

Chapter 8

Stack and Recursive Functions

Z. Gu

Fall 2025

Recursive Functions

- ▶ A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.
- ▶ An effective tactic is to
 - ▶ divide a problem into sub-problems of the same type as the original,
 - ▶ solve those sub-problems, and
 - ▶ combine the results

Defining Factorial(n)

Product of the first n numbers

$$1 \times 2 \times 3 \times \dots \times n$$

$$\text{factorial}(0) = 1$$

$$\text{factorial}(1) = 1 = 1 \times \text{factorial}(0)$$

$$\text{factorial}(2) = 2 \times 1 = 2 \times \text{factorial}(1)$$

$$\text{factorial}(3) = 3 \times 2 \times 1 = 3 \times \text{factorial}(2)$$

$$\text{factorial}(4) = 4 \times 3 \times 2 \times 1 = 4 \times \text{factorial}(3)$$

$$\text{factorial}(n) = n \times (n-1) \times \dots \times 1 = n \times \text{factorial}(n-1)$$



Classic Example: Factorial

- ▶ Factorial is the classic example:
 - ▶ $6! = 6 \times 5!$
 - ▶ $6! = 6 \times 5 \times 4!$
 - ...
 - ▶ $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- ▶ The factorial function can be easily written as a recursive function:

```
int Factorial(int n) {  
  
    if (n < 2)  
        return 1; /* base case */  
  
    return (n * Factorial(n - 1));  
  
}
```

Classic Example: Fibonacci Numbers

$$f(n) = f(n-1) + f(n-2)$$

$$f(0) = 1$$

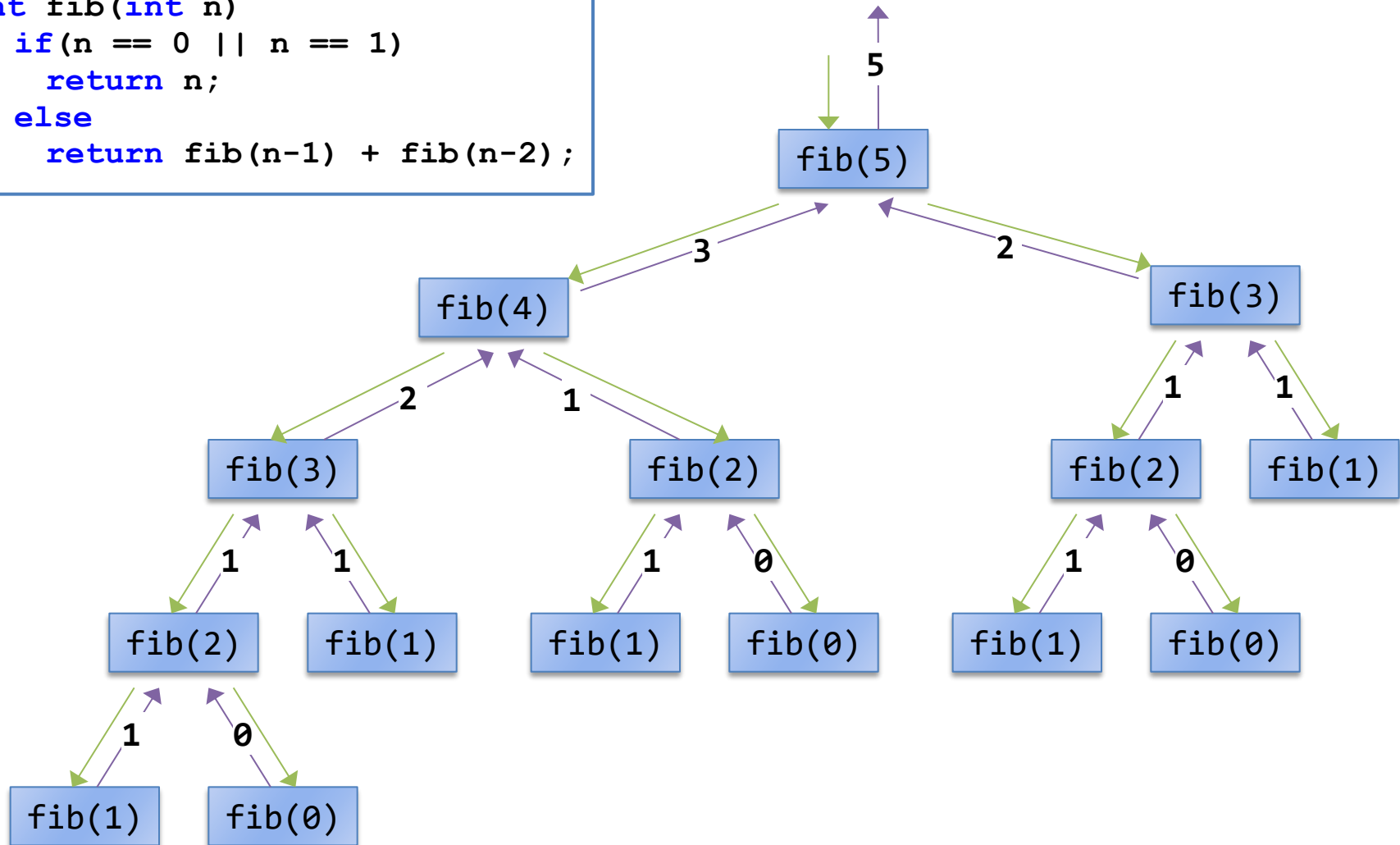
$$f(1) = 1$$

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

```
int Fibonacci(int n) {  
  
    if (n <= 1)  
        return 1;    /* base case */  
  
    return (Fibonacci(n-1) + Fibonacci(n-2));  
  
}
```

Analysis of fib(5)

```
int fib(int n)
  if (n == 0 || n == 1)
    return n;
  else
    return fib(n-1) + fib(n-2);
```



Example of Recursive Function: Testing Palindrome

```
bool isPalindrome(char* s, int len) {  
    if(len < 2)  
        return TRUE;  
    else  
        return s[0] == s[len-1] && isPalindrome(&s[1], len-2);  
}
```

Recursion vs Iteration

Any problem that can be solved **recursively** can also be solved **iteratively** (using loop).

Recursive functions vs Iterative functions

▶ Cons:

- ▶ Recursive functions are slow
- ▶ Recursive function take more memory

▶ Pros

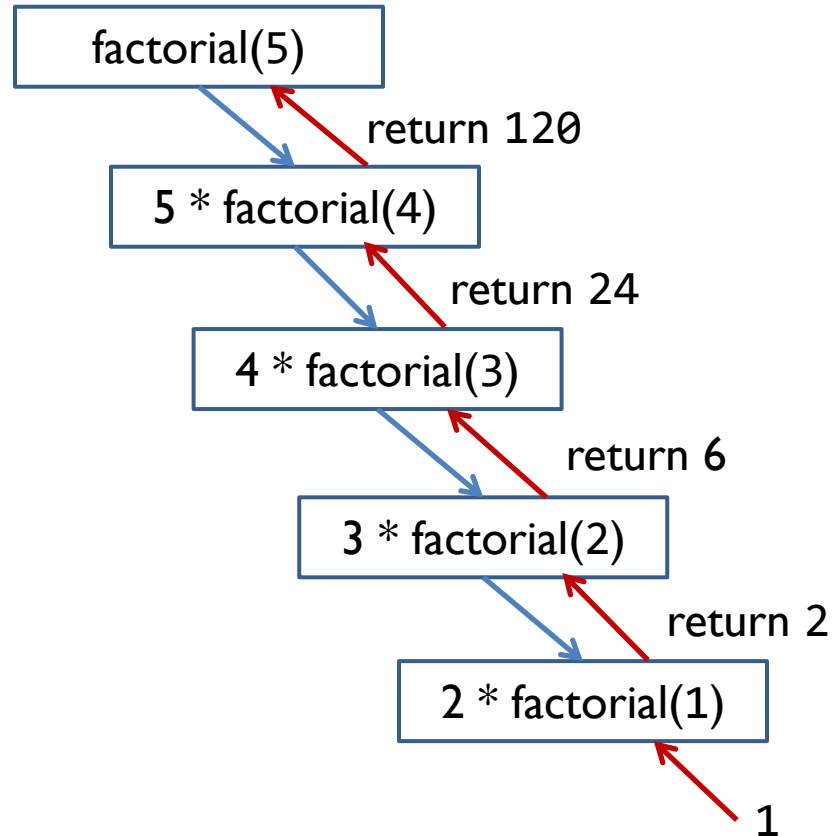
- ▶ Recursive functions resembles the problem more naturally
- ▶ Recursive function are easier to program, and debug.

