

Chapter 8

Stack and Recursive Functions

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Recursive Functions

- ▶ A recursive function is one that solves its task by calling itself on smaller pieces of data.
- ▶ An effective tactic is to
 - ▶ divide a problem into sub-problems of the same type as the original,
 - ▶ solve those sub-problems, and
 - ▶ combine the results

Defining Factorial(n)

Product of the first n numbers

$$1 \times 2 \times 3 \times \dots \times n$$

$$\text{factorial}(0) = 1$$

$$\text{factorial}(1) = 1 = 1 \times \text{factorial}(0)$$

$$\text{factorial}(2) = 2 \times 1 = 2 \times \text{factorial}(1)$$

$$\text{factorial}(3) = 3 \times 2 \times 1 = 3 \times \text{factorial}(2)$$

$$\text{factorial}(4) = 4 \times 3 \times 2 \times 1 = 4 \times \text{factorial}(3)$$

$$\text{factorial}(n) = n \times (n-1) \times \dots \times 1 = n \times \text{factorial}(n-1)$$

Classic Example: Factorial

- ▶ Factorial is the classic example:
 - ▶ $6! = 6 \times 5!$
 - ▶ $6! = 6 \times 5 \times 4!$
 - ...
 - ▶ $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- ▶ The factorial function can be easily written as a recursive function:

```
int Factorial(int n) {  
  
    if (n < 2)  
        return 1; /* base case */  
  
    return (n * Factorial(n - 1));  
}
```

Classic Example: Fibonacci Numbers

$$f(n) = f(n-1) + f(n-2)$$

$$f(0) = 1$$

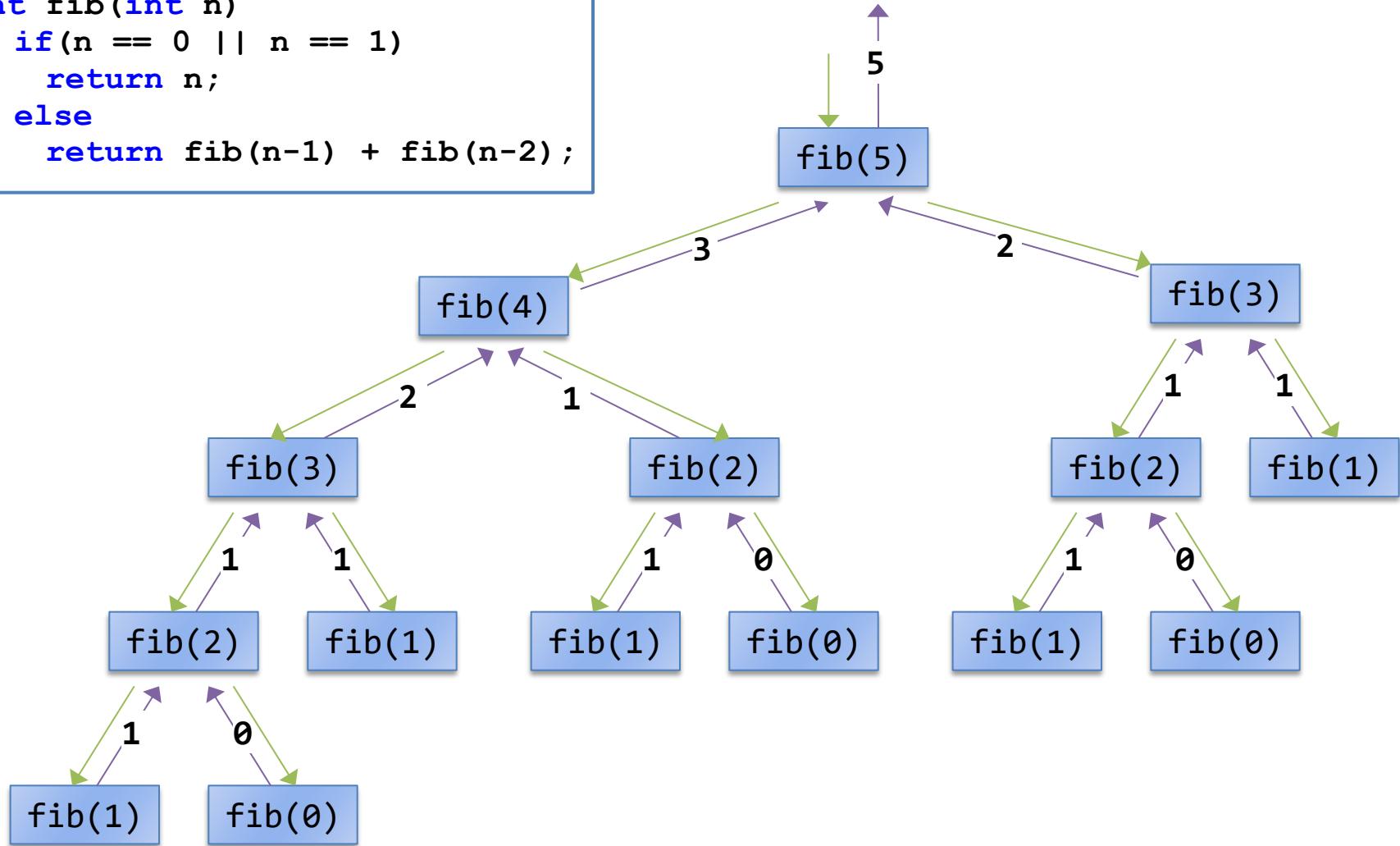
$$f(1) = 1$$

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

```
int Fibonacci(int n) {  
  
    if (n <= 1)  
        return 1; /* base case */  
  
    return (Fibonacci(n-1) + Fibonacci(n-2));  
}
```

Analysis of fib(5)

```
int fib(int n)
    if(n == 0 || n == 1)
        return n;
    else
        return fib(n-1) + fib(n-2);
```



Example of Recursive Function: Testing Palindrome

```
bool isPalindrome(char* s, int len) {  
    if(len < 2)  
        return TRUE;  
    else  
        return s[0] == s[len-1] && isPalindrome(&s[1], len-2);  
}
```

Recursion vs Iteration

Any problem that can be solved **recursively** can also be solved **iteratively** (using loop).

Recursive functions vs Iterative functions

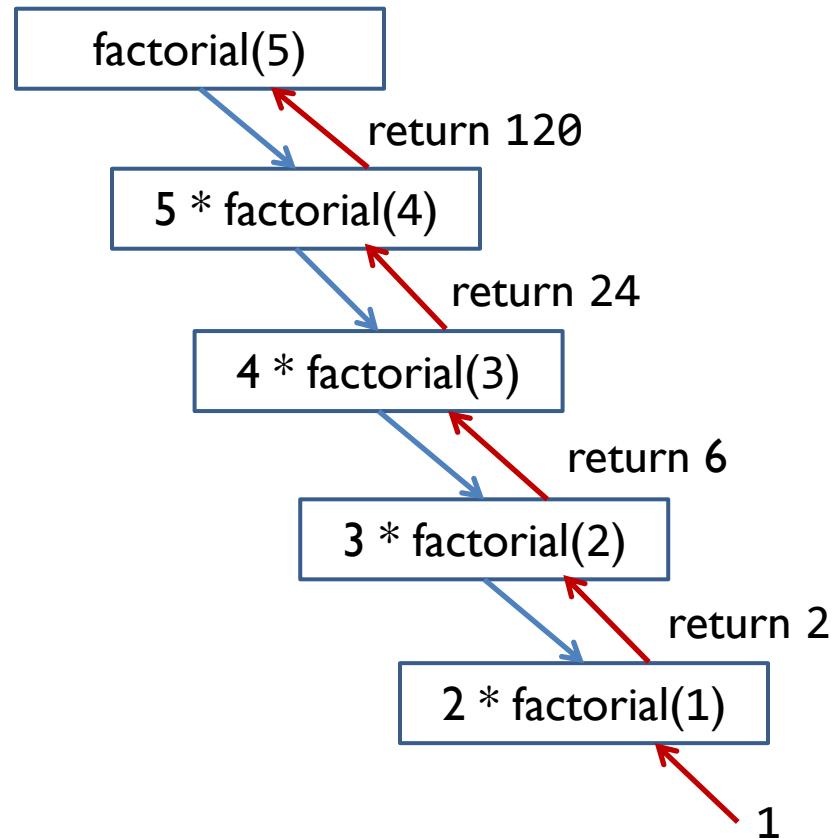
- ▶ Cons:
 - ▶ Recursive functions are slow
 - ▶ Recursive function take more memory
- ▶ Pros
 - ▶ Recursive functions resembles the problem more naturally
 - ▶ Recursive function are easier to program, and debug.

