

1) la place

$$a) \begin{vmatrix} 1 & 2 \\ -4 & 3 \end{vmatrix}$$

$$\begin{aligned} \det A &= a_{11} \cdot \tilde{a}_{11} + a_{12} \cdot \tilde{a}_{12} \\ &= 1 \cdot (-1)^{1+1} \det(\tilde{A}_{11}) + 2 \cdot (-1)^{1+2} \det(\tilde{A}_{12}) \\ &= 1 \cdot (3) + (-2) \cdot (-4) = 11 \end{aligned}$$

$$b) \begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix}$$

$$\begin{aligned} \det A &= a_{21} \cdot \tilde{a}_{21} + a_{22} \cdot \tilde{a}_{22} \\ &= 2 \cdot (-1)^{2+1} \det(\tilde{A}_{21}) + \sqrt{3} \cdot (-1)^{2+2} \det(\tilde{A}_{22}) \\ &= -2 \cdot 3\sqrt{6} + \sqrt{3} \cdot \sqrt{2} = -6\sqrt{6} + \sqrt{6} = -5\sqrt{6} \end{aligned}$$

$$c) \begin{vmatrix} 1 & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\begin{aligned} \det A &= a_{31} \cdot \tilde{a}_{31} + a_{32} \cdot \tilde{a}_{32} + a_{33} \cdot \tilde{a}_{33} \\ &= 1 \cdot (-1)^{3+1} \det(\tilde{A}_{31}) + 0 \cdot (-1)^{3+2} \det(\tilde{A}_{32}) + 0 \cdot (-1)^{3+3} \det(\tilde{A}_{33}) \\ &= \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = a_{11} \cdot \tilde{a}_{11} \det(\tilde{A}_{11}) + a_{12} \cdot \tilde{a}_{12} \det(\tilde{A}_{12}) \\ &= 0 \cdot (-1)^{1+1} \det(\tilde{A}_{11}) + 2 \cdot (-1)^{1+2} \det(\tilde{A}_{12}) \\ &= 2 \cdot (-1) = -2 \end{aligned}$$

$$d) \begin{vmatrix} -2 & 1 & -11 \\ 1 & 5 & 4 \\ 3 & 4 & 2 \end{vmatrix}$$

$$\begin{aligned} \det A &= a_{11} \cdot \tilde{a}_{11} \det(\tilde{A}_{11}) + a_{12} \cdot \tilde{a}_{12} \det(\tilde{A}_{12}) + a_{13} \cdot \tilde{a}_{13} \det(\tilde{A}_{13}) \\ &= -2 \cdot (-1)^{1+1} \begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1 \cdot (-1)^{1+2} \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} + (-11) \cdot (-1)^{1+3} \begin{vmatrix} 1 & 5 \\ 3 & 4 \end{vmatrix} \\ &= \begin{vmatrix} -10 & -8 \\ -8 & -4 \end{vmatrix} + \begin{vmatrix} -1 & -4 \\ -3 & -2 \end{vmatrix} + \begin{vmatrix} -1 & -5 \\ -3 & -4 \end{vmatrix} \\ &= \begin{vmatrix} -12 & -17 \\ -14 & -10 \end{vmatrix} = -14 \cdot (-11)^{2+1} \det(\tilde{A}_{21}) + (-10) \cdot (-1)^{2+2} \det(\tilde{A}_{22}) \\ &= 14 \cdot (-17) + (-10) \cdot (-12) \\ &= 238 + 120 = 358 \end{aligned}$$



$$e) \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{vmatrix}$$

$$\det A = 0 \cdot \tilde{a}_{11}(\tilde{A}_{11}) + 2 \cdot \tilde{a}_{12}(\tilde{A}_{12}) + 0 \cdot \tilde{a}_{13}(\tilde{A}_{13})$$

