CSC 112: Computer Operating Systems
Lecture 6

Real-Time Scheduling
Exercises ANS

Department of Computer Science,

Hofstra University

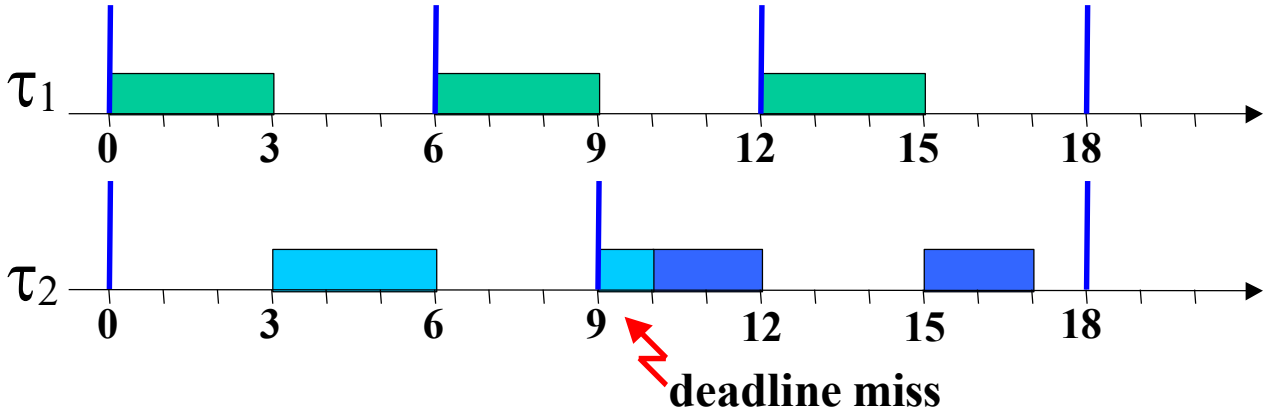
Q1. Schedulability under RM or EDF

- Determine schedulability of the following tasksets under (1) Rate Monotonic (RM) scheduling, using Utilization Bound test and/or Response Time Analysis (RTA) to determine taskset schedulability. (2) Earliest Deadline First (EDF) scheduling, using Utilization Bound test. We use the notation $\tau_i\,(C_i, T_i, D_i)$ to denote task τ_i with WCET C_i Period T_i, Deadline D_i (c.f. Slide 33 in Lecture 6)
- 1) Taskset $\tau_1\,(3, 6, 6)$, $\tau_2\,(4, 9, 9)$
- 2) Taskset $\tau_1\,(3, 6, 6)$, $\tau_2\,(3, 9, 9)$
- 3) Taskset $\tau_1\,(3, 6, 6)$, $\tau_2\,(2, 9, 9)$
- 4) Taskset $\tau_1\,(2, 4, 4)$, $\tau_2\,(4, 8, 8)$
- 5) Taskset $\tau_1\,(2, 5, 5)$, $\tau_2\,(4, 7, 7)$
- 6) Taskset $\tau_1\,(1, 2, 2)$, $\tau_2\,(2.5, 5, 5)$

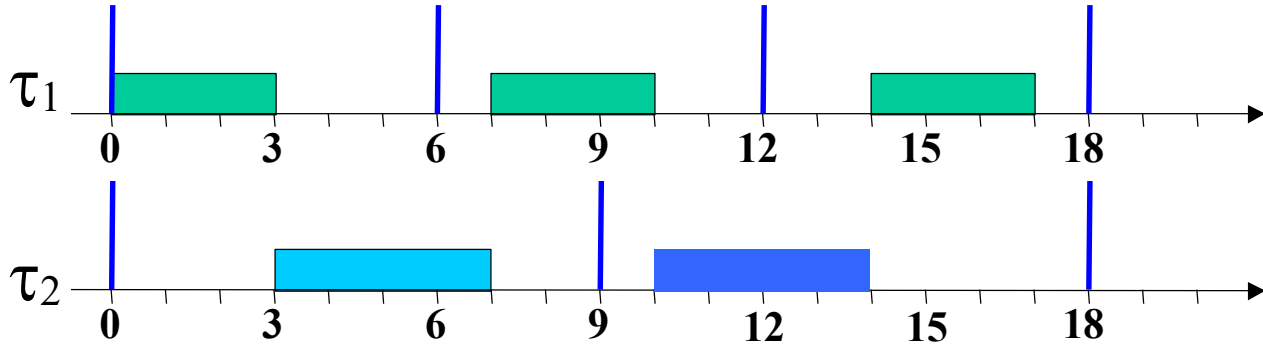
# Tasks	RM Util Bound
1	1.00
2	0.828
3	0.780

Q1. Schedulability under RM or EDF ANS

- 1) Taskset $\tau_1(3, 6, 6)$, $\tau_2(4, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.944 > 0.828$ (UB for 2 tasks under RM). Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA analysis to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 3 \leq D_1 = 6$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{6} \right\rceil \cdot 3$, solving it iteratively gives $R_2 = 10 > D_2 = 9$, hence τ_2 is not schedulable
- This taskset is unschedulable under RM.
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.944 \leq 1$ (UB under EDF), hence this taskset is schedulable under EDF
- (You are not required to draw the Gantt charts below, they are FYI only.)