Recursive Factorial in C

```
int factorial(int n);

int main(void){
    factorial(5);
    return 0;
}

int factorial(int n) {
    int f;
    if(n==1)
        return 1;
    else
        f = n * factorial(n-1);
    return f;
}
```



Recursive Functions

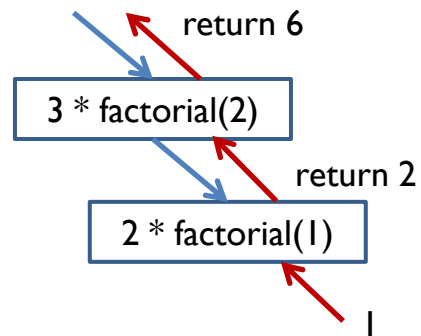
- ▶ **PUSH LR** (& working registers) onto stack before nested call
- ▶ **POP LR** (& working registers) off stack after nested return

Recursive Factorial in Assembly

```
AREA main, CODE, READONLY
    EXPORT __main
    ENTRY
__main PROC
    MOV    r0, #0x03                ; Set argument n = 3 in r0 (r0 holds first arg)
    BL     factorial                ; Call factorial(n); LR gets return address;
                                        ; result returns in r0
stop B     stop                    ; Halt by branching to self (infinite loop)
ENDP

; --- Recursive factorial(n) ---
factorial
    PUSH   {r4, lr}                ; Save callee-used r4 and return address LR on stack
    MOV    r4, r0                  ; Preserve n in r4 across the recursive call
    CMP    r4, #0x01               ; Check base case: n == 1 ?
    BNE    NZ                      ; If n != 1, branch to NZ for recursive case
    MOVS   r0, #0x01               ; Base case: return 1 in r0
loop POP   {r4, pc}                ; restore r4, and return by popping PC
NZ SUBS   r0, r4, #1               ; Prepare argument r0 = n - 1 for recursive call
    BL     factorial                ; r0 <- factorial(n-1) after return
    MUL    r0, r4, r0              ; r0 = n * factorial(n-1)
    B      loop
END
```

Trace Function Execution for this Call Sequence



Recursive Factorial in Assembly

```
AREA main, CODE, READONLY
EXPORT __main

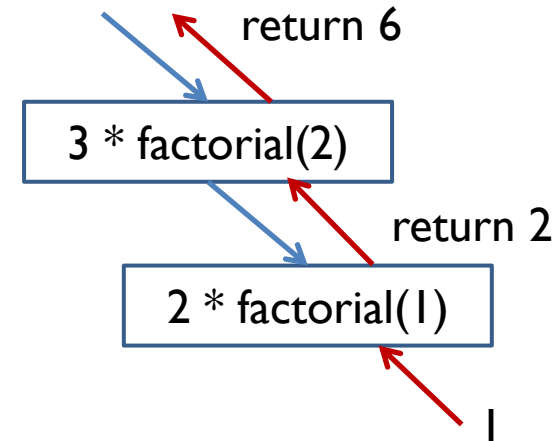
ENTRY

__main PROC
    MOV r0, #0x03
0x08000130    BL factorial
0x08000134 stop B    stop
                ENDP

factorial

0x08000136    PUSH {r4, lr}
0x08000138    MOV  r4, r0
0x0800013A    CMP  r4, #0x01
0x0800013C    BNE  NZ
0x0800013E    MOVS r0, #0x01
0x08000140 loop POP  {r4, pc}
0x08000142 NZ   SUBS r0, r4, #1
0x08000144    BL   factorial
0x08000148    MUL  r0, r4, r0
0x0800014C    B    loop

                END
```



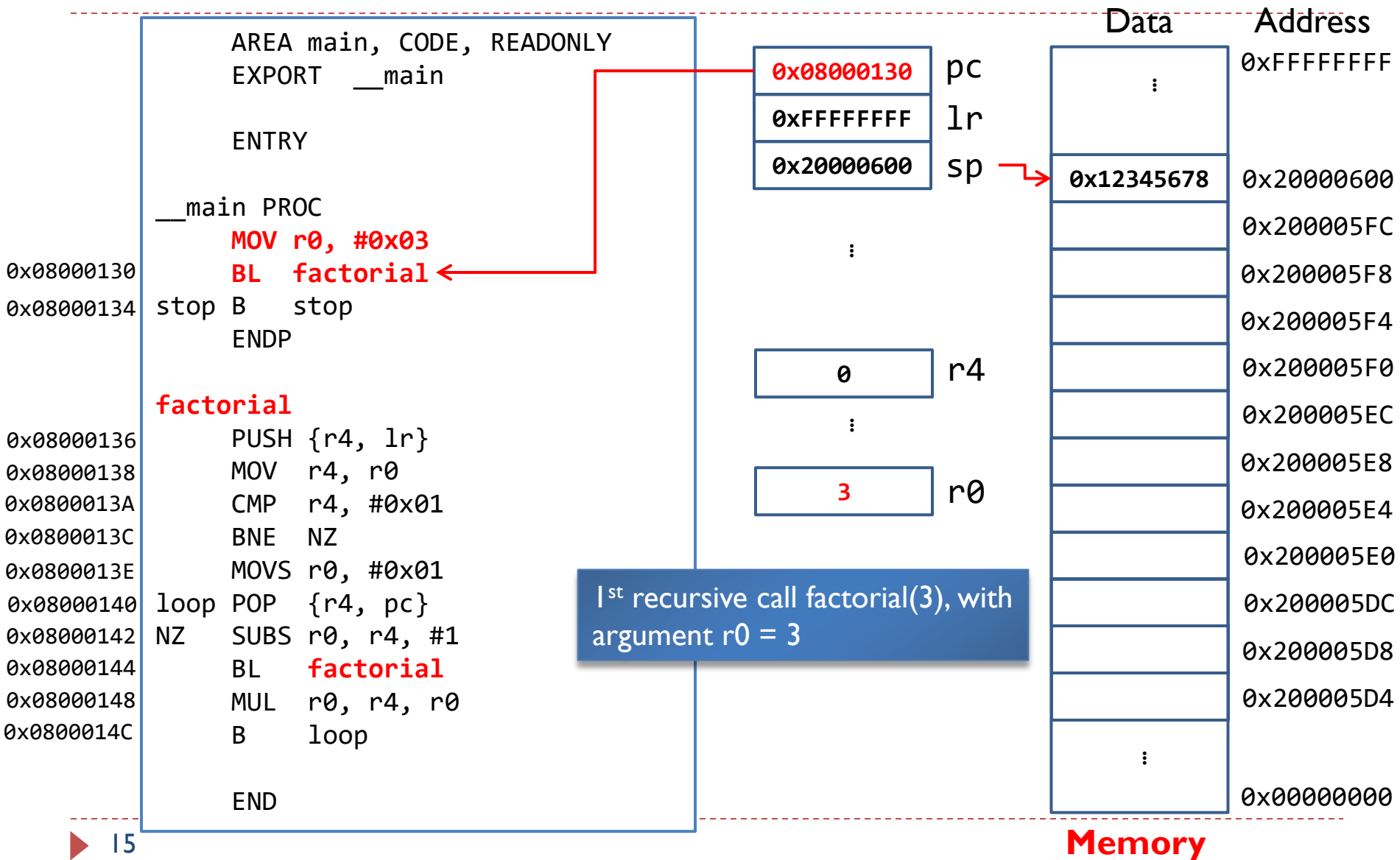
Recursive Factorial in Assembly

```
AREA main, CODE, READONLY
    EXPORT __main
    ENTRY
__main PROC
    MOV    r0, #0x03                ; Set argument n = 3 in r0 (r0 holds first arg)
    BL     factorial                ; Call factorial(n); LR gets return address;
                                        ; result returns in r0

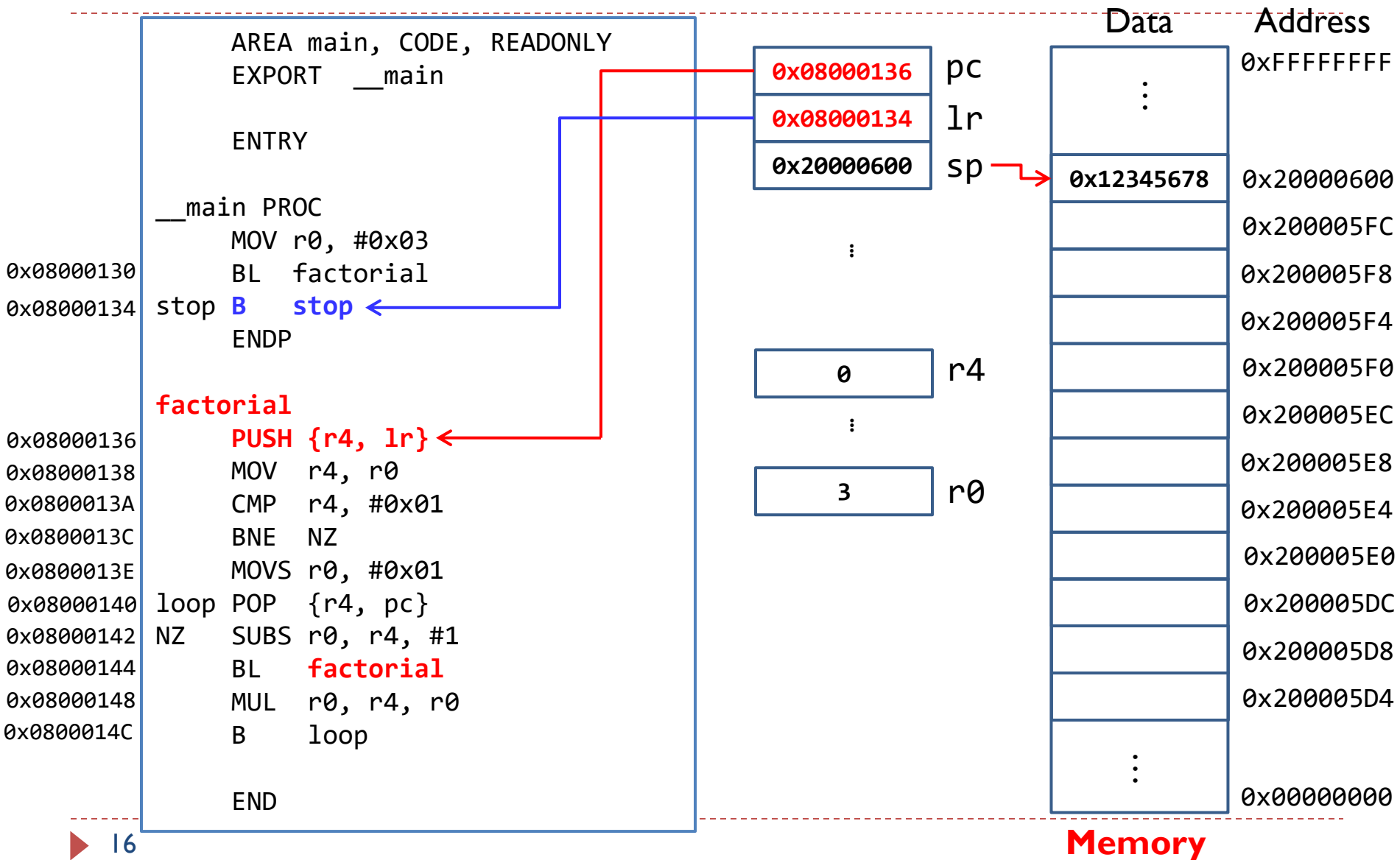
stop
    B      stop                    ; Halt by branching to self (infinite loop)
ENDP
; --- Recursive factorial(n) ---
factorial
    PUSH   {r4, lr}                ; Save callee-used r4 and return address LR on
stack
    MOV    r4, r0                  ; Preserve n in r4 across the recursive call
    CMP    r4, #0x01               ; Check base case: n == 1 ?
    BNE    NZ                      ; If n != 1, branch to NZ for recursive case
    MOVS   r0, #0x01               ; Base case: return 1 in r0

loop
    POP    {r4, pc}                ; restore r4, and return by popping PC
NZ
    SUBS   r0, r4, #1               ; Prepare argument r0 = n - 1 for recursive call
    BL     factorial                ; r0 <- factorial(n-1) after return
    MUL    r0, r4, r0              ; r0 = n * factorial(n-1)
    B      loop
14 END
```

