CSC 112: Computer Operating Systems
Lecture 6

Real-Time Scheduling
Exercises ANS

Department of Computer Science,

Hofstra University

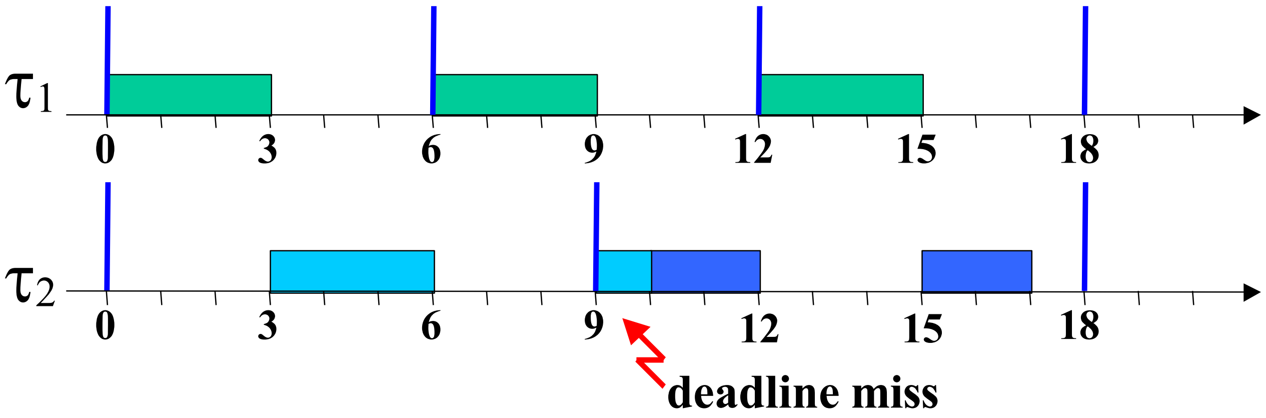
Q1. Schedulability under RM or EDF

- Determine schedulability of the following tasksets under (1) Rate Monotonic (RM) scheduling, using Utilization Bound test and/or Response Time Analysis (RTA) to determine taskset schedulability. (2) Earliest Deadline First (EDF) scheduling, using Utilization Bound test. We use the notation $\tau_i (C_i, T_i, D_i)$ to denote task τ_i with WCET C_i Period T_i, Deadline D_i (c.f. Slide 33 in Lecture 6)
- 1) Taskset $\tau_1 (3, 6, 6)$, $\tau_2 (4, 9, 9)$
- 2) Taskset $\tau_1 (3, 6, 6)$, $\tau_2 (3, 9, 9)$
- 3) Taskset $\tau_1 (3, 6, 6)$, $\tau_2 (2, 9, 9)$
- 4) Taskset $\tau_1 (2, 4, 4)$, $\tau_2 (4, 8, 8)$
- 5) Taskset $\tau_1 (2, 5, 5)$, $\tau_2 (4, 7, 7)$
- 6) Taskset $\tau_1 (1, 2, 2)$, $\tau_2 (2.5, 5, 5)$

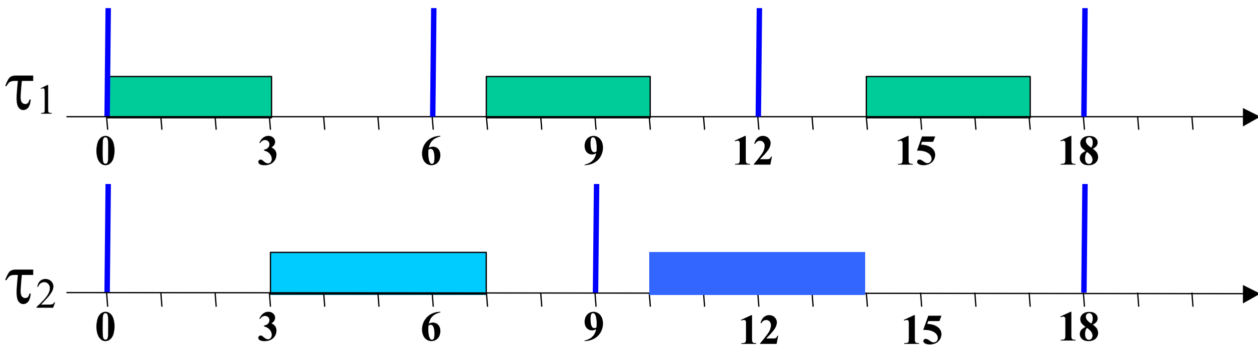
# Tasks	RM Util Bound
1	1.00
2	0.828
3	0.780

Q1. Schedulability under RM or EDF ANS

- 1) Taskset $\tau_1(3, 6, 6)$, $\tau_2(4, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.944 > 0.828$ (UB for 2 tasks under RM). Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 3 \leq D_1 = 6$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{6} \right\rceil \cdot 3$, solving it iteratively gives $R_2 = 10 > D_2 = 9$, hence τ_2 is not schedulable
- This taskset is unschedulable under RM.
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.944 \leq 1$ (UB under EDF), hence this taskset is schedulable under EDF
- (You are not required to draw the Gantt charts below, they are FYI only.)