Recursive Factorial in C

```
int factorial(int n);

int main(void){
    factorial(5);
    return 0;
}

int factorial(int n) {
    int f;
    if(n==1)
        return 1;
    else
        f = n * factorial(n-1);
    return f;
}
```



Recursive Functions

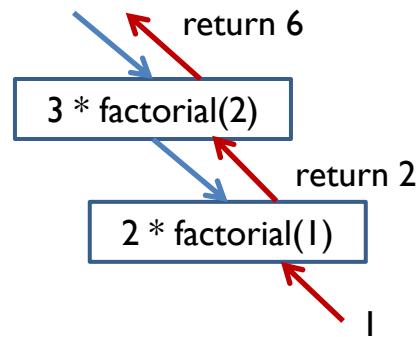
- ▶ PUSH LR (& working registers) onto stack before nested call
- ▶ POP LR (& working registers) off stack after nested return

Recursive Factorial in Assembly

```
AREA main, CODE, READONLY
EXPORT __main
ENTRY
_main PROC
    MOV r0, #0x03          ; Set argument n = 3 in r0 (r0 holds first arg)
    BL factorial            ; Call factorial(n); LR gets return address;
                            ; result returns in r0
stop B stop                ; Halt by branching to self (infinite loop)
ENDP

; --- Recursive factorial(n) ---
factorial
    PUSH {r4, lr}           ; Save callee-used r4 and return address LR on stack
    MOV r4, r0                ; Preserve n in r4 across the recursive call
    CMP r4, #0x01             ; Check base case: n == 1 ?
    BNE NZ                   ; If n != 1, branch to NZ for recursive case
    MOVS r0, #0x01            ; Base case: return 1 in r0
loop POP {r4, pc}           ; restore r4, and return by popping PC
NZ SUBS r0, r4, #1           ; Prepare argument r0 = n - 1 for recursive call
BL factorial                ; r0 <- factorial(n-1) after return
MUL r0, r4, r0               ; r0 = n * factorial(n-1)
B loop
END
```

Trace Function Execution for this Call Sequence



Recursive Factorial in Assembly

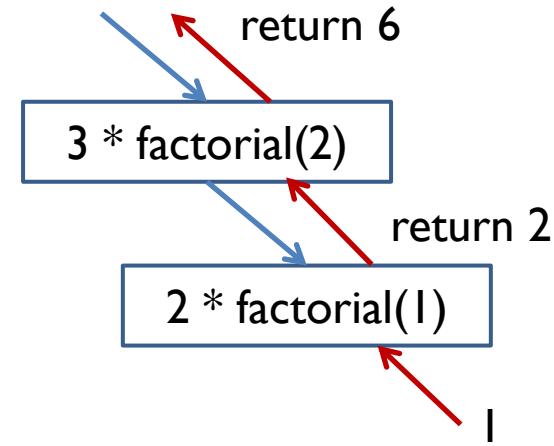
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ENTRY

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ENDP

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PUSH {r4, lr}
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CMP r4, #0x01
BNE NZ
MOVS r0, #0x01
loop POP {r4, pc}
NZ SUBS r0, r4, #1
BL factorial
MUL r0, r4, r0
B loop

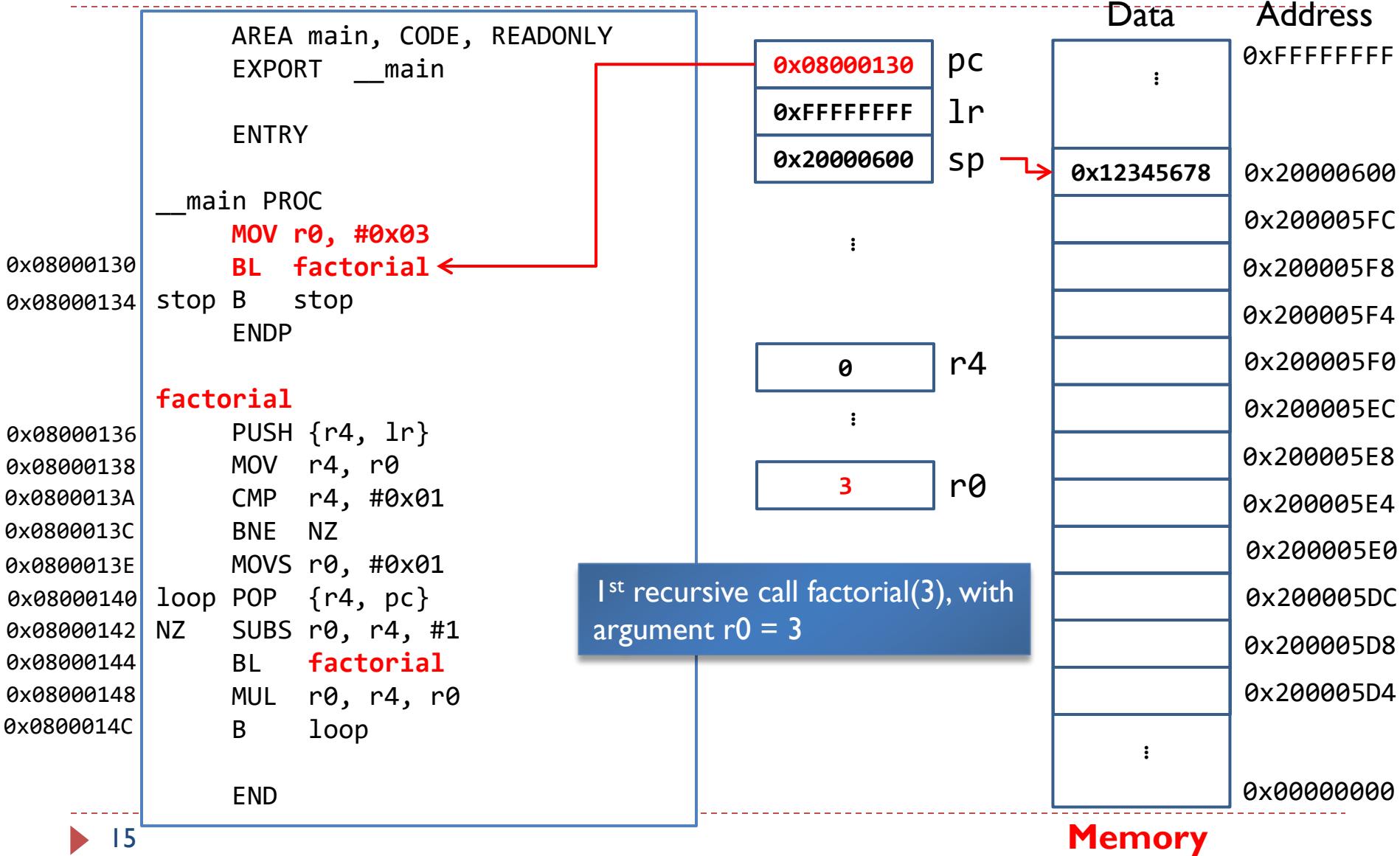
END
```



Recursive Factorial in Assembly

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EXPORT __main
ENTRY
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    B stop                ; Halt by branching to self (infinite loop)
ENDP
; --- Recursive factorial(n) ---
factorial
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stack
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    CMP r4, #0x01           ; Check base case: n == 1 ?
    BNE NZ                 ; If n != 1, branch to NZ for recursive case
    MOVS r0, #0x01          ; Base case: return 1 in r0
loop
    POP {r4, pc}            ; restore r4, and return by popping PC
NZ
    SUBS r0, r4, #1          ; Prepare argument r0 = n - 1 for recursive call
    BL factorial            ; r0 <- factorial(n-1) after return
    MUL r0, r4, r0           ; r0 = n * factorial(n-1)
    B loop
END
```