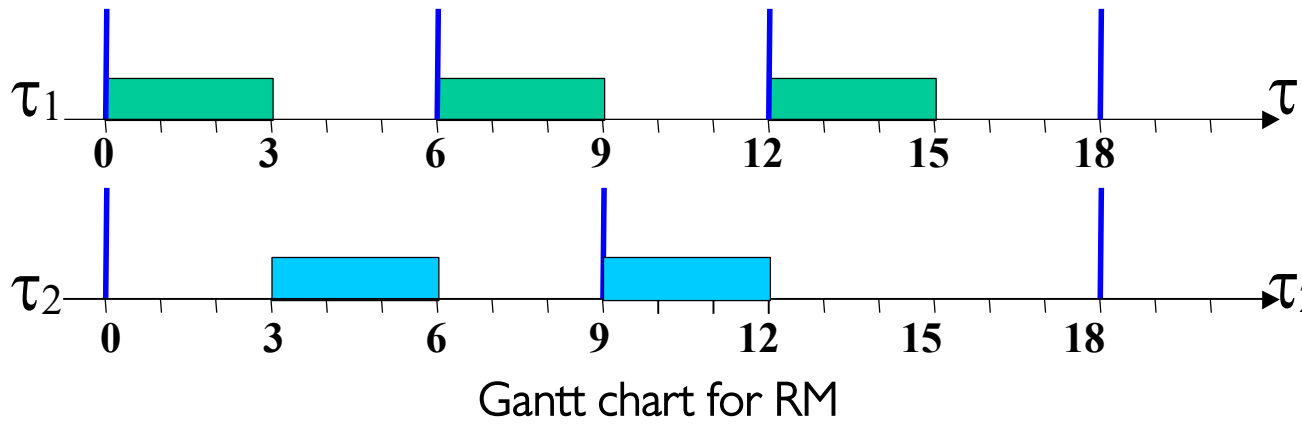
Gantt chart for RM



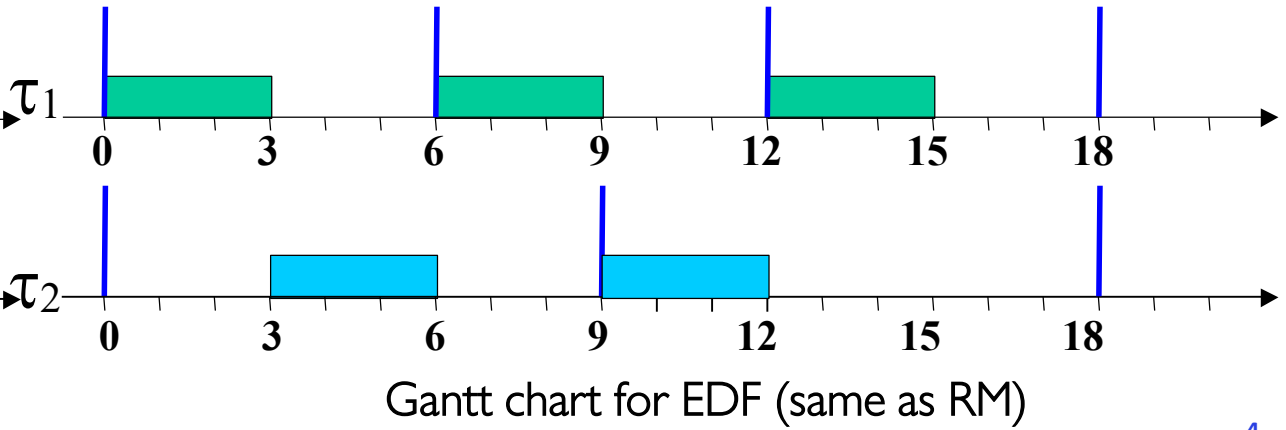
Gantt chart for EDF

Q1. Schedulability under RM or EDF ANS

- 2) Taskset $\tau_1(3, 6, 6), \tau_2(3, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{3}{9} = 0.833 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA analysis to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task $\tau_1, R_1 = C_1 = 3 \leq D_1 = 6$, hence τ_1 is schedulable
- For lower-priority (larger period) task $\tau_2, R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 3 + \left\lceil \frac{R_2}{6} \right\rceil \cdot 3$, solving it iteratively gives $R_2 = 6 \leq D_2 = 9$, hence τ_2 is schedulable
  - $R_2 = 9$ is another possible solution for the recursive equation, but we consider the minimum fixed-point solution of $R_2 = 6$
- We determine this taskset to be schedulable under RM.
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.833 \leq 1$, hence this taskset is schedulable under EDF