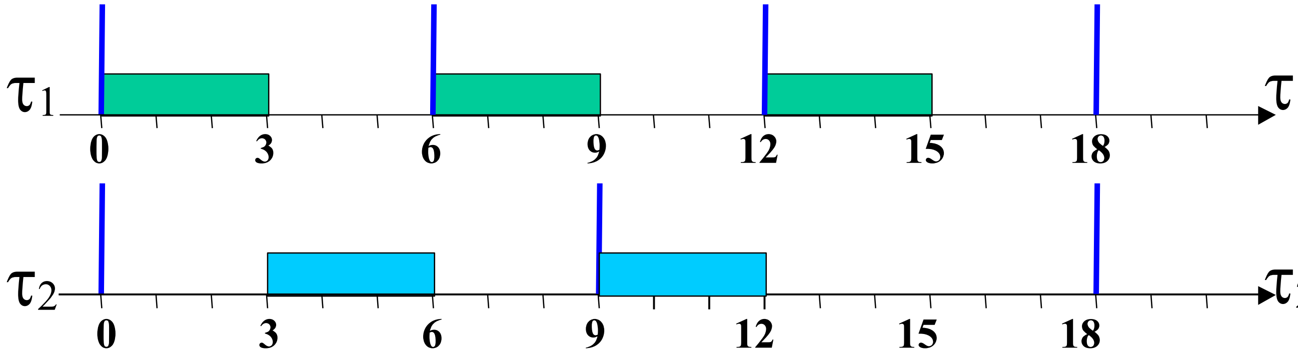
Gantt chart for RM



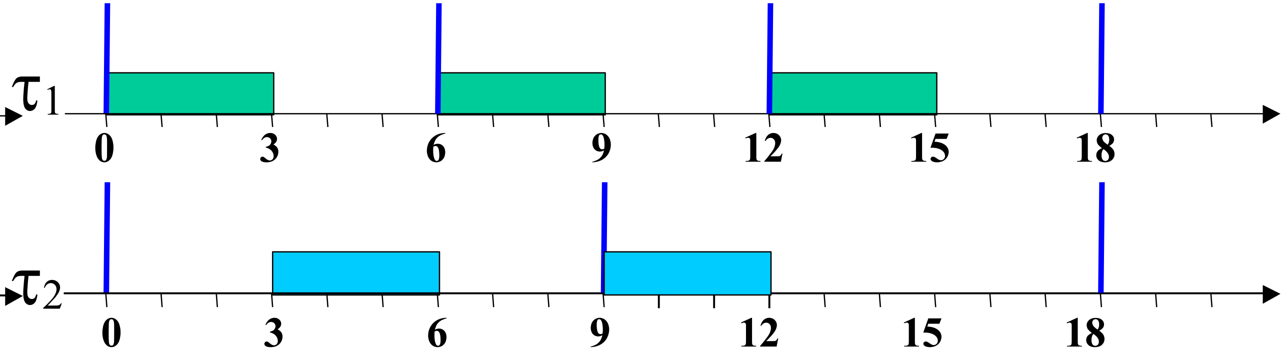
Gantt chart for EDF

Q1. Schedulability under RM or EDF ANS

- 2) Taskset $\tau_1\,(3, 6, 6)$, $\tau_2\,(3, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{3}{9} = 0.833 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 3 \le D_1 = 6$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 3 + \left\lceil \frac{R_2}{6} \right\rceil \cdot 3$, solving it iteratively gives $R_2 = 6 \le D_2 = 9$, hence τ_2 is schedulable
  - $R_2 = 9$ is another possible solution for the recursive equation, but we consider the minimum fixed-point solution of $R_2 = 6$
- We determine this taskset to be schedulable under RM.
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.833 \le 1$, hence this taskset is schedulable under EDF



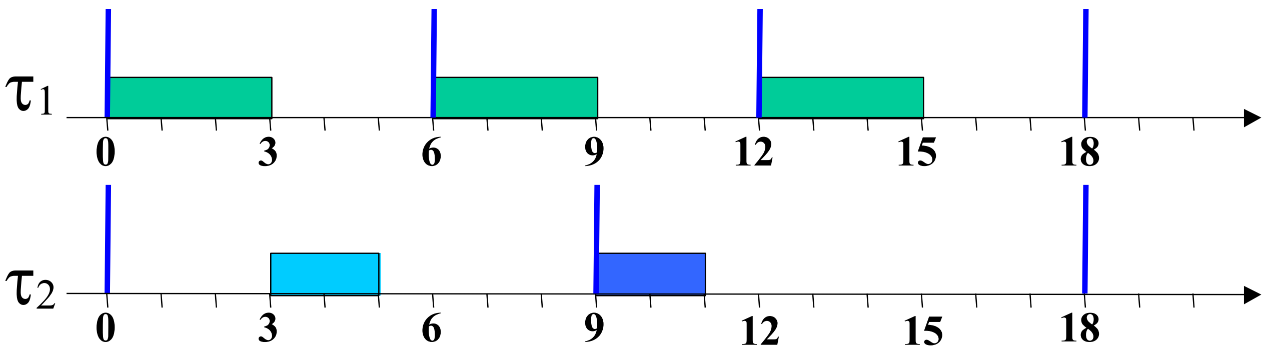
Gantt chart for RM



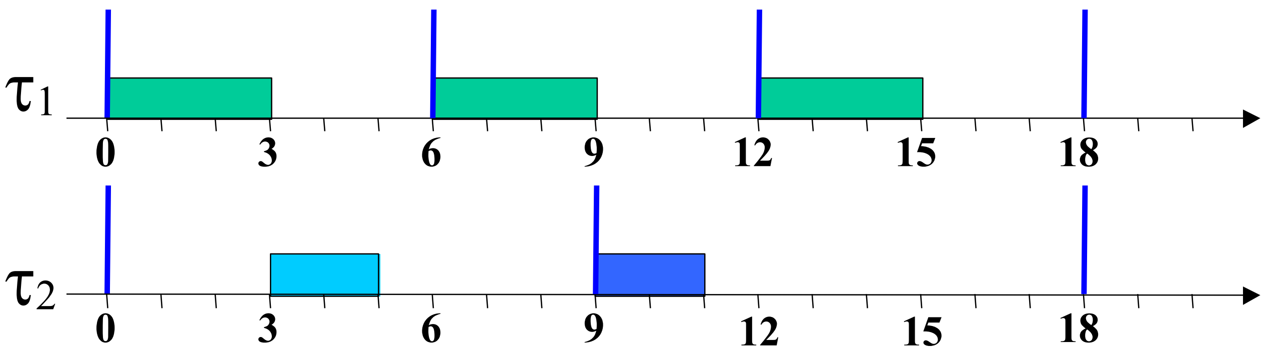
Gantt chart for EDF (same as RM)

Q1. Schedulability under RM or EDF ANS

- 3) Taskset $\tau_1 (3, 6, 6)$, $\tau_2 (2, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{2}{9} = 0.722 \leq 0.828$. Since utilization is within the RM bound, we determine this taskset to be schedulable under RM, without the need for RTA
- System utilization $U = \frac{3}{6} + \frac{2}{9} = 0.722 \leq 1$, hence this taskset is schedulable under EDF