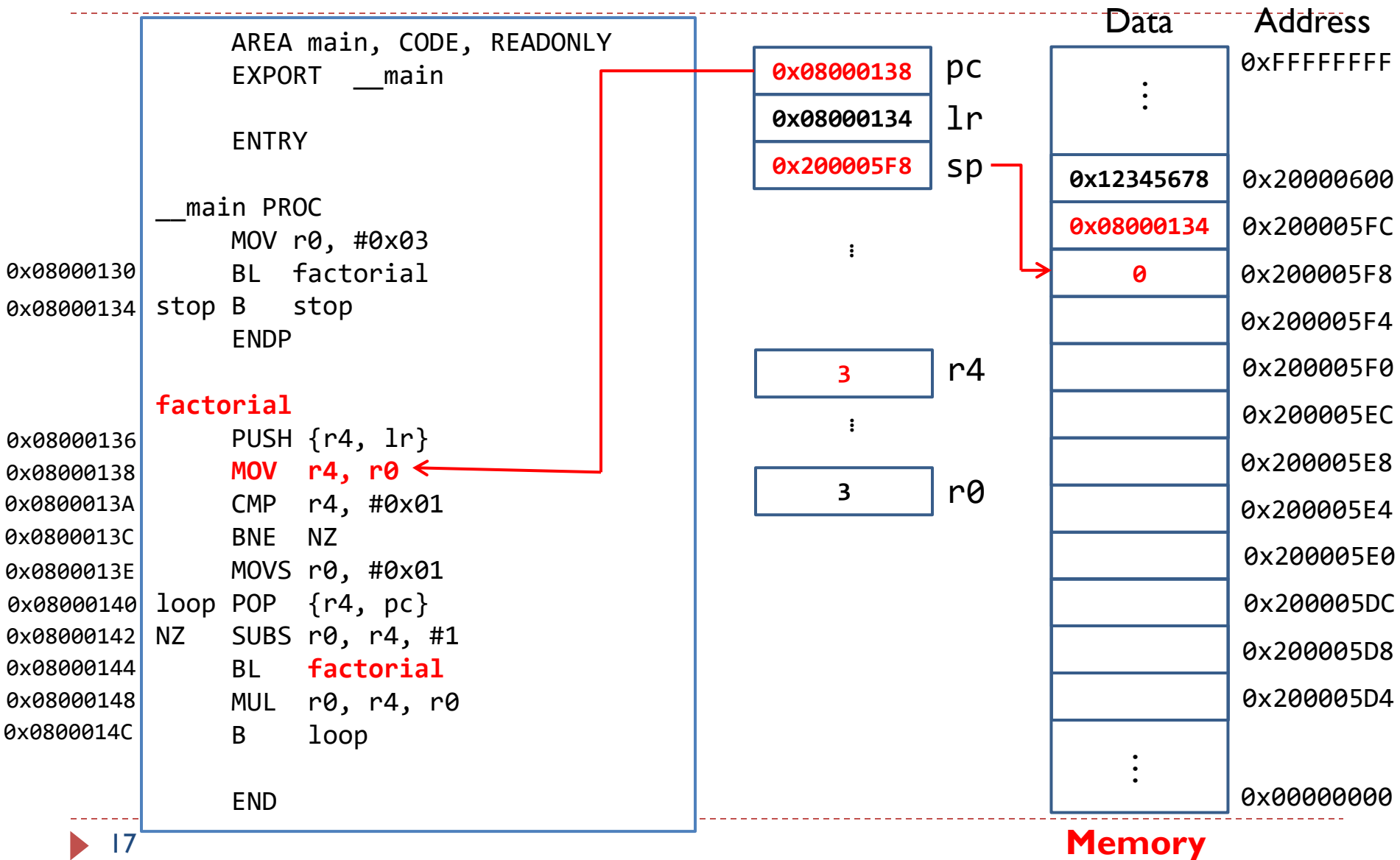
Recursive Factorial in Assembly



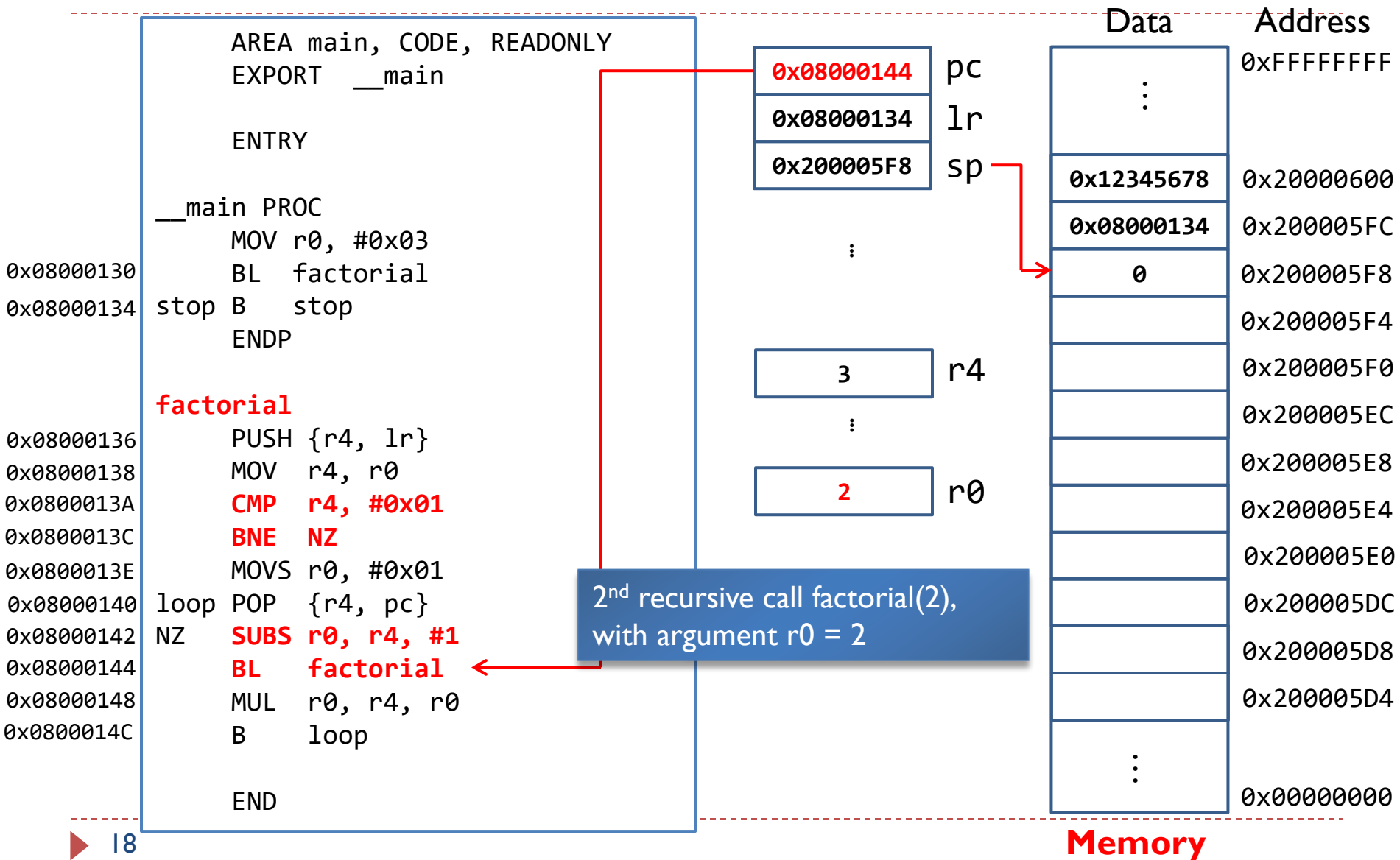
Recursive Factorial in Assembly



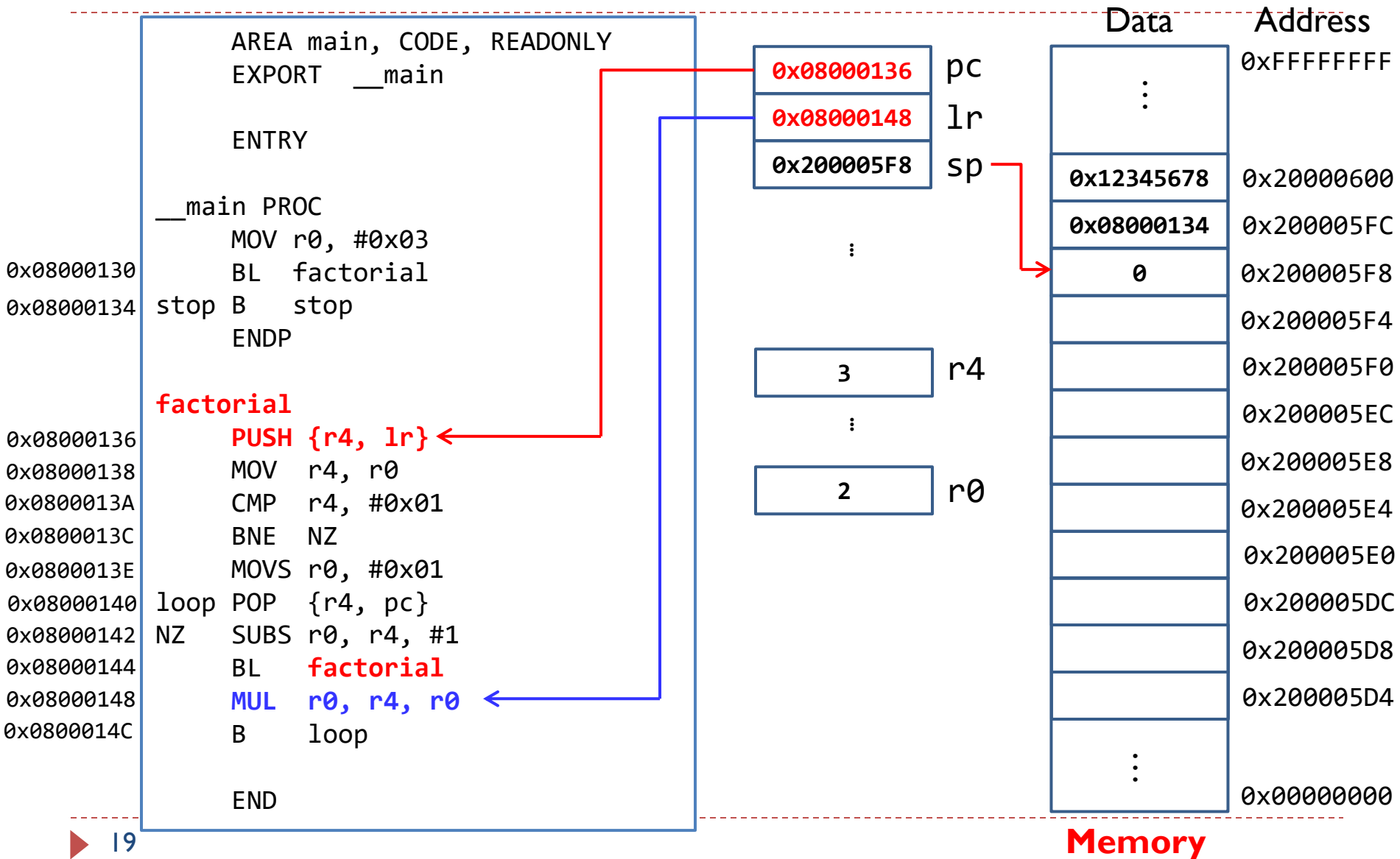