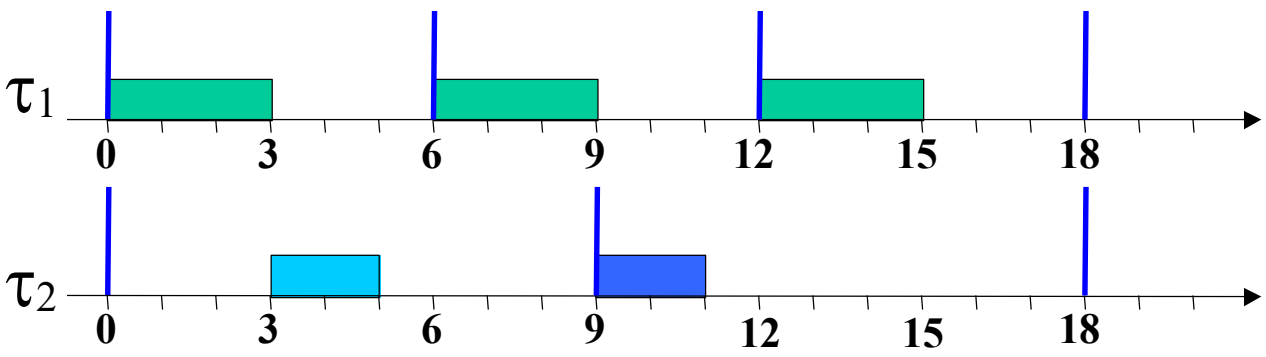
Gantt chart for RM



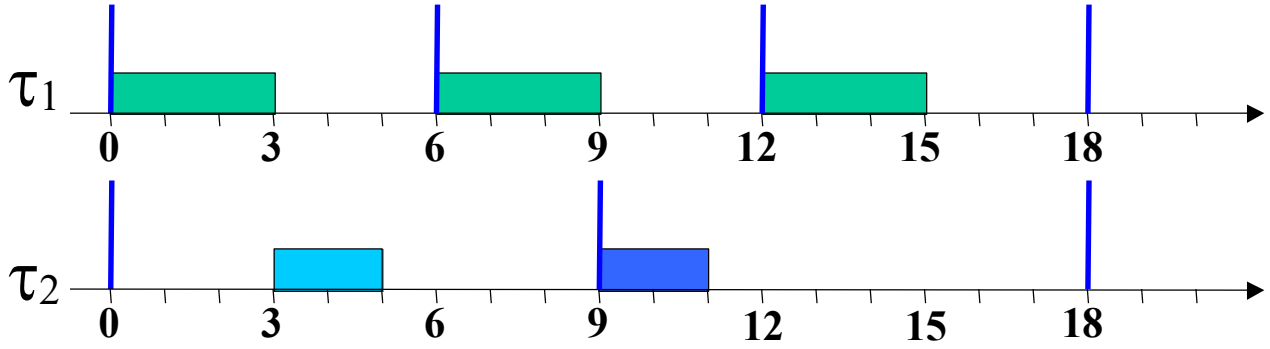
Gantt chart for EDF (same as RM)

Q1. Schedulability under RM or EDF ANS

- 3) Taskset $\tau_1(3, 6, 6), \tau_2(2, 9, 9)$
- System utilization $U = \frac{3}{6} + \frac{2}{9} = 0.722 \leq 0.828$. Since utilization is within the RM bound, we determine this taskset to be schedulable under RM, without the need for RTA analysis
- System utilization $U = \frac{3}{6} + \frac{4}{9} = 0.833 \leq 1$, hence this taskset is schedulable under EDF



