Lecture 12
Minimum Spanning Trees

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Lecture Goals

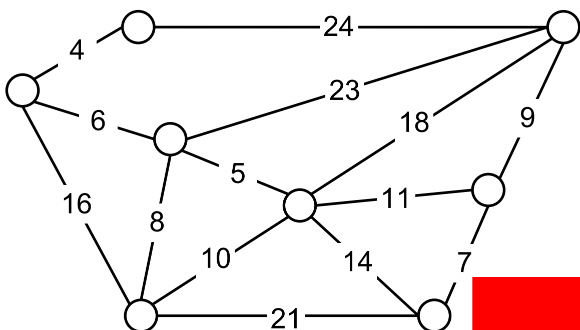
- In this lecture we study the minimum spanning tree problem.
- We consider two classic algorithm for the problem—Kruskal's algorithm and Prim's algorithm.

Minimum Spanning Tree (MST)

Given. Undirected graph G with positive edge weights (connected).

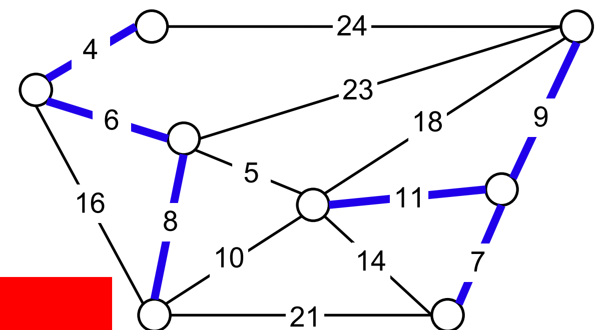
Def. A spanning tree of G is a subgraph T that is both a tree (connected and acyclic) and spanning (includes all of the vertices).

Goal. Find a min weight spanning tree.

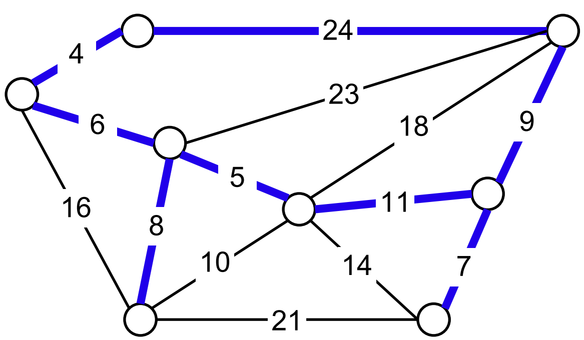


graph G

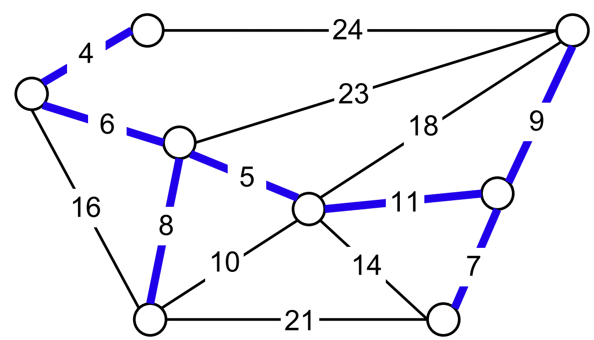
Brute force.
Try all spanning trees?