$$= 2 \cdot (-1)^{1+2} \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} = -2 \begin{vmatrix} -2 & -10 \\ 4 & -4 \end{vmatrix} = -4 \cdot (-1)^{2+1} \det(A_{11}) + (-4) \cdot (-1)^{2+2} \det(A_{12})$$

$$= 4 \cdot (-10) + (+4) \cdot (-2)$$

$$-40 + 8 = -32$$

$$f) \begin{vmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}$$

$$\det A = 3 \cdot (-1)^{1+1} \cdot (\tilde{A}_{11})$$

$$= 3 \cdot \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 0 & 3 \\ -3 & 3 & 0 \\ 0 & 3 & -3 \end{vmatrix}$$

$$\det A = 0 \cdot (-1)^{1+2} \cdot (\tilde{A}_{12}) + 3 \cdot (-1)^{2+2} \cdot (\tilde{A}_{22}) + 3 \cdot (-1)^{3+2} \cdot (\tilde{A}_{32})$$

$$= 3 \begin{vmatrix} 3 & 3 \\ 0 & -3 \end{vmatrix} + (-3) \begin{vmatrix} 3 & 3 \\ -3 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 9 & 9 \\ 0 & -9 \end{vmatrix} + \begin{vmatrix} -9 & -9 \\ 9 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 \\ 0 & -9 \end{vmatrix} \xrightarrow{D_2} \det A = 0$$

$$g) \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix}$$

$$\det A = 1 \cdot (-1)^{11} \cdot (\tilde{A}_{11})$$

$$= \begin{vmatrix} 2 & 5 & 3 \\ 2 & \sqrt{5} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{vmatrix}$$

$$\det A = 3 \cdot (-1)^{14} \cdot (\tilde{A}_{14})$$

$$= -3 \begin{vmatrix} 2 & \sqrt{5} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -6 & -3\sqrt{5} & 0 \\ 9 & -18 & -3 \\ 9 & 0 & 0 \end{vmatrix}$$

$$\det A = -3 \cdot (-1)^{2+3} \cdot (\tilde{A}_{23})$$

$$= 3 \begin{vmatrix} -6 & -3\sqrt{5} \\ 9 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -18 & -9\sqrt{5} \\ 127 & 0 \end{vmatrix}$$

$$= \det A = 27 \cdot (-1)^{2+1} \cdot (\tilde{A}_{21})$$

$$= -27 \cdot (-9\sqrt{5}) = 243\sqrt{5}$$

$$h) \begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$$

$$\det A = 3 \cdot (-1)^{11} \cdot (\tilde{A}_{11})$$

$$= 3 \begin{vmatrix} 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{vmatrix}$$

na outra  
folha!



$$\det A = \begin{vmatrix} 0 & 0 & 0 & -6 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ -6 & 0 & 0 & 0 \end{vmatrix}$$

$$\det A = 3 \cdot (-1)^{3+3} \cdot (\tilde{A}_{33})$$

$$= 3 \begin{vmatrix} 0 & 0 & -6 \\ 0 & 6 & 0 \\ -6 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -18 \\ 0 & 18 & 0 \\ -18 & 0 & 0 \end{vmatrix}$$

$$\det A = 18 \cdot (-1)^{2+2} \cdot (\tilde{A}_{22})$$

$$= 18 \begin{vmatrix} 0 & -18 \\ -18 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -324 \\ -324 & 0 \end{vmatrix} \Rightarrow \det A = -324 \cdot (-1)^{2+1} \cdot (\tilde{A}_{21})$$
$$= 324 \cdot (-324) \Rightarrow 104,976$$

$$2) A = \begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & 9 & 6 \end{pmatrix}; B = \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix}; C = \begin{pmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{pmatrix}$$

$$a) \det(A+B)$$

$$\det \begin{vmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & 10 & 2 \end{vmatrix}$$

$$\det = 28 - 80 + 420 - 112 - 700 + 12$$
$$= -432$$

$$b) \det(AB)$$

$$\det \begin{vmatrix} 12 & -15 & 49 \\ -4 & 0 & 10 \\ 3 & 9 & -24 \end{vmatrix}$$