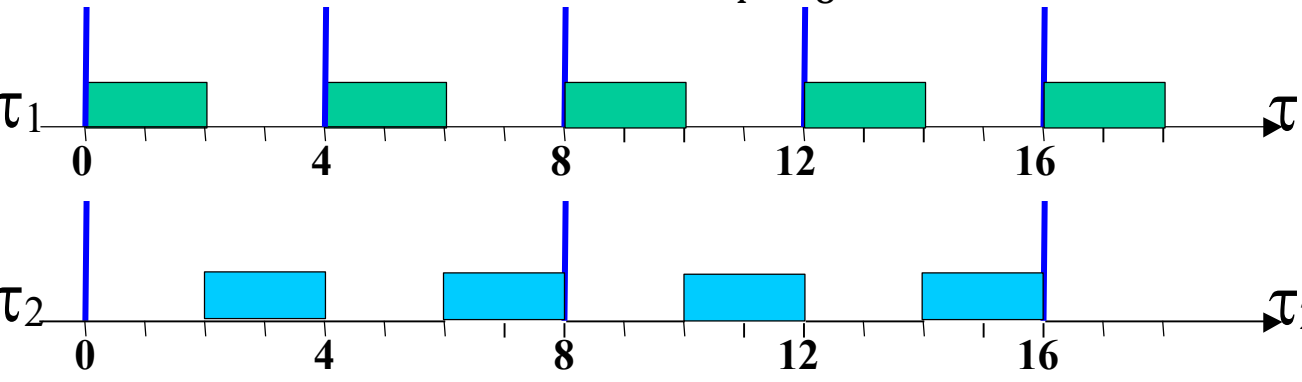
Gantt chart for RM



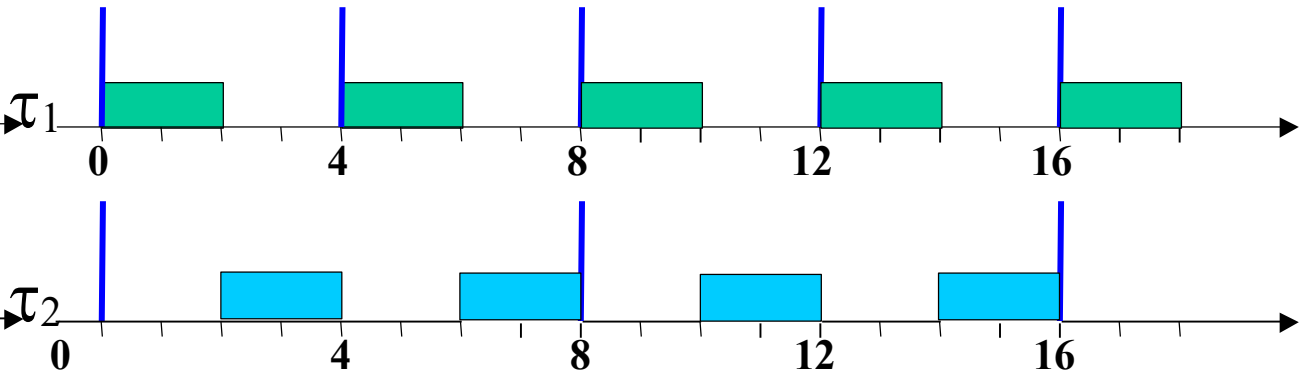
Gantt chart for EDF (same as RM)

Q1. Schedulability under RM or EDF ANS

- 4) Taskset $\tau_1\,(2, 4, 4)$, $\tau_2\,(4, 8, 8)$
- System utilization $U = \frac{2}{4} + \frac{4}{8} = 1.0 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA analysis to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task τ_1, $R_1 = C_1 = 2 \leq D_1 = 4$, hence τ_1 is schedulable
- For lower-priority (larger period) task τ_2, $R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{4} \right\rceil \cdot 2$, solving it iteratively gives $R_2 = 8 \leq D_2 = 8$, hence τ_2 is schedulable
- We determine this taskset to be schedulable under RM.
  - We can also skip RTA, and use this condition to this taskset to be schedulable under RM. "If periods are harmonic (larger periods divisible by smaller periods), then utilization bound is 1."
- System utilization $U = \frac{2}{4} + \frac{4}{8} = 1.0 \leq 1$, hence this taskset is schedulable under EDF



