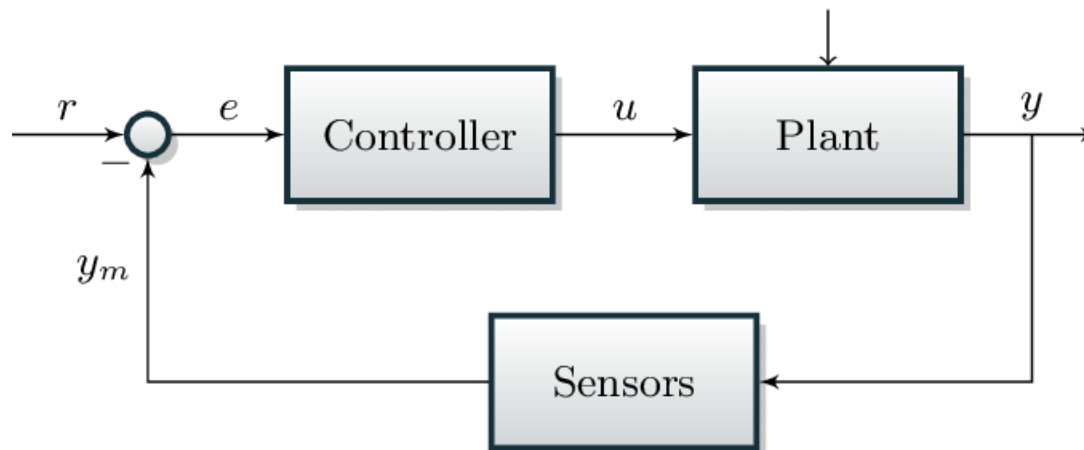


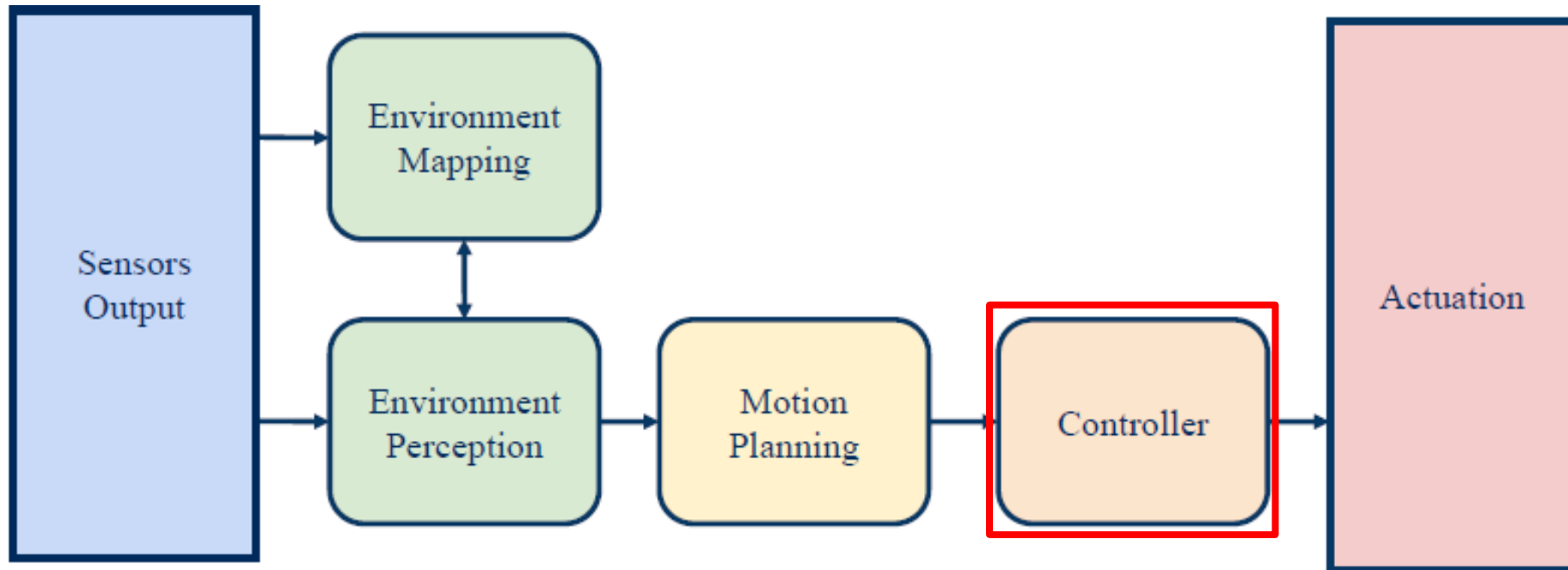
L6 Control

Zonghua Gu, 2021

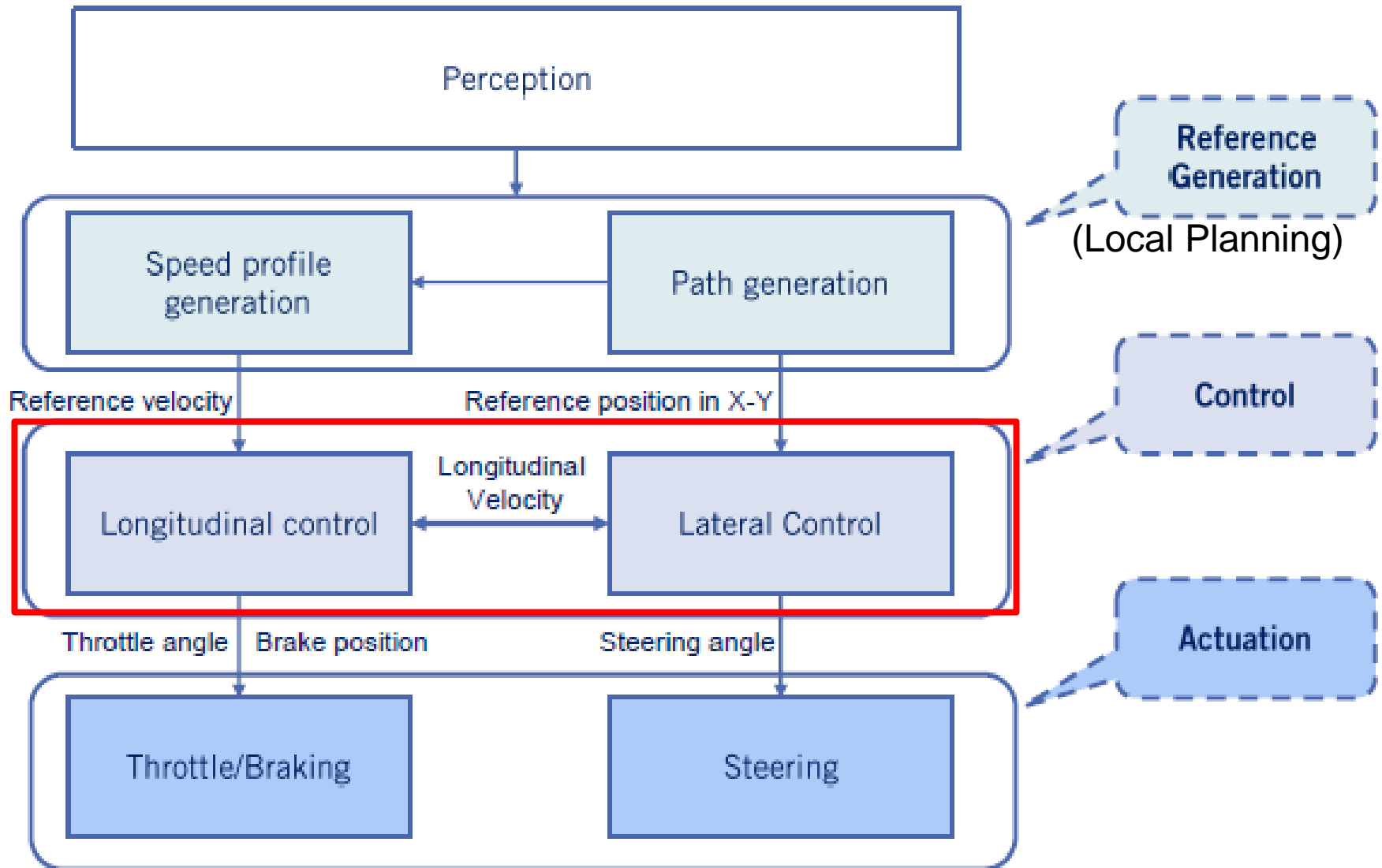


Controller Design

- We need to design a controller to follow the trajectory output from motion planning.
 - Many control algorithms. We mainly discuss PID and MPC, the two most widely-used control algorithms for AVs.
 - We only give the basic intuition and avoid mathematical details.

