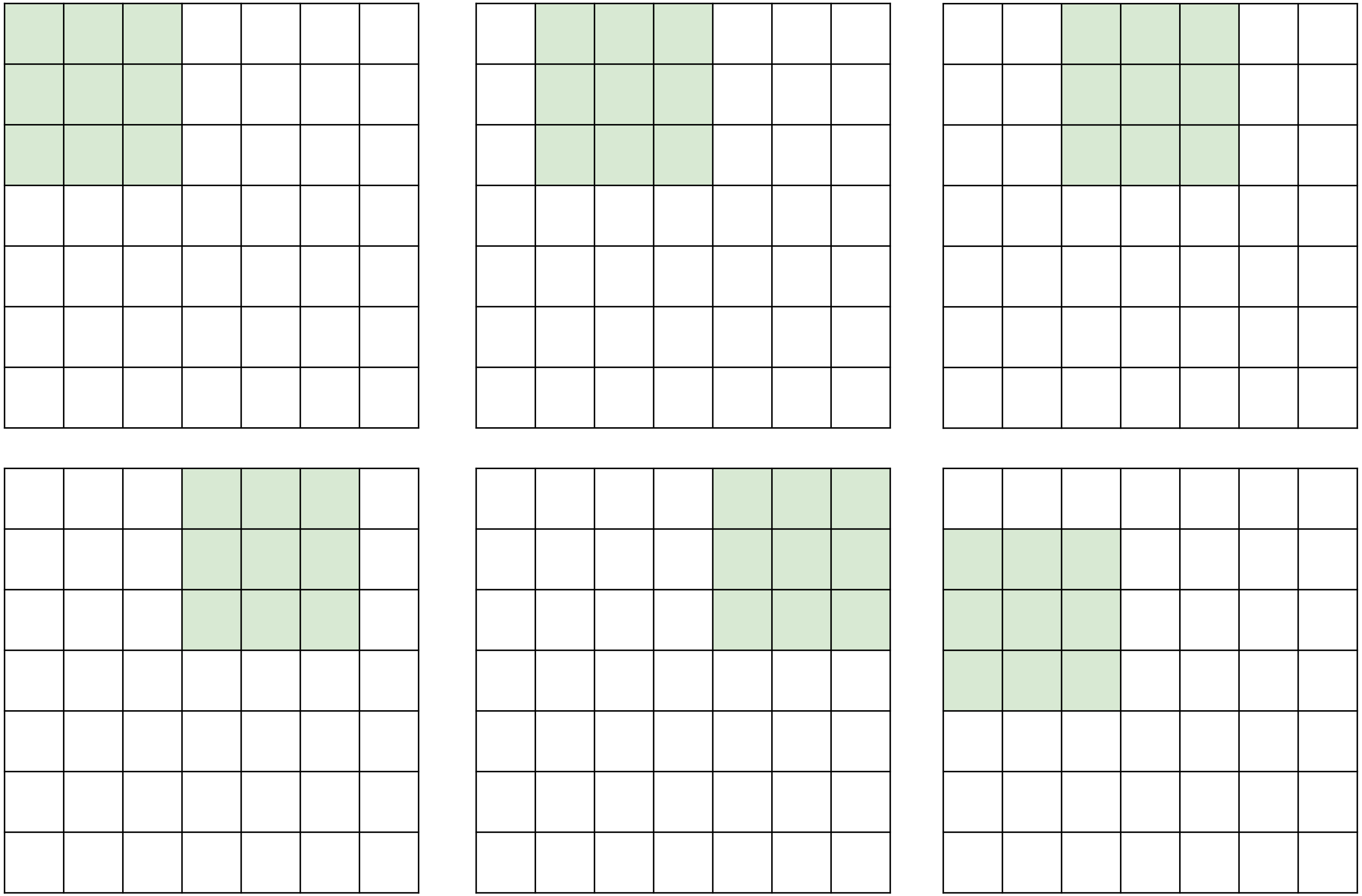
3 filters/kernels, 3 output feature/activation map



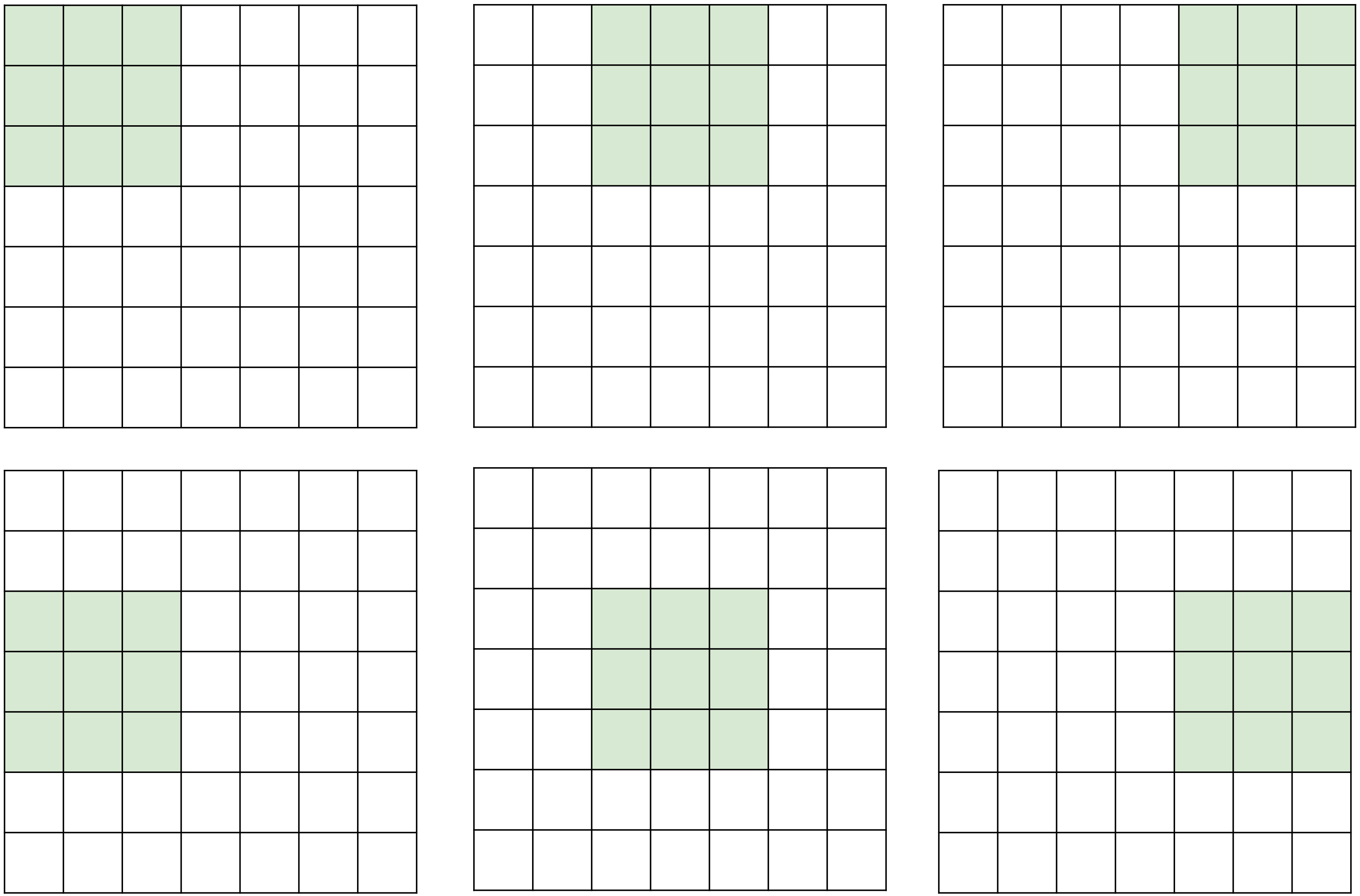
Filters and feature maps Example



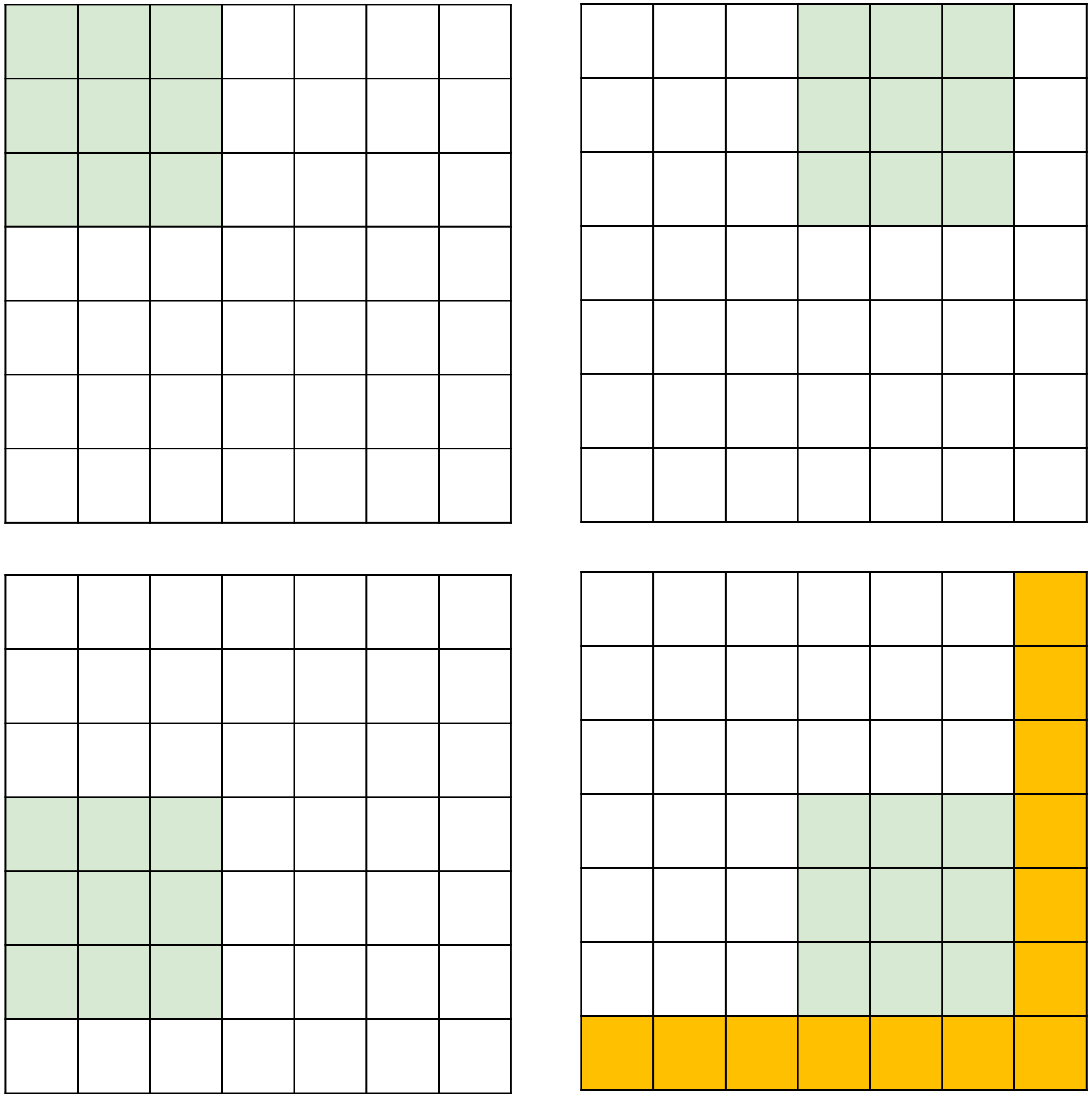
7x7 input, 3x3 filter, stride=1 ⇒ output: 5x5



7x7 input, 3x3 filter, stride=2 ⇒ output: 3x3



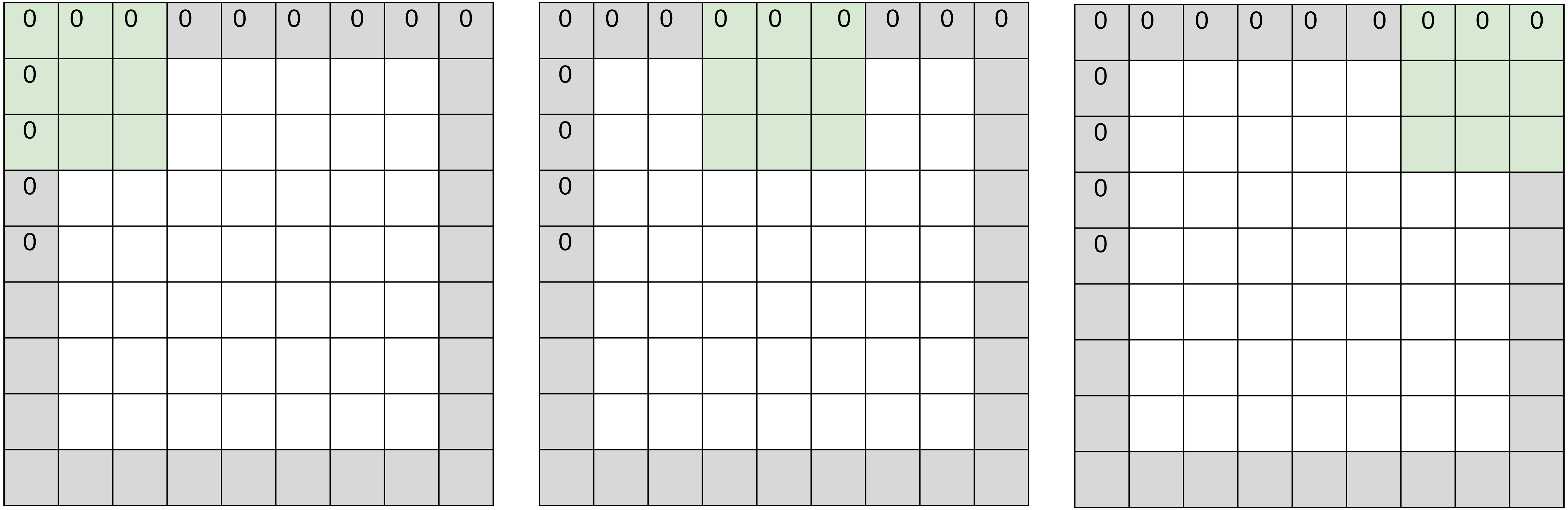
7x7 input, 3x3 filter, stride=3 ⇒ output: ???



The rightmost and bottom columns are not processed!