Recursive Factorial in Assembly



Recursive Factorial in Assembly

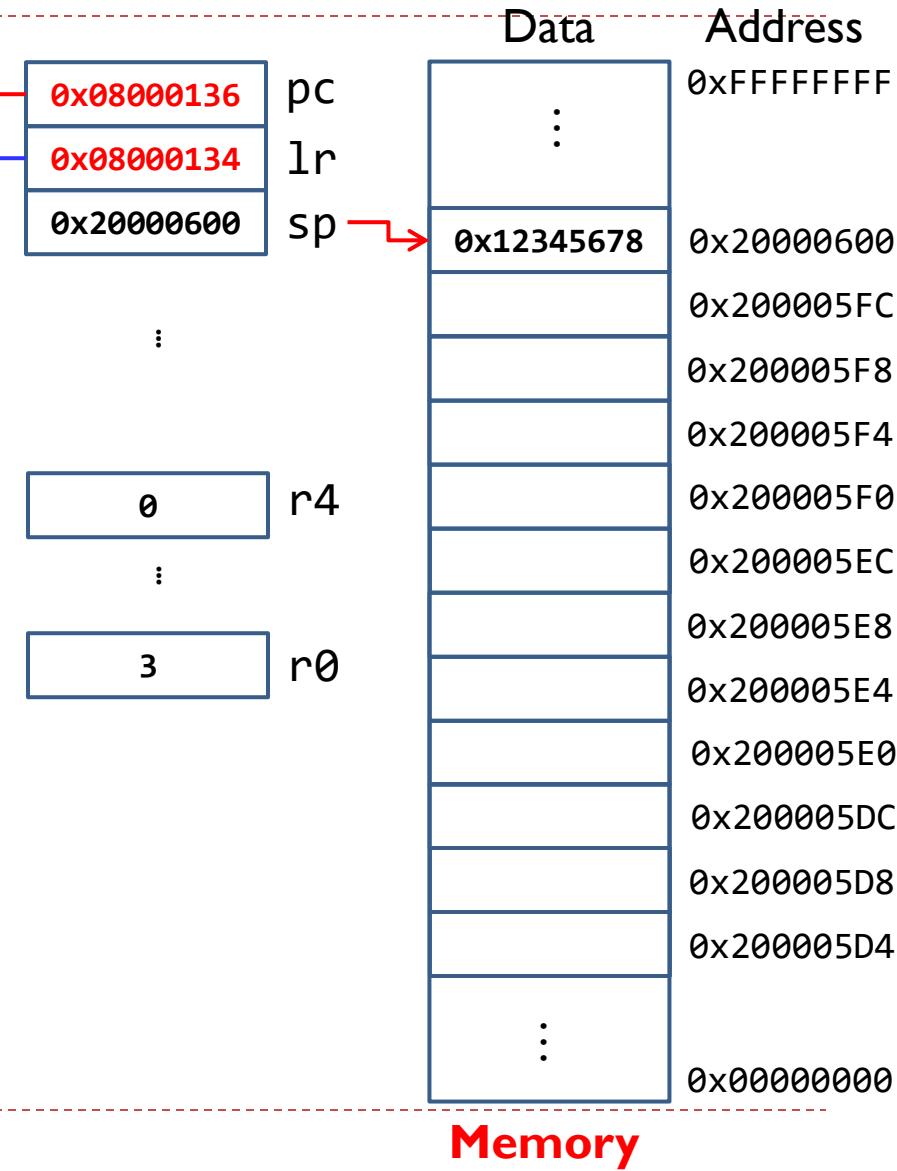
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END
```



Recursive Factorial in Assembly

