Marra Assignment

November 25, 2019

[1]:
```
import numpy as np
import numpy.random as rand
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.datasets import make_blobs, make_moons, make_circles
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import OneHotEncoder
from numpy.linalg import norm
```

[2]:
```
np.random.seed(7)
```

1 Perceptrons and Back propagation

2 Submit to moodle by the 25th November - Please submit a PDF version of your notebook

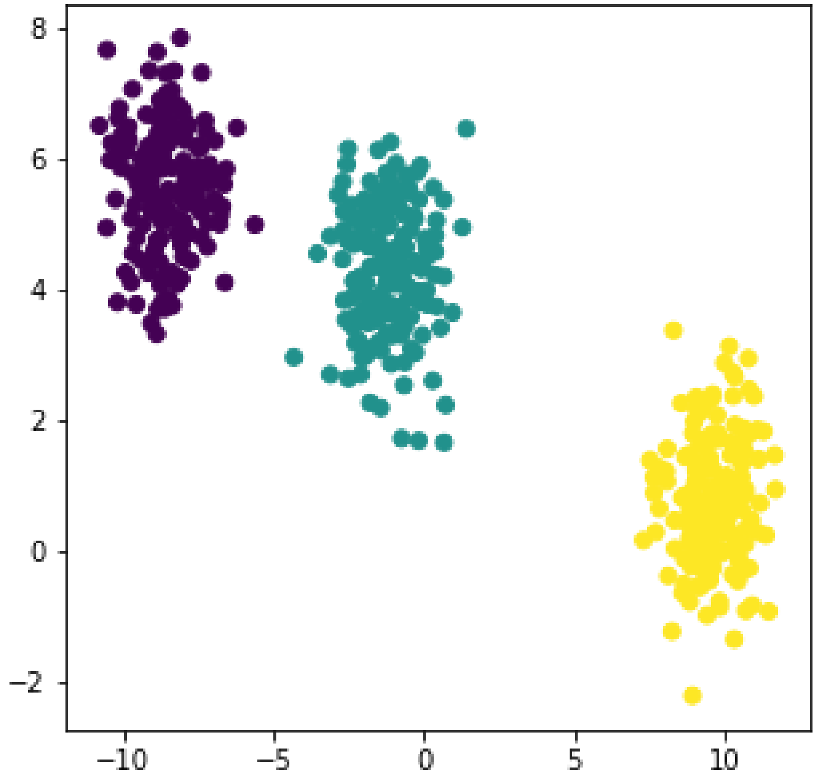
2.1 One layer perceptron - Linear separation

We start by generating *blobs*. In this setting data is linearly separable, we can thus use a single layer perceptron.

$$\hat{y} = W^\top x + b$$
$$\hat{P}(y_j = 1) = Softmax(\hat{y})_j$$

With $W \in M_{N,C}(\mathbb{R})$ and b a vector of size C. N, C the dimension of the input and the number of classes.

[3]:
```
X, Y = make_blobs(n_samples=500, n_features=2)
plt.figure(figsize=(5,5))
plt.scatter(X[:,0], X[:,1], c=Y)
plt.show()
Y = OneHotEncoder(categories='auto').fit_transform(Y.reshape(-1,1)).toarray()
```




```
[4]: X_train, X_test, Y_train, Y_test = train_test_split(X, Y, random_state=42,␣
     ↪test_size=0.4)
```

2.1.1 Task 1: Implement softmax and the forward pass of a single layer percepton.

```
[5]: def softmax(x):
         """
         Takes the output of a layer as input and returns a probability distribution
         input:
             x (np.array)

         returns:
             x (np.array)
         """
         exp = np.exp(x)
         return exp/np.sum(exp)

     def forward_one_layer(W, b, x):
         """
         Computes the forward pass of a single layer perceptron
         input:
```

```
        W (np.array): (INPUT_SHAPE, N_CLASSES) The weight matrix of the␣
↪perceptron
        b (np.array): (N_CLASSES, 1) The bias matrix of the perceptron
        x (np.array): (INPUT_SHAPE, 1) The input of the perceptron

    returns:
        (np.array) (C, 1)
    """
    return softmax(W.T @ x + b)
```