Solution: Add padding

- 7x7 input, 3x3 filter, stride=3, zero padding $P = 1$ ⇒ output: 3x3



Important

Computation of CONV Layer Sizes

Summary. To summarize, the Conv Layer:

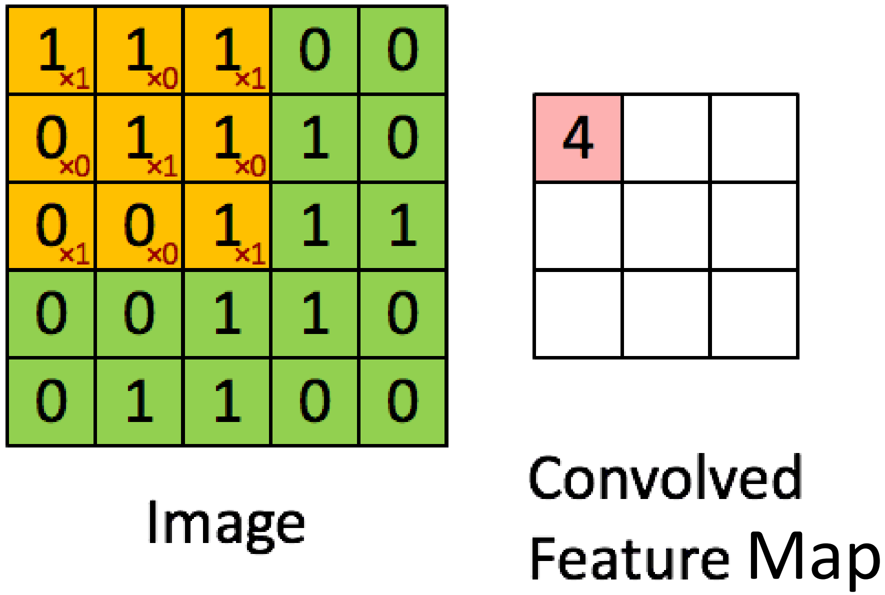
- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

Common settings:
K = (powers of 2, e.g. 32, 64, 128, 512)
- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 1, S = 1, P = 0

- A filter typically has square shape $F \times F$. A filter has the same depth D_1 as its input volume, and the number of filters K equals the depth D_2 of its output volume
- Same Padding: stride $S = 1$, filter size $F \times F$, and zero-padding $P = \frac{1}{2}(F - 1)$. Then output feature map has same spatial size as input
  - $W_2 = \frac{1}{S}(W_1 + 2P - F) + 1 = \frac{1}{1}(W_1 + F - 1 - F) + 1 = W_1$; similarly, $H_2 = H_1$
  - e.g., $F < 3 \Rightarrow P = 0$; $F = 3 \Rightarrow P = 1$; $F = 5 \Rightarrow P = 2$

CONV Example 1: No Padding

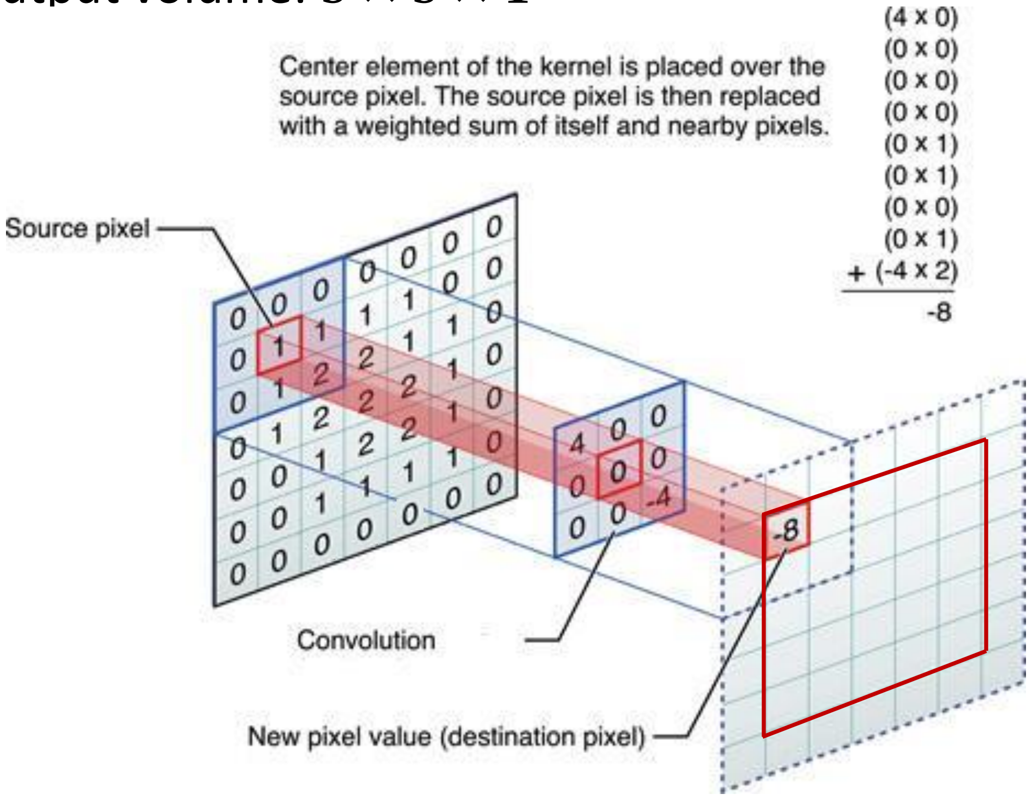
- Input volume: $5 \times 5 \times 1$ ($W_1 = H_1 = N_1 = 5, D_1 = 1$)
- A $3 \times 3 \times 1$ filter ($K = 1, F = 3$) w. stride $S = 1$, no padding $P = 0$
- Output feature map:
  - Spatial size: $\mathrm{W}_2 = \mathrm{H}_2 = \mathrm{N}_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(5 + 0 - 3) + 1 = 3$
  - Depth: $D_2 = K = 1$
- Output volume: $3 \times 3 \times 1$

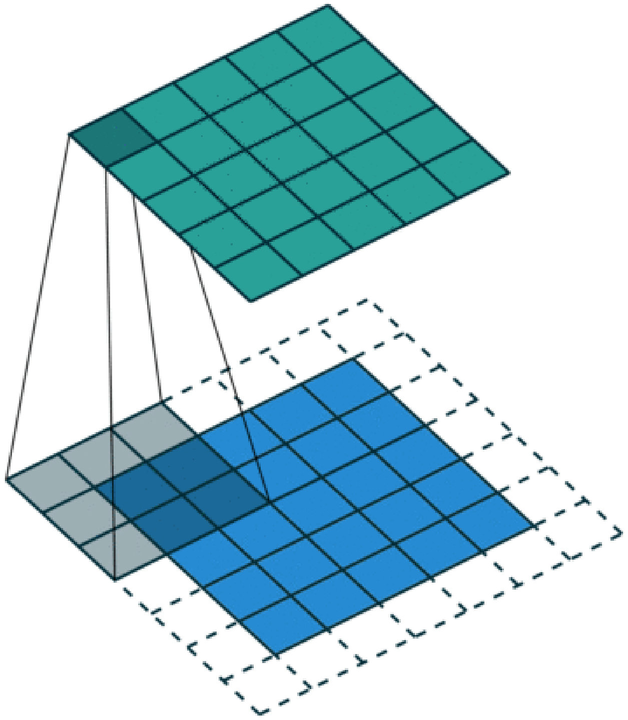


3 × 3 Filter $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

CONV Example 2: Same Padding

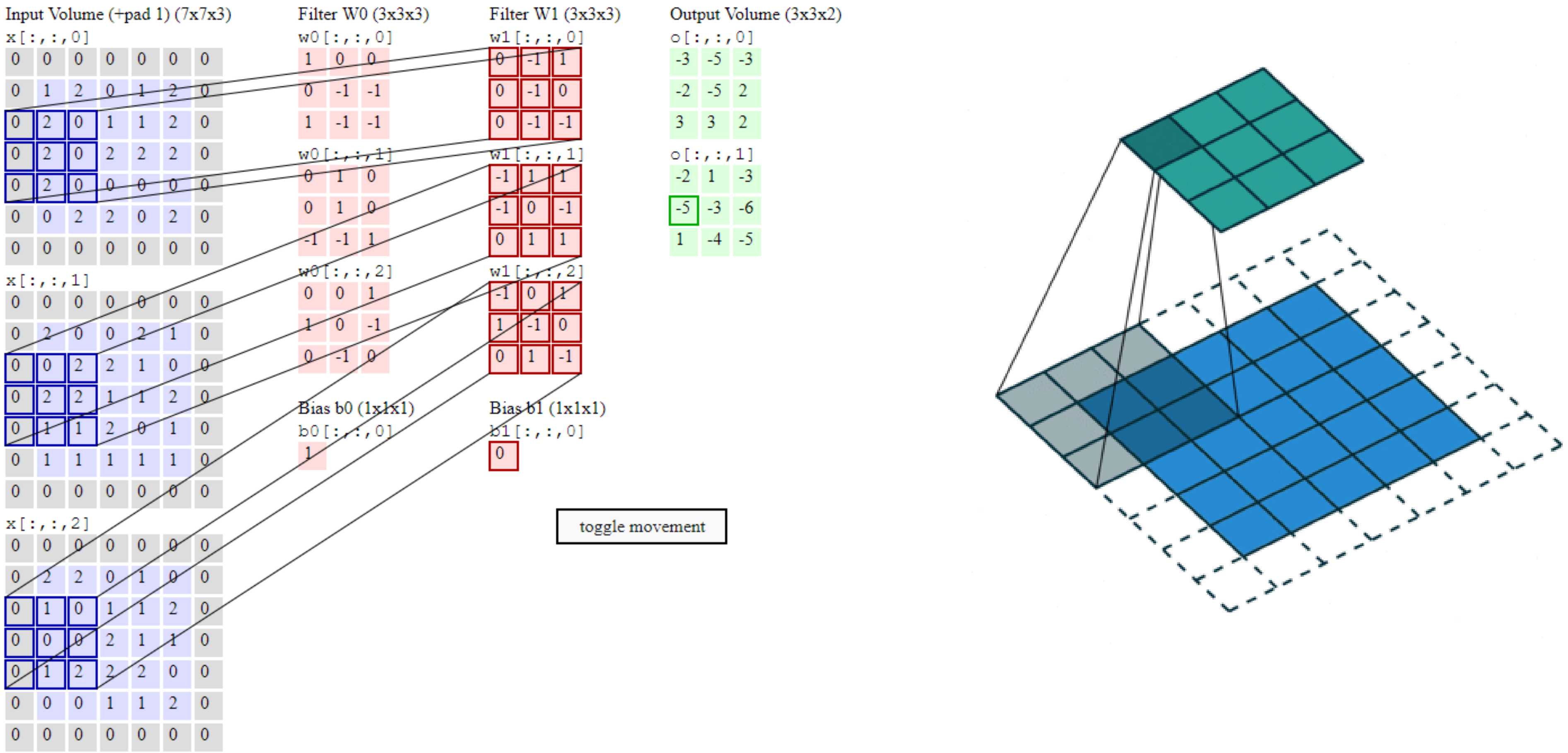
- Input volume: $5 \times 5 \times 1$
- A $3 \times 3 \times 1$ filter ($K = 1, F = 3$) w. stride $S = 1$, padding $P = 1$
- Output feature map:
  - Spatial size: $\mathrm{W}_2 = \mathrm{H}_2 = \mathrm{N}_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(5 + 2 - 3) + 1 = 5$ (same as input)
  - Depth: $D_2 = K = 1$
- Output volume: $5 \times 5 \times 1$





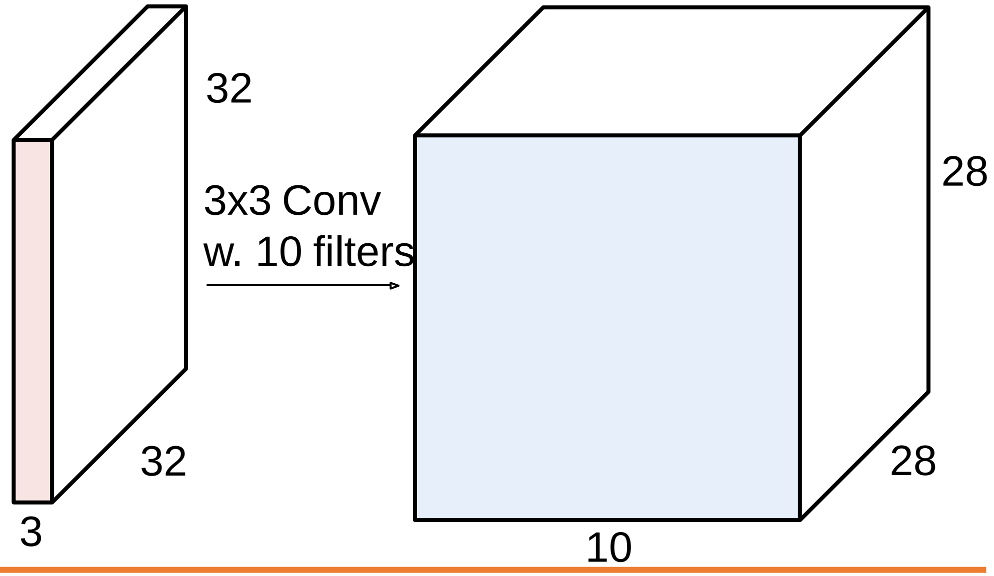
CONV Example 3: Stride $S = 2$

- Input volume: $5 \times 5 \times 3$
- Two $3 \times 3 \times 3$ filters ($K = 2, F = 3$) w. stride $S = 2$, padding $P = 1$
- Output volumes: Two $3 \times 3 \times 1$ (since $\frac{1}{2}(5 + 2 * 1 - 3) + 1 = 3$)
  - Convolution Demo: https://cs231n.github.io/convolutional-networks/

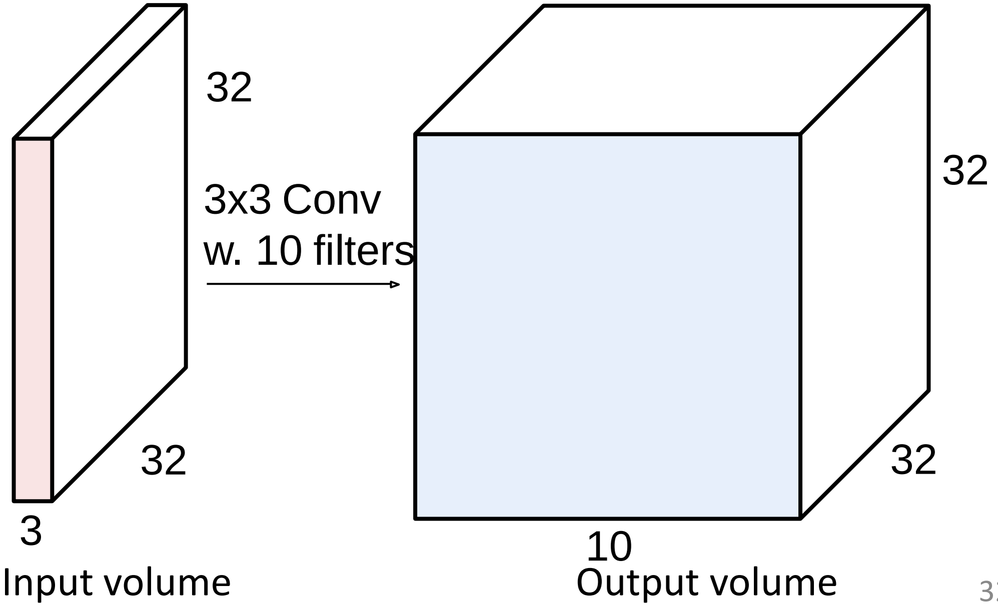


CONV Example 4: No Padding vs. Same Padding

- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$)
- 10 $5 \times 5 \times 3$ filters ($K = 10, F = 5$) w. stride $S = 1$, no padding $P = 0$
- Each output feature map:
  - Spatial size: $\mathrm{W}_2 = \mathrm{H}_2 = \mathrm{N}_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(32 - 5) + 1 = 28$
  - Depth: $D_2 = K = 10$
- Output volume: $28 \times 28 \times 10$
- No. params (incl. weights and Bias terms) in this layer: each filter has $5 * 5 * 3 + 1 = 76$ params, so 10 filters add up to $76 * 10 = 760$ params

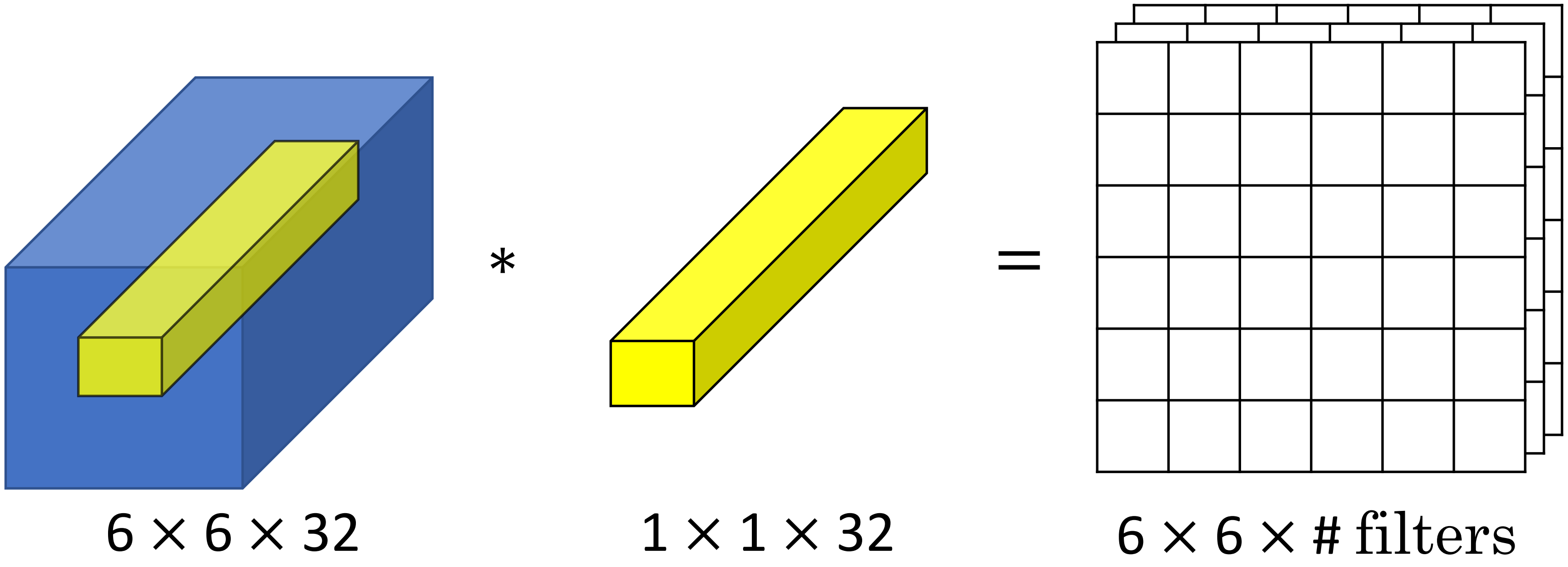


- Input volume: $32 \times 32 \times 3$ ($W_1 = H_1 = N_1 = 32, D_1 = 3$)
- 10 $5 \times 5 \times 3$ filters ($K = 10, F = 5$) w. stride $S = 1$, padding $P = 2$
- Each feature map:
  - Spatial size: $\mathrm{W}_2 = \mathrm{H}_2 = \mathrm{N}_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(32 + 2 * 2 - 5) + 1 = 32$
  - Depth: $D_2 = K = 10$
- Output volume: $32 \times 32 \times 10$
- No. params: same as above