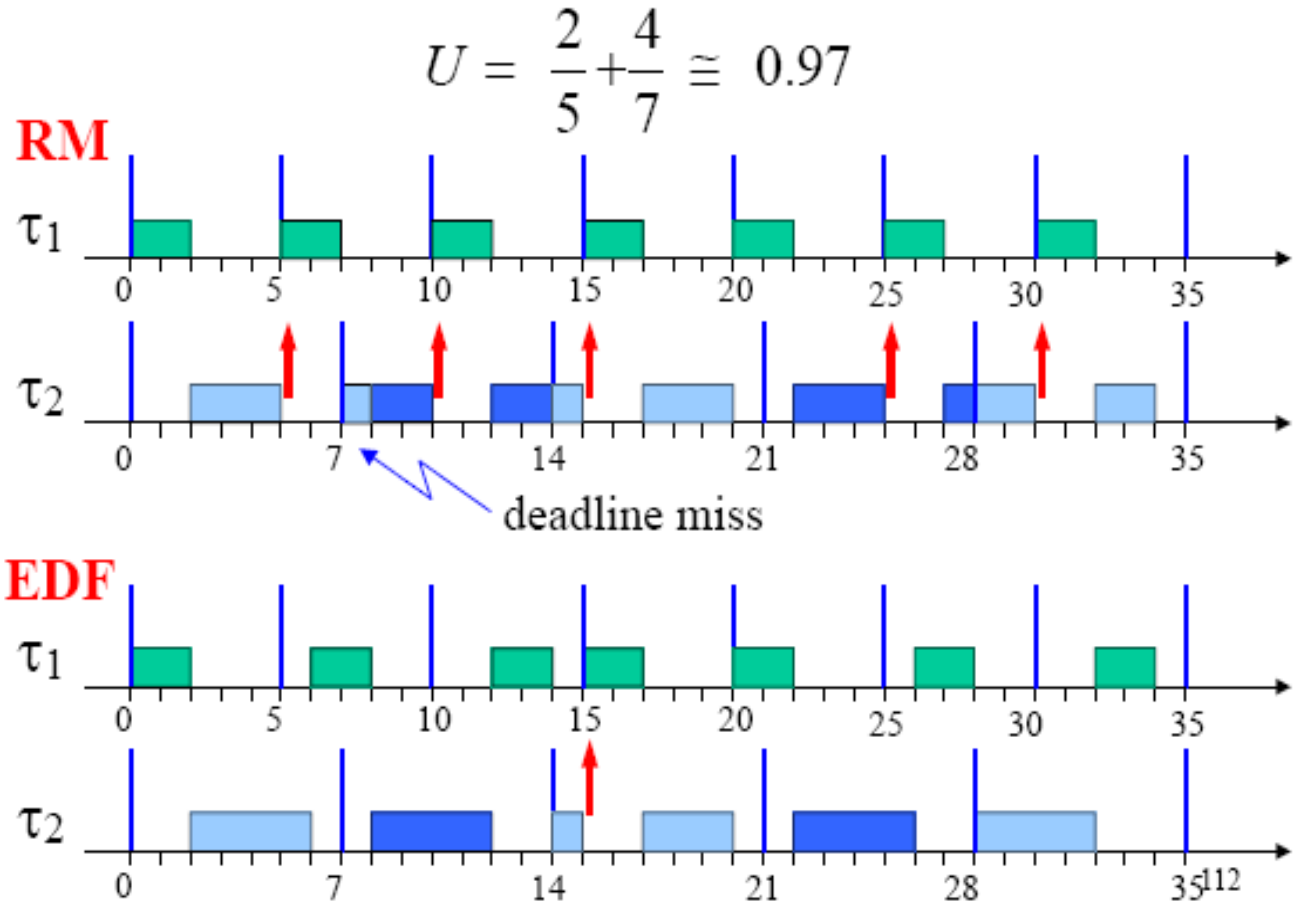
Gantt chart for RM



Gantt chart for EDF (same as RM)

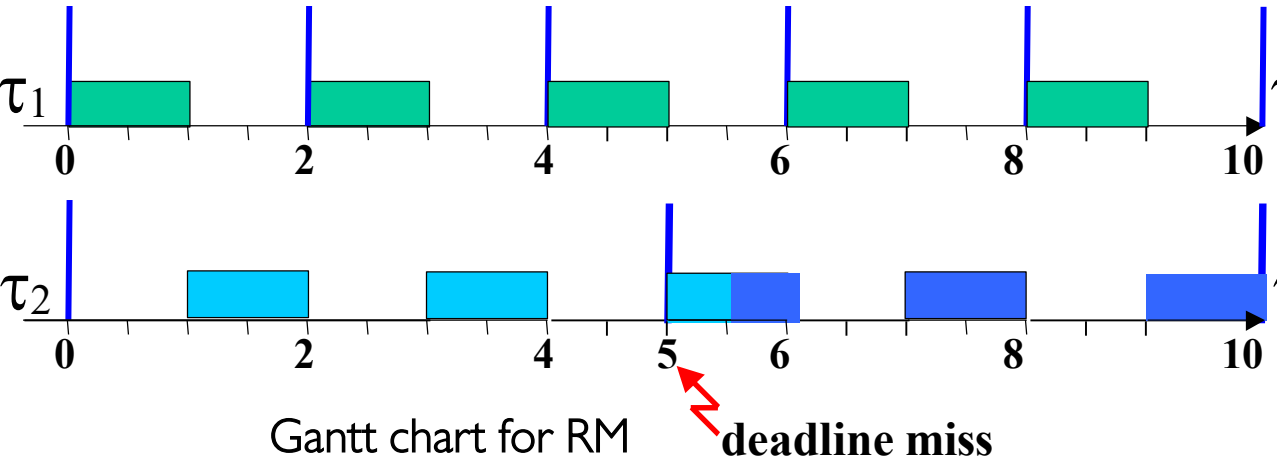
Q1. Schedulability under RM or EDF ANS

- 5) Taskset $\tau_1(2, 5, 5), \tau_2(4, 7, 7)$
- System utilization $U = \frac{2}{5} + \frac{4}{7} = 0.97 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA analysis to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task $\tau_1, R_1 = C_1 = 2 \leq D_1 = 5$, hence τ_1 is schedulable
- For lower-priority (larger period) task $\tau_2, R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{R_2}{5} \right\rceil \cdot 2$, solving it iteratively gives $R_2 = 8 > D_2 = 7$, hence τ_2 is not schedulable
- This taskset is unschedulable under RM
- System utilization $U = \frac{2}{5} + \frac{4}{7} = 0.97 \leq 1$, hence this taskset is schedulable under EDF