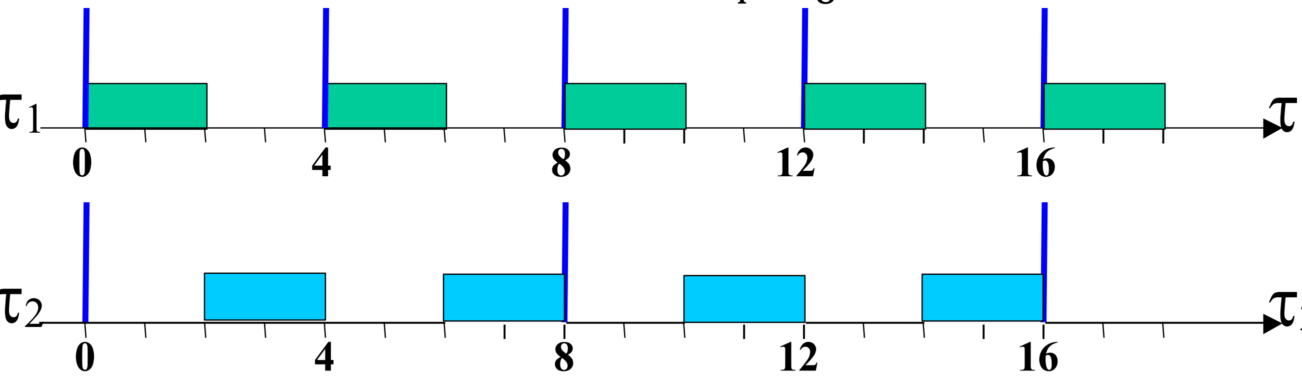
Gantt chart for RM



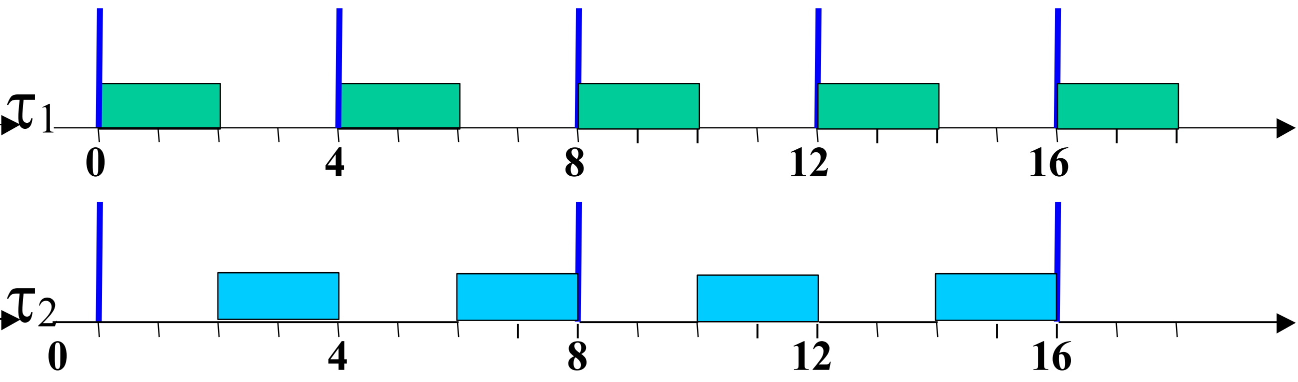
Gantt chart for EDF (same as RM)

Q1. Schedulability under RM or EDF ANS

- 4) Taskset $\tau_1(2, 4, 4)$, $\tau_2(4, 8, 8)$
- System utilization $U = \frac{2}{4} + \frac{4}{8} = 1.0 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 2 \leq D_1 = 4$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{4} \right\rceil \cdot 2,$ solving it iteratively gives $R_2 = 8 \leq D_2 = 8$, hence τ_2 is schedulable
- We determine this taskset to be schedulable under RM.
  - We can also skip RTA, and use this condition to this taskset to be schedulable under RM. "If periods are harmonic (larger periods divisible by smaller periods), then utilization bound is 1."
- System utilization $U = \frac{2}{4} + \frac{4}{8} = 1.0 \leq 1$, hence this taskset is schedulable under EDF



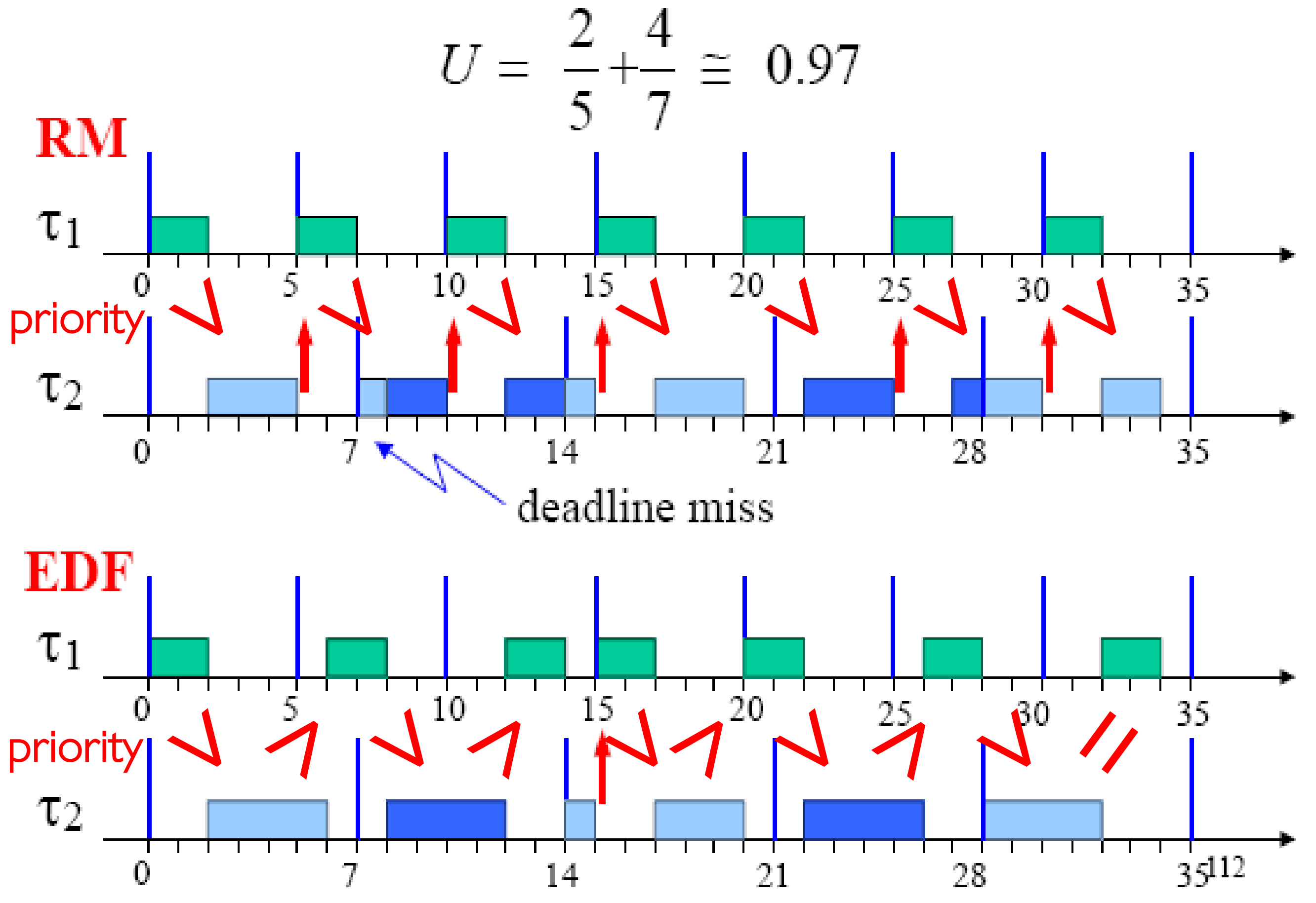
Gantt chart for RM



Gantt chart for EDF (same as RM)

