CSC 112: Computer Operating Systems
Lecture 6

Real-Time Scheduling

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Hofstra University

Acknowledgement: Lecture slides based on Buttazzo, Hard Real-Time Computing Systems

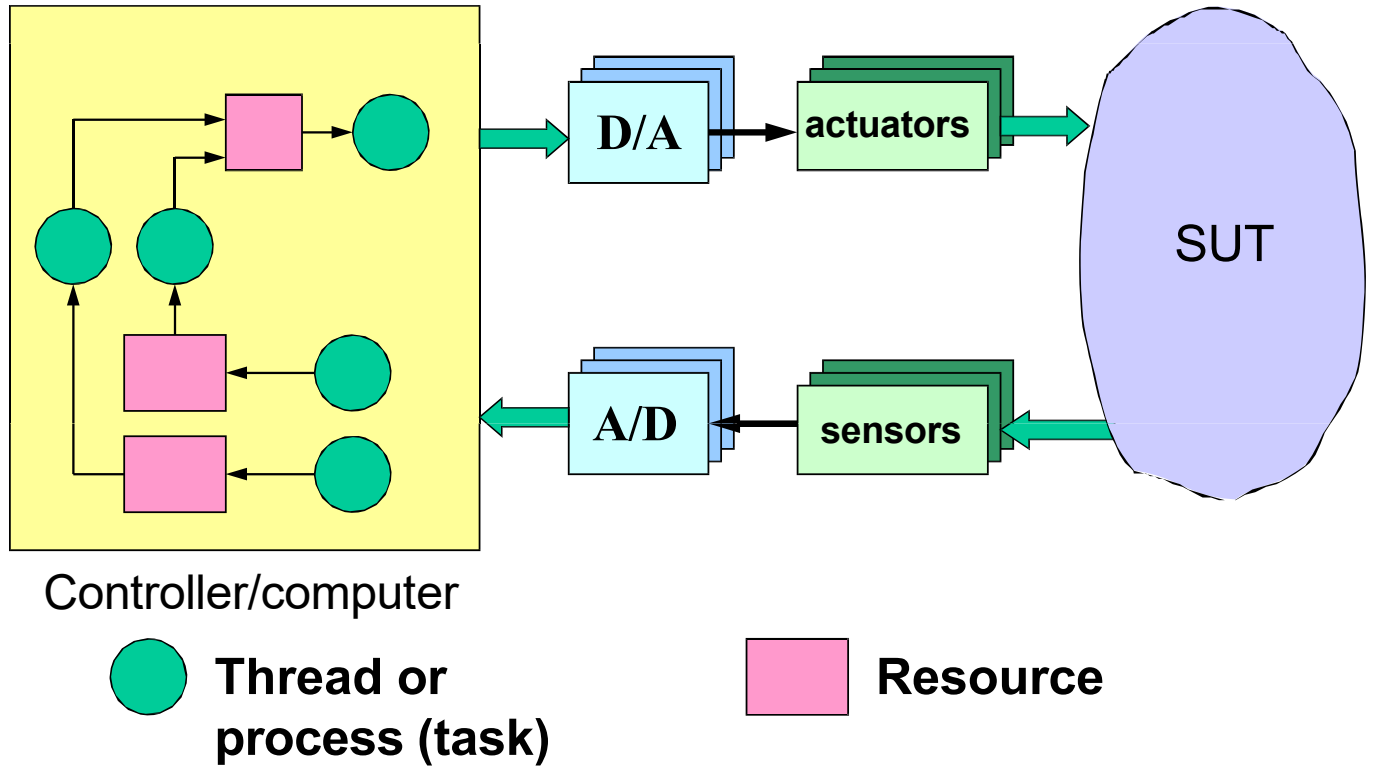
Outline

- Introduction to RTOS and Real-Time Scheduling
- Fixed-Priority Scheduling
- Earliest Deadline First Scheduling
- Least Laxity First (LLF) Scheduling
- Preemptive vs. Non-Preemptive Scheduling
- Multiprocessor Scheduling
- Resource Synchronization Protocols (for Fixed-Priority Scheduling)

Introduction to RTOS and Real-Time Scheduling

Embedded Control Systems

- An embedded control system co´nsists of:
  - The system-under-control (SUT)
    - » may include sensors and actuators
  - The controller/computer
    - » sends signals to the system according to a predetermined control objective
- In the old days, each control task runs on a dedicated CPU
  - No RTOS, bare metal
  - No need for scheduling
  - Just make sure that task execution time < deadline
- Now, multiple control tasks share one CPU
  - Multitasking RTOS
  - Need scheduling to make sure all tasks meet deadlines



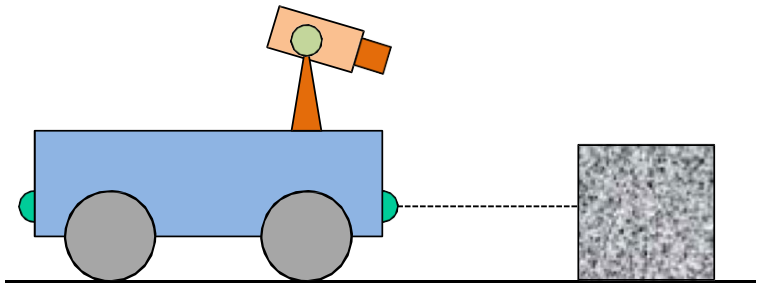
Requirements

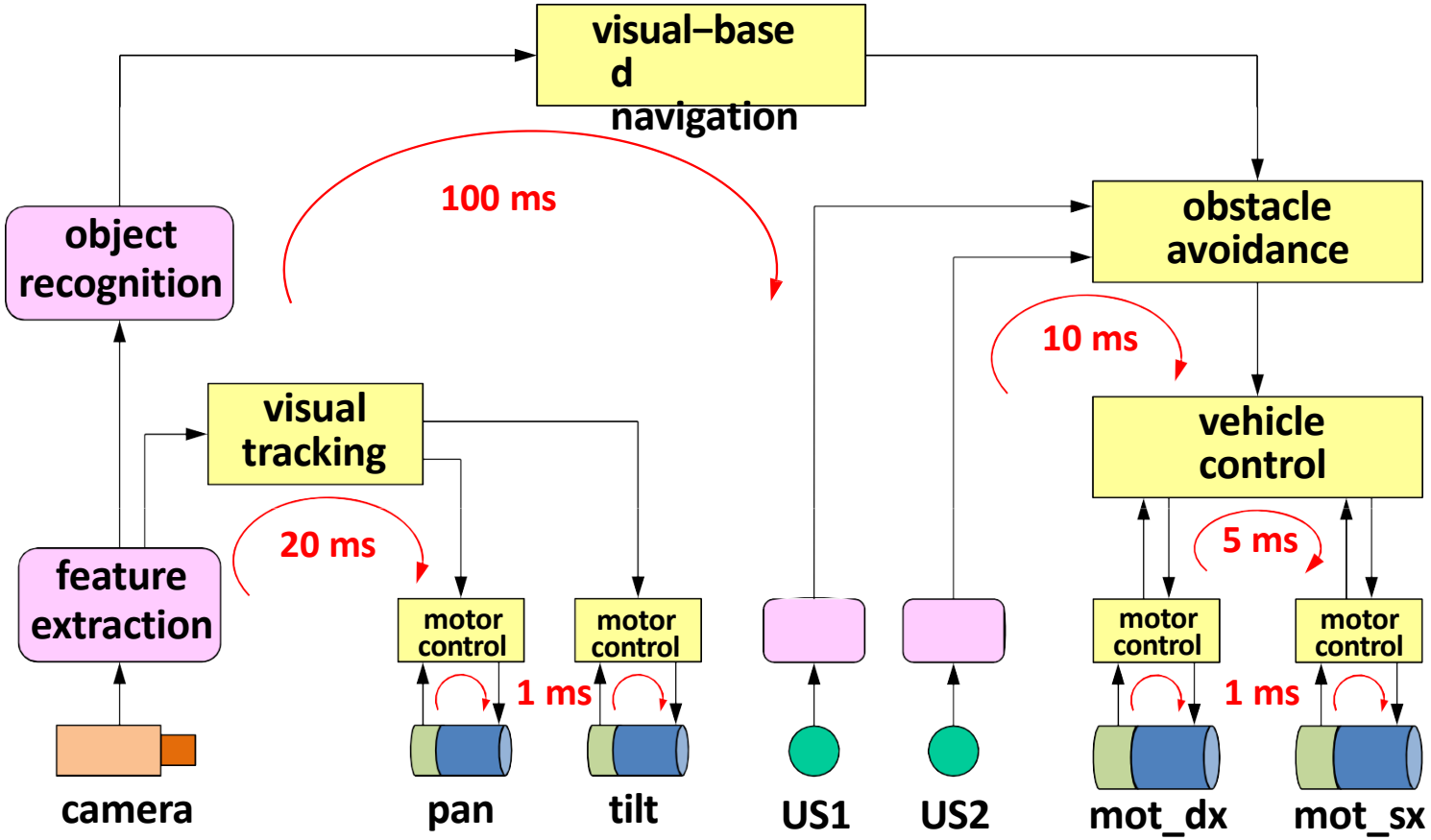
- The tight interaction with the environment requires the system to react to events within precise timing constraints
- Timing constraints are imposed by the dynamics of the environment

- The real-time operating system (RTOS) must be able to execute tasks within timing constraints

A Robot Control Example

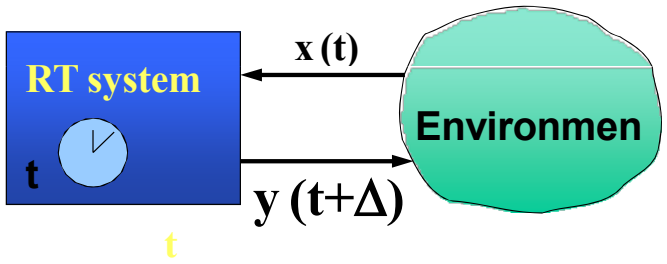
- Consider a robot equipped with:
  - two actuated wheels
  - two proximity (US) sensors
  - a mobile (pan/tilt) camera
  - a wireless transceiver
- Goal:
  - follow a path based on visual feedback
  - avoid obstacles

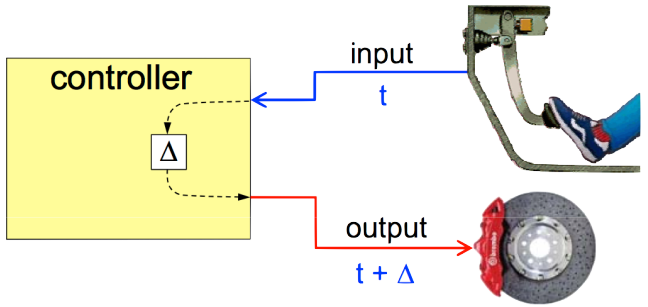


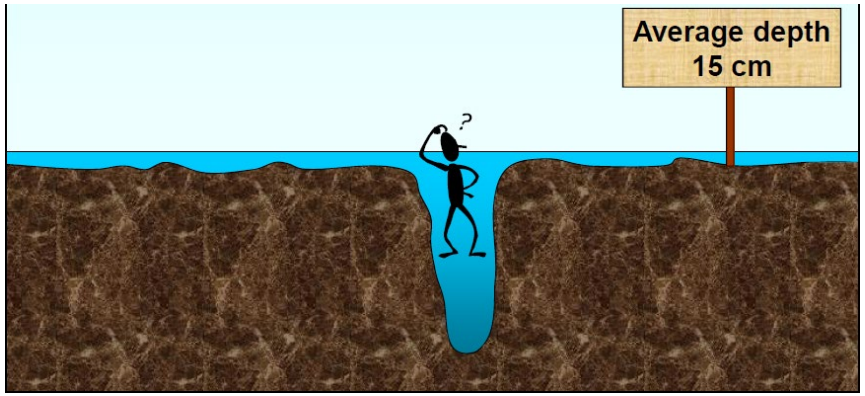


Real-Time Systems

- A computer system that is able to respond to events within precise timing constraints
- A system where the correctness depends not only on the output values, but also on the time at which results are produced
- A real-time system is not a necessarily a real fast system
  - Speed is always relative to a specific environment
  - Running faster is good, but does not guarantee hard real-time constraints
- The objective of a real-time system is to guarantee the worst-case timing behaviour of each individual task
- The objective of a fast system is to optimize the average-case performance
  - A system with fast average-case performance may not meet worst-case timing requirements
  - Analogy: there was a person who drowned in a river with average depth of 15 cm







RTOS Requirements

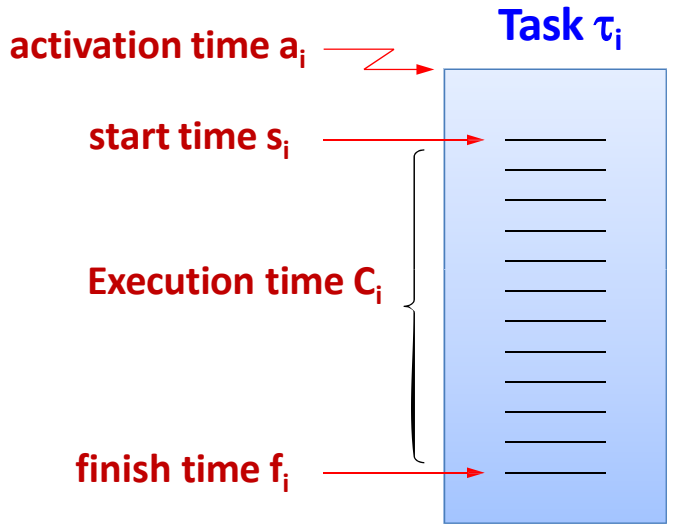
- Timeliness: results must be correct not only in their value but also in the time domain
  - provide kernel mechanism for time management and for handling tasks with explicit timing constraints and different criticality
- Predictability: system must be analyzable to predict the consequences of any scheduling decision
  - if some task cannot be guaranteed within time constraints, system must notify this in advance, to handle the exception (plan alternative actions)
- Efficiency: operating system should optimize the use of available resources (computation time, memory, energy)
- Robustness: must be resilient to peak-load conditions
- Fault tolerance: single software/hardware failures should not cause the system to crash
- Maintainability: modular architecture to ensure that modifications are easy to perform

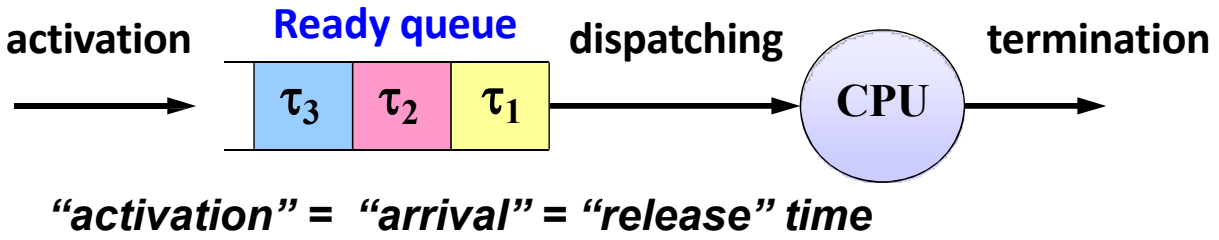
Sources of Nondeterminism

- Architecture
  - cache, pipelining, interrupts, DMA
- Operating System (our focus in this lecture)
  - scheduling, synchronization, communication
- Language
  - lack of explicit support for time
- Design Methodologies
  - lack of analysis and verification techniques

Task

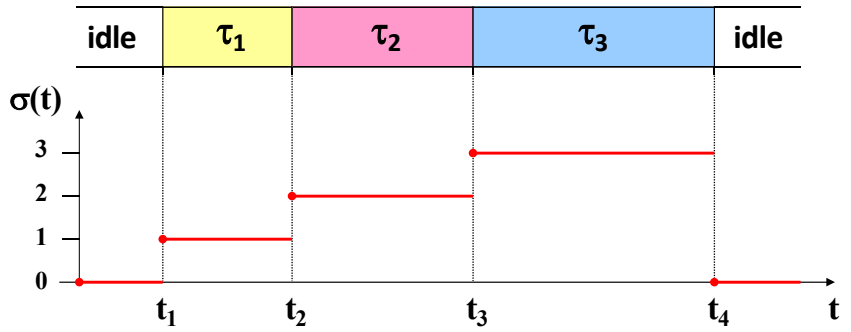
- The concept of concurrent tasks reflects the intuition about the functionality of embedded systems.
  - Task here can refer to either process or thread, depending on the underlying RTOS support
- Tasks help us manage timing complexity:
  - multiple execution rates
    - » multimedia
    - » automotive
  - asynchronous input
    - » user interfaces
    - » communication systems





Schedule

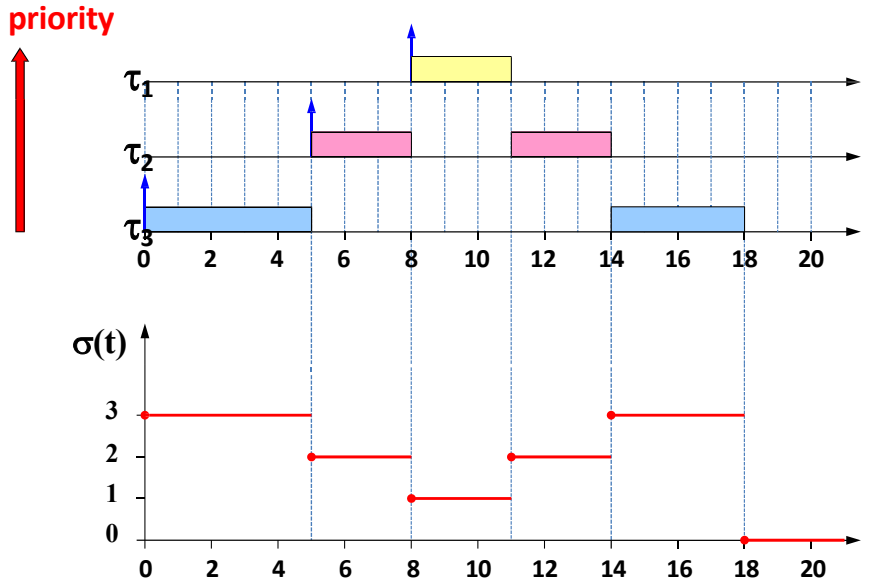
- A specific assignment of tasks to the processor that determines the task execution sequence. Formally:
- Given a task set $\Gamma = \{\tau_1, \ldots, \tau_n\}$, a schedule is a function $\sigma: R^+ \rightarrow N$ that associates an integer k to each time slice $[t_i, t_{i+1})$ with the meaning:
  - $k = 0$: in $[t_i, t_{i+1})$ the processor is idle
  - $k > 0$: in $[t_i, t_{i+1})$ the processor executes τ_k



