1) $AX = B$

$$X \begin{vmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ -3 \end{vmatrix}$$

$$\begin{cases} x_1 + 3x_2 + 4x_3 = 1 \\ -x_1 + 2x_2 + 5x_3 = 2 \cdot (-1) \\ -2x_1 - x_2 + 3x_3 = -3 \end{cases}$$

$$\begin{cases} \cancel{x_1} + 3\cancel{x_2} + 4x_3 = 1 \\ \cancel{x_1} - 2x_2 - 5x_3 = -2 \quad + \\ -2\cancel{x_1} - \cancel{x_2} + 3x_3 = -3 \end{cases}$$

$2x_3 = -4$

$\boxed{x_3 = -2}$

$$\begin{cases} 5x_2 + 9x_3 = 3 \\ 5x_2 + 9(-2) = 3 \end{cases}$$

$5x_2 = 21$

$\boxed{x_2 = 21/5}$

$$\begin{cases} -x_1 + 2x_2 + 5x_3 = 2 \\ -2x_1 - x_2 + 3x_3 = -3 \cdot 2 \end{cases}$$

$$\begin{cases} -x_1 + 2x_2 + 5x_3 = 2 \\ -4x_1 - 2x_2 + 6x_3 = -6 \end{cases} +$$

$-5x_1 + 11x_3 = -4$

$-5x_1 - 22 = -4$

$\boxed{x_1 = -18/5}$

$$X = \begin{pmatrix} -18/5 \\ 21/5 \\ -2 \end{pmatrix}$$

2) $X = A^{-1}B$

a) $\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} X = \begin{vmatrix} 2 \\ -1 \end{vmatrix} \Rightarrow X = \begin{bmatrix} -3/5 & 4/5 \\ 2/5 & -1/5 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\cdot \det(A) = 3 - 8 = -5$

$\cdot \operatorname{cof}(A) = \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}$

$\cdot \operatorname{adj}(A) = \begin{pmatrix} 1 & -2 \\ -4 & 3 \end{pmatrix}$

$\cdot A^{-1} = -\frac{1}{5} \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3/5 & 4/5 \\ 2/5 & -1/5 \end{bmatrix}$

$X = \begin{bmatrix} -6/5 + 3/5 & 8/5 - 4/5 \\ 4/5 - 2/5 & -2/5 + 1/5 \end{bmatrix}$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} -3/5 & 4/5 \\ 2/5 & -1/5 \end{bmatrix}$

$\boxed{x_1 = 1/5}$ $\boxed{x_2 = 1/5}$

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b) $\overbrace{\begin{pmatrix}2 & 3\\ 5 & 5\end{pmatrix}}^{C} + \overbrace{\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}}^{A} Y = \begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$

$Y = B \cdot A^{-1} \cdot C^{-1}$

$\cdot\, A^{-1} = \frac{1}{-1} = -1 \begin{bmatrix}5 & -2\\ -3 & 1\end{bmatrix}$ $\qquad \cdot\, C^{-1} = -\frac{1}{5}\begin{bmatrix}5 & -3\\ -5 & 2\end{bmatrix}$

$A^{-1} = \begin{bmatrix}-5 & 2\\ 3 & -1\end{bmatrix}$ $\qquad \cdot\, C^{-1} = \begin{bmatrix}-1 & 3/5\\ 1 & -2/5\end{bmatrix}$

$$Y = \left(\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}\begin{pmatrix}-5 & 2\\ 3 & -1\end{pmatrix}\right) \cdot \begin{pmatrix}-1 & 3/5\\ 1 & -2/5\end{pmatrix}$$

$$= \begin{pmatrix}-5+3 & -15+9\\ 4-2 & 14-7\end{pmatrix} \cdot \begin{pmatrix}-1 & 3/5\\ 1 & -2/5\end{pmatrix}$$

$$= \begin{pmatrix}-2 & -6\\ 2 & 7\end{pmatrix}\begin{pmatrix}-1 & 3/5\\ 1 & -2/5\end{pmatrix}$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix} = \begin{pmatrix}2-2 & 6-6\\ 6/5-4/5 & 21/5-14/5\end{pmatrix} = \begin{pmatrix}0 & 0\\ 2/5 & 7/5\end{pmatrix}$$