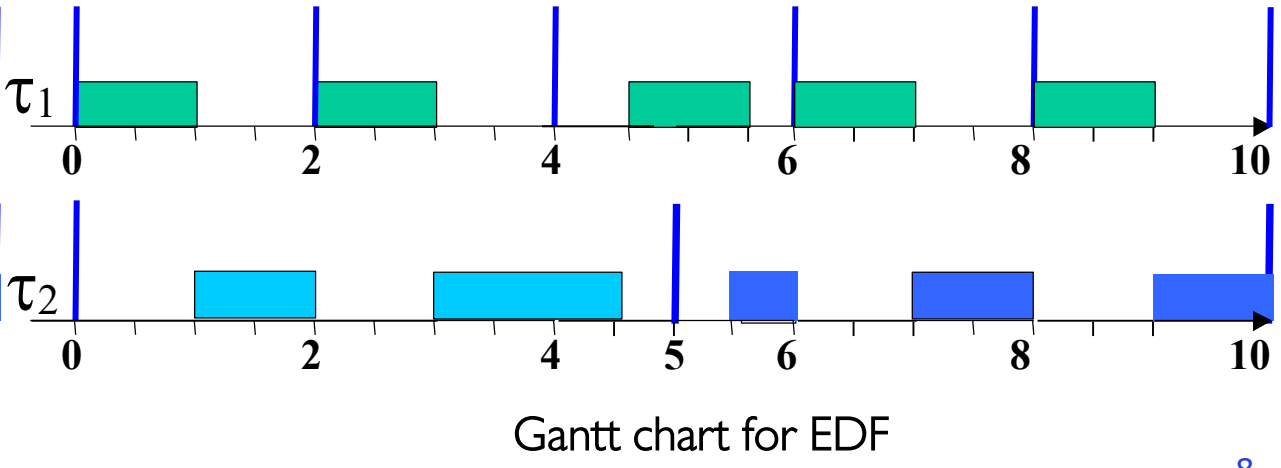


Q1. Schedulability under RM or EDF ANS

- 6) Taskset $\tau_1(1, 2, 2), \tau_2(2.5, 5, 5)$
- System utilization $U = \frac{1}{2} + \frac{2.5}{5} = 1 > 0.828$. Since utilization exceeds the RM bound, we cannot determine its schedulability under RM, so we perform RTA analysis to compute WCRT of each task, by solving $R_i = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$
- For higher-priority (smaller period) task $\tau_1, R_1 = C_1 = 1 \leq D_1 = 2$, hence τ_1 is schedulable
- For lower-priority (larger period) task $\tau_2, R_2 = C_2 + \left\lceil \frac{R_2}{T_1} \right\rceil C_1 = 2.5 + \left\lceil \frac{R_2}{2} \right\rceil \cdot 1$, solving it iteratively gives $R_2 = 5.5 > D_2 = 5$, hence τ_2 is not schedulable
- This taskset is unschedulable under RM
- System utilization $U = \frac{1}{2} + \frac{2.5}{5} = 1 \leq 1$, hence this taskset is schedulable under EDF



