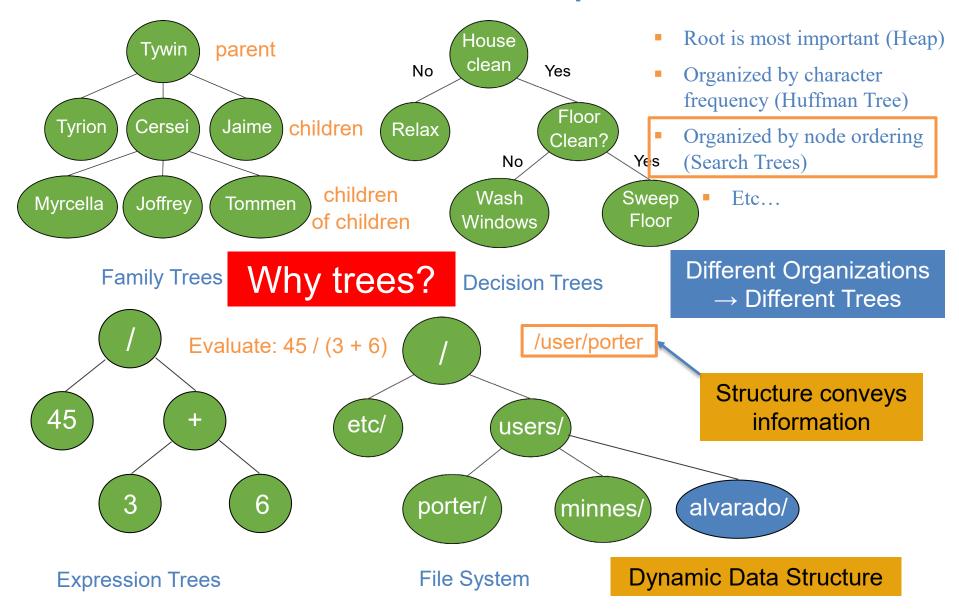
# Lecture 8 Binary Search Tree and Trie

Department of Computer Science Hofstra University

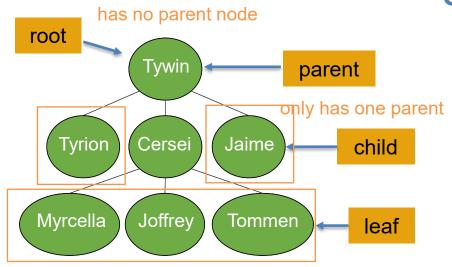
### **Lecture Goals**

- Describe the value of trees and their data structure
- Explain the need to visit data in different orderings
- Perform pre-order, in-order, post-order and level-order traversals
- Define a Binary Search Tree
- Perform search, insert, delete in a Binary Search Tree
- Explain the running time performance to find an item in a BST
- Compare the performance of linked lists and BSTs
- Explain what a trie data structure is

# Different Trees in Computer Science



# **Defining Trees**

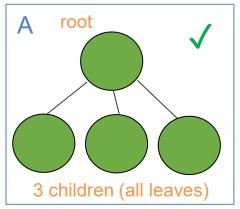


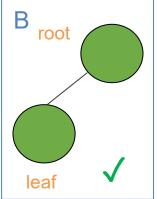
What defines a tree?

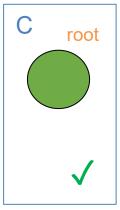
- Single root
- Each node can have only one parent (except for root)
- No cycles in a tree

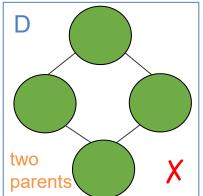
Family Trees nodes without children

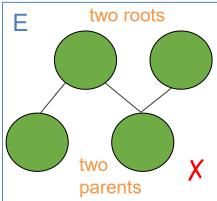
Which are trees?







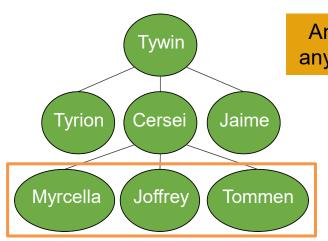




Cycle: two different paths between a pair of nodes

# **Binary Trees**

#### Generic Tree



Any Parent can have any number of children

How would a general tree node differ?

A general tree would just have a list for children

#### A tree just needs a root node

like the head and tail for linked list

Each node needs:

1. A value

2. A parent

3. A left child

4. A right child

#### **Binary Tree**

Tyrion Cersei How

Joffrey Tommen

Any Parent can have at most two children

How do we construct a tree?

Like Linked Lists, Trees have a "Linked Structure"

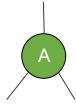
nodes are connected by references

# Write Code for Binary Tree

```
public class BinaryTree<E> {
    TreeNode<E> toot;
    // more methods
}
```



```
public class TreeNode<E> {
      private E value;
      private TreeNode<E> parent;
      private TreeNode<E> left;
      private TreeNode<E> right;
      public TreeNode(E val, TreeNode<E> par) {
            this.value = val;
                                        For root: TreeNode(val, null)
            this.parent = par;
            this.left = null;
            this.right = null;
      public TreeNode<E> addLeftChild(E val) {
            this.left = new TreeNode<E>(val, this);
            return this left;
```



Let's write a constructor together

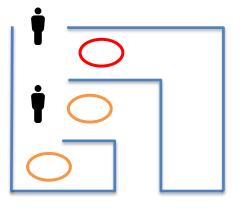
Next Step is to able to set/get children

#### **Tree Traversal - Motivation**

Warning: These first examples are really graphs. We'll visit graphs in detail in the next course. Here they are used as motivating examples

#### start

Strategy: go until hit a dead end, then retrace steps and try again



Imagine this is a hedge maze

What's my next step?