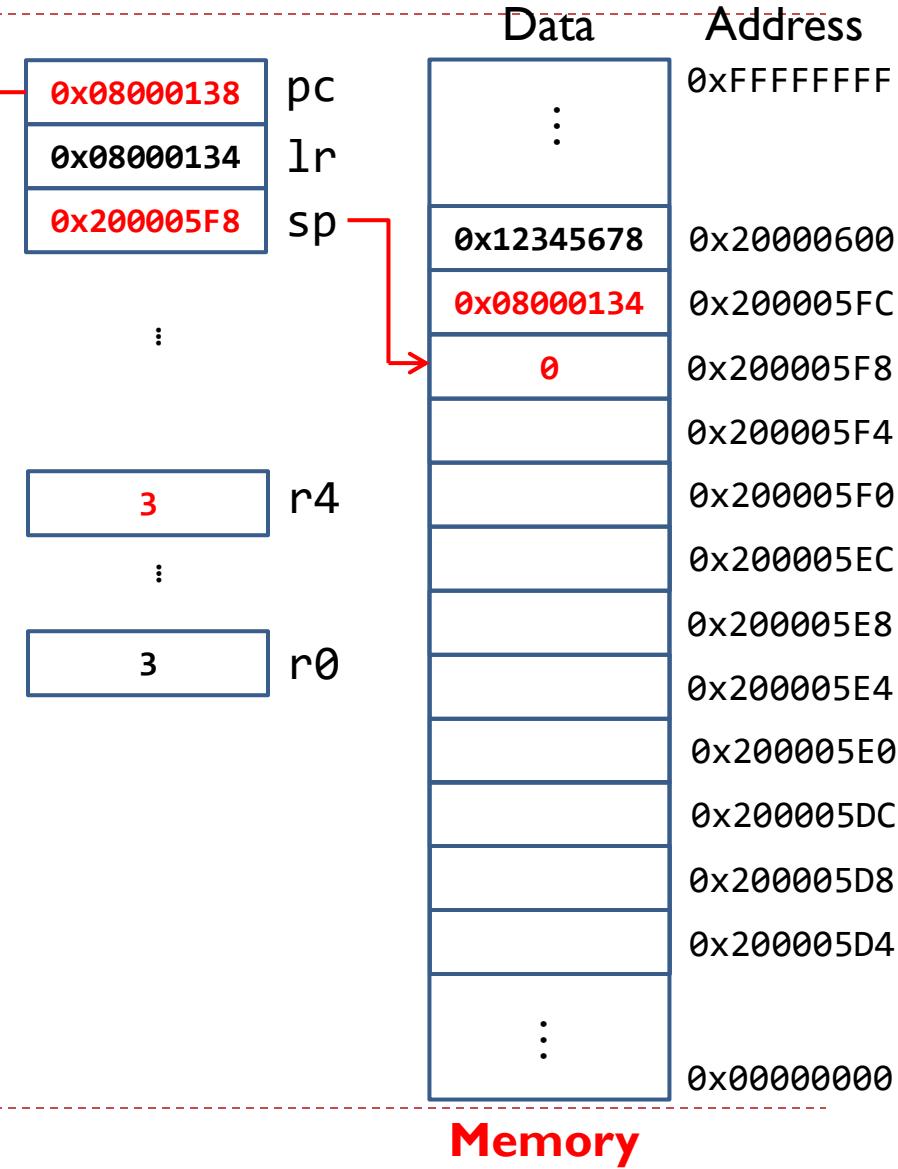
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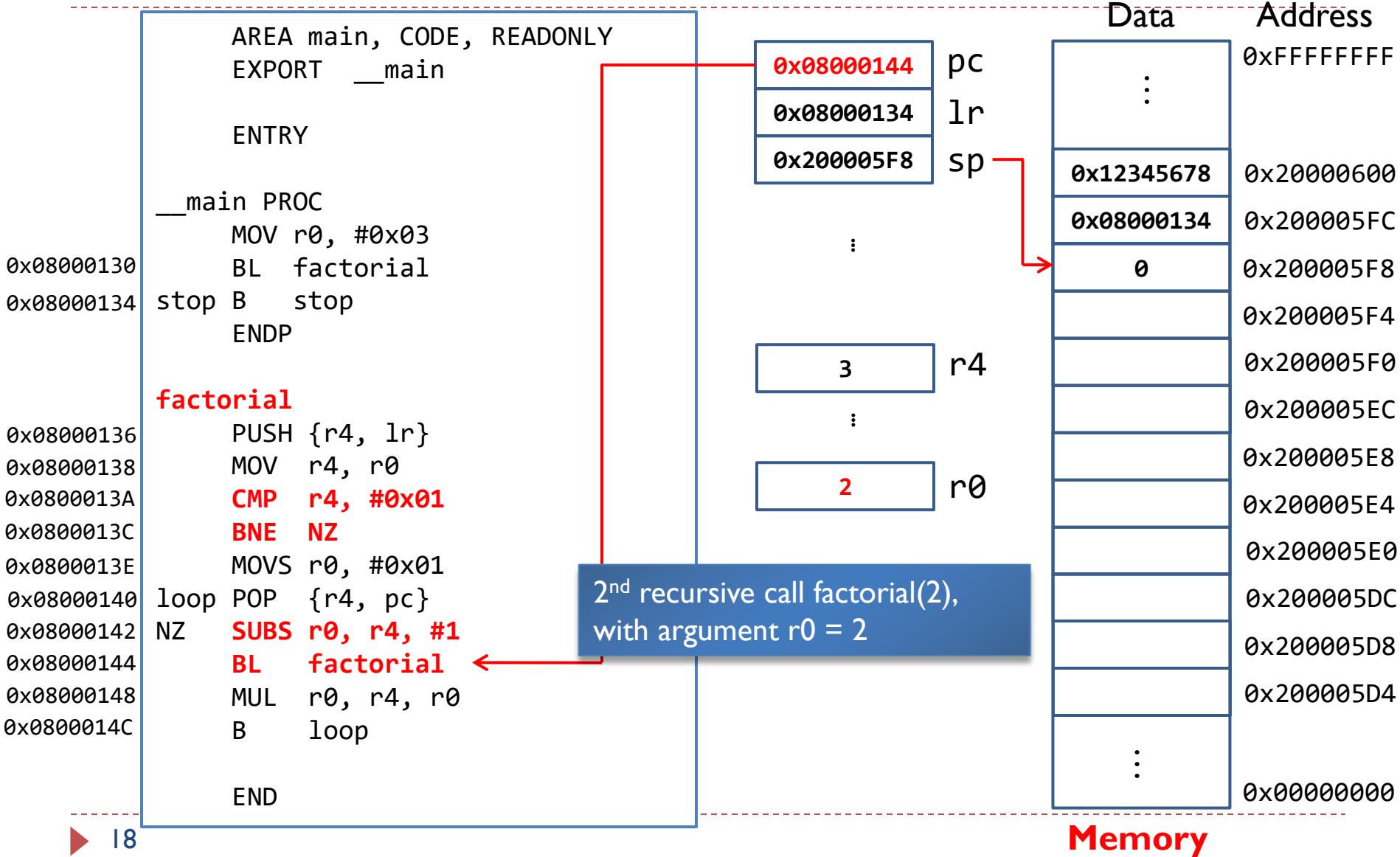
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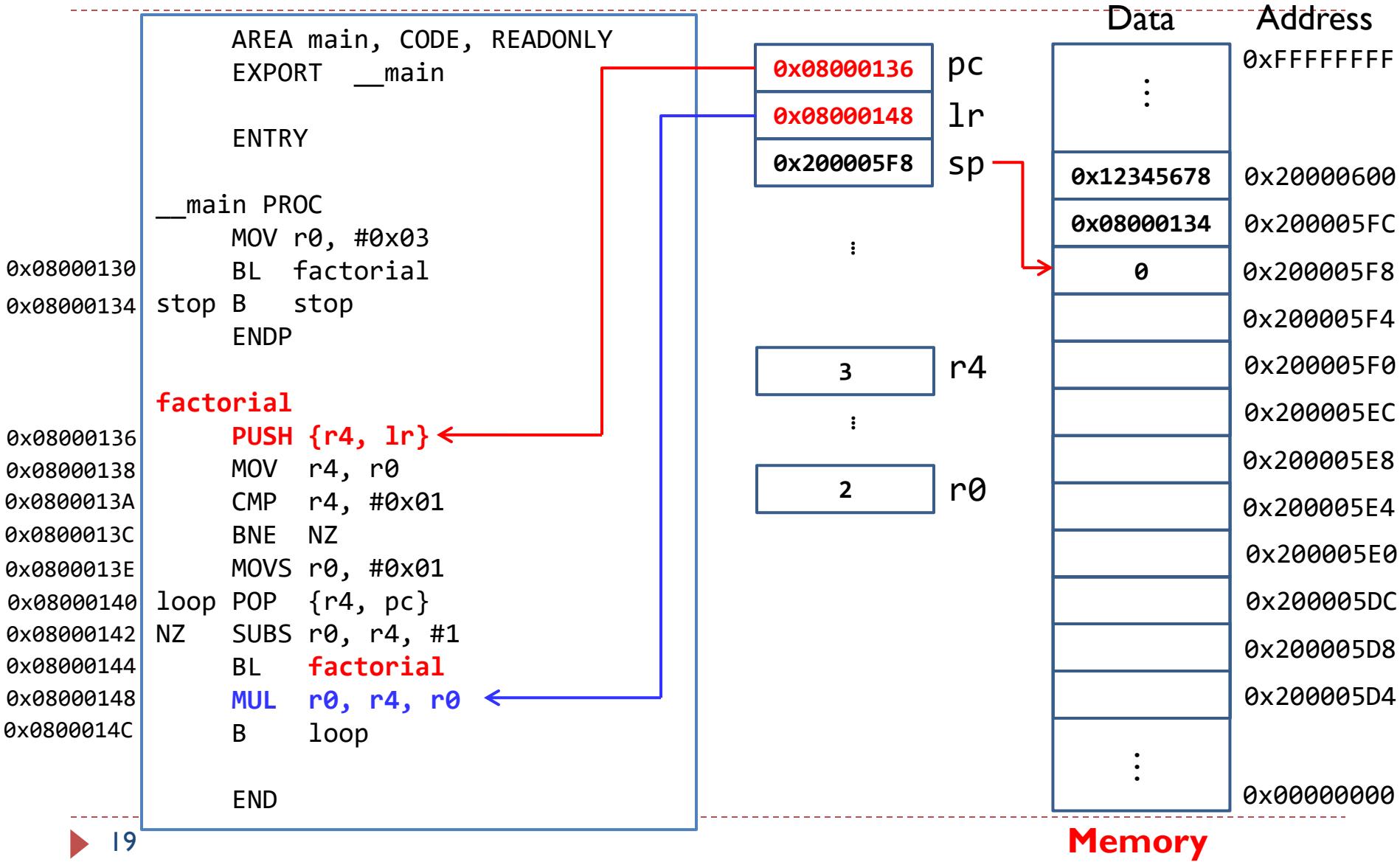
END
```



