L1 (CHAPTER 2)

Data Representation Exercises

Decimal, Binary and Hex

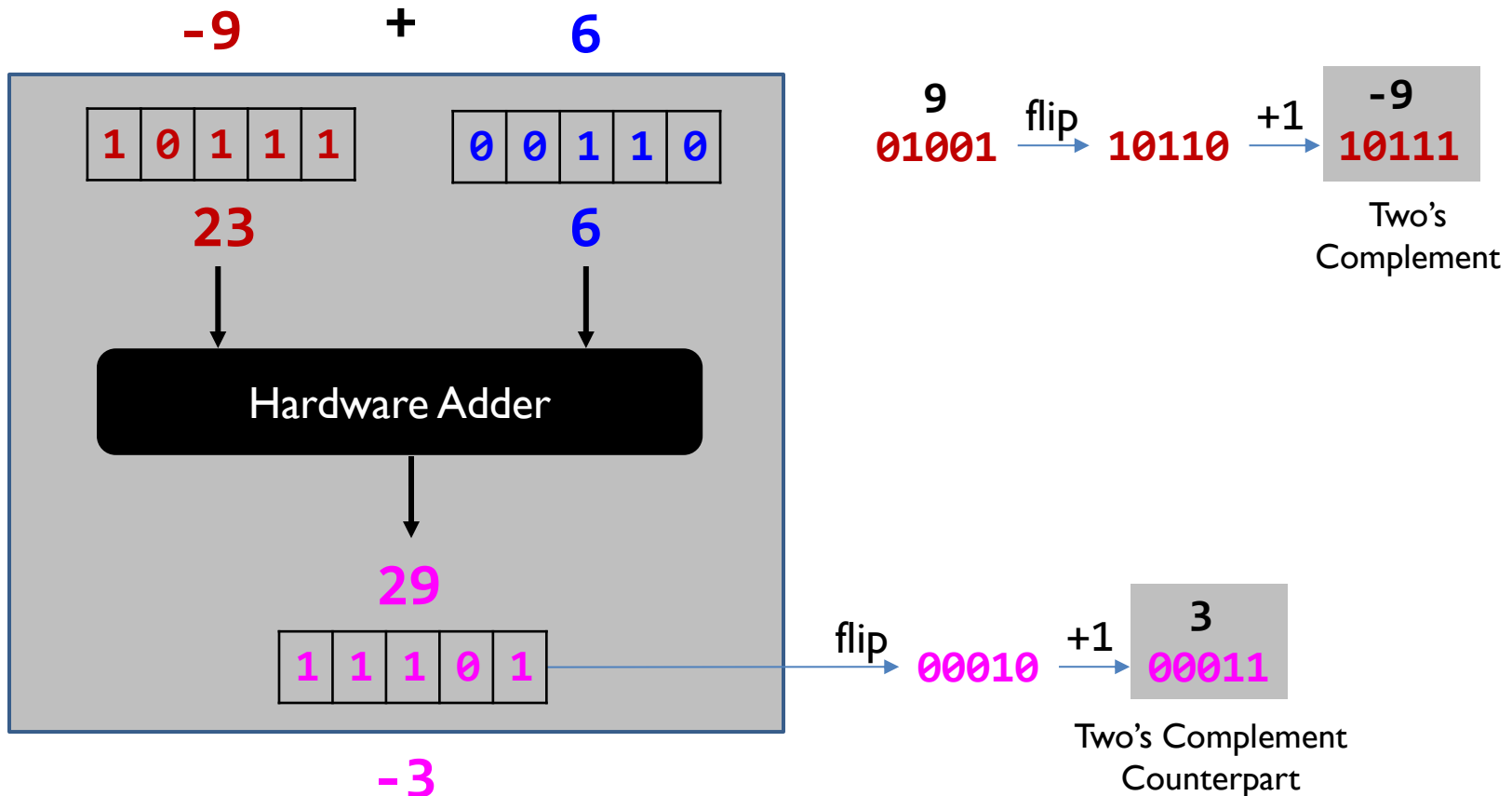| Decimal	Binary	Hex
0	0000	0x0
1	0001	0x1
2	0010	0x2
3	0011	0x3
4	0100	0x4
5	0101	0x5
6	0110	0x6
7	0111	0x7
8	1000	0x8
9	1001	0x9
10	1010	0xA
11	1011	0xB
12	1100	0xC
13	1101	0xD
14	1110	0xE
15	1111	0xF