Mazes benefit from "Depth First Traversals"

finish

**Maze Traversal** 

Suppose you have a list of your friends and each of your friends have lists

Bottom line: Order we visit matters and we'll make choices based on our needs

How closely are you connected with D?

What's my next step?

Strategy: look at all of your friends first, and then branch out.

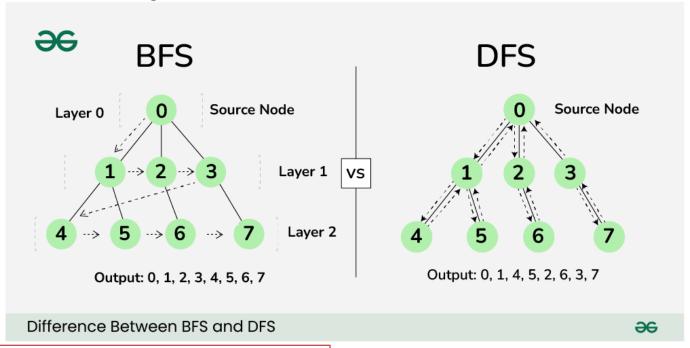
C D

This problem benefits from "Breadth First Traversals"

**Social Network** 

### BFS vs. DFS

- Breadth-First Search (BFS) and Depth-First Search (DFS) are two fundamental algorithms used for traversing or searching graphs and trees
  - BFS traversal explores all the neighboring nodes at the present depth prior to moving on to the nodes at the next depth level.
  - DFS uses backtracking. The deepest node is visited and then backtracks to its parent node if no sibling of that node exists



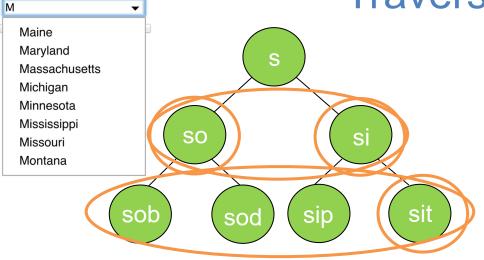
Breadth First Search (BFS) Animations
<a href="https://www.youtube.com/watch?v=QUfEOCOEKkc">https://www.youtube.com/watch?v=QUfEOCOEKkc</a>
Depth First Search (DFS) Animations
<a href="https://www.youtube.com/watch?v=3">https://www.youtube.com/watch?v=3</a> NMDJkmvLo

Tree Traversal Algos // Michael Sambol <a href="https://www.youtube.com/watch?v=iaBEKo5sM7w">https://www.youtube.com/watch?v=iaBEKo5sM7w</a>

## Traversal Order for Binary Trees

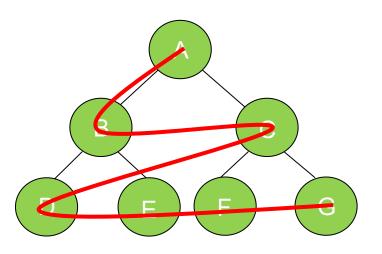
- Breadth First Traversal with BFS
  - Level Order Traversal
- Depth First Traversals with DFS
  - Pre-order Traversal (Root-Left-Right)
  - In-order Traversal (Left-Root-Right)
  - Post-order Traversal (Left-Right-Root)

# Graph traversal with BFS: Level-order Traversal



- You've typed "s" What words should we suggest?
- Most frequent?
- How about "closest"?

"Breadth First Traversal"



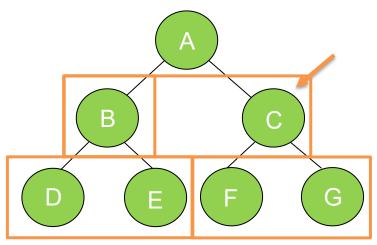
Visit: Level-order

ABCDEFG

Level-order is "Breadth First Traversal"

Pre/In/Post Order are: "Depth First Traversals"

# Graph traversal with BFS: Level-order Traversal (Contd.) Visit:



Visit: A B C D E F G

List: A B C D E F C

We used this list like a "Queue"

- Add to the end
- Remove from the front
- First-In, First-Out (FIFO)

ABCDEFG

Challenging: When we finish B, how do we go to C next?

Idea: Keep a list and keep adding to it and removing from start.



Summary: Nested | Field | Constr | Method Detail: Field | Constr | Method

#### Interface Queue<E>

iava.util

	Throws exception
Insert	add(e)
Remove	remove()
Examine	element()

## Level-order Traversal Implementation

Linkedlist implements both

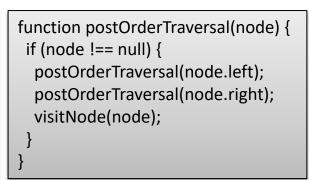
```
list and queue interfaces
public class BinaryTree<E> {
     TreeNode<E> root;
     public void levelOrder() {
           Queue<TreeNode<E>> q = new LinkedList<TreeNode<E>>();
           q.add(root);
          while(!q.isEmpty()) {
                TreeNode<E> curr = q.remove();
           if(curr != null) {
                curr.visit();
                q.add(curr.getLeftChild());
                q.add(curr.getRightChild());
```

Could also check for null children before adding

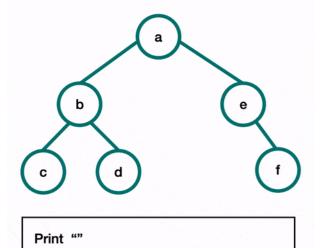
# Graph traversal with DFS: pre-order, inorder, post-order

```
function preOrderTraversal(node) {
  if (node !== null) {
    visitNode(node);
    preOrderTraversal(node.left);
    preOrderTraversal(node.right);
  }
}
```

```
function inOrderTraversal(node) {
  if (node !== null) {
    inOrderTraversal(node.left);
    visitNode(node);
    inOrderTraversal(node.right);
  }
}
```