Q1. Schedulability under RM or EDF ANS

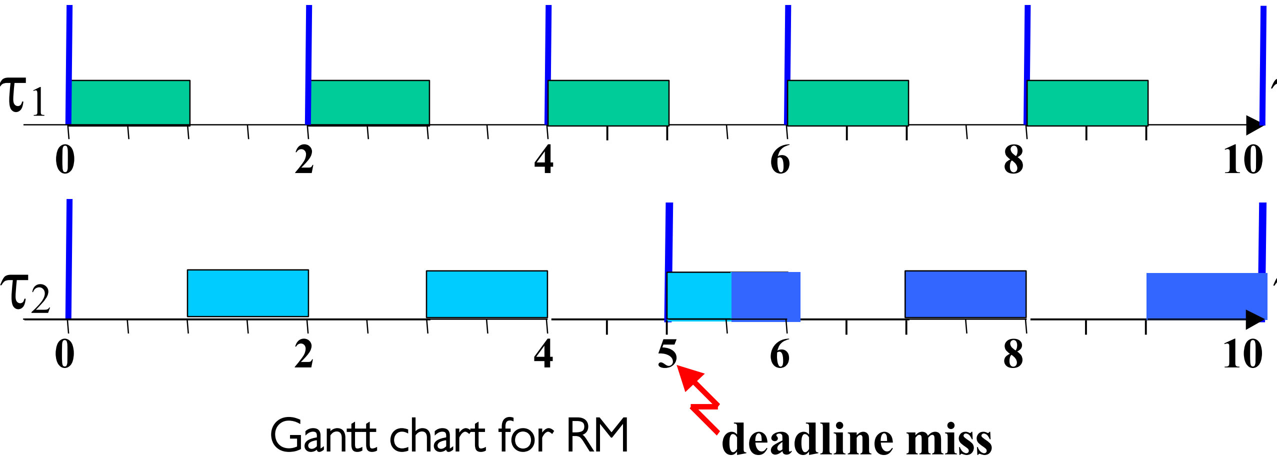
- 5) Taskset τ1 (2, 5, 5), τ2 (4, 7, 7)
- System utilization $U = \frac{2}{5} + \frac{4}{7} = 0.97 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ1, $R_1 = C_1 = 2 \le D_1 = 5$, hence τ1 is schedulable
- For lower-priority (larger period) task τ2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{5} \right\rceil \cdot 2$, solving it iteratively gives $R_2 = 8 > D_2 = 7$, hence τ2 is not schedulable
- This taskset is unschedulable under RM
- System utilization $U = \frac{2}{5} + \frac{4}{7} = 0.97 \le 1$, hence this taskset is schedulable under EDF



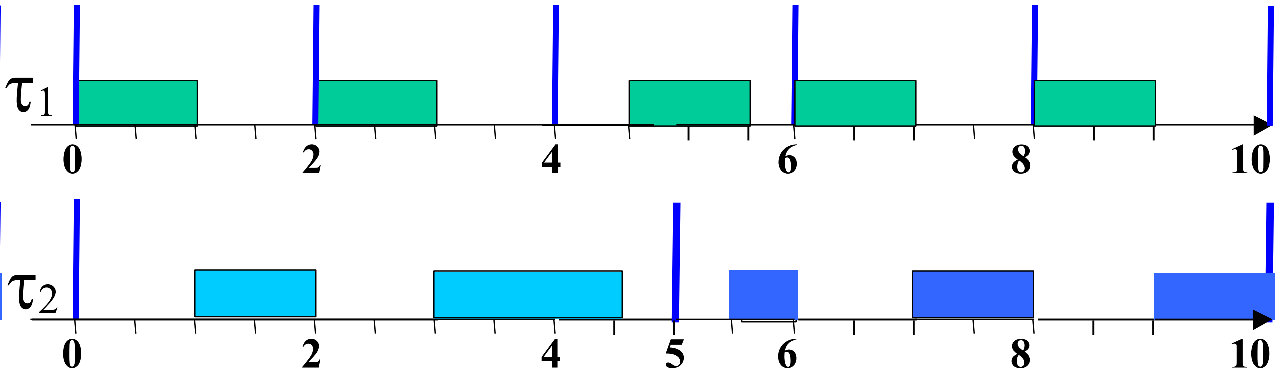
Red arrows indicate preemption

Q1. Schedulability under RM or EDF ANS

- 6) Taskset $\tau_1(1, 2, 2)$, $\tau_2(2.5, 5, 5)$
- System utilization $U = \frac{1}{2} + \frac{2.5}{5} = 1 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 1 \leq D_1 = 2$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 2.5 + \left\lceil \frac{R_2}{2} \right\rceil \cdot 1$, solving it iteratively gives $R_2 = 5.5 > D_2 = 5$, hence τ_2 is not schedulable
- This taskset is unschedulable under RM
- System utilization $U = \frac{1}{2} + \frac{2.5}{5} = 1 \leq 1$, hence this taskset is schedulable under EDF



