

1)

$$a) \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} = \tilde{M}; \tilde{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cdot \text{posto de } \tilde{M} = 2 \left\{ \begin{array}{l} p = 2 \quad m = 2 \quad / \quad m = p \\ \text{posto de } \tilde{A} = 2 \end{array} \right. \therefore \text{SPD}$$

$$x = 2 \quad \Delta y = -1$$

$$b) \begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = \tilde{M}; \tilde{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\cdot \text{posto de } \tilde{M} = 4 \left\{ \begin{array}{l} p = 4 \quad m = 4 \quad / \quad m = p \\ \text{posto de } \tilde{A} = 4 \end{array} \right. \therefore \text{SPD}$$

$$x = 4; y = 3; z = 2; w = 1$$

$$c) \begin{pmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} = \tilde{M}; \tilde{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\cdot \text{posto de } \tilde{M} = 3 \left\{ \begin{array}{l} p = 3 \quad m = 4 \quad / \quad m > p \\ \text{posto de } \tilde{A} = 3 \end{array} \right. \therefore \text{SPI}$$

$$x = 6; y = 3; z + w = 2$$

$$d) \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{pmatrix} = \tilde{M}; \tilde{A} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\cdot \text{posto de } \tilde{M} = 2 \left\{ \begin{array}{l} p = 2 \quad m = 3 \quad / \quad m > p \\ \text{posto de } \tilde{A} = 2 \end{array} \right. \therefore \text{SPI}$$

$$\begin{cases} x + 3z = 1 \\ y - z = 2 \end{cases} //$$

$$c) \begin{pmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{pmatrix} = \tilde{M}, \tilde{A} = \begin{pmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} \text{posto de } \tilde{M} &= 3 \\ \text{posto de } \tilde{A} &= 3 \end{aligned} \left\{ \begin{array}{l} p=3 \quad m=4 / m > p \\ \therefore \text{SPI} \end{array} \right.$$

$$\begin{cases} x - 7w = 8 \\ y + 3w = 2 \\ z + w = -5 \end{cases} //$$

$$d) \begin{pmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \tilde{M}, \tilde{A} = \begin{pmatrix} 1 & -6 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} \text{posto de } \tilde{M} &= 3 \\ \text{posto de } \tilde{A} &= 3 \end{aligned} \left\{ \begin{array}{l} p=3 \quad m=5 / m > p \\ \therefore \text{SPI} \end{array} \right.$$

$$\begin{cases} x - 6y + 3\alpha = -2 \\ z + 4\alpha = 7 \\ w + 5\alpha = 8 \end{cases} //$$

2)

$$a) \begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases}$$

$$M = \left| \begin{array}{cc|c} 3 & -4 & 1 \\ 1 & 3 & 9 \end{array} \right|$$

2)

$$a) \begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases}$$

$$\begin{array}{ccc|c} 3 & -4 & 1 & l_1 \leftarrow 3l_2 \\ 1 & 3 & 9 & l_2 \leftarrow 4l_2 \end{array}$$

$$\begin{array}{ccc|c} 9 & -12 & 3 & l_1 \leftarrow l_1 + l_2 \\ 4 & -12 & 36 & \end{array}$$

$$\begin{array}{ccc|c} 13 & 0 & 39 & \end{array}$$

$$\begin{array}{ccc|c} 4 & -12 & 36 & l_2 \leftarrow l_2 - 4 \end{array}$$

$$\begin{array}{ccc|c} 13 & 0 & 39 & l_1 \leftarrow l_1 / 13 \end{array}$$

$$\begin{array}{ccc|c} 0 & -8 & 32 & l_2 \leftarrow l_2 / -8 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 3 & \\ 0 & 1 & -4 & \end{array} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$b) \begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$$

$$\begin{array}{ccc|c} 5 & 8 & 34 & l_1 \leftarrow -2l_1 + l_2 \\ 10 & 16 & 50 & \end{array}$$

