1) a) · vetor diretor de s: $z = 3x$; $z = 3t$ e $y = \frac{3t+16}{2}$

eq. parametrica: $x = \left(t, \frac{3t+16}{2}, 3t\right)$

$\cdot \vec{v_s} = (1, 3/2, 3)$

$\cdot \vec{v_r} = (1/2, 1, 1)$

$\cdot \vec{v_r} \cdot \vec{v_s} = 1/2 \cdot 1 + 1 \cdot 3/2 + 1 \cdot 3 = 5$

$\|v_r\| = \sqrt{(1/2)^2 + 1 + 1} = 3/2$

$\|v_s\| = \sqrt{1 + (3/2)^2 + 9} = \sqrt{49}/2 = 7/2$

$\cdot \cos\alpha = \frac{5}{3/2 \cdot 7/2} = \frac{20}{21}$ $\quad \cdot \operatorname{sen}\alpha = \sqrt{1 - \left(\frac{20}{21}\right)^2} = \frac{\sqrt{41}}{21}$

b) $z = 4$ e $x - y = -3$; se $y = t$ ∴ $x = t - 3$

$\cdot x = (t - 3, t, 4)$

$\cdot \vec{v_s} = (1, 1, 0)$

$\cdot \vec{v_r} = (0, -1, 1)$

$\cdot \vec{v_r} \cdot \vec{v_s} = 0 \cdot 1 + (-1) \cdot 1 + 1 + 1 \cdot 0 = -1$

$\|\vec{v_r}\| = \sqrt{0 + 1 + 1} = \sqrt{2}$

$\|\vec{v_s}\| = \sqrt{1 + 1 + 0} = \sqrt{2}$

$\cdot \cos\alpha = -1/2$ $\quad \cdot \operatorname{sen}\alpha = \sqrt{1 - (-1/2)^2} = \sqrt{3}/2$

c) · para r: se $z = t$ ∴ $x = 7 - 3t$ ⇒ $\vec{v_r} = (-3, 0, 1)$

· para s: " ∴ $x = 5 + 2t$ ⇒ $\vec{v_s} = (2, 0, 1)$

$\vec{v_r} \cdot \vec{v_s} = (-3) \cdot 2 + 0 \cdot 0 + 1 \cdot 1 = -5$

$\|\vec{v_r}\| = \sqrt{9 + 0 + 1} = \sqrt{10}$

$\|\vec{v_s}\| = \sqrt{4 + 0 + 1} = \sqrt{5}$

$\cdot \cos\alpha = \frac{-5}{\sqrt{10}\sqrt{5}} = \frac{-5}{\sqrt{50}} = -\frac{\sqrt{2}}{2}$

d) r: se $x = t$, ∴ $y = 1 - 2t$ e $z = 3t$

$\vec{v_r} = (1, -2, 3)$

$\vec{v_s} = (3, 1, -5) \cdot (1, -2, 3) = (-7, -14, -7) ⇒ (1, 2, 1)$

$\vec{v_r} \cdot \vec{v_s} = 1 \cdot 1 + (-2) \cdot 2 + 3 \cdot 1 = 0$ ∴ $\cos\alpha = 0$

2) $P: (0, 2+\lambda, 0)$; $Q: (1, 2, \mu)$
vetor $\vec{PQ} = (1, -\lambda, \mu)$
r: $\cos 45° = \frac{\vec{PQ} \cdot \vec{v_r}}{\|\vec{PQ}\| \cdot \|\vec{v_r}\|} \Rightarrow \frac{-\lambda}{\sqrt{1+\lambda^2+\mu^2}} = -\frac{\sqrt{2}}{2}$
s: $\cos 60° = \frac{\vec{PQ} \cdot \vec{v_s}}{\|\vec{PQ}\| \cdot \|\vec{v_s}\|} \Rightarrow \frac{\mu}{\sqrt{1+\lambda^2+\mu^2}} = \frac{1}{2}$
dos sistemas: $-\lambda = \frac{\sqrt{2}}{2}\sqrt{1+\lambda^2+\mu^2}$ e $\mu = \frac{1}{2}\sqrt{1+\lambda^2+\mu^2}$

$\lambda = -1$; $\mu = \sqrt{2}$
$\therefore$ $P: (0, 1, 0)$ e $Q: (1, 2, \sqrt{2})$