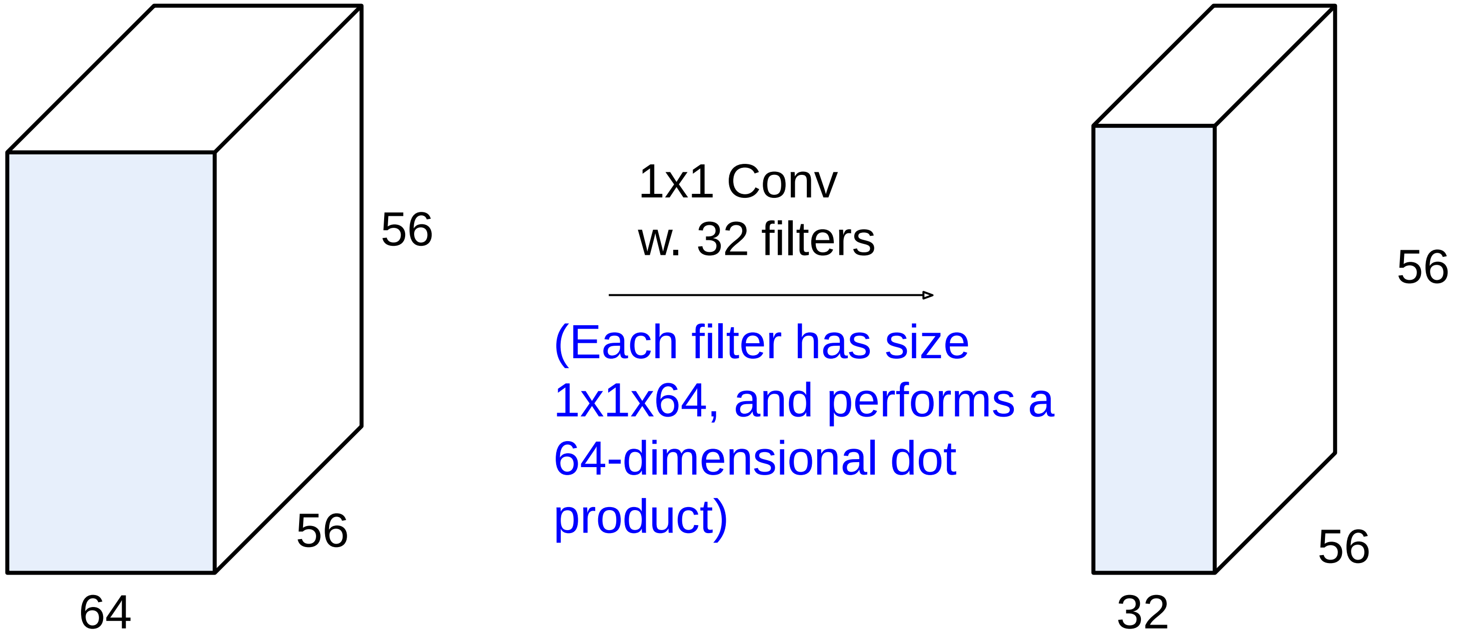
Pointwise Convolution with 1×1 Filter

- A 1×1 filter performs "mixing" of the input channels, then applies a non-linear activation function
- Can be used to reduce the number of channels (volume depth); the non-linear activation function also helps increase model capacity



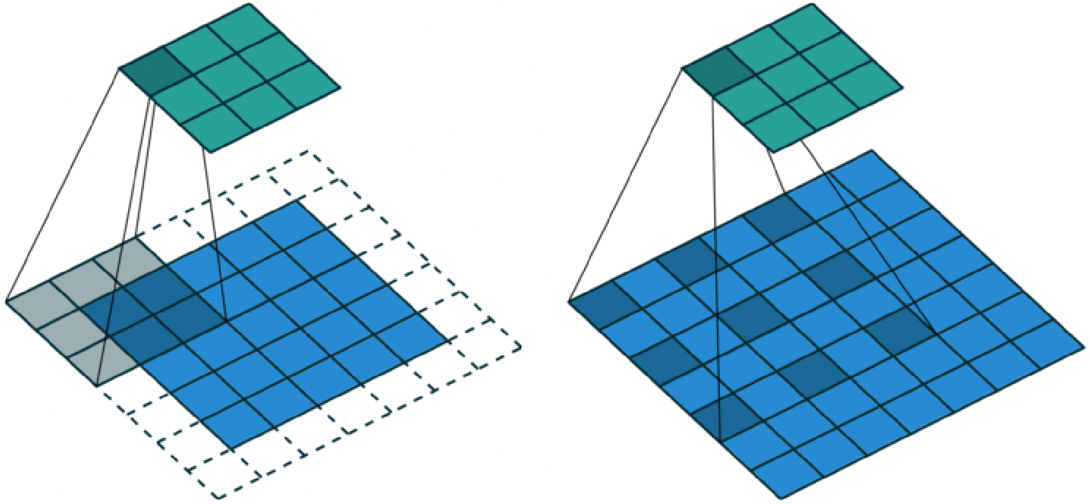
1×1 Filter Example

- Input volume: $56 \times 56 \times 64$ $(W_1 = H_1 = N_1 = 56, D_1 = 64)$
- 32 $1 \times 1 \times 64$ filters $(K = 32, F = 1)$ w. stride $S = 1$, no padding
- Each feature map:
  - Spatial size: $\mathrm{W}_2 = \mathrm{H}_2 = \mathrm{N}_2 = \frac{1}{S}(N_1 + 2P - F) + 1 = \frac{1}{1}(56 - 1) + 1 = 56$
  - Depth: $D_2 = K = 32$
- Output volume:$56 \times 56 \times 32$
- No. params: each filter has $1 * 1 * 64 + 1 = 65$ params, so 32 filters add up to $65 * 32 = 2080$ params



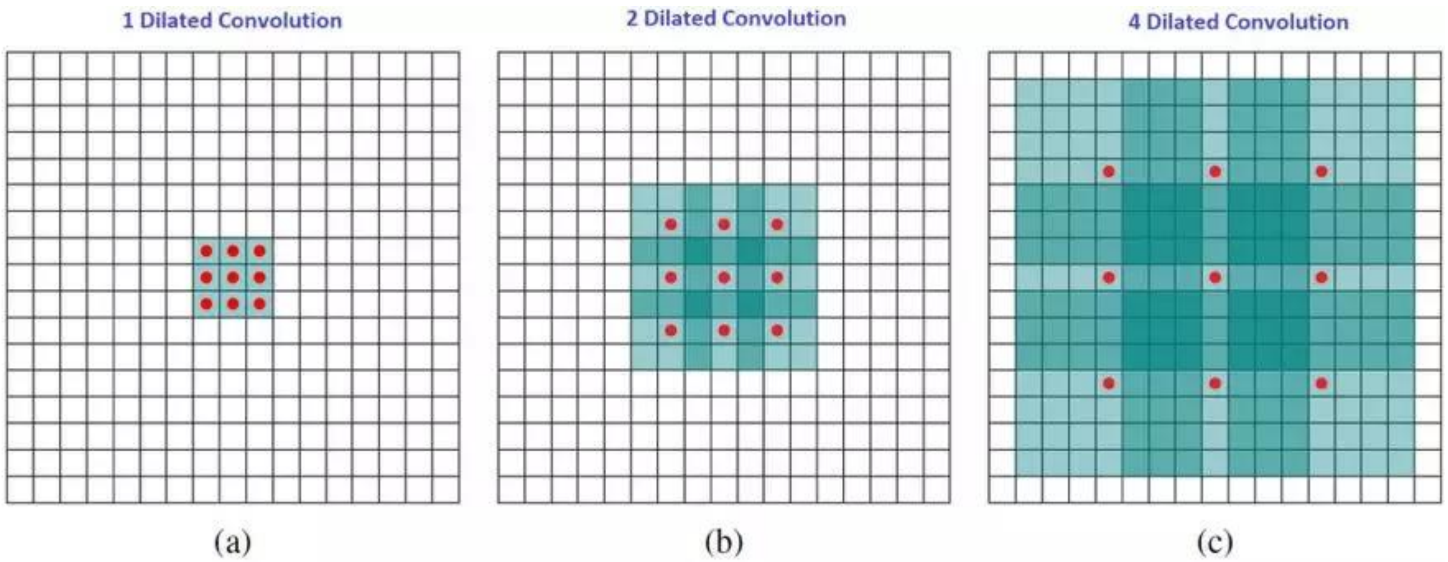
Dilated Convolution

- Insert 0s between input elements to increase receptive field size without increasing # params



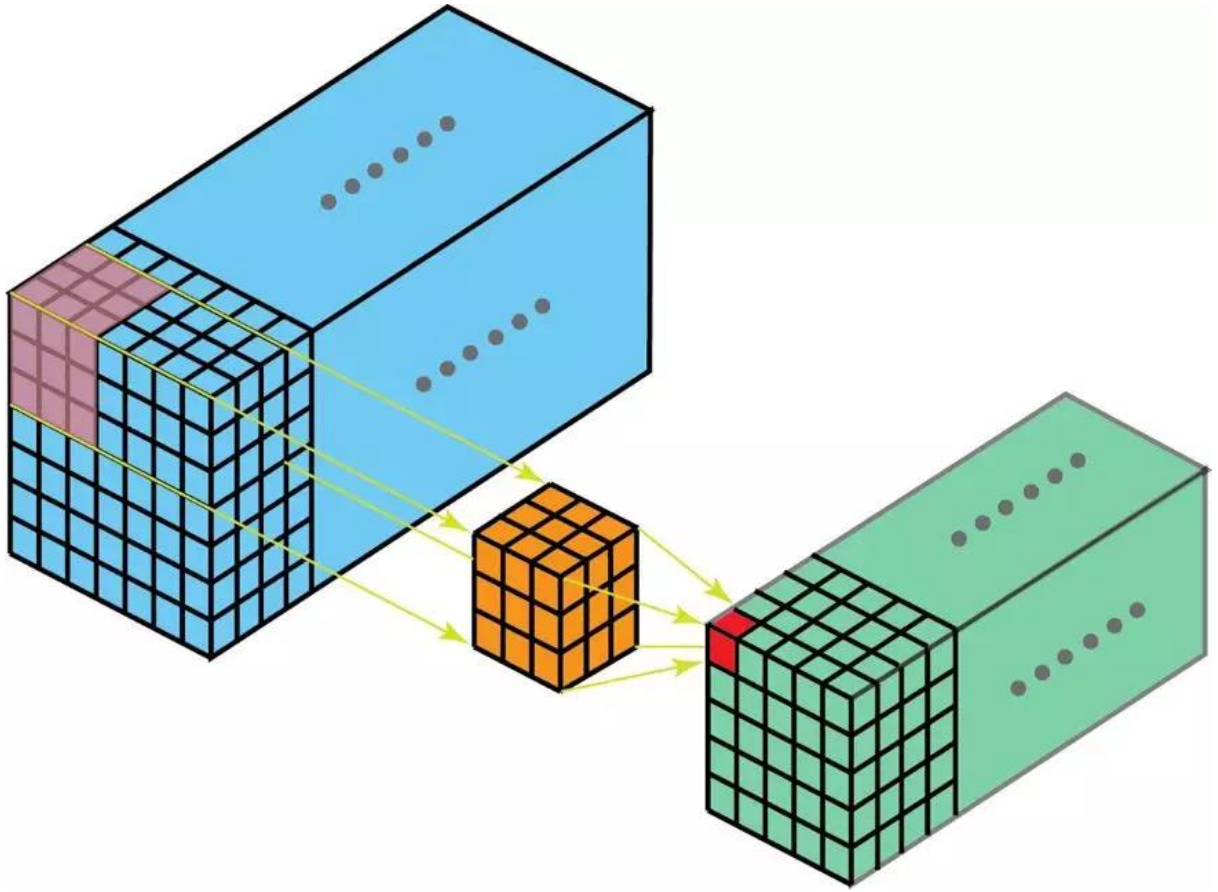
Regular convolution (1-dilated)

2-dilated convolution



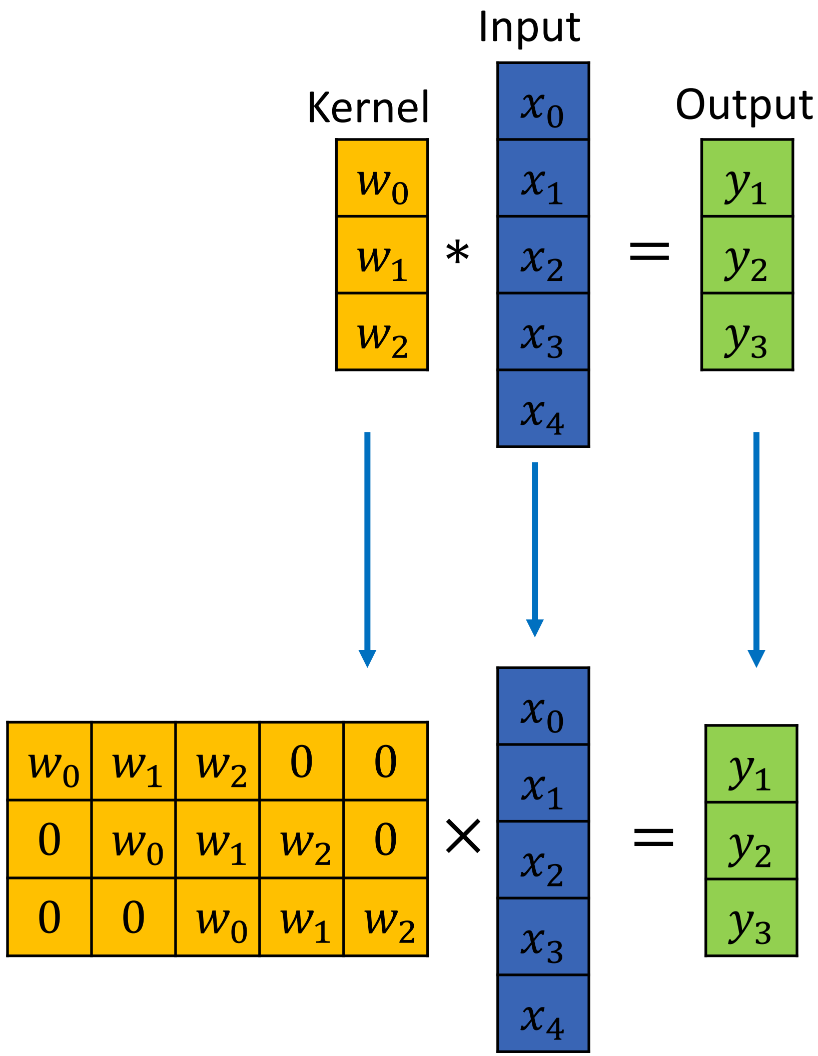
3D Convolution

- 3D filter slides along all 3 axes (width, height, depth). Very computation intensive
- Useful for 3D images such as medical CT/MRI images, or Point Clouds from Lidar