Gantt chart for RM



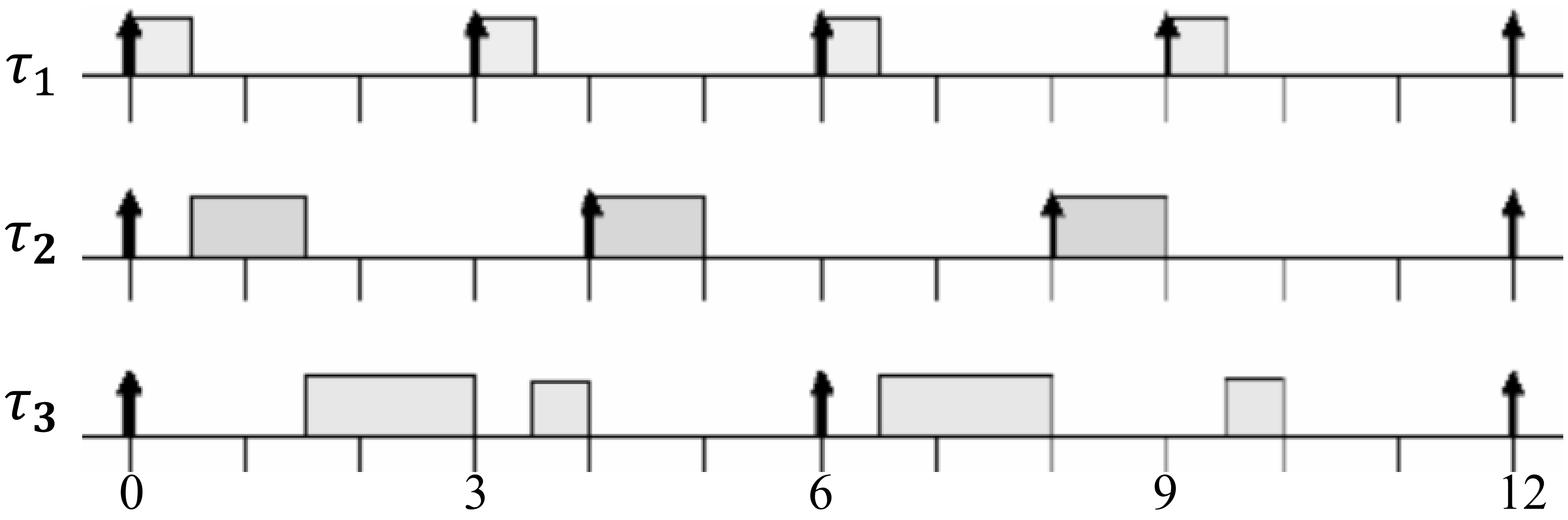
Gantt chart for EDF

Q2. Schedulability under RM, DM, or EDF

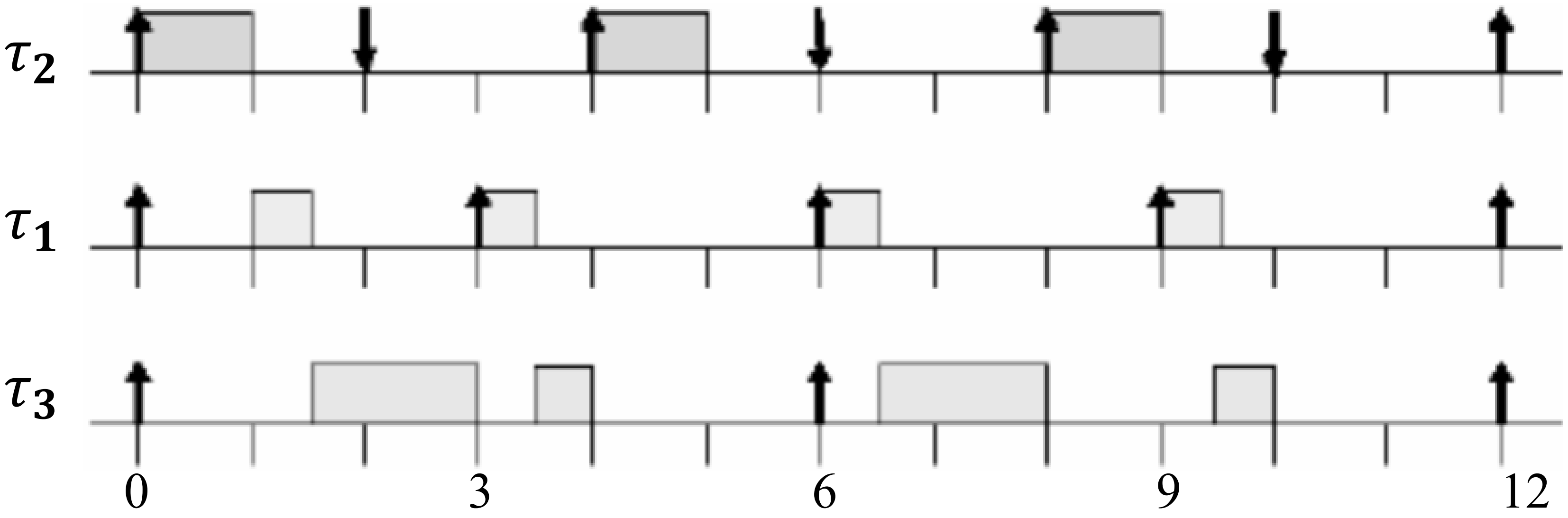
- Determine schedulability of the following tasksets under (1) Rate Monotonic (RM) scheduling, using Utilization Bound test and/or Response Time Analysis (RTA) to determine taskset schedulability. (2) Deadline Monotonic (DM) scheduling (3) Earliest Deadline First (EDF) scheduling, using Utilization Bound test.
- 1) Taskset $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
- 2) Taskset $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- 3) Taskset $\tau_1 = (1, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$

Recall: RM vs. DM Example

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
- Under RM (or DM), priority ordering $\tau_1 > \tau_2 > \tau_3$



- Three tasks with τ_2 assigned a smaller deadline of $D_2 = 2$: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- Under DM, priority ordering $\tau_2 > \tau_1 > \tau_3$



Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
  - For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
    - » System utilization $U = \frac{0.5}{3} + \frac{1}{4} + \frac{2}{6} = 0.75 \leq 0.780$ (UB for 2 tasks under RM), hence the taskset is schedulable under RM
  - For DM: priority ordering $\tau_1 > \tau_2 > \tau_3$
    - » Since $D_i = T_i$, DM scheduling is the same as RM scheduling, hence it is also schedulable under DM
  - For EDF: System utilization $U = \frac{0.5}{3} + \frac{1}{4} + \frac{2}{6} = 0.75 \leq 1$ (UB for EDF), hence the taskset is schedulable under EDF

Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- No Utilization Bound test for RM or DM, for taskset with $D_i < T_i$; need to use Response Time Analysis (RTA)
- For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
  - $R_1 = C_1 + 0 = 0.5 + 0 = 0.5 \leq D_1 = 3$
  - $R_2 = C_2 + \left\lceil \frac{R_1}{T_1} \right\rceil \cdot C_1 = 1 + \left\lceil \frac{R_1}{3} \right\rceil \cdot 0.5 = 1.5 \leq D_2 = 2$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 = 2 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 0.5 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 = 4 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For DM: priority ordering $\tau_2 > \tau_1 > \tau_3$
  - $R_2 = C_2 + 0 = 1 + 0 = 1 \leq D_2 = 2$
  - $R_1 = C_2 + \left\lceil \frac{R_1}{T_2} \right\rceil \cdot C_2 = 0.5 + \left\lceil \frac{R_1}{4} \right\rceil \cdot 1 = 1.5 \leq D_1 = 3$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 = 2 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 0.5 = 4 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For EDF:
  - System density $\Delta = \frac{0.5}{3} + \frac{1}{2} + \frac{2}{6} = 1.0 \leq 1$, hence this taskset is schedulable under EDF

Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (1,3,3), \tau_2 = (1,4,2), \tau_3 = (2,6,6)$
- No Utilization Bound test for RM or DM, for taskset with $D_i < T_i$; need to use Response Time Analysis (RTA)
- For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
  - $R_1 = C_1 + 0 = 1 + 0 = 1 \leq D_1 = 3$
  - $R_2 = C_2 + \left\lceil \frac{R_1}{T_1} \right\rceil \cdot C_1 = 1 + \left\lceil \frac{R_1}{3} \right\rceil \cdot 1 = 2 \leq D_2 = 2$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 = 2 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 = 6 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For DM: priority ordering $\tau_2 > \tau_1 > \tau_3$
  - $R_2 = C_2 + 0 = 1 + 0 = 1 \leq D_2 = 2$
  - $R_1 = C_2 + \left\lceil \frac{R_1}{T_2} \right\rceil \cdot C_2 = 1 + \left\lceil \frac{R_1}{4} \right\rceil \cdot 1 = 2 \leq D_1 = 3$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 = 2 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 1 = 6 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- Three tasks: $\tau_1 = (1,3,3), \tau_2 = (1,4,2), \tau_3 = (2,6,6)$ under EDF
  - System density $\Delta = \frac{1}{3} + \frac{1}{2} + \frac{2}{6} = 1.17 > 1$, hence we CANNOT determine this taskset's schedulability under EDF

Q3 RM, EDF, LLF

- Consider the set of 2 periodic tasks whose period, deadline and WCET parameters are given.
- 1. For each scheduling algorithm (RM, EDF, LLF), draw the Gantt chart by filling in the table with the task ID that runs in each time slot until time 10, and calculate the WCRT for each task.
- 2. Under RM scheduling, use utilization bound and Response Time Analysis (RTA) to determine taskset schedulability.