$$\begin{array}{ccc|c} 0 & 0 & -14 & \neq \text{ sistema inconsistente} \\ 10 & 16 & 50 & \end{array}$$

$$c) \begin{cases} x + 2y = 5 \\ 2x + 3y = -4 \end{cases}$$

$$\begin{array}{ccc|c} 1 & 2 & 5 & l_1 \leftarrow -3l_1 \\ 2 & 3 & -4 & l_2 \leftarrow 2l_2 \end{array}$$

$$\begin{array}{ccc|c} -3 & -6 & -15 & l_1 \leftarrow l_1 + l_2 \\ 4 & 6 & -8 & \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & -23 & l_1 \leftarrow -4l_1 \\ 4 & 6 & -8 & \end{array}$$

$$\begin{array}{ccc|c} -4 & 0 & 92 & \end{array}$$

$$\begin{array}{ccc|c} 4 & 6 & -8 & l_2 \leftarrow l_2 + l_1 \end{array}$$

$$\begin{array}{ccc|c} -4 & 0 & 92 & l_1 \leftarrow l_1 / (-4) \end{array}$$

$$\begin{array}{ccc|c} 0 & 6 & 84 & l_2 \leftarrow l_2 / 6 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & -23 & \end{array}$$

$$\begin{array}{ccc|c} 0 & 1 & 14 & \end{array}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23 \\ 14 \end{pmatrix}$$



$$d) \begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$$

$$\begin{array}{ccc|c} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{array} \quad l_1 \leftarrow l_1 + l_2$$

$$\begin{array}{ccc|c} 2 & -4 & -2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & -2 & -3 & -4 \end{array}$$

$$\begin{array}{ccc|c} 4 & 0 & -8 & 4 \end{array}$$

$$\begin{array}{ccc|c} 2 & -4 & -2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & -2 & -3 & -4 \end{array} \quad l_3 \leftarrow -2l_3 + l_2$$

$$\begin{array}{ccc|c} 4 & 0 & -8 & 4 \end{array}$$

$$\begin{array}{ccc|c} 2 & -4 & -2 & -4 \end{array} \quad l_2 \leftarrow -2l_2 + l_1$$

$$\begin{array}{ccc|c} 0 & 0 & 4 & 4 \end{array}$$

$$\begin{array}{ccc|c} 4 & 0 & -8 & 4 \end{array}$$

$$\begin{array}{ccc|c} 0 & 8 & -4 & 12 \end{array} \quad l_2 \leftarrow l_2 + l_3$$

$$\begin{array}{ccc|c} 0 & 0 & 4 & 4 \end{array}$$

$$\begin{array}{ccc|c} 4 & 0 & 0 & 4 \end{array} \quad l_1 \leftarrow l_1/4$$

$$\begin{array}{ccc|c} 0 & 8 & 0 & 18 \end{array} \quad l_2 \leftarrow l_2/8$$

$$\begin{array}{ccc|c} 0 & 0 & 4 & 4 \end{array} \quad l_3 \leftarrow l_3/4$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 9/4 \\ 0 & 0 & 1 & 1 \end{array} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 9/4 \\ 1 \end{pmatrix}$$

$$e) \begin{cases} 2x - 6y = -4 \\ x + 3y = 1 \\ 4x + 12y = 2 \end{cases}$$

$$\begin{array}{cc|c} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{array}$$