At times t_1, t_2, …: context switch to a different task

Preemptive vs. Nonpreemptive Scheduling

- A scheduling algorithm is:
  - preemptive: if the active job can be temporarily suspended to execute a more important job
  - non-preemptive: if the active job cannot be suspended, i.e., always runs to completion



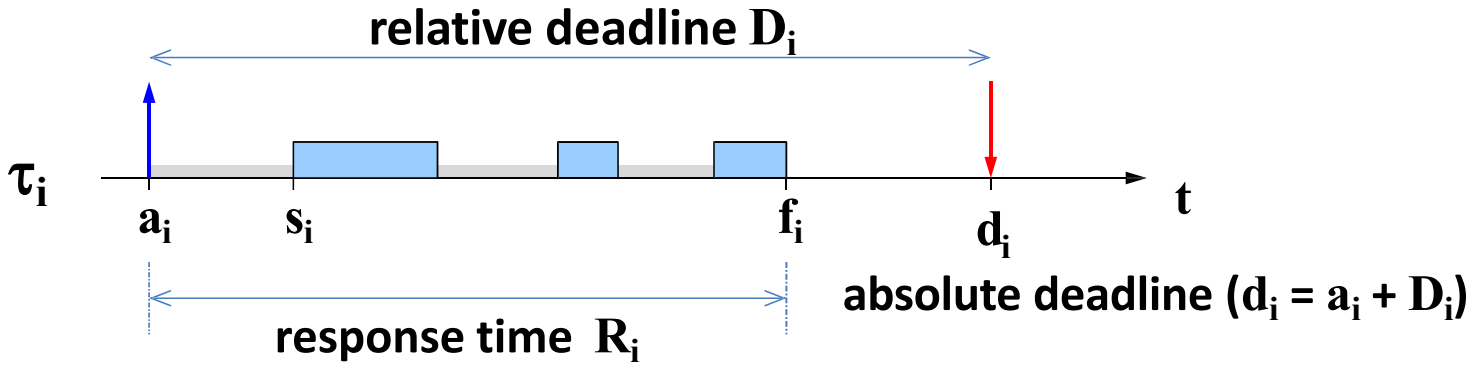
Preemptive scheduling example

Definitions

- Feasible schedule
  - A schedule σ is said to be feasible if all the tasks can complete according to a set of specified constraints.
- Schedulable set of tasks
  - A set of tasks Γ is said to be schedulable if there exists at least one algorithm that can produce a feasible schedule for it.
- Hard real-time task: missing deadline may have catastrophic consequences, so deadline violations are not permitted. A system able to handle hard real-time tasks is a hard real-time system
  - sensory acquisition
  - low-level control
  - sensory-motor planning
- Soft real-time task: missing deadlines causes Quality-of-Service(QoS)/performance degradation, so deadline violations are expected and permitted
  - reading data from the keyboard—user command interpretation
  - message displaying
  - graphical activities

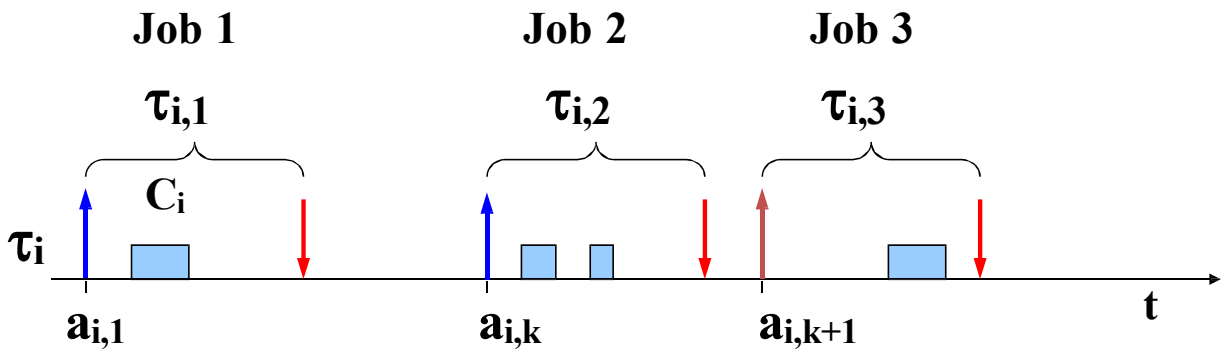
Real-Time Task

- A task characterized by a timing constraint on its response time, called deadline:
  - relative deadline D_i: part of task attribute definition, measured from task arrival time ai
  - Absolute deadline $d_i = a_i + D_i$: measured from some absolute reference time point 0
  - Gantt chart convention: upwards arrows denote job arrival/release times; downwards arrows denote deadlines
- Definition: feasible task
  - A real-time task τ_i is said to be feasible if it completes within its absolute deadline, that is, if $f_i \leq d_i$, or, equivalently, if $R_i \leq D_i$



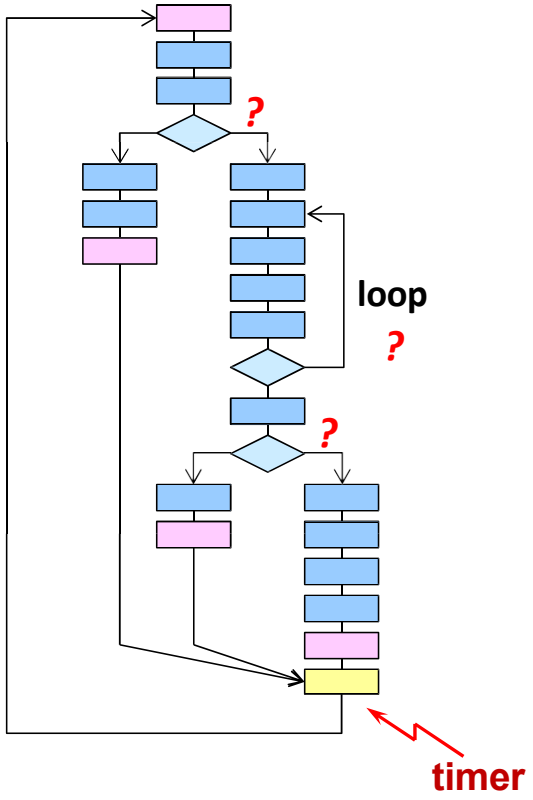
Tasks and Jobs

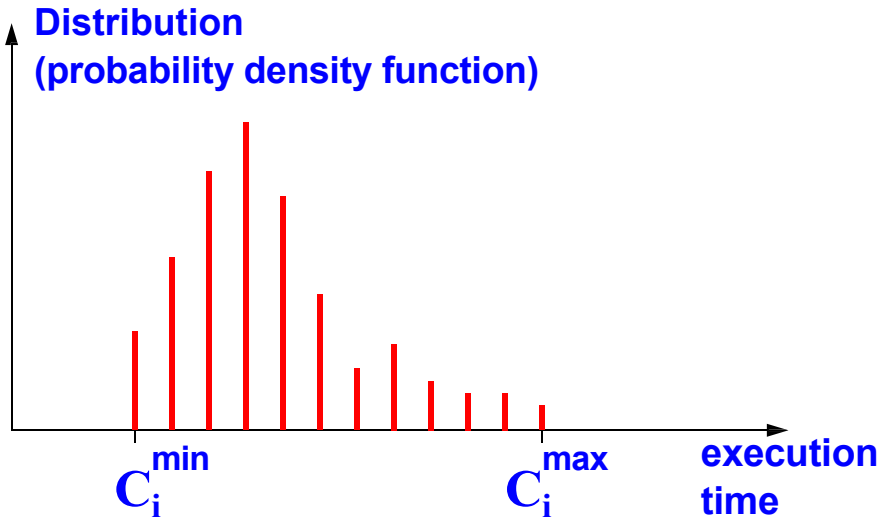
- A task running several times on different input data generates a sequence of instances (jobs)
  - Upwards arrow: task arrival or release times; downwards arrow: task deadlines
- Activation mode:
  - Periodic tasks: the task is activated by the operating system at predefined time intervals
  - Aperiodic tasks: the task is activated at an event arrival



Estimating WCET is Not Easy

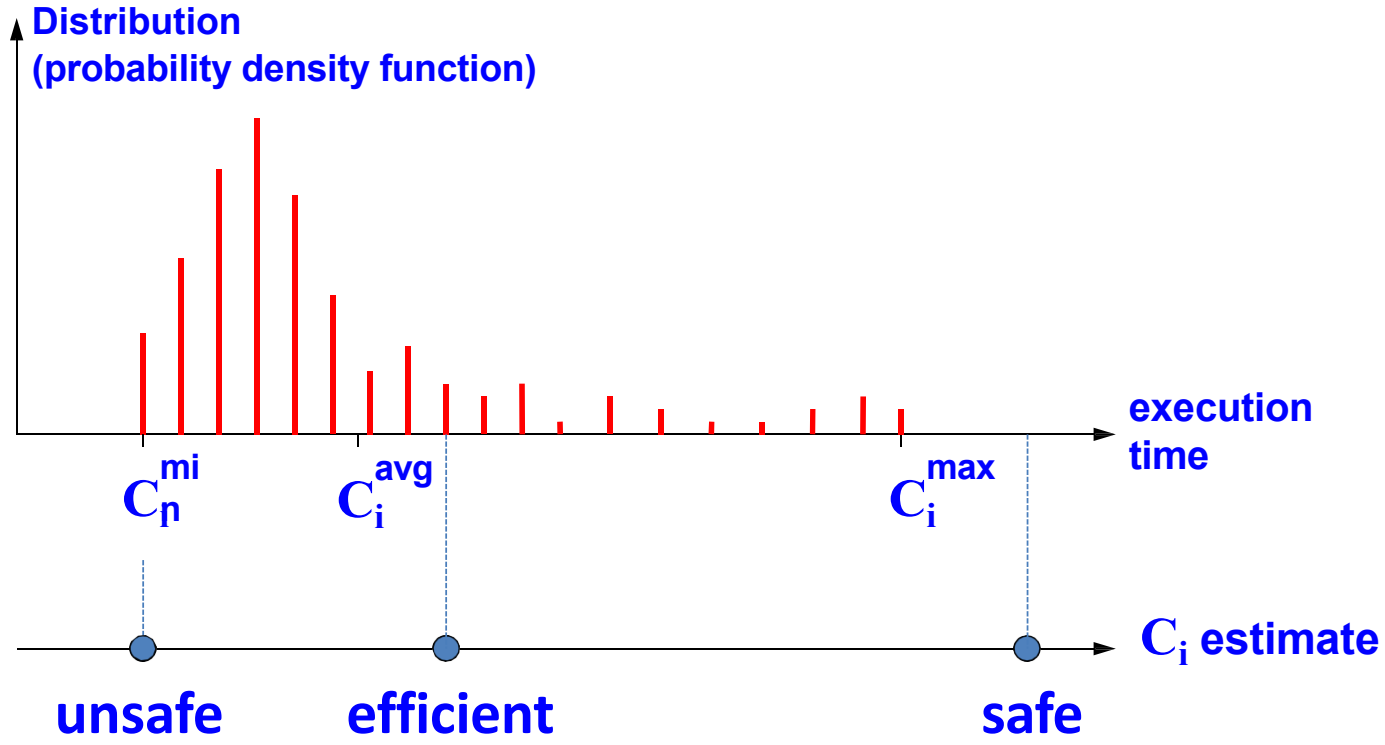
- Each job operates on different data and can take different paths.
- Even for the same data, computation time depends on processor state (cache state, number of preemptions).
- We use C_i to denote C_i^{max} Worst-Case Execution Time (WCET) in this lecture, and assume it is given as part of task parameters.





Predictability/Safety vs. Efficiency

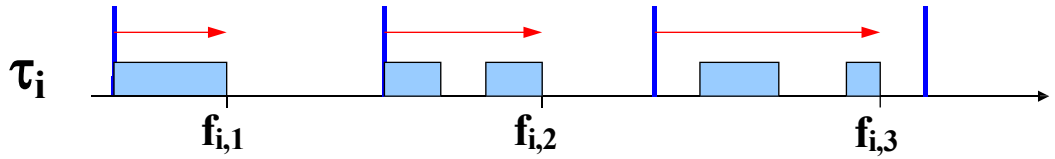
- Tradeoff between safety and efficiency in estimating the WCET C_i
  - Setting a large C_i achieves high predictability and safety, since it is unlikely to be exceeded at runtime; but it hurts efficiency, since the system needs to reserve more CPU time for the task. Suitable for hard real-time tasks.
  - Setting a small C_i achieves high efficiency, but hurts safety, since the task may execute for more than its C_i estimate. Suitable for soft real-time tasks.



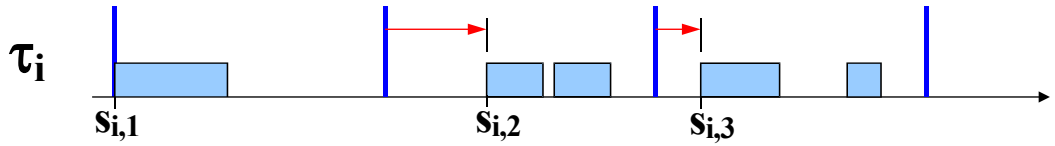
Jitter

- It is a measure of the time variation of a periodic event:

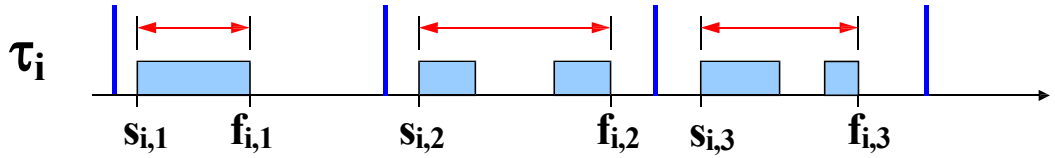
Finish-time Jitter



Start-time Jitter

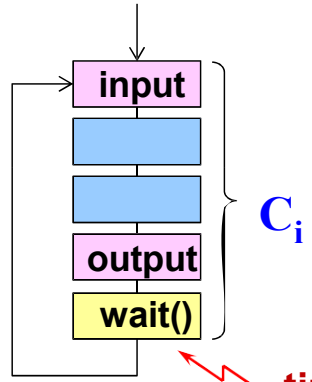


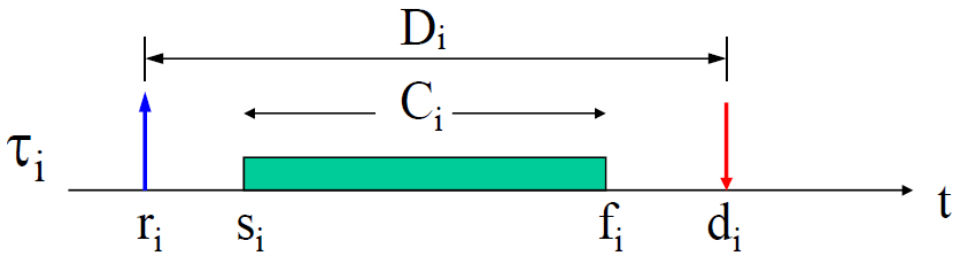
Completion-time Jitter (I/O Jitter)



Periodic Task

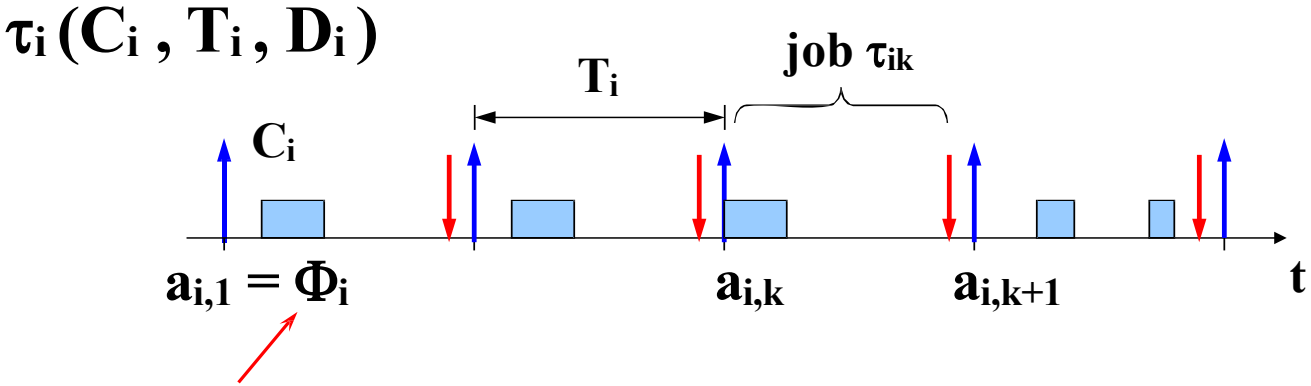
- A periodic task τ_i has a tuple of 3 attributes (C_i, T_i, D_i):
  - Worst-Case Execution Time (WCET) C_i; Period T_i; Relative Deadline D_i
  - Implicit deadline if $D_i = T_i$; Constrained deadline if $D_i \leq T_i$
- It generates an infinite sequence of jobs in every period: $\tau_{i,1}, \tau_{i,1}, \ldots, \tau_{i,k}, \ldots$





$\mathbf{r_i}$	release time (arrival time $\mathbf{a_i}$)
$\mathbf{s_i}$	start time
$\mathbf{C_i}$	worst-case execution time (wcet)
$\mathbf{d_i}$	absolute deadline
$\mathbf{D_i}$	relative deadline
$\mathbf{f_i}$	finishing time