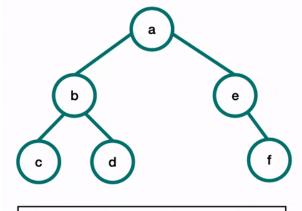


**Pre-Order Traversal** 



abcdef

In-Order Traversal

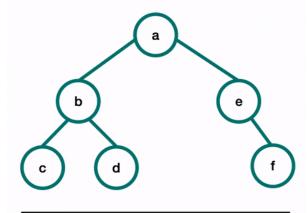


cbdaef

Print ""

Inorder Traversal in Binary Tree Animations <a href="https://www.youtube.com/watch?v=ne5o">https://www.youtube.com/watch?v=ne5o</a></a>
<a href="https://www.youtube.com/watch?v=ne5o">OmYdWGw</a>

**Post-Order Traversal** 



Print ""

cdbfea

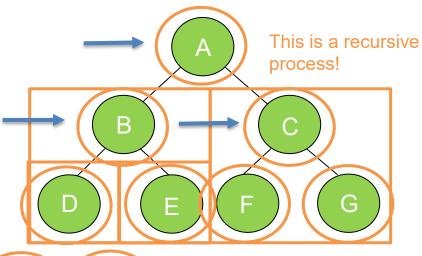
Postorder Traversal in Binary Tree Animations <a href="https://www.youtube.com/watch?v=a8kmbu">https://www.youtube.com/watch?v=a8kmbu</a> Nm8Uo

Preorder Traversal in Binary Tree Animations <a href="https://www.youtube.com/watch?v=gLx7Px7IE">https://www.youtube.com/watch?v=gLx7Px7IE</a>
Zg

## Graph traversal with DFS: pre-order, inorder, post-order

- Pre-order Traversal Algorithm | Tree Traversal | Visualization,
   Code, Example
  - https://www.youtube.com/watch?v=8xue-ZBlTKQ
- In-order Traversal Algorithm | Tree Traversal | Visualization, Code, Example
  - https://www.youtube.com/watch?v=4\_UDUj1j1KQ
- Post-order Traversal Algorithm | Tree Traversal | Visualization,
   Code, Example
  - https://www.youtube.com/watch?v=4Xo-GtBiQN0

# Pre-order Traversal (Recursively)



#### Idea:

- Visit yourself
- Then visit all your left subtree
- Then visit all your right subtree

#### Visited:

ABDECFG

What's the order in which you think the nodes will be visited?

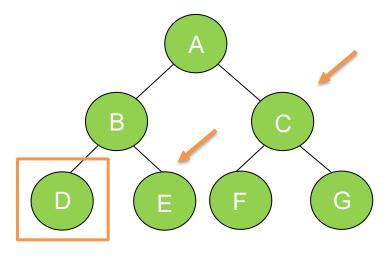
Recursion will help us do this!

This can be done iteratively

```
public class BinaryTree<E> {
    TreeNode<E> root;
    private void preOrder(TreeNode<E> node) {
        if(node!= null) {
            node.visit();
            preOrder(node.getLeftChild());
            preOrder(node.getRightChild());
        }
    }
    public void preOrder() {
        this.preOrder(root);
    }
}
```

Iterative algorithm not covered in exam

# Pre-order Traversal (Iteratively)



Visit: A B D E C F G

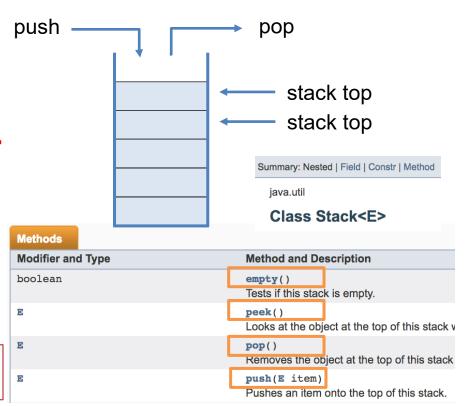
List: <del>A C B E D C F</del>

We used this list like a "Stack"

- Add to the top
- Remove from the top
- Last-In, First-Out (LIFO)

Challenging: When we finish D, how do we go to E and C next?

Idea: Keep a list and keep adding to it and removing from end.



PREORDER TRAVERSAL USING A STACK <a href="https://www.youtube.com/watch?v=zvleLiQn-1">https://www.youtube.com/watch?v=zvleLiQn-1</a>

# Pre-order Traversal (Iteratively)

```
public class BinaryTree<E> {
      TreeNode<E> root;
      void iterativePreorder(TreeNode<E> par) {
            if (par == null) { return; }
      Stack<TreeNode<E>> nodeStack = new Stack<TreeNode<E>>();
      nodeStack.push(par);
            while (nodeStack.empty() == false) {
                   TreeNode<E> node = nodeStack.peek();
            node.visit();
            nodeStack.pop();
            if (node.right != null) {
                         nodeStack.push(node.right);
            if (node.left != null) {
                                                                  1) Create an empty stack nodeStack and push
            nodeStack.push(node.left);
                                                                  root node to stack.
                                                                  2) Do following while nodeStack is not empty.
                                                                  ....a) Pop an item from stack and print it.
                                                                  ....b) Push right child of popped item to stack
  void iterativePreorder() {
                                                                  ....c) Push left child of popped item to stack
    iterativePreorder(root);
                                                                  Right child is pushed before left child to make
```

sure that left subtree is processed first.