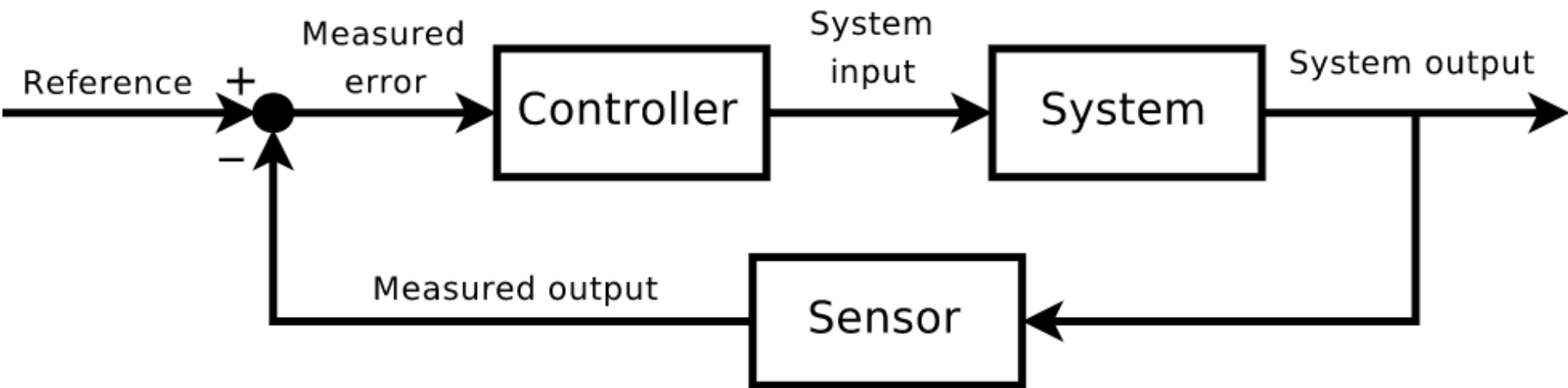


Vehicle Control Architecture



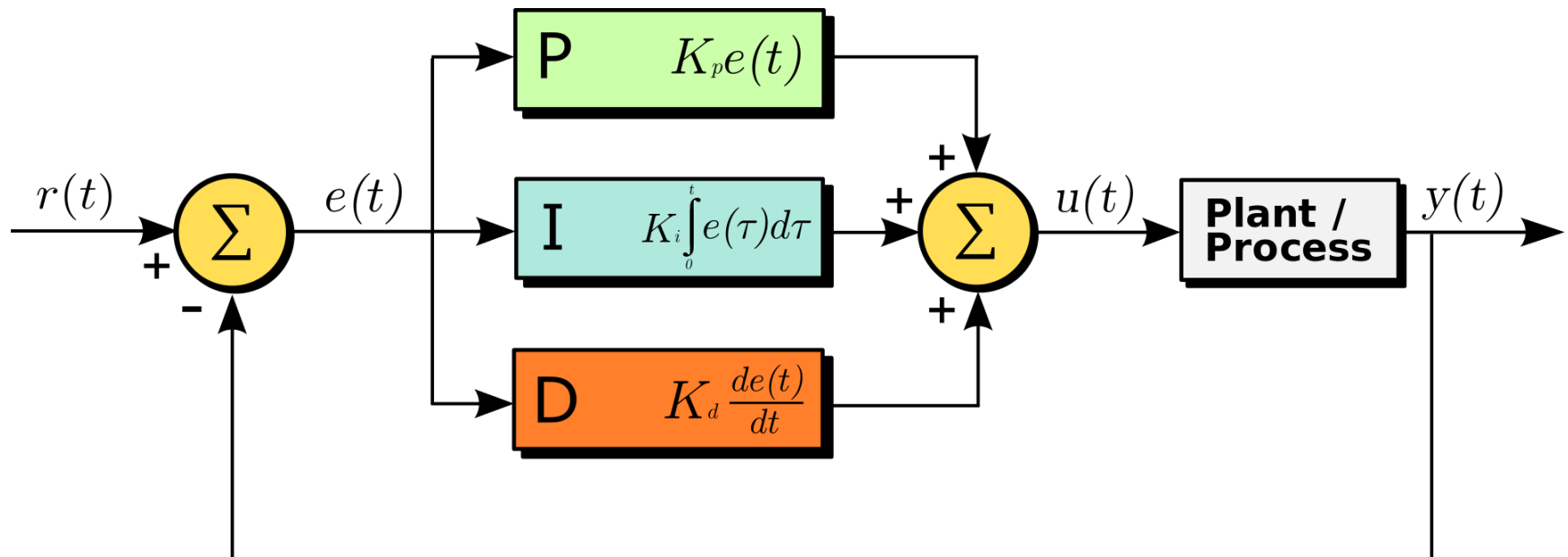
The Feedback Control Problem

- Given a system and a reference signal, find a control law such that the closed loop system is stable and follows the reference signal.



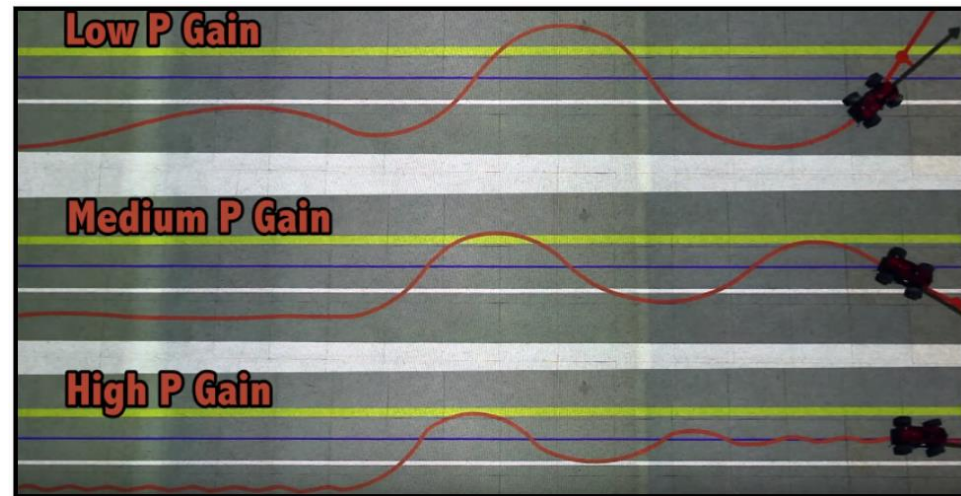
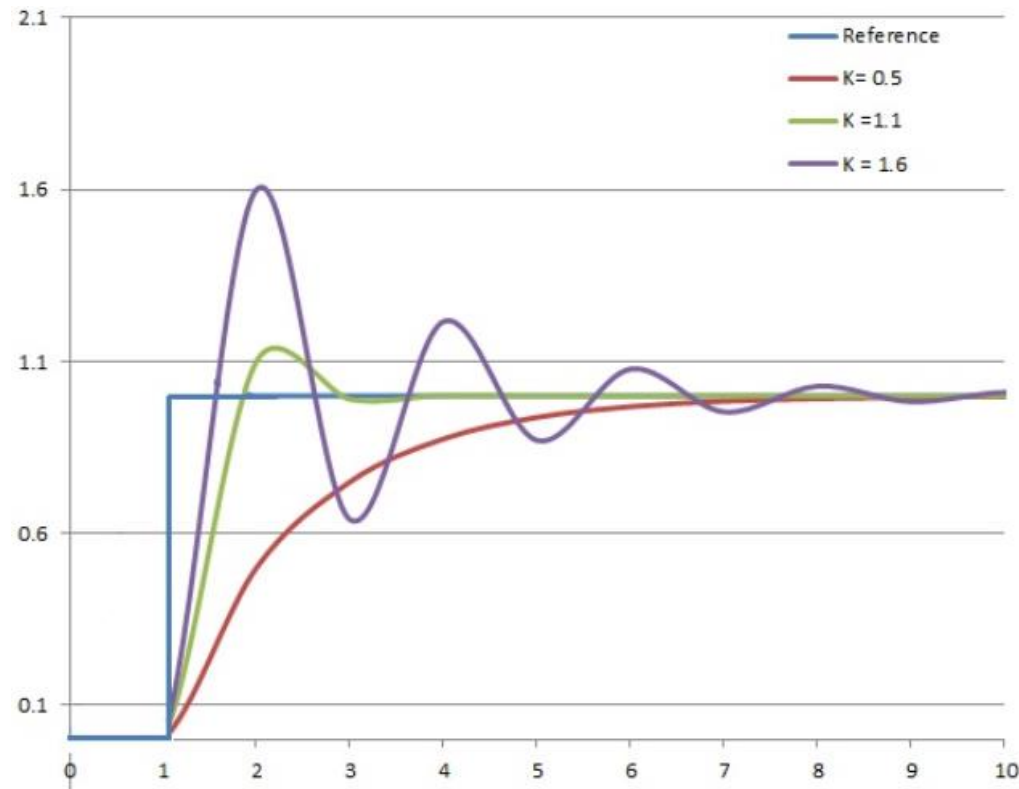
PID Control

- Tracking Error: $e(t) = r(t) - y(t)$
- Control input: $u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \dot{e}(t)$