A job of task τ_i



$$a_{i,k} = \Phi_i + (k-1)\,T_i$$
$$d_{i,k} = a_{i,k} + D_i$$

often $D_i = T_i$

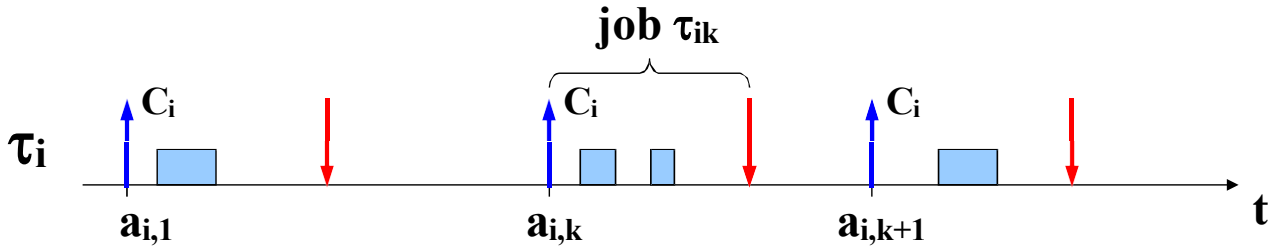
Multiple jobs released by task τ_i

Aperiodic & Sporadic Task

- Aperiodic task: jobs may arrive at arbitrary time instants
- Sporadic task: arrival times with a minimum interarrival time constraint

- **Aperiodic:** $a_{i,k+1} > a_{i,k}$
- **Sporadic:** $a_{i,k+1} \geq a_{i,k} + T_i$

minimum interarrival time

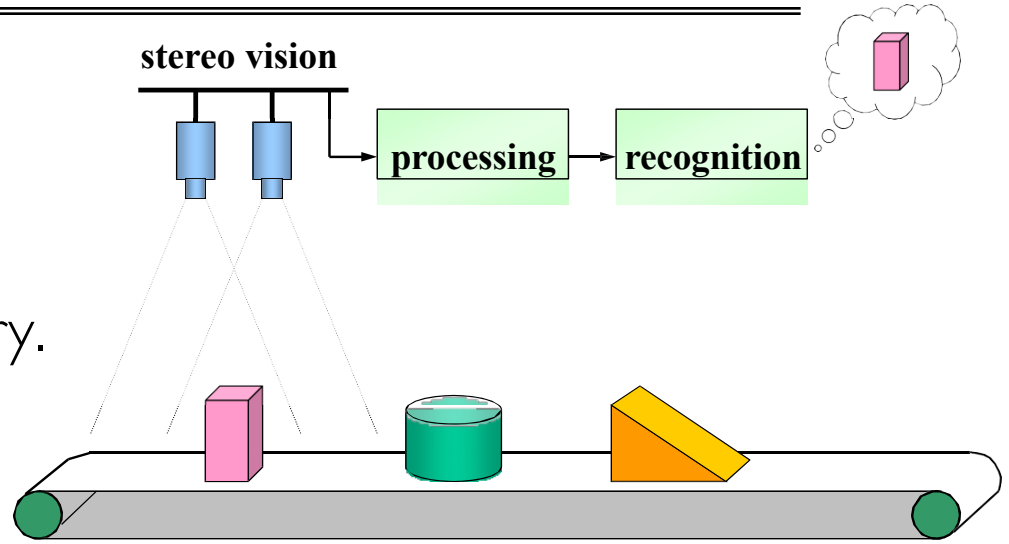


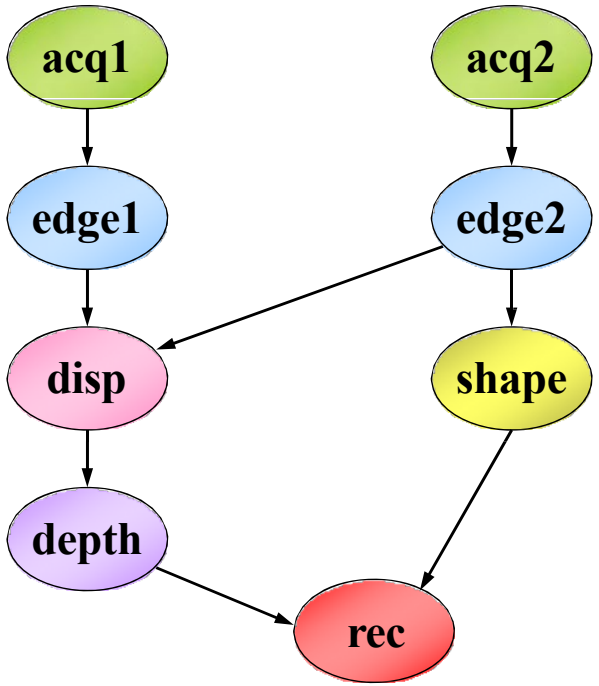
Types of Constraints

- Timing constraints
  - Deadline, jitter
- Precedence constraints
  - Relative ordering among task executions
- Resource constraints
  - Synchronization when accessing mutually-exclusive resources (shared data)

Precedence Constraints

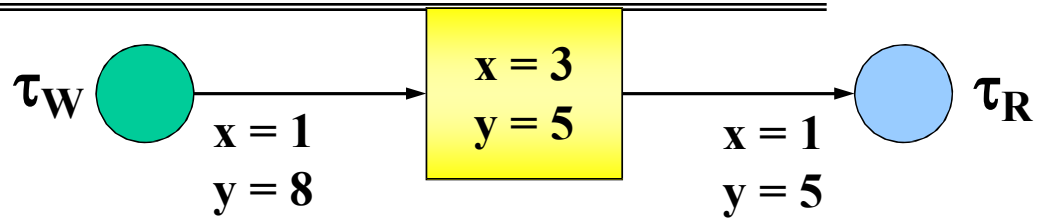
- Tasks must be executed with specific precedence relations, specified by a Directed Acyclic Graph (Precedence Graph)
- Example application of parts inspection in a factory. Tasks:
  - Image acquisition (acq1, acq2)
  - Edge detection (edge1, edge2)
  - Shape detection (shape), pixel disparities (disp)
  - Height determination (height), recognition (rec)

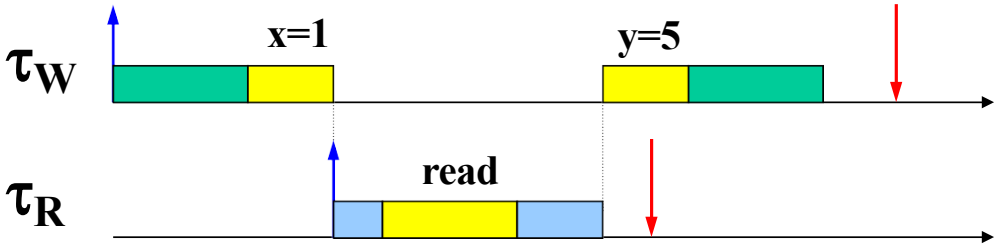




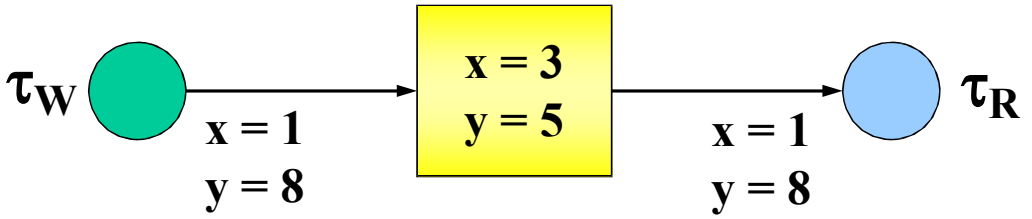
Resource Constraints

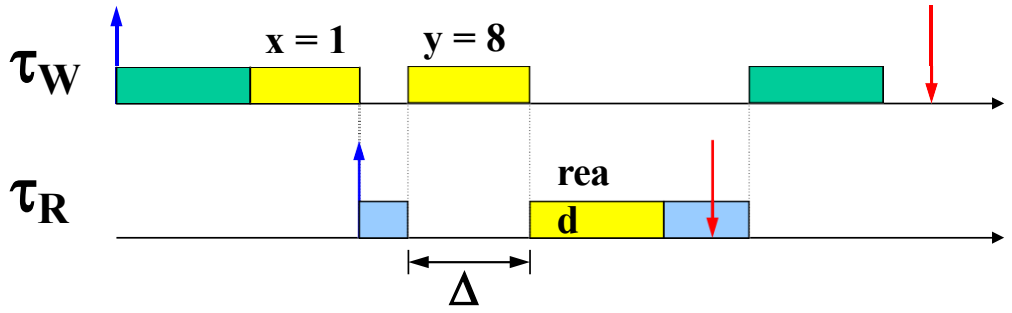
- To ensure data consistency, shared data must be accessed in mutual exclusion
- Example: the writer task τ_W writes to variables x and y; the reader task τ_R reads x and y. The pair of variables (x, y) should be updated atomically, i.e., τ_R should read either $(x, y) = (1,8)$ or $(x, y) = (3,5)$.
- Left upper: an erroneous scenario when τ_R reads a set of inconsistent values $(x, y) = (3,5)$.
- Left lower: protecting the critical section (yellow parts) with a mutex lock ensures atomicity.





Erroneous scenario with no lock protection

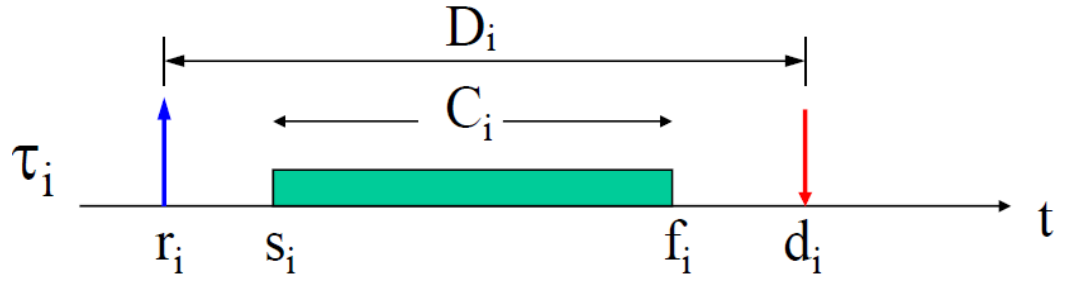




Correct scenario with lock protection

Scheduling Metrics

- Lateness $L_i = f_i - d_i$ represents the delay of a task completion with respect to its deadline; if a task completes before the deadline, its lateness is negative.
- Tardiness or exceeding time $E_i = \max(0, L_i)$ is the time a task stays active after its deadline; if a task completes before the deadline, its tardiness is 0.



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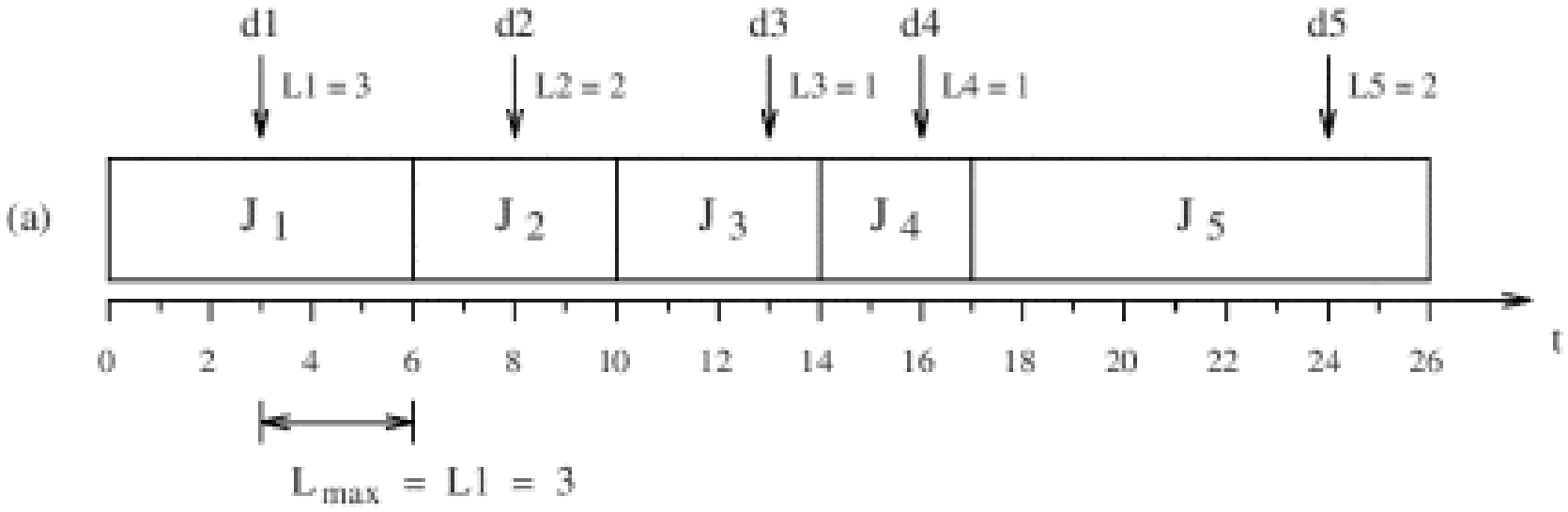
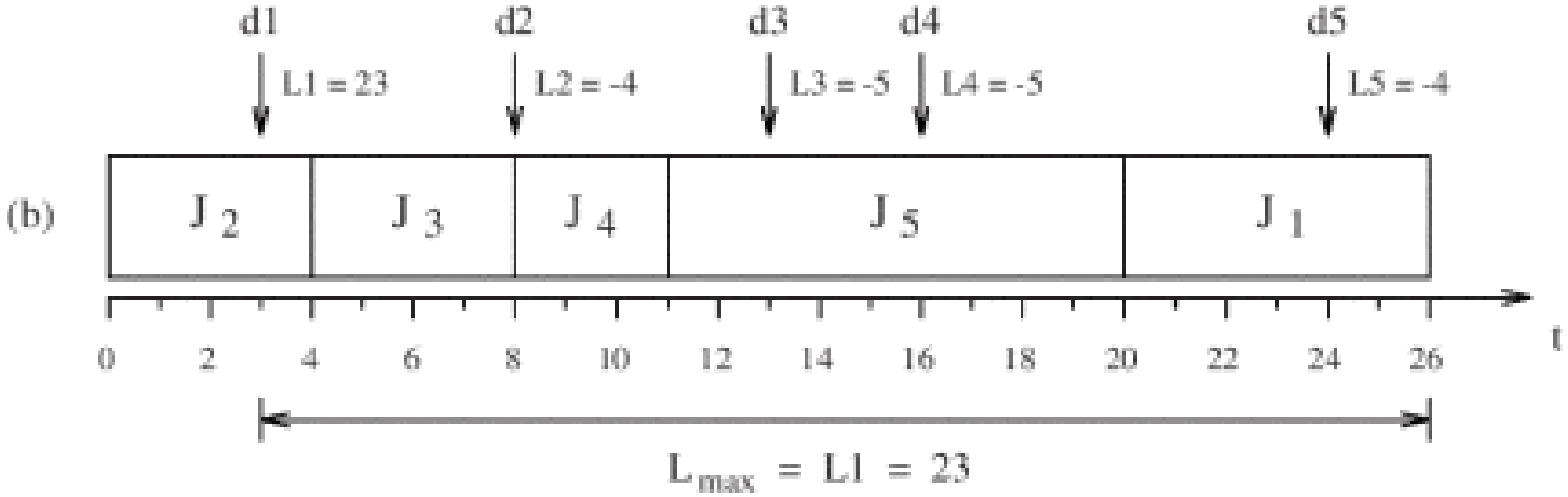
$\mathbf{s_i}$	start time
$\mathbf{C_i}$	worst-case execution time (wcet)
$\mathbf{d_i}$	absolute deadline
$\mathbf{D_i}$	relative deadline
$\mathbf{f_i}$	finishing time

Example: Lateness

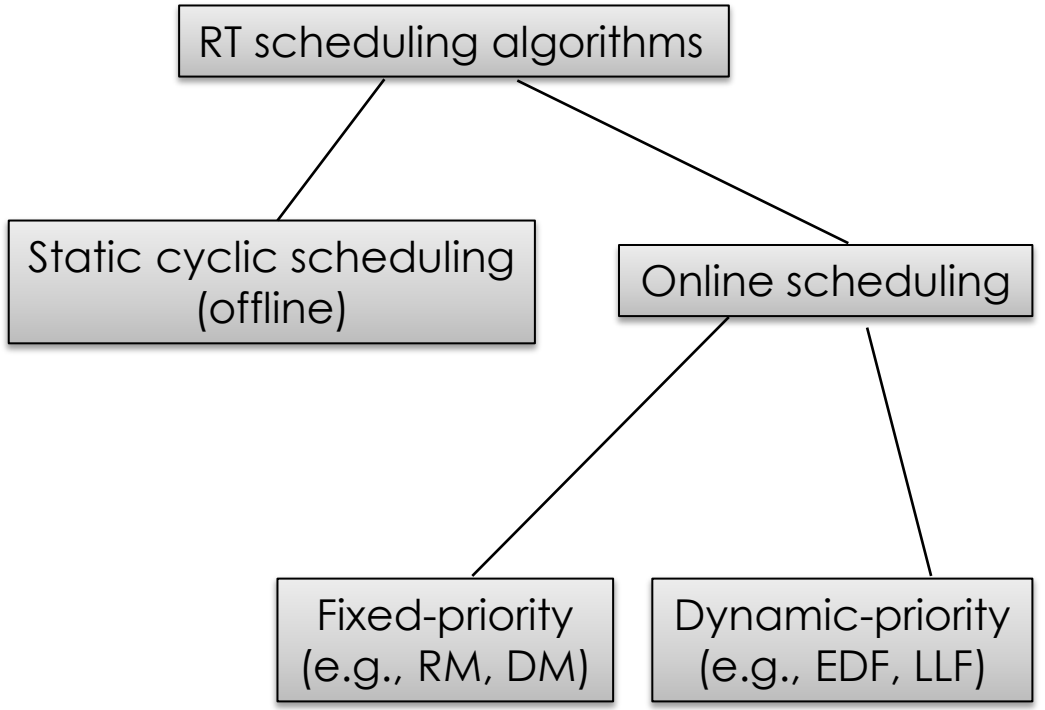
- Which schedule is better depends on application requirements:
- In (a), the maximum lateness is minimized with $L_{max} = 3$, but all jobs J_1 to J_5 miss their deadlines.
- In (b), the maximal lateness is larger with $L_{max} = 23$, but only one job J_1 misses its deadline.