Recursive Factorial in Assembly



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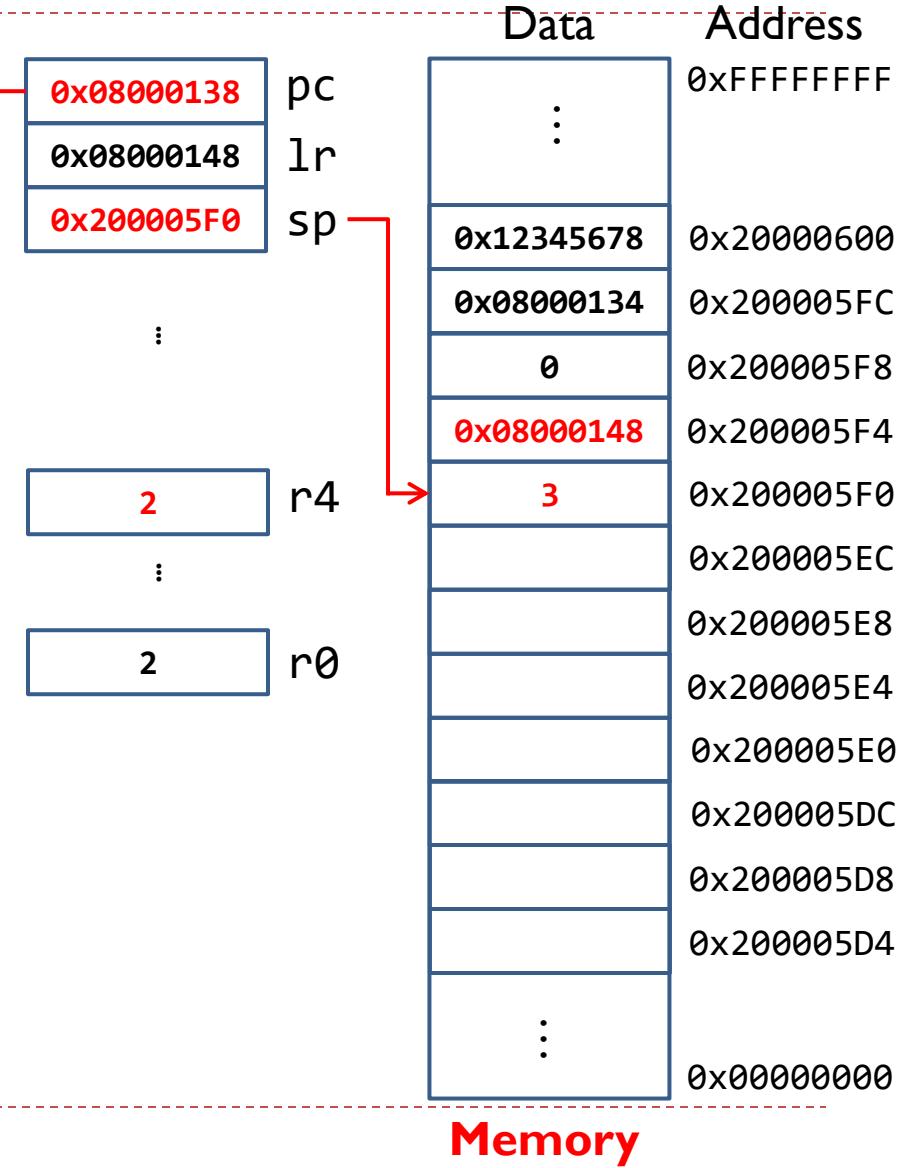
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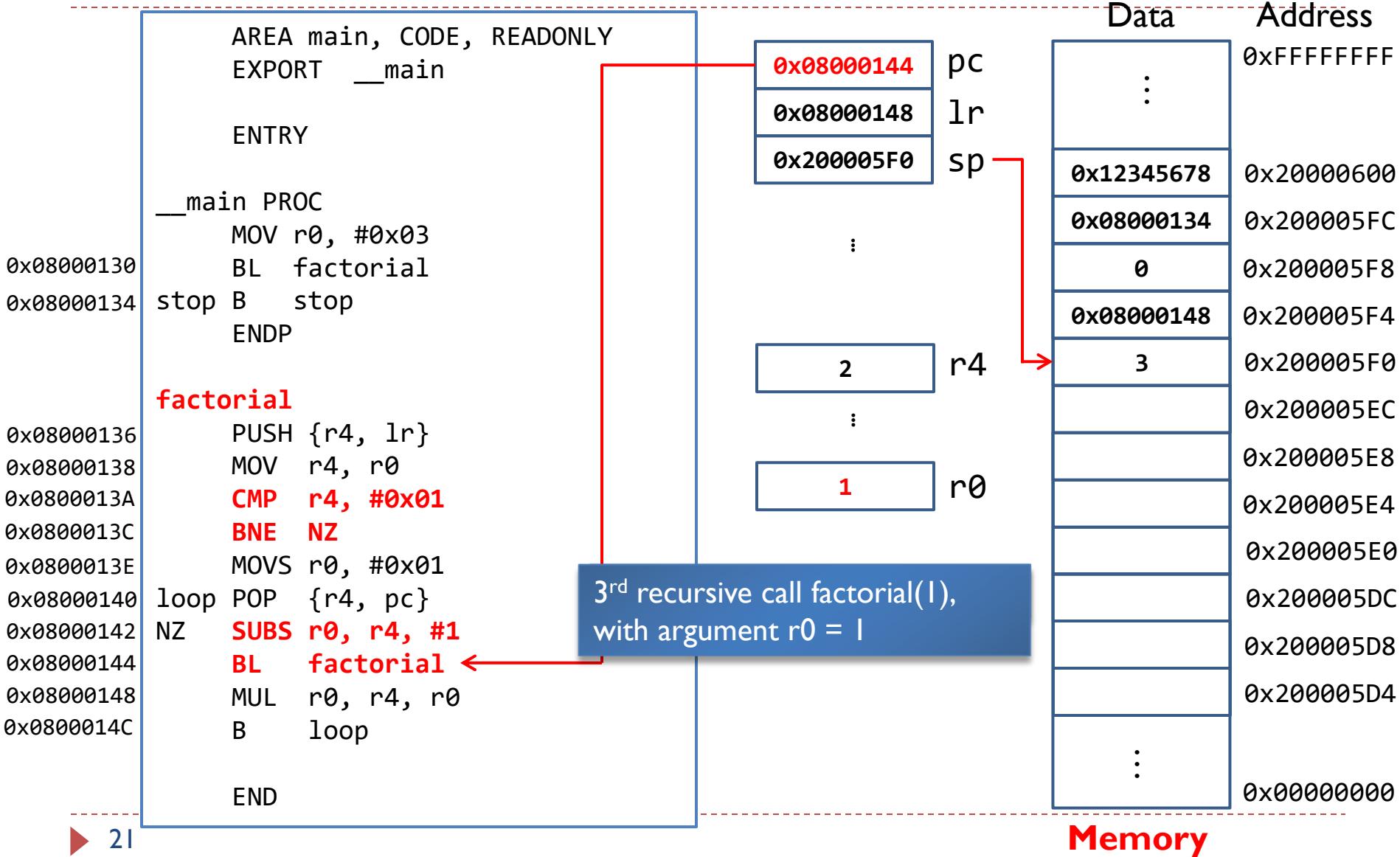
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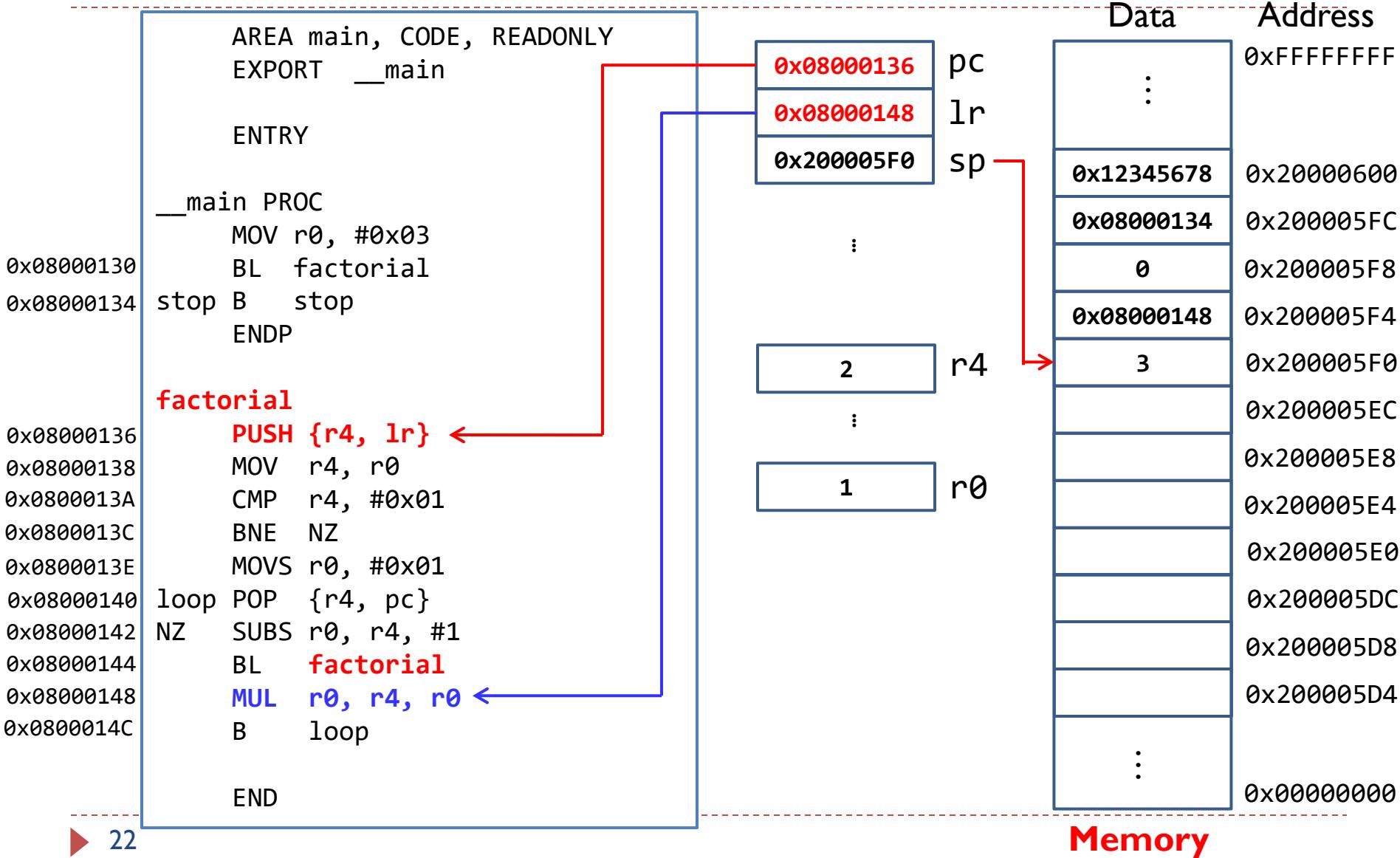