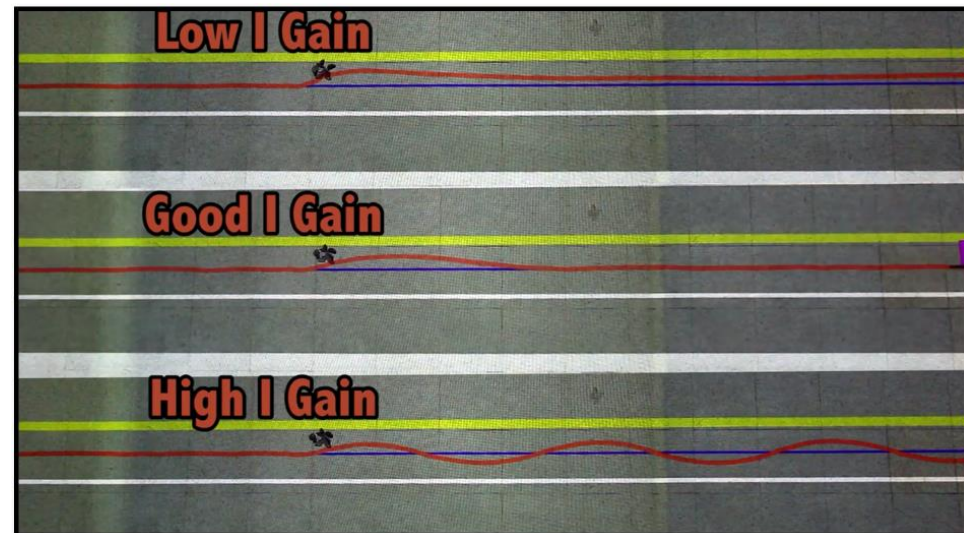
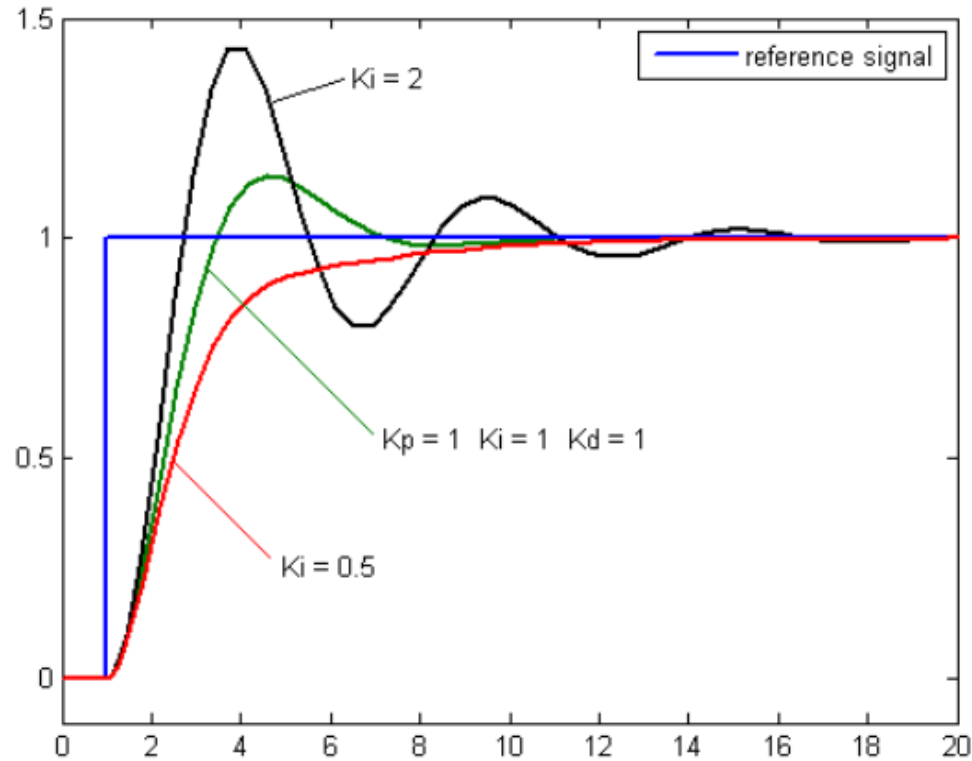
Proportional Term K_p

- Increasing K_p causes:
 - Faster response
 - Bigger overshoot, oscillations
 - System may become unstable
 - Smaller but non-zero steady state error



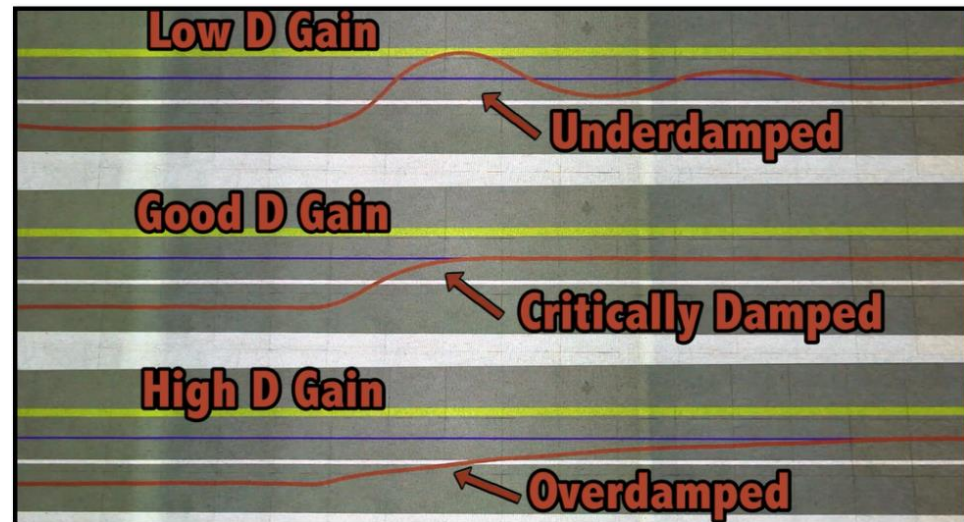
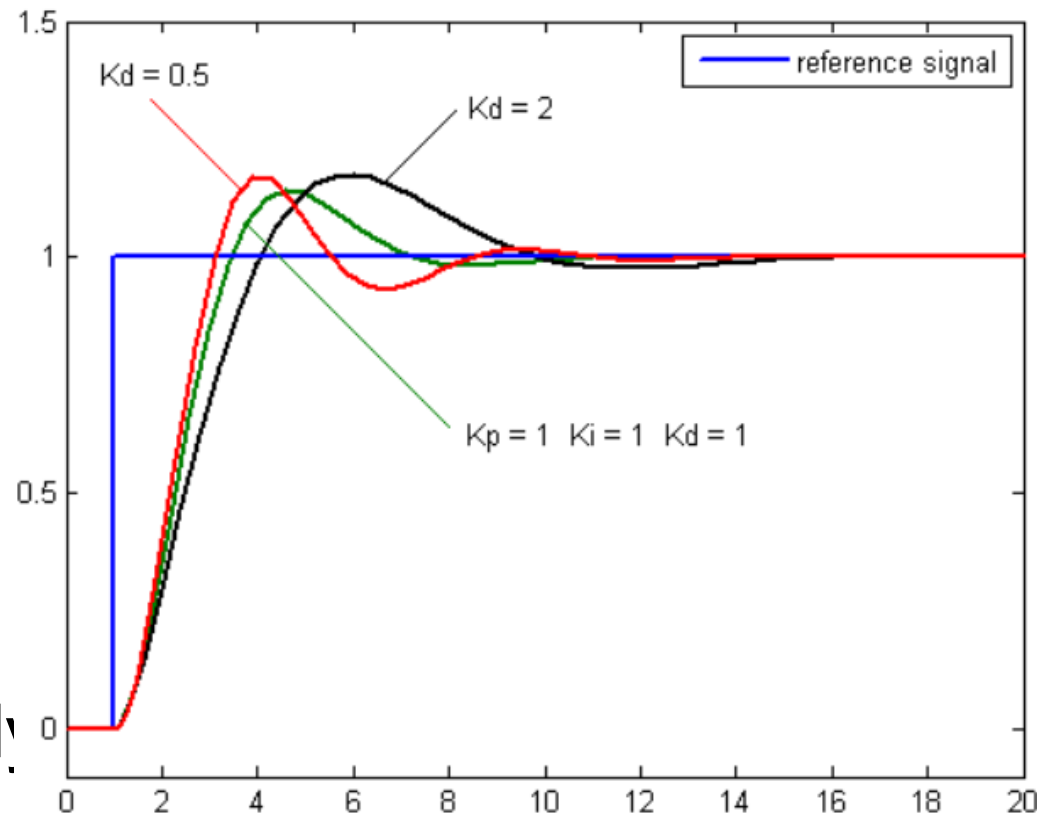
Integral Term K_i

- K_i takes into account history of tracking error, and eliminates steady state error
- Increasing K_i causes:
 - Increases overshoot
 - More robust to disturbances



Derivative Term K_d

- Increasing K_d causes:
 - Reduced overshoot
 - Faster response
 - Little effect on steady state
 - More sensitive to measurement noise

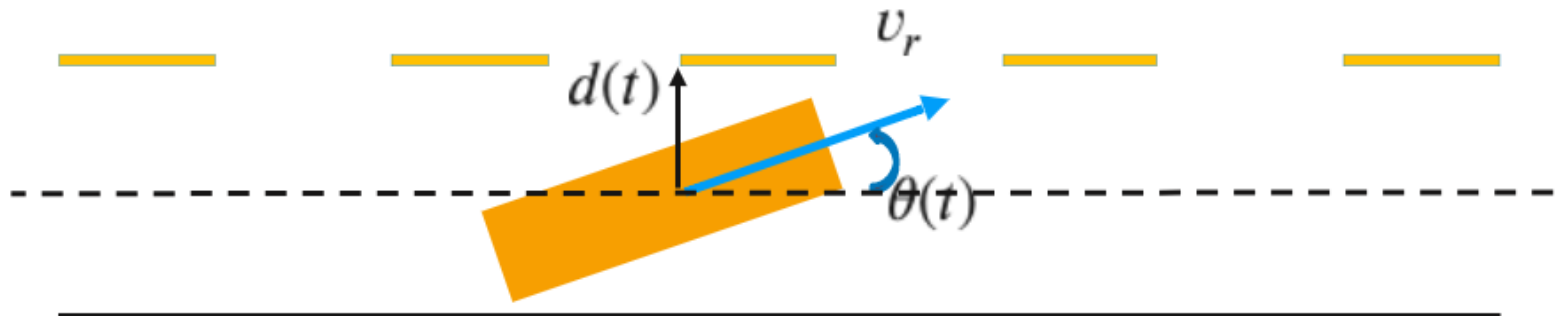
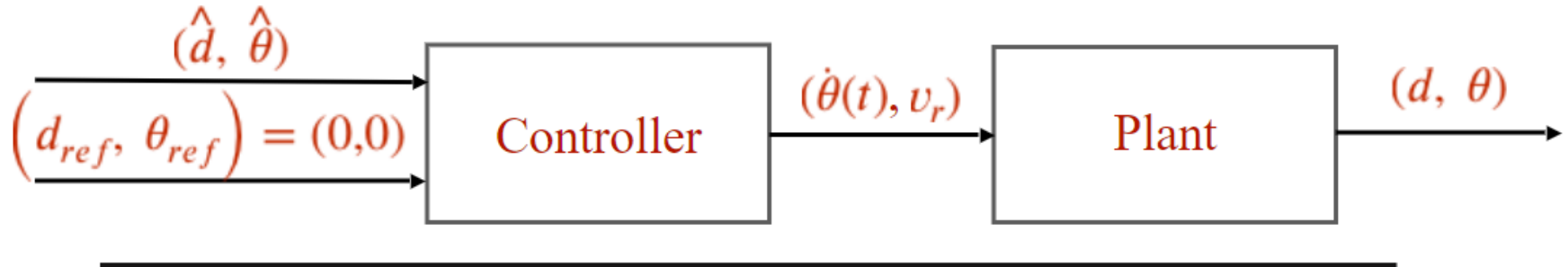


