1)

a) $A + 2B$

$$\begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix} + \begin{pmatrix} 0 & 10 \\ 6 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 10 \\ 8 & 3 \end{pmatrix}_{//}$$

b) $AB - BA$

$$\begin{pmatrix} 1 & 0 \\ 6 & -14 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 6 & -14 \end{pmatrix} = 0$$

c) $2C - D$

$$\begin{pmatrix} -4 & 6 & -14 \\ 14 & -6 & -4 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix}$$

não há possibilidade de resolução devido as leis de formação das matrizes serem diferentes,

d) $2D^t - 3E^t$

$$\begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix}$$

$$\begin{pmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ -9 & 20 & 7 \end{pmatrix}_{//}$$

e) $D^2 + DE$

$$\begin{pmatrix} 6 & 4 & 0 \\ 1 & 1 & 16 \\ 4 & 0 & 4 \end{pmatrix} + \begin{pmatrix} -6 & 8 & 0 \\ -1 & 0 & -16 \\ 12 & 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 12 & 0 \\ 0 & 1 & 0 \\ 16 & 0 & 2 \end{pmatrix}_{//}$$

f) $C^t A$

$$\begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}$$

matrizes incompatíveis,,

g) $E - AC$ → não há como resolver a multiplicação de matrizes dada sua incompatibilidade

$$\begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}$$

h) $F^t E$

mesmo fazendo a F^t, ainda haverá incompatibilidade entre as matrizes

i) BCF

não há compatibilidade entre as matrizes

2)

a) $A_{2\times3} B_{3\times4} = C_{2\times3}$

$B_{3\times4} A_{2\times3}$ = mão está definido para $q \neq p$

b) $A_{4\times1} B_{1\times2} = C_{1\times1}$

$B_{1\times2} A_{4\times1}$ = mão definido para $q \neq p$

c) $A_{1\times2} B_{3\times1}$ = mão definido, $p \neq q$

$B_{3\times1} A_{1\times2} = C_{1\times1}$

d) $A_{5\times2} B_{2\times3} = C_{2\times2}$

$B_{2\times3} A_{5\times2}$ = mão definido, $p \neq q$

e) $A_{4\times4} B_{3\times3} = p \neq q$

$B_{3\times3} A_{4\times4} = q \neq p$

f) $A_{4\times2} B_{2\times4} = C_{2\times2}$

$B_{2\times4} A_{4\times2} = C_{4\times4}$

g) $A_{2\times1} B_{1\times3} = C_{1\times1}$

$B_{1\times3} A_{2\times1} = q \neq p$

h) $A_{2\times2} B_{2\times2} = C_{2\times2}$

$B_{2\times2} A_{2\times2} = C_{2\times2}$

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h) verificação a partir de valores aleatórios

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 5 & 12 \\ 21 & 24 \end{pmatrix}$$

3)

a) $(a_{ij})_{2\times3}$; $a_{ij} : 3i - 2j$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$$

b) $(b_{ij})_{3\times3}$; $b_{ij} = 3i + j \, (i = j)$; $i^2 - j \, (i \neq j)$

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 11 \end{pmatrix}$$

c) $(c_{ij})_{1\times4}$, $c_{ij} = j^i$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$$

d) $(d_{ij})_{4\times4}$, $d_{ij} = i^2 + j^2 \, (i = j)$; $2ij \, (i \neq j)$

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$$

4) $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix}$ e $B = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & 7 \end{pmatrix}$

a) $[BA]_{23}$

$\begin{pmatrix} 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \Rightarrow 2 - 2 + 20 = 20_{//}$

b) $[AB]_{23}$

$\begin{pmatrix} -2 & -3 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} \Rightarrow -6 - 12 + 14 = -4_{//}$

c) $[B^2]_{31}$

$\left[\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & 7 \end{pmatrix} \right]_{31}$

$\begin{pmatrix} 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \Rightarrow 1 + 2 - 9 = -6_{//}$

d) $tr(A) = a_{11} + a_{22} + a_{33}$

$= 1 - 3 + 5 = 6_{//}$

e) $tr(B^t) = [B^t]_{11} + [B^t]_{22} + [B^t]_{33}$

$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & 7 \end{pmatrix} \Rightarrow 1 - 1 + 7 = 7_{//}$

f) $tr(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33}$

$$\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 4 \\ 0 & -4 & 6 \\ 0 & 3 & 12 \end{pmatrix} \Rightarrow 0 - 4 + 12 = 8_{//}$$

g) $tr(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33}$

$$\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & 7 \end{pmatrix}$$

$\cdot [AB]_{11}$ $\qquad$ $\cdot [AB]_{22}$ $\qquad$ $\cdot [AB]_{33}$

$$(1\ 2\ 1) \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 1 \qquad (-2\ -3\ 2) \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = 2 \qquad (1\ 4\ 5) \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$$

$3 + 16 + 35 = 54_{//}$

$\cdot [AB]_{11} + [AB]_{22} + [AB]_{33} = 1 + 2 + 54 \Rightarrow 57_{//}$

5) $A = \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$; $B = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$; $C = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

a) $2X + A = 3B + C$

$\frac{1}{2} \cdot \ 2X = 3B + C - A$

$X = \frac{1}{2}(3B + C - A)$

$= \frac{1}{2}\left[\begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}\right]$