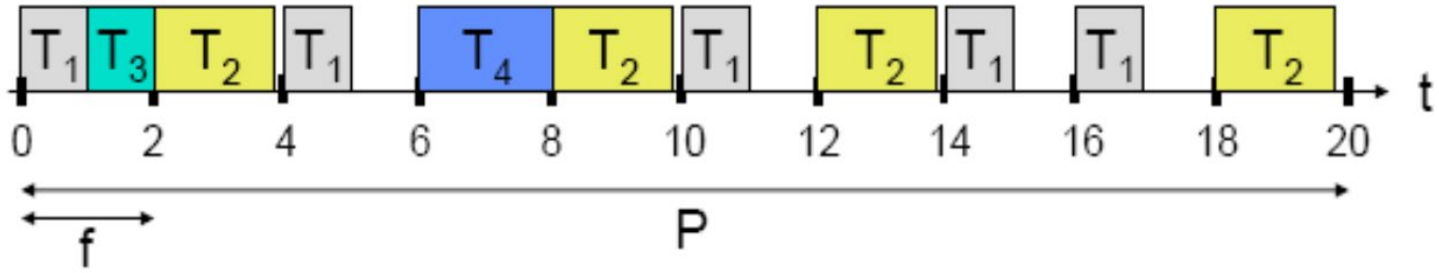
Scheduling Algorithms

- Static cyclic scheduling (offline)
  - All task invocation times are computed offline and stored in a table; Runtime dispatch is a simple table lookup
- Online scheduling;
  - Fixed priority scheduling (also called static-priority scheduling)
    - » Each task is assigned a fixed priority; Runtime dispatch is priority-based, e.g., Rate Monotonic (RM), Deadline Monotonic (DM)
  - Dynamic priority scheduling
    - » Task priorities are assigned dynamically at runtime, e.g., Earliest Deadline First (EDF), Least-Laxity First (LLF)
  - Non-real-time scheduling, e.g., round-robin, multi-level queue…



Static Cyclic Scheduling

- The same schedule is executed once during each hyper-period (least common multiple of all task periods in a taskset).
  - The hyper-period is partitioned into frames of length f.
    - » If a task's WCET exceeds f, then programmer needs to cut it to fit within a frame, and save/restore program state manually
  - The schedule is computed offline and stored in a table. Runtime task dispatch is a simple table lookup.
- Pros:
  - Deals with precedence, exclusion, and distance constraints
  - Efficient, low-overhead for runtime task dispatch
  - Lock-free at runtime
- Cons:
  - Task table can get very large if task periods are relatively prime
  - Maintenance nightmare: complete redesign when new tasks are added, or old tasks are deleted
- Not widely used
  - Except in certain safety-critical systems such as avionic systems



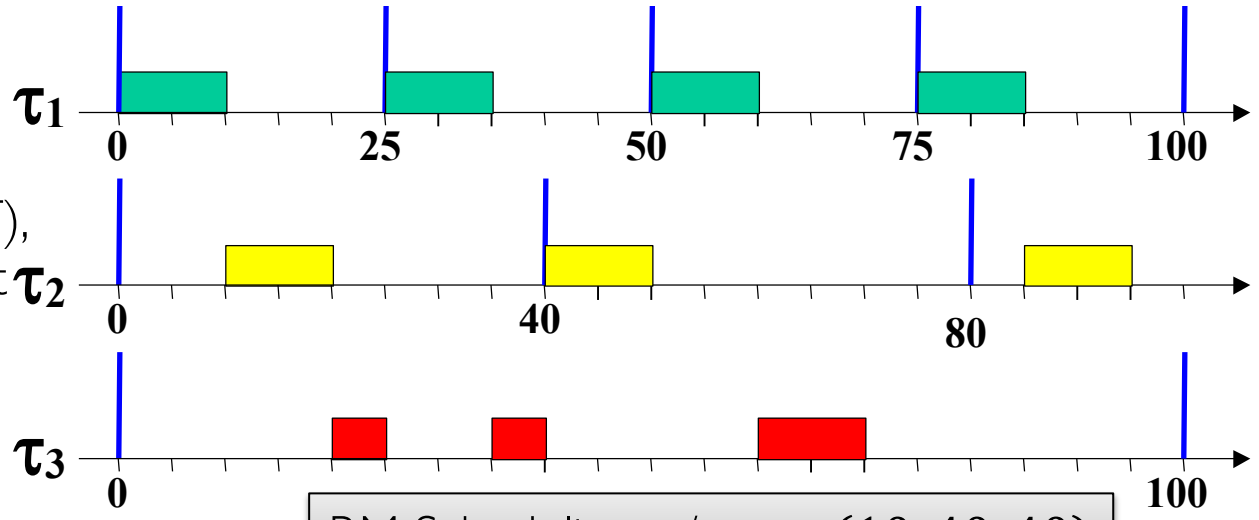
Fixed-Priority Scheduling

Fixed Priority Scheduling

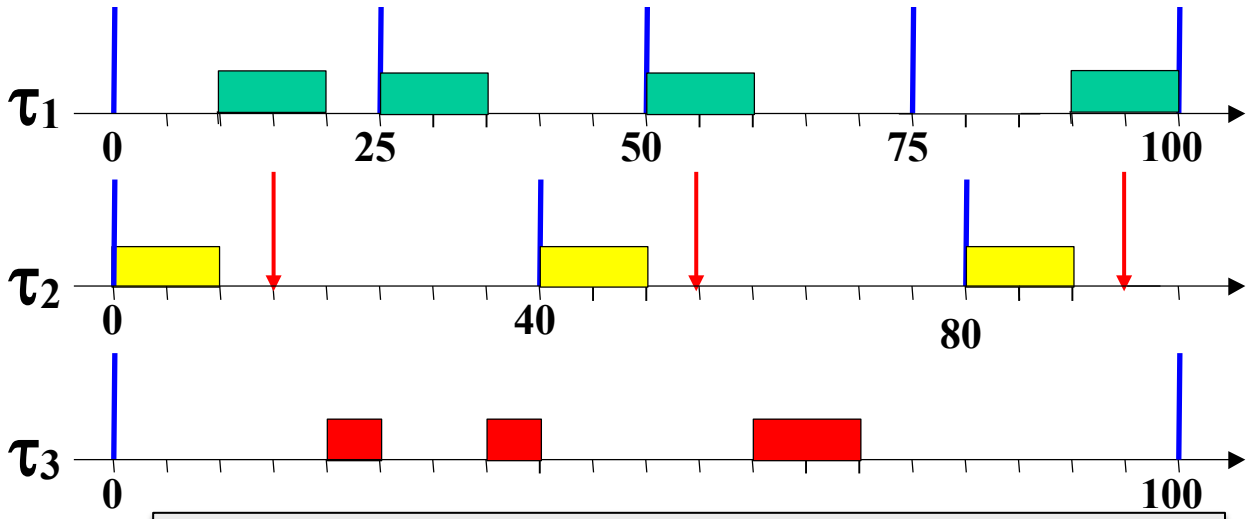
- Each task is assigned a fixed priority for all its invocations
- Pros:
  - Predictability
  - Low runtime overhead
  - Temporal isolation during overload
- Cons:
  - Cannot achieve 100% utilization in general, except when task periods are harmonic
- Widely used in most commercial RTOSes and CAN bus

Rate Monotonic & Deadline Monotonic Scheduling

- Rate Monotonic (RM)
  - Assign higher priority to task with smaller period
  - For implicit deadline tasksets (deadline D = period T), RM is the optimal priority assignment, i.e., if a taskset is not schedulable with RMS priority assignment, then it is not schedulable with any other fixed priority assignment
- Deadline Monotonic (DM)
  - Assign higher priority to task with smaller relative deadline
  - For constrained deadline tasksets (D ≤ T), DM is the optimal priority assignment
- Why do we want D < T?
  - Some events happen infrequently, but need to be handled urgently
- Example taskset: $\tau_1 = (10, 25, 25), \tau_2 = (10, 40, 40)$ or $(10, 40, 15), \tau_3 = (20, 100, 100)$



RM Scheduling w/ $\tau_2 = (10, 40, 40)$



DM Scheduling w/ $\tau_2 = (10, 40, 15)$
(D_2=15 indicated by red downward arrow for τ_2)

Two Schedulability Analysis Approaches

IMPORTANT

- Utilization bound test
  - Calculate total CPU utilization and compare it to a known bound
  - Polynomial time complexity
  - Pessimistic: sufficient but not necessary condition for schedulability

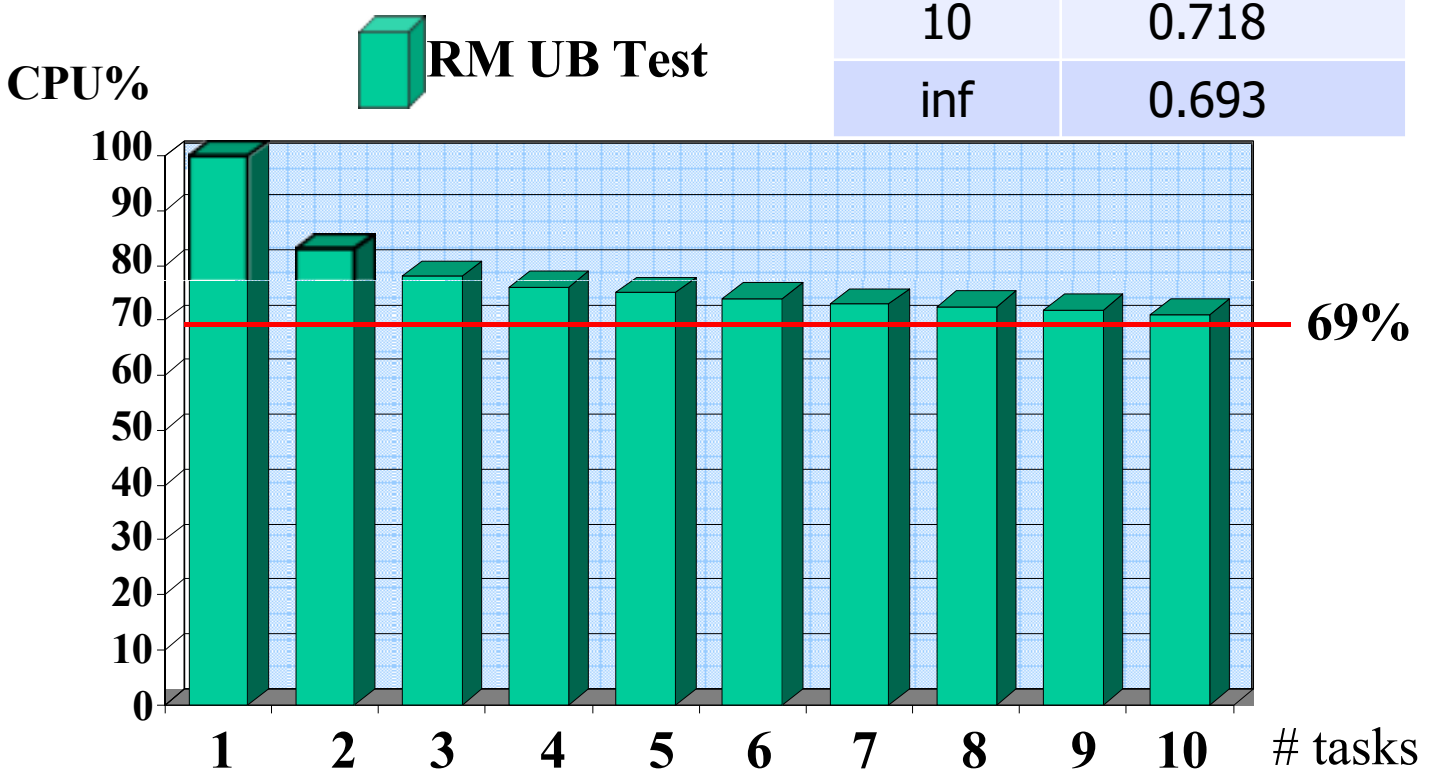
- Response Time Analysis (RTA)
  - Calculate Worst-Case Response Time R_i for each task Tau_i and compare it to its deadline D_i
  - Pseudo-polynomial time complexity
    - » An algorithm runs in pseudo-polynomial time if its running time is polynomial in the numeric value of the input (which is exponential in the length of the input – its number of digits).
  - Accurate: necessary and sufficient condition for schedulability

Utilization Bound Test

IMPORTANT

- A taskset is schedulable under RM scheduling if system utilization $U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq N(2^{1/N} - 1)$
  - $U \rightarrow 0.69$ as $N \rightarrow \infty$
  - Assumptions: task period equal to deadline ($P_i = D_i$); task with smaller period P_i is assigned higher priority (RM priority assignment); tasks are independent (no resource sharing)
- Sufficient but not necessary condition
  - Guaranteed to be schedulable if test succeeds
  - May still be schedulable even if test fails
- Special case: if periods are harmonic (larger periods divisible by smaller periods), then utilization bound is 1 (necessary and sufficient condition)

# Tasks	RM Util Bound
1	1.00
2	0.828
3	0.780
4	0.757
5	0.743
10	0.718
inf	0.693

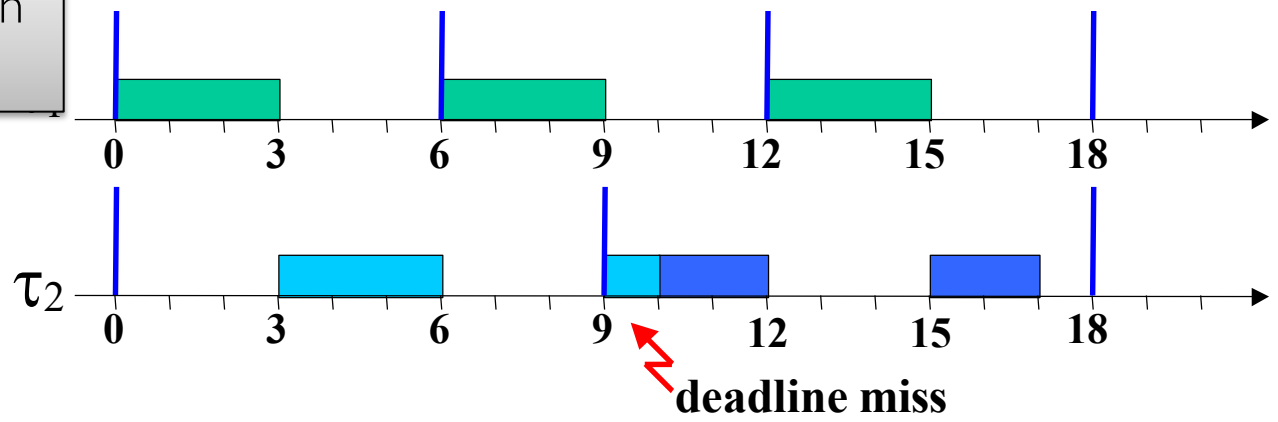


Utilization Bound Test Examples

We use the notation $\tau_i\,(C_i, T_i, D_i)$ to denote task τ_i with WCET C_i Period T_i, Deadline D_i

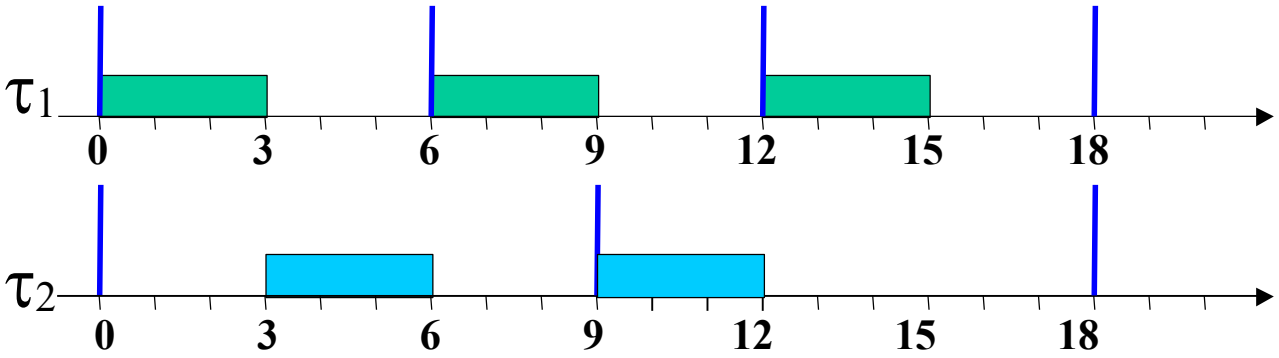
Taskset $\tau_1\,(3, 6, 6), \tau_2\,(4, 9, 9)$ unschedulable

$$U = \frac{3}{6} + \frac{4}{9} = 0.944 > 0.828$$



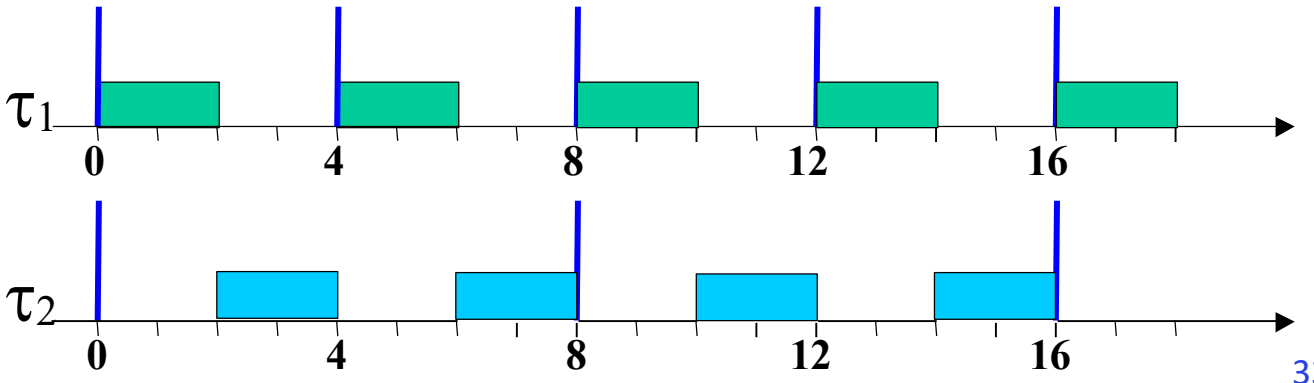
Taskset $\tau_1\,(3, 6, 6), \tau_2\,(3, 9, 9)$ schedulable (UB test is sufficient but not necessary condition)