Summary of Effects of PID Gains

Closed-Loop Response	Rise Time	Overshoot	Settling Time	Steady State Error
Increase K_p	Decrease	Increase	Small change	Decrease
Increase K_i	Decrease	Increase	Increase	Eliminate
Increase K_d	Small change	Decrease	Decrease	Small change

Example: Vehicle Lateral Control

- State (d, θ)
 - d : distance to center of lane; θ : heading angle.
- Reference trajectory: $(d_{ref}, \theta_{ref}) = (0, 0)$
 - The vehicle travels straight ahead at center of lane.
- Vehicle has constant speed v_r , so the only control input is $u(t) = \dot{\theta}(t)$, the angular velocity.
- Assume perfect sensor state estimation: $(\hat{d}, \hat{\theta}) = (d, \theta)$



System and Controller Modeling

- Linear system dynamics:

- $\begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & v_r \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

- $\dot{d}(t) = v_r \sin \theta(t) \approx v_r \theta(t)$ assuming $\theta(t)$ is small. This is a linearized kinematic model.

- P Controller:

- $u(t) = \begin{bmatrix} K_d & K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$

- Closed-loop dynamics is obtained by plugging $u(t)$ into system dynamics:

- $\begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & v_r \\ K_d & K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$

- (Please note symbol – above is NOT a minus sign!)

System and Controller Modeling

- Ref trajectory: $\begin{bmatrix} d_{ref} \\ \theta_{ref} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

- Error signal:

$$- \begin{bmatrix} e_d(t) \\ e_\theta(t) \end{bmatrix} = \begin{bmatrix} d_{ref} \\ \theta_{ref} \end{bmatrix} - \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix} = - \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$$

- Error dynamics:

$$- \begin{bmatrix} \dot{e}_d(t) \\ \dot{e}_\theta(t) \end{bmatrix} = - \begin{bmatrix} \dot{d}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -v_r \\ -K_d & -K_\theta \end{bmatrix} \begin{bmatrix} d(t) \\ \theta(t) \end{bmatrix}$$

- We need to design K_d, K_θ to drive $\begin{bmatrix} e_d(t) \\ e_\theta(t) \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Control Design with Pole Placement (not covered in this course)

- Closed-loop poles:

- $0 = \det(\lambda I - \begin{bmatrix} 0 & -v_r \\ -K_d & -K_\theta \end{bmatrix}) = \lambda^2 + K_\theta \lambda - v_r K_d$

- Solution $\lambda_{1,2} = -\frac{K_\theta}{2} \pm \frac{1}{2} \sqrt{K_\theta^2 + 4v_r K_d}$

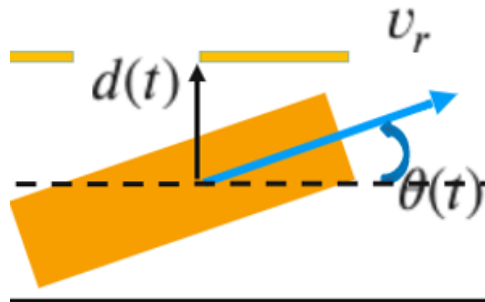
- Critically-damped dynamics (repeated real roots):

- $\sqrt{K_\theta^2 + 4v_r K_d} = 0 \Rightarrow K_d = -\frac{K_\theta^2}{4v_r}$

- K_θ chosen empirically

Control Design cont'd

- Linear systems dynamics relies on the small angle approximation $\sin \theta(t) \approx \theta(t)$. To keep it valid, i.e., $|\theta(t)| < \theta_{th}, \forall d(t)$,
- Closed-loop dynamics $\dot{\theta}(t) = K_d d(t) + K_\theta \theta(t)$
 - Change to $\dot{\theta}(t) = K_d \text{sat}(d(t), d_{th}) + K_\theta \theta(t)$
 - where: $\text{sat}(d(t), d_{th}) = \begin{cases} -d_{th} & d(t) < -d_{th} \\ d(t) & d(t) \in [-d_{th}, d_{th}] \\ d_{th} & d(t) > d_{th} \end{cases}$
- For example: to have $\theta_{th} = \frac{\pi}{6}$, set $d_{th} = \left| \frac{K_\theta \theta_{th}}{K_d} \right|$
 - Even if the vehicle is very far from center of lane, do not change heading angle $\theta(t)$ too fast (keep $\dot{\theta}(t)$ small).
 - (I think it is a rule of thumb, not a hard guarantee.)

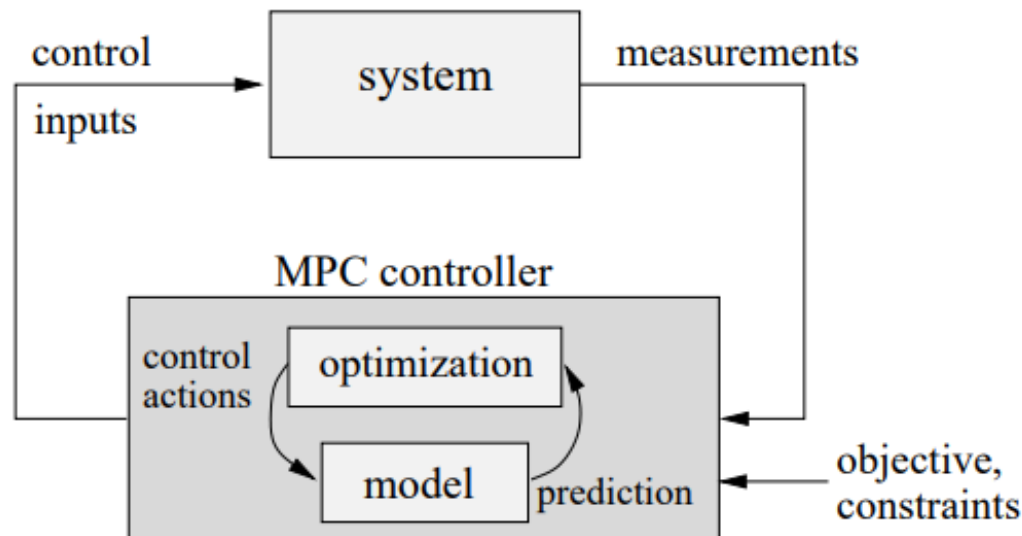


Simplifying Assumptions We Made

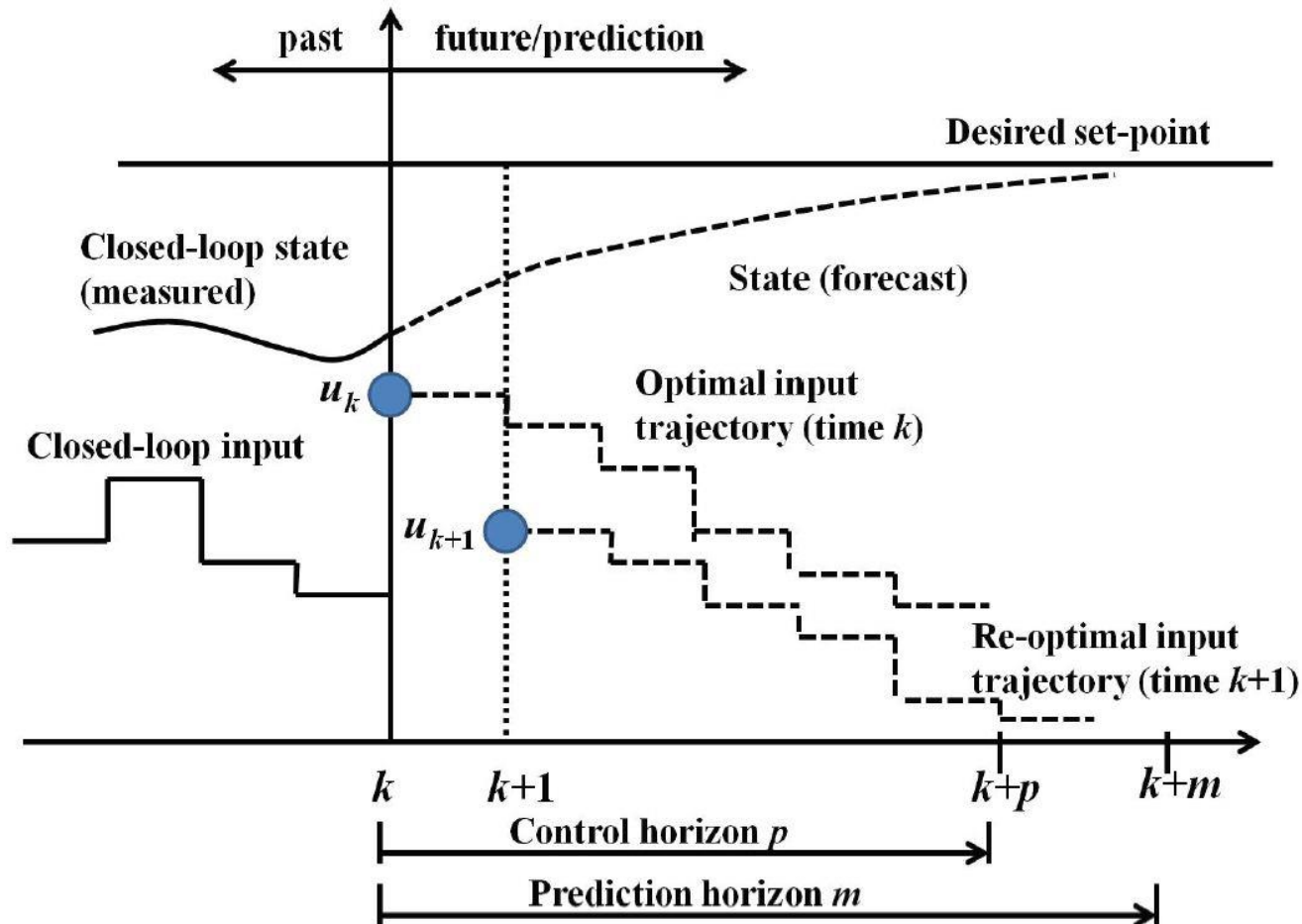
- We did not consider:
 - Estimation uncertainty due to measurement noise
 - We assume perfect state estimation $(\hat{d}, \hat{\theta}) = (d, \theta)$
 - Estimation latency:
 - Time from measurements (d, θ) to state estimation $(\hat{d}, \hat{\theta})$ availability to the controller
 - Constraints (e.g., actuator limits)
 - We may need to impose a maximum curvature radius to simulate a real car.
 - Discrete time (multi-rate), non-uniform sampling:
 - Our controller is continuous time, but the actual implementation runs in discrete time.
 - Sampling rate of the estimator (slower) may be different than that of actuation (faster), or may be variable.