$= \frac{1}{2}\begin{pmatrix} 7 & -1 \\ 11 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} 7/2 & -1/2 \\ 11/2 & 2 \end{pmatrix}_{//}$

b) $Y + A = \frac{1}{2}(B - C)^t$

$Y = \frac{1}{2}(B - C)^t - A$

$= \frac{1}{2}\left[\begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix}^t - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}\right]$

$= \frac{1}{2}\left[\begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}\right]$

$= \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -3 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{3}{2} & -2 \\ -\frac{3}{2} & -\frac{3}{2} \end{pmatrix}$ //

c) $3X + A = B - X$

$4X = B - A \cdot \frac{1}{4}$

$X = \frac{1}{4}(B - A)$

$X = \frac{1}{4}\begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$ //

d) $\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases}$ +

$2X = 3A + 2B + C \cdot \frac{1}{2}$

$X = \frac{1}{2}(3A + 2B + C)$

$= \frac{1}{2}\left[\begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}\right]$

$= \frac{1}{2}\begin{pmatrix} -1 & 25 \\ 15 & 24 \end{pmatrix} \Rightarrow \begin{pmatrix} -\frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}$ //

6) $A^2 = A \cdot A = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 + 1 & \frac{1}{x} + \frac{1}{x} \\ x + x & 1 + 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

$= 2\begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = 2A$

$* \; A^2 = 2A$

· multiplicando os lados por A

$$A^3 = 2A^2$$

$$A^3 = 4A_{//}$$

· $A^4 = A \cdot A^3 = A \cdot 4A = 4A^2 = 8A$

$\therefore A^m = 2^{m-1} A$, para $m \geq 1$

7) $X = AB$; $Y = AC$ a partir produto AB e AC

a) $A(B+C) = AB + AC \Rightarrow X + Y$

b) $B^t A^t = (AB)^t \Rightarrow (X)^t_{//}$

c) $C^t A^t = (AC)^t \Rightarrow (Y)^t_{//}$

d) $(ABA)C = (XA)C = X(AC) \Rightarrow XY_{//}$

8)

a)
$$\underset{A}{\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}} = \underset{A^t}{\begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}}$$

· $a_{11} \Rightarrow 4 = 4$ · $a_{12} \Rightarrow x+2 = 2x+3$

$$x = -1_{//}$$

substituindo

$$\underset{A}{\begin{pmatrix} 4 & -1 \\ -5 & 0 \end{pmatrix}} = \underset{A^t}{\begin{pmatrix} 4 & -5 \\ -1 & 0 \end{pmatrix}}$$

$\Downarrow$

$$\begin{vmatrix} 4 & -1 \\ -5 & 0 \end{vmatrix} \Rightarrow 4 + 5 = 9$$

$\therefore$

$x = -1$ e $A = 9$

b)

$$\underset{-B}{\begin{pmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{pmatrix}} = \underset{B^t}{\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{pmatrix}}$$

$\cdot\ a_{12} \Rightarrow x = -4$ // $\cdot\ a_{13} \Rightarrow y = 2$ //

$\cdot\ a_{32} \Rightarrow 2z = 1 - z \Rightarrow z = \frac{1}{3}$ //

$\therefore\ \underline{x = -4}\ ;\ \underline{y = 2}\ ;\ \underline{z = \frac{1}{3}}$

9)

$$3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix} = \begin{pmatrix} 4x & 6+x+y \\ -1+z+t & 2t+3 \end{pmatrix}$$

$\cdot\ 3x = 4x$ $\cdot\ 3y = 6 + x + y$

$x = 0$ // $2y = 6 \Rightarrow y = 3$ //

$\cdot\ 3z = -1 - z + t$ $\cdot\ 3t = 2t + 3$

$2z = 2$ $t = 3$ //

$z = 1$ //

$\therefore\ \underline{x = 0}\ ;\ \underline{y = 3}\ ;\ z = 1\ ;\ \underline{t = 3}$

10)

a) $$\underset{A}{\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}} \cdot \underset{A^t}{\begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}} = \underset{I_n}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} (\cos\theta)^2 + (\operatorname{sen}\theta)^2 & -\cos(\theta)\cdot\operatorname{sen}(\theta) + \operatorname{sen}\theta\cdot\cos\theta \\ -\operatorname{sen}(\theta)\cos(\theta) + \cos\theta\cdot\operatorname{sen}\theta & (\operatorname{sen}\theta)^2 + (\cos\theta)^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ //

b) $\quad A \cdot A^t = I_m$

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1/\sqrt{2} & y \\ 0 & 1/\sqrt{2} & z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ x & y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 + x^2 & xy & xz \\ xy & \frac{1}{2} + y^2 & \frac{1}{2} + yz \\ xz & \frac{1}{2} + yz & \frac{1}{2} + z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\cdot\ 1 + x = 1$

$x = 0$

$\cdot\ \frac{1}{2} + y^2 = 1$

$y = \pm\sqrt{\frac{1}{2}}$

$\cdot\ \frac{1}{2} + z^2 = 1$

$z = \pm\sqrt{\frac{1}{2}}$

$\therefore\ x = 0;\ y = \pm\sqrt{\frac{1}{2}};\ z = \pm\sqrt{\frac{1}{2}}$

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