The loss typically associated with a classification problem is the cross entropy loss:

$$loss = -\log(\hat{P}(y_j = 1))$$

Question 1 (2 points): Derive the gradients of loss with respect to W and b.

$$\nabla_W loss = ?$$

$$\nabla_b loss = ?$$

To calculate the gradient we start by applying the chain rule on the simplified form of the loss function, which depends only on y:

$$\begin{aligned}
\frac{\partial loss}{\partial W_{i,k}} &= \frac{\partial loss}{\partial y_k}\frac{\partial y_k}{\partial W_{i,k}} = \frac{\partial loss}{\partial y_k} x_i \\
\frac{\partial loss}{\partial b_k} &= \frac{\partial loss}{\partial y_k}\frac{\partial y_k}{\partial b_k} = \frac{\partial loss}{\partial y_k}
\end{aligned}$$

Simplifying the loss function:

$$\begin{aligned}
loss &= -\log\left(\hat{P}(y_j = 1)\right) \\
&= -\log\left(Softmax(\hat{y})_j\right) \\
&= -\log\left(\frac{e^{y_j}}{\sum_{k=1}^{C} e^{y_k}}\right) \\
&= \log\left(\sum_{k=1}^{C} e^{y_k}\right) - \log e^{y_j} \\
&= \log\left(\sum_{k=1}^{C} e^{y_k}\right) - y_j
\end{aligned}$$

where $y_k = W^T_{(k,\cdot)}x + b_k = \sum_{i=1}^{C} w_{i,k}x_i + b_k$.

Then: - For $k \neq j$:

$$\begin{aligned}
\frac{\partial loss}{\partial W_{i,k}} &= \frac{e^{y_k}}{\sum_{p=1}^{C} e^{y_p}} x_i \\
&= Softmax(\hat{y})_k x_i
\end{aligned}$$

$$\begin{aligned}
\frac{\partial loss}{\partial b_k} &= \frac{e^{y_k}}{\sum_{p=1}^{C} e^{y_p}} \\
&= Softmax(\hat{y})_k
\end{aligned}$$

- For $k = j$:

$$\begin{aligned}\frac{\partial loss}{\partial W_{i,j}} &= (\frac{e^{y_j}}{\sum_{p=1}^{C} e^{y_p}} - 1)x_i \\ &= (Softmax(\hat{y})_j - 1)\, x_i \\ \frac{\partial loss}{\partial b_j} &= \frac{e^{y_j}}{\sum_{p=1}^{C} e^{y_p}} - 1 \\ &= Softmax(\hat{y})_j - 1\end{aligned}$$

Our final result for the gradient is:

$$\begin{aligned}\nabla_W loss &= \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \left(\begin{bmatrix} Softmax(\hat{y})_1 & \cdots & Softmax(\hat{y})_C \end{bmatrix} - \begin{bmatrix} 0 & \cdots & 0 & 1_{jth} & 0 & \cdots \end{bmatrix} \right) \\ &= x\,(Softmax(\hat{y}) - y)^T\end{aligned}$$

$$\begin{aligned}\nabla_b loss &= \begin{bmatrix} Softmax(\hat{y})_1 \\ \vdots \\ Softmax(\hat{y})_C \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1_{jth} \\ 0 \\ \vdots \end{bmatrix} \\ &= Softmax(\hat{y}) - y\end{aligned}$$

2.1.2 Task 2: Implement the gradients

[6]:
```python
def compute_grads_one_layer(softmaxed, x, y):
    """
    inputs:
        softmaxed (np.array): (N_CLASSES, 1)
        y (np.array): (N_CLASSES, 1)
        x (np.array): (INPUT_SHAPE, 1)

    returns:
        d_W (np.array): (INPUT_SHAPE, N_CLASSES) Gradient of the loss with␣
↪respect to the weight matrix
        d_b (np.array): (N_CLASSES, 1) Gradient of the loss with respect to the␣
↪bias matrix
    """
    d_b = softmaxed - y
    d_W = x.reshape(-1,1) @ d_b.reshape(1,-1)

    return  d_W, d_b

def compute_loss(softmaxed, y):
    """
    inputs:
```

```
        softmaxed (np.array): (N_CLASSES, 1)
        y (np.array): (N_CLASSES, 1)

    returns:
        (float)
    """
    return float(-np.log( max(softmaxed.T @ y, 1e-10) ))
```

2.1.3 Question 2 (1 points): As a sanity check, we want to compare the gradients we calculated to approximated gradients. How could we do this ?