MPC (Model-Predictive Control)

- Also called Receding Horizon Control
- Choose prediction horizon m and control horizon p
- At each time step k :
 - Set initial state to predicted state $x[k]$
 - Solve a constrained optimization problem over lookahead window $[k, k + m]$, to get a sequence of control inputs u , while in the time interval $[k - 1, k]$
 - Apply 1st control command $u[k]$ at time step k

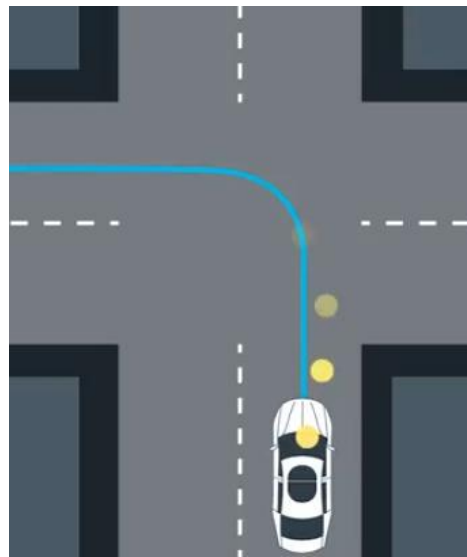


MPC Illustration

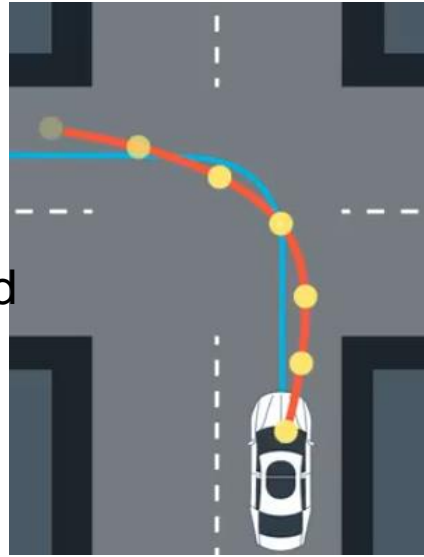
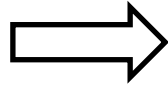


- Control horizon p and prediction horizon m may be different, but often the same (denoted as lookahead horizon T in next slides).
- “input trajectory” refers to computed control inputs $U = \{u_k, \dots, u_{k+p-1}\}$, not state trajectory.

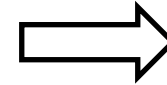
MPC Illustration Con't



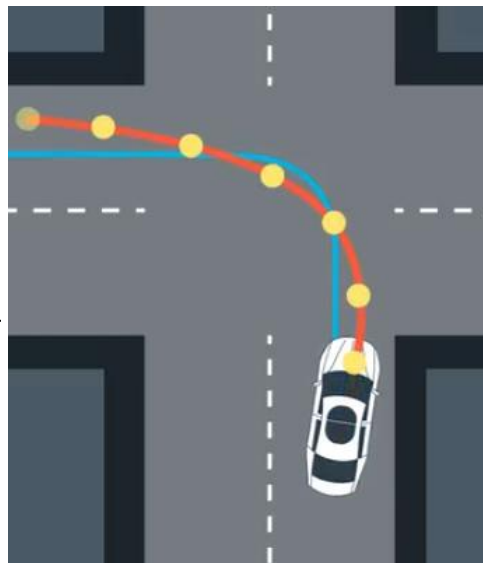
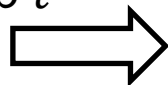
Solve for opt
control inputs
 $U =$
 $\{u_t, \dots, u_{t+T-1}\}$
within lookahead
horizon T to
track the blue
desired traj



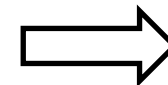
Execute
control
input u_t
at 1st
timestep



Replan a
new set of
control
inputs $U =$
 $\{u_t, \dots, u_{t+T-1}\}$
at next time
step t



Execute
control
input u_t at
1st
timestep



...

MPC Illustration Con't

- **Yellow:** reference trajectory from planner
- **Green:** trajectory from running control inputs $u[0, \dots, T - 1]$ computed by MPC based on system model for lookahead horizon T



MPC Formulations

- Linear MPC (no constraints)

- $\min_{U=\{u_t, \dots, u_{t+T-1}\}} J(x(t), U) = x_{t+T}^T Q_f x_{t+T} + \sum_{j=t}^{t+T-1} (x_j^T Q x_j + u_j^T R u_j)$
- s.t. for $t \leq j \leq t + T - 1$
- $x_{j+1} = A x_j + B u_j$
- x_j is the difference between actual state and ref state, which should be minimized with the term $x_{t+T}^T Q_f x_{t+T}$
- u_j is control input, which should be minimized with the term $u_j^T R u_j$ (e.g., to save fuel)
- Relative magnitudes of Q and R encode relative importance of the two objectives
- Can be solved analytically $u_t = -K x_t$ with Linear Quadratic Regulator (LQR)

- Nonlinear MPC (with constraints) $\min_{U=\{u_t, \dots, u_{t+T-1}\}} J(x(t), U) = \sum_{j=t}^{t+T} C(x_j, u_j)$

- s.t. for $t \leq j \leq t + T - 1$
- $x_{j+1} = f(x_j, u_j)$
- $x_{min} \leq x_{j+1} \leq x_{max}$
- $u_{min} \leq u_j \leq u_{max}$
- $g(x_j, u_j) \leq 0$
- $h(x_j, u_j) = 0$
- Both objective $J(x(t), U)$ and system dynamics $f(x_j, u_j)$ may be nonlinear.

- Note: in x_{t+T}^T , superscript T denotes “vector transpose”; subscript T denotes “lookahead horizon”;

MPC Pros and Cons

- Pros

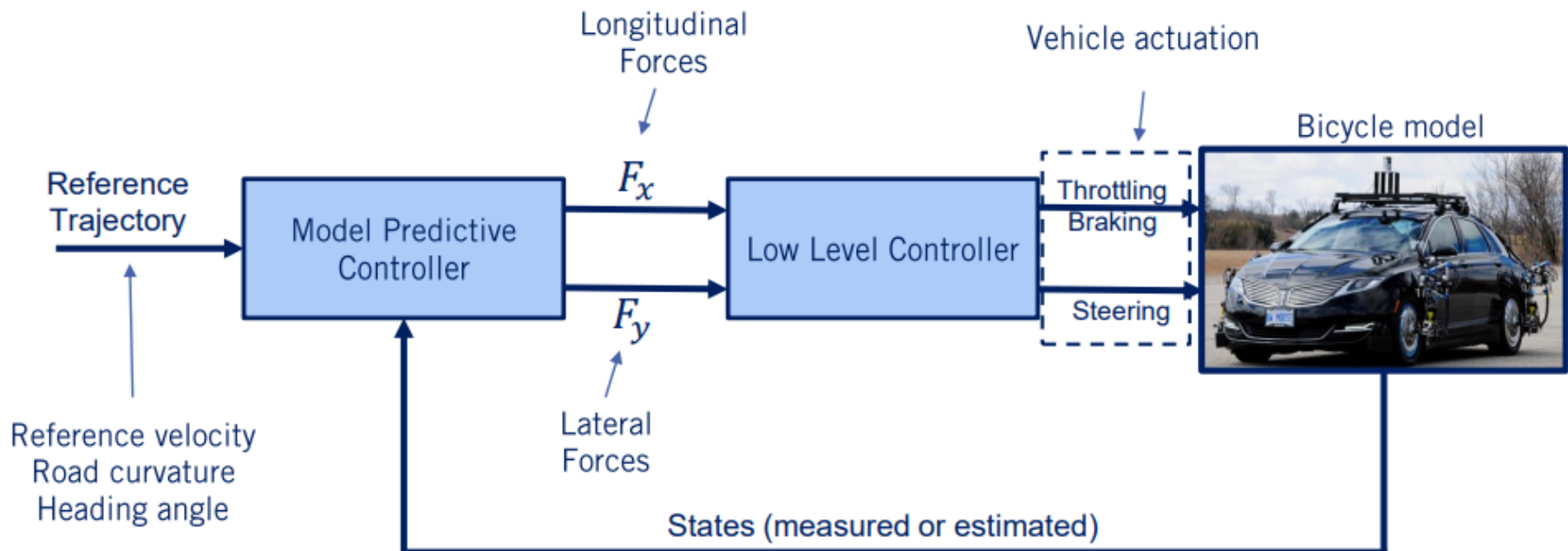
- Predictive control with lookahead.
 - PID control is like driving your car by looking in the rearview mirror.
- Handles constraints explicitly.
 - PID control cannot handle constraints.
- Applicable to both linear and nonlinear systems.
 - Pole placement for PID control design is applicable to linear systems only.

