1)

a) $\vec{u} = (1,1,1)$

$\|\vec{u}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

b) $\vec{u} = 3\vec{i} + 4\vec{k}$

$\|\vec{u}\| = \sqrt{3^2+0^2+4^2} = \sqrt{25} = 5$

c) $\vec{u} = -\vec{i} + \vec{j}$

$\|\vec{u}\| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{2}$

d) $\vec{u} = 4\vec{i} + 3\vec{j} - \vec{k}$

$\vec{u} = \sqrt{4^2+3^2+(-1)^2} = \sqrt{26}$

2)

a) $\vec{e_1} = \vec{DH}$, $\vec{e_2} = \vec{DC}$, $\vec{e_3} = \vec{DA}$ são mutuamente perpendiculares e possuem norma 1, portanto, E é ortonormal

b) • $\vec{u} = \vec{CD} + \vec{CB} = -\vec{e_2} + \vec{e_3} = (0,-1,1)_E$

• $\vec{v} = \vec{DC} + \vec{CB} = \vec{e_2} + \vec{e_3} = (0,1,1)_E$

• $\vec{w} = \vec{GC} = \vec{e_1} + \vec{e_2} = (1,1,0)_E$

c) • $\|\vec{u}\| = \sqrt{0+1+1} = \sqrt{2}$

• $\|\vec{v}\| = \sqrt{0+1+1} = \sqrt{2}$

• $\vec{f_1} = \dfrac{\vec{u}}{\|\vec{u}\|} = \dfrac{1}{\sqrt{2}}(0,-1,1)$

$\vec{f_2} = \dfrac{\vec{v}}{\|\vec{v}\|} = \dfrac{1}{\sqrt{2}}(0,1,1)$

$\vec{f_3} = \vec{w} = (1,1,0)$

$\vec{f_1} \cdot \vec{f_2} = \frac{1}{2}(0 + (-1)(1) + 1(1)) \Rightarrow 0$

$\vec{f_1} \cdot \vec{f_3} = \frac{1}{\sqrt{2}}(0 + (-1)(1) + 1(0)) \Rightarrow -\frac{1}{\sqrt{2}} \neq 0$

e) $\vec{HB} = \vec{HC} + \vec{CB} = -\vec{e_1} + \vec{e_3} = (-1,0,1)_E$

FORONI

3)

a) • $\vec{AB} = B - A = (3, -3, -6)$

• $\vec{BC} = C - B = (-5, -4, 4)$

• $\vec{CA} = A - C = (2, 7, 2)$

b) • $\|\vec{AB}\| = \sqrt{9+9+36} = \sqrt{54} = 3\sqrt{6}$

• $\|\vec{BC}\| = \sqrt{25+16+16} = \sqrt{57}$

• $\|\vec{CA}\| = \sqrt{4+49+4} = \sqrt{57}$ } triângulo isósceles

c) • pontos médios

• AB: $\left(\frac{7}{2}, \frac{5}{2}, 0\right)$

• BC: $\left(\frac{5}{2}, -1, -1\right)$

• CA: $\left(1, \frac{1}{2}, 2\right)$

d) $\cos\theta = \dfrac{\vec{CB}\cdot\vec{CA}}{\|\vec{CB}\|\cdot\|\vec{CA}\|} = \dfrac{5\cdot 2 + 4\cdot 7 + (-4)\cdot 2}{\sqrt{57}} = \dfrac{30}{57}$

$\theta = \arccos\left(\dfrac{30}{57}\right)$

e) $\vec{AB} + \vec{BC} + \vec{CA} = (3-5+2, -3-4+7, -6+4+2) = (0,0,0)$

4)

a) $|\vec{u}\cdot\vec{v}| = \|\vec{u}\|\,\|\vec{v}\|\,|\cos\theta| \leq \|\vec{u}\|\,\|\vec{v}\|$

• igualdade $\Leftrightarrow$ $|\cos\theta| = 1$, ou seja, quando $\vec{u}$ e $\vec{v}$ são paralelos

b) $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u}\cdot\vec{v} \leq \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\|\vec{u}\|\,\|\vec{v}\|$

$= (\|\vec{u}\| + \|\vec{v}\|)^2$

• Tomando raiz quadrada $= \|\vec{u}+\vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$

c) $\|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) - (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v})$

$= 4\,\vec{u}\cdot\vec{v}$ //

FORON:

5)

a) $\vec{u} = (1, 0, 1)$, $\vec{w} = (-2, 10, 2)$

° prod. esc.