Recursive Factorial in Assembly



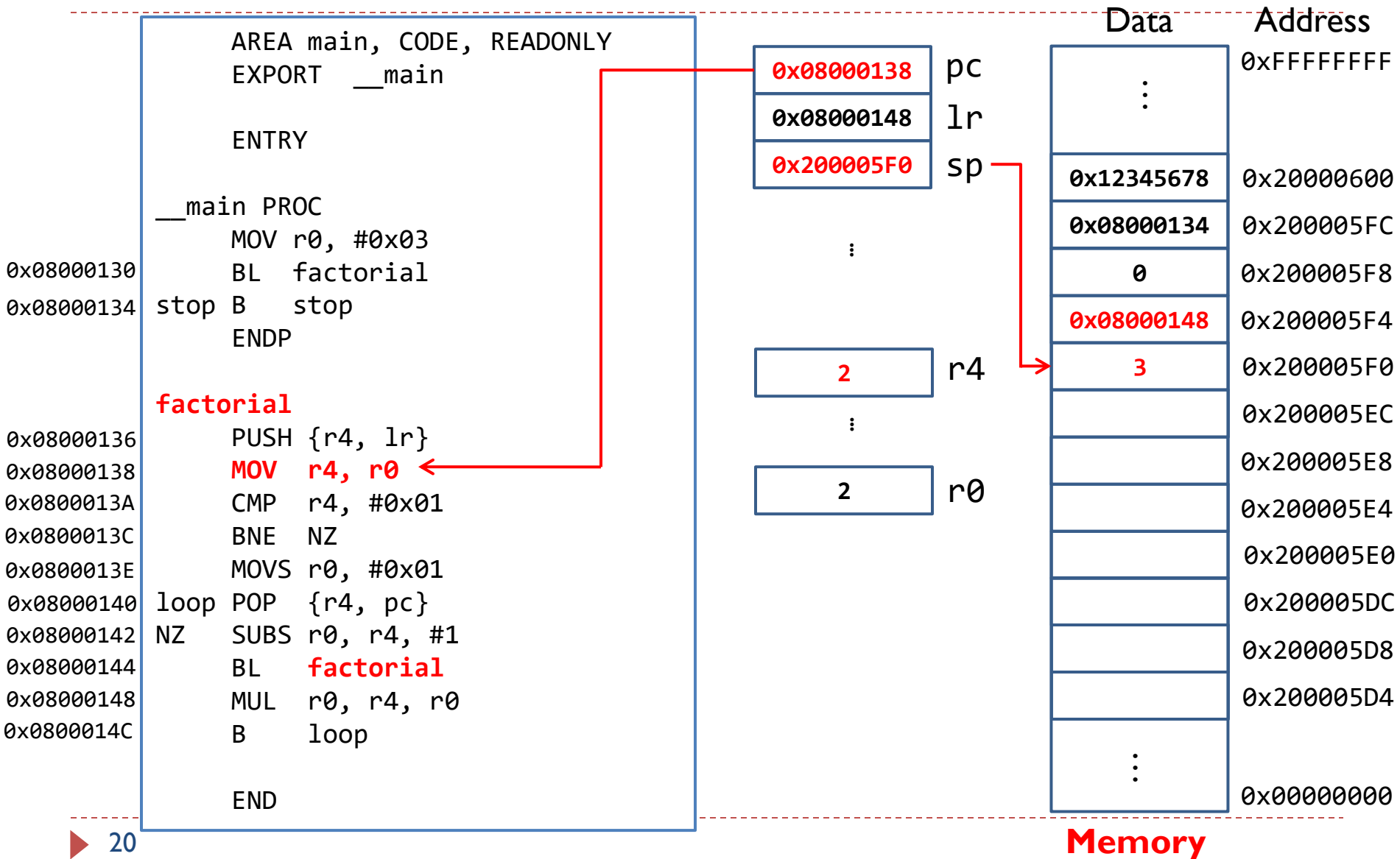
Recursive Factorial in Assembly



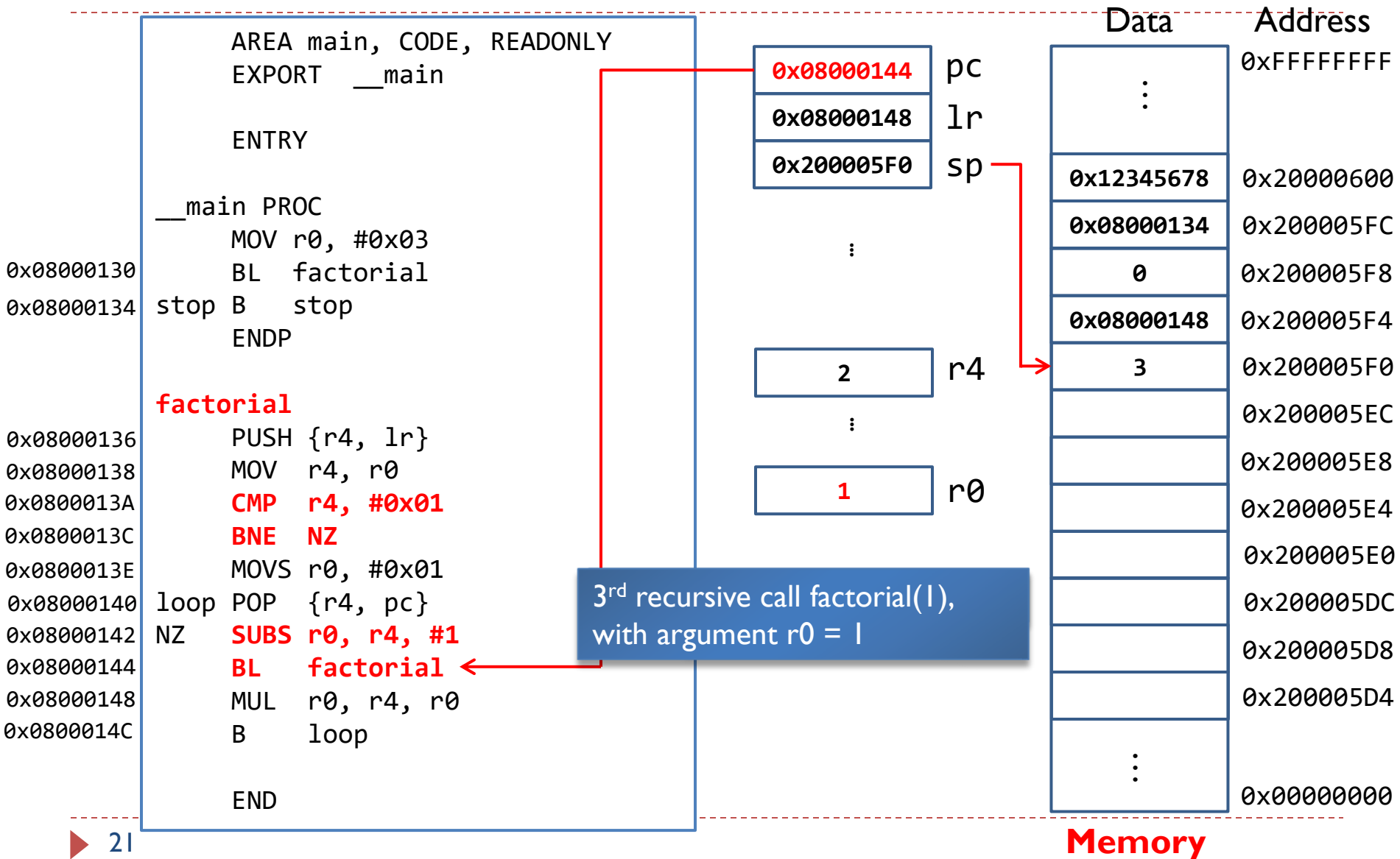
Recursive Factorial in Assembly



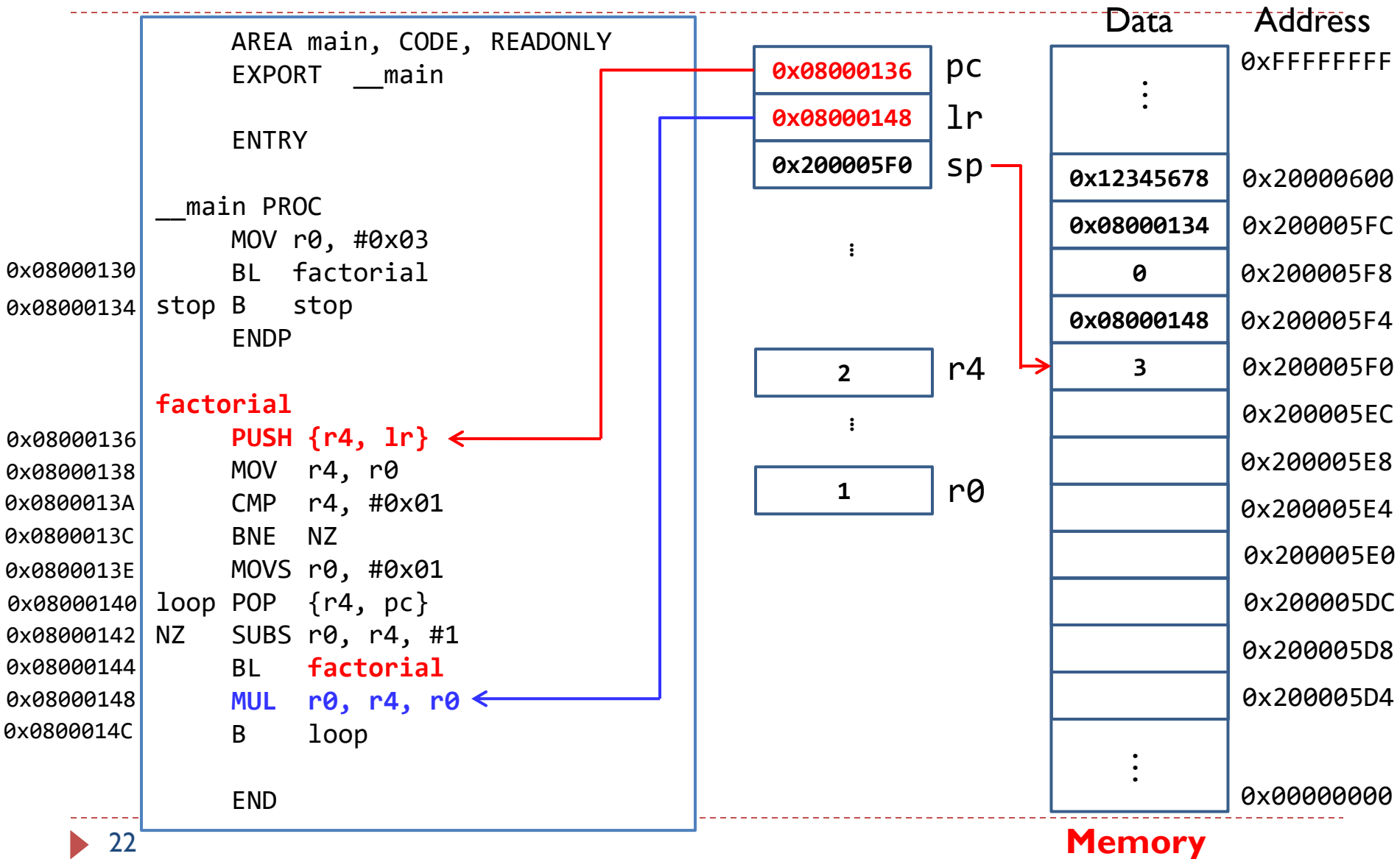
