How do we perform regression on multiple input dimensions?

Ex.

| $x_1$ (Quiz 1) | $x_2$ (Quiz 2) | $x_3$ (Midterm) | $y$ (final) |
|---|---|---|---|
| 73 | 80 | 75 | 152 |
| 93 | 88 | 93 | 185 |
| 89 | 91 | 90 | 180 |
| 96 | 98 | 100 | 196 |
| 73 | 66 | 70 | 142 |

Hypothesis:

- $H(x) = Wx + b$ single
- $H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$ multi

Cost function:

- $Cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} (H(x_i) - y_i)^2$
- $Cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} (H(x_{i1}, x_{i2}, x_{i3}) - y_i)^2$
    - $x_{in}$ refers to $x_n$ of $i^{th}$ entry
    - Notice that cost is still a function w/ parameters $W, b$ no matter the dimension of the input.

How do we deal w/ super large input dimensions?

- $H(x_1, x_2, x_3, \dots, x_n) = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n + b$

Solution: Matrices!

Recall:

$$(x_1 \quad x_2 \quad x_3) \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = x_1 w_1 + x_2 w_2 + x_3 w_3$$

$$H(X) = XW \quad (\text{Hypothesis of } X \text{ is } XW)$$

☆ Each row of $x$ input is called an instance.

So every matrix $(x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_n)$ is an "instance".

Note:

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

☆ The instances can all be put in one matrix and the hypotheses can be all returned in one matrix

Note on Hypotheses:

- Theory: $H(x) = Wx + b$
- Practice (TensorFlow): $XW$ ← No $+b$

we can add the $b$ as a column matrix later