$$\begin{array}{cc|c} 2 & -6 & -4 \end{array} \quad l_1 \leftarrow l_1/2 + l_2$$

$$\begin{array}{cc|c} 1 & 3 & 1 \end{array}$$

$$\begin{array}{cc|c} 4 & 12 & 2 \end{array} \quad l_3 \leftarrow l_3/2 + l_1$$

$$\begin{array}{cc|c} 1 & 0 & -1 \end{array} \quad l_1 \leftarrow 4l_1$$

$$\begin{array}{cc|c} 1 & 3 & 1 \end{array} \quad l_2 \leftarrow l_2 - l_1$$

$$\begin{array}{cc|c} 4 & 0 & -3 \end{array}$$

$$\begin{array}{cc|c} 4 & 0 & -4 \end{array}$$

$$\begin{array}{cc|c} 0 & 3 & 2 \end{array}$$

$$\begin{array}{cc|c} 4 & 0 & -3 \end{array} \quad l_3 \leftarrow l_3 - l_1$$

$$\begin{array}{cc|c} 4 & 0 & -4 \end{array} \quad l_1 \leftarrow l_1/4$$

$$\begin{array}{cc|c} 0 & 3 & 2 \end{array} \quad l_2 \leftarrow l_2/3$$

$$\begin{array}{cc|c} 0 & 0 & 7 \end{array}$$

$$\begin{array}{cc|c} 1 & 0 & -1 \end{array}$$

$$\begin{array}{cc|c} 0 & 1 & 2/3 \end{array}$$

$$\begin{array}{cc|c} 0 & 0 & 7 \end{array}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2/3 \end{pmatrix}$$

FORONI:



$$f) \begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$$

$$\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{array} \quad \begin{array}{l} \\ l_2 + l_2 - 2l_1 \\ l_3 + l_3 - 3l_1 \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{array} \quad \begin{array}{l} \\ l_2 + l_2 / (-5) \\ \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & -3 \end{array} \quad \begin{array}{l} l_1 + l_1 - 2l_2 \\ \\ l_3 + l_3 + 3l_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -6 \end{array} \quad \begin{array}{l} \\ \\ l_3 + l_3 / (-2) \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array} \quad \begin{array}{l} l_1 + l_1 - l_3 \\ l_2 + l_2 + l_3 \\ \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \quad \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$g) \begin{cases} x + 3y = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$$

$$\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{array} \quad \begin{array}{l} l_1 + 5l_1 + l_3 \\ \\ l_3 + l_3 - 2l_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ -1 & 0 & -5 & 18 \end{array} \quad \begin{array}{l} l_1 + 2l_1 + l_2 \\ \\ l_3 + 2l_3 + l_2 \end{array}$$

$$\begin{array}{ccc|c} 18 & 0 & 0 & -32 \\ 2 & -4 & 0 & -4 \\ 0 & -4 & -10 & 32 \end{array} \quad \begin{array}{l} l_2 + 3l_2 = l_1 \\ l_2 + l_2 = l_2 \\ l_1 + l_1 / 18 \\ l_2 + l_2 / 36 \\ l_3 + l_3 / (-10) \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 16/9 \\ 0 & 1 & 0 & 1/18 \\ 0 & 0 & 1 & 3,6 \end{array}$$

$$(x, y, z) = (16/9, 1/18, 3,6)$$

FORONI



$$h) \begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 2x + 2y + 3z = 13 \end{cases}$$

$$\begin{array}{ccc|c|l} 1 & 2 & 3 & 10 & \\ 3 & 4 & 6 & 23 & l_2 + l_1 - 3l_1 \\ 2 & 2 & 3 & 13 & l_3 + l_1 - 2l_1 \end{array}$$

$$\begin{array}{ccc|c|l} 1 & 2 & 3 & 10 & l_1 + l_1 + l_2 \end{array}$$

$$\begin{array}{ccc|c|l} 0 & -2 & -3 & -7 & \end{array}$$

$$\begin{array}{ccc|c|l} 0 & -2 & -3 & -7 & l_3 + l_3 - l_2 \end{array}$$

$$\begin{array}{ccc|c|l} 1 & 0 & 0 & 3 & \end{array}$$

$$\begin{array}{ccc|c|l} 0 & -2 & -3 & -7 & l_2 + l_2 / (-2) \end{array}$$

$$\begin{array}{ccc|c|l} 0 & 0 & 0 & 0 & \end{array}$$

$$\begin{array}{ccc|c|l} 1 & 0 & 0 & 3 & \\ 0 & 1 & -3/2 & -7/2 & \\ 0 & 0 & 0 & 0 & \end{array} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 7/2 \\ 0 \end{pmatrix}$$