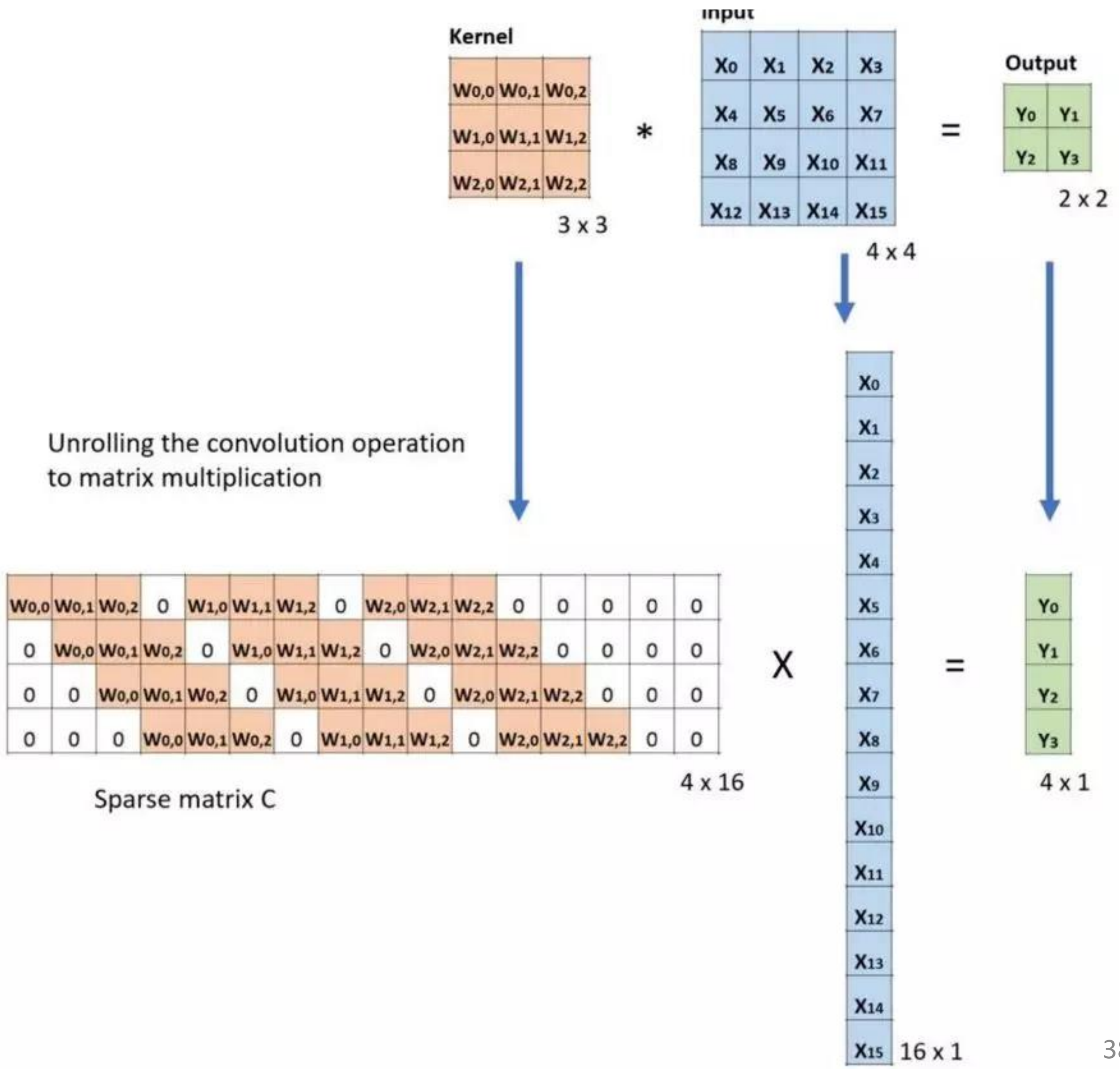
Converting Convolution to Matrix Multiplication: 1D CONV Example

- Since parallel hardware (GPU, FPGA…) can handle matrix multiplication efficiently, this conversion increases computation efficiency at the expense of increased memory size for storing the weights (bias terms are not shown in fig)



Converting Convolution to Matrix Multiplication: 2D CONV Example

- An Illustrated Explanation of Performing 2D Convolutions Using Matrix Multiplications https://medium.com/@_init_/an-illustrated-explanation-of-performing-2d-convolutions-using-matrix-multiplications-1e8de8cd2544

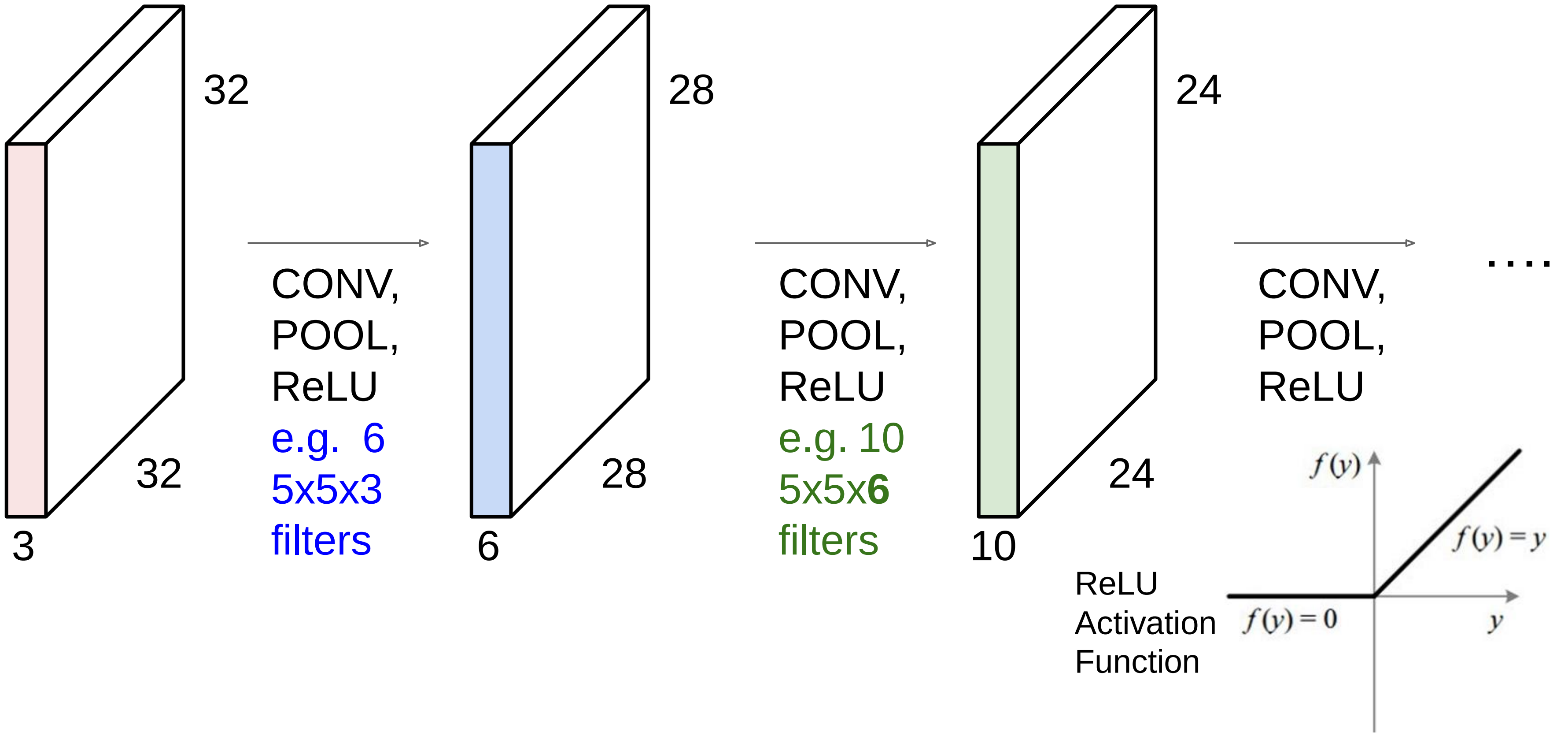


Outline

- Convolution layers
- Pooling and Fully-Connected layers
- Well-known CNN architectures

Typical CNN Architecture

- Multiple layers, each consisting of CONV, POOL and non-linear activation functions (e.g., ReLU), are stacked into a deep network
  - Many variants possible, e.g., multiple CONV layers can be stacked without POOL and activation functions in-between



Important

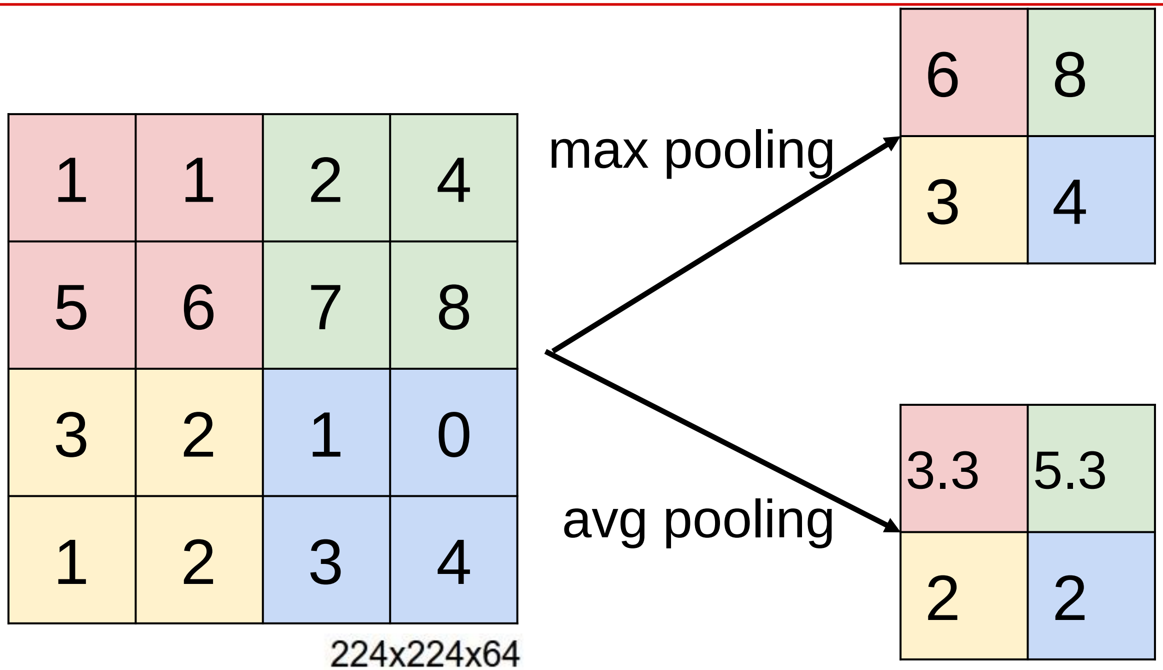
Pooling (Sub-Sampling) Layer

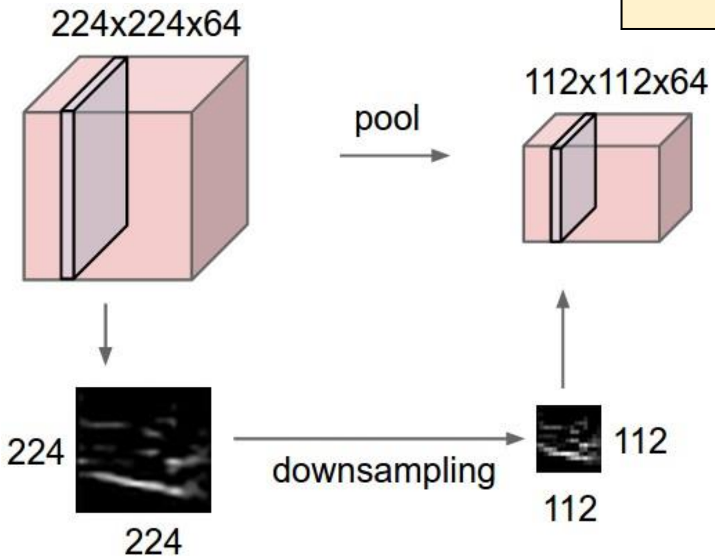
- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - their spatial extent F,
  - the stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- For Pooling layers, it is not common to pad the input using zero-padding.

- A pooling filter has depth 1, and operates over each feature map independently, hence the input volume and output volume have the same depth $D_1 = D_2$
  - In contrast, a CONV filter has the same depth D_1 as its input volume, and the number of filters K equals the depth D_2 of its output volume
  - Common settings: $F = 2, S = 2$, or $F = 3, S = 2$
- Example: pooling w. a 2×2 filter w. stride $S = 2$, no padding. Output volume: $\frac{W_1}{2} \times \frac{H_1}{2} \times \mathrm{D}_1$(since $\frac{1}{2}(W_1 - 2) + 1 = \frac{W_1}{2}, \frac{1}{2}(H_1 - 2) + 1 = \frac{H_1}{2}$)

Max Pooling w. Examples

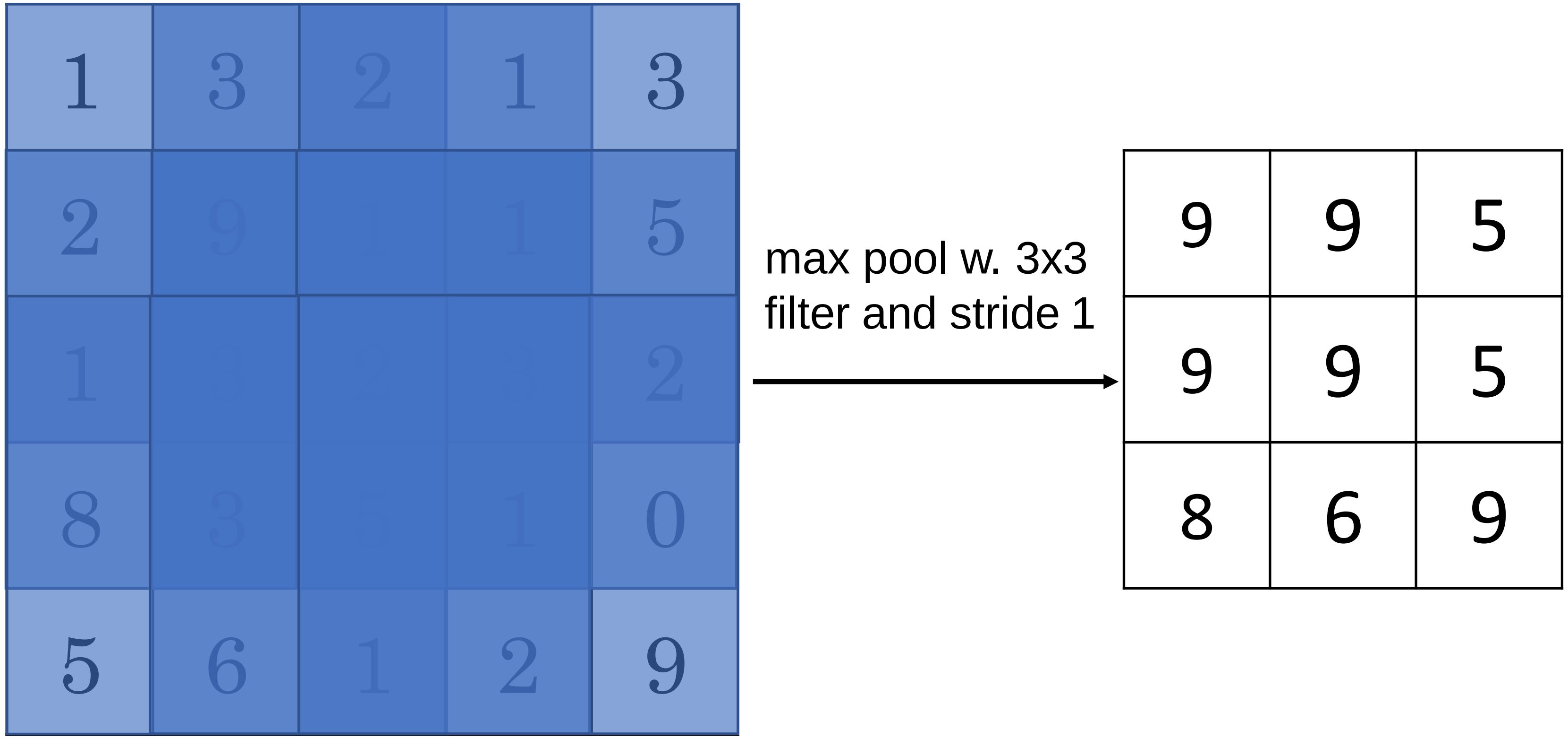
- Max pooling: take the max element among the $F * F$ elements in each $F \times F$ patch of each input feature map to reduce its dimension ($F = 2, S = 2$)
- Alternative: average pooling is less commonly used
- Pooling is also called subsampling or downsampling
  - Max pooling selects the brighter pixels from the image. It is useful when the background of the image is dark and we are interested in only the lighter pixels of the image.
  - Average pooling method smooths out the image and hence the sharp features may not be identified when this pooling method is used