$$\det = 0 - 450 - 1764 - 0 - 1080 + 1440$$

$$\text{FORONI: } \det = -1854$$

c)  $\det(B^0 A^0)$

$$\det \begin{pmatrix} 4 & -1 & 3 \\ 3 & 0 & 1 \\ 7 & 2 & -4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 & 1 \\ -5 & 2 & -9 \\ 7 & 8 & 6 \end{pmatrix}$$

$$\det \begin{vmatrix} 12 & -4 & 3 & 12 & -4 \\ -15 & 0 & -9 & -15 & 0 \\ 49 & 16 & -24 & 49 & 16 \end{vmatrix}$$

$$\det = 0 - 1728 - 1440 - 0 - 1764 + 720$$

$$\det = -4212 //$$

d)  $\det(3A - 2C + B)$

$$\begin{pmatrix} 9 & -15 & 21 \\ 12 & 6 & 24 \\ 3 & -27 & 18 \end{pmatrix} - \begin{pmatrix} 4 & 6 & -2 \\ 12 & 18 & -4 \\ 16 & 24 & -6 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix}$$

$$\det \begin{vmatrix} 9 & -18 & 30 & 9 & -18 \\ -1 & -12 & 30 & -1 & -12 \\ -10 & 0 & 20 & -10 & 0 \end{vmatrix}$$

$$\det = -2160 + 5400 - 0 - 52 - 0 - 360$$

$$\det = 2828 //$$

e)  $\det(AC^0)$

$$\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} \begin{pmatrix} 2 & 6 & 8 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{pmatrix}$$

$$\det \begin{vmatrix} 6 & -30 & 56 & 6 & -30 \\ 12 & 18 & 96 & 12 & 18 \\ -1 & 18 & -18 & -1 & 18 \end{vmatrix}$$

$$\det = -1944 + 2880 + 12096 + 1008 + 10368 + 6480$$

$$\det = 30888 //$$



$$3) \det(A) = -2, A_{4 \times 4}$$

$$a) \det(A^0)$$

$$\det(A^0) = \det(A) = -2$$

$$b) \det(6A)$$

$$\cdot \det(cA) = c^m \det(A)$$

$$\therefore \det(6A) = 6^4 \cdot (-2) = -2592$$

$$c) \det(A^7)$$

$$\cdot \det(A^7) = (-2)^7 = -128$$

$$d) \det(A^{-1})$$

$$\cdot \det(A^{-1}) = \frac{1}{\det(A)} = -\frac{1}{2}$$

$$4) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$$

$$a) \begin{vmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{vmatrix} = -3 \cdot 4 = -12$$

$$l_3 = 4 \cdot l_1$$

$$b) \begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = -3 \cdot 3 \cdot (-2) = 18$$

$$l_2 = 3l_1 \quad c_3 = -2c_3$$

$$c) \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \xrightarrow{1^2 \text{ troca}} \begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} \xrightarrow{2^2 \text{ troca}} -3 \cdot (-1) \neq -3$$

$$d) \begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} \xrightarrow{1^2 \text{ troca}} \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \xrightarrow{2^2 \text{ troca}} \det(A) = -3$$

$$e) \begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} \neq 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq \det(A) \cdot 2 \cdot (-3) = -6$$

$$f) \begin{vmatrix} ka+a & kb+b & kc+c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\begin{vmatrix} a(k+1) & b(k+1) & c(k+1) \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$(k+1) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq \det(A) = -3(k+1) = -3k+3$$