Task ID	T=D	C	RM Resp. Time	EDF Resp. Time	LLF Resp. Time
1	8	3			
2	10	4			

Time	τ1 Laxity	τ2 Laxity	Running Task
t=0			
t=1			
t=2			
t=3			
t=4			
t=5			
t=6			
t=7			
t=8			
T=9			

RM										
EDF										
LLF										
Time	0	1	2	3	4	5	6	7	8	9	10

Gantt Chart

Q3 RM, EDF, LLF ANS

- Consider the set of 2 periodic tasks whose period, deadline and WCET parameters are given.
- 1. For each scheduling algorithm (RM, EDF, LLF), draw the Gantt chart by filling in the table with the task ID that runs in each time slot until time 10, and calculate the WCRT for each task.
- 2. Under RM scheduling, determine taskset schedulability using utilization bound and/or Response Time Analysis (RTA) to

Task ID	T=D	C	RM Resp. Time	EDF Resp. Time	LLF Resp. Time
1	8	3	3	3	5
2	10	4	7	7	7

RM	1	1	1	2	2	2	2	X	1	1
EDF	1	1	1	2	2	2	2	X	1	1
LLF	1	1	2	2	1	2	2	X	1	1

Time 0 1 2 3 4 5 6 7 8 9 10

Gantt Chart

Time	T1 Laxity	T2 Laxity	Running Task
t=0	8-0-3=5	10-0-4=6	1
t=1	8-1-2=5	10-1-4=5	1 (tie)
t=2	8-2-1=5	10-2-4=4	2
t=3	8-3-1=4	10-3-3=4	2 (tie)
t=4	8-4-1=3	10-4-2=4	1
t=5	T1 done	10-5-2=3	2
t=6	T1 done	10-6-1=3	2
t=7	T1 done	T2 done	X
t=8	16-8-3=5	T2 done	1
T=9	16-9-2=5	T2 done	1

Q3 RM, EDF, LLF ANS

- System utilization $U = \frac{3}{8} + \frac{4}{10} = \mathbf{0.775} \leq \mathbf{0.828}$. Since utilization is within the RM bound, we determine this taskset to be schedulable under RM, without the need for RTA
- System utilization $U = \frac{3}{8} + \frac{4}{10} = \mathbf{0.775} \leq \mathbf{1.0}$, hence this taskset is schedulable under EDF and LLF

Recall: PCP Blocking Time

A given task i is blocked (or delayed) by at most one critical section of any lower priority task locking a semaphore with priority ceiling greater than or equal to the priority of task i. We can explain that mathematically using the notation:

$$B_i = \max_{\{k,s \,\mid\, k \in lp(i) \,\wedge\, s \in used\_by(k) \,\wedge\, ceil(s) \geq pri(i)\}} cs_{k,s} \qquad (4.5)$$

- Consider all lower-priority tasks ($k \in lp(i)$), and the semaphores they can lock (s)
- Select from those semaphores (s) with ceiling higher than or equal to $pri(i) = P_i$
- Take max length of all tasks (k)'s critical sections that lock semaphores (s)
- (Note: this formula applies only when task i requests some semaphore. If task i does not require any semaphores itself, then it does not experience any blocking time, i.e., $B_i = 0$.)

Q5. Schedulability with Shared Resources

- Consider the set of 8 periodic tasks scheduled with Deadline Monotonic (DM) scheduling, with period, deadline, priority (larger number denotes higher priority), and WCET parameters given in the table. The tasks may require one or more of the 5 semaphores.
- 1) Calculate priority ceilings of the semaphores
- 2) Determine taskset schedulability

- ANS: Since some tasks have deadline less than period D<T, we cannot use utilization bound test, and must use RTA, by calculating worst-case blocking time B_i, and WCRT R_i of all tasks based on RTA $R_i = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Q5. Schedulability with Shared Resources

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1		
B	500	200	50	7	s_3	4		
C	800	400	90	6	s_1	30		
D	800	800	20	5	s_1, s_2, s_4	9,3,3		
E	1000	1000	50	4	s_3	4		
F	2000	2000	10	3	s_5	7		
G	2000	2000	10	2	/	/		
H	2000	2000	30	1	s_2, s_5	13,7		

sem	Ceiling
s_1	
s_2	
s_3	
s_4	
s_5	

Q5. Schedulability with Shared Resources ANS

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	/	/	4	158
D	800	800	20	5	s_1,s_2,s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2,s_5	13,7	0	288

sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Schedulability with Shared Resources ANS

- Each semaphore is assigned a ceiling, equal to maximum priority of all tasks that require it: $C(s_k) = \max\{P_j : \tau_j \text{ uses } s_k\}$. Hence we can fill in the semaphore ceiling table.

sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task A

- Consider task A:
- The set of lower priority tasks lp(A) includes tasks B, C, D, E, F, G, H
- The set of semaphores used/required by these tasks includes s_1, s_2, s_3, s_4, and s_5
- Ceilings $C(s_4)=8 \geq prio(A)=8$
- Maximum blocking time of task A is B_A=cs(D, s4)=3
  - cs(D, s4)=3 means that Task D has a CS with length 3, associated with s_4
- $R_A = C_A + B_A = 14 + 3 = 17 \leq D_A = 50$
- Hence task A is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1,s_2,s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2,s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task B

- Consider task B:
- The set of lower priority tasks lp(B) includes tasks C, D, E, F, G, H
- The set of semaphores used/required by these tasks includes s_1, s_2, s_3, s_4, and s_5
- Ceilings $C(s_3)=7$, $C(s_4)=8 \geq prio(B)=7$
- Maximum blocking time of task B is $B_B=\max\{cs(D, s4), cs(E, s3)\}=\max(3, 4)=4$
- $R_B = C_B + B_B + \left\lceil \frac{R_B}{T_A} \right\rceil C_A = 50 + 4 + \left\lceil \frac{R_B}{250} \right\rceil 14 = 68 \leq D_B = 200$
- Hence task B is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task C