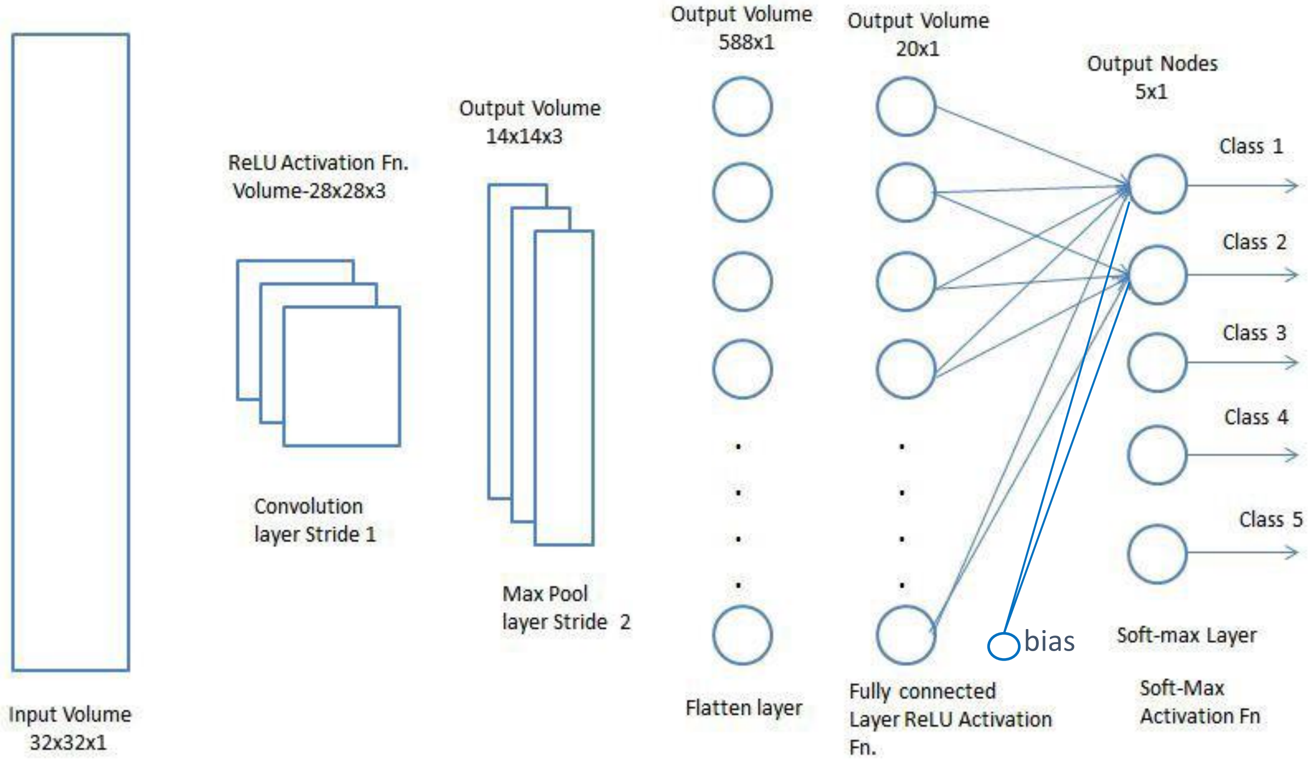
Overlapping Pooling

- Input volume: $N \times N \times \mathrm{D}_1$
- A 3×3 pooling filter w. stride $S = 1,$ no padding
- Output volume: $(N-2) \times (N-2) \times \mathrm{D}_1$(since $\frac{1}{1}(N-3)+1 = N-2$)
  - In practice, it is more common to have $F = 3, S = 2$ for overlapping pooling



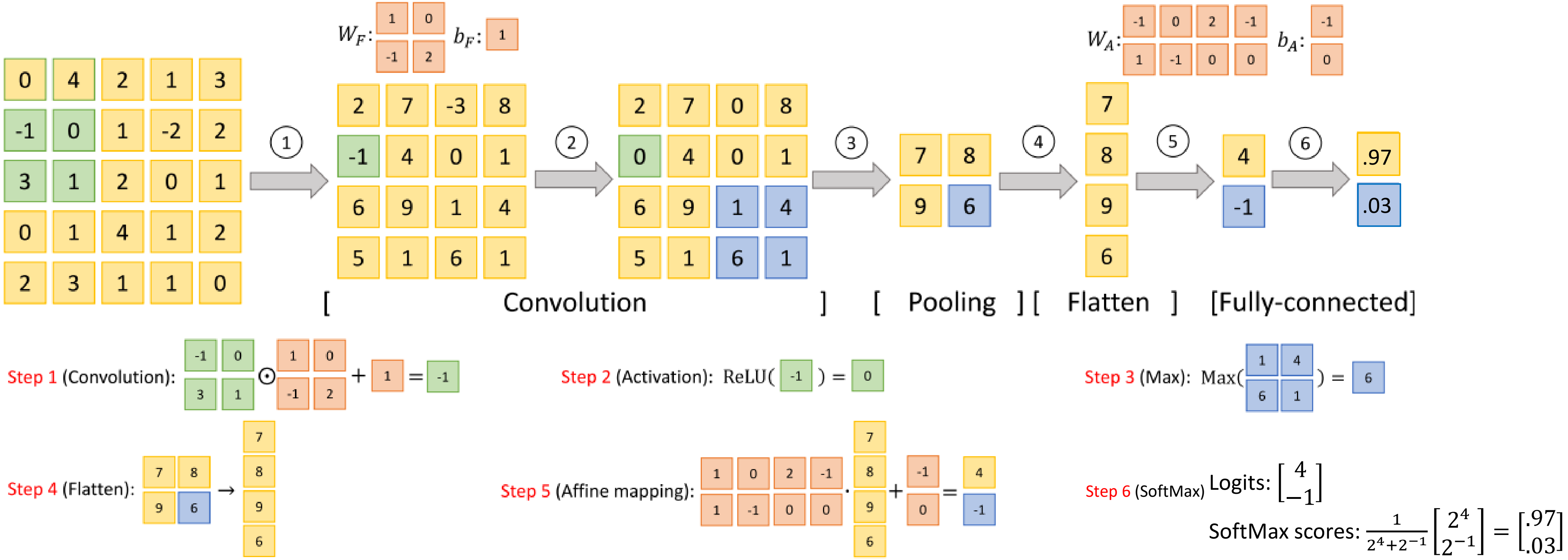
Fully-Connected (FC) Layer

- Each neuron connects to the entire input volume w. no weight sharing
  - No. params for FC layer of size N_{out} connected to input layer of size N_{in} is $(N_{in} + 1) * N_{out}$ ($= (20 + 1) * 5 = 105$ for the example)



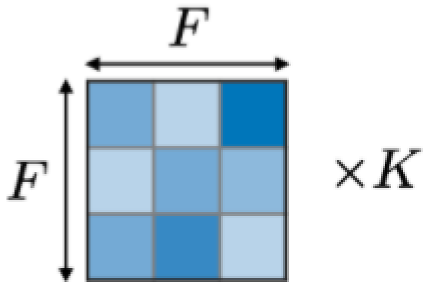
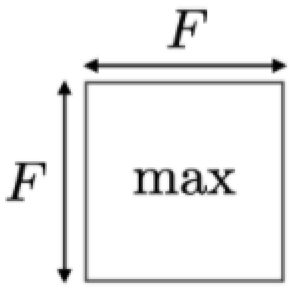
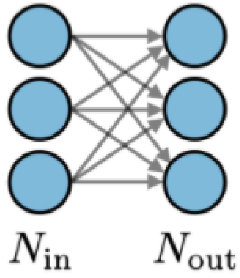
A Complete CNN Example

- A CNN with a CONV layer, a FC layer and a SoftMax layer



Huang C, Fan J, Chen X, et al. Divide and slide: Layer-wise refinement for output range analysis of deep neural networks[J]. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2020, 39(11): 3323-3335.

Summary of 3 Types of CNN Layers

	CONV	POOL	FC
			
Input volume	$W_1 \times H_1 \times D_1$	$W_1 \times H_1 \times D_1$	N_{in}
Output volume	$W_2 \times H_2 \times K$	$W_2 \times H_2 \times D_1$	N_{out}
No. params	$(F * F * D_1 + 1) * K$	0	$(N_{in} + 1) * N_{out}$
No. MULs	$(F * F * D_1 + 1) * K * W_2 * H_2$	0	$(N_{in} + 1) * N_{out}$

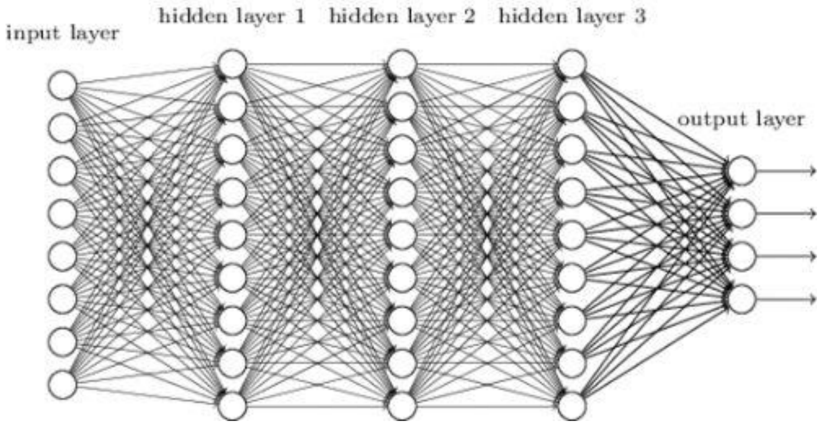
(1) No. MULs for CONV layer: $(F * F * D_1 + 1)$ MULs to compute each output element; $K * W_2 * H_2$ output elements
(2) Bias term of $+1$ is often omitted

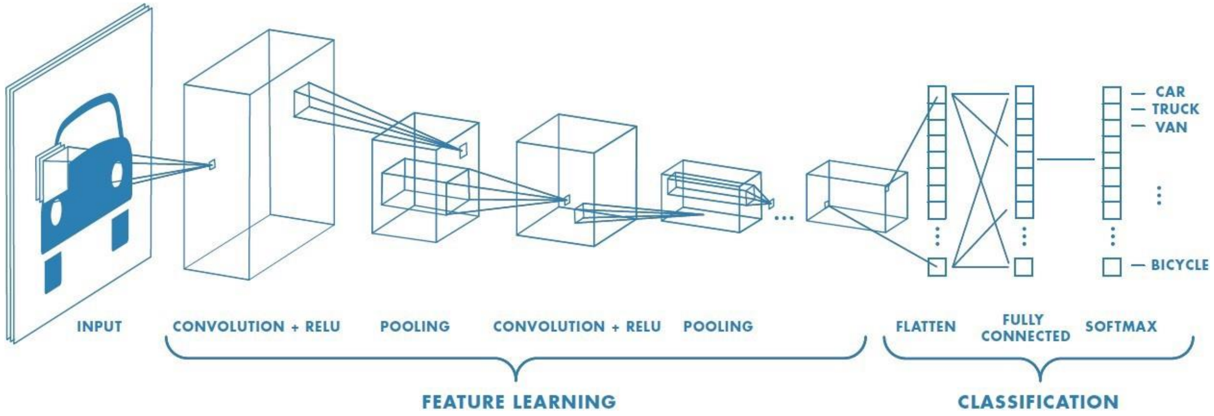
https://stanford.edu/~shervine/teaching/cs-230/cheatsheet-convolutional-neural-networks

MLP vs. CNN

- In a Multi-Layer Perceptron (MLP), all layers are FC layers
- Cannot alter input image size
- No translation invariance
- No. params can grow very large, prune to overfitting

- In a CNN, only the last few (typically <=3) layer(s) are FC
- CONV layers can handle images of arbitrary size
- Translation invariance
- Fewer params than MLP

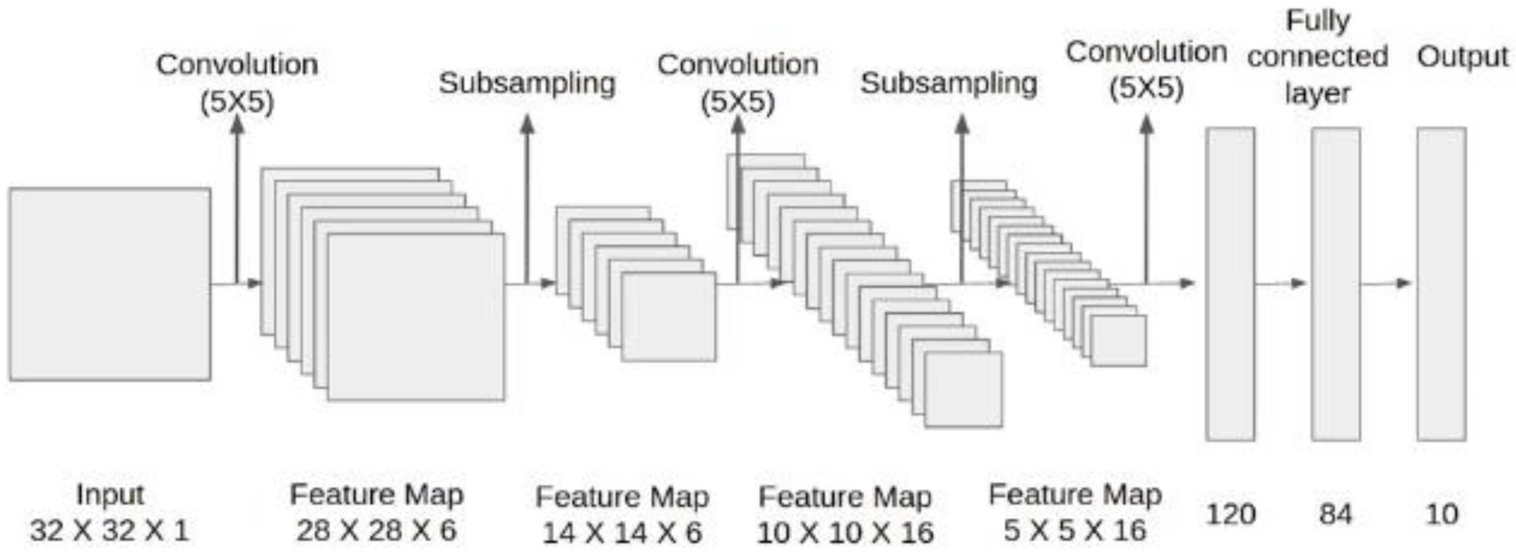




Outline

- Convolution layers
- Pooling and Fully-Connected layers
- Well-known CNN architectures

LeNet-5 (LeCun et al. 1989)



Layer	Input $W_1 \times H_1 \times D_1$	No. Filters	Filter $K \times K \times D/S$	Output $W_2 \times H_2 \times D_2$	No. params
C1:CONV	$32 \times 32 \times 1$	6	$5 \times 5 \times 1$	$28 \times 28 \times 6$	156
S2:POOL	$28 \times 28 \times 6$	6	$2 \times 2 \times 1/2$	$14 \times 14 \times 6$	0
C3:CONV	$14 \times 14 \times 6$	16	$5 \times 5 \times 6$	$10 \times 10 \times 16$	2416
S4:POOL	$10 \times 10 \times 16$	16	$2 \times 2 \times 1/2$	$5 \times 5 \times 16$	0
C5:CONV	$5 \times 5 \times 16$	120	$5 \times 5 \times 16$	$1 \times 1 \times 120$	48120
F6	FC	-	$-$	84	10164
Output	FC			10	850