$$U = \frac{3}{6} + \frac{3}{9} = 0.833 > 0.828$$



Taskset $\tau_1\,(2, 4, 4), \tau_2\,(4, 8, 8)$ schedulable (periods are harmonic)

$$U = \frac{2}{4} + \frac{4}{8} = 1.0 > 0.828$$

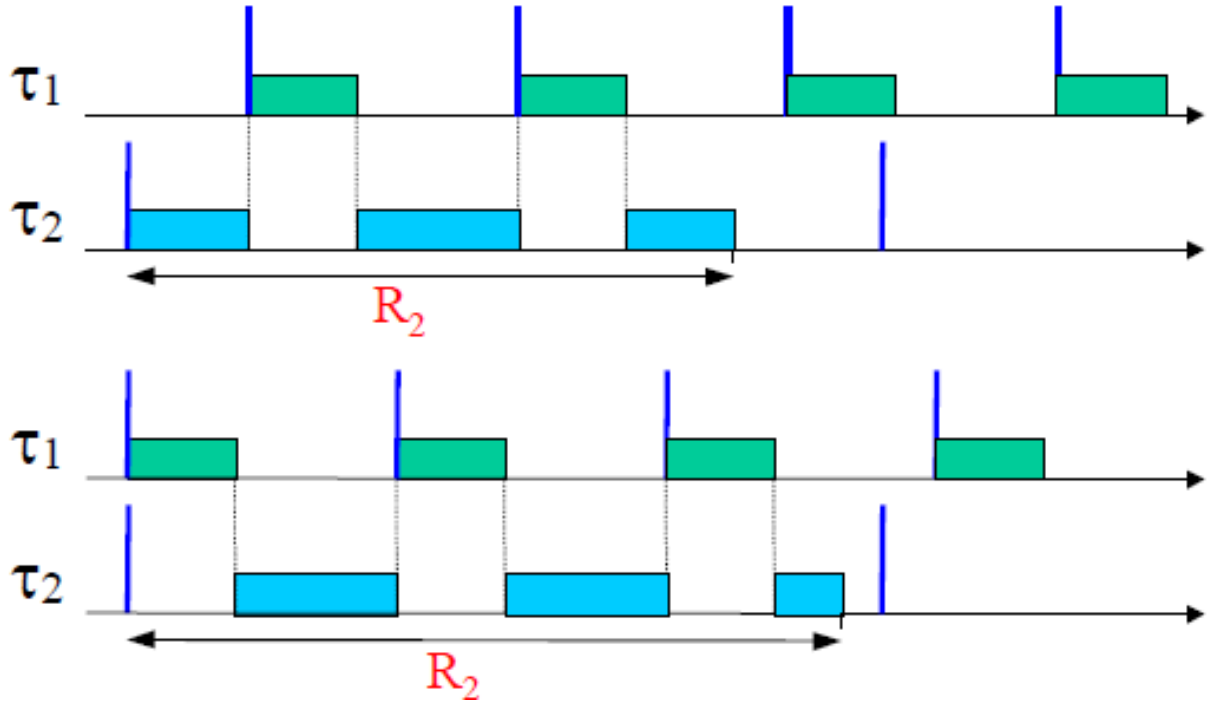


Response Time Analysis (RTA)

- Assumptions:
  - Consider the synchronous taskset: *all tasks are initially released at time 0 simultaneously*, called the *critical instant*. This is the worst-case when each task experiences maximum amount of interference from higher priority tasks: if the taskset is schedulable with this assumption, then it will be schedulable for any other release offset.
  - No resource sharing (no critical sections)
  - Figure shows task τ_2 has the worst-case response time R_2 if it is initially released at time 0, simultaneously with higher priority task τ_1 (lower figure)

τ_1, τ_2 initially released with a non-zero offset, not all at time 0. τ_2 experiences 2 preemptions by τ_1 and has shorter response time

τ_1, τ_2 initially released at time 0 simultaneously, the critical instant. τ_2 experiences 3 preemptions by τ_1 and has longer response time



Response Time Analysis (RTA)

IMPORTANT

- For each task τ_i, compute its Worst-Case Response Time (WCRT) R_i and compare to its deadline D_i. τ_i is schedulable iff $R_i \leq D_i$. The taskset is schedulable if all tasks are schedulable (necessary and sufficient condition. "iff" stands for "if and only if").
- Task τ_i's WCRT R_i is computed by solving the following recursive equation to find the *minimum fixed-point solution*:
- $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
  - where $hp(i)$ is the set of tasks with higher priority than task τ_i.
  - $\lceil \rceil$ is the ceiling operator, e.g., $\lceil 1.1 \rceil = 2, \lceil 1.0 \rceil = 1$
  - $\left\lceil \frac{R_i}{T_j} \right\rceil$ denotes the number of times HP task τ_j pre-empts τ_i during its one job execution; $\left\lceil \frac{R_i}{T_j} \right\rceil C_j$ denotes the total preemption delay caused by HP task τ_j to τ_i during its one job execution

An Example Taskset

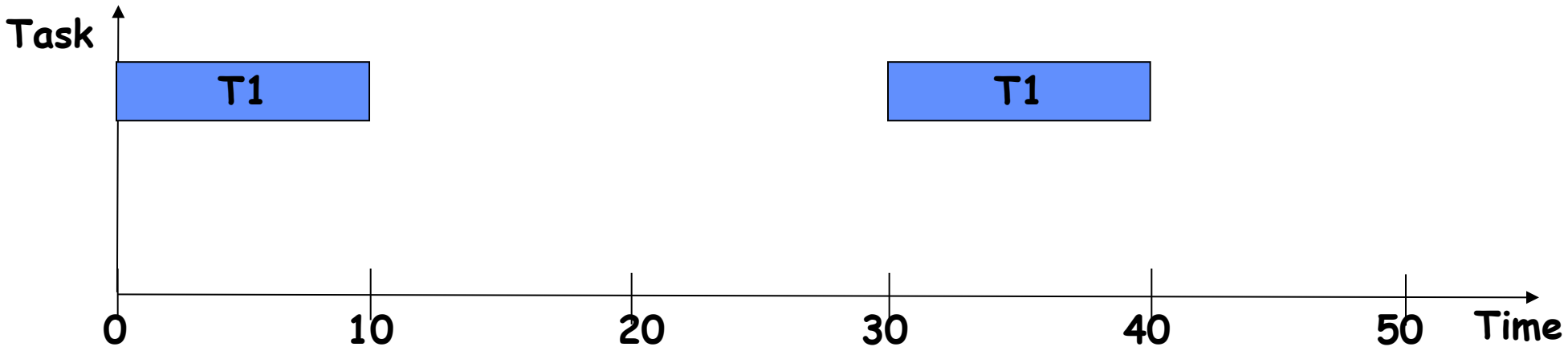
- Consider a taskset of 3 task with (C_i, T_i, D_i) of $(10, 30, 30), (10, 40, 40), (12, 52, 52)$. Under RM, task priorities are assigned to be High for T1, Medium for T2, and Low for T3
- System Utilization $U = \sum_{i=1}^{3} \frac{C_i}{T_i} = \frac{10}{30} + \frac{10}{40} + \frac{12}{52} = 0.81 > 0.78$
  - Utilization Bound $(N = 3) = 3 * \left(2^{1/3} - 1\right) = 0.78$
- Utilization bound test fails, but taskset is actually schedulable

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

Task T1

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

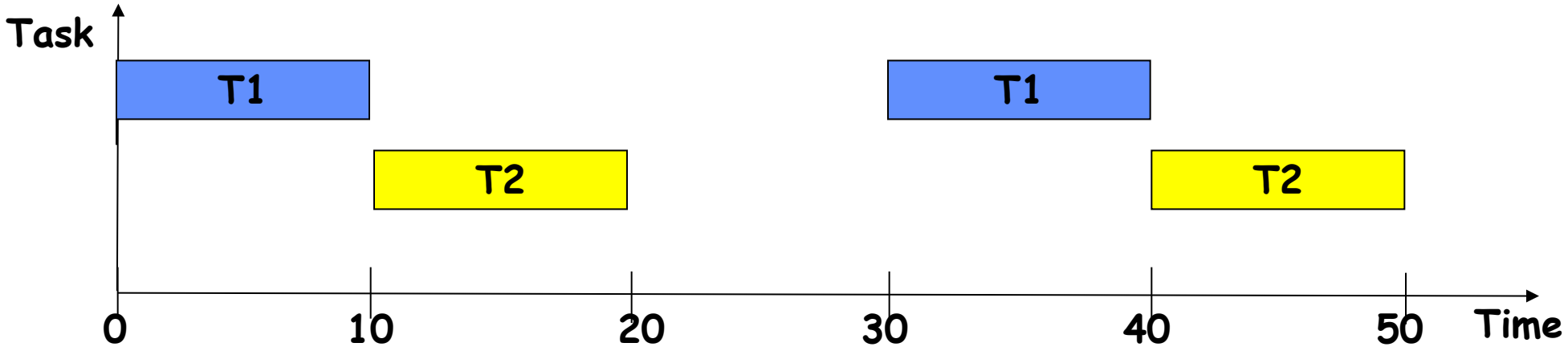
- T1 is the highest priority task, with no interference from other tasks $hp(1) = \emptyset$
- $R_1 = C_1 + 0 = 10$
- $R_1 < D_1$, so T1 is schedulable



Task T2

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

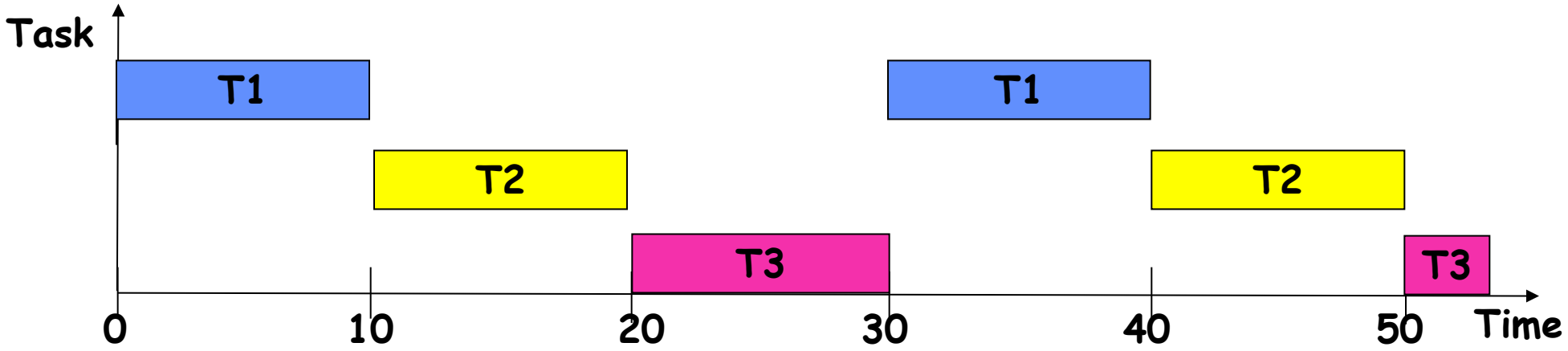
- T2 is the medium priority task, with interference from higher priority Task 1 $hp(2) = 1$
- $R_2 = C_2 + \lceil \frac{R_2}{T_1} \rceil * C_1 = 10 + \lceil \frac{R_2}{30} \rceil * 10$
- Solve for $R2$ iteratively, starting with initial value $R_2 = C_2 = 10$:
  - Iteration 1: $R_2 = 10 + \left\lceil \frac{10}{30} \right\rceil * 10 = 10 + 1 * 10 = 20$
  - Iteration 2: $R_2 = 10 + \left\lceil \frac{20}{30} \right\rceil * 10 = 10 + 1 * 10 = 20$
- Hence $R_2 = 20 < D_2 = 40$, so T2 is schedulable



Task T3

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

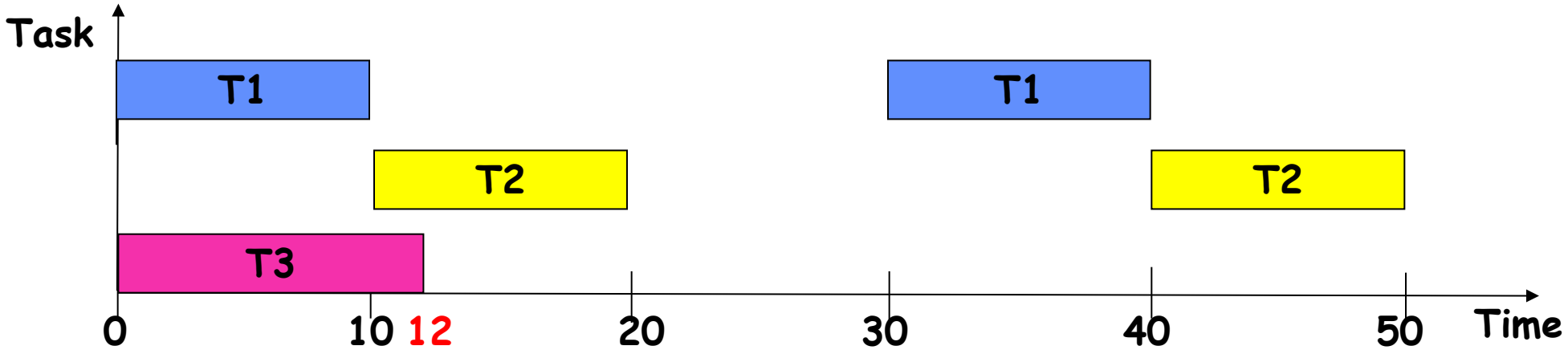
- T3 is the lowest priority task, with interference from higher priority tasks $hp(3) = \{1,2\}$
- $R_3 = C_3 + \lceil \frac{R_3}{T_1} \rceil * C_1 + \lceil \frac{R_3}{T_2} \rceil * C_2 = 12 + \lceil \frac{R_3}{30} \rceil * 10 + \lceil \frac{R_3}{40} \rceil * 10$
- Solve for R_3 iteratively, starting with initial value $R_3 = C_3 = 12$:
  - Iteration 1: $R_3 = 12 + \lceil 12/30 \rceil * 10 + \lceil 12/40 \rceil * 10 = 32$
  - Iteration 2: $R_3 = 12 + \lceil 32/30 \rceil * 10 + \lceil 32/40 \rceil * 10 = 42$
  - Iteration 3: $R_3 = 12 + \lceil 42/30 \rceil * 10 + \lceil 42/40 \rceil * 10 = 52$
  - Iteration 4: $R_3 = 12 + \lceil 52/30 \rceil * 10 + \lceil 52/40 \rceil * 10 = 52$
- Hence $R_3 = 52 \leq D_3 = 52$, so T3 is schedulable



RTA for T3: Initial Condition

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

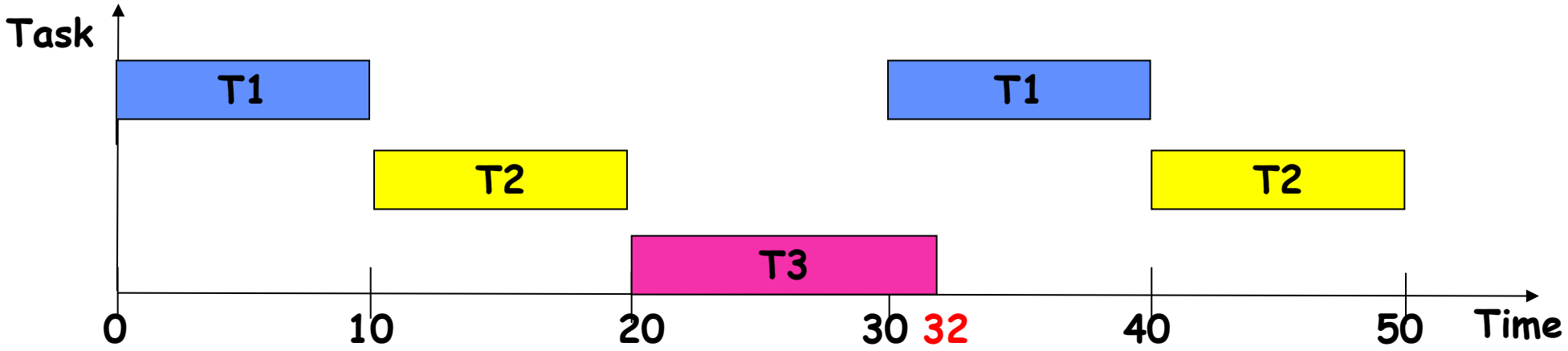
- Initially $R_3 = 12$
- We have not taken into account any preemption delays from higher priority tasks T1 and T2 yet



RTA for Task 3: Iteration 1

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

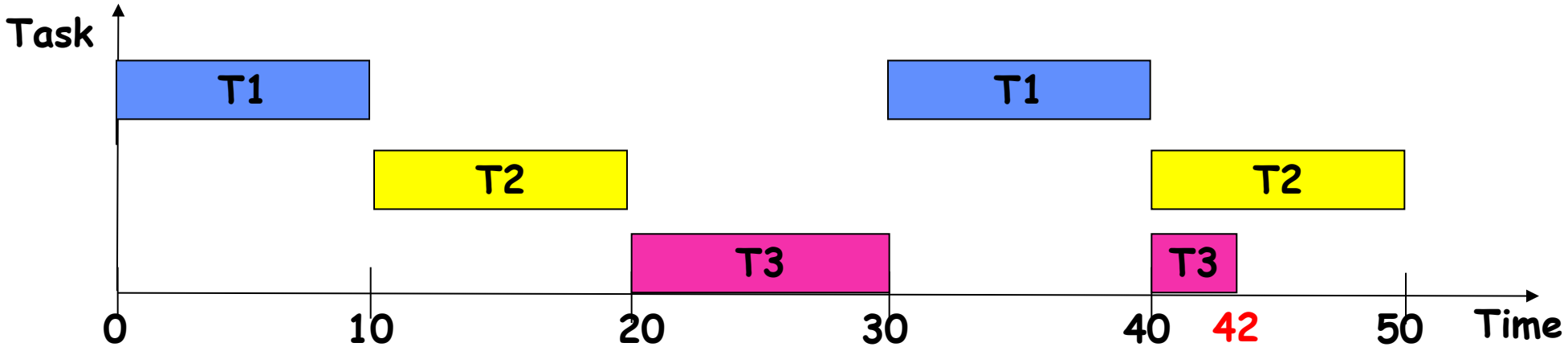
- $R_3 = 12 + \left\lceil \frac{12}{30} \right\rceil * 10 + \left\lceil \frac{12}{40} \right\rceil * 10$
- $= 12 + 1 * 10 + 1 * 10 = 32$
- T1 preempts T3 once, and T2 preempts T3 once
  - since all 3 tasks are released at time 0 (synchronous release time assumption), and T1 and T2 have higher priority than T3