END
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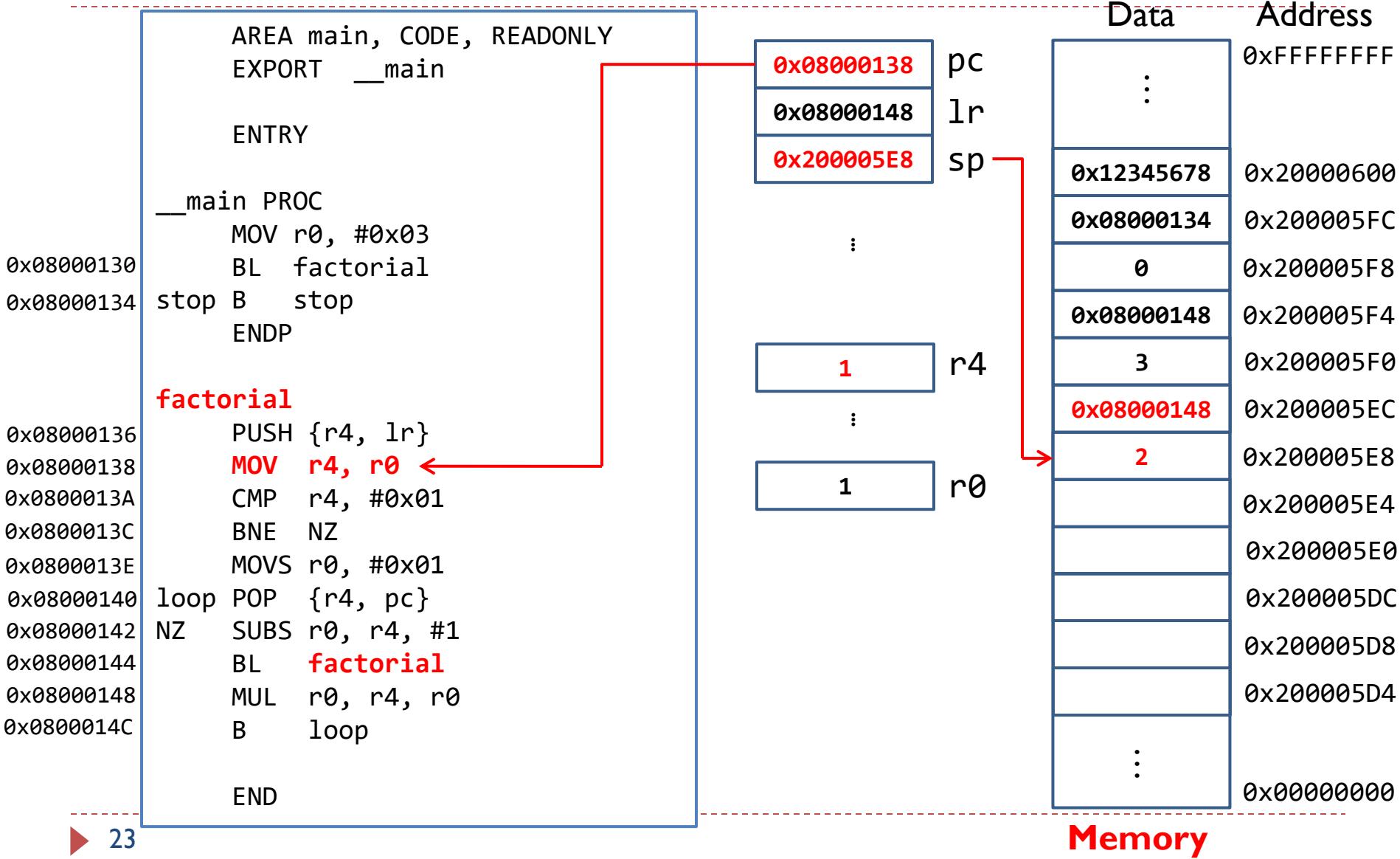
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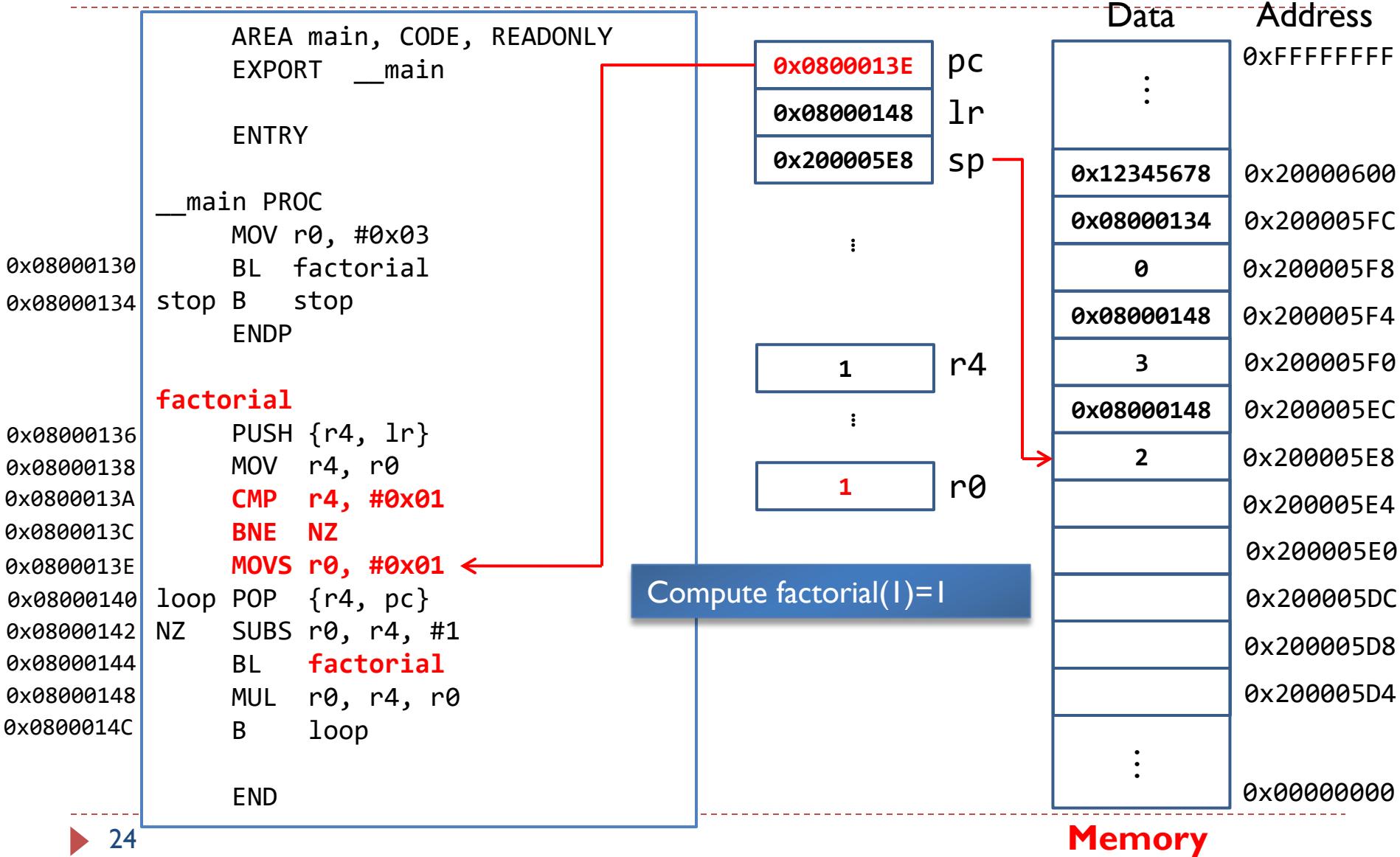
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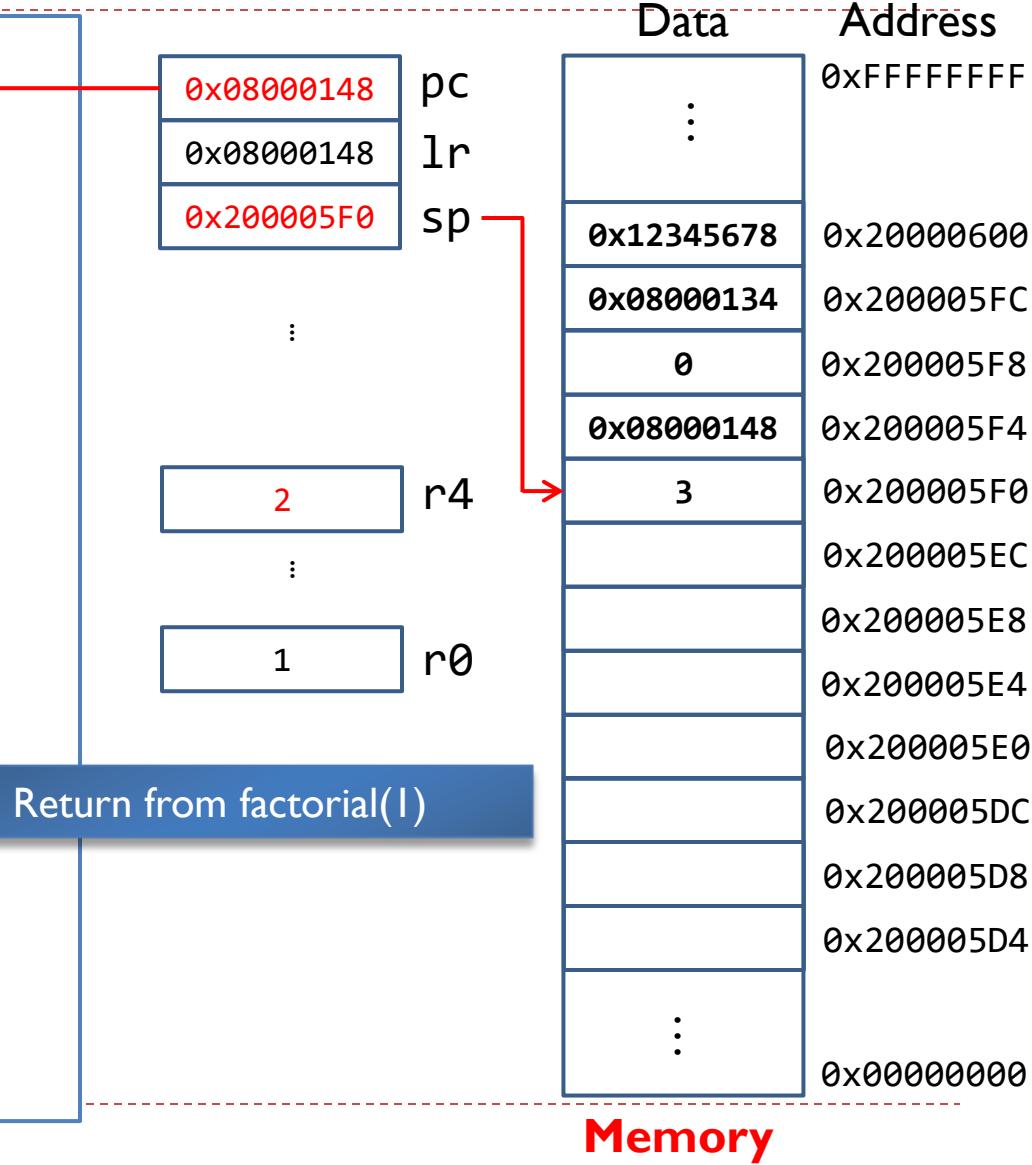