Recursive Factorial in Assembly



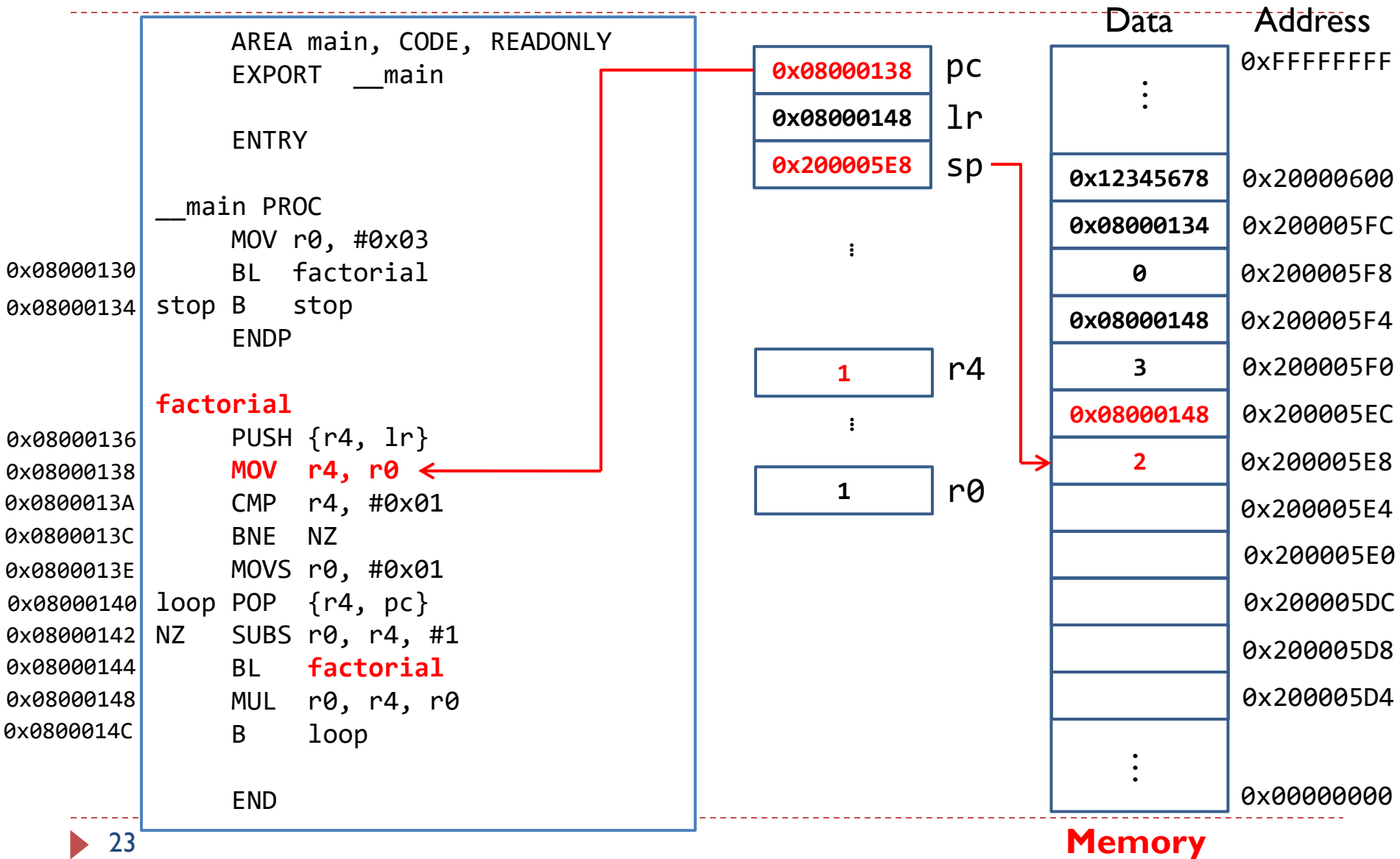
Recursive Factorial in Assembly



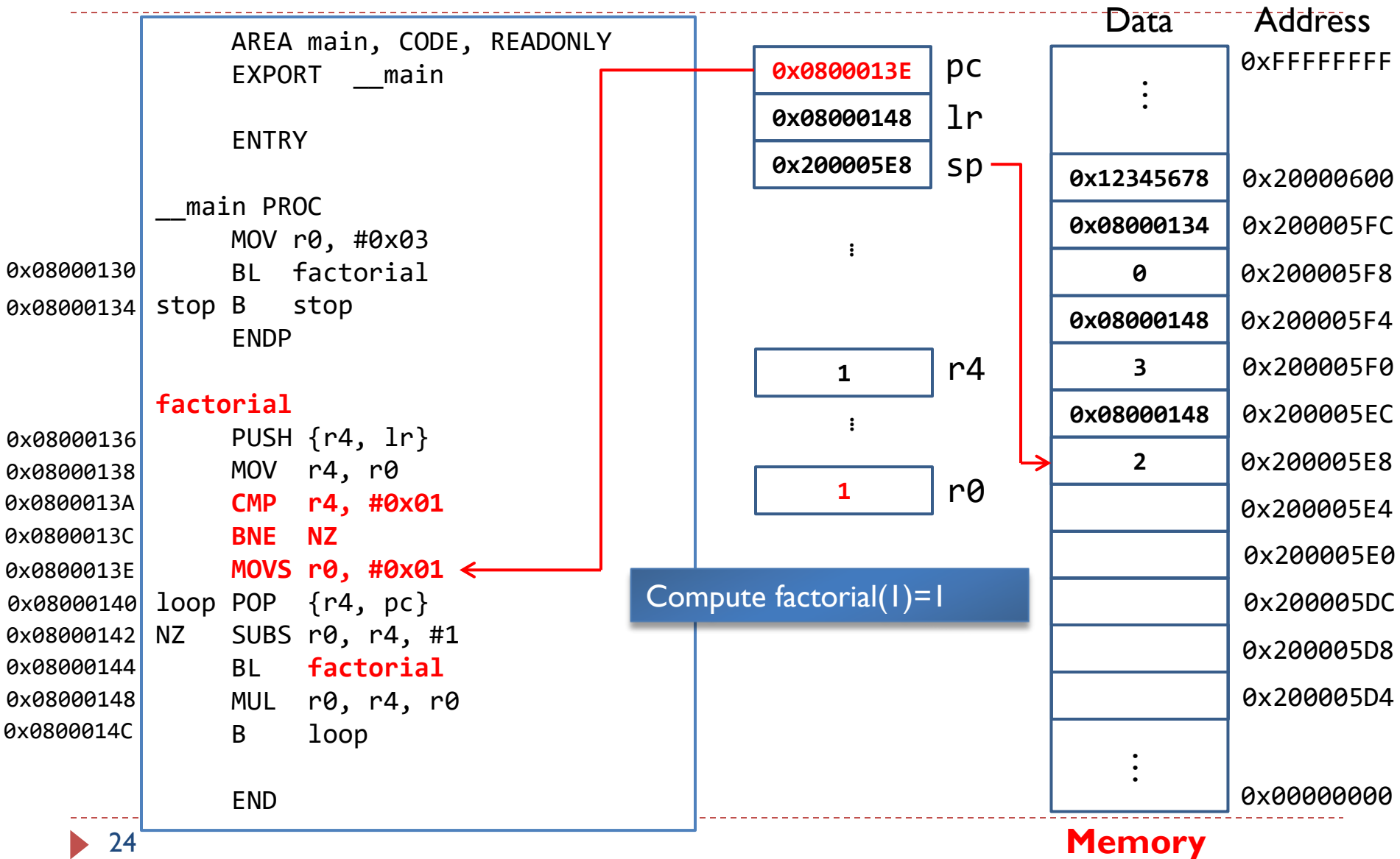
Recursive Factorial in Assembly