3) a) r: $\vec{v_r} = (0, 1, 1)$ · ângulo θ: $\sin\theta = \frac{|\vec{v_r} \cdot \vec{n_\pi}|}{\|\vec{v_r}\|} = \frac{1}{\sqrt{2}}$
π: $\vec{n_\pi} = (0, 0, 1)$ · $\theta = \pi/4$

b) · $\vec{v_r} = (-1, -1, 2)$ · $\sin\theta = \frac{|\vec{v_r} \cdot \vec{n_\pi}|}{\|\vec{v_r}\| \|\vec{n_\pi}\|} = \frac{|-2+1|}{\sqrt{6} \cdot \sqrt{5}} = \frac{1}{\sqrt{30}}$
· $\vec{n_\pi} = (2, -1, 0)$
· $\theta = \arcsin \frac{1}{\sqrt{30}}$

c) $\vec{v_r} = (1, 1, -2)$ · $\sin\theta = \frac{|1+1+2|}{\sqrt{6}\sqrt{3}} = \frac{4}{3\sqrt{2}}$
$\vec{n_\pi} = (1, 1, -1)$
· $\theta = \arcsin\left(\frac{4}{3\sqrt{2}}\right)$

4) $\vec{w} = (a, b, c)$
$a + b + c = 0$; $\frac{|a-b|}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$
$\vec{w} = (1, -1, 0)$ $\rightsquigarrow$ normalizado
vetor unitário: $\frac{(1, -1, 0)}{\sqrt{2}}$

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5) a) $\vec{n}_1 = (2,1,-1)$; $\vec{n}_2 = (1,-1,3)$

$\vec{n}_1 \cdot \vec{n}_2 = 2\cdot 1 + 1\cdot(-1) + (-1)\cdot 3 = -2$

$\|\vec{n}_1\| = \sqrt{4+1+1} = \sqrt{6}$, $\|\vec{n}_2\| = \sqrt{1+1+9} = \sqrt{11}$

$\cdot \cos\theta = \dfrac{|\vec{n}_1 \cdot n_2|}{\|\vec{n}_1\|\cdot\|\vec{n}_2\|} = \dfrac{2}{\sqrt{66}}$ $\quad \cdot \theta = \arccos\left(\dfrac{2}{\sqrt{66}}\right)$

b) π_1 : $\vec{u} = (1,0,1)$, $\vec{v} = (-1,0,0)$

$\vec{n}_1 = \vec{u}\cdot\vec{v} = (0,-1,0)$

$\cdot \pi_2$: $\vec{n}_2 = (1,1,1)$

$\cdot \vec{n}_1 \cdot \vec{n}_2 = 0\cdot 1 + (-1)\cdot 1 + 0\cdot 1 = -1$

$\|\vec{n}_1\| = 1$; $\|\vec{n}_2\| = \sqrt{3}$

$\cdot \cos\theta = 1/\sqrt{3}$ $\quad \cdot \theta = \arccos(1/\sqrt{3})$

c) $\cdot \pi_1$: $\vec{u} = (1,0,0)$, $\vec{v} = (1,1,1) \Rightarrow \vec{n}_1 = \vec{u}\cdot\vec{v} = (0,-1,1)$

$\cdot \pi_2$: $\vec{u} = (-1,2,0)$, $\vec{v} = (0,1,0) \Rightarrow \vec{n}_2 = \vec{u}\cdot\vec{v} = (0,0,-1)$