$$z = a, a \in \mathbb{R}$$

$$\begin{cases} x = 3 \\ y = 7/2 - 3/2 a \\ z = a \end{cases}$$

$$i) \begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$$

$$\begin{array}{cccc|c|l} 1 & -3 & 4 & -1 & 2 & \\ 2 & -1 & 3 & -2 & 19 & l_2 + l_1 - 2l_1 \end{array}$$

$$\begin{array}{cccc|c|l} 1 & -3 & 4 & -1 & 2 & \\ 0 & 5 & -5 & 0 & 15 & l_2 + l_2 / 5 \end{array}$$

$$\begin{array}{cccc|c|l} 1 & -3 & 4 & -1 & 2 & l_1 + l_1 + 3l_2 \\ 0 & 1 & -1 & 0 & 3 & \end{array}$$

$$\begin{array}{cccc|c|l} 1 & 0 & 1 & -1 & 11 & \\ 0 & 1 & -1 & 0 & 3 & \end{array}$$

$$\begin{cases} x + z - w = 11 \\ y - z = 3 \end{cases}$$

$$z = a, w = b, a, b \in \mathbb{R}$$

$$x = 11 - a + b$$

$$y = 3 + a$$

$$z = a$$

$$w = b$$

3)

a)

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{pmatrix} \begin{array}{l} \\ l_2 + l_1 - l_1 \\ l_3 + l_3 - l_1 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{pmatrix} \begin{array}{l} \\ \\ l_3 + l_3 + 2l_2 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix} \begin{array}{l} \\ \\ l_3 + l_3 / (-7) \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 - 3l_3 \\ l_2 + l_2 + 3l_3 \\ \end{array}$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 - 2l_2 \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \\ \end{array} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{pmatrix} \begin{array}{l} \\ \\ l_3 + l_3 - l_2 \end{array}$$

$$\begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{pmatrix} \begin{array}{l} l_1 + l_1 + l_3 \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{pmatrix} \begin{array}{l} l_1 + l_1 + 2l_3 \\ \\ l_3 + 2l_3 + l_2 \end{array}$$

$$\begin{pmatrix} 1 & -2 & 0 & -3 & -3 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 + l_2 \\ l_2 + l_2 + l_3 \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 1 & -2 & 0 \\ 0 & 2 & 0 & -2 & 4 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 - l_3 \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 2 & 0 & -2 & 4 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{array}{l} \\ l_2 + l_2 / 2 \\ l_3 + -l_3 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{pmatrix} \begin{array}{l} \\ \\ \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$



$$\begin{array}{l|cccc|l}
 c) & 1 & 2 & 3 & 0 & \\
 & 1 & 1 & 1 & 0 & l_2 \leftarrow l_2 - l_1 \\
 & 1 & 1 & 2 & 0 & l_3 \leftarrow l_3 - l_1 \\
 & 1 & 3 & 3 & 0 & l_4 \leftarrow l_4 - l_1 \\
 & 1 & 2 & 3 & 0 & \\
 & 0 & -1 & -2 & 0 & l_2 \leftrightarrow l_4 \\
 & 0 & -1 & -1 & 0 & \\
 & 0 & 1 & 0 & 0 & \\
 & 1 & 2 & 3 & 0 & l_1 \leftarrow l_1 - 2l_2 \\
 & 0 & 1 & 0 & 0 & \\
 & 0 & -1 & -1 & 0 & l_3 \leftarrow l_3 + l_2 \\
 & 0 & -1 & -2 & 0 & l_4 \leftarrow l_4 + l_2 \\
 & 1 & 0 & 3 & 0 & l_1 \leftarrow l_1 - 3l_3 \\
 & 0 & 1 & 0 & 0 & \\
 & 0 & 0 & -1 & 0 & l_3 \leftarrow -l_3 \\
 & 0 & 0 & -2 & 0 & l_4 \leftarrow l_4 - 2l_3 \\
 & 1 & 0 & 0 & 0 & \\
 & 0 & 1 & 0 & 0 & \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
 & 0 & 0 & 1 & 0 & \\
 & 0 & 0 & 0 & 0 & 
 \end{array}$$