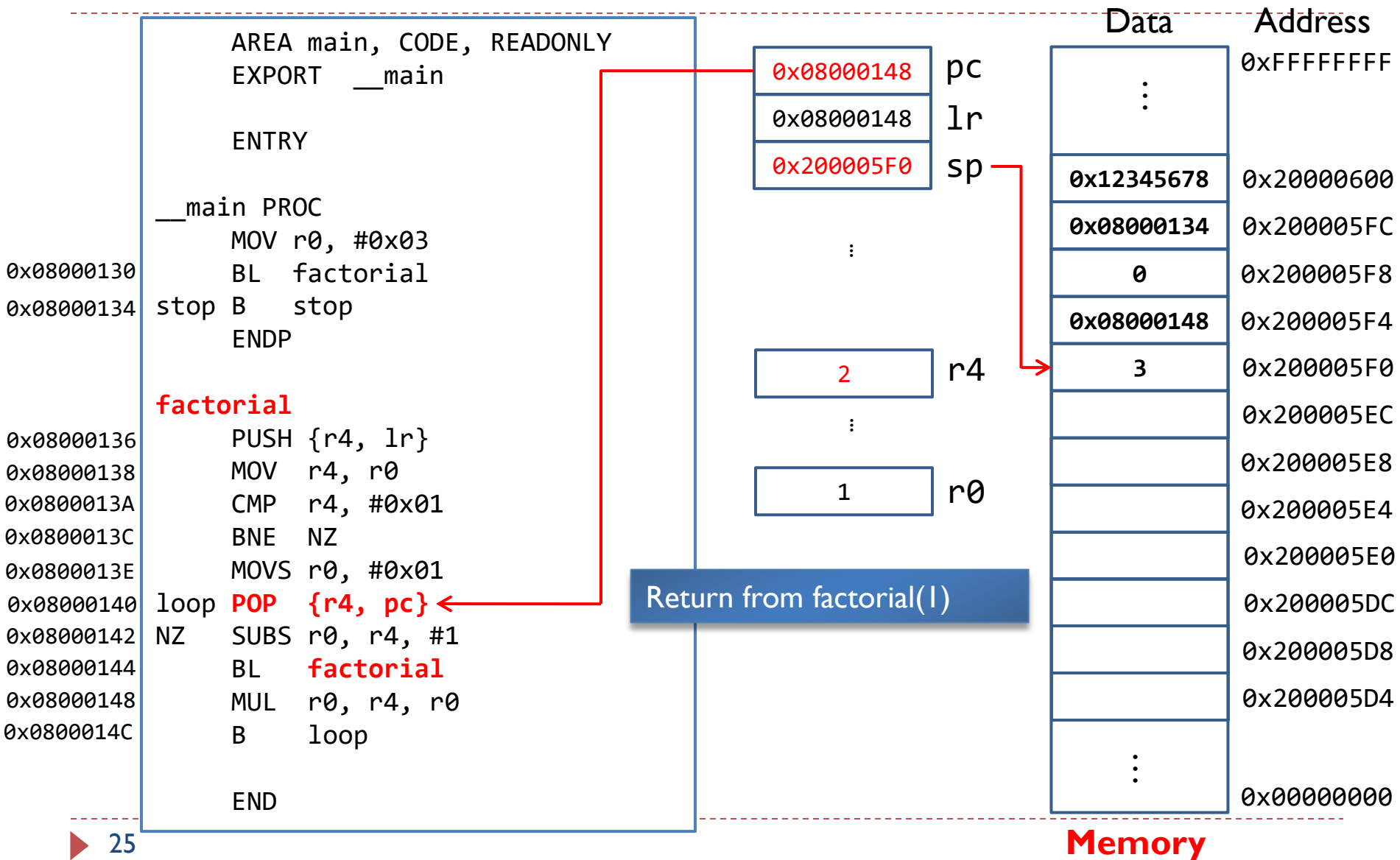
Recursive Factorial in Assembly



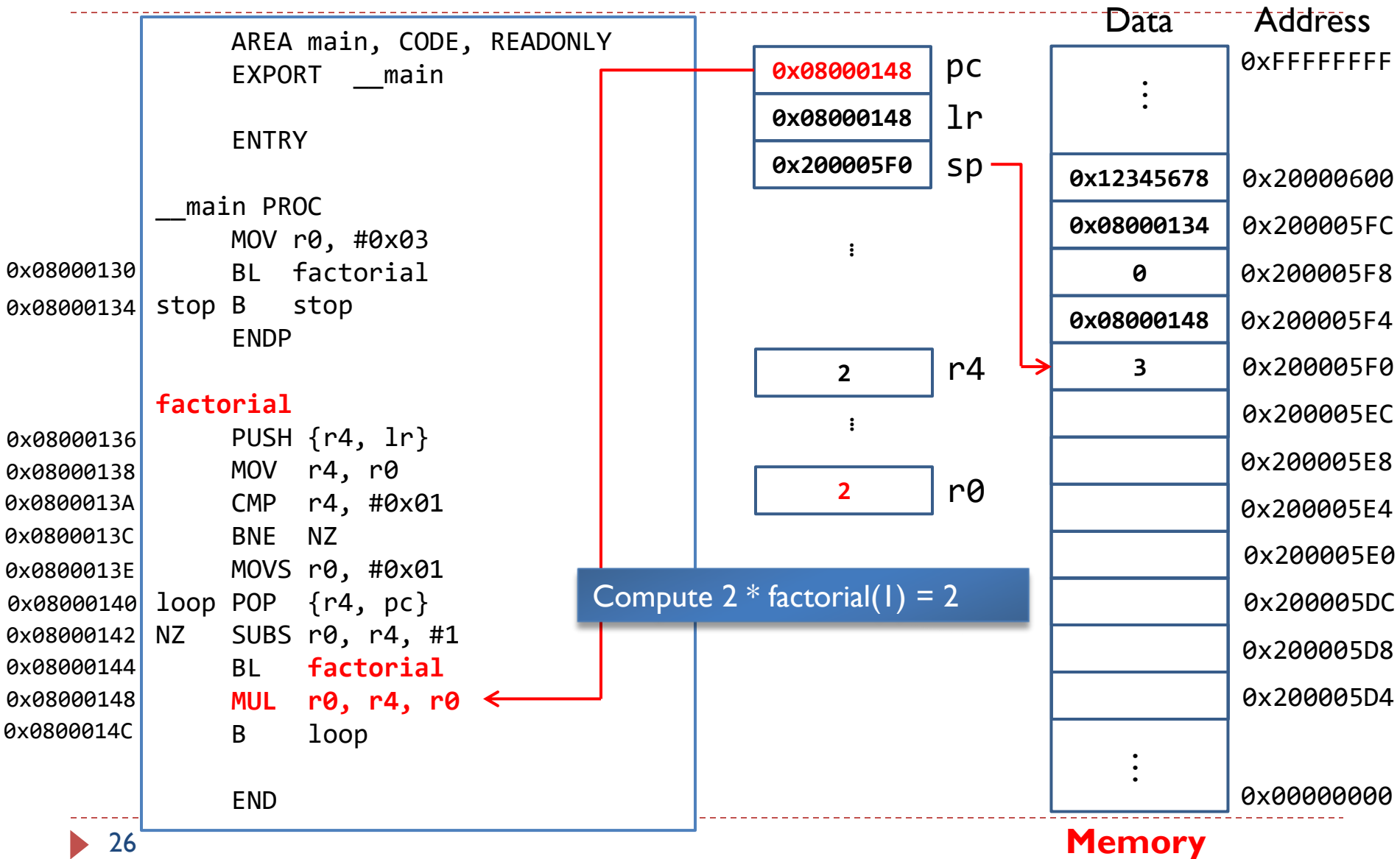
Recursive Factorial in Assembly



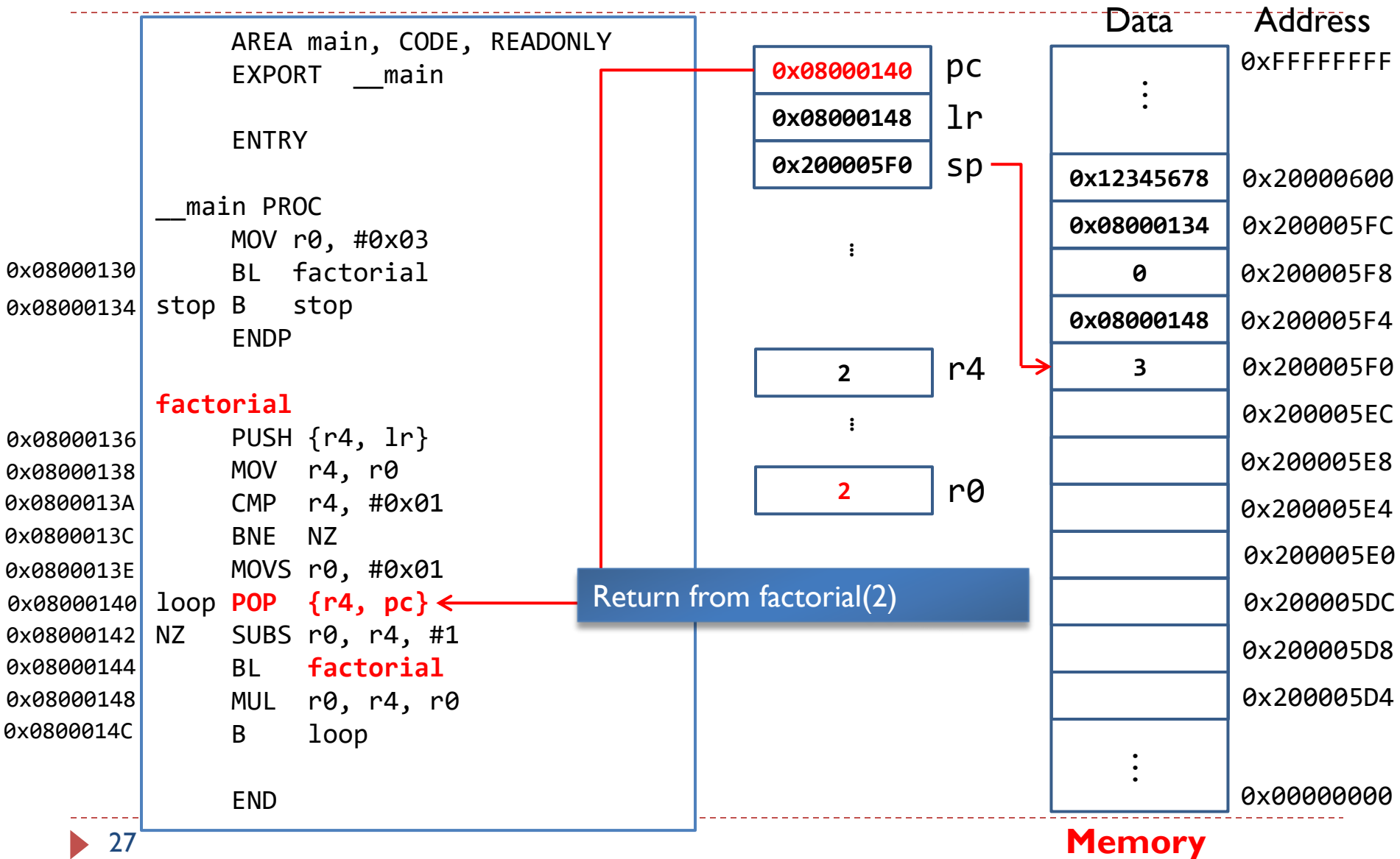
Recursive Factorial in Assembly