- Cons

- Optimizer computation may be expensive.
 - Especially for non-linear MPC.
 - Select lookahead horizon T to tradeoff between control performance and computation overhead; Larger $T \rightarrow$ better control performance but higher overhead.
- Requires accurate yet efficient system model.

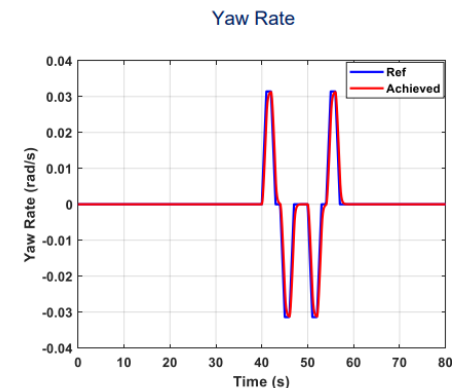
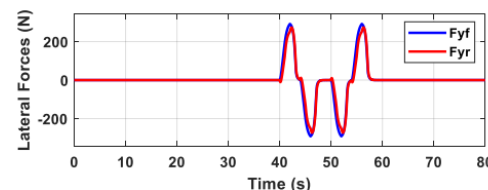
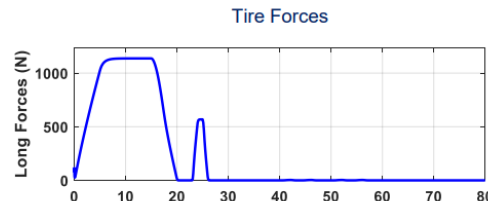
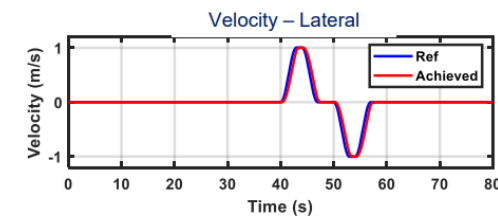
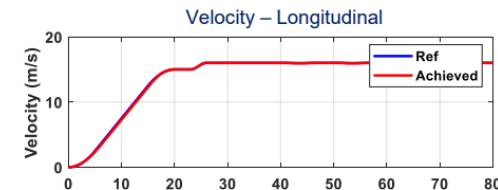
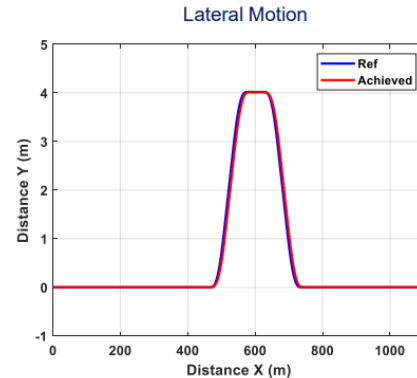
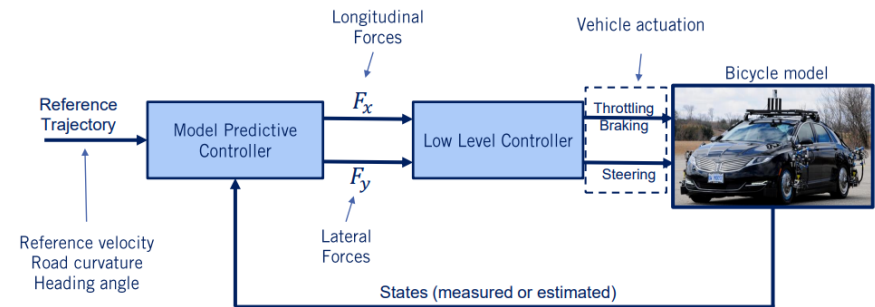
MPC Example: Vehicle Lateral Control

- MPC cost function: minimize
 - Deviation from desired reference trajectory
 - Minimization of control effort (to save fuel)
- MPC constraints:
 - System dynamics (longitudinal and lateral)
 - Tire force limits
- Low-level controller takes into account:
 - Engine map (a lookup table)
 - Actuator models
 - Tire force models
- It is possible to integrate low-level controller into MPC, so the overall MPC has more complex models and constraints, with increased optimizer computation load.



MPC Example Performance

- This is a passing maneuver: ego vehicle turns left to pass a front vehicle, and then turns right to return to ego-lane.
- MPC controller performance
 - Actual vehicle state trajectories (lateral position y , longitudinal and lateral velocities, yaw rate (angular velocity)) (blue) track ref trajectories (ref) well
- Low-level controller performance
 - Actual tire forces (blue) track ref tire forces F_x, F_y (ref) well



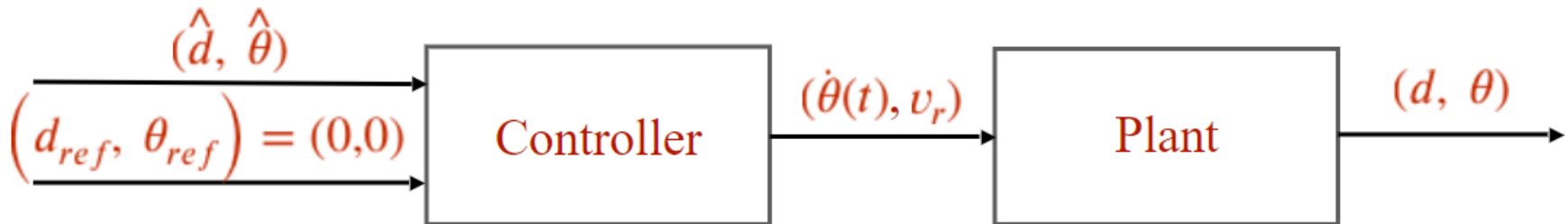
MPC Quiz

- Which from the below statement about MPC are true?
 - 1) Horizon is a finite window of time
 - 2) Prediction horizon keeps being shifted at each time step
 - 3) Full optimization over the time horizon is performed at each iteration
 - 4) Only the first control action from the optimization is applied at time t
 - 5) All of the above
- ANS: 5

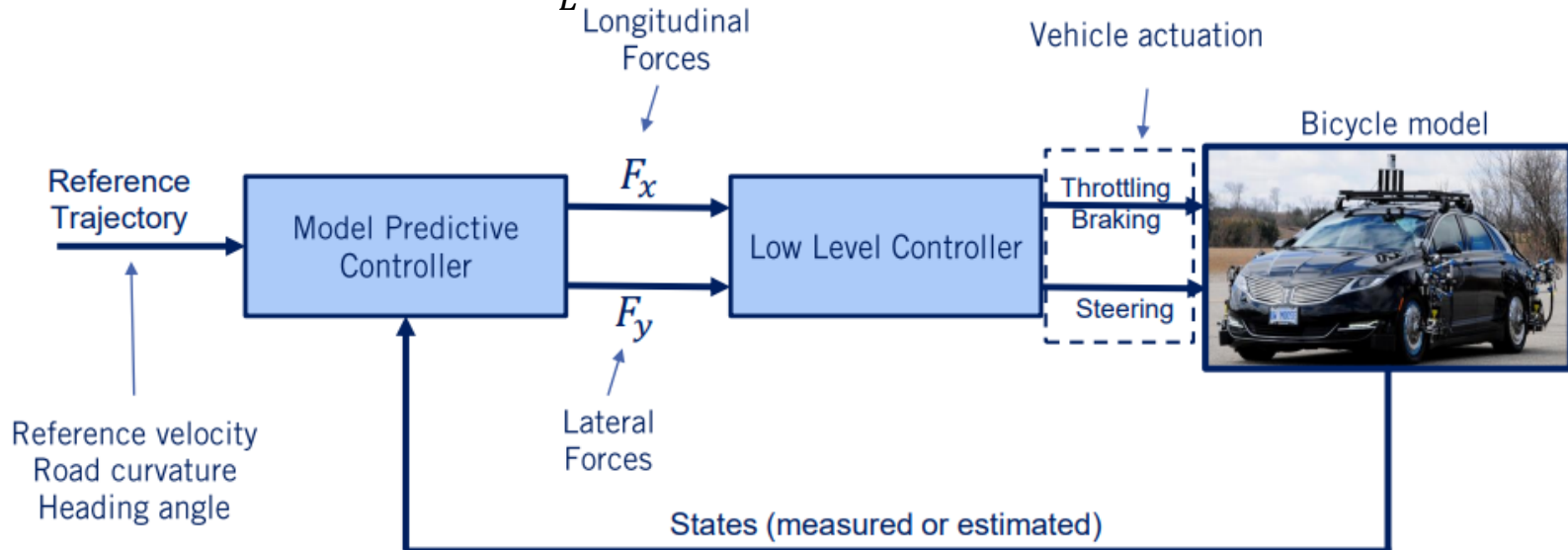
Kinematics vs. Dynamics

- Kinematics is study of motion without considering the forces that affect the motion. It deals with the geometric relationships that govern the system
 - A kinematic model: $\dot{x} = v, \dot{v} = \dot{x} = a$
 - Uses position, velocity, acceleration (and/or further derivatives) as control input (e.g. the kinematic bicycle model)
- Dynamics is the study of motion taking into account the forces that affect it. It is described by the equations of motion.
 - A dynamic model: $F = M\ddot{x} + B\dot{x}$ (Newton's law with friction)
 - Uses force and torque as control input, and takes mass and inertia into consideration.
 - e.g., vehicle dynamics model (longitudinal and lateral)
- Consider two vehicles with the same geometry but different mass/weight turning a tight corner
 - They may have the same kinematic model, but different dynamic model due to different mass M .
 - If both are controllable kinematically by control input a :
 - The light vehicle may be controllable dynamically by control input F .
 - The heavy vehicle may not be controllable dynamically by control input F . Due to large M , F may exceed the max actuator limit.
- Orthogonal issue from control algorithm
 - A control algorithm, e.g., PID or MPC, may be either kinematic or dynamic control

Kinematic vs. Dynamic Control



PID controller for kinematic bicycle model with heading angle rate ($\dot{\theta}$) and velocity (v_r , constant here) as control input. ($\dot{\theta}$ should be controlled indirectly by controlling steering angle δ , with $\dot{\theta} = \frac{v \tan \delta}{L}$.)