Answer One can estimate the gradient of W using an approximation of the formal definition of the derivative.

$$(\nabla_W loss)_{i,j} = \lim_{h \to 0} \frac{loss(W_{i,j} + h) - loss(W_{i,j})}{h}$$

2.1.4 Task 3: Implement the approx gradient function for the weight matrix

[7]:
```
def approx_grad_W(W, b, x, y, h=0.0001):
    """
    Approximates the gradient with respect to W
    input:
        W (np.array): (INPUT_SHAPE, N_CLASSES) The weight matrix of the␣
↪perceptron
        b (np.array): (N_CLASSES, 1) The bias matrix of the perceptron
        x (np.array): (INPUT_SHAPE, 1) The input of the perceptron
        x (np.array): (N_CLASSES, 1) The input of the perceptron
        h (float): variation

    returns:
        d_W_approx (np.array): (INPUT_SHAPE, N_CLASSES)
    """
    n_features, n_classes = W.shape
    d_W_approx = np.zeros((n_features, n_classes))
    previous_softmaxed = forward_one_layer(W,b,x)

    for i in range (n_features):
        for j in range (n_classes):


            W[i][j]+=h
            new_softmaxed = forward_one_layer(W,b,x)

            d_W_approx[i][j] = ( compute_loss(new_softmaxed, y) -␣
↪compute_loss(previous_softmaxed, y) )/h
            W[i][j]-=h
```

```
    return d_W_approx
```

The following function trains the perceptron

[8]:
```
def train_one_layer(X_train, Y_train, X_test, Y_test, lr,
                    n_it=1000, test_freq=10, random_seed=42):

    INPUT_SHAPE = X_train.shape[1]
    N_CLASSES = Y_train.shape[1]

    # Initialise metrics lists
    loss = []
    acc = []
    test_acc = []
    approx_grads = []
    grads = []

    # Initialisation of the weigths
    np.random.seed(random_seed)
    b = rand.normal(size=(N_CLASSES, 1))
    W = rand.normal(size=(INPUT_SHAPE, N_CLASSES))

    # Shuffling data
    indexes = rand.randint(X_train.shape[0], size=n_it)
    # training loop
    for it, i in enumerate(indexes):
        x = X_train[i,:].reshape(-1,1)
        y = Y_train[i,:].reshape(-1,1)

        # Forward passs
        softmaxed = forward_one_layer(W, b, x)
        # Back probagaiton
        d_W, d_b = compute_grads_one_layer(softmaxed, x, y)
        W -= lr * d_W
        b -= lr * d_b

        # Recording approximate gradients
        grads.append(d_W)
        approx_grads.append(approx_grad_W(W, b, x, y))

        # Metrics recording
        loss.append(compute_loss(softmaxed, y))
        acc.append(np.argmax(softmaxed) == np.argmax(y))
```