$$\begin{array}{l|cccc|l}
 4) & 1 & -2 & 1 & 1 & 2 & \\
 & 2 & -5 & 1 & -2 & -1 & l_2 \leftarrow l_2 - 2l_1 \\
 & 3 & -7 & 2 & 1 & 2 & l_3 \leftarrow l_3 - 3l_1 \\
 & 1 & -2 & 1 & 1 & 2 & \\
 & 0 & -1 & -1 & -4 & -5 & l_2 \leftarrow -l_2 \\
 & 0 & -1 & -1 & -4 & -4 & \\
 & 1 & -2 & 1 & 1 & 2 & l_1 \leftarrow l_1 + 2l_2 \\
 & 0 & 1 & 1 & 4 & 5 & \\
 & 0 & -1 & -1 & -4 & -4 & l_3 \leftarrow l_3 + l_2 \\
 & 1 & 0 & 3 & 9 & 12 & \\
 & 0 & 1 & 1 & 4 & 5 & \\
 & 0 & 0 & 0 & 0 & 1 & 
 \end{array}
 \rightarrow \begin{cases} x + 3z = 9 \\ y + z = 4 \\ z = \lambda, \lambda \in \mathbb{R} \\ x = 9 - 3\lambda \\ y = 4 - \lambda \\ z = \lambda \end{cases} \rightarrow a$$

b) não possui solução  
 $l_3 \neq 0 \Rightarrow 1$   
 $\text{A}$



5)  
a)  $(A + 4I_3)X \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 5x + 5z = 0 \\ x + 5y + z = 0 \\ y = 0 \end{cases} \Rightarrow \begin{cases} 5x + 5z = 0 \\ x + z = 0 \end{cases} \quad (x = -z)$$

solução geral:  $(x, y, z) = (-z, 0, z) = z(-1, 0, 1), z \in \mathbb{R}$

b)  $AX = 2X \Leftrightarrow (A - 2I_3)X = \vec{0}$

$$\left[ \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{cases} x - 5z = 0 \\ y - 6z = 0 \end{cases} \Leftrightarrow \begin{cases} x = 5z \\ y = 6z \end{cases}$$

- matriz ampliada

$$M = \left[ \begin{array}{ccc|c} -1 & 0 & 5 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 \end{array} \right] \begin{matrix} \\ l_2 + l_1 + l_3 \\ \end{matrix}$$

$$\begin{cases} z = \lambda, \lambda \in \mathbb{R} \\ x = 5\lambda \\ y = 6\lambda \\ z = \lambda \end{cases}$$

$$\left[ \begin{array}{ccc|c} -1 & 0 & 5 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 1 & -6 & 0 \end{array} \right] \begin{matrix} \\ l_3 + l_2 + l_3 \\ \end{matrix}$$

$$\left[ \begin{array}{ccc|c} -1 & 0 & 5 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} l_1 + -l_1 \\ l_2 + -l_2 \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