Prefix 0x denotes hex

Question: Number Conversion

▶ Q: Convert 0x3A56E2F8 into binary

▶ Q: Convert binary number 111010 into hex

Adding two integers

-9 **+** **6**

1	0	1	1	1

23

0	0	1	1	0

6

Hardware Adder

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1	1	1	0	1

-3

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01001 — flip → **10110** — +1 → **-9** **10111**

Two's Complement

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flip → **00010** — +1 → **3** **00011**

Two's Complement Counterpart

▸ Same bit patterns, different interpretation.
   ▸ Unsigned addition: 23+6=29
   ▸ Signed addition: -9+6=-3
▸ This example shows that the hardware adder for adding unsigned numbers, also works correctly for adding signed numbers.
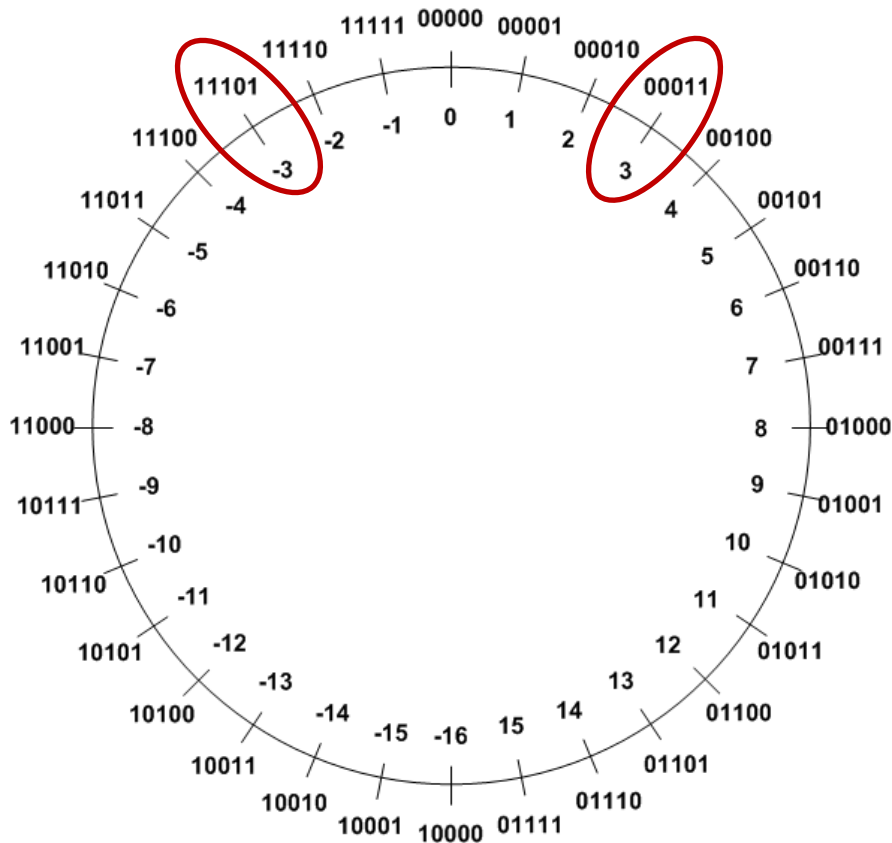
Signed Integers
Method 3: Two's Complement

Two's Complement ($\overline{\alpha}$):

$$\alpha + \overline{\alpha} = 2^n$$

TC of a number can be obtained by its bitwise NOT plus one.