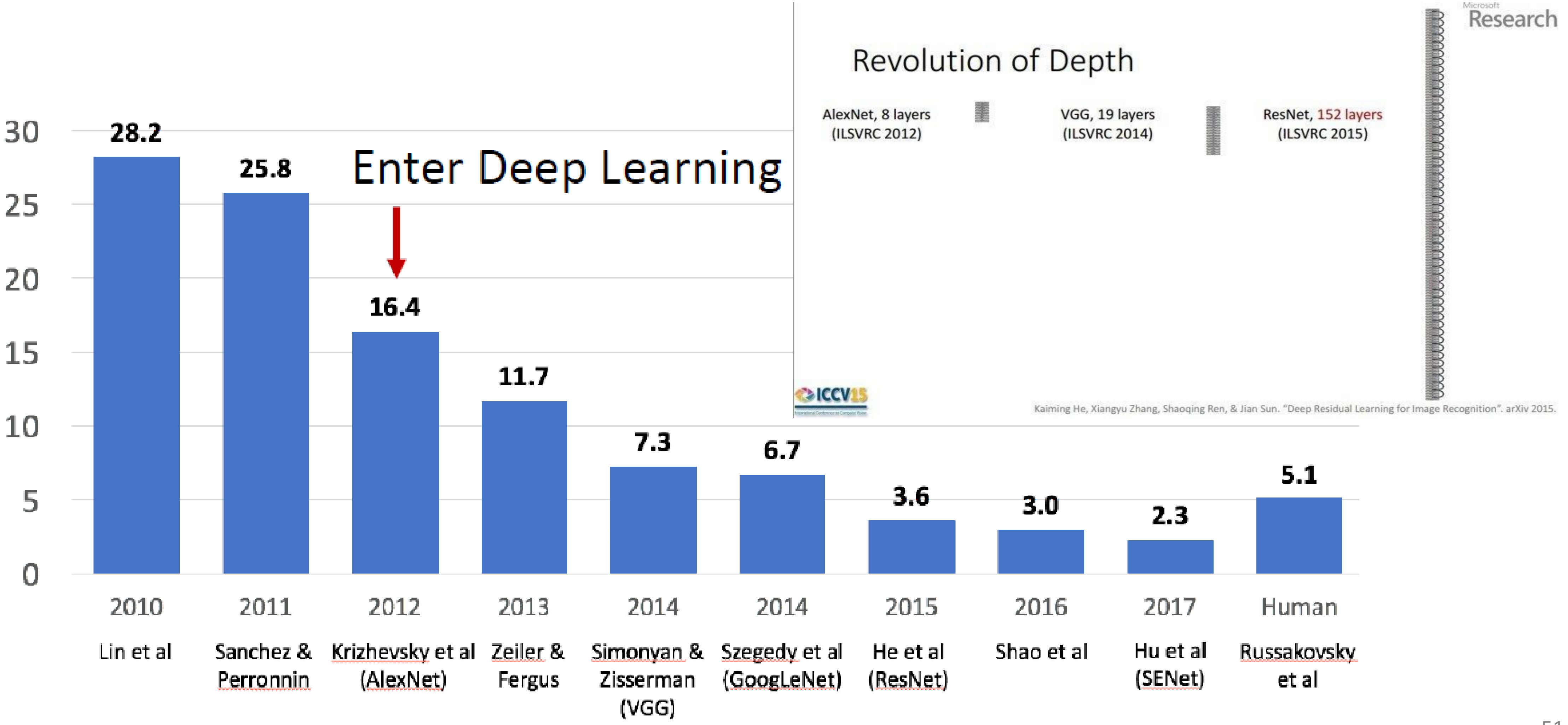
LeNet-5 Details

- Input image: $32 \times 32 \times 1$ (grey-scale images of hand-written digits w. size 32×32 pixels)
- Conv filters $5 \times 5 \times 1$ w. stride 1; Pooling filters 2×2 w. stride 2
- Conv layer C1 maps from input volume $32 \times 32 \times 1$ to 6 feature maps w. volume $28 \times 28 \times 6$ (since $\frac{1}{1}(32 - 5) + 1 = 28$). No params: $(5 * 5 * 1 + 1) * 6 = 156$
- Pooling layer S2 maps from input volume $28 \times 28 \times 6$ to 6 feature maps w. volume $14 \times 14 \times 6$ (since $\frac{1}{2}(28 - 2) + 1 = 14$).
- Conv layer C3 maps from input volume $14 \times 14 \times 6$ to 16 feature maps w. volume $10 \times 10 \times 16$ (since $\frac{1}{1}(14 - 5) + 1 = 10$). No params: $(5 * 5 * 6 + 1) * 16 = 2416$
- Pooling layer S4 maps from input volume $10 \times 10 \times 16$ to 16 feature maps w. volume $5 \times 5 \times 16$ (since $\frac{1}{2}(10 - 2) + 1 = 5$)
- Conv layer C5 maps from input volume $5 \times 5 \times 16$ to 120 feature maps w. volume $1 \times 1 \times 120$ (since $\frac{1}{1}(5 - 5) + 1 = 1$). No params: $(5 * 5 * 16 + 1) * 120 = 48120$
  - You can also view it as an equivalent Fully-Connected layer that maps from the flattened input of size 400×1 ($5 * 5 * 16 = 400$) to output of size 120×1. For details, refer to L4.2 “Turning FC layer into CONV Layers”
- FC layer F6 maps from input of size 120×1 to output of size 84×1. No params: $(120 + 1) * 84 = 10164$
- Output layer (SoftMax) maps from input of size 84×1 to output of size 10. No params: $(84 + 1) * 10 = 850$

https://www.analyticsvidhya.com/blog/2021/03/the-architecture-of-lenet-5/
https://towardsdatascience.com/understanding-lenet-a-detailed-walkthrough-17833d4bd155

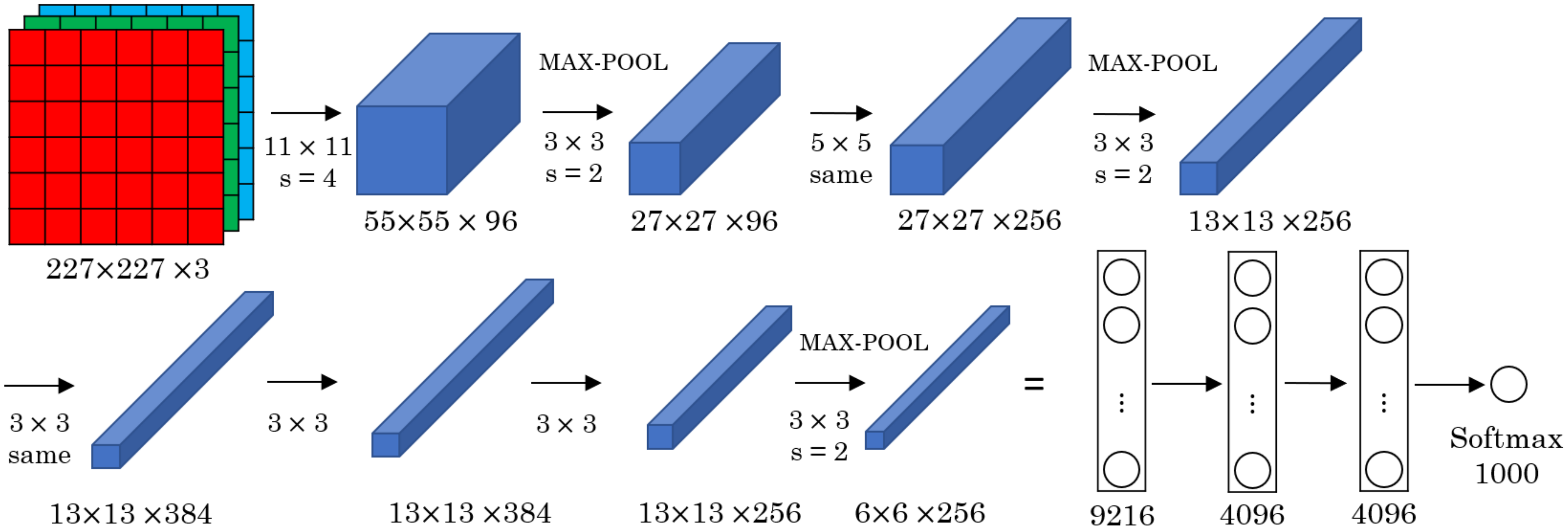
ImageNet Large Scale Visual Recognition Challenge

- 1,000 object classes, 1.4 M labeled images



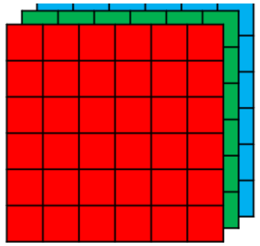
AlexNet [Krizhevsky et al. 2012]

- Input image: $227 \times 227 \times 3$
- 1st layer (CONV1): $96\ 11 \times 11$ filters w. stride $S = 4$, w. ReLU activation function
- Output volume: $55 \times 55 \times 96$ (since $\frac{1}{4}(227 - 11) + 1 = 55$).
- 2nd layer (POOL1): 3×3 filters w. stride $S = 2$ (overlapping)
- Output volume: $27 \times 27 \times 96$ (since $\frac{1}{2}(55 - 3) + 1 = 27$)
- …
- Total # params: 60M
- Introduced ReLU activation function



VGGNet [Simonyan 2014] (the best performing variant VGG-16)

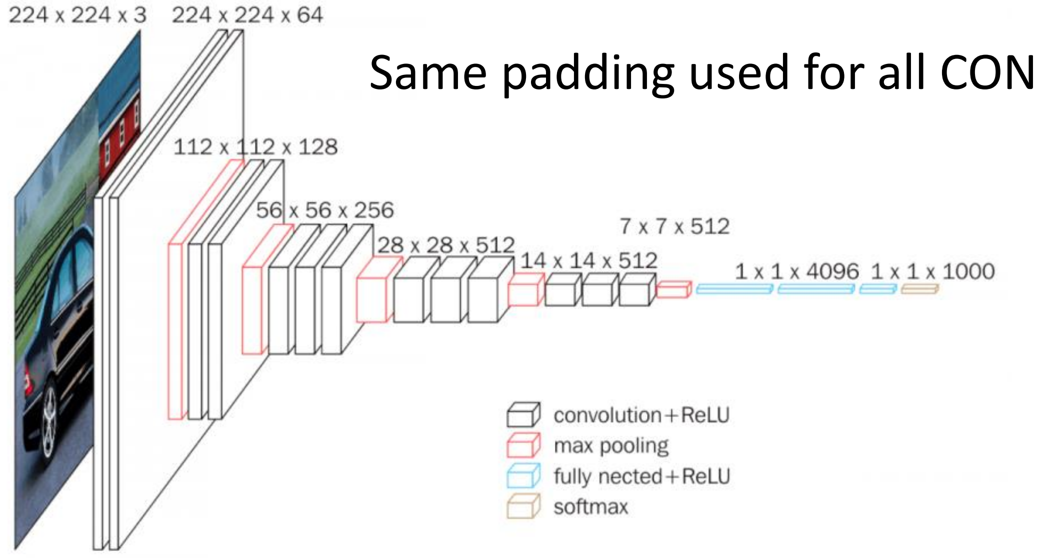
CONV = 3×3 filter. s = 1. same   MAX-POOL = 2×2 . s = 2



224×224 ×3 → [CONV 64] ×2 → 224×224×64 → POOL → 112×112 ×64 → [CONV 128] ×2 → 112×112 ×128 → POOL → 56×56 ×128

→ [CONV 256] ×3 → 56×56 ×256 → POOL → 28×28 ×256 → [CONV 512] × 3 → 28×28 ×512 → POOL → 14×14×512

→ [CONV 512] ×3 → 14×14 ×512 → POOL → 7×7×512 → FC 4096 → FC 4096 → Softmax 1000



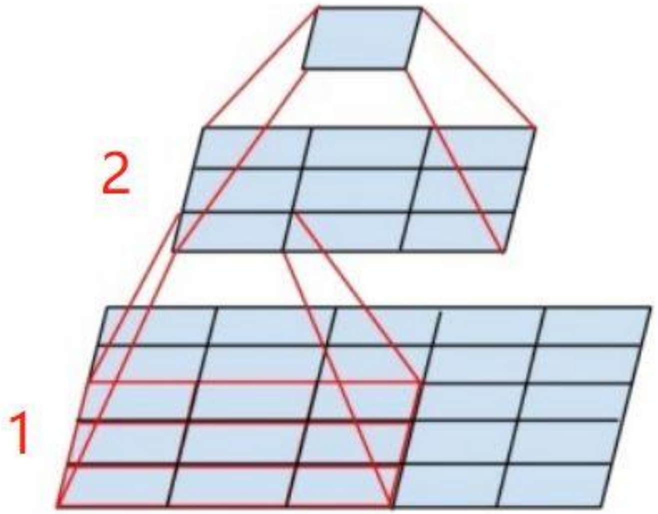
Same padding used for all CONV layers

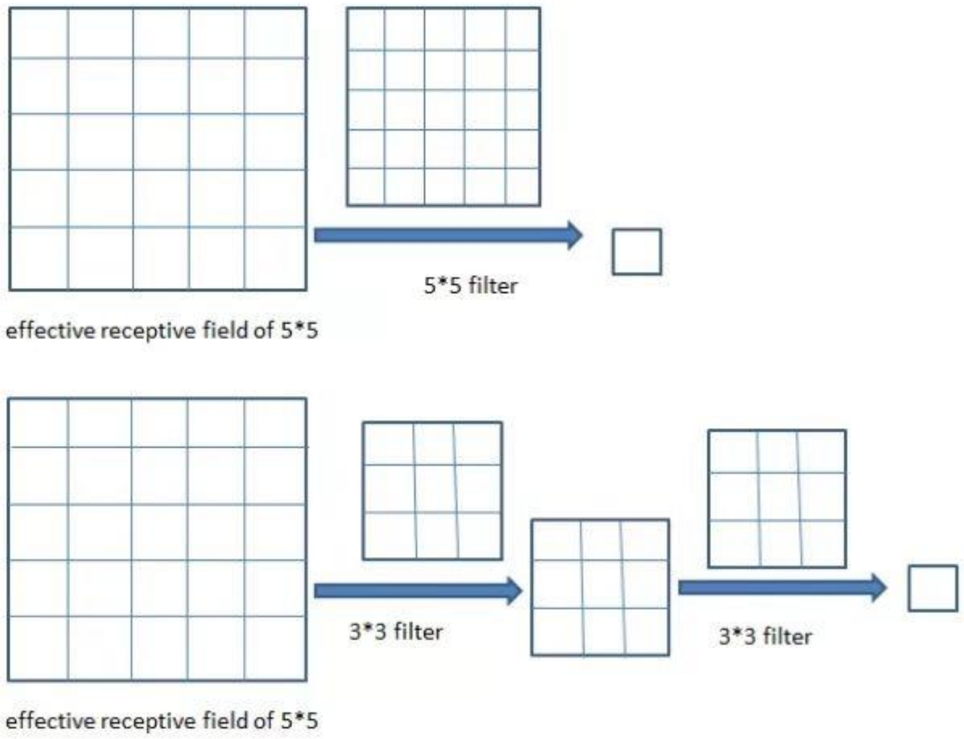
VGG-16 Details

- VGG-16 has 16 weight layers, not including POOL layers w. 0 weight
- Input image: $224 \times 224 \times 3$
- 1st and 2nd CONV layers: $64\ 3 \times 3$ filters w. stride $S = 1$, padding $P = 1$
  - Output volume: $224 \times 224 \times 64$ (since $\frac{1}{1}(224 + 2 * 1 - 3) + 1 = 224$)
- 3rd POOL layer: 2×2 filters w. stride $S = 2$
  - Output volume: $112 \times 112 \times 64$ (since $\frac{1}{2}(224 - 2) + 1 = 112$)
- 4th and 5th CONV layers: $128\ 3 \times 3$ filters w. stride $S = 1$, padding $P = 1$
  - Output volume: $112 \times 112 \times 128$ (since $\frac{1}{1}(112 + 2 * 1 - 3) + 1 = 112$)
- 6th POOL layer: 2×2 filters w. stride $S = 2$
  - Output volume: $56 \times 56 \times 128$ (since $\frac{1}{2}(112 - 2) + 1 = 56$)
- Total # params: 60M
- ImageNet top 5 error: 7.3%

Stacked 3×3 CONV Layers

- 2 stacked 3×3 CONV layers w. padding $P = 1$ have the same effective receptive field as a 5×5 CONV layer; 3 stacked 3×3 CONV layers w. padding $P = 1$ have RF of 7×7; L stacked 3×3 CONV layers w. padding $P = 1$ have RF of $1 + 2L$. Benefits:
  - Fewer params. Suppose all volumes have the same depth D, then a 7×7 CONV layer has $(7 * 7 * D + 1) * D \approx 49D^2$ params, while three stacked 3x3 CONV layers have only $(3 * 3 * D + 1) * D * 3) \approx 27D^2$ params
  - Two layers of non-linear activation functions increases CNN depth, hence larger model capacity