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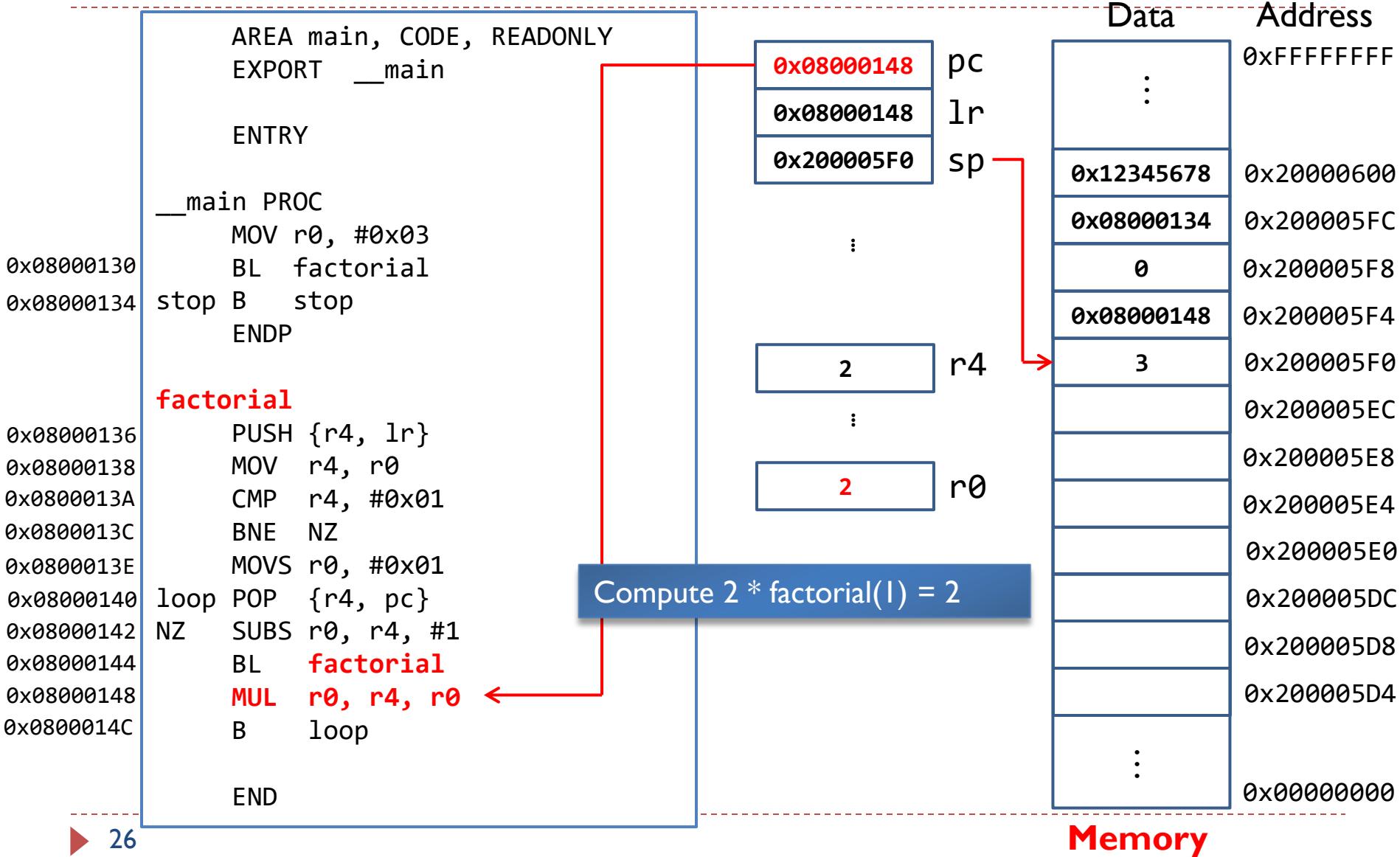
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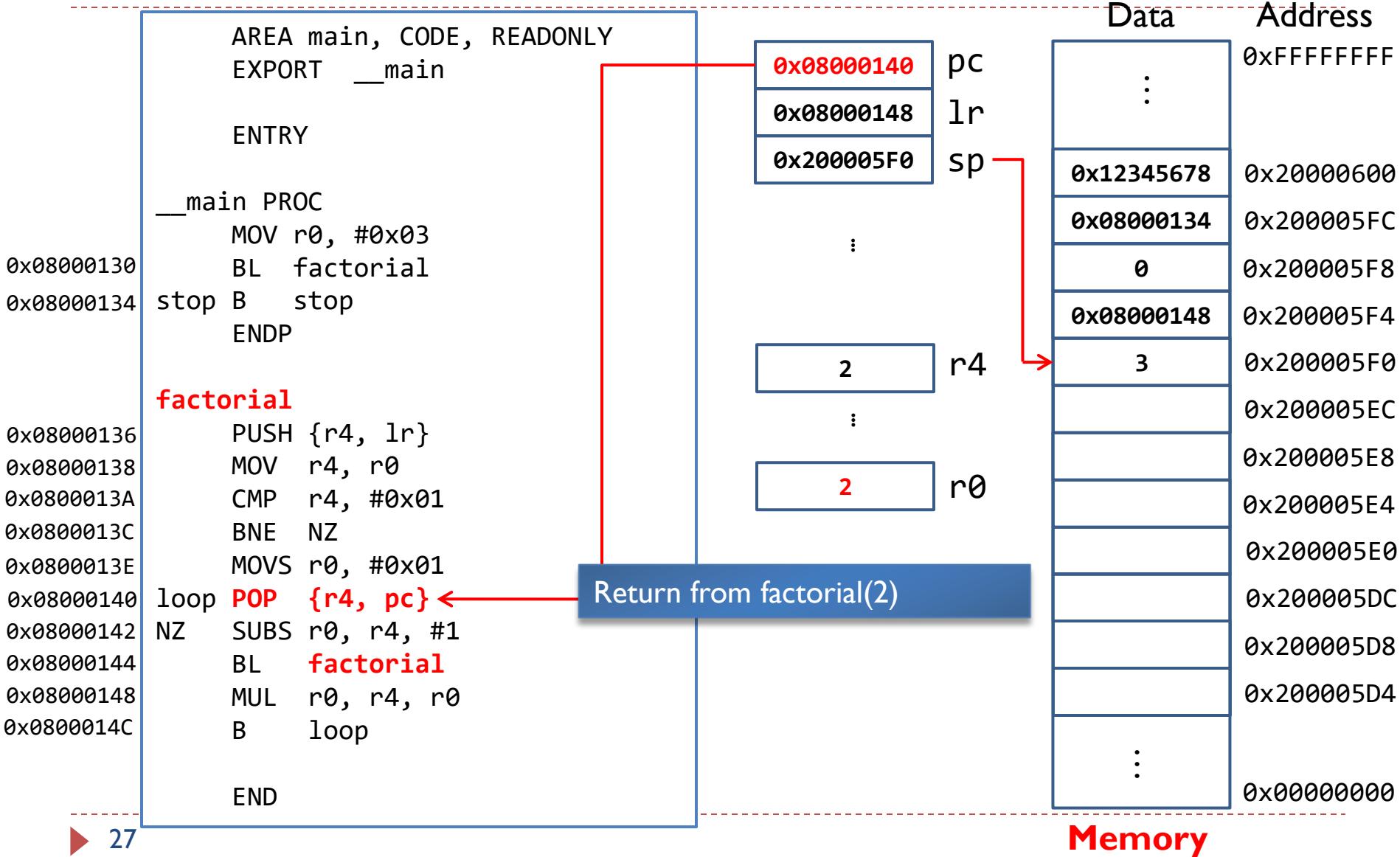
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Recursive Factorial in Assembly