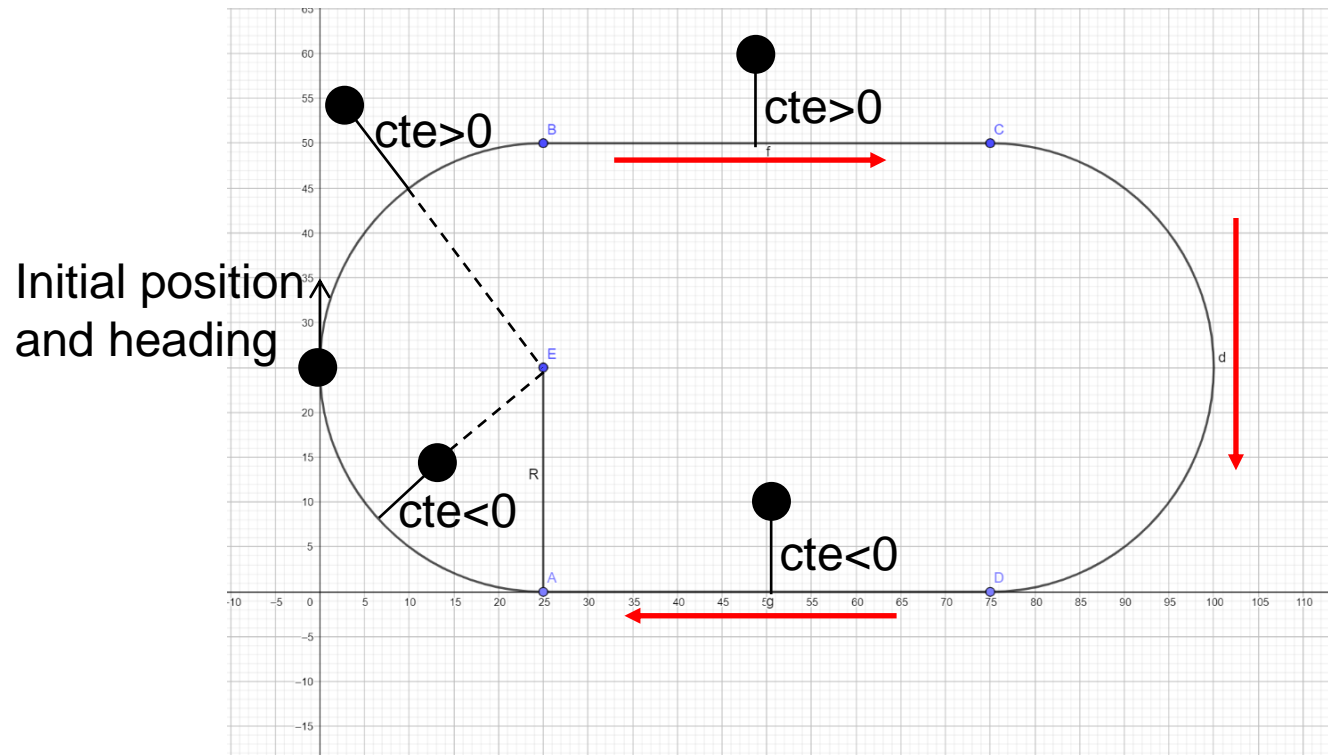
MPC for dynamic vehicle model (high-level controller) with forces (longitudinal and lateral) as control input.

PID Control Lab Setup

- Based on Udacity course Lesson AI for Robotics, Lesson 15: PID Control
 - <https://classroom.udacity.com/courses/cs373/>
- Lateral control of a car to run along a race track (either straight line or circular) with fixed speed.

Udacity: Racetrack Control

- Lesson 16: Problem Set 5, 4. Quiz: Racetrack control.
- Cross-track error (cte): lateral distance (of rear wheel) to desired trajectory, defined as:
 - if $x \in [radius, 3 * radius]$: deviation from the straight horizontal track.
 - Otherwise: deviation from each semi-circle track.
- For clockwise traversal, $cte > 0$ if car is outside of the track region; $cte < 0$ if inside.

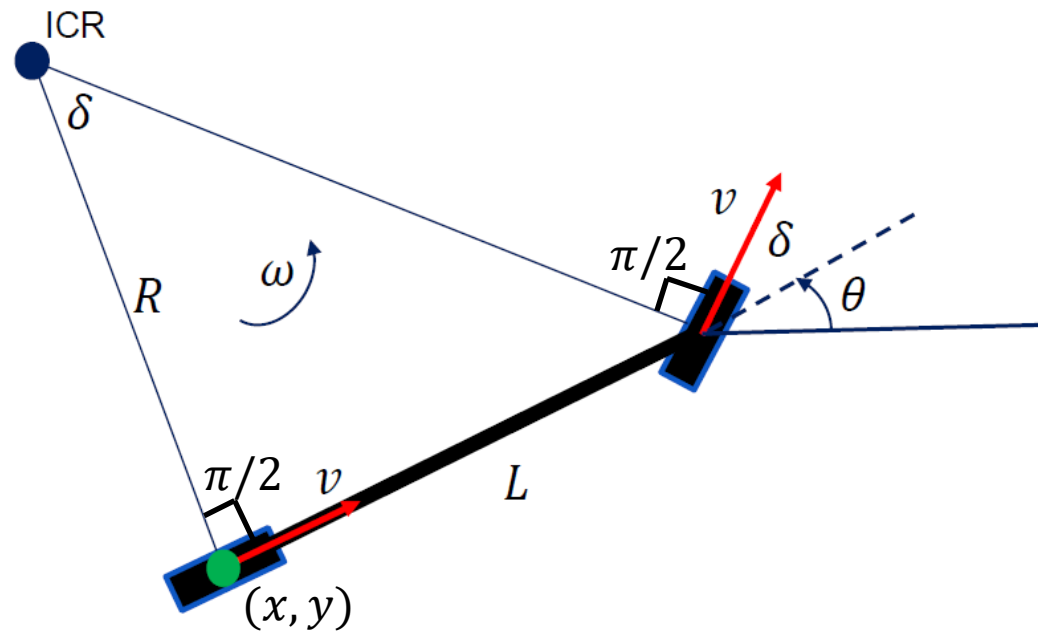


Kinematic Bicycle Model

- Front wheel steering. Assuming here rear wheel as reference point (may also use front wheel or center of gravity).
- State vector: $[x \ y \ \theta]^T$: vehicle pose includes its position (x, y) and heading angle θ .
- Control inputs: $[\delta \ v]^T$: steering angle δ and vehicle speed v (assumed to be constant).
- $\tan \delta = \frac{L}{R}$
 - L : vehicle length (distance between 2 wheels); R : rotation radius of Instantaneous Center of Rotation (ICR), equal to distance between ICR and rear wheel. Curvature $\kappa = \frac{1}{R}$.
 - Line from ICR to each wheel is perpendicular to it.
- $\dot{\theta} = \omega = \frac{v}{R} = \frac{v \tan \delta}{L}$
 - Angular velocity is speed v divided by rotation radius R
- Typically, angles δ, θ are based on counter-clockwise convention w.r.t reference direction.
 - In the fig, $\delta \approx \frac{\pi}{6}$ w.r.t ref direction v ; $\theta \approx \frac{\pi}{6}$ w.r.t ref direction east (horizontal); $\dot{\theta} > 0$ for counter-clockwise rotation.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{\tan \delta}{L} \end{bmatrix}$$

(Non-linear in δ ;
Linear in v)



State Update Equation

- Circular motion around ICR (accurate):

- $$\begin{bmatrix} x(t + dt) \\ y(t + dt) \\ \theta(t + dt) \end{bmatrix} = \begin{bmatrix} x(t) - R \sin \theta + R \sin(\theta + \dot{\theta} dt) \\ y(t) + R \cos \theta - R \cos(\theta + \dot{\theta} dt) \\ \theta + \dot{\theta} dt \end{bmatrix}$$

- $R = \frac{L}{\tan \delta} = \frac{v}{\dot{\theta}}$

- Straight-line motion for small $\dot{\theta} dt$ (approximate):

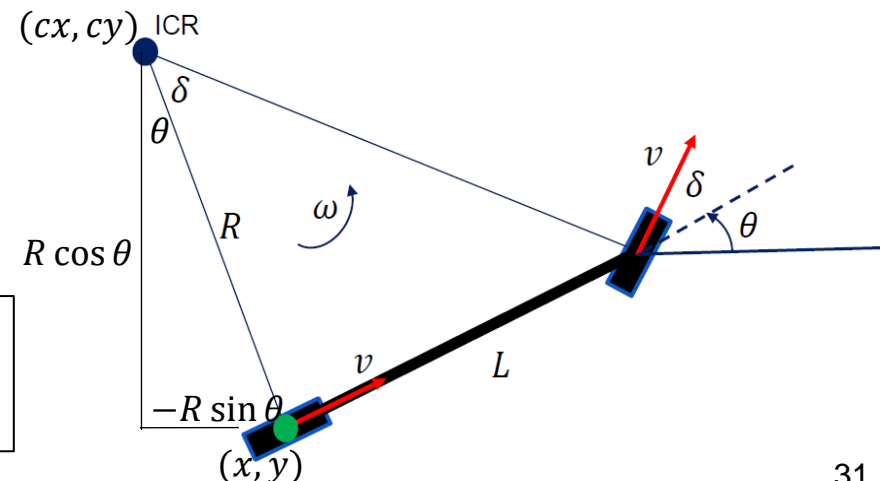
- $$\begin{bmatrix} x(t + dt) \\ y(t + dt) \\ \theta(t + dt) \end{bmatrix} = \begin{bmatrix} x(t) + v dt \cos \theta \\ y(t) + v dt \sin \theta \\ \theta + \dot{\theta} dt \end{bmatrix}$$