$\boxed{y_1 = 0}$ $\qquad \boxed{y_2 = 9/5}$

c) $\begin{pmatrix}1 & 0 & 0\\ 2 & -1 & 0\\ 2 & 3 & 1\end{pmatrix} W = \begin{pmatrix}5\\ 7\\ 2\end{pmatrix}$ $\qquad A^{-1} = -\frac{1}{1} = -1 \begin{vmatrix}1 & 0\\ -3 & -1\end{vmatrix} = \begin{vmatrix}-1 & 0\\ 3 & 1\end{vmatrix}$

$\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix} W = \begin{pmatrix}5\\ 7\\ 2\end{pmatrix}$

$$W = \overbrace{\begin{pmatrix}-1 & 0\\ 3 & 1\end{pmatrix}}^{A^{-1}} \begin{pmatrix}5\\ 7\\ 2\end{pmatrix}$$

$$= \begin{pmatrix}-5-7-2 & 0\\ 15+21+6 & 5+7+2\end{pmatrix}$$

$$\begin{pmatrix}w_1\\ w_2\end{pmatrix} = \begin{pmatrix}-14 & 0\\ 42 & 14\end{pmatrix} \;\therefore\; \boxed{w_1 = -14} \;\; \boxed{w_2 = 56}$$

3)

a) $AXB = C$

$X = CA^{-1}B^{-1}$

b) $A(B + X) = A$

$AB + AX = A$

$AX = A - AB \Rightarrow X = \overbrace{A^{-1}A}^{I} - AB$

$X = -AB$ //

c) $ACXB = C$

$CX = A^{-1}CB^{-1}$

$X = C^{-1}A^{-1}CB^{-1}$

d) $(AB)^{-1}(AX) = CC^{-1}$

$B^{-1}\underbrace{A^{-1}A}_{I}X = \underbrace{CC^{-1}}_{I}$

$X = B$ //

e) $AB^{t}XB^{-1} = A^{t}$

$(AB^{t})^{-1}AB^{t}X\underbrace{B^{-1}B}_{I} = (AB^{t})^{-1}A^{t}B$

$X = (AB^{t})^{-1}A^{t}B$ //

f) $2AX - X = 3B$

$(2A - I_m)X = (2A - I_m)^{-1} \cdot 3B$

$\underbrace{(2A - I_m)^{-1}(2A - I_m)}_{I_m}X = (2A - I_m)^{-1} \cdot 3B$

$\therefore X = (2A - I_m)^{-1} \cdot 3B$

4) regra de Cramer

a) $\begin{cases} 3x - 4y = 1 \\ 2x + 6y = 18 \end{cases}$

$\begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} = \begin{vmatrix} 1 \\ 18 \end{vmatrix}$

$\det(A) = 18 + 8 = 26$

$\det(A_x) = \begin{vmatrix} 1 & -4 \\ 18 & 6 \end{vmatrix} = 6 + 72 = 78$

$\det(A_y) = \begin{vmatrix} 3 & 1 \\ 2 & 18 \end{vmatrix} = 54 - 2 = 52$

$x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{78}{26} = 3$

$y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{52}{26} = 2$

$(x, y) = (3, 2)$

b) $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$

$\begin{vmatrix} 5 & 8 \\ 10 & 16 \end{vmatrix} = \begin{vmatrix} 34 \\ 50 \end{vmatrix}$