$\vec{u} \cdot \vec{w} = 1(-2) + 0 \cdot 10 + 1 \cdot 2 = 0$

° normas

$\cdot \|\vec{u}\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$

$\cdot \|\vec{w}\| = \sqrt{(-2)^2 + 10^2 + 2^2} = \sqrt{108} = 6\sqrt{3}$

· cálculo do ângulo

$\cos\theta = \dfrac{0}{\sqrt{2} \cdot 6\sqrt{3}} = 0 \Rightarrow \theta = \pi/2$ rad

b) $\vec{u} = (-1, 1, 1)$, $\vec{w} = (1, 1, 1)$

° prod. esc

$\vec{u} \cdot \vec{w} = -1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 = 1$

° normas

$\cdot \|\vec{u}\| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$

$\cdot \|\vec{w}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

· calc. ang.

$\cos\theta = \dfrac{1}{\sqrt{3} \cdot \sqrt{3}} = \dfrac{1}{3}$

$\theta = \arccos \frac{1}{3}$ rad

c) $\vec{u} = (3, 3, 0)$, $\vec{w} = (2, 1, -2)$

° prod. esc

$\vec{u} \cdot \vec{w} = 3 \cdot 2 + 3 \cdot 1 + 0 = 9$

° normas

$\|\vec{u}\| = \sqrt{3^2 + 3^2 + 0^2} = \sqrt{18} = 3\sqrt{2}$

$\|\vec{w}\| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{9} = 3$

° calc. ang.

$\cos\theta = \dfrac{9}{3\sqrt{2} \cdot 3} = \dfrac{9}{9\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\theta = \pi/4$ rad

a) $\vec{u} = (\sqrt{3}, 1, 0)$; $\vec{v} = (\sqrt{3}, 1, 2\sqrt{3})$

° prod. esc.

· $\vec{u} \cdot \vec{v} = \sqrt{3} \cdot \sqrt{3} + 1 + 0 = 4$

· calc. ang.

$\cos\theta = \frac{4}{2 \cdot 4} = \frac{1}{2}$

$\theta = \pi/3$ rad

° normas

· $\|\vec{u}\| = \sqrt{(\sqrt{3})^2 + 1^2 + 0^2} = \sqrt{4} = 2$

· $\|\vec{v}\| = \sqrt{(\sqrt{3})^2 + 1^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$

6)

a) $\vec{u} = (x+1, 1, 2)$, $\vec{v} = (x-1, -1, -2)$

$\vec{u} \cdot \vec{v} = (x+1)(x-1) + (1)(-1) + (2)(-2) = 0$

$x^2 - 1 - 1 - 4 = 0 \Rightarrow x^2 - 6 = 0 \Rightarrow x = \pm\sqrt{6}$

b) $\vec{u} = (x, x, 4)$, $\vec{v} = (4, x, 1)$

$\vec{u} \cdot \vec{w} = x(4) + x(x) + 4 \cdot 1 = 0$

$= 4x + x^2 + 4 = 0 \Rightarrow x^2 + 4x + 4 = 0 \Rightarrow (x+2)^2 = 0 \Rightarrow x = -2$

7)

a) $\vec{v} \cdot \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{vmatrix} = (7, -7, -7) = 7(1, -1, -1)$

$\vec{u} \cdot (1,1,1) = k(1-1-1) = -k = -1 \Rightarrow k = 1$

$\therefore \vec{u} = (1, -1, -1)$

b) $\vec{v} \cdot \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} = (14, -14, -14) = 14(1, -1, 1)$

- 2 possibilidades

$\vec{u} = \pm 3\sqrt{3} \cdot \frac{1}{\sqrt{3}}(1, -1, -1) = \pm 3(1, -1, 1)$

produto escalar = positivo

$\underbrace{3(1,-1,-1)}_{\text{vetor que forma ângulo agudo}} \cdot (1,0,0) = 3 > 0$ e $-3(1,-1,-1) \cdot (1,0,0) = -3 < 0$

FORONI:

c) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$

$\dfrac{\sqrt{2}}{2} = \dfrac{\vec{u}\cdot\vec{v}}{\sqrt{5}\cdot 1} \Rightarrow \vec{u}\cdot\vec{v} = \dfrac{\sqrt{10}}{2}$

• $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u}\cdot\vec{v} = 5+1+2\cdot\sqrt{10}/2 = 6+\sqrt{10}$

• $\|\vec{u}-\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\| - 2\vec{u}\cdot\vec{v} = 5+1-\sqrt{10} = 6-\sqrt{10}$

• $(\vec{u}+\vec{v})(\vec{u}-\vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2 = 5-1 = 4$