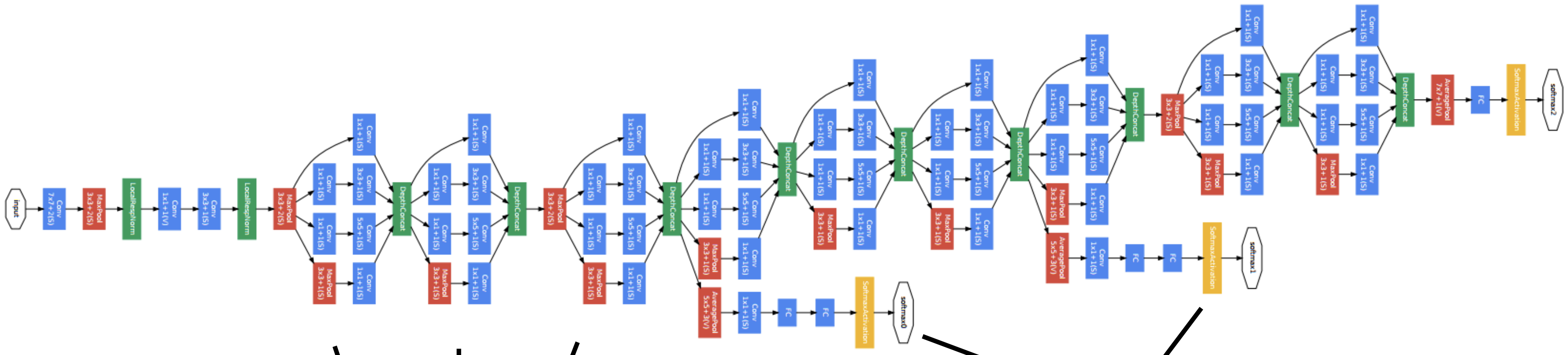
https://zhuanlan.zhihu.com/p/79258431

VGGNet Variants

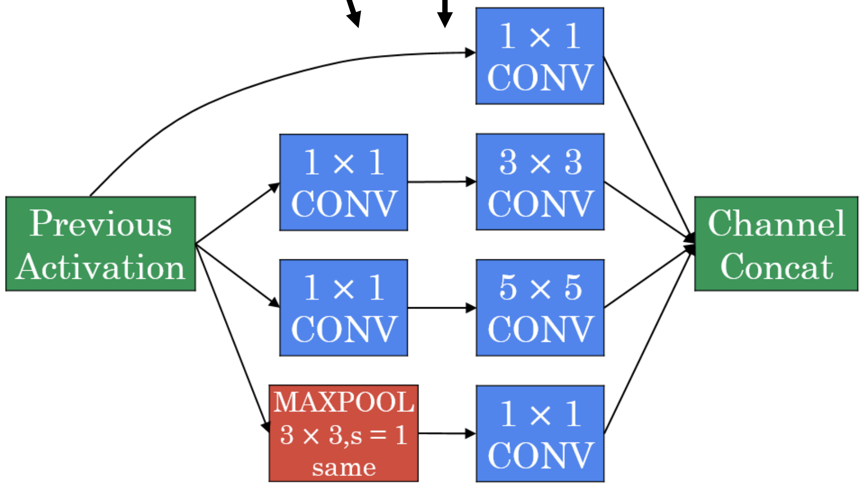
Best performing variant
VGG-16

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

GoogLeNet [Szegedy et al., 2014]



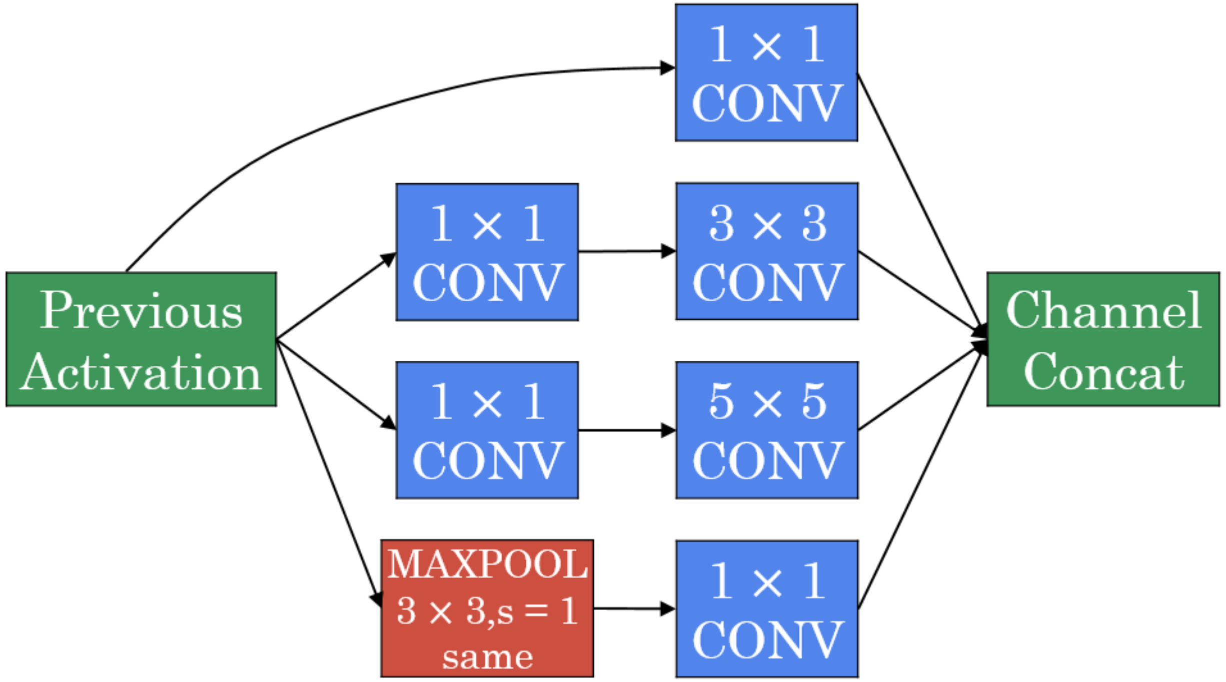
Additional classification heads for regularization

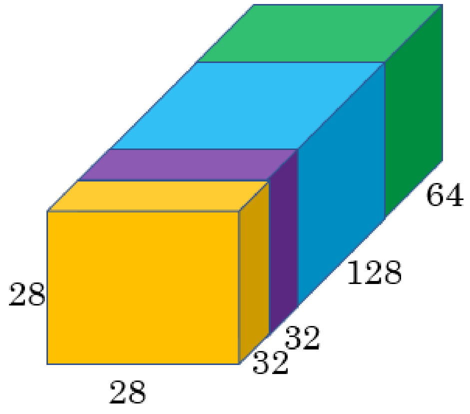


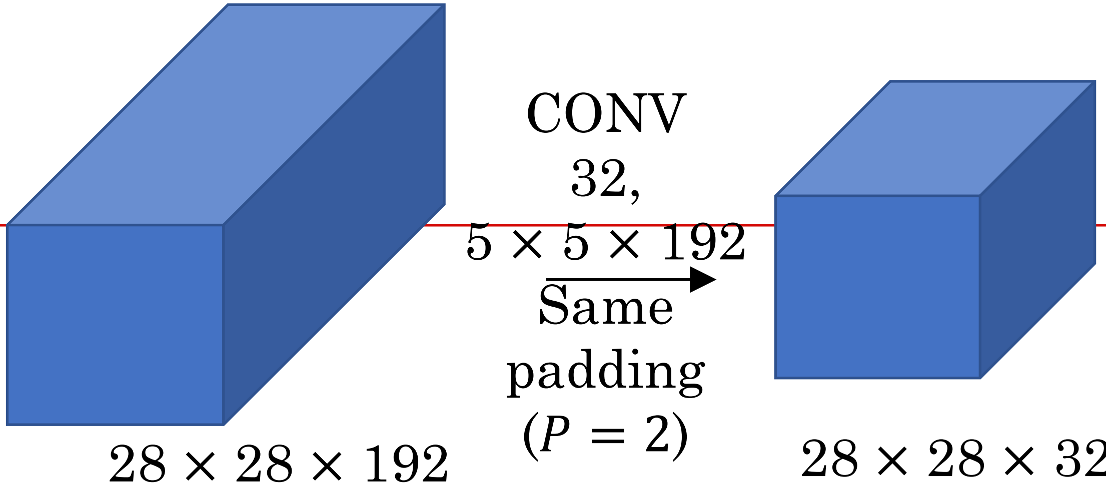
Inception Module

Inception Module

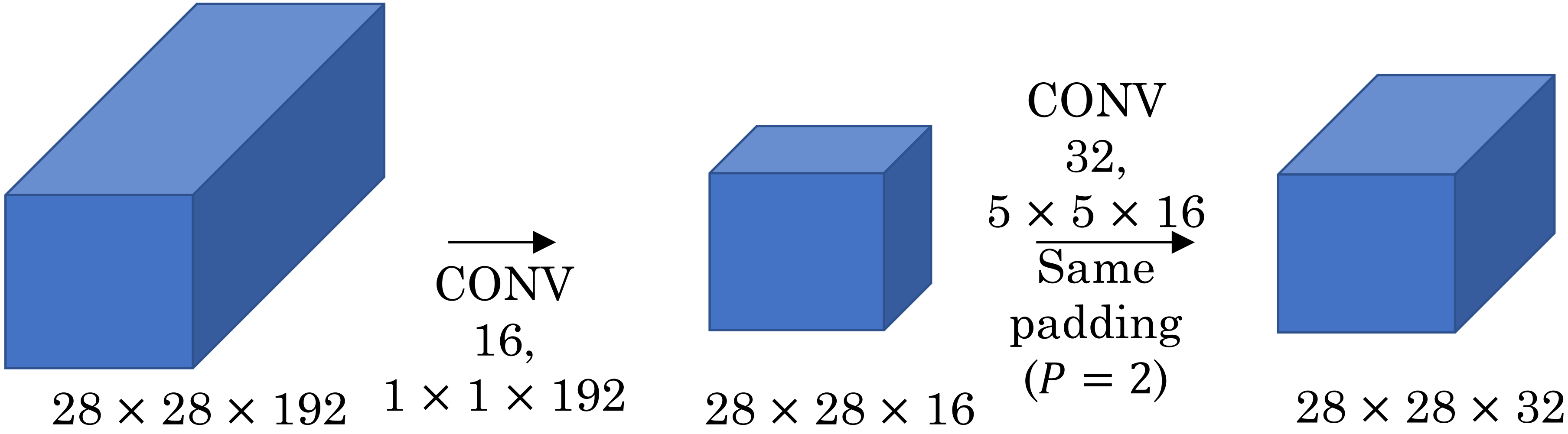
- Can’t make up your mind about filter size? Have them all in the Inception Module!
    - But this increases computation load
- Additional 1×1 CONV layers serve as bottleneck to reduce number of parameters and computation load







- Without the bottleneck layer: No. params: $5 * 5 * 192 * 32 = 153600$; No. MULs: $(5 * 5 * 192) * (32 * 28 * 28) = 120\text{M}$



- With the bottleneck layer: No. params: $1 * 1 * 192 * 16 + 5 * 5 * 16 * 32 = 15872$; No. MULs: $(1 * 1 * 192) * (16 * 28 * 28) + (5 * 5 * 16) * (32 * 28 * 28) = 12.4\text{M}$

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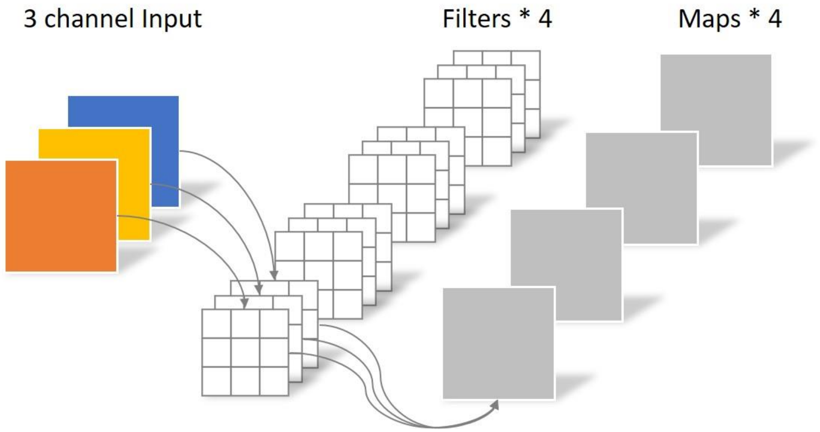
GoogLeNet Size

- Compared to AlexNet:
  - 12x less params (only 5M, due to no FC layers), 2x more compute (due to more CONV layers)

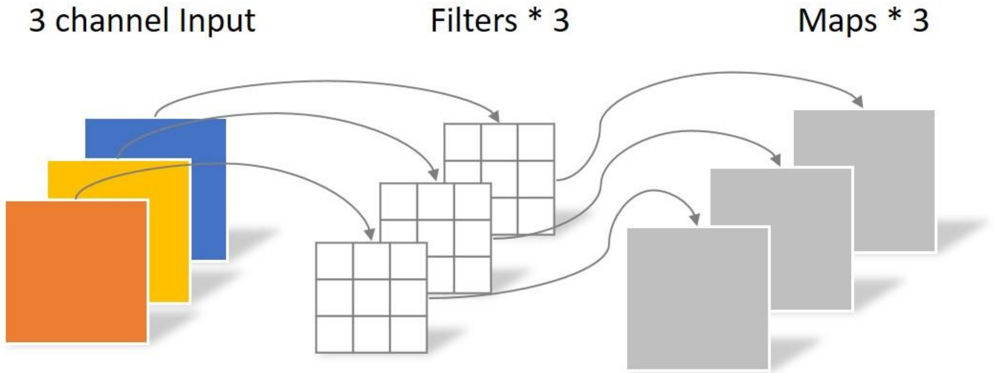
type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Xception [Chollet 2017] MobileNets [Howard et al. 2017] : Depthwise Separable Convolution

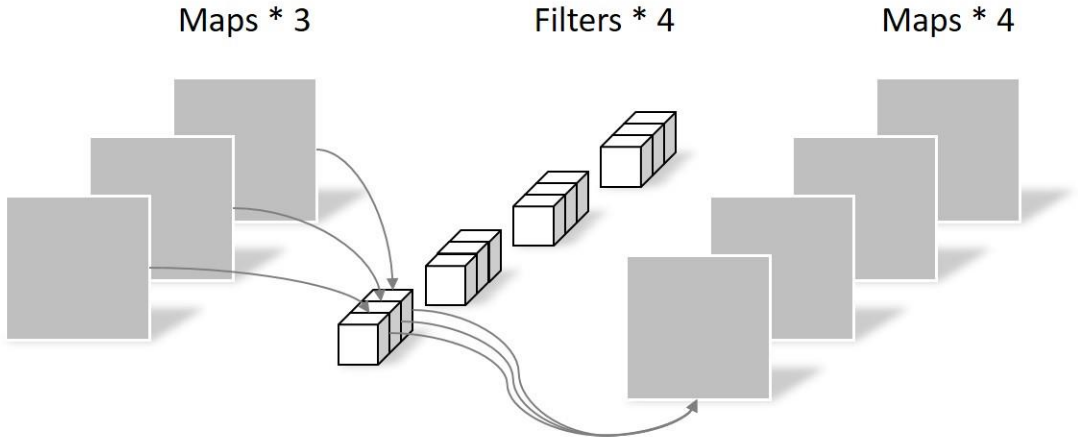


Each filter is convolved with all input channels

Regular Convolution



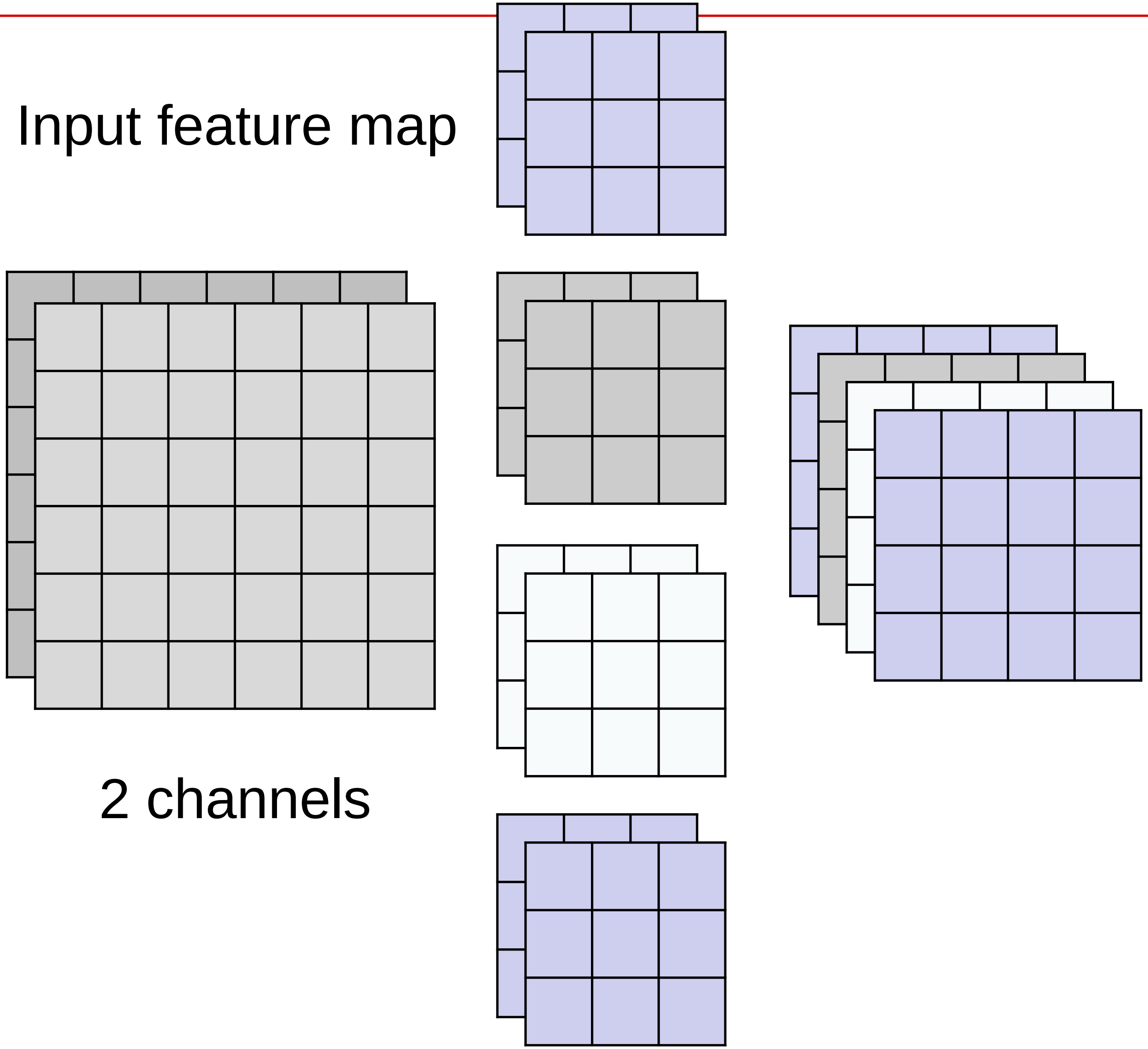
Each filter is convolved with one input channel