not connected



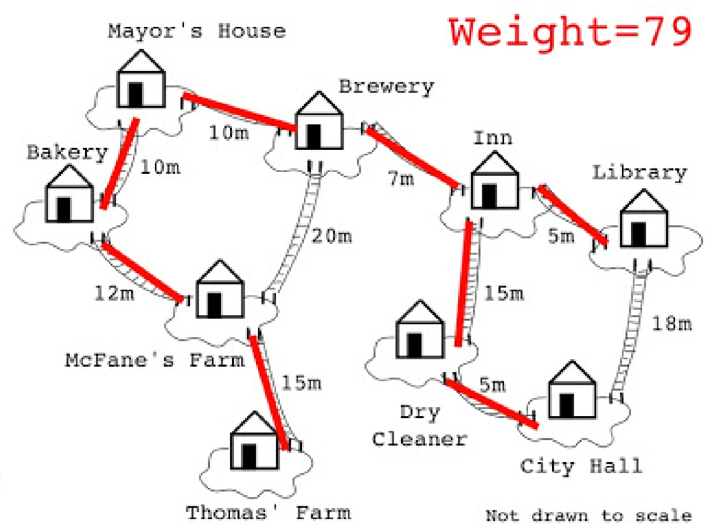
not acyclic

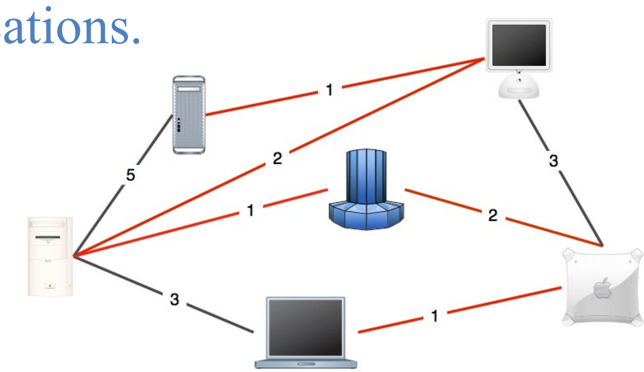


spnning tree T: cost = 50 = 4 + 6 + 8 + 5 + 11 + 9 + 7

MST Applications

- One example would be a telecommunications company trying to lay cable in a new neighborhood. If it is constrained to bury the cable only along certain paths (e.g. roads), then there would be a graph containing the points (e.g. houses) connected by those paths.
- Some of the paths might be more expensive, because they are longer, or require the cable to be buried deeper; these paths would be represented by edges with larger weights.
- A *MST* would be one with the lowest total cost, representing the least expensive path for laying the cable.
- Network design.
- Cluster analysis.
- Indirect applications.



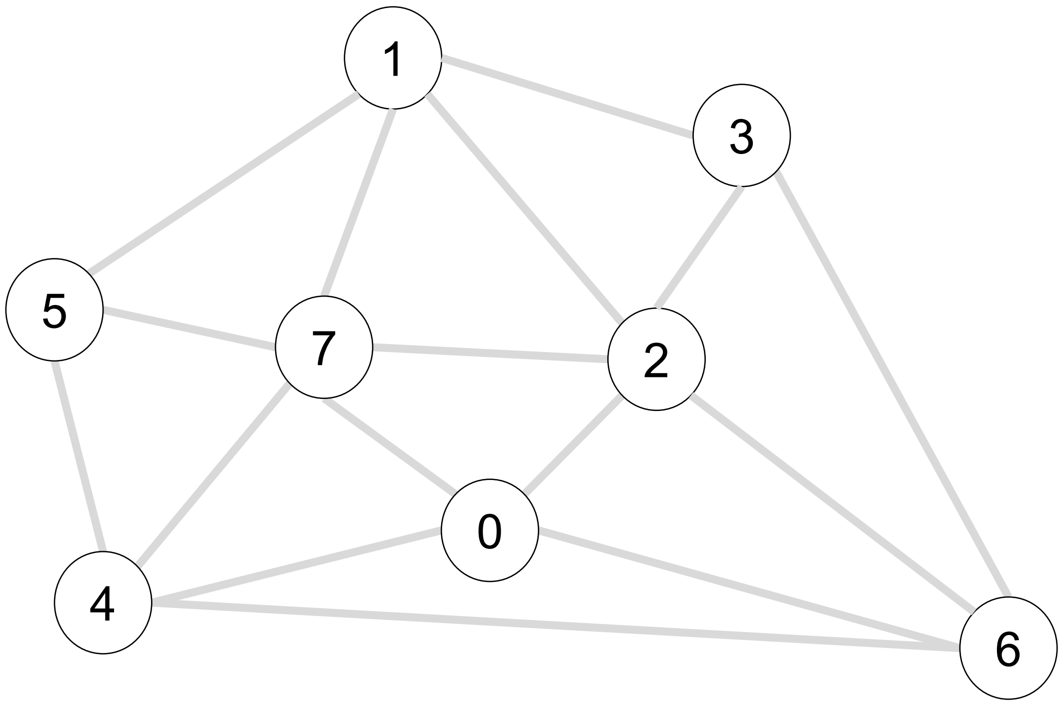


Kruskal's Algorithm

- Consider edges in ascending order of weight.
- Add next edge to tree T unless doing so would create a cycle.