- $\delta \rightarrow 0, \dot{\theta} dt \rightarrow 0$

Udacity: State Update based on Kinematic Bicycle Model

- # apply noise
- steering2 = random.gauss(steering, self.steering_noise)
- distance2 = random.gauss(distance, self.distance_noise)
- # apply steering drift
- steering2 += self.steering_drift
- # noise and drift are all set to 0, so steering2 is steering angle δ , distance2 is distance traveled per time step vdt
- # Execute motion
- turn = np.tan(steering2) * distance2 / self.length ($\dot{\theta}dt = \frac{vdt \tan \delta}{L}$)
- **if abs(turn) < tolerance:** (with small $\dot{\theta}dt$)
- # approximate by straight line motion
- self.x += distance2 * np.cos(self.orientation) ($x += vdt \cos \theta$)
- self.y += distance2 * np.sin(self.orientation) ($y += vdt \sin \theta$)
- self.orientation = (self.orientation + turn) % (2.0 * np.pi) ($\theta = (\theta + \dot{\theta}dt) \%(2\pi)$)

- **else:** (with large $\dot{\theta}dt$)
- # Circular motion around ICR
- radius = distance2 / turn ($R = \frac{v}{\dot{\theta}}$; can also use $R = \frac{L}{\tan \delta}$)
- # compute ICR's coordinates (cx, cy)
- cx = self.x - (np.sin(self.orientation) * radius) ($cx = x - R \sin \theta$)
- cy = self.y + (np.cos(self.orientation) * radius) ($cy = y + R \cos \theta$)
- self.orientation = (self.orientation + turn) % (2.0 * np.pi) ($\theta = (\theta + \dot{\theta}dt) \%(2\pi)$)
- # compute vehicle's x, y coordinate after rotation around ICR
- self.x = cx + (np.sin(self.orientation) * radius) ($x = cx + R \sin \theta$)
- self.y = cy - (np.cos(self.orientation) * radius) ($y = cy - R \cos \theta$)



$\%(2\pi)$ (modulo 2π) keeps angles to be within 2π ; It can be omitted since it does not affect results of cos, sin functions (assuming no numeric overflow).

Straight Line vs. Circular Motion

- Straight line motion (with steering angle $\delta = 0$) is a special case of circular motion with radius ∞ .
 - $\delta \rightarrow 0 \Rightarrow R = \frac{L}{\tan \delta} \rightarrow \infty, \dot{\theta} = \frac{v}{R} \rightarrow 0$ (small steering angle δ leads to slow angular velocity $\dot{\theta}$.)
 - We use this special case to improve computational efficiency for small $\delta, \dot{\theta}$, and avoid division by 0.
- $x(t + dt) = x(t) - R \sin \theta + R \sin(\theta + \dot{\theta} dt) = x + R(\sin(\theta + \dot{\theta} dt) - \sin \theta) = x + \frac{v}{\dot{\theta}} * \cos \theta * \dot{\theta} dt = x + v dt \cos \theta$
 - $\sin(\theta + \dot{\theta} dt) - \sin \theta \approx \frac{d}{dt} \sin \theta * \dot{\theta} dt = \cos \theta * \dot{\theta} dt$, for small $\dot{\theta} dt$.
- $y(t + dt) = y(t) + R \cos \theta - R \cos(\theta + \dot{\theta} dt) = y - R(\cos(\theta + \dot{\theta} dt) - \cos \theta) = y + \frac{v}{\dot{\theta}} * \sin \theta * \dot{\theta} dt = y + v dt \sin \theta$
 - $\cos(\theta + \dot{\theta} dt) - \cos \theta \approx \frac{d}{dt} \cos \theta * \dot{\theta} dt = -\sin \theta * \dot{\theta} dt$, for small $\dot{\theta} dt$.
- Four special cases of straight line motion:
 - $\theta = 0 \Rightarrow x' = x + v dt, y' = y$ (east)
 - $\theta = \pi \Rightarrow x' = x - v dt, y' = y$ (west)
 - $\theta = \frac{\pi}{2} \Rightarrow x' = x, y' = y + v dt$ (north)
 - $\theta = \frac{3\pi}{2} \Rightarrow x' = x, y' = y - v dt$ (south)

run()

- Implements the PID controller for with PID gains
 - $\text{params}[0]$ as K_p ; $\text{params}[1]$ as K_d ; $\text{params}[2]$ as K_i . (Minus signs can be removed without affecting correctness.)
 - Returns actual trajectory as arrays of $x_trajectory[]$, $y_trajectory[]$; and average error, defined as sum of squared cte.
- For clockwise travel direction:
 - If $\text{cte} > 0$, car is outside of track region; steering angle δ should decrease (turn right according to the counter-clockwise convention).
 - If $\text{cte} < 0$, car is inside of track region; steering angle δ should increase (turn left).
 - Based on the above, K_p should be positive (this can be a sanity check for your final solution).

```
def run(robot, params, n=100, speed=1.0):
    x_trajectory = []
    y_trajectory = []
    err = 0
    # TODO: your code here
    prev_cte = robot.y
    int_cte = 0
    for i in range(2 * n):
        cte = robot.y
        diff_cte = cte - prev_cte
        int_cte += cte
        prev_cte = cte
        steer = -params[0] * cte - params[1] * diff_cte - params[2] * int_cte
        robot.move(steer, speed)
        x_trajectory.append(robot.x)
        y_trajectory.append(robot.y)
        if i >= n:
            err += cte ** 2
    return x_trajectory, y_trajectory, err / n
```

This is for straight line track; need to call `myrobot.cte()` for circular track.

PID control law

For General dt

- The Python code assumes controller time step size $dt = 1$.
 - Also, v is constant. δ 's range is unconstrained (steering wheel can be turned to arbitrary angle).
- For general dt , the following needs to be modified:
 - Call `myrobot.move(steer, speed*dt)`, to match its definition `move(self, steering, distance,...)`
 - In the PID control law:
 - Differential of error $diffcte = \frac{cte - prevcte}{dt}$
 - Integral of error $intcte += cte * dt$

twiddle()

Adjust each $p[i]$ in turn.

Adjust $p[i]$ to $p[i] + dp[i]$ and `run()`. If error decreases, we adjusted in the right direction, keep the original adjustment $p[i] + dp[i]$, and increase step size to $1.1 * dp[i]$.

If error increases, we adjusted in the wrong direction, adjust in the opposite direction to $p[i] - 2 * dp[i]$. Run again.

If error decreases, we adjusted in the right direction, keep the original adjustment $p[i] + dp[i]$, and increase step size to $1.1 * dp[i]$.

If error increases, we adjusted in the wrong direction, reduce step size to $.9 * dp[i]$. We do not reverse direction here to avoid oscillation around current $p[i]$ with no progress.

```
def twiddle(tol=0.2):  
    # TODO: Add code here  
    # Don't forget to call 'make_robot' before you call 'run'!  
    p = [0.0, 0.0, 0.0]  
    dp = [1.0, 1.0, 1.0]  
    robot = make_robot()  
    x_trajectory, y_trajectory, best_err = run(robot, p)  
  
    it = 0  
    while sum(dp) > tol:  
        # print("Iteration {}, best error = {}".format(it, best_err))  
        for i in range(len(p)):  
            p[i] += dp[i]  
            robot = make_robot()  
            x_trajectory, y_trajectory, err = run(robot, p)  
  
            if err < best_err:  
                best_err = err  
                dp[i] *= 1.1  
            else:  
                p[i] -= 2 * dp[i]  
                robot = make_robot()  
                x_trajectory, y_trajectory, err = run(robot, p)  
  
                if err < best_err:  
                    best_err = err  
                    dp[i] *= 1.1  
                else:  
                    p[i] += dp[i]  
                    dp[i] *= 0.9  
  
            it += 1  
    return p, best_err
```

twiddle() vs. Pole Placement

- “Gradient descent” approach to tuning PID controller (model-free), by adjusting each PID gain parameter systematically, gradually decreasing or increasing step of adjustment $dp[i]$ (the gradient) until convergence to minimum `best_err`.
- Pole placement for PID controller design is model-based, but requires a linear model

$$- \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos \theta \\ \sin \theta \\ \frac{\tan \delta}{L} \end{bmatrix} \approx \begin{bmatrix} v \\ v\theta \\ u \end{bmatrix}$$

– With simplifying assumptions:

- θ is small
- Simplified kinematic model: $u = \dot{\theta}$ as control input instead of δ .

Changing to P Control

- Setting $dparams[1] = dparams[2] = 0$ turns it into a P controller, since K_d, K_i are initialized to 0 and never changed. P_p is adjusted in `twiddle()`
- `twiddle()` is a local optimization algorithm, so perf is dependent on parameter initialization. Here are initialized to 0 for simplicity.
 - <https://classroom.udacity.com/courses/cs373/lessons/48743150/concepts/f9fe06f9-9b1c-40b1-b9ad-312ca92be287>

ENTER CODE BELOW THIS LINE

```
n_params = 3
dparams   = [1.0 for row in range(n_params)]
params    = [0.0 for row in range(n_params)]
dparams[2] = 0.0
dparams[1] = 0.0
```

```
best_error = run(params)
n = 0
while sum(dparams) > tol:
    for i in range(len(params)):
```