Followed by pointwise convolution

- Intermediate feature maps serve as bottleneck to reduce number of parameters and computation load
    - A Basic Introduction to Separable Convolutions https://towardsdatascience.com/a-basic-introduction-to-separable-convolutions-b99ec3102728
    - Depthwise Separable Convolution - A FASTER CONVOLUTION! https://www.youtube.com/watch?v=T7o3xvJLuHk

Example: Regular Convolution

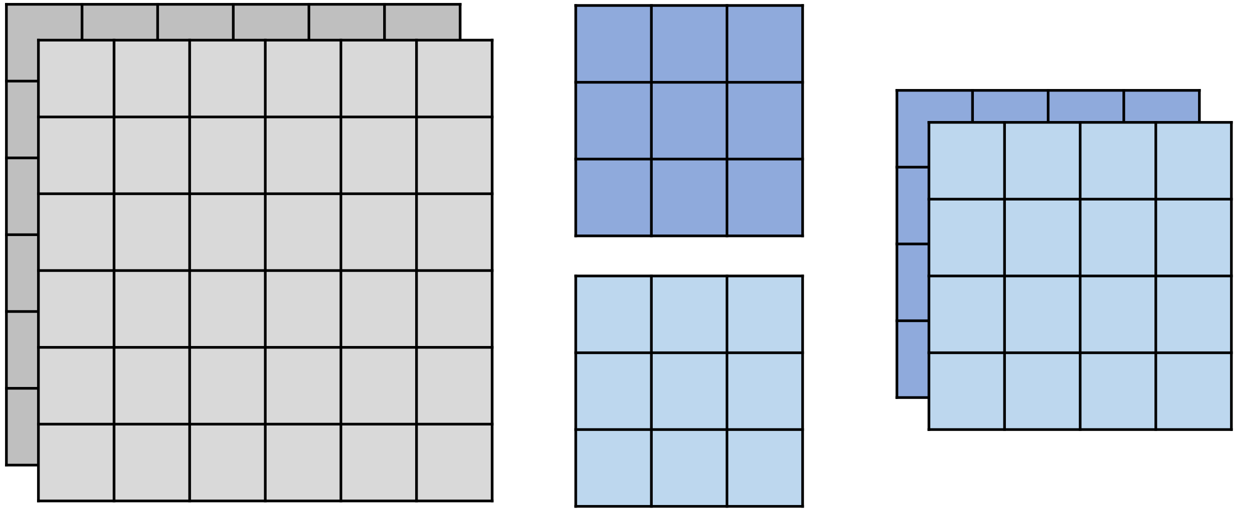


No. params: $3 * 3 * 2 * 4 = 72$ (Four $3 \times 3 \times 2$ filters) (not counting Bias terms)

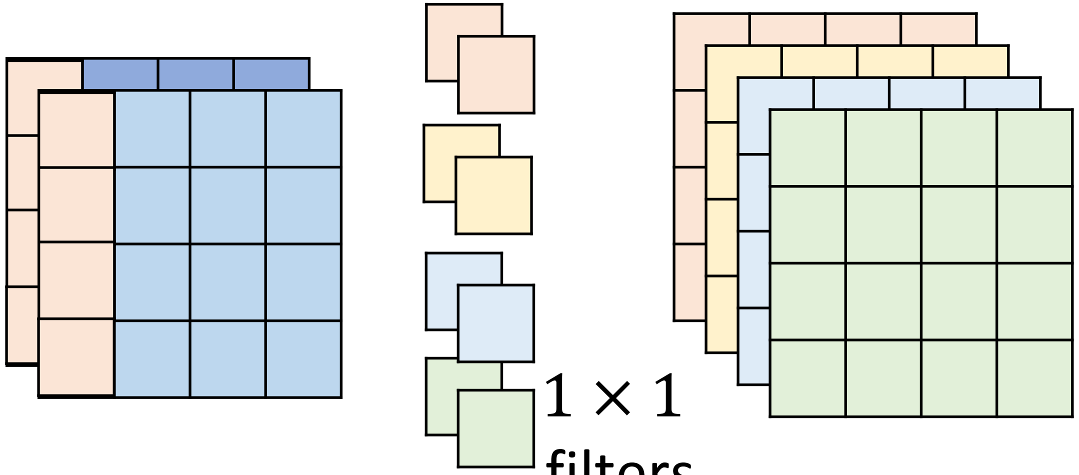
No. MULs: $(3 * 3 * 2) * (4 * 4 * 4) = 1152$ $(3 * 3 * 2$ MULs to compute each output element; $4 * 4 * 4$ output elements)

Example: Depthwise Separable Convolution

1. Depthwise Convolution



2. Pointwise Convolution

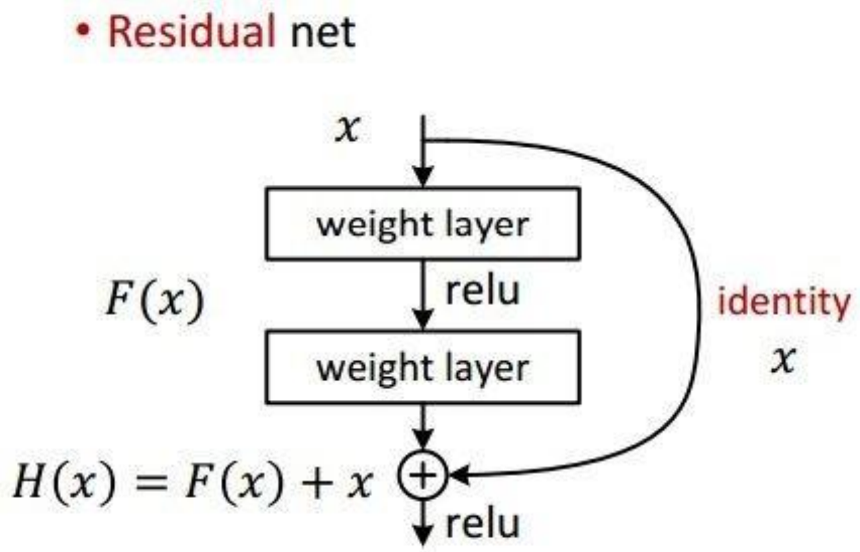


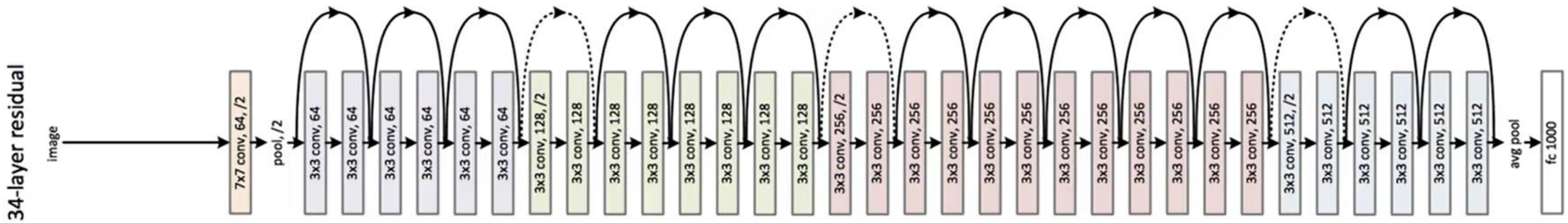
No. params: $3 * 3 * 2 + 2 * 4 = 26$ (Two $3 \times 3 \times 1$ filters and four $1 \times 1 \times 2$ filters) (not counting Bias terms)

No. MULs: $(3 * 3 * 1) * (2 * 4 * 4) + (1 * 1 * 2) * (4 * 4 * 4) = 416$ (Depthwise Conv: $3 * 3 * 1$ MULs to compute each output element; $2 * 4 * 4$ output elements; Pointwise Conv: $1 * 1 * 2$ MULs to compute each output element; $4 * 4 * 4$ output elements)

Residual Networks (ResNet) [He et al. 2015]

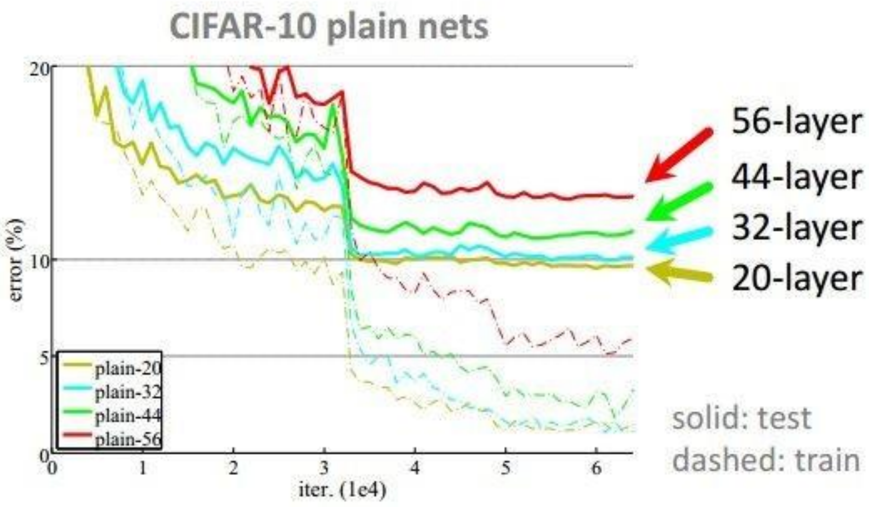
- In a standard NN, output from a given layer is $F(x)$
- In ResNet w. skip connection, output from a given layer is $H(x) = F(x) + x$
- Benefits:
  - Residual connections help in handling the vanishing gradient problem in very deep NNs
  - If identify mapping is close to optimal, then weights can be small to capture minor differences only, in other words, “unnecessary layers” can learn to be identity mapping. This allows stacking many layers (e.g., 152) without overfitting

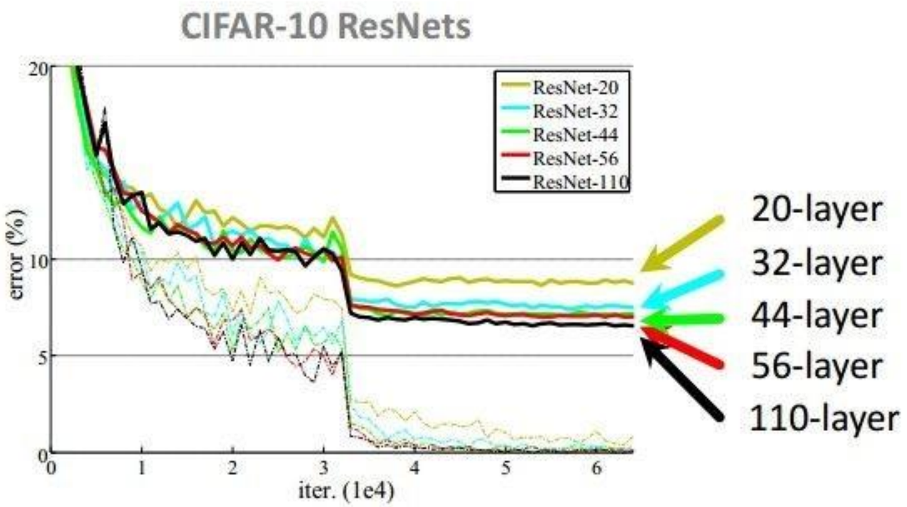




Deeper Nets have Better Performance

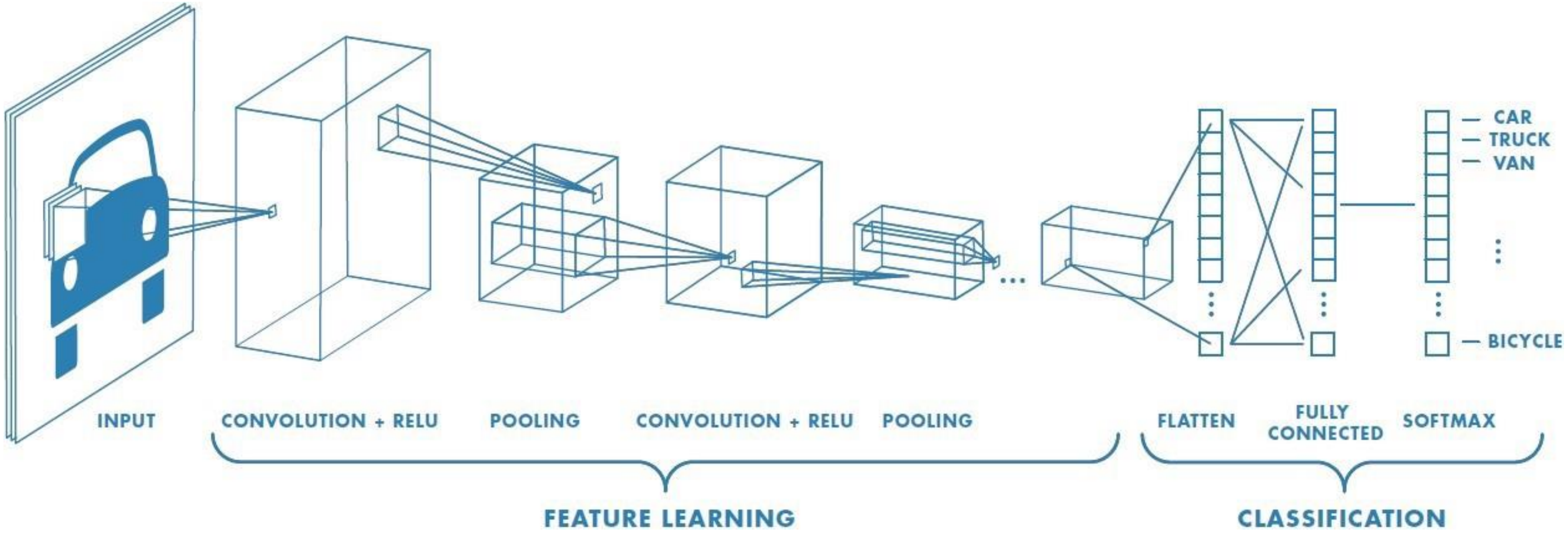
CIFAR-10 experiments





CNN Layer Patterns

- A typical CNN architecture looks like: `INPUT->[[CONV->RELU]*N->POOL?]*M->[FC->RELU]*K->FC`
  - where `*` indicates repetition, and `POOL?` indicates an optional pooling layer. $N \geq 0$ (usually $N \leq 3$), $M \geq 0$, $K \geq 0$ (and usually $K < 3$)
- Some common architectures:
  - `INPUT->FC`, implements a linear classifier. Here N = M = K = 0.
  - `INPUT->CONV->RELU->FC`
  - `INPUT->[CONV->RELU->POOL]*2->FC->RELU->FC` (fig below). There is a single CONV layer between every POOL layer.
  - `INPUT->[CONV->RELU->CONV->RELU->POOL]*3->[FC->RELU]*2->FC` There are two `CONV` layers stacked before every `POOL` layer, e.g., two stacked 3×3 CONV Layers. This is generally a good idea for larger and deeper networks, because multiple stacked `CONV` layers can develop more complex features of the input volume before the destructive pooling operation



Transfer Learning

- Instead of training your CNN from scratch, start from a pre-trained CNN (e.g., ResNet) and fine-tune it for your task
- First, replace SoftMax layer (classification head) with your own
- Next, train the CNN while keeping parameters frozen for
  - all CONV layers and only train the FC layer
  - or part of the earlier CONV layers close to the input layer (since earlier layers extract lower-level features that are more likely to be common among different tasks)
  - or none of the layers
  - The decision depends on how much training data you have, and how similar your task is to that of the pre-trained CNN