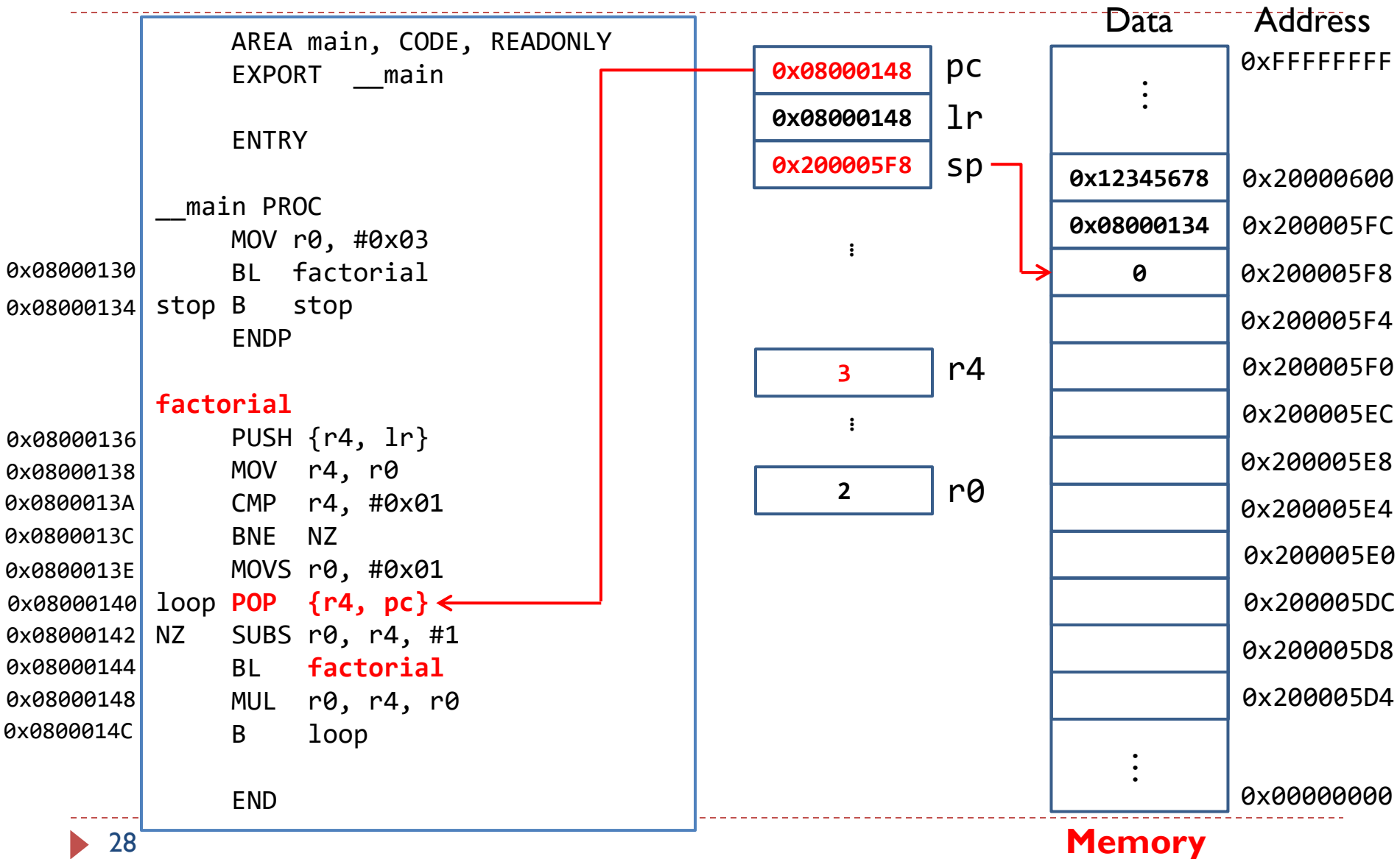
Recursive Factorial in Assembly



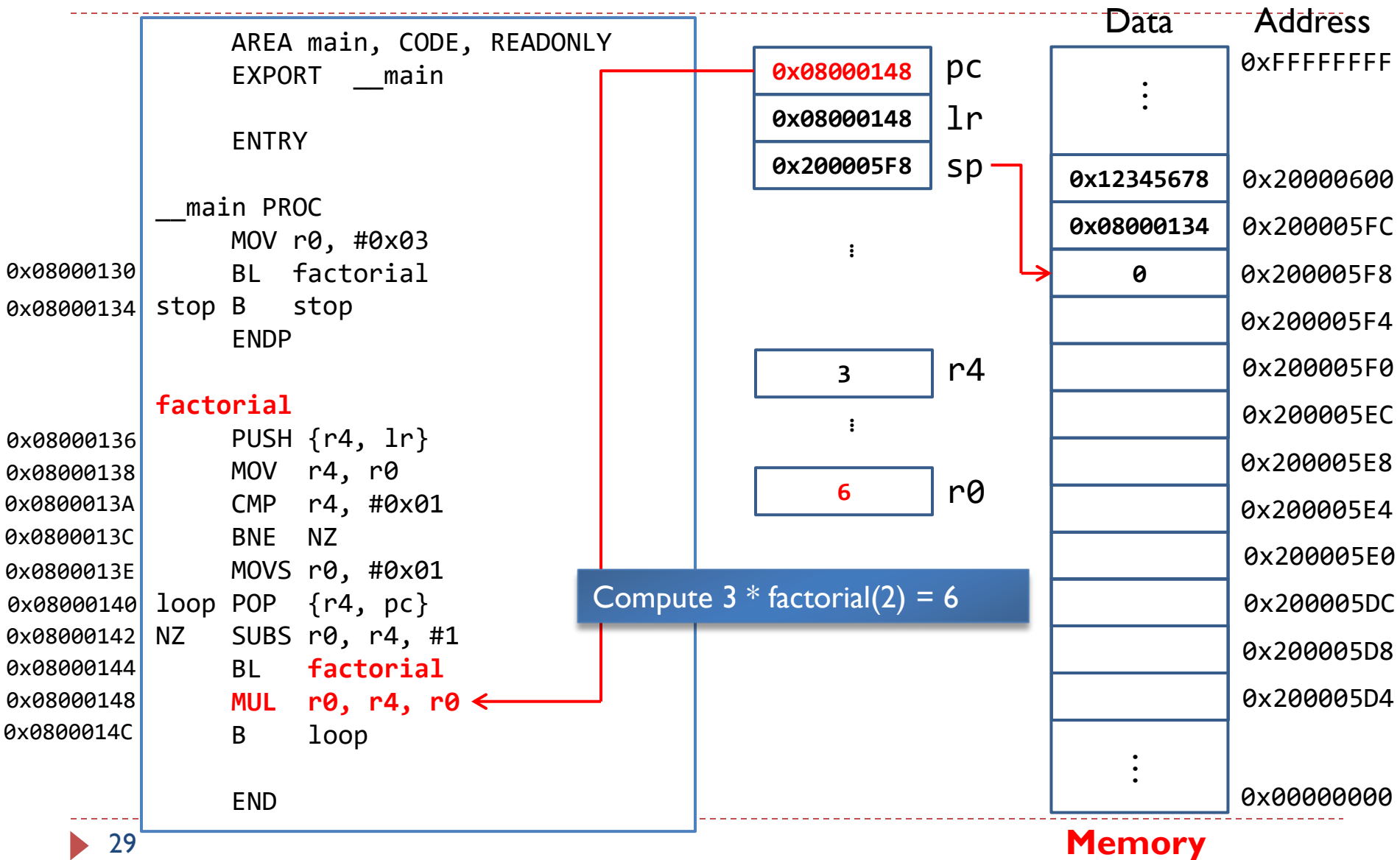
Recursive Factorial in Assembly



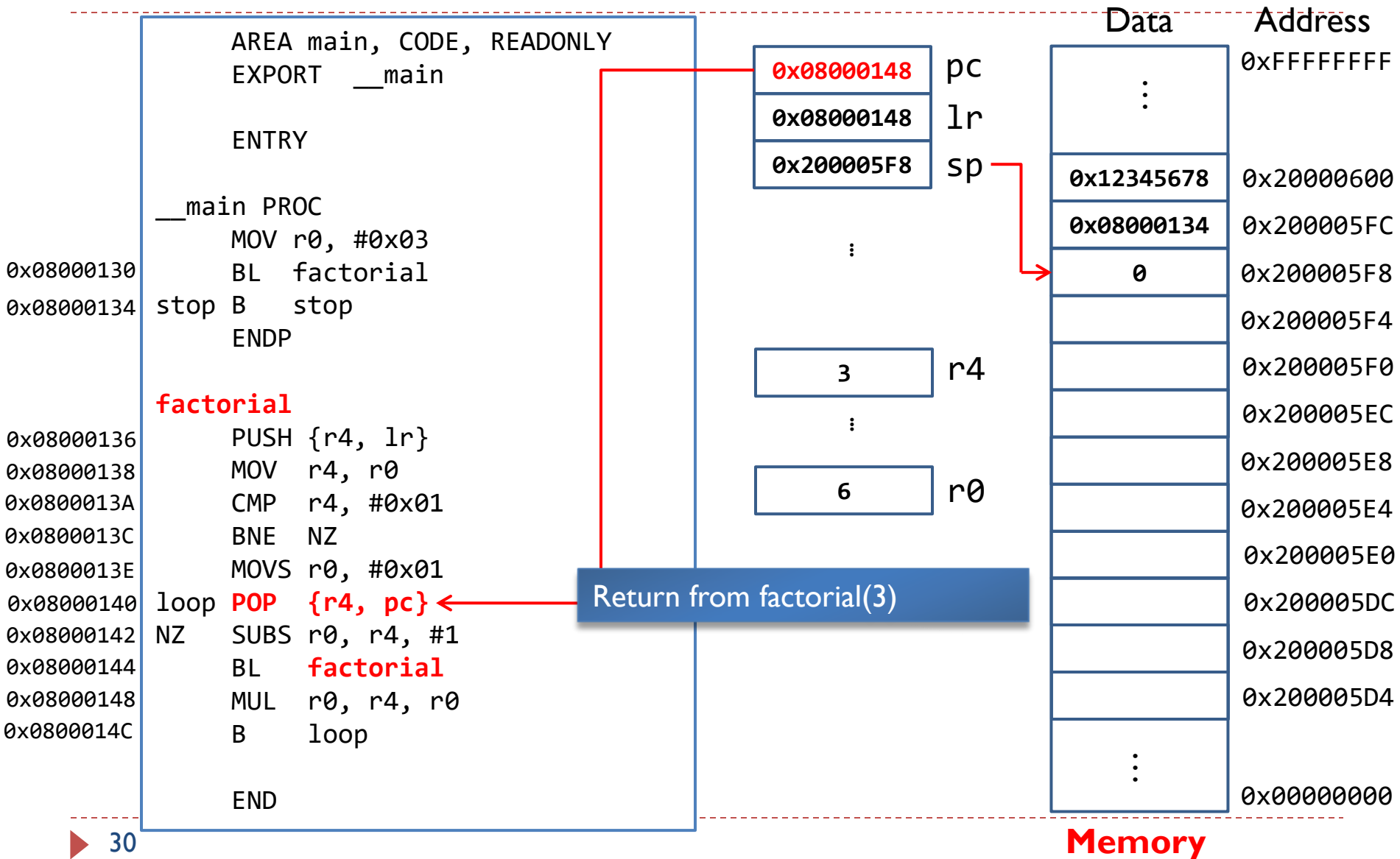