### In-order Traversal (Recursively and Iterativ

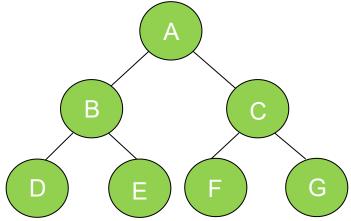
Iterative algorithm not covered in exam

```
public class BinaryTree<E> {
    TreeNode<E> root;

public void Inorder(TreeNode<E> node) {
    if (node == null)
        return;

Inorder(node.left);
    node.visit();
    Inorder(node.right);
    }
    void Inorder() { Inorder(root); }
}
```

- 1) Create an empty stack S.
- 2) Initialize current node as root
- 3) If current is not NULL, push the current node to S and set current = current->left. Repeat until current is NULL
- 4) If current is NULL and stack is not empty then
- ....a) Pop the top item from stack.
- ....b) Print the popped item, set current = popped\_item->right
- ....c) Go to step 3.
- 5) If current is NULL and stack is empty then we are done.

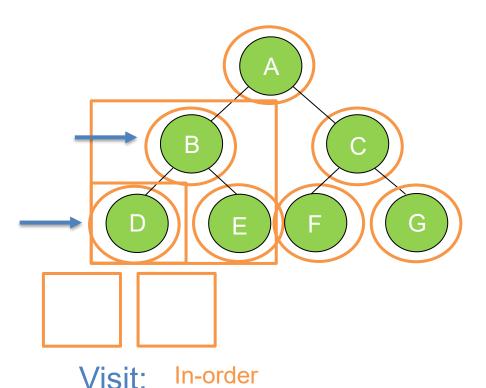


Visit: D B E A F C G

Stack: A B D E C F G

```
public class BinaryTree<E> {
                                                                      Iterative
  TreeNode<E> root;
  public void iterativeInorder() {
    if (root == null)
      return;
    Stack<TreeNode<E>> s = new Stack<TreeNode<E>>();
    TreeNode<E> curr = root;
    while (curr != null | | s.empty() == false) {
      while (curr != null) {
         s.push(curr);
         curr = curr.left:
      curr = s.pop();
      curr.visit();
      curr = curr.right;
```

### Post-order and In-order Traversal



Visit: Post-order

DEBFGCA

#### **REARRANGE:**

- ? Visit yourself
- ? Visit all your left subtree
- ? Visit all your right subtree
- Visit all your left subtree
- Visit all your right subtree
- Visit yourself

#### What does this do?

- Visit all your left subtree
- Visit yourself
- Visit all your right subtree

Fill in the Blank:

A. ABCDEFG

B. DBEAFCG

C. DBAEFCG

Recursion will help us do these!

They can also be done iteratively with Stack.

### Post-order Traversal (Recursively and Iterati

Iterative algorithm not covered in exam

```
public class BinaryTree<E> {
    TreeNode<E> root;

public void Postorder(TreeNode<E> node) {
    if (node == null)
        return;

    Postorder(node.left);
    Postorder(node.right);
    node.visit();
    }
    void Posterorder() {Postorder(root); }
}
```

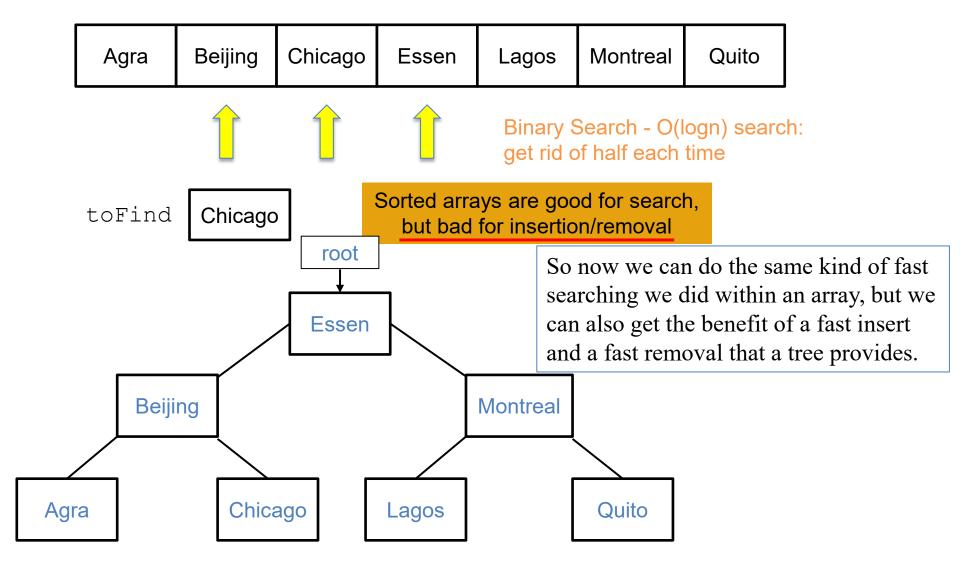
For <u>iterative</u> version, the idea is to push reverse postorder traversal to a stack. Then, we can just pop all items one by one from the stack and visit them. To get reversed postorder elements in a stack – the second stack is used for this purpose. We can observe that this sequence is very similar to the preorder traversal. The only difference is that the right child is visited before left child.

- 1. Push root to first stack.
- 2. Loop while first stack is not empty
- ....2.1 Pop a node from first stack and push it to second stack
- ....2.2 Push left and right children of the popped node to first stack
- 3. Visit contents of second stack

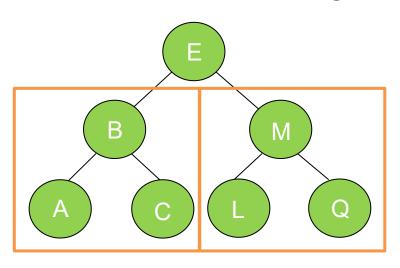
```
Visit: D
Stack 2: A C
Stack 1: A B C F
```

```
public class BinaryTree<E> {
                                                                       Iterative
 TreeNode<E> root;
 public void iterativePostorder() {
 Stack<TreeNode<E>> s1 = new Stack<TreeNode<E>>();
 Stack<TreeNode<E>> s2 = new Stack<TreeNode<E>>();
   ii (root == nuii)
      return;
   s1.push(root);
    while (!s1.isEmpty()) {
     TreeNode<F> temp = s1.pop();
      s2.push(temp);
      if (temp.left != null)
       s1.push(temp.left);
      if (temp.right != null)
        s1.push(temp.right);
   while (!s2.isEmpty()) {
         TreeNode<E> temp = s2.pop();
         temp.visit();
                              visit all elements of second stack
```