RTA for Task 3: Iteration 2

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

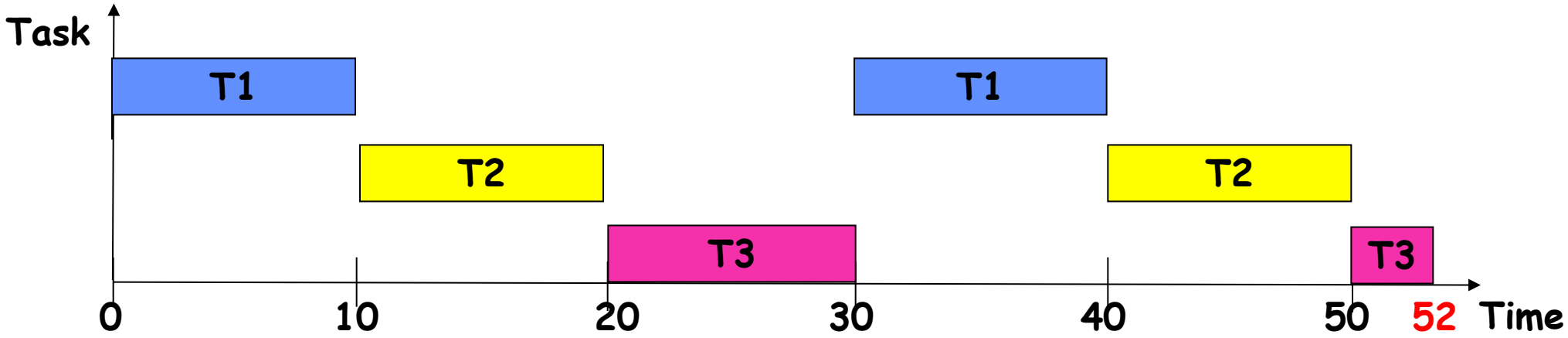
- $R_3 = 12 + \left\lceil \frac{32}{30} \right\rceil * 10 + \left\lceil \frac{32}{40} \right\rceil * 10$
- $= 12 + 2 * 10 + 1 * 10 = 42$
- T1 preempts T3 twice, and T2 preempts T3 once
  - Since T3 has not finished execution at time 30, and another job of higher priority task T1 is released at time 30 and preempts T3



RTA for Task 3: Iteration 3

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

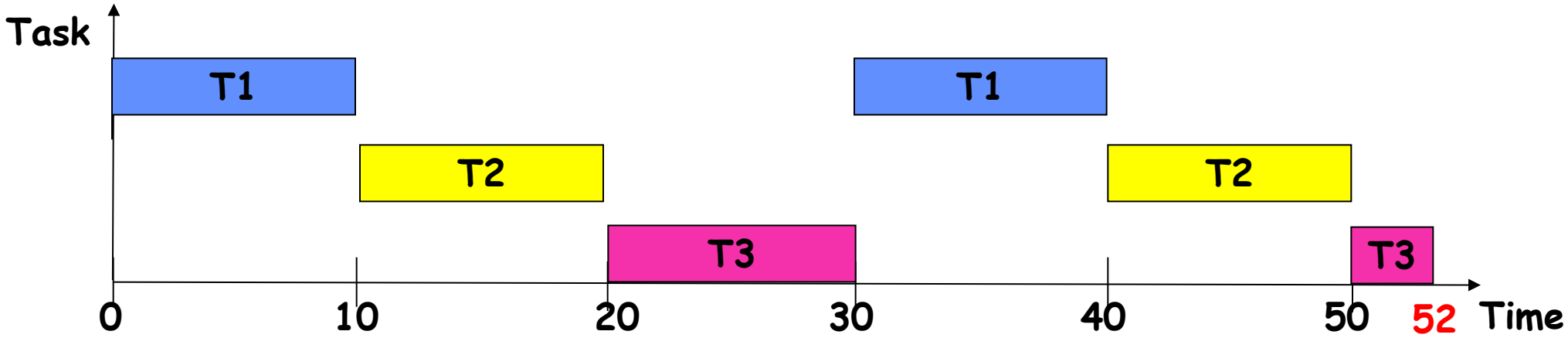
- $R_3 = 12 + \left\lceil \frac{42}{30} \right\rceil * 10 + \left\lceil \frac{42}{40} \right\rceil * 10$
- $= 12 + 2 * 10 + 2 * 10 = 52$
- T1 preempts T3 twice, and T2 preempts T3 twice
  - Since T3 has not finished execution at time 40, and another job of higher priority task T2 is released at time 40 and preempts T3



RTA for Task 3: Iteration 4

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	52	12	L

- $R_3 = 12 + \left\lceil \frac{52}{30} \right\rceil * 10 + \left\lceil \frac{52}{40} \right\rceil * 10 = 12 + 2 * 10 + 2 * 10 = 52$
- T1 preempts T3 twice, and T2 preempts T3 twice
  - Since T3 has finished execution at time 52, and the next arrivals of T1 and T2 are at time 60 and 80, respectively, so T3 will not experience additional preemptions from T1 and T2
- Now the recursive equation has converged, and we have obtained the WCRT of T3 $R_3 = 52 \leq D_3 = 52$



When T3 is Unschedulable

- The recursive equation may not converge, i.e., a task's WCRT may be infinity, e.g., suppose we change T2's WCET to be 20, then:

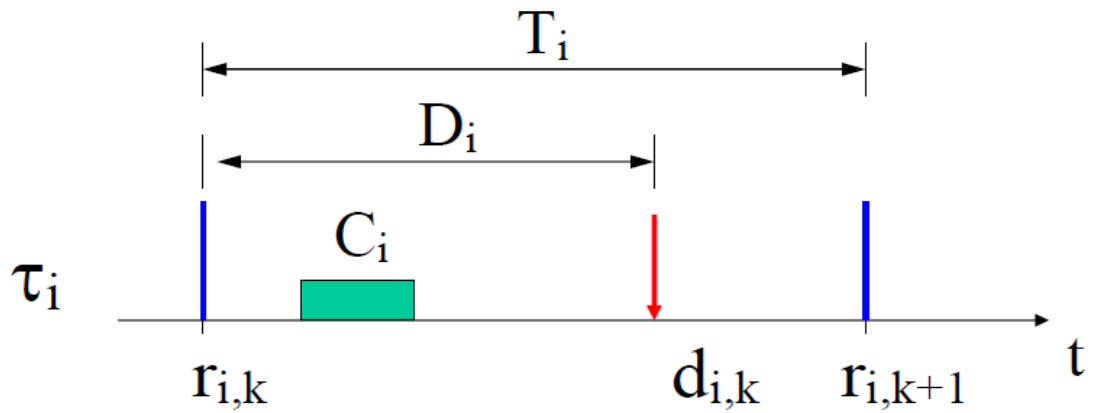
Task	T=D	C	Prio
T1	30	10	H
T2	40	20	M
T3	52	12	L

- $R_3 = C_3 + \lceil \frac{R_3}{T_1} \rceil * C_1 + \lceil \frac{R_3}{T_2} \rceil * C_2 = 12 + \lceil \frac{R_3}{30} \rceil * 10 + \lceil \frac{R_3}{40} \rceil * 20$
- Solve for R_3 iteratively, starting with initial value $R_3 = C_3 = 12$:
  - Iteration 1: $R_3 = 12 + \lceil 12/30 \rceil * 10 + \lceil 12/40 \rceil * 20 = 42$
  - Iteration 1: $R_3 = 12 + \lceil 42/30 \rceil * 10 + \lceil 42/40 \rceil * 20 = 72$
  - Iteration 3: $R_3 = 12 + \lceil 72/30 \rceil * 10 + \lceil 72/40 \rceil * 20 = 82$
  - Iteration 4: $R_3 = 12 + \lceil 82/30 \rceil * 10 + \lceil 82/40 \rceil * 20 = 102$
  - ...
- Hence $R_3 \rightarrow \infty$. This means that T3's first job never finishes execution due to interferences by higher priority tasks, hence T3 is unschedulable
- It is also possible for T3 to be unschedulable if R_3 converges but it exceeds its deadline D_3, e.g., if we set $D_3 = 50$, then $R_3 = 52 > D_3 = 50$ (another job of T3 is released at time 50, but RTA for the current job is not affected by the newly-released job.)

Task	T=D	C	Prio
T1	30	10	H
T2	40	10	M
T3	50	12	L

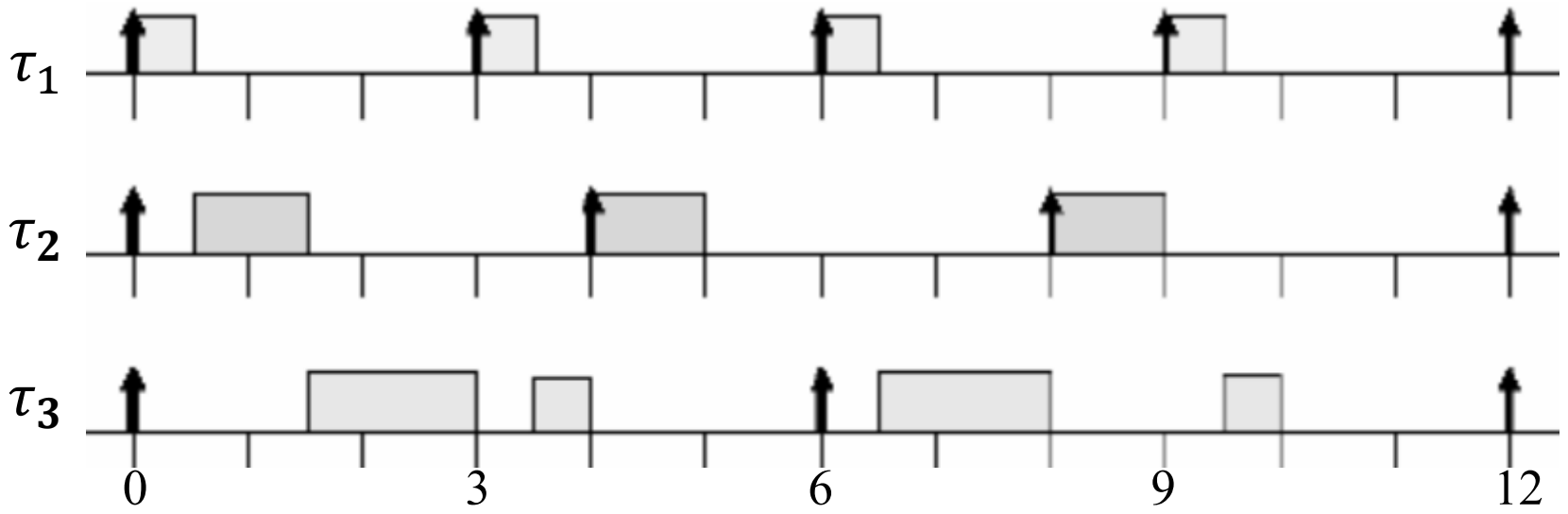
DM for Constrained Deadline Tasksets (D ≤ T)

- Deadline monotonic (Fixed Priority):
  - A task with smaller relative deadline gets higher priority $P_i \propto 1/D_i$
  - For constrained deadline tasksets (D ≤ T), DM is the optimal priority assignment
  - No Utilization Bound test for RM or DM, for tasksets with D ≤ T; must use Response Time Analysis (RTA)
  - Consider a taskset with two tasks both with $(C_i, T_i, D_i) = (1, 2, 1)$. Using RTA, assuming τ_1 has higher priority (since task periods are equal, we can assign either task higher priority), we can determine $R_1 = 1 \leq D_2 = 1, R_2 = 2 > D_2 = 1$, hence it is unschedulable

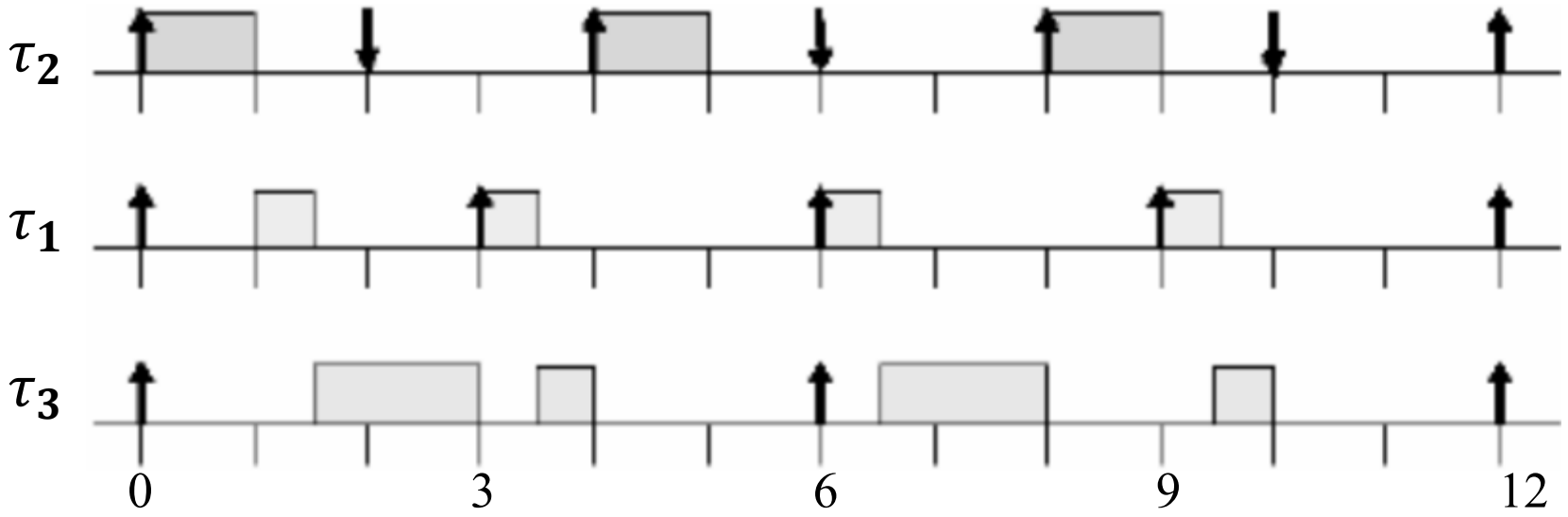


RM vs. DM Example

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
- Under RM (or DM), priority ordering $\tau_1 > \tau_2 > \tau_3$



- Three tasks with τ_2 assigned a smaller deadline of $D_2 = 2$: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- Under DM, priority ordering $\tau_2 > \tau_1 > \tau_3$



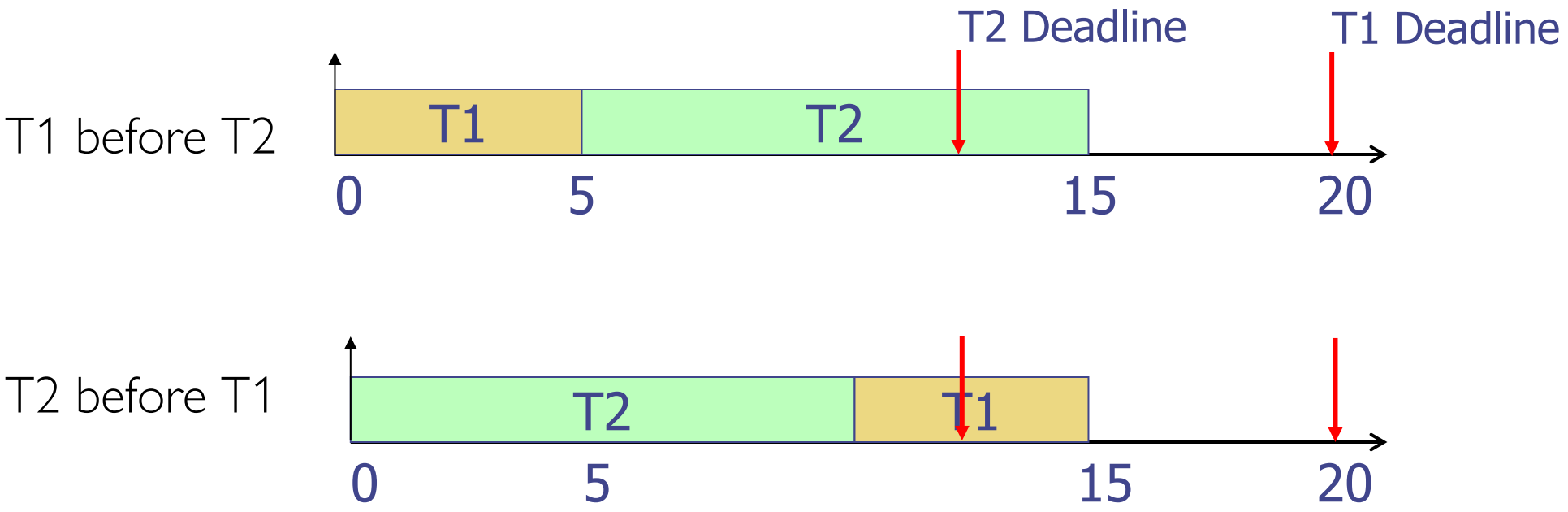
Earliest Deadline First (EDF) Scheduling

Earliest Deadline First (EDF)

- As each jon enters the system, it is assigned a deadline, and its priority is determined by its absolute deadline d_i
  - The job with the earlier deadline is assigned the higher priority
  - This priority assignment is dynamic because a periodic task's priority changes for each job released by the task (vs. fixed-priority scheduling, where a periodic task is assigned a fixed priority for all its jobs)
- Pros:
  - Optimal: can achieve 100% CPU utilization
- Cons:
  - Poor temporal isolation during overload
  - c.f. RM vs. EDF: Robustness under Overload