• $\cos\phi = \dfrac{(\vec{u}+\vec{v})(\vec{u}-\vec{v})}{\|\vec{u}+\vec{v}\|\,\|\vec{u}-\vec{v}\|} = \dfrac{4}{\sqrt{6+\sqrt{10}}\cdot\sqrt{6-\sqrt{10}}} = \dfrac{4}{\sqrt{26}}$

$\phi = \arccos\left(4/\sqrt{26}\right)$ rad

8)

a) $\vec{v} = (1,-1,2)$, $\vec{u} = (3,-1,1)$

$proj_{\vec{u}}\vec{v} = \left(\dfrac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \left(\dfrac{3+1+2}{9+1+1}\right)(3,-1,1)$

$= \dfrac{6}{11}(3,-1,1) = \left(\dfrac{18}{11}, -\dfrac{6}{11}, \dfrac{6}{11}\right)$

b) $\vec{v} = (1,3,5)$, $\vec{u} = (-3,1,0)$

$proj_{\vec{u}}\vec{v} = \left(\dfrac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$

$\vec{v}\cdot\vec{u} = 1(-3)+3\cdot1+5\cdot0 = -3+3 = 0$

$\vec{u}\cdot\vec{u} = (-3)^2+1^2+0^2 = 9+1+0 = 10$

$proj_{\vec{u}}\vec{v} = (0/10)\,\vec{u} = \vec{0}$

c) $\vec{v} = (-1,1,1)$, $\vec{u} = (-2,1,2)$

$proj_{\vec{u}}\vec{v} = \left(\dfrac{2+1+2}{4+1+4}\right)(-2,1,2) = 5/9\,(-2,1,2) = \left(-\dfrac{10}{9}, \dfrac{5}{9}, \dfrac{10}{9}\right)$

d) $\vec{v} = (1, 2, 4)$, $\vec{u} = (-2, -4, -8)$
$\cdot \vec{v} \cdot \vec{u} = 1(-2) + 2(-4) + 4(-8) = -42$

$\vec{u} \cdot \vec{u} = (-2)^2 + (-4)^2 + (-8)^2 = 84$

$proj_{\vec{u}} \vec{v} = \left(\frac{-42}{84}\right) \vec{u} = -\frac{1}{2} \vec{u} = -\frac{1}{2} (-2, -4, -8)$

9)

a) · prod esc.
$\cdot \vec{v} \cdot \vec{u} = 3 \cdot 2 + (-6)(-2) + 0 = 18$

· norma
$\vec{u} = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$

· proj
$\left(\frac{18}{9}\right) \vec{u} = 2(2, -2, 1) = (4, -4, 2)$

$proj_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right) \vec{v}$

· norma
$\vec{v} = 3^2 + (-6)^2 + 0^2 = 45$

$proj = \left(\frac{18}{45}\right)(3, -6, 0) = \left(\frac{2}{5}\right)(3, -6, 0) = \left(\frac{6}{5}, -\frac{12}{5}, 0\right)$

b) $\vec{q} = \vec{v} - \vec{p} = (3, -6, 0) - (4, -4, 2) = (-1, -2, -2)$
$\vec{p} \cdot \vec{q} = 4(-1) + (-4)(-2) + 2(-2) = 0 \rightsquigarrow$ são ortogonais

c) $\vec{u} \cdot \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{vmatrix} = \vec{i}(0+6) - \vec{j}(0-3) + k(-12+6)$
$= (6, 3, -6)$

$\|\vec{u} \cdot \vec{v}\| = \sqrt{6^2 + 3^2 + (-6)^2} = \sqrt{81} = 9$ unidades de área

FORONI:

10)

a) $\vec{u} = 3\vec{i} + 3\vec{j}$, $\vec{w} = 5\vec{i} + 4\vec{j}$

$$\vec{u} \cdot \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} = (0, 0, 12 - 15) = (0, 0, -3)_{//}$$

$\|\vec{u} \cdot \vec{w}\| = 3_{//}$

b) $\vec{u} = (7, 0, -5)$, $\vec{v} = (1, 2, -1)$

$$\vec{u} \cdot \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} = (10, 2, 14)_{//}$$

$\|\vec{u} \cdot \vec{v}\| = \sqrt{10^2 + 2^2 + 14^2} = \sqrt{100 + 4 + 196} = \sqrt{300} = 10\sqrt{3}_{//}$

c) $\vec{u} = (1, -3, 1)$, $\vec{v} = (1, 1, 4)$

$$\vec{u} \cdot \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = (-13, -3, 4)_{//}$$

$\|\vec{u} \cdot \vec{w}\| = \sqrt{(-13)^2 + (-3)^2 + 4^2} = \sqrt{194}_{//}$

d) $\vec{u} = (2, 1, 2)$, $\vec{w} = (4, 2, 4)$

$$\vec{u} \cdot \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{vmatrix} = (0, 0, 0)_{//}$$