$\vec{n}_1 \cdot \vec{n}_2 = 0\cdot 0 + (-1)\cdot 0 + 1\cdot(-1) = -1$

$\|\vec{n}_1\| = \sqrt{2}$; $\|\vec{n}_2\| = 1$

$\cos\theta = 1/\sqrt{2}$ $\quad \theta = \arccos(1/\sqrt{2}) = \pi/4$

6) $\vec{n}_1 = (2,-1,1)$; $\vec{n}_2 = (1,-2,1)$

$\vec{n}_1 \cdot \vec{n}_2 = 2\cdot 1 + (-1)\cdot(-2) + 1\cdot 1 = 5$

$\|\vec{n}_1\| = \sqrt{6}$; $\|\vec{n}_2\| = \sqrt{6}$

$\cos\theta = 5/6$ $\quad \theta = \arccos(5/6)$

7) a) $x = 1 + t$; $y = t/2$, $z = t \Rightarrow X = (1+t, t/2, t)$

$d(X, A) = d(X, B)$

$\sqrt{t^2 + (t/2 - 1)^2 + t^2} = \sqrt{(1+t)^2 + (t/2 - 1)^2 + (t-1)^2}$

simplificando: $t = 0$ ou $t = -4/3$

pontos: $(1,0,0)$ e $(-1/3, -2/3, -4/3)$

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b) $x = (4a, 2a, 4-3a)$

$$\sqrt{(4a-2)^2+(2a-2)^2+(-1-3a)^2} = \sqrt{(4a)^2+(2a)^2+(3-3a)^2}$$

$a = \frac{1}{2}$ ∴ ponto $(2, 1, \frac{5}{2})$

c) $x = (2+a, 3+a, -3+a)$

$$\sqrt{(1+a)^2+(2+a)^2+(-3+a)^2} = \sqrt{a^2+(1+a)^2+(-7+a)^2}$$

$a = 2$ ∴ ponto $(4, 5, -1)$

8) a) $\vec{v_r} = (3, 2, 1)$

$\vec{AP}$: $A = (1, -2, 0)$; $\vec{AP} = (3, 2, 1)$

$\vec{AP} \cdot \vec{v_r} = (0, 6, -12)$

$\|\vec{AP} \cdot \vec{v_r}\| = \sqrt{0+36+144} = \sqrt{180} = 6\sqrt{5}$

$$d = \frac{6\sqrt{5}}{\sqrt{9+4+1}} = \frac{6\sqrt{5}}{14} = \frac{6\sqrt{70}}{14} = \frac{3\sqrt{70}}{7}$$

b) $x = (2, 0, 1) + a(4, -3, -2)$

$\vec{AP}$: $A = (2, 0, 1)$; $\vec{AP} = (-1, -1, 3)$

$\vec{AP} \cdot \vec{v_r} = (-8, -10, -1)$

$\|\vec{AP} \cdot \vec{v_r}\| = \sqrt{64+100+1} = \sqrt{165}$

$$d = \frac{\sqrt{165}}{\sqrt{16+9+4}} = \frac{\sqrt{165}}{\sqrt{29}}$$

c) $x = t$ ∴ $y = t + 3/2$, $z = t + 1/2$

$x = (t, t+3/2, t+1/2)$

$\vec{v_r} = (1, \frac{1}{2}, \frac{1}{2})$

ponto A em r, p/ $t = 0$, $A = (0, 3/2, 1/2)$

$\vec{AP} = (0, -\frac{5}{2}, -\frac{1}{2})$

$\vec{AP} \cdot \vec{v_r} = (-\frac{1}{2}, \frac{1}{2}, 5/4)$

$\|\vec{AP} \cdot \vec{v_r}\| = \sqrt{1/4 + 1/4 + 25/16} = \sqrt{33/16} = \sqrt{33}/4$

$$d = \frac{\sqrt{33}/4}{\sqrt{1+1/4+1/4}} = \frac{\sqrt{33}/4}{\sqrt{3/2}} = \frac{\sqrt{33}}{4} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{66}}{4\sqrt{3}} = \frac{\sqrt{22}}{4}$$

10) a) $\vec{m_r} = (1,0,0) \cdot (-1,0,3) = (0,-3,0)$

eq. geral $= -3y = 0$

$d = \frac{|-3 \cdot 3|}{\sqrt{0+9+0}} = \frac{9}{3} = 3$

b) $d = \frac{|0-0+12-6|}{\sqrt{1+4+4}} = \frac{6}{3} = 2$

c) $d = \frac{|2-1+2-3|}{\sqrt{4+1+4}} = \frac{0}{3} = 0$

11) se $x = t$, $\therefore$ $y = 2-t$; $z = t-2 \Rightarrow x = (t, 2-t, t-2)$

$d = \frac{|(2-t) - 2t - (2-2t) - 1|}{\sqrt{1^2+(-2)^2+(-1)^2}} = \frac{|2-t-2t-2+2t-1|}{\sqrt{6}} = \frac{|-t-1|}{\sqrt{6}}$

• $\frac{|-t-1|}{\sqrt{6}} = \sqrt{6} \Rightarrow |-t-1| = 6 \Rightarrow |t+1| = 6$