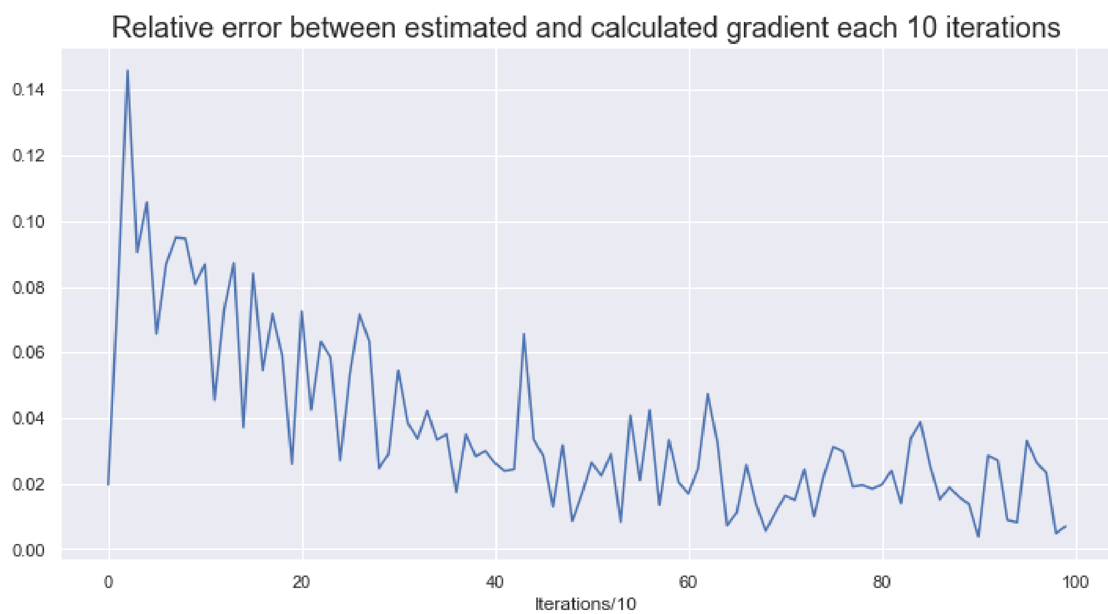
```
        # Test loop
        if it % test_freq == 0:
            acc_temp = []
            for i in range(X_test.shape[0]):
                x = X_train[i,:].reshape(-1,1)
                y = Y_train[i,:].reshape(-1,1)
                softmaxed = forward_one_layer(W, b, x)
                acc_temp.append(np.argmax(softmaxed) == np.argmax(y))

            test_acc.append(np.mean(acc_temp))

    return W, b, loss, acc, test_acc, np.stack(grads, -1), np.
 ↪stack(approx_grads, -1)
```

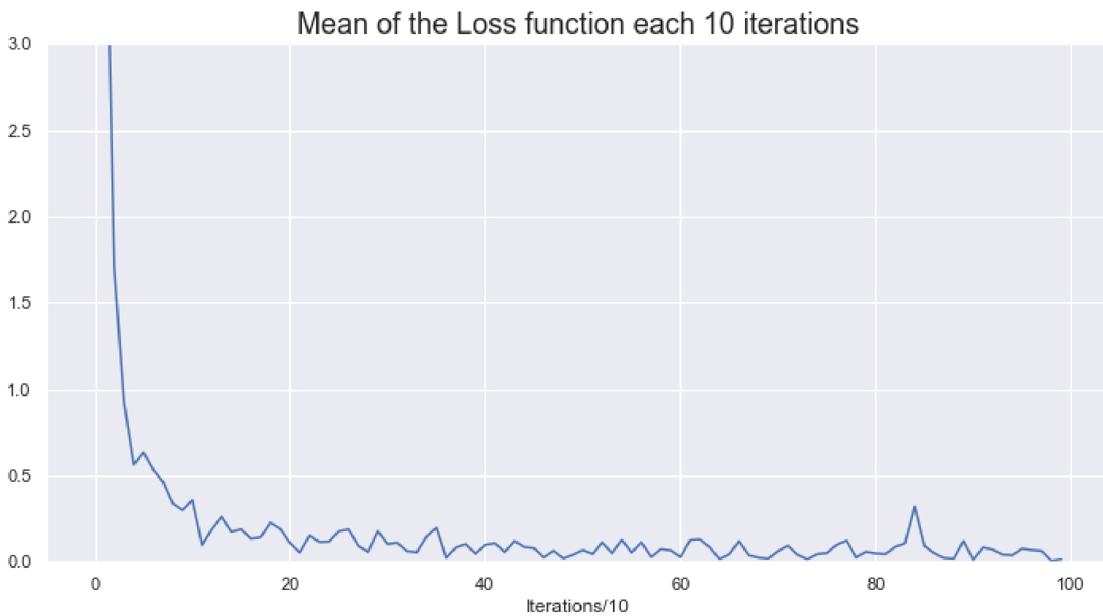
2.1.5 Task 4 (4 points): Using the previous function train the model. By producing an appropriate set of plots validate that it trained correctly and that you computed the correct gradients. Remember that a plot should be self explanatory.