- Consider task C:
- The set of lower priority tasks lp(C) includes tasks D, E, F, G, H
- The set of semaphores used/required by these tasks includes s_1, s_2, s_3, s_4, and s_5
- Ceilings $C(s_3)=7$, $C(s_4)=8 \geq prio(C)=6$
- Maximum blocking time of task C is $B_C=\max\{cs(D, s4), cs(E, s3)\}=\max(3, 4)=4$
- $R_C = C_C + B_C + \left\lceil \frac{R_C}{T_A} \right\rceil C_A + \left\lceil \frac{R_C}{T_B} \right\rceil C_B = 90 + 4 + \left\lceil \frac{R_C}{250} \right\rceil 14 + \left\lceil \frac{R_C}{500} \right\rceil 50 = 158 \leq D_C = 400$
- Hence task C is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task D

- Consider task D:
- The set of lower priority tasks lp(D) includes tasks E, F, G, H
- The set of semaphores used/required by these tasks includes s_2, s_3 and s_5
- Ceilings of $C(s_2)=5$, $C(s_3)=7 \geq prio(D)=5$
- Maximum blocking time of task D is $B_D=\max\{cs(E, s3), cs(H, s2)\}=\max(4, 13)=13$
  - Task E has a CS with length 4, associated with s_3
  - Task H has a CS with length 13, associated with s_2
  - (Note that task B has higher priority than D, so it is not included even though it also requires s_3)
- $R_D = C_D + B_D + \left\lceil \frac{R_D}{T_A} \right\rceil C_A + \left\lceil \frac{R_D}{T_B} \right\rceil C_B + \left\lceil \frac{R_D}{T_C} \right\rceil C_C = 20 + 13 + \left\lceil \frac{R_D}{250} \right\rceil 14 + \left\lceil \frac{R_D}{500} \right\rceil 50 + \left\lceil \frac{R_D}{800} \right\rceil 90 = 187 \leq D_D = 800$
- Hence task D is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task E

- Consider task E:
- The set of lower priority tasks lp(E) includes tasks F, G, H
- The set of semaphores used/required by these tasks includes s_2 and s_5
- Ceiling $C(s_2)=5 \geq prio(E)=4$
- Maximum blocking time of task E is $B_E=cs(H, s2)=13$
- $R_E = C_E + B_E + \left\lceil \frac{R_E}{T_A} \right\rceil C_A + \left\lceil \frac{R_E}{T_B} \right\rceil C_B + \left\lceil \frac{R_E}{T_C} \right\rceil C_C + \left\lceil \frac{R_E}{T_D} \right\rceil C_D = 50 + 13 + \left\lceil \frac{R_E}{250} \right\rceil 14 + \left\lceil \frac{R_E}{500} \right\rceil 50 + \left\lceil \frac{R_E}{800} \right\rceil 90 + \left\lceil \frac{R_E}{800} \right\rceil 20 = 237 \leq D_E = 1000$
- Hence task E is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task F

- Consider task F:
- The set of lower priority tasks lp(F) includes tasks G, H
- The set of semaphores used/required by these tasks includes s_2 and s_5
- Ceilings $C(s_2)=5$, $C(s_5)=3 \geq prio(F)=3$
- Maximum blocking time of task E is $B_E=\max(cs(H, s2), cs(H, s5))=\max(13, 7)=13$
- $R_F = C_F + B_F + \left\lceil \frac{R_F}{T_A} \right\rceil C_A + \left\lceil \frac{R_F}{T_B} \right\rceil C_B + \left\lceil \frac{R_F}{T_C} \right\rceil C_C + \left\lceil \frac{R_F}{T_D} \right\rceil C_D + \left\lceil \frac{R_F}{T_E} \right\rceil C_E = 10 + 13 + \left\lceil \frac{R_E}{250} \right\rceil 14 + \left\lceil \frac{R_E}{500} \right\rceil 50 + \left\lceil \frac{R_E}{800} \right\rceil 90 + \left\lceil \frac{R_E}{800} \right\rceil 20 + \left\lceil \frac{R_E}{1000} \right\rceil 50 = 247 \leq D_F = 2000$
- Hence task F is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task G

- Consider task G:
- The set of lower priority tasks lp(G) includes task H
- The set of semaphores used/required by these tasks includes s_2 and s_5
- Ceilings $C(s_2)=5$, $C(s_5)=3 \geq prio(G)=2$
- Maximum blocking time of task E is $B_E=\max(cs(H, s2), cs(H, s5))=\max(13, 7)=13$
- $R_G = C_G + \left\lceil \frac{R_G}{T_A} \right\rceil C_A + \left\lceil \frac{R_G}{T_B} \right\rceil C_B + \left\lceil \frac{R_G}{T_C} \right\rceil C_C + \left\lceil \frac{R_G}{T_D} \right\rceil C_D + \left\lceil \frac{R_G}{T_E} \right\rceil C_E + \left\lceil \frac{R_G}{T_F} \right\rceil C_F = 10 + 13 + \left\lceil \frac{R_G}{250} \right\rceil 14 + \left\lceil \frac{R_G}{500} \right\rceil 50 + \left\lceil \frac{R_G}{800} \right\rceil 90 + \left\lceil \frac{R_G}{800} \right\rceil 20 + \left\lceil \frac{R_G}{1000} \right\rceil 50 + \left\lceil \frac{R_G}{2000} \right\rceil 10 = 271 \leq D_G = 2000$
- Hence task G is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1,s_2,s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2,s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q5. Task H

- Consider task H:
- Task H is the lowest priority task, so it does not experience any blocking B_H=0
- $R_H = C_H + \left\lceil \frac{R_H}{T_A} \right\rceil C_A + \left\lceil \frac{R_H}{T_B} \right\rceil C_B + \left\lceil \frac{R_H}{T_C} \right\rceil C_C + \left\lceil \frac{R_H}{T_D} \right\rceil C_D + \left\lceil \frac{R_H}{T_E} \right\rceil C_E + \left\lceil \frac{R_H}{T_F} \right\rceil C_F + \left\lceil \frac{R_H}{T_G} \right\rceil C_G = 30 + \left\lceil \frac{R_H}{250} \right\rceil 14 + \left\lceil \frac{R_H}{500} \right\rceil 50 + \left\lceil \frac{R_H}{800} \right\rceil 90 + \left\lceil \frac{R_H}{800} \right\rceil 20 + \left\lceil \frac{R_H}{1000} \right\rceil 50 + \left\lceil \frac{R_H}{2000} \right\rceil 10 + \left\lceil \frac{R_H}{2000} \right\rceil 10 = 288 \leq D_H = 2000$
- Hence task H is schedulable

Task	T	D	C	Prio	sems	CS Len	B	R
A	250	50	14	8	s_4	1	3	17
B	500	200	50	7	s_3	4	4	68
C	800	400	90	6	s_1	30	4	158
D	800	800	20	5	s_1, s_2, s_4	9,3,3	13	187
E	1000	1000	50	4	s_3	4	13	237
F	2000	2000	10	3	s_5	7	13	247
G	2000	2000	10	2	/	/	13	271
H	2000	2000	30	1	s_2, s_5	13,7	0	288