I $t+1 = 6 \Rightarrow t = 5$

II $t+1 = -6 \Rightarrow t = -7$

I) $P = (2-5, 5, 2-2\cdot 5)$

$P = (-3, 5, -8)$

II) $(2+7, -7, 2-2(-7))$

$P = (9, -7, 16)$

12) a) $x = t$ $\therefore$ $y = 2t-1$; $z = -3t+1 \Rightarrow x = (t, 2t-1, -3t+1)$

$\vec{w_r} = (1,-1,1)$; $\vec{w_s} = (1,2,-3)$

$\vec{w_r} \cdot \vec{w_s} = (1,4,3)$

$\vec{AB} = (-2,0,1)$

$d = \frac{|-2+0+3|}{\sqrt{1+16+9}} = \frac{1}{\sqrt{26}}$

b) $\vec{w_r} = (3,4,-2)$; $\vec{w_s} = (6,-4,-1)$

$\vec{w_r} \cdot \vec{w_s} = (-12,-9,-36)$

$\vec{AB} = (25,-5,7)$

$d = \frac{|25(-12)+(-5)(-9)+7(-36)|}{\sqrt{144+81+1296}} = \frac{543}{\sqrt{1521}} = \frac{543}{39}$

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c) $x = (1 - 2t, t/2, t)$

$\vec{v_r} = (-2, 1/2, 1)$; $\vec{v_s} = (-4, 1, 2)$

$\vec{v_r} \cdot \vec{v_s} = (0,0,0)$

13) a) $\vec{n_\pi} = (1,0,0) \cdot (0,1,0) = (0,0,1)$

eq. geral: $z = 9$

$\vec{v_r} \cdot \vec{n_\pi} = (3,3,3) \cdot (0,0,1) = 3 \neq 0$

b) $z = t$ ∴ $x = 4/3$; $y = 4/3 + t \Rightarrow x = (4/3, 4/3 + t, t)$

$\vec{v_r} = (0,1,1)$; $\vec{n_\pi} = (0,1,-1)$

$\vec{v_r} \cdot \vec{n_\pi} = 0 \cdot 0 + 1 \cdot 1 + 1 \cdot (-1) = 0$

$d = \frac{|4/3 - 0 - 4|}{\sqrt{0 + 1 + 1}} = \frac{8/3}{\sqrt{2}} = \frac{4\sqrt{2}}{3}$

c) $x = (t, t+1, t-3)$

$\vec{v_r} = (1,1,1)$, $\vec{n_\pi} = (2,1,-3)$

$\vec{v_r} \cdot \vec{n_\pi} = 2 + 1 - 3 = 0$

$d = \frac{|0 + 1 + 9 - 10|}{\sqrt{4 + 1 + 9}} = \frac{0}{\sqrt{14}} = 0$

14) a) $\vec{n_1} = (2,-1,2)$, $\vec{n_2} = (4,-2,4) = 2\vec{n_1}$

$d = \frac{|0 + 0 + 0 - 21|}{\sqrt{16 + 4 + 16}} = \frac{21}{6} = \frac{7}{2}$

b) $\vec{n_2} = (-1,0,3) \cdot (1,1,0) = (-3,3,-1)$

eq. geral: $-3x + 3y - z + 1 = 0$

$\vec{n_1} = (2,2,2)$, $\vec{n_2} = (-3,3,-1) \not\Rightarrow \vec{n_1} = k\vec{n_2}$

c) $\vec{n_1} = (1,1,1)$, $\vec{n_2} = (2,1,1) \not\Rightarrow$ não paralelos

15) $r: (t, -4, 5-t)$

$\pi: \vec{w}_1 = (1, 0, -1)$, $\vec{v}_2 = (4, 2, -3)$

$\vec{n} = \vec{w}_1 \cdot \vec{w}_2 = (2, -1, 2)$

eq. geral $\pi: 2x - y + 2z + k = 0$

$(4, 1, 1) = 8 - 1 + 2 + k = 0 \therefore k = -9$

$\pi: 2x - y + 2z - 9 = 0$

paralelo a π: $2x - y + 2z + C = 0$

$d = \dfrac{|2 \cdot 4 - 1 + 2 \cdot 1 + C|}{\sqrt{4+1+4}} = 2 \Rightarrow \dfrac{|9 + C|}{3} = 2$ $C = -3$ ou $C = -15$

planos:

$2x - y + 2z - 3 = 0$ e $2x - y + 2z - 15 = 0$.