does not create a cycle

creates a cycle



an edge-weighted graph

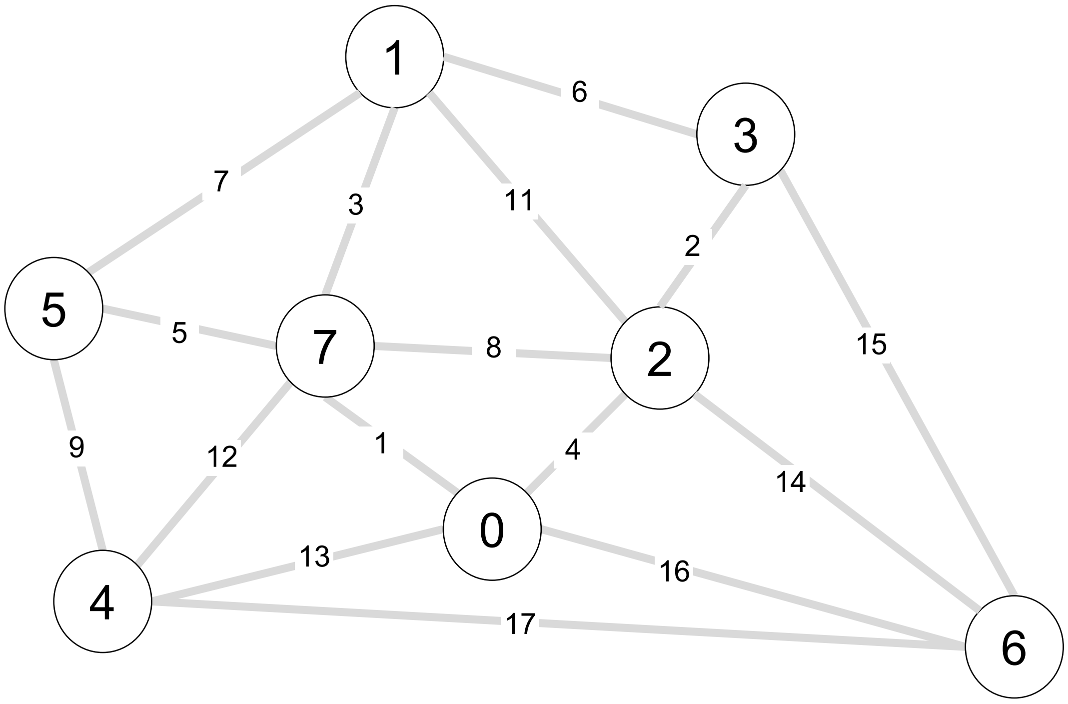
Kruskal's algorithm in 2 minutes
https://www.youtube.com/watch?v=71UQH7Pr9kU

graph edges sorted by weight

0 - 7	1
2 - 3	2
1 - 7	3
0 - 2	4
5 - 7	5
1 - 3	6
1 - 5	7
2 - 7	8
4 - 5	9
1 - 2	10
4 - 7	11
0 - 4	12
2 - 6	13
3 - 6	14
0 - 6	15
4 - 6	16

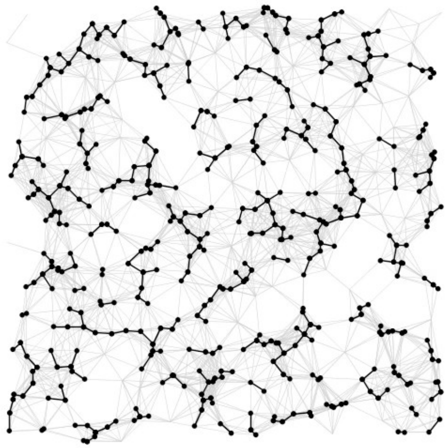
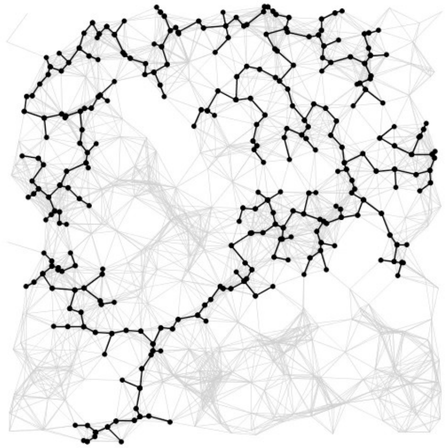
Prim’s Algorithm

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V – 1 edges.



Prim's algorithm in 2 minutes
https://www.youtube.com/watch?v=cplfcGZmX7I

Summary

algorithm	visualization	bottleneck	running time
Kruskal		sorting union-find	$E \log V$
Prim		priority queue	$E \log V$

https://www.youtube.com/watch?v=vmWSnkBVvQ0