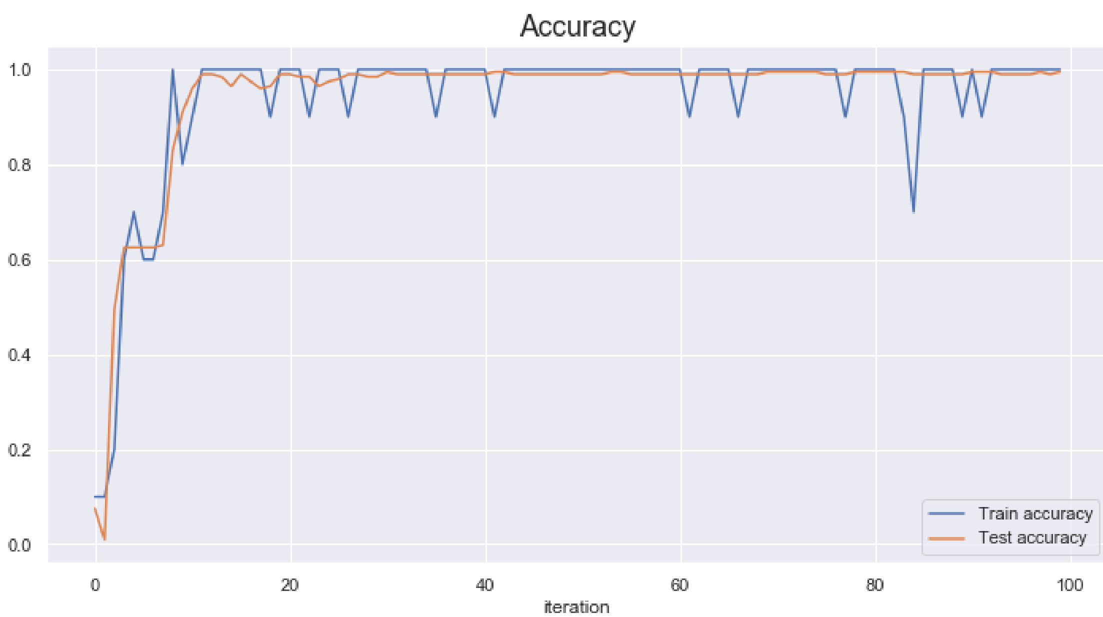
[9]:
```
W, b, loss, acc, test_acc, grads, approx_grads = train_one_layer(X_train,␣
 ↪Y_train, X_test, Y_test, 0.01)
```

[10]:
```
sns.set()
n_iterations = len(acc)



#Function that returns the mean of 10 itens within a list
get_mean_list = lambda myList, i: ( np.sum([myList[i:i+10]]) / 10)



#GRADS DIFF RELATIVE ERROR
get_relative_error = lambda i: (norm(grads[:,:,i] - approx_grads[:,:,i]) /␣
 ↪(norm(grads[:,:,i]) + norm(approx_grads[:,:,i])))

gradsRelativeError = [get_relative_error(i) for i in range(n_iterations)]
gradsRelativeError = [get_mean_list(gradsRelativeError,i) for i in range(0,␣
 ↪n_iterations, 10)]



plt.figure(figsize = (12,6))
plt.plot(gradsRelativeError)
plt.title("Relative error between estimated and calculated gradient each 10␣
 ↪iterations", fontsize=18)
plt.xlabel("Iterations/10")
plt.show()
```

```python
#LOSS
Loss = [get_mean_list(loss,i) for i in range(0, n_iterations, 10)]

plt.figure(figsize = (12,6))
plt.plot(Loss)
plt.ylim(0,3)
plt.title("Mean of the Loss function each 10 iterations", fontsize=18)
plt.xlabel("Iterations/10")
plt.show()


#ACCURACY
# Training data accuracy
Acc = [get_mean_list(acc,i) for i in range(0, n_iterations, 10)]

plt.figure(figsize = (12,6))
plt.plot(Acc, label="Train accuracy")
plt.plot(test_acc, label="Test accuracy")
plt.title("Accuracy", fontsize=18)
plt.xlabel("iteration")
plt.legend()
plt.show()
```