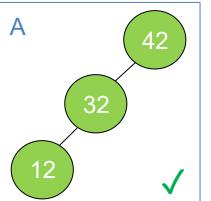
# Motivation for Binary Search Tree



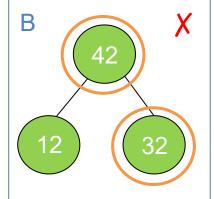
# Binary Search Trees



Left subtree's values must be lesser

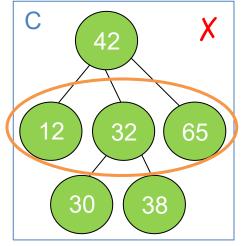


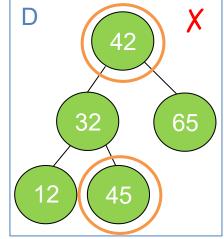
Right subtree's values must be greater



- Ordered, or sorted, binary trees.
- Each node can have 2 subtrees.
- Items to the left of a given node are smaller.
- Items to the right of a given node are larger.

#### Which of these are binary search trees?



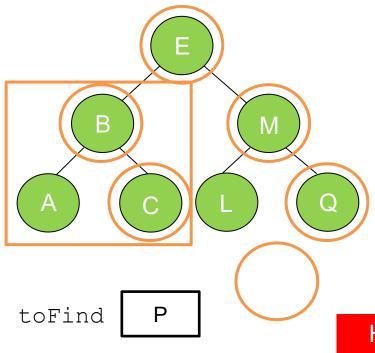


Binary Search Tree Animations | Data Structure | Visual How <a href="https://www.youtube.com/watch?v=ymGjUOiR8Jg">https://www.youtube.com/watch?v=ymGjUOiR8Jg</a>

### **BST Video Tutorials**

- Binary Search Tree : Overview
  - https://www.youtube.com/watch?v=6I3evyt9ApA
- Binary Search Tree : Insert Overview
  - https://www.youtube.com/watch?v=KkEnuK-2Ymc
- Binary Search Tree: Deletion Overview
  - https://www.youtube.com/watch?v=DkOswl0k7s4

# Searching a BST



Same fundamental idea as binary search of an array

Found it!

toFind

С

Compare: E and C

Compare: B and C

Compare: C and C

How to implement this?

You could solve this with recursion.

You could also solve it with iteration by keeping track of your current node.

Not Found!

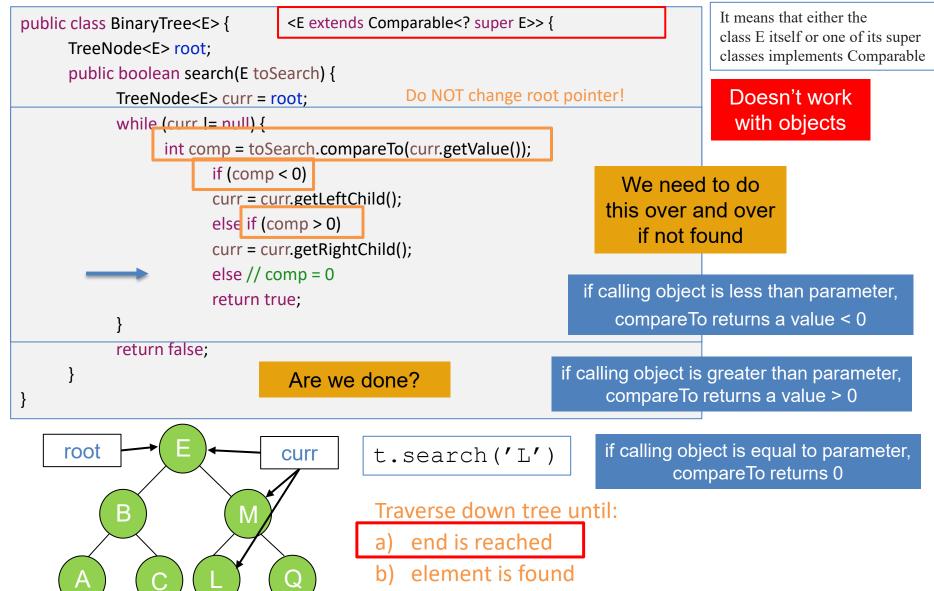
Node is null

Compare: E and P

Compare: M and P

Compare: Q and P

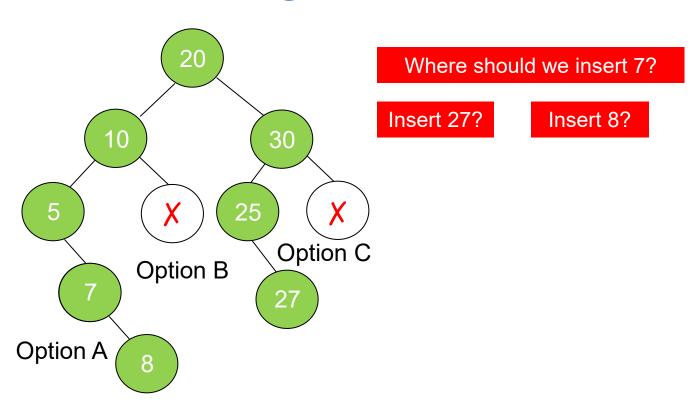
Searching a BST Iteratively



# Searching a BST Recursively

```
public class BinaryTree<E extends Comparable<? super E>> {
  TreeNode<E> root;
                                                  Root of the tree we look at
     private boolean search(TreeNode<=> p, E toSearch) {
          if (p == null)
                                       Tree is empty
                return false:
          int comp = toSearch.compareTo(p.getValue());
          if (comp == 0)
                                       Found it!
                return true;
          else if (comp < 0)
                                                                look left
                return search(p.left, toSearch);
          else // comp > 0
                                                                 look right
                return search(p.right, toSearch);
     public boolean search(E toSearch) {
                                                               root
          return search(root, toSearch);
                                                                     В
                                 t.search('L')
```

# Inserting into a BST



Option D: Either Option A or Option B are fine.