6)

$$a) \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array} \begin{array}{l} \\ l_2 + l_2 - 2l_1 \\ l_3 + l_3 - 2l_1 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array} \begin{array}{l} \\ \\ l_3 + l_3 - l_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}$$

solução única

$$a^2 - 3 \neq 0$$

$$a \neq \pm\sqrt{3} \therefore a \in \mathbb{R} \setminus (\sqrt{3}, -\sqrt{3})$$

infinitas soluções:

$$a^2 - 3 = 0 \Rightarrow a = \pm\sqrt{3} \Rightarrow \text{não existe valor}$$

$$a - 4 = 0 \Rightarrow a = 4$$

nenhuma solução

$$a^2 - 3 = 0 \quad / \quad a = \sqrt{3} \Rightarrow a - 4 = \sqrt{3} - 4 \neq 0$$

$$a - 4 = 0 \quad / \quad a = -\sqrt{3} \Rightarrow a - 4 = -\sqrt{3} - 4 \neq 0$$

$$\therefore a = \sqrt{3} \text{ ou } a = -\sqrt{3}$$

$$b) \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array} \begin{array}{l} \\ l_2 + l_2 - 3l_1 \\ l_3 + l_3 - 4l_1 \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array} \begin{array}{l} \\ \\ l_3 + l_3 - l_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}$$

solução única

$$a^2 - 16 \neq 0$$

$$\therefore a \neq 4 \text{ e } a \neq -4 \Rightarrow S = \{a \in \mathbb{R} / (4, -4)\}$$

infinitas soluções

$$a^2 - 16 = 0 \Rightarrow a = 4 \text{ ou } a = -4 \therefore S = \{4\}$$

$$a - 4 = 0 \Rightarrow a = 4$$

nenhuma solução

$$a^2 - 16 = 0$$

$$a - 4 \neq 0 \Rightarrow -4 - 4 \neq 0 \Rightarrow -8 \neq 0 \therefore S = \{-4\}$$

$$a = -4$$

f)

$$a) \left( \begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} l_1 + l_1 + 2l_2 \\ \end{array}$$

$$\left( \begin{array}{cc|cc} 8 & 0 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{array} \right)$$

$$l_2 + 8l_2 - 3l_1$$

$$\left( \begin{array}{cc|cc} 8 & 0 & 1 & 2 \\ 0 & 8 & -3 & 2 \end{array} \right) \begin{array}{l} l_1 + l_1/8 \\ l_2 + l_2/8 \end{array}$$

$$\left( \begin{array}{cc|cc} 1 & 0 & 1/8 & 1/4 \\ 0 & 1 & -3/8 & 1/4 \end{array} \right)$$

$$= [I_2 | A^{-1}]$$

$$\hookrightarrow \begin{bmatrix} 1/8 & 1/4 \\ -3/8 & 1/4 \end{bmatrix}$$

$$c) \left( \begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array} \right)$$

$$l_2 + 3l_2 - l_1$$

$$\left( \begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 0 & 1 & -1 & 3 \end{array} \right) \begin{array}{l} l_1 + l_1 - 5l_2 \\ \end{array}$$

$$\left( \begin{array}{cc|cc} 3 & 0 & 6 & -15 \\ 0 & 1 & -1 & 3 \end{array} \right)$$

$$l_1 + l_1/3$$

$$\left( \begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array} \right)$$

$$\left( \begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array} \right)$$

$$= [I_2 | C^{-1}]$$

$$\hookrightarrow \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$



$$b) \begin{pmatrix} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 + l_2 \\ l_2 + l_2 + l_3 \end{array}$$

$$\begin{pmatrix} 3 & 0 & 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_2 + 3l_2 - l_1 \end{array}$$

$$\begin{pmatrix} 3 & 0 & 1 & 1 & 1 & 0 \\ 0 & 3 & -1 & -1 & 2 & 3 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 + l_1 + l_2 \\ l_2 + l_2 + l_3 \\ l_3 + l_3 \end{array}$$

$$\begin{pmatrix} 3 & 1 & 0 & 1 & 1 & 1 \\ 0 & 10 & 0 & -1 & 2 & 4 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 + 10l_1 - l_2 \\ l_3 + 10l_3 + l_2 \end{array}$$