5)

$$\det(A) = \begin{vmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 1 \\ 3 & -6 & -30 & 12 \end{vmatrix} \begin{matrix} \rightarrow 2 \\ \rightarrow 2 \\ \\ \rightarrow 3 \end{matrix}$$

$$\det(A) = 12 \cdot \begin{vmatrix} 5 & 4 & 20 & -1 \\ 2 & 3 & 10 & -2 \\ -5 & -7 & -30 & 1 \\ 1 & -2 & -10 & 4 \end{vmatrix}$$

$$= 120 \cdot \begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 11 & -2 & -1 & 4 \end{vmatrix}$$

$$= -1 \cdot \begin{vmatrix} 4 & 2 & -1 \\ 3 & 1 & -2 \\ -7 & 3 & 1 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 5 & 2 & -1 \\ 2 & 1 & -2 \\ -5 & 3 & 1 \end{vmatrix} + \begin{vmatrix} 5 & 4 & -1 \\ 2 & 3 & -2 \\ -5 & -7 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 5 & 4 & 2 \\ 2 & 3 & 1 \\ -5 & -7 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} -4 & -2 & 1 \\ -3 & -1 & 2 \\ 7 & -3 & -1 \end{vmatrix} + \begin{vmatrix} -10 & -4 & 2 \\ -4 & -2 & 4 \\ 10 & -6 & -2 \end{vmatrix} + \begin{vmatrix} 5 & 4 & -1 \\ 2 & 3 & -2 \\ -5 & -7 & 1 \end{vmatrix} + \begin{vmatrix} 20 & 16 & 8 \\ 8 & 12 & 4 \\ -20 & -18 & 12 \end{vmatrix}$$

$$= 120 \cdot \begin{vmatrix} 11 & -18 & 10 \\ 3 & 12 & 8 \\ -8 & -44 & 10 \end{vmatrix} \begin{matrix} 11 & -18 \\ 3 & 12 \\ -8 & -44 \end{matrix}$$

$$\det(A) = 120 \cdot (1320 + 1152 - 1320 + 360 - 3872 + 540) \\ = 120 \cdot (-1220) = -146400$$



6)

$$a) \begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128$$

$$2 \begin{vmatrix} 4 & 3 & x \\ 7 & 2 & 2x \\ 5 & 1 & -x \end{vmatrix} = 128$$

$$2 \cdot (-6x + 30x + 7x - 10x - 8x + 21x) = 128$$

$$68x = -128 \Rightarrow x \approx -1,89$$

$$b) \begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix} = 39$$

$$21x + 60x + 84x - 28x - 54x - 70x = 39$$

$$13x = 39 \Rightarrow x = 3$$

$$c) \begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7$$

$$(x+3) \begin{vmatrix} 5 & 3 \\ 10 & 7 \end{vmatrix} - (x+1) \begin{vmatrix} 4 & 3 \\ 9 & 7 \end{vmatrix} + (x+4) \begin{vmatrix} 4 & 5 \\ 9 & 10 \end{vmatrix}$$

$$(x+3) \cdot (-5) - (x+1) \cdot (-5) + (x+4) \cdot (-5) = -7$$

$$5x + 15 - x - 5 - 5x - 20 = -7$$

$$-x = -1 \Rightarrow x = 1$$



6)  $\rightarrow$  singular  $\therefore \det = 0$

$$d) \begin{vmatrix} x & x+2 \\ 1 & x \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 2+2 \\ 1 & 2 \end{vmatrix} = 0$$

$$x^2 - x - 2 = 0 \quad 4 - 4 = 0$$

$$(-x-2)(-x+0) \quad 0 = 0$$

$$\hookrightarrow x = -2 \quad \hookrightarrow x = 0 \quad \therefore x = 2 //$$

$$2^2 - 2 - 2 = 0$$

$$0 = 0$$

$$e) \begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{vmatrix} \neq 0 \quad \rightarrow \text{é invertível}$$

$$3 \begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 1 & 0 \end{vmatrix} \neq 0$$