$\det(A) = 80 - 80 = 0$

$S = \{\varnothing\}$

$\left.\begin{array}{l} \det(A_x) = \dfrac{\det(A_x)}{0} \\ \text{e} \\ \det(A_y) = \dfrac{\det(A_y)}{0} \end{array}\right\} \nexists$

c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$

$$\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = \begin{vmatrix} 5 \\ -4 \end{vmatrix}$$

$\det(A) = -3 - 4 = -7$

$\det(A_x) = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = -15 + 8 = -7$

$\det(A_y) = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4 - 10 = -14$

$x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-7}{-7} = 1$

$y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-14}{-7} = 2$

$(x, y) = (1, 2)$

d) $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$

$$\begin{vmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{vmatrix} \begin{matrix} 3 & 2 \\ 2 & -4 \\ 1 & -2 \end{matrix} = \begin{vmatrix} 8 \\ -4 \\ -4 \end{vmatrix}$$

$\det(A) = 36 - 4 + 20 - (20 + \cancel{12} - \cancel{12})$

$= 36 - 4 + \cancel{20} - \cancel{20} = 32$

$\det(A_x) = \begin{vmatrix} 8 & 2 & -5 \\ -4 & -4 & -2 \\ -4 & -2 & -3 \end{vmatrix} \begin{matrix} 8 & 2 \\ -4 & -4 \\ -4 & -2 \end{matrix}$

$= 96 + 16 - 40 - (-80 - 32 + 24)$

$= 96 + 16 - 40 + 80 + 32 - 24 = 160$

$\det(A_y) = \begin{vmatrix} 3 & 8 & -5 \\ 2 & -4 & -2 \\ 1 & -4 & -3 \end{vmatrix} \begin{matrix} 3 & 8 \\ 2 & -4 \\ 1 & -4 \end{matrix}$

$= 36 - 16 + 40 - (20 + 24 - 48)$

$= 36 - 16 + 40 - 20 - 24 + 48 = 64$

$\det(A_z) = \begin{vmatrix} 3 & 2 & 8 \\ 2 & -4 & -4 \\ 1 & -2 & -4 \end{vmatrix} \begin{matrix} 3 & 2 \\ 2 & -4 \\ 1 & -2 \end{matrix}$

$= 48 - 8 - 32 - (-32 + 24 - 16)$

 $= 48 - 8 - \cancel{32} + \cancel{32} - 24 + 16 = 32$

$\cdot\ x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{160}{32} = 5$

$\cdot\ y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{64}{32} = 2 \qquad z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{32}{32} = 1$

$(x, y, z) = (5, 2, 1)$

e) $\begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$

$$\left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = \left|\begin{array}{c} 2 \\ 9 \\ 3 \end{array}\right|$$

$\cdot \det(A) = 2 + 18 - 6 - (3 + 9 - 8)$

$= 2 + \cancel{18} - \cancel{6} - \cancel{3} - \cancel{9} + 8 = 10$

$\cdot \det(A_x) = \left|\begin{array}{ccc|cc} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right|$

$= 4 + 18 - 27 - (3 + 18 - 36)$

$= 4 + 18 - 27 - 3 - 18 + 36 = 10$

$\cdot \det(A_y) = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right|$

$= -18 + 18 - 6 - (-27 + 9 - 8)$

$= -\cancel{18} + \cancel{18} - 6 + 27 - 9 + 8 = 20$

$\cdot \det(A_z) = \left|\begin{array}{ccc|cc} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{array}\right|$

$= -3 + 54 + 12 - (-6 + 27 + 12)$

$= -3 + 54 + \cancel{12} + 6 - 27 - \cancel{12} = 30$

$\cdot\ x = \dfrac{10}{10} = 1 \qquad \cdot\ y = \dfrac{20}{10} = 2 \qquad \cdot\ z = \dfrac{30}{10} = 3$

$(x, y, z) = (1, 2, 3)$

f) $\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$

$$\left|\begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{array}\right| = \left|\begin{array}{c} -8 \\ -4 \\ 26 \end{array}\right|$$