$$\begin{pmatrix} 30 & 0 & 0 & 11 & 8 & 6 \\ 0 & 10 & 0 & -1 & 2 & 4 \\ 0 & 0 & 10 & -1 & 2 & 4 \end{pmatrix} \begin{array}{l} l_1 + l_1/30 \\ l_2 + l_2/10 \\ l_3 + l_3/10 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 11/30 & 4/15 & 2/5 \\ 0 & 1 & 0 & -1/10 & 1/5 & 2/5 \\ 0 & 0 & 1 & -1/10 & 1/5 & 2/5 \end{pmatrix} = [I_3 | B^{-1}]$$

$$d) \begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 \leftrightarrow l_2 \end{array}$$

$$\begin{pmatrix} 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_3 + 2l_3 - l_1 \end{array}$$

$$\begin{pmatrix} 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & -1 & 2 \end{pmatrix} \begin{array}{l} l_3 + 2l_3 + l_2 \end{array}$$

$$\begin{pmatrix} 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & -1 & 2 \end{pmatrix} \begin{array}{l} l_3 + 3l_1 + l_3 \end{array}$$



$$\left( \begin{array}{ccc|ccc} 6 & 0 & 0 & 2 & 2 & 2 \\ -2 & -2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 2 & -1 & 2 \end{array} \right) \quad l_2 + 3l_3 - l_1$$

$$\left( \begin{array}{ccc|ccc} 6 & 0 & 0 & 2 & 2 & 2 \\ 0 & -3 & 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 2 & -1 & 2 \end{array} \right) \quad \begin{array}{l} l_1 \leftarrow l_1/6 \\ l_2 \leftarrow l_2/3 \\ l_3 \leftarrow l_3/3 \end{array}$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array} \right) = [I_3 | D^{-1}]$$

$$e) \left( \begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \quad l_2 \leftarrow l_2 + l_1$$

$$\left( \begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \quad l_4 \leftarrow l_4 + l_3$$

$$\left( \begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 & 0 & 0 & 1 & 0 \end{array} \right) \quad \begin{array}{l} l_1 + 3l_1 = l_4 \\ l_3 + l_3 = l_4 \end{array}$$

$$\left( \begin{array}{cccc|cccc} 5 & -3 & 0 & 0 & 3 & 0 & -1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 & 0 & 0 & 1 & 0 \end{array} \right) \quad \begin{array}{l} l_1 + l_1 + 3l_2 \\ l_2 + 5l_2 - 2l_1 \end{array}$$

$$\left( \begin{array}{cccc|cccc} 11 & 0 & 0 & 0 & 3 & 0 & 2 & 0 \\ 0 & 11 & 0 & 0 & -6 & 0 & 7 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \end{array} \right) \quad l_4 + 11l_4 - l_1$$



e) 
$$\left( \begin{array}{cccc|cccc} 11 & 0 & 0 & 0 & 3 & 0 & 2 & 0 \\ 0 & 11 & 0 & 0 & -6 & 0 & 7 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 33 & -3 & 0 & -2 & 0 \end{array} \right) \begin{array}{l} l_1 + l_1/11 \\ l_2 + l_2/11 \\ l_3 + l_3/2 \\ l_4 + l_4/33 \end{array}$$

$$\left( \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 3/11 & 0 & 2/11 & 0 \\ 0 & 1 & 0 & 0 & -6/11 & 0 & 7/11 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2/11 & 0 & -2/33 & 0 \end{array} \right) = [I_4 | A^{-1}]$$



8)

·  $c$  = preço unitário da calça

·  $u$  = " " short

·  $b$  = " " blusa

$$\begin{cases} c + 2u + 3b = 26 \quad (-2) \\ 2c + 5u + 6b = 60 \\ 2c + 3u + 4b = 40 \end{cases}$$

$$2c + 5u + 6b = 60$$

$$2c + 3u + 4b = 40$$

$$\begin{cases} -2c - 4u - 6b = 52 \uparrow + \Rightarrow u = 8 \\ 2c + 5u + 6b = 60 \\ 2c + 3u + 4b = 40 \end{cases}$$