# Deleting from a BST

12 30 15 25

Delete 7

For the smallest value in a node's right subtree, its left child is null

For the largest value in a node's left subtree, its right child is null.

If leaf node: Delete parent's link 7

Delete 5

If only one child, hoist child

Delete 10

When a deleted node has two children, this gets tricky.

Find smallest value in right subtree

Find largest value in left subtree

Replace deleted element with it

OR Replace deleted element with it

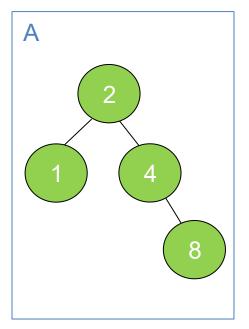
Then delet right subtree duplicate (12)

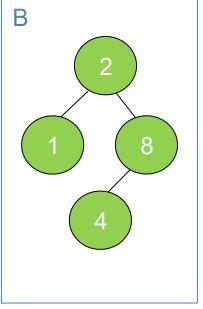
Then delete left subtree duplicate (7)

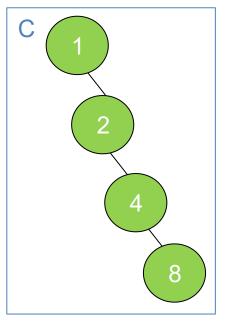
# Binary Search Tree Shape

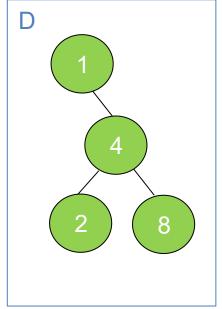
Which of the following Binary Search Trees could be the result of adding elements: 1, 2, 4, and 8 in some order.

#### These are all valid binary search trees!

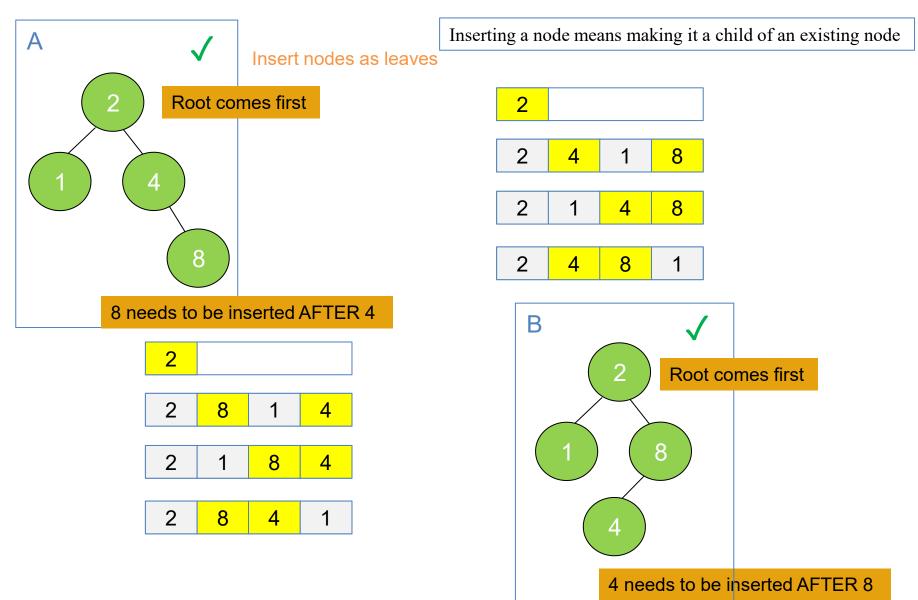




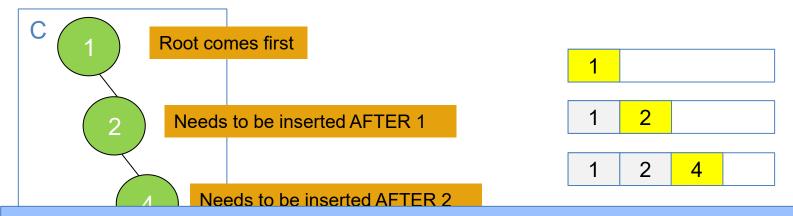




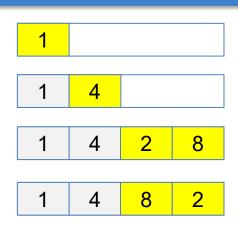
# Binary Search Tree Shape (Contd.)

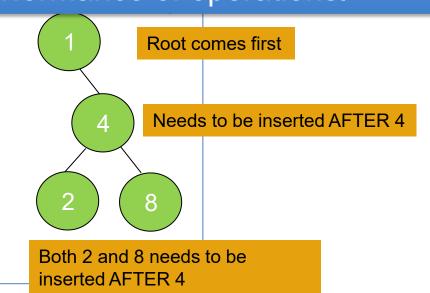


# Binary Search Tree Shape (Contd.)



The order in which we put elements into a BST impacts the shape, and what you'll see is that the shape of BST will have a huge impact on the performance of operations.