$det(A) = 20 + 0 - 12 - (-36 + 0 + 0)$

$= 20 + 0 - 12 + 36 = 44$

$\cdot det(A_x) = \left|\begin{array}{ccc|cc} -8 & 0 & 3 & -8 & 0 \\ -4 & -4 & 0 & -4 & -4 \\ 26 & -2 & -5 & 26 & -2 \end{array}\right|$

$= -160 + 0 + 24 - (-312 - 0 - 0)$

$= -160 + 24 + 312 = 176$

$\cdot det(A_y) = \left|\begin{array}{ccc|cc} 1 & -8 & 3 & 1 & -8 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & 26 & 5 & 3 & 26 \end{array}\right|$

$= -20 + 0 + 156 - (-36 - 0 - 80)$

$= -20 + 156 + 36 + 80 = 252$

$\cdot det(A_z) = \left|\begin{array}{ccc|cc} 1 & 0 & -8 & 1 & 0 \\ 2 & -4 & -4 & 2 & -4 \\ 3 & -2 & 26 & 3 & -2 \end{array}\right|$

$= -104 + 0 + 32 - (+96 + 8 - 0)$

$= -104 + 32 - 96 - 8 = -176$

$\cdot x = \frac{176}{44} = 4 \qquad y = \frac{252}{44} = 8 \qquad z = \frac{-176}{44} = -4$

$(x, y, z) = (4, 8, -4)$

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g) $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 3x + 2y + 3z = 10 \end{cases}$

$$\left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 3 & 4 & 6 & 3 & 4 \\ 3 & 2 & 3 & 3 & 2 \end{array}\right| = \left|\begin{array}{c} 10 \\ 23 \\ 10 \end{array}\right|$$

$det(A) = 12 + 36 + 18 - (36 + 12 + 18)$
$= 0$

$\cdot\ x = \dfrac{det(A_x)}{det(A)} = \dfrac{det(A_x)}{0} \quad \cdot\ y = \dfrac{det(A_y)}{det(A)} = \dfrac{det(A_y)}{0} \quad \cdot\ z = \dfrac{det(A_z)}{det(A)} = \dfrac{det(A_z)}{0}$

$\nexists \therefore S = \{\emptyset\}$

5)

a) $\begin{cases} 3x_1 - 4x_2 = 0 \\ -6x_1 + 8x_2 = 0 \end{cases} \cdot (-2) \rightarrow$ SPI, as duas equações representam a mesma reta com infinitos pontos

b) $\begin{cases} x + y + z = 0 \\ 2x + 2y + 4z = 0 \\ x + y + 3z = 0 \end{cases}$ det = 0 ∴ SPD, há apenas uma única solução

$$\left|\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 4 & 2 & 2 \\ 1 & 1 & 3 & 1 & 1 \end{array}\right|$$

$det(A) = 6 + 4 + 2 - 2 - 4 - 6 = 0$

c) $\begin{cases} x + y + 2z = 0 \\ x - y - 3z = 0 \\ x + 4y = 0 \end{cases}$

$$\left|\begin{array}{ccc|cc} 1 & 1 & 2 & 1 & 1 \\ 1 & -1 & -3 & 1 & -1 \\ 1 & 4 & 0 & 1 & 4 \end{array}\right|$$

$det = 0 - 3 + 8 - (-2 - 12 \cdot 0)$
$det = -3 + 8 + 2 + 12 = 17$ ∴ SPD

6)

a) $\begin{cases} 3x + my = 2 \\ x - y = 1 \end{cases}$

$\begin{vmatrix} 3 & m \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} 2 \\ 1 \end{vmatrix}$

$det(A) = -3 - m$

$det \neq 0 \Rightarrow -3 - m \neq 0$

$m \neq -3$ $\therefore$ SPD p/ todos os valores, exceto (-3)

b) $\begin{cases} 3x + 2(m-1)y = 1 \\ mx - 4y = 0 \end{cases}$

$\begin{vmatrix} 3 & 2m-2 \\ m & -4 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$det(A) = -12 - 2m^2 + 2m = 2m + 2m - 12 \neq 0$