$$-3 \begin{vmatrix} x-4 & 3 \\ 2 & x-9 \end{vmatrix} \neq 0$$

$$-3(x^2 - 13x + 36 - 6) \neq 0$$

$$-3x^2 + 39x - 90 \neq 0 : (-3)$$

$$x^2 - 13x + 30 \neq 0 \cdot (-1)$$

$$x^2 - 13x + 30 \neq 0$$

$$(x-3)(x-10) = 0$$

$$\hookrightarrow x = 3 \quad \hookrightarrow x = 10$$

enquanto  $x$  não tiver esses valores, a matriz continuará invertível

$$7) A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

a) matriz da inversa é

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\det(A^{-1}) = (ad - bc) \therefore (1/(ad - bc))$$

cond. existência:  $ad - bc \neq 0$

$$b) A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}; B = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$$

•  $A^{-1}$

$$\det(A^{-1}) = \frac{1}{1} = 1$$

•  $B^{-1}$

$$\det(B^{-1}) = \frac{1}{1} = 1$$

$$\cdot (AB)^{-1} = B^{-1}A^{-1}$$

$$\therefore 1 \cdot 1 \Rightarrow \det(B^{-1}A^{-1}) = 1$$

8) motores capacitores

$$a) A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$$

$$\text{cof}(A) = \begin{pmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot (-2) \\ (-1)^{2+1} \cdot 3 & (-1)^{2+2} \cdot 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 2 \\ -3 & 1 \end{pmatrix}$$

$$b) B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\text{cof}(B) = -1 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}$$

$$\text{cof}(B) = \begin{vmatrix} -2 & 0 \\ -1 & -1 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ -1 & -2 \end{vmatrix}$$

$$\text{cof}(B) = \begin{vmatrix} 0 & 2 \\ 0 & -3 \end{vmatrix} \Rightarrow 0$$



9) matriz adjunta p/ encontrar inversa

$$a) A = \begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix}$$

$$\text{cof}(A) = \begin{vmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot (-2) \\ (-1)^{2+1} \cdot 3 & (-1)^{2+2} \cdot 1 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ -3 & 1 \end{vmatrix}$$

$$\text{adj}(A) = [\text{cof}(A)]^T \Rightarrow \begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix}$$

$$b) B = \begin{vmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$B = -1 \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 0 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix}$$

$$\text{cof}(B) = \begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix}$$

$$\text{adj}(B) = [\text{cof}(B)]^T \Rightarrow \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix}$$

$$c) C = \begin{vmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$C = 1 \cdot \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix}$$

$$\text{cof}(C) = \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix}$$

$$\text{adj}(C) = [\text{cof}(C)]^T = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$$



$$d) D = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{vmatrix}$$

$$D = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 5 \\ -1 & -1 \end{vmatrix}$$

$$\text{cof}(D) = \begin{vmatrix} 4 & -5 \\ 1 & -1 \end{vmatrix} ; \text{adj} = [\text{cof}(B)]^T \Rightarrow \begin{vmatrix} 4 & 1 \\ -5 & -1 \end{vmatrix}$$

10) eq. matricial:  $2A^T = C - XB$  (quadrado e  $3 \times 3$ )

$$a) XB = C - 2A^T$$

$$\underbrace{(XB)}_{\text{Im}} B^{-1} = (C - 2A^T) B^{-1}$$

$$X = (C - 2A^T) B^{-1} \quad \text{impossível} \Rightarrow \det(B) \neq 0$$

$$b) X = \left( \begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -1 & 0 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{pmatrix} \right) \cdot \begin{pmatrix} 3 & -6 & 4 \\ 1 & -3 & 3 \\ -1 & 2 & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & -6 & 4 \\ 1 & -3 & 3 \\ -1 & 2 & -1 \end{pmatrix}$$

$$X = 2IB^{-1} \Rightarrow 2B^{-1}$$

$$X = \begin{pmatrix} 6 & -12 & 8 \\ 2 & -6 & 6 \\ -2 & 4 & -2 \end{pmatrix}$$