$$2c + 5u + 6b = 60$$

$$2c + 3u + 4b = 40$$

$$I + III = -u - 2b = -12$$

subst. (2) na equação  $(-u - 2b = -12)$

$$-8 - 2b = -12$$

$$-2b = -4$$

$$b = 2$$

subst. (2) e (3) na eq. 1

$$c + 16 + 6 = 26$$

$$c + 22 = 26 \Rightarrow c = 4$$

calça: R\$ 4; short: R\$ 8; blusa: R\$ 2

9)

$$\begin{cases} 5u + 2c + 6b = 2200 \rightarrow 5(2b) + 2(3b) + 6b = 2200 \\ c = 3b \\ c = u + b \end{cases}$$

$$c = 3b$$

$$10b + 6b + 6b = 2200$$

$$c = u + b$$

$$22b = 2200$$

subst. II na III

$$b = 100$$

$$3b = u + b$$

$$u = 2b$$

subst. (b) na eq. (C) e (2)

$$c = 3(100) = 300$$

$$u = 2(100) = 200$$

$$\cdot \text{sumoe } (u) = 200$$

$$\cdot \text{casquinhas } (c) = 300$$

$$\text{banana} = 100$$

$$\text{split } (b)$$

FORONI:

10)

$$\begin{cases} 4t + 3l + p = 700 \\ 2t + 4l + 3p = 600 \\ t + 2l + 4p = 500 \end{cases}$$

I e II

$$\begin{aligned} -12t - 9l - 3p &= -2100 \\ -2t + 4l + 3p &= 600 \quad + \\ \hline -10t - 5l &= -1500 \quad :(-5) \\ 2t + l &= 300 \quad (\text{eq. IV}) \end{aligned}$$

I e III

$$\begin{aligned} -16t - 12l - 4p &= -2800 \quad + \\ t + 2l + 4p &= 500 \\ \hline -15t - 10l &= -2300 \quad :(-5) \\ 3t + 2l &= 460 \quad (\text{eq. V}) \end{aligned}$$

usist eq. IV e V

$$\begin{aligned} l &= 300 - 2t \\ 3t + 2l &= 460 \rightarrow 3t + 2(300 - 2t) = 460 \\ 3t + 600 - 4t &= 460 \end{aligned}$$

usubst ① em l

$$l = 300 - 2(140)$$

$$l = 300 - 280$$

$$l = 20$$

$$-t = -140$$

$$t = 140$$

usubst ① e ② na eq. I

$$4 \cdot (140) + 3 \cdot 20 + p = 700$$

$$560 + 60 + p = 700$$

$$p = 80$$

$$\text{Torta de carne (t)} = R\$ 140$$

$$\text{salada (l)} = R\$ 20$$

$$\text{pizza} = R\$ 80$$



11)

$$2a + 3b + c = 8420$$

$$a + 2b + 2c = 7940$$

$$4a + 3b + c = 8110$$

I - III

$$2a + 3b + c = 8420$$

$$4a + 3b + c = 8110$$

$$-2a = 310$$

$$2a = -310$$

$$a = -155$$

substitua (a) em I e II

- I

$$2(-155) + 3b + c = 8420$$

$$-310 + 3b + c = 8420$$

$$3b + c = 8730 \text{ (eq. IV)}$$

- II

$$(-155) + 2b + 2c = 7940$$

$$2b + 2c = 8095 \text{ (eq. V)}$$

IV + V

$$3b + c = 8730 \quad (-2)$$

$$2b + 2c = 8095$$

$$-6b - 2c = -17460$$

$$2b + 2c = 8095$$

$$-4b = -9365$$

$$b = 2341,25$$

substitua (b) na eq. 4

$$3 \cdot 2341,25 + c = 8730$$

$$7023,75 + c = 8730$$

$$c = 1706,25$$

maior comprimento = 2341,25 m (b)

menor comprimento = 155 m (a)

diferença

$$2341,25 - 155 = 2186,25 \text{ m}$$