$= m^2 - m + 6 \neq 0$

$\Delta = 1 - 24 = -23$ $\therefore$ SPD p/ todos os valores diferentes de 0

c) $\begin{cases} x - y = 2 \\ x + my = -z \\ -x + y - z = 4 \end{cases}$

$\begin{vmatrix} 1 & -1 & 0 \\ 1 & m & 0 \\ -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 2 \\ -z \\ 4 \end{vmatrix}$

$\therefore det \neq 0$

$-m + 1 \neq 0$

$m \neq 1$

$det = (-1) \cdot (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 1 & m \end{vmatrix}$

$\therefore$ SPD p/ todos os valores, exceto 1

$det = \begin{vmatrix} -1 & 1 \\ -1 & m \end{vmatrix} \Rightarrow -m + 1$

d) $\begin{cases} mx + y - z = 4 \\ x + my + z = 0 \\ x - y = 2 \end{cases}$

$$\left|\begin{matrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{matrix}\right|\begin{matrix} m & 1 \\ 1 & m \\ 1 & -1 \end{matrix} = \left|\begin{matrix} 4 \\ 0 \\ 2 \end{matrix}\right|$$

$det = 0 + 1 + 1 - (-m - m + 0)$
$= 2 + 2m$

$det \neq 0$
$2+2m \neq 0$
$2m \neq -2$
$m \neq -1$

$\therefore$ SPD p/ todos os valores, exceto (-1)

7)

· x = peças corretas
· y = peças com defeito

$\begin{matrix} I - \\ II - \end{matrix} \begin{cases} x + y = 225 \Rightarrow y = 225 - x \\ 6x - 2y = 750 \end{cases}$

subst. I em II

$6x - 2(225 - x) = 750$
$6x - 450 + 2x = 750$
$8x = 1200$
$x = 150$ peças corretas

8)

· x = km com carro
· y = km com moto

$\begin{matrix} I - \\ II - \end{matrix} \begin{cases} x + y = 540 \Rightarrow y = 540 - x \\ 0,6x + 0,2y = 300 \end{cases}$

subst I em II

$0,6x + 0,2(540 - x) = 300$
$0,6x + 108 - 0,2x = 300$
$0,4x = 192$
$x = 480$ km c/ carro

$y = 540 - 480$
$y = 60$ km com moto

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9)

- x = cédulas de R$ 2,00
- y = " de R$ 5,00
- z = " de R$ 10,00

$$\begin{cases} x+y+z = 92 & \text{I} \\ x = z & \text{II} \\ 2x+5y+10z = 500 & \text{III} \end{cases}$$

subst. II em I em III

I) $x+y+x = 92$ → III) $2x+5y+10x = 500$

$2x+y = 92$ — $12x+5y = 500$

$y = 92-2x$ — subst.

$12x+5(92-2x) = 500$

$12x+460-10x = 500$

$2x = 40$

$x = 20$

- $y = 92 - 2 \cdot 20$

$y = 52$ cédulas de R$ 5,00

10)

- $k + a = 109$ kg
- $k + t = 142$ kg
- $t + a = 97$ kg

- x = kiba
- y = akamaru
- z = tamaki

$$\begin{cases} x+y = 109 \Rightarrow x = 109-y \\ x+z = 142 \Rightarrow x = 142-z \\ z+y = 97 \end{cases}$$

$109 - y = 142 - z$

$z - y = 33 \Rightarrow z = y+33$

subst na III

$y+33+y = 97$

$2y = 64$

$y = 32$

- $z = y+33 = 65$

$x = 109 - y = 77$

kiba = 77 kg

akamaru = 32 kg

tamaki = 65 kg