Gantt chart for RM



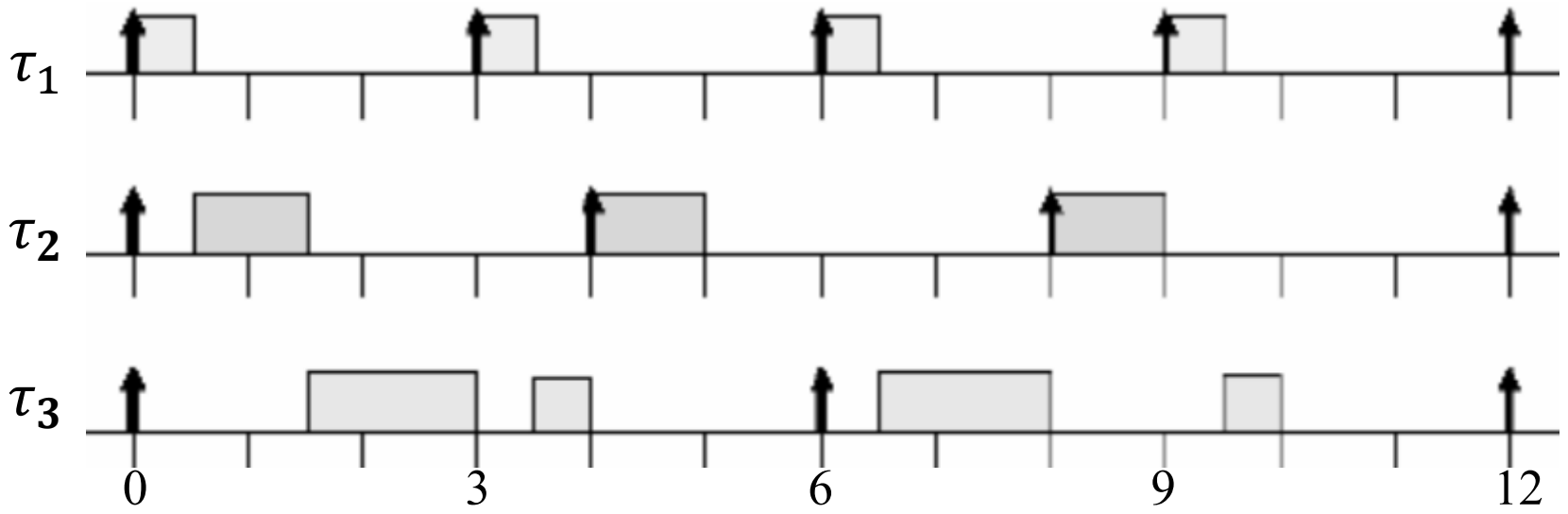
Gantt chart for EDF

Q2. Schedulability under RM, DM, or EDF

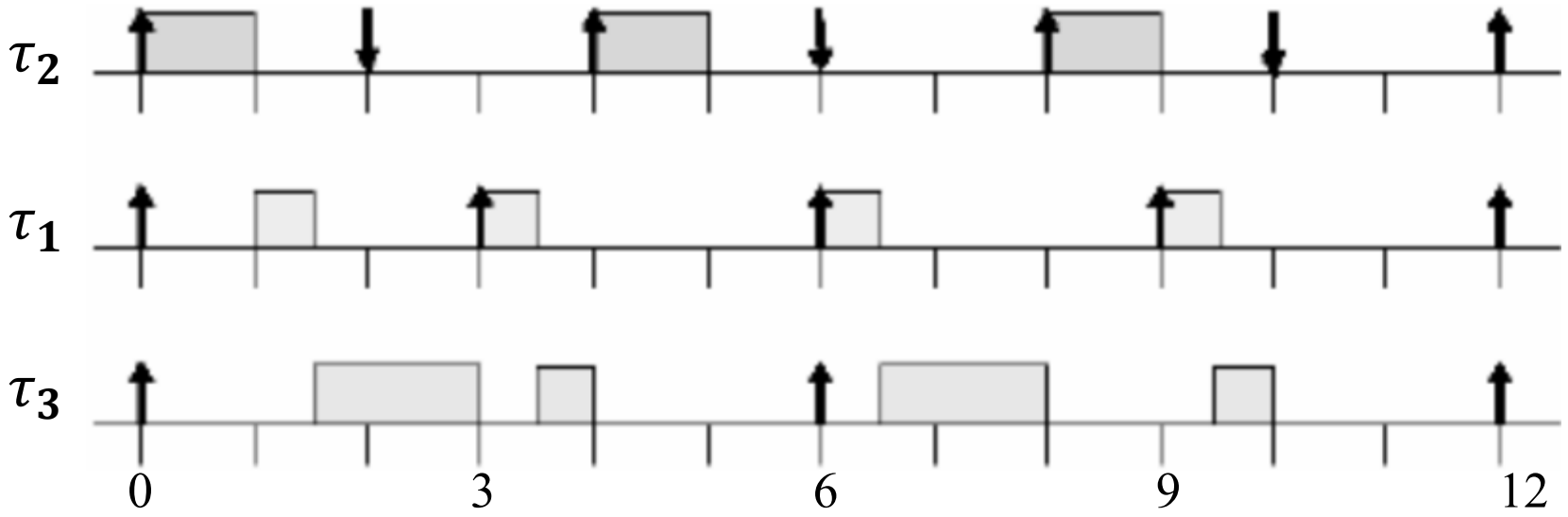
- Determine schedulability of the following tasksets under (1) Rate Monotonic (RM) scheduling, using Utilization Bound test and/or Response Time Analysis (RTA) to determine taskset schedulability. (2) Deadline Monotonic (DM) scheduling (3) Earliest Deadline First (EDF) scheduling, using Utilization Bound test.
- 1) Taskset $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
- 2) Taskset $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- 3) Taskset $\tau_1 = (1, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$

Recall: RM vs. DM Example

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
- Under RM (or DM), priority ordering $\tau_1 > \tau_2 > \tau_3$



- Three tasks with τ_2 assigned a smaller deadline of $D_2 = 2$: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- Under DM, priority ordering $\tau_2 > \tau_1 > \tau_3$



Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 4), \tau_3 = (2, 6, 6)$
  - For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
    - » System utilization $U = \frac{0.5}{3} + \frac{1}{4} + \frac{2}{6} = 0.75 \leq 0.780$ (UB for 2 tasks under RM), hence the taskset is schedulable under RM
  - For DM: priority ordering $\tau_1 > \tau_2 > \tau_3$
    - » Since $D_i = T_i$, DM scheduling is the same as RM scheduling, hence it is also schedulable under DM
  - For EDF: System utilization $U = \frac{0.5}{3} + \frac{1}{4} + \frac{2}{6} = 0.75 \leq 1$ (UB for EDF), hence the taskset is schedulable under EDF

Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (0.5, 3, 3), \tau_2 = (1, 4, 2), \tau_3 = (2, 6, 6)$
- No Utilization Bound test for RM or DM, for taskset with $D_i < T_i$; need to use Response Time Analysis (RTA)
- For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
  - $R_1 = C_1 + 0 = 0.5 + 0 = 0.5 \leq D_1 = 3$
  - $R_2 = C_2 + \left\lceil \frac{R_1}{T_1} \right\rceil \cdot C_1 = 1 + \left\lceil \frac{R_1}{3} \right\rceil \cdot 0.5 = 1.5 \leq D_2 = 2$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 = 2 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 0.5 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 = 4 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For DM: priority ordering $\tau_2 > \tau_1 > \tau_3$
  - $R_2 = C_2 + 0 = 1 + 0 = 1 \leq D_2 = 2$
  - $R_1 = C_2 + \left\lceil \frac{R_1}{T_2} \right\rceil \cdot C_2 = 0.5 + \left\lceil \frac{R_1}{4} \right\rceil \cdot 1 = 1.5 \leq D_1 = 3$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 = 2 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 0.5 = 4 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For EDF:
  - System density $\Delta = \frac{0.5}{3} + \frac{1}{2} + \frac{2}{6} = 1.0 \leq 1$, hence this taskset is schedulable under EDF