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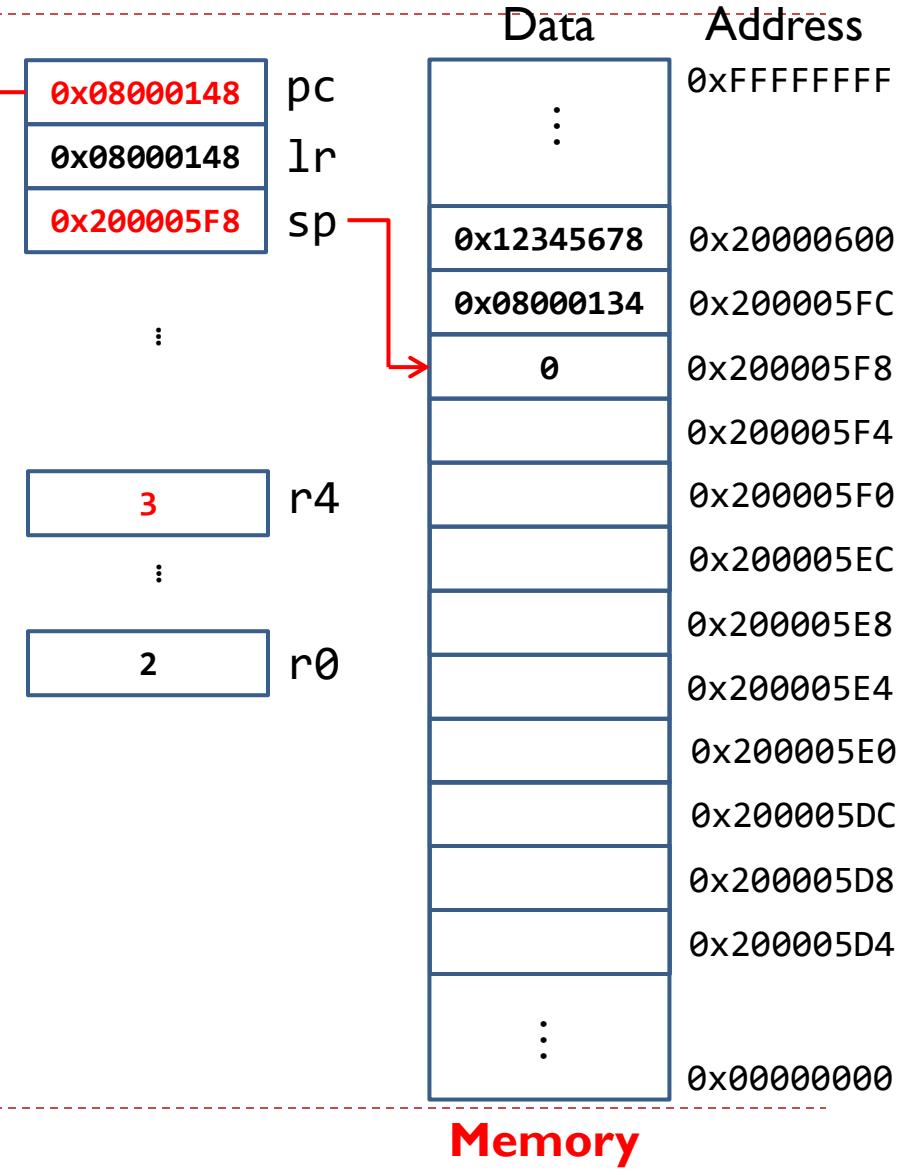
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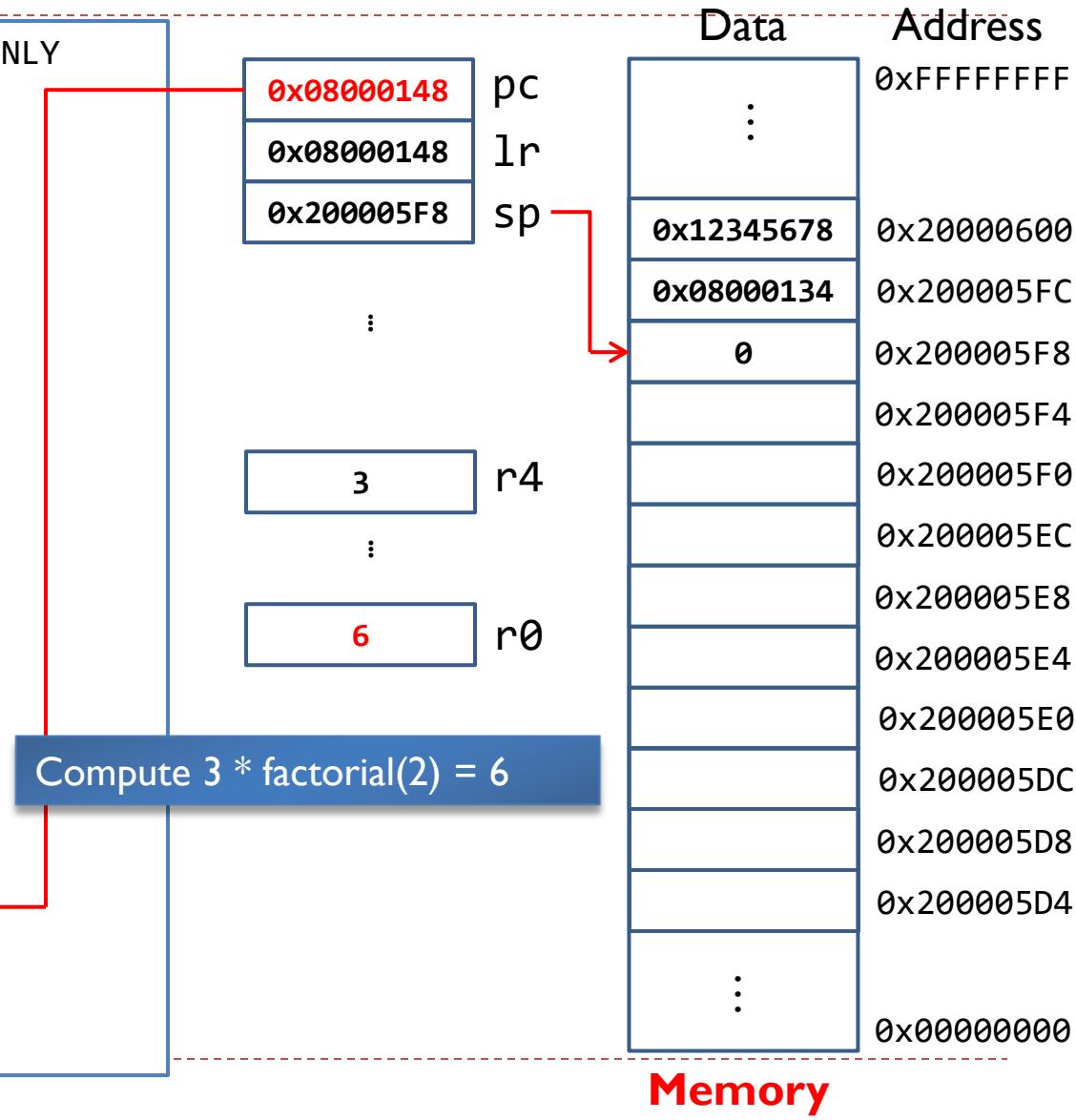
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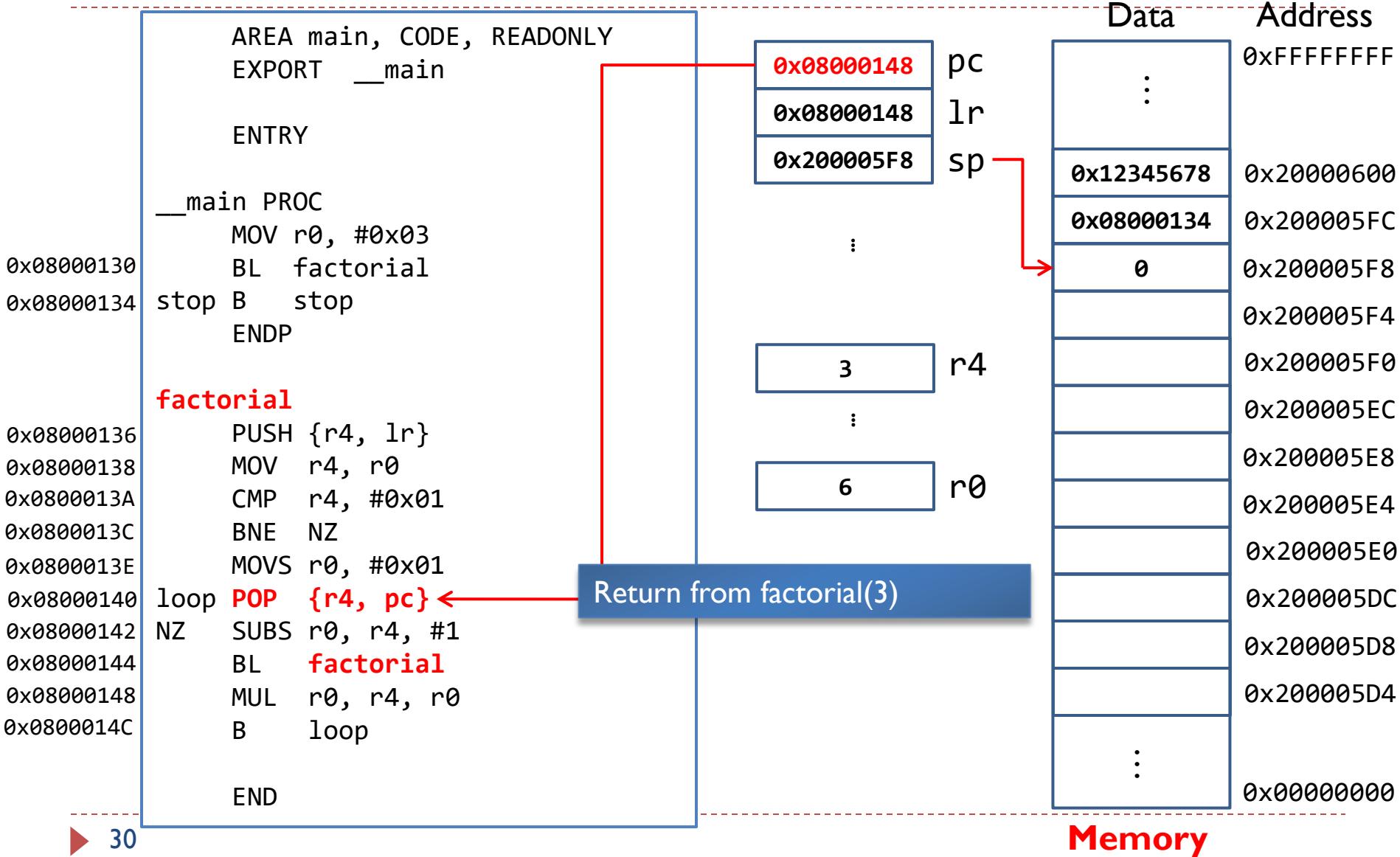
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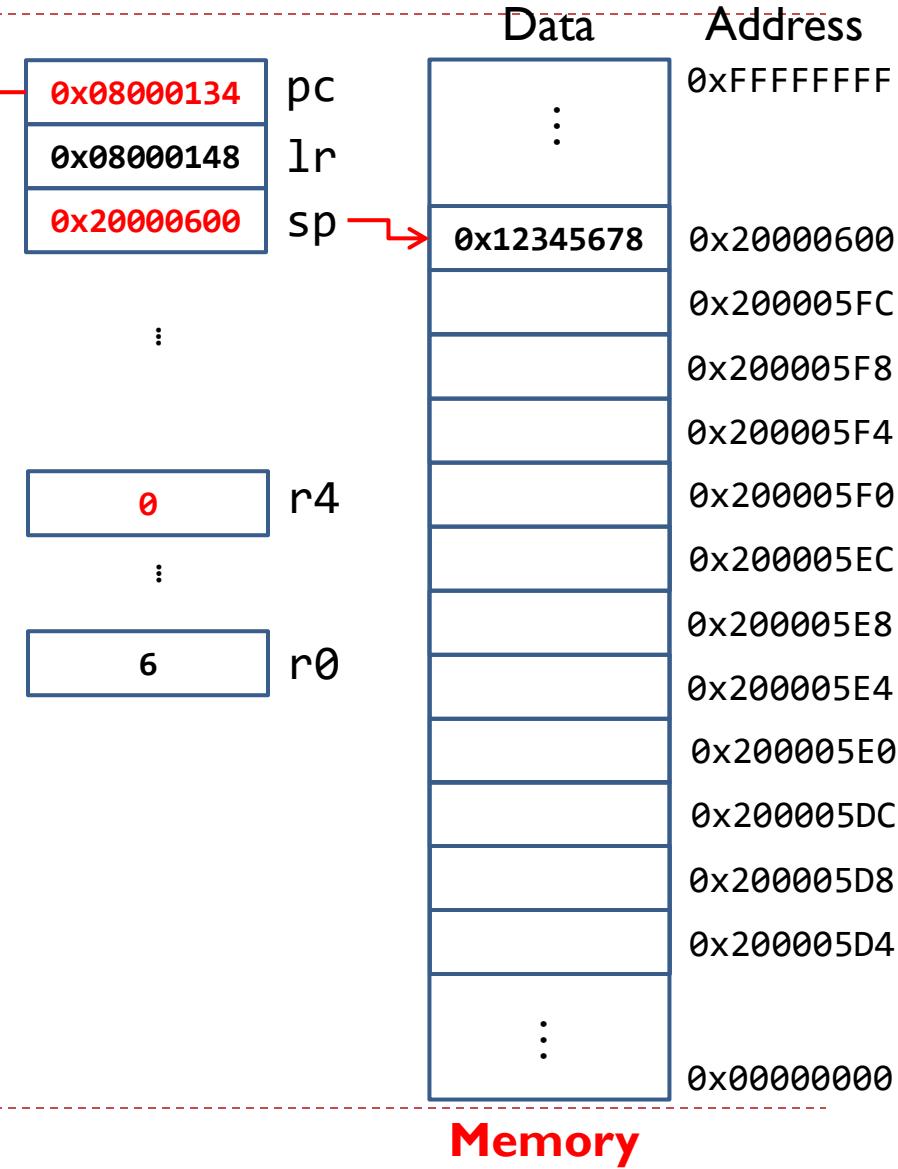
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