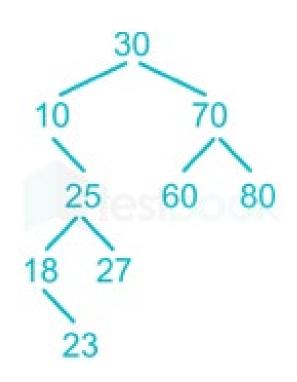


#### Video Tutorial

- Binary Search Trees (BST) Explained in Animated Demo
  - https://www.youtube.com/watch?v=mtvbVLK5xDQ

### In Order Traversal on a BST

 When we perform in order traversal on a binary search tree, we get the ascending order array.



#### Pre-order traversal:

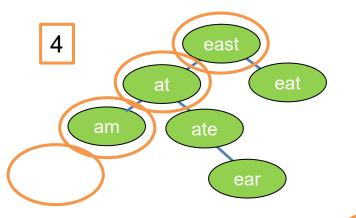
- Traversal sequence: 30, 10, 25, 18, 23, 27, 70, 60, 80
- In-order traversal:
- Traversal Sequence: 10, 18, 23, 25, 27, 30, 60, 70, 80
- Post-order traversal:
- Traversal sequence: 23, 18, 27, 25, 10, 60, 80, 70, 30

# Performance Analysis of BST

Storing a dictionary as a BST

{ am, at, ate, ear, eat, east }

Structure of a BST depends on the order of insertion

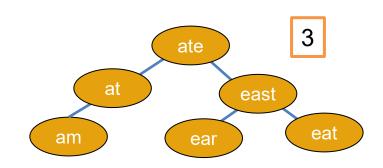


isWord(east)

Best case: O(1)

isWord(a)

Compared with 3 out of 7 words



Performance also depends on the actual structure of the BST

at

6

am

isWord(a)

eat

Compared with all

east

Worst case: O(n)

ear

words

ate

How does the performance of isWord relate to input size n?

#### isWord(String wordToFind)

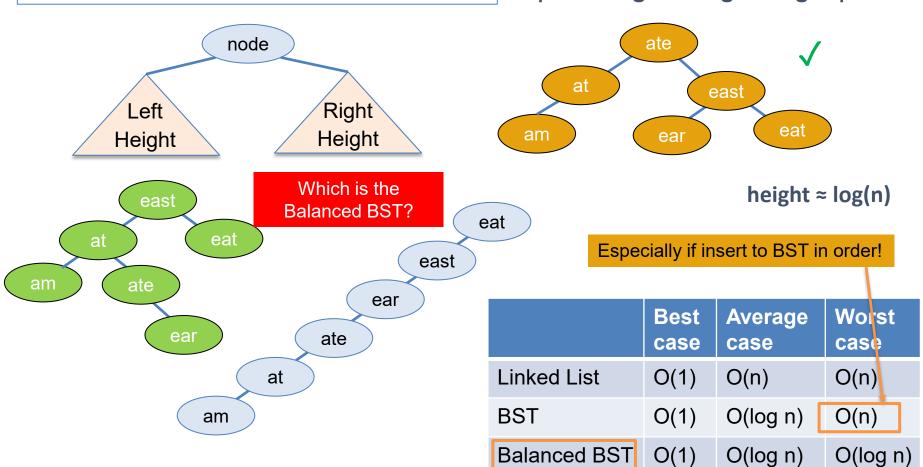
- 1. Start at root
- 2. Compare word to current node
  - 1. If current node is null, return false
  - 2. If wordToFind is less than word at current node, continue searching in left subtree
  - 3. If wordToFind is greater than word at current node, continue searching in right subtree
  - 4. If wordToFind is equal to word at current node, return true

To optimize the worst case, we can modify the tree to control the max distance until leaf height

### **Balanced BST**

We want to keep the height down as much as we can while still maintaining the same number of nodes.

| LeftHeight - RightHeight | <=1



How to keep balanced? TreeSet and TreeMap in Java API

isWord(String wordToFind)

### BST vs. Hash Table

#### Time Complexity

- Average case:
  - Hash Tables generally offer O(1) average time complexity for insertion, deletion, and search operations.
  - BSTs provide O(log n) time complexity for these operations, assuming the tree is balanced.
- Worst case
  - Hash Tables can degrade to O(n) performance in cases of poor hash function design or many collisions.
  - BSTs maintain O(log n) performance even in the worst-case for self-balancing BST.

#### Ordered Operations

- BSTs excel at operations requiring ordered data
  - In-order traversal yields sorted elements.
  - Efficient range searches and finding closest elements.
- Hash Tables do not inherently maintain order, making these operations more difficult.

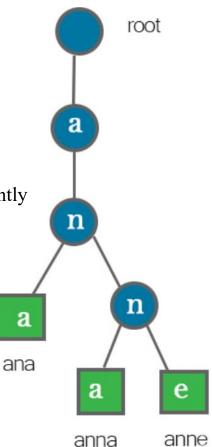
### Tree vs. Trie

#### Structure and Purpose

- Trees:
  - General-purpose data structure for representing hierarchical relationships
  - Each node can contain any type of data
  - Nodes typically have a value and references to child nodes
- Tries:
  - Specialized tree structure for storing and retrieving strings efficiently
  - Also known as a prefix tree
  - Optimized for operations on strings or sequences

#### Node Content

- Trees:
  - Each node stores a value directly
- Tries:
  - Nodes typically do not store complete strings
  - The path from the root to a node represents a string or prefix
  - Characters are stored along the edges between nodes



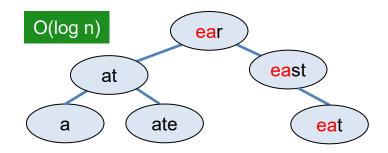
https://romankurnovskii.com/en/posts/tree-vs-trie-data-structures/