Q2. Schedulability under RM, DM, or EDF ANS

- Three tasks: $\tau_1 = (1,3,3), \tau_2 = (1,4,2), \tau_3 = (2,6,6)$
- No Utilization Bound test for RM or DM, for taskset with $D_i < T_i$; need to use Response Time Analysis (RTA)
- For RM: priority ordering $\tau_1 > \tau_2 > \tau_3$
  - $R_1 = C_1 + 0 = 1 + 0 = 1 \leq D_1 = 3$
  - $R_2 = C_2 + \left\lceil \frac{R_1}{T_1} \right\rceil \cdot C_1 = 1 + \left\lceil \frac{R_1}{3} \right\rceil \cdot 1 = 2 \leq D_2 = 2$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 = 2 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 = 6 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- For DM: priority ordering $\tau_2 > \tau_1 > \tau_3$
  - $R_2 = C_2 + 0 = 1 + 0 = 1 \leq D_2 = 2$
  - $R_1 = C_2 + \left\lceil \frac{R_1}{T_2} \right\rceil \cdot C_2 = 1 + \left\lceil \frac{R_1}{4} \right\rceil \cdot 1 = 2 \leq D_1 = 3$
  - $R_3 = C_3 + \left\lceil \frac{R_3}{T_2} \right\rceil \cdot C_2 + \left\lceil \frac{R_3}{T_1} \right\rceil \cdot C_1 = 2 + \left\lceil \frac{R_3}{4} \right\rceil \cdot 1 + \left\lceil \frac{R_3}{3} \right\rceil \cdot 1 = 6 \leq D_3 = 6$
  - Since all tasks meet their deadlines, the taskset is schedulable
- Three tasks: $\tau_1 = (1,3,3), \tau_2 = (1,4,2), \tau_3 = (2,6,6)$ under EDF
  - System density $\Delta = \frac{1}{3} + \frac{1}{2} + \frac{2}{6} = 1.17 > 1$, hence we CANNOT determine this taskset's schedulability under EDF

Q3 RM, EDF, LLF

- Consider the set of 2 periodic tasks whose period, deadline and WCET parameters are given.
- 1. For each scheduling algorithm (RM, EDF, LLF), draw the Gantt chart by filling in the table with the task ID that runs in each time slot until time 10, and calculate the WCRT for each task.
- 2. Under RM scheduling, use utilization bound and Response Time Analysis (RTA) to determine taskset schedulability.

Task ID	T=D	C	RM Resp. Time	EDF Resp. Time	LLF Resp. Time
1	8	3			
2	10	4			

RM										
EDF										
LLF										
Time	0	1	2	3	4	5	6	7	8	9	10

Gantt Chart

Time	T_1 Laxity	T_2 Laxity	Running Task
t=0			
t=1			
t=2			
t=3			
t=4			
t=5			
t=6			
t=7			
t=8			
T=9			

Q3 RM, EDF, LLF ANS

- Consider the set of 2 periodic tasks whose period, deadline and WCET parameters are given.
- 1. For each scheduling algorithm (RM, EDF, LLF), draw the Gantt chart by filling in the table with the task ID that runs in each time slot until time 10, and calculate the WCRT for each task.
- 2. Under RM scheduling, determine taskset schedulability using utilization bound and/or Response Time Analysis (RTA) to

Task ID	T=D	C	RM Resp. Time	EDF Resp. Time	LLF Resp. Time
1	8	3	3	3	5
2	10	4	7	7	7

RM	1	1	1	2	2	2	2	X	1	1
EDF	1	1	1	2	2	2	2	X	1	1
LLF	1	1	2	2	1	2	2	X	1	1

Time 0 1 2 3 4 5 6 7 8 9 10

Gantt Chart

Time	T_1 Laxity	T_2 Laxity	Running Task
t=0	8-0-3=5	10-0-4=6	1
t=1	8-1-2=5	10-1-4=5	1 (tie)
t=2	8-2-1=5	10-2-4=4	2
t=3	8-3-1=4	10-3-3=4	2 (tie)
t=4	8-4-1=3	10-4-2=4	1
t=5	T1 done	10-5-2=3	2
t=6	T1 done	10-6-1=3	2
t=7	T1 done	T2 done	X
t=8	16-8-3=5	T2 done	1
T=9	16-9-2=5	T2 done	1

Q3 RM, EDF, LLF ANS

- System utilization $U = \frac{3}{8} + \frac{4}{10} = 0.775 \leq 0.828$. Since utilization is within the RM bound, we determine this taskset to be schedulable under RM, without the need for RTA analysis
- System utilization $U = \frac{3}{8} + \frac{4}{10} = 0.775 \leq 1.0$, hence this taskset is schedulable under EDF and LLF