$\|\vec{u} \cdot \vec{w}\| = 0_{//}$

11)

b) $\|\vec{u} \cdot \vec{v}\|^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$

$= 1 \cdot 25 - 9 = 16 \Rightarrow \|\vec{u} \cdot \vec{v}\| = 4_{//}$

c) $\frac{1}{2} \|\vec{AB} \cdot \vec{AC}\| = \frac{\sqrt{3}}{4} l^2$

$\|\vec{AB} \cdot \vec{AC}\| = \frac{\sqrt{3}}{2} l^2$

$\|\vec{AB} \cdot \vec{AC}\| = \|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \sin\theta$

$= l^2 \cdot \sin(60°)$

$= l^2 \cdot \frac{\sqrt{3}}{2}$

$\|\vec{AB} \cdot \vec{AC}\| = \frac{\sqrt{3}}{2} l^2_{//}$

FORONI

12)

a) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix} = -2\vec{i} + 0\vec{j} + 2\vec{k}$

$\begin{cases} -b - c = 2 \\ a + c = 0 \Rightarrow a = -c \\ a + b = 2 \Rightarrow b = 2 + c \end{cases}$

$-(2 + c) - c = -2$

$-2 - 2c = -2 \Rightarrow c = 0$

$\Rightarrow a = 0,\ b = 2,\ c = 0$

$(0,2,0)(2,3,4) = 6 \neq 9$

sistema inconsistente

b) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 1 & 0 & 1 \end{vmatrix} = (2, 2, -2)$

$\begin{cases} b = 2 \\ -a + c = 2 \Rightarrow c = 2 + a \\ -b = -2 \Rightarrow b = 2 \end{cases}$

$a^2 + 2^2 + (a+2)^2 = 6$

$2a^2 + 4a + 8 = 6 \Rightarrow 2a^2 + 4a + 2 = 0$

$a = \dfrac{-4 \pm \sqrt{16 - 16}}{4} = -1 \Rightarrow \boxed{\begin{matrix} a = -1 \\ c = 1 \end{matrix}}$

$\vec{x} = (-1, 2, 1)$

c) $\vec{u} \cdot \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 0 & 3 \\ 2 & -2 & 0 \end{vmatrix} = 6(1,1,1) \Rightarrow k(1,1,1)$

$\|k(1,1,1)\| = \sqrt{3} \Rightarrow |k|\sqrt{3} = \sqrt{3} \Rightarrow k = \pm 1$

$\cos\theta = \dfrac{\vec{x} \cdot \vec{j}}{\|\vec{x}\|} = \dfrac{k}{\sqrt{3}}$

$\therefore$ âng. obtuso $\Rightarrow \cos\theta < 0 \therefore k = -1$

$\vec{x} = (-1, -1, -1)$

13)

a) $\bullet\ \vec{AD} = D - A = (5-3, 3-2, 3+1) = (2, 1, 4)$

$\vec{AB} \cdot \vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{vmatrix} = (5, -6, -1)$

área $= \|(5, -6, -1)\| = \sqrt{25 + 36 + 1} = \sqrt{62}$ //

b) $\vec{AB} \cdot \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = (3, 3, -1)$

área $= \frac{1}{2} \|(3,3,-1)\| = \frac{1}{2}\sqrt{9+9+1} = \sqrt{19}/2$

$\|\vec{BC}\| = \|\vec{AC} - \vec{AB}\| = \|(1, 0, 3)\| = \sqrt{1+0+9} = \sqrt{10}$

área $= \frac{1}{2} \cdot$ base $\cdot$ altura

$\frac{\sqrt{19}}{2} = \frac{1}{2} \cdot 10 \cdot h \Rightarrow h = \frac{\sqrt{19}}{\sqrt{10}} = \frac{\sqrt{190}}{10}$ //

14)

a) $\vec{u}(\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v}) \cdot \vec{w} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix} = 12$

$\circ\ [\vec{u}, \vec{w}, \vec{v}] = -[\vec{u}, \vec{v}, \vec{w}] = -12$

$\circ\ [\vec{v}, 2\vec{w}, \vec{u}] = 2[\vec{v}, \vec{w}, \vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}] = 24$

$\circ\ [\vec{u}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{u}]$

$3[\vec{u}, \vec{v}, \vec{w}] + 9[\vec{u}, \vec{v}, \vec{u}] - 6[\vec{u}, \vec{u}, \vec{u}] - 2[\vec{u}, \vec{u}, \vec{w}]$

. Termos repetidos $= 0$

$\therefore\ 3 \cdot 12 = 36$