Sem	Ceiling
s_1	5
s_2	5
s_3	7
s_4	8
s_5	3

Q4. Schedulability with Shared Resources

- Consider the set of 3 periodic tasks scheduled with Rate Monotonic (RM) scheduling, with period, deadline, priority, and WCET parameters given in the table. Tasks 1 and 3 both require semaphore s_1.
- 1) Calculate priority ceilings of the semaphore s_1;
- 2) Determine taskset schedulability under PCP
- ANS: Since some tasks have deadline less than period D<T, we cannot use utilization bound test, and must use RTA, by calculating worst-case blocking time B_i, and WCRT R_i of all tasks based on RTA $R_i = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1	3		
2	200	100	50	M	/	/		
3	300	300	100	L	s_1	30		

sem	Ceiling
s_1	

Q4. Schedulability with Shared Resources ANS

- Ceiling $C(s_1) = \max(P_1, P_3) = H$
- Blocking times:
  - Task 1: $B_1 = 30$ (CS length of LP Task 3, associated with semaphore s_1, since $P_1 \leq C(s_1) = H$). WCRT $R_1 = C_1 + B_1 = 25 + 30 = 55 > D_1 = 50$. Hence Task 1 is unschedulable
  - Task 2: $B_2 = 30$ (CS length of LP Task 3, associated with semaphore s_1, since $P_2 \leq C(s_1) = H$). WCRT $R_2 = C_2 + B_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 50 + 30 + \left\lceil \frac{R_2}{100} \right\rceil 25 = 130 > D_2 = 100$. Hence Task 2 is unschedulable
  - Task 3: $B_3 = 0$ (Task 3 is the lowest priority task, so it does not experience any blocking delay). WCRT $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil C_2 = 100 + \left\lceil \frac{R_3}{100} \right\rceil 25 + \left\lceil \frac{R_3}{200} \right\rceil 50 = 200 \leq D_3 = 300$. Hence Task 3 is schedulable
- The taskset is unschedulable with shared resources under PCP

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1	3	30	55
2	200	100	50	M	/	/	30	130
3	300	300	100	L	s_1	30	0	200

sem	Ceiling
s_1	H

Q5. Schedulability with Shared Resources

- Consider the set of 3 periodic tasks scheduled with Rate Monotonic (RM) scheduling, with period, deadline, priority, and WCET parameters given in the table. Tasks 1 and 3 both require semaphore s_1.
- 1) Calculate priority ceilings of the semaphore s_1;
- 2) Determine taskset schedulability under PCP
- ANS: Since some tasks have deadline less than period D<T, we cannot use utilization bound test, and must use RTA, by calculating worst-case blocking time B_i, and WCRT R_i of all tasks based on RTA $R_i = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1	3		
2	200	100	50	M	s_2	10		
3	300	300	100	L	s_1, s_2	30, 40		

sem	Ceiling
s_1	
s_2	

Q5. Schedulability with Shared Resources ANS

- Ceiling $C(s_1) = \max(P_1, P_3) = H, C(s_2) = \max(P_2, P_3) = M$
- Blocking times:
  - Task 1: $B_1 = 30$ (CS length of LP Task 3, associated with semaphore s_1, since $P_1 \leq C(s_1) = H$). WCRT $R_1 = C_1 + B_1 = 25 + 30 = 55 > D_1 = 50$. Hence Task 1 is unschedulable
  - Task 2: $B_2 = \max(30, 40) = 40$ (max CS length of LP Task 3, associated with semaphores s_1, s_2) since $P_2 \leq C(s_1) = H, P_2 \leq C(s_2) = M$. WCRT $R_2 = C_2 + B_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 50 + 40 + \left\lceil \frac{R_2}{100} \right\rceil 25 = 140 > D_2 = 100$. Hence Task 2 is unschedulable
  - Task 3: $B_3 = 0$ (Task 3 is the lowest priority task, so it does not experience any blocking delay). WCRT $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil C_2 = 100 + \left\lceil \frac{R_3}{100} \right\rceil 25 + \left\lceil \frac{R_3}{200} \right\rceil 50 = 200 \leq D_3 = 300$. Hence Task 3 is schedulable
- The taskset is unschedulable with shared resources under PCP

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1	3	30	55
2	200	100	50	M	s_2	10	40	140
3	300	300	100	L	s_1, s_2	30, 40	0	200

sem	Ceiling
s_1	H
s_2	M

Q6. Schedulability with Shared Resources

- Consider the set of 3 periodic tasks scheduled with Rate Monotonic (RM) scheduling, with period, deadline, priority, and WCET parameters given in the table. Tasks 1 and 3 both require semaphore s_1.
- 1) Calculate priority ceilings of the semaphore s_1;
- 2) Determine taskset schedulability under PCP
- ANS: Since some tasks have deadline less than period D<T, we cannot use utilization bound test, and must use RTA, by calculating worst-case blocking time B_i, and WCRT R_i of all tasks based on RTA $R_i = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1, s_2	3, 4		
2	200	100	50	M	s_2	10		
3	300	300	100	L	s_1, s_2	30, 40		

sem	Ceiling
s_1	
s_2	

Q6. Schedulability with Shared Resources ANS

- Ceiling $C(s_1) = \max(P_1, P_3) = H, C(s_2) = \max(P_1, P_2, P_3) = H$
- Blocking times:
  - Task 1: $B_1 = \max(10, 30, 40) = 40$ (max CS length of LP Tasks 2 and 3, associated with semaphores s_1, s_2, since $P_1 \leq C(s_1) = H, P_1 \leq C(s_2) = H$). WCRT $R_1 = C_1 + B_1 = 25 + 40 = 65 > D_1 = 50$. Hence Task 1 is unschedulable
  - Task 2: $B_2 = \max(30, 40) = 40$ (max CS length of LP Task 3, associated with semaphores s_1 s_2) since $P_2 \leq C(s_1) = H, P_2 \leq C(s_2) = H$. WCRT $R_2 = C_2 + B_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 50 + 40 + \left\lceil \frac{R_2}{100} \right\rceil 25 = 140 > D_2 = 100$. Hence Task 2 is unschedulable
  - Task 3: $B_3 = 0$ (Task 3 is the lowest priority task, so it does not experience any blocking delay). WCRT $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil C_2 = 100 + \left\lceil \frac{R_3}{100} \right\rceil 25 + \left\lceil \frac{R_3}{200} \right\rceil 50 = 200 \leq D_3 = 300$. Hence Task 3 is schedulable
- The taskset is unschedulable with shared resources under PCP

Task	T	D	C	Prio	sems	CS Len	B	R
1	100	50	25	H	s_1, s_2	3, 4	40	65
2	200	100	50	M	s_2	10	40	140
3	300	300	100	L	s_1, s_2	30, 40	0	200

sem	Ceiling
s_1	H
s_2	H