Recursive Factorial in Assembly



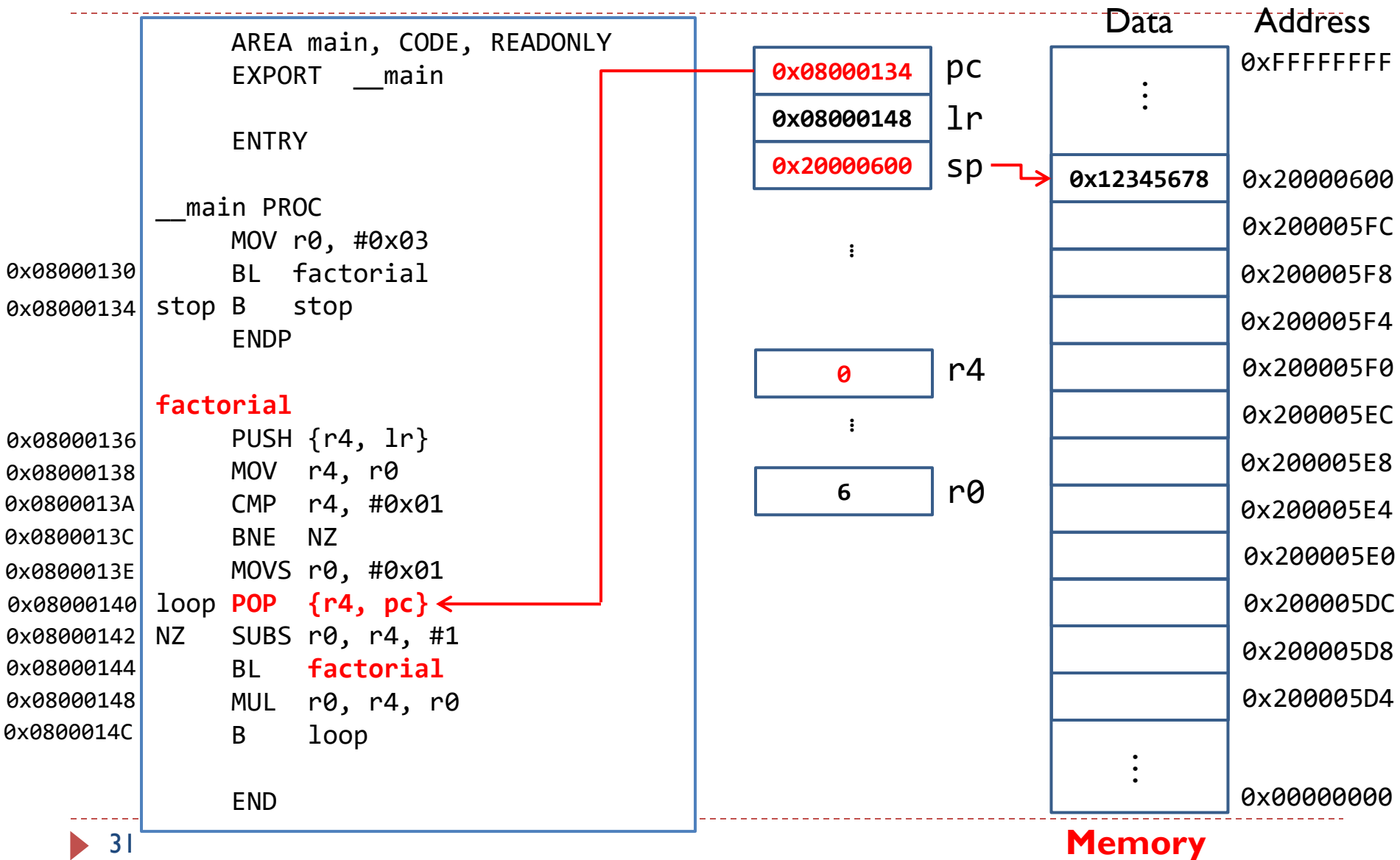
Recursive Factorial in Assembly



Recursive Factorial in Assembly



Recursive Factorial in Assembly



Recursive Factorial in Assembly