### Trie Data Structure

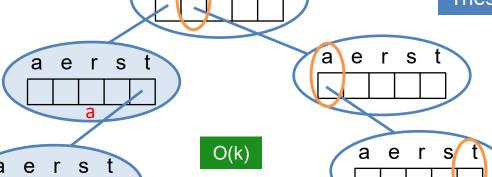
re(TRIE)ve

Storing a dictionary as a (balanced) BST

BSTs don't take advantage of shared structure



Tries: Use the key to navigate the search

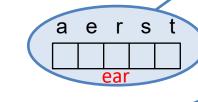


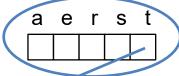
S

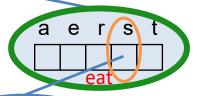
Finding "eat"

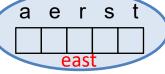
Adding "eats"

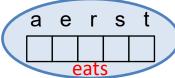
- Not all nodes represent words
- Nodes can have more than 2 children











Trie Data Structure (EXPLAINED)

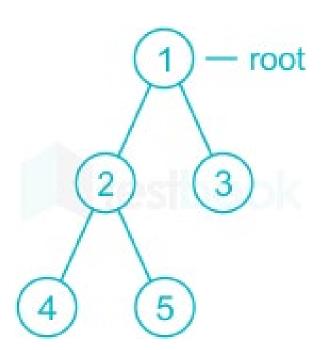
aerst

https://www.youtube.com/watch?v=-urNrIAQnNo

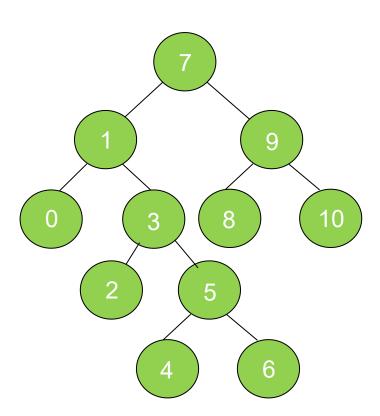
 $\log_2(250000) \approx 18$ 

### Additional Resources

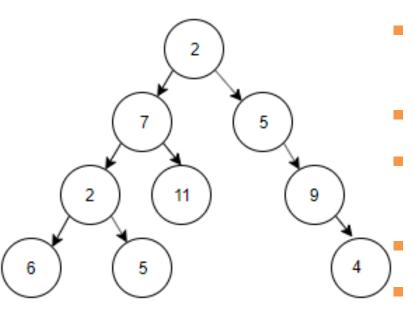
- Trees and Binary Search Trees
  - <u>https://www.geeksforgeeks.org/bfs-vs-dfs-binary-tree/</u> BFS vs DFS for Binary Tree
  - <a href="http://www.openbookproject.net/thinkcs/archive/java/english/chap17.ht">http://www.openbookproject.net/thinkcs/archive/java/english/chap17.ht</a>
     <a href="mailto:m">m -- explains trees</a>, how to build and traverse it
  - <u>http://algs4.cs.princeton.edu/32bst/</u> -- about binary search trees
  - Data structures: Binary Search Tree
    - https://www.youtube.com/watch?v=pYT9F8\_LFTM
- Tries
  - https://www.toptal.com/java/the-trie-a-neglected-data-structure -explains with solid example
  - https://www.topcoder.com/community/data-science/data-sciencetutorials/using-tries/ -- explains as well as providing code



- Pre-order traversal:
- Traversal sequence: 12453
- In-order traversal:
- Traversal Sequence: 42513
- Post-order traversal:
- Traversal sequence: 45231



- Pre-order traversal:
- Begins at the root (7), ends at the rightmost node (10)
- Traversal sequence: 7, 1, 0, 3, 2, 5, 4, 6, 9, 8, 10
- In-order traversal:
- Begins at the left-most node (0), ends at the rightmost node (10)
- Traversal Sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
- Post-order traversal:
- Begins with the left-most node (0), ends with the root (7)
- Traversal sequence: 0, 2, 4, 6, 5, 3, 1, 8, 10, 9, 7



- Pre-order traversal:
- Traversal sequence: 2, 7, 2, 6, 5, 11,5, 9, 4
- In-order traversal:
- Traversal Sequence: 6, 2, 5, 7, 11, 2, 5,9, 4
  - **Post-order traversal:**
  - Traversal sequence: 6, 5, 2, 11, 7, 4, 9, 5, 2

- 1. \*\*What is the sequence of nodes visited in a Preorder traversal?\*\*
- a) Left, Root, Right
- b) Root, Left, Right
- c) Left, Right, Root
- d) Right, Root, Left

ANS: b

- 2. \*\*Which traversal method is used to get nodes of a Binary Search Tree in ascending order?\*\*
- a) Preorder
- b) Inorder
- c) Postorder
- d) Level order

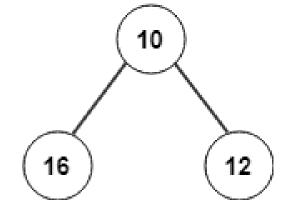
• 3. \*\*In a Postorder traversal, when is the root node visited?\*\*

- a) First
- b) After visiting the left subtree
- c) After visiting both left and right subtrees
- d) Before visiting any subtree

ANS: c

ANS: c

- The following given tree is an example for?
- a) Binary tree
- b) Binary search tree
- c) Fibonacci tree
- d) AVL tree



Explanation: The given tree is an example for binary tree since has got two children and the left and right children do not satisfy binary search tree's property, Fibonacci and AVL tree.

ANS: a

- How many common operations are performed in a binary tree?
- **a**) 1

ANS: c

- **b**) 2
- **c**) 3
- **d)** 4

Explanation: Three common operations are performed in a binary tree- they are insertion, deletion and traversal.