Example **1**: TC(3)

	Binary	Decimal
Original number	00011	3
Step 1: Invert every bit	11100	
Step 2: Add 1	+ 00001	
Two's complement	11101	-3

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Signed Integer Representation Overview

- Three ways to represent signed binary integers:
  - Signed magnitude
    - $value = (-1)^{sign} \times Magnitude$
  - One's complement ($\widetilde{\alpha}$)
    - $\alpha + \widetilde{\alpha} = 2^n - 1$
  - Two's complement ($\overline{\alpha}$)
    - $\alpha + \overline{\alpha} = 2^n$

	Sign-and-Magnitude	One's Complement	Two's Complement
Range	$[-2^{n-1} + 1, 2^{n-1} - 1]$	$[-2^{n-1} + 1, 2^{n-1} - 1]$	$[-2^{n-1}, 2^{n-1} - 1]$
Zero	Two zeroes (± 0)	Two zeroes (± 0)	One zero
Unique Numbers	$2^n - 1$	$2^n - 1$	2^n

Question: 2's Complement

- For each of the following binary numbers, give the corresponding binary number of the negative of its value, for 2's-complement system
- (a) x=01010101
- (b) x=10101010
- (c) x=10000000

Question: Number Conversion

- Q: What is the decimal value of binary number x=10100111 as either unsigned int, or signed int in 2's complement representation?
- What about x=11100001?
- What about x=10000000?

Question: Number Conversion

- Q: Which number is larger: 1001 or 0011 in binary?
- Q: Which number is larger: 0xFFFFFFFF or 0x00000001 in hex?

Question: Number Range

- Which range of decimals can be expressed with a 6-bit number (assuming Two's complement representation)?

Answer	Range
A	-32 … 32
B	-64 … 63
C	-31 … 32
D	-16 … 15
E	-32 … 31

Question: Number Range

- Which range of decimals can be expressed with a 6-bit unsigned integer?

Answer	Range
A	-32 … 32
B	-64 … 63
C	-31 … 32
D	-16 … 15
E	-32 … 31

Question: Integer arithmetic

- Q: Consider a 4-bit system. What is the result of 1001 + 0011?

Summary of Carry and Overflow Flags

Bit	Name	Meaning after add or sub
N	negative	result is negative
Z	zero	result is zero
V	overflow	signed overflow
C	carry	unsigned overflow

Carry flag C = 1 upon an **<u>unsigned</u>** addition if the answer is wrong (true result > 2^n-1)

Carry flag C = 0 (Borrow flag = 1) upon an <u>unsigned</u> subtraction if the answer is wrong (true result < 0)

Overflow flag V =1 upon a **<u>signed</u>** addition if the answer is wrong (true result > 2^{n-1}-1 or true result < -2^{n-1})

31   0

N	Z	C	V	

CPSR (Current Program Status Register)

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Signed or unsigned

▸ Whether the carry flag or the overflow flag should be used depends on the programmer's intention.

```
uint a;
uint b;
…
c = a + b
…
```

C Program

Check the carry flag
for unsigned addition

```
int a;
int b;
…
c = a + b
…
```

C Program

Check the overflow flag
for signed addition

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Question: Addition

▸ Q: Consider a 4-bit system. What is the result of addition 1011+0110, assuming either unsigned integers, or signed integers in 2's-complement representation?

Question: Subtraction

- Q: Q: Consider a 4-bit system. What is the result of subtraction 1011-0110, assuming either unsigned integers, or signed integers in 2's-complement representation?

Question: Subtraction

▶ Q: Consider a 4-bit system. What is the result of subtraction 0110-1011, assuming either unsigned integers, or signed integers in 2's-complement representation?

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Question: True or False

- 1. Borrow=1 is impossible when subtracting one unsigned number from another.

- 2. Overflow=1 is impossible when subtracting two signed operands of the same sign.

- 3. There are two representations of zero in 2's complement representation.

- 4. In 2's complement, the absolute values of full-scale negative and full-scale positive are identical

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