EDF Scheduling Example

- Say you have two tasks, both released at time 0
  - T1 has WCET 5 ms, with deadline of 20 ms
  - T2 has WCET 10 ms, with deadline of 12 ms
- Non-EDF scheduling: T1 before T2, T2 misses its deadline at 12
- EDF scheduling: T2 before T1, both tasks meet their deadlines

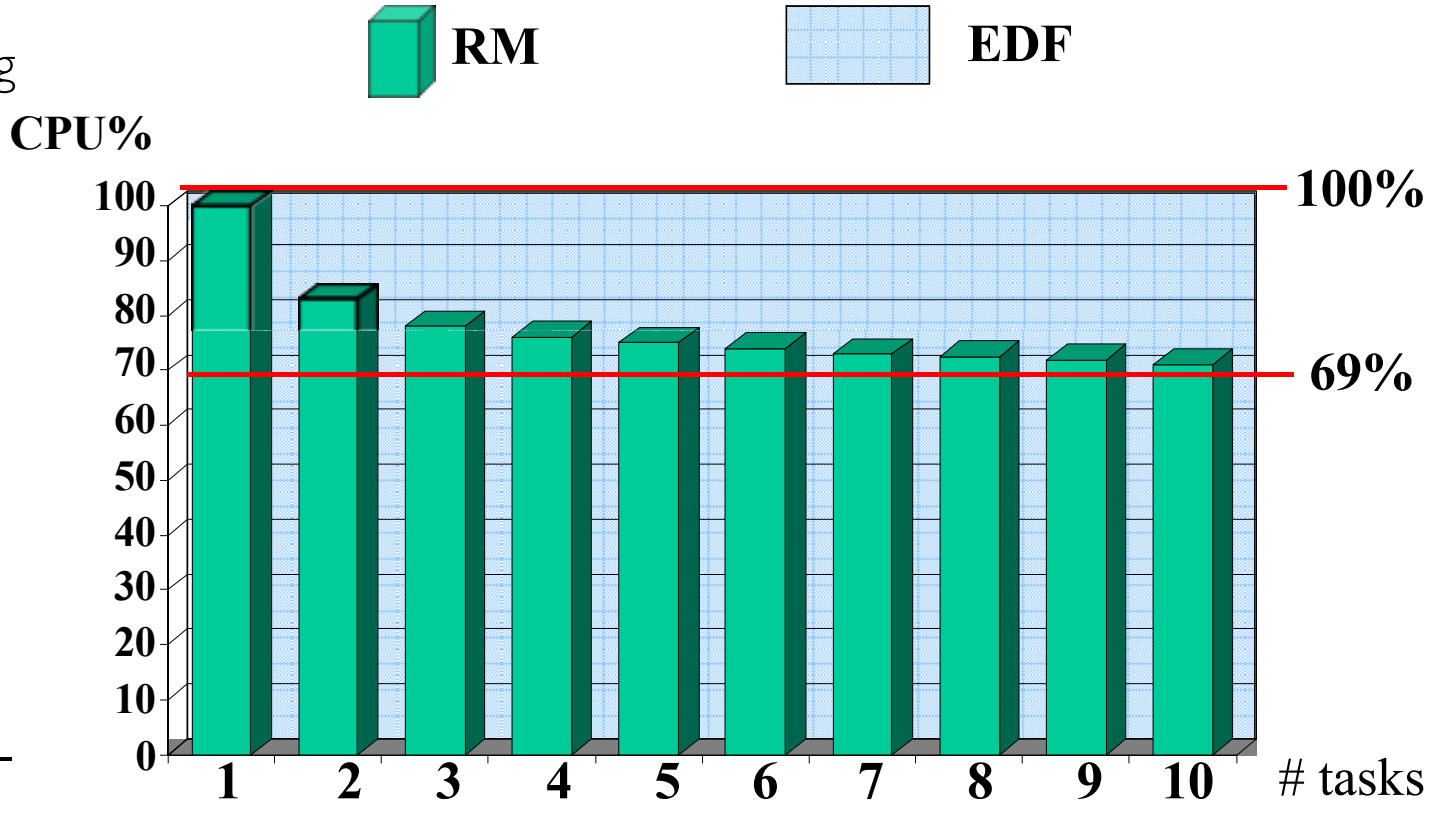


Convention: Upwards arrows indicate arrival time; Downwards arrows indicate deadline

Schedulable Utilization Bound: EDF vs. RM

IMPORTANT

- The schedulable utilization bound for EDF Scheduling is 1 (necessary and sufficient condition):
  - A taskset is schedulable under EDF scheduling iff system utilization does not exceed 1 $U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq 1$
    - » "iff" stands for "if and only if"
  - Assumptions: task period equal to deadline ($P_i = D_i$); tasks are independent (no resource sharing)
- Recall: schedulable utilization bound for Fixed-Priority scheduling (sufficient but not necessary condition):
  - A taskset is schedulable under RM scheduling if system utilization $U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq N(2^{1/N} - 1)$
  - $U \to 0.69$ as $N \to \infty$

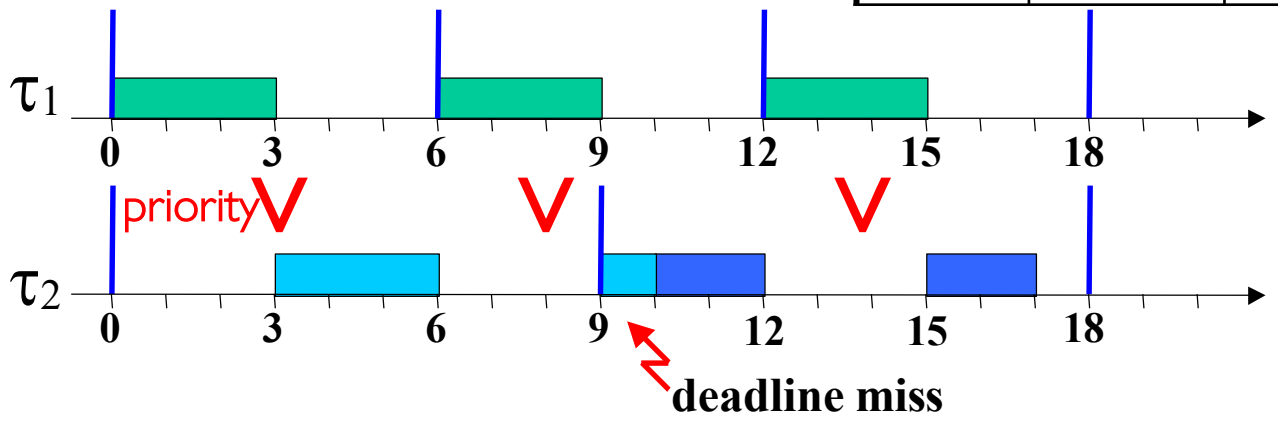


RM vs. EDF Example

Task	T=D	C
τ_1	6	3
τ_2	9	4

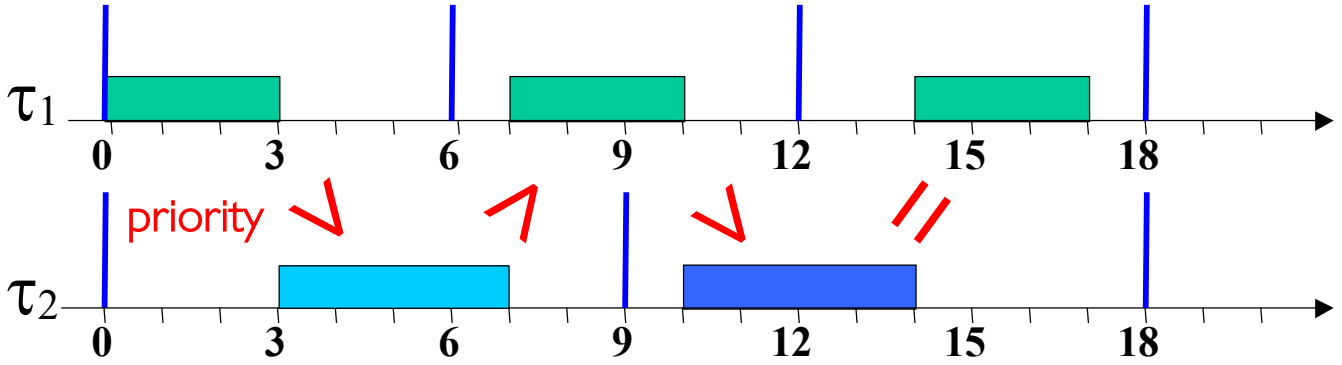
Under RM (Fixed-Priority scheduling), all jobs of τ_1 (with smaller period T=6) have higher priority than all jobs of τ_2 (with larger period T=9). Taskset unschedulable with RM

$$U = \frac{3}{6} + \frac{4}{9} = 0.944 > 0.828$$



Under EDF (Dynamic Priority scheduling), different jobs of τ_1 and τ_2 may have different priorities, depending on their absolute deadlines d_i, which is different for each newly-released job every period. Taskset schedulable with EDF

$$U = \frac{3}{6} + \frac{4}{9} = 0.944 < 1.0$$



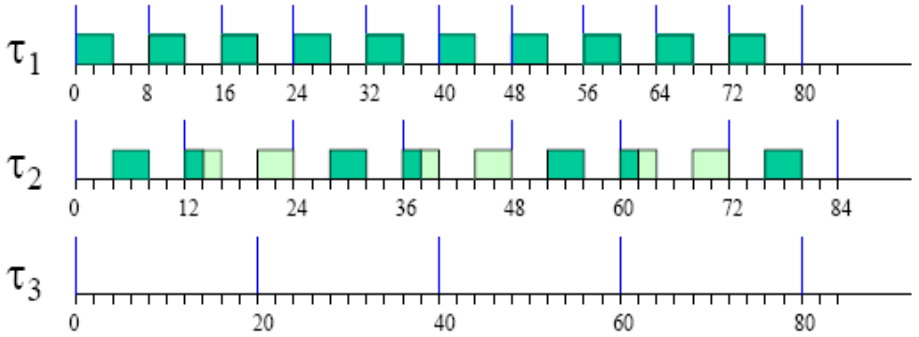
When two jobs have equal priority, the newly arrived job does not preempt the running job

RM vs. EDF: Robustness under Overload

- Under permanent overload, with CPU utilization U > 1
  - Under EDF, all tasks execute at a slower rate with "period rescaling", i. e., all tasks are delayed evenly
  - Under RM, higher priority tasks are protected while lower priority tasks are delayed or complete blocked
  - Recall Slide 25 Example Lateless
- Under transient overload, when some job overruns (executes longer than expected temporarily)
  - Under EDF, task overruns can cause deadline miss of arbitrary task
  - Under RM: task overruns only affect lower priority tasks
- Conclusion: RM offers better temporal isolation for higher priority tasks, at the expense of lower priority tasks

RM under permanent overload

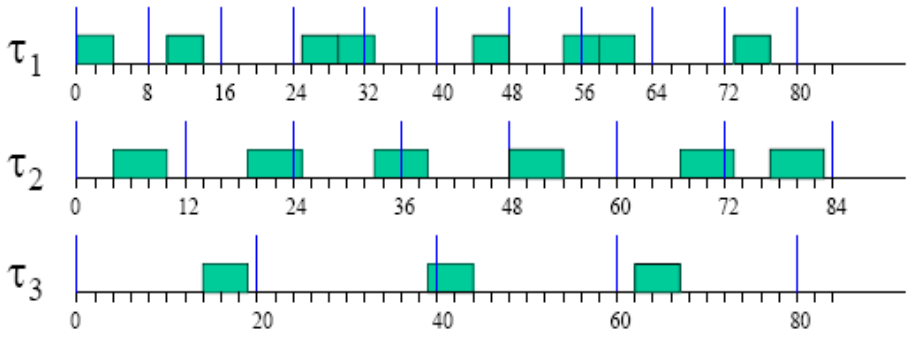
$$U = \frac{4}{8} + \frac{6}{12} + \frac{5}{20} = 1.25$$



- High priority tasks execute at the proper rate
- Low priority tasks are completely blocked

EDF under permanent overload

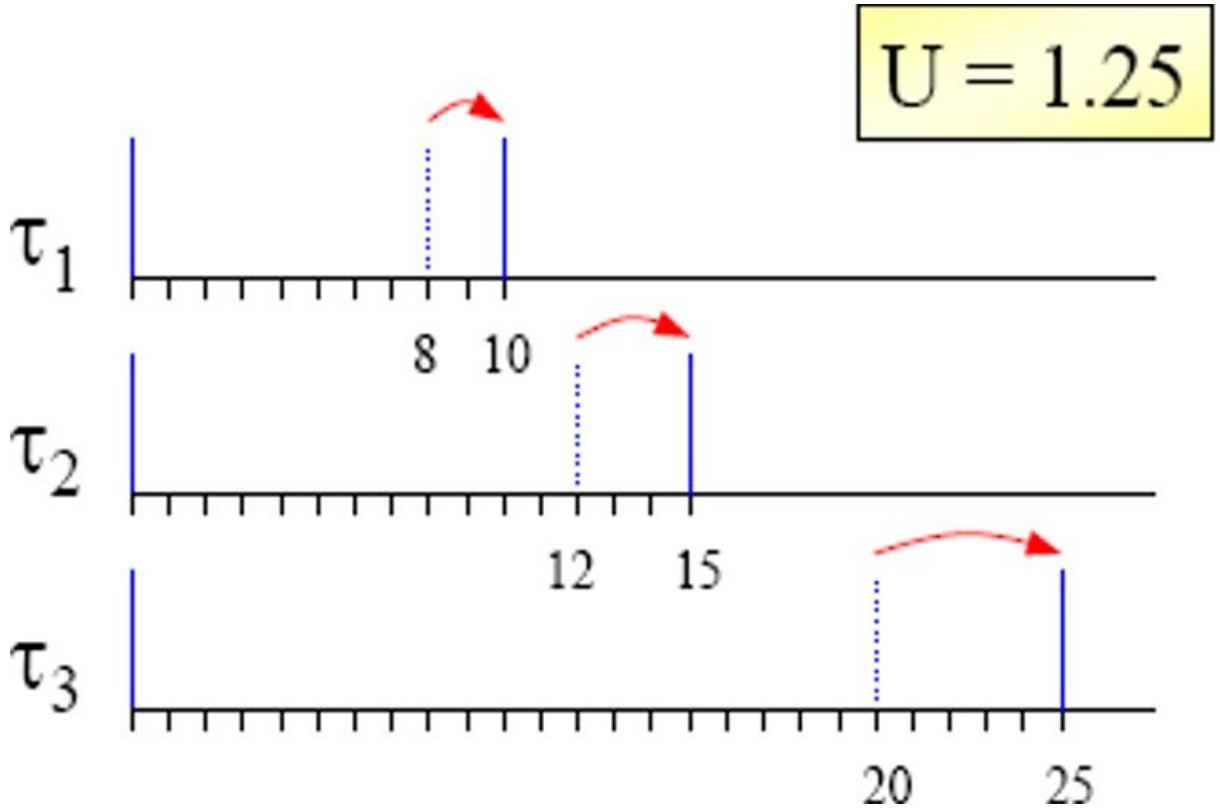
$$U = \frac{4}{8} + \frac{6}{12} + \frac{5}{20} = 1.25$$



- All tasks execute at a slower rate
- No task is blocked

EDF Period Rescaling

- Theorem on Period Rescaling [Cervin et al. 2003]:
  - If system utilization $U > 1$, tasks are executed with an average period $T_i' = T_i U$ under EDF scheduling

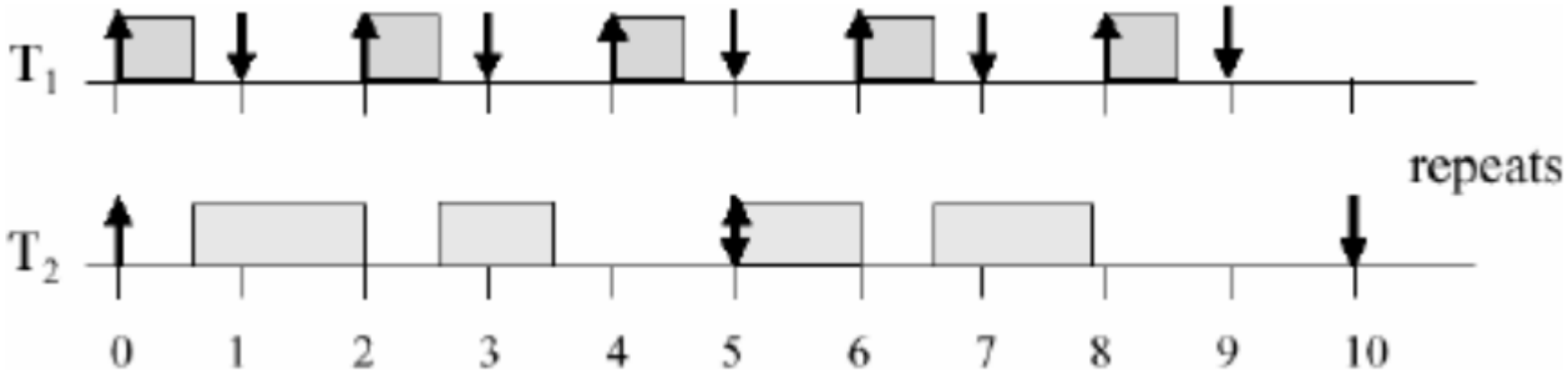


	T_i	T'_i
τ_1	8	10
τ_2	12	15
τ_3	20	25

EDF for Constrained Deadline Tasksets (D ≤ T)

IMPORTANT

- Earliest Deadline First (Dynamic-Priority):
  - A task with smaller absolute deadline gets higher priority $P_i \propto 1/d_i$
  - EDF is still optimal, but instead of Utilization Bound, we use Density Bound to determine schedulability
  - Density of task τ_i is defined as $\delta_i = \frac{C_i}{\min(D_i, T_i)}$. Taskset is schedulable if system density does not exceed 1: $\Delta = \sum_i \delta_i \leq 1$ (sufficient but not necessary condition)
    - » (Demand Bound Function can be used as necessary and sufficient condition (not covered))
  - Consider a taskset with two tasks both with $(C_i, T_i, D_i) = (1, 2, 1)$. It is obviously unschedulable under any scheduling algo. System utilization is $U = \frac{1}{2} + \frac{1}{2} = 1$; System density $\Delta = \frac{1}{1} + \frac{1}{1} = 2$. But we cannot determine schedulabiity based on $\Delta > 1$.
  - Consider a taskset with two tasks $\tau_1 = (0.6, 2, 1), \tau_2 = (2.3, 5, 5)$. $\Delta = \frac{0.6}{1} + \frac{2.3}{5} = 1.06$. Yet the taskset is schedulable under EDF:



Summary of Schedulability Analysis Algorithms

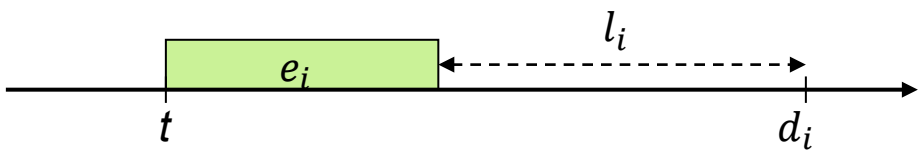
IMPORTANT

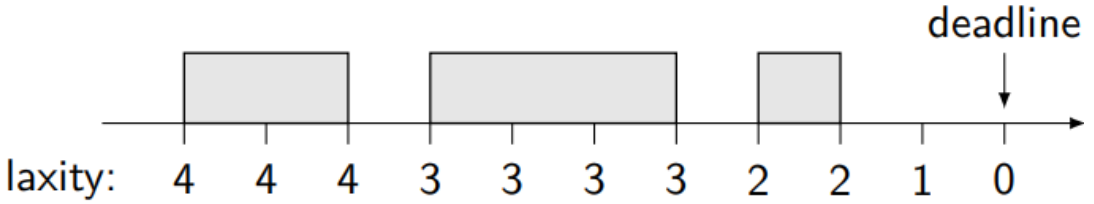
	Fixed-Priority Scheduling		Dynamic Priority Scheduling	
Optimal Scheduling Algorithm	**Rate Monotonic (RM)** Scheduling for implicit deadline taskset (D=T)	**Deadline Monotonic (DM)** Scheduling for constrained deadline taskset (D≤T)	**Earliest Deadline First (EDF)** Scheduling for implicit deadline taskset (D=T)	**Earliest Deadline First (EDF)** Scheduling for constrained deadline taskset (D≤T)
Schedulability Analysis Algorithm	Utilization Bound (UB) test $U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq N(2^{1/N} - 1)$ (sufficient condition) or Response Time Analysis (RTA) (necessary and sufficient) $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j \leq D_i$	RTA Response Time Analysis (RTA) (necessary and sufficient) $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j \leq D_i$	Utilization Bound (UB) test $U = \sum_{i=1}^{N} \frac{C_i}{T_i} \leq 1$ (necessary and sufficient)	Density Bound test $\Delta = \sum_i \frac{C_i}{\min(D_i, T_i)} \leq 1$ (sufficient condition) or Demand Bound Function (not covered)

Least Laxity First (LLF) Scheduling

Least Laxity First (LLF) Scheduling

- LLF assigns priority to jobs dynamically based on their current laxity (slack)
  - With absolute deadline d_i and remaining execution time e_i, laxity of τ_i's job at time t is $l_i = d_i - t - e_i$. Job with the smallest laxity has the highest priority
  - While an active job waits and does not run, its laxity decreases and its priority increases until it becomes the highest priority job and starts to run
  - If an active job runs in the previous time slot, then its laxity remains the same, as t is incremented by 1, and e_i is decremented by 1
  - If an active job does not run in the previous time slot, then its laxity is decremented by 1, as t is incremented by 1, and e_i remains the same
- Analogy: suppose you have an assignment that is due in 5 hours at 12:00, and it takes e_i=3 hours to complete. Current time is t=7:00, so the current laxity is $l_i = d_i - t - e_i$=12−7−3=2.
  - If you work for an hour until t=8:00, then the laxity remains the same: $l_i = d_i - t - e_i$=12−8−2=2, since the remaining execution time is decremented by 1: e_i=3-1=2
  - If you sleep for an hour until t=8:00, then the laxity is decremented by 1: $l_i = d_i - t - e_i$=12−8−3=1, since the remaining execution time does not change: e_i=3
  - If you sleep for 2 hours until t=9:00, then the laxity is now 0: $l_i = d_i - t - e_i$=12−9−3=0. Your must give the assignment the highest priority and start working on it right away, otherwise you will miss the deadline

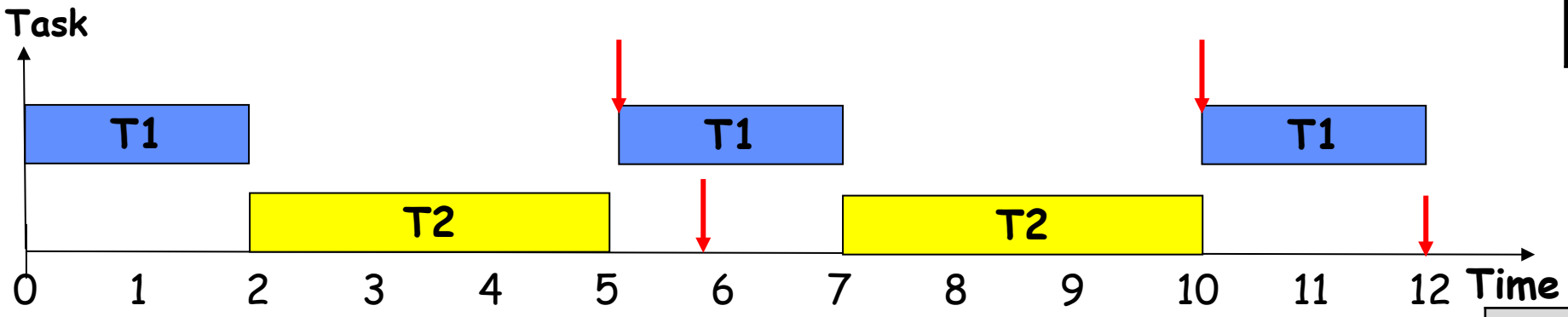




- EDF and LLF are both optimal scheduling algorithms, i.e., they both have schedulable utilization bound of 1
  - LLF incurs frequent context switches, hence is less practical than EDF

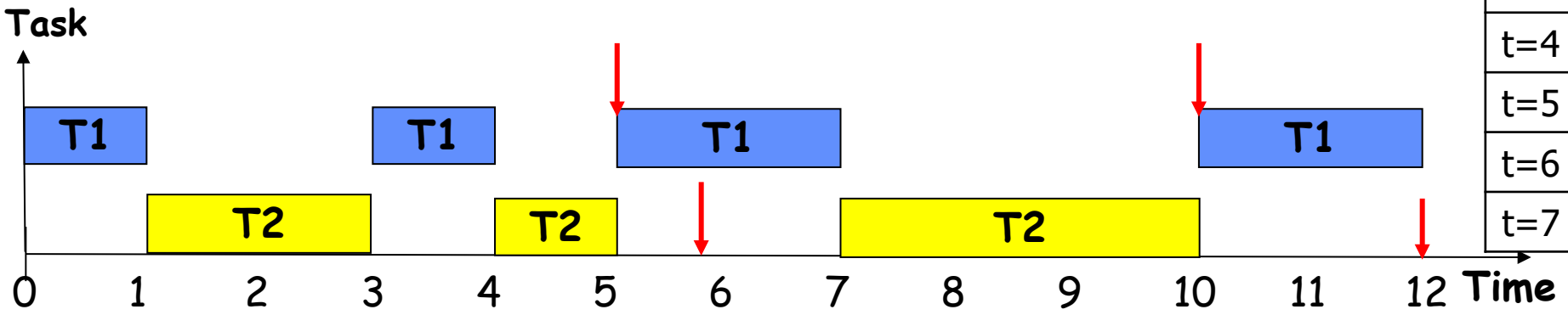
RM, EDF, LLF Example

Task	T=D	C
T1	5	2
T2	6	3



EDF and RM have the same schedule

Time	T_1 Laxity	T_2 Laxity	Running Task
t=0	5-0-2=3	6-0-3=3	τ_1 (tie)
t=1	5-1-1=3	6-1-3=2	τ_2
t=2	5-2-1=2	6-2-2=2	τ_2 (tie)
t=3	5-3-1=1	6-3-1=2	τ_1
t=4	τ_1 done	6-4-1=1	τ_2
t=5	10-5-2=3	τ_2 done	τ_1
t=6	10-6-1=3	12-6-3=3	τ_1 (tie)
t=7	τ_1 done	12-7-3=2	τ_2



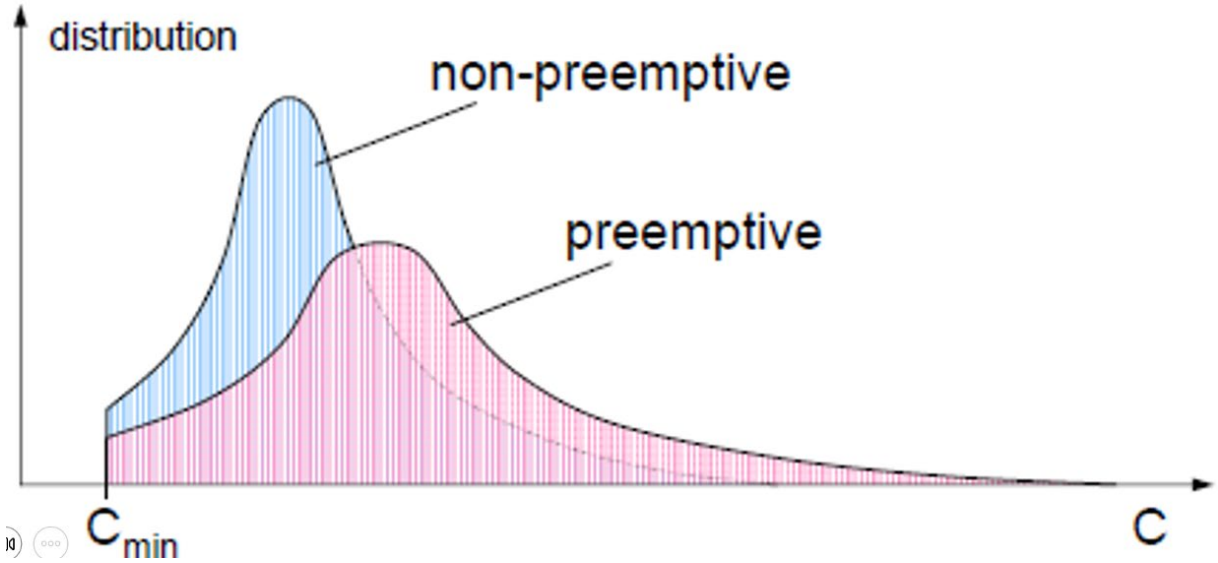
LLF has more frequent context switches

Preemptive vs. Non-Preemptive Scheduling

Preemptive vs. Non-Preemptive Scheduling

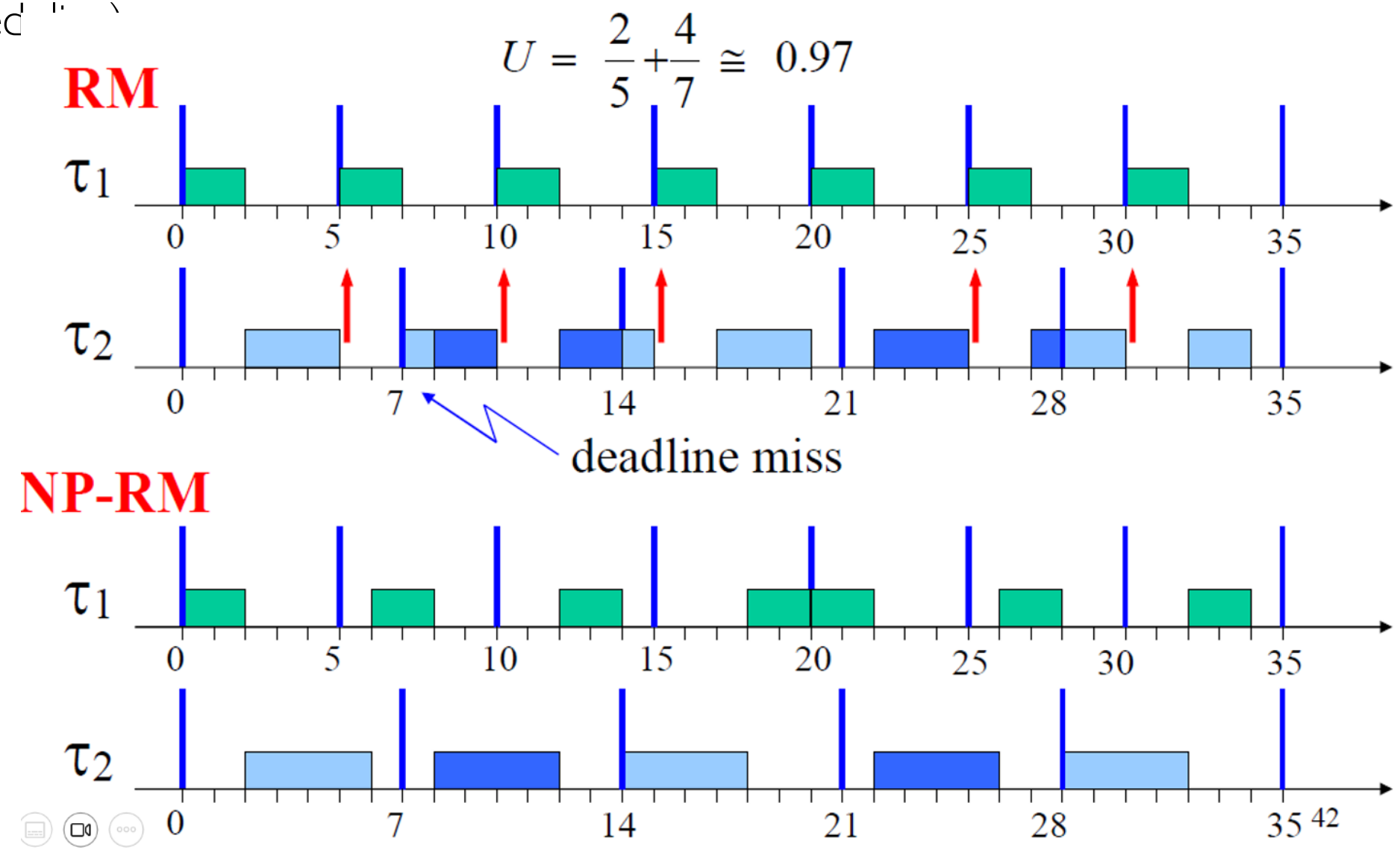
- Non-preemptive scheduling pros:
  - It reduces runtime overhead
    - Less context switches
    - No mutex locks needed for critical sections
  - It preserves program locality, improving the effectiveness of CPU cache
    - As a result, task WCET becomes smaller and execution time distribution becomes more predictable (shown on right)
  - Sometimes NP scheduling can improve schedulability
- Cons:
  - Reduced schedulability
  - Scheduling anomalies

- Preemptive scheduling pros:
  - Better schedulability (higher CPU utilization)
- Cons:
  - Runtime overhead due to frequent context-switches
  - Destroys program locality so task WCET becomes larger



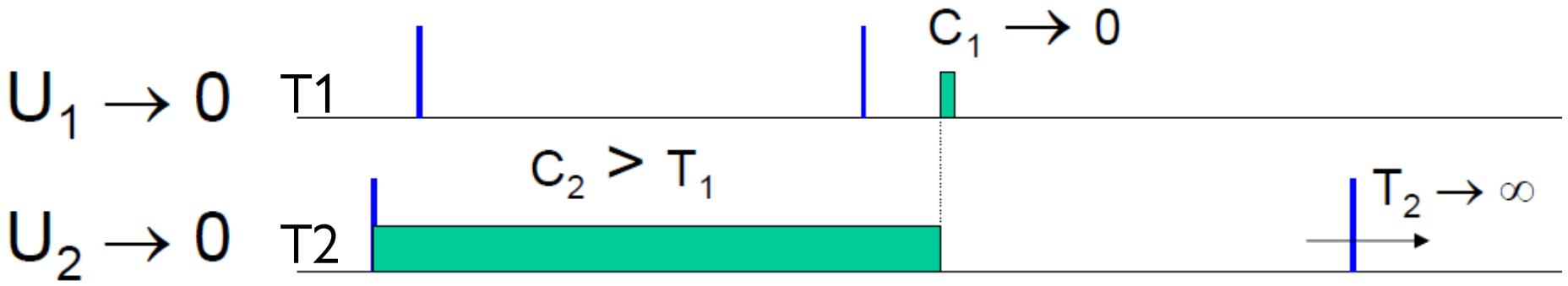
Sometimes NP Scheduling Improves Schedulability

- An example where NP scheduling improves schedulability (for fixed-priority sched



Disadvantage of NP Scheduling: Reduced Schedulability

- In general, NP scheduling reduces schedulability. The utilization bound under NP scheduling drops to zero due to blocking time
- An example with with two tasks T1 and T2, CPU utilization of nearly 0, yet unschedulable.
  - If C_2 (WCET of T2) $\geq T_1$ (period of T1), then the taskset is unschedulable with arbitrarily small system CPU utilization $\frac{C_1}{T_1} + \frac{C_2}{T_2} \rightarrow \frac{0}{T_1} + \frac{C_2}{\infty}$ (when C_1 goes to 0 and T_2 goes to infinity)
  - This example is valid whether τ_1 or τ_2 has higher priority: even if τ_1 has higher priority, it may be released very shortly after τ_2 is released at time 0, and it has to wait for τ_2 to finish due to NP scheduling



Disadvantage of NP Scheduling: Scheduling Anomalies

- Scheduling anomaly: three tasks under NP fixed-priority scheduling witjh priority ordering $\tau_1 > \tau_2 > \tau_3$ and NP
- Doubling the processor speed (reducing task execution times by half) makes task τ_1 miss its deadline, since τ_3 starts earlier before τ_1 is released, causing a long delay to it due to NP scheduling (this anomaly does not occur for preemptive scheduling)