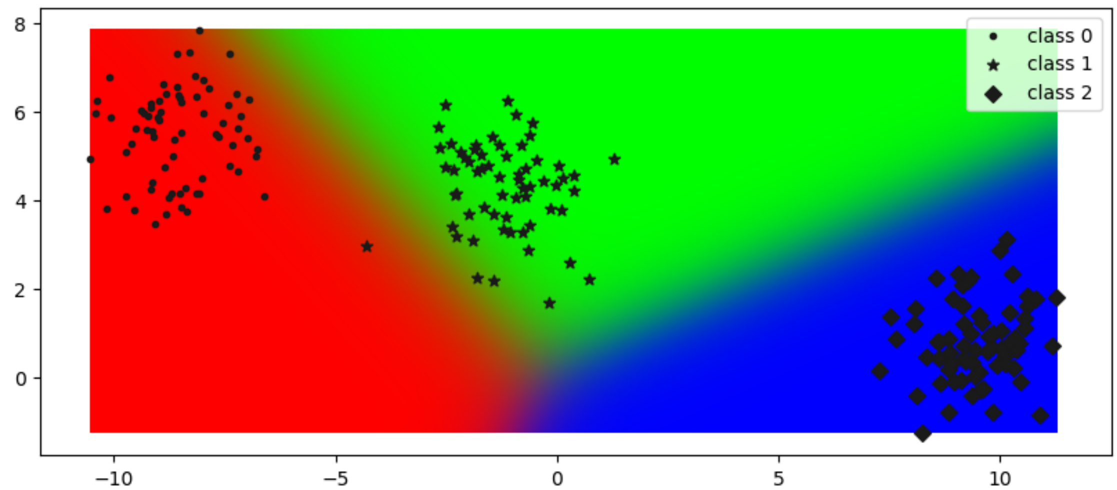
[ ]:
```
sns.reset_orig()
```

3 Non linearity

Now that the perceptron is train we can visualize its decusion funciton.

[12]:
```python
def plot_decision(X, Y, forward, figure=None):
    """Plots the decision function of a perceptron with respect to a forward␣
↪funciton

    input:
        X, Y (np.array): Test data
        forward (function): only accepts x as input (Ex: lambda x:␣
↪forward_one_layer(W, b, x))
        figure (plt.figure): optional, usefull if you dont want to generate any␣
↪new figure,
                        in the case of suplots.
    """
    markers=[".", "*", "D"]
    low0, high0 = np.min(X[:,0]), np.max(X[:,0])
    low1, high1 = np.min(X[:,1]), np.max(X[:,1])
    data = np.zeros((100,100,Y.shape[1]))
    for i1, x1 in enumerate(np.linspace(low0,high0,100)):
        for i2, x2 in enumerate(np.linspace(low1,high1,100)):
            x = np.array([x1, x2]).reshape(-1, 1)
            softmaxed = forward(x)
            data[i2, i1, :] = softmaxed.reshape(-1)
    if Y.shape[1] < 3:
        data = data[:,:,0]

    if figure is None:
        plt.figure(figsize=(10,10))

    plt.imshow(data, extent=(low0,high0,low1,high1), origin='lower',␣
↪interpolation='gaussian')
    for c in range(Y.shape[1]):
        plt.scatter(X[np.argmax(Y, 1) == c, 0], X[np.argmax(Y, 1) == c, 1],␣
↪c='k',
                    marker=markers[c], label="class %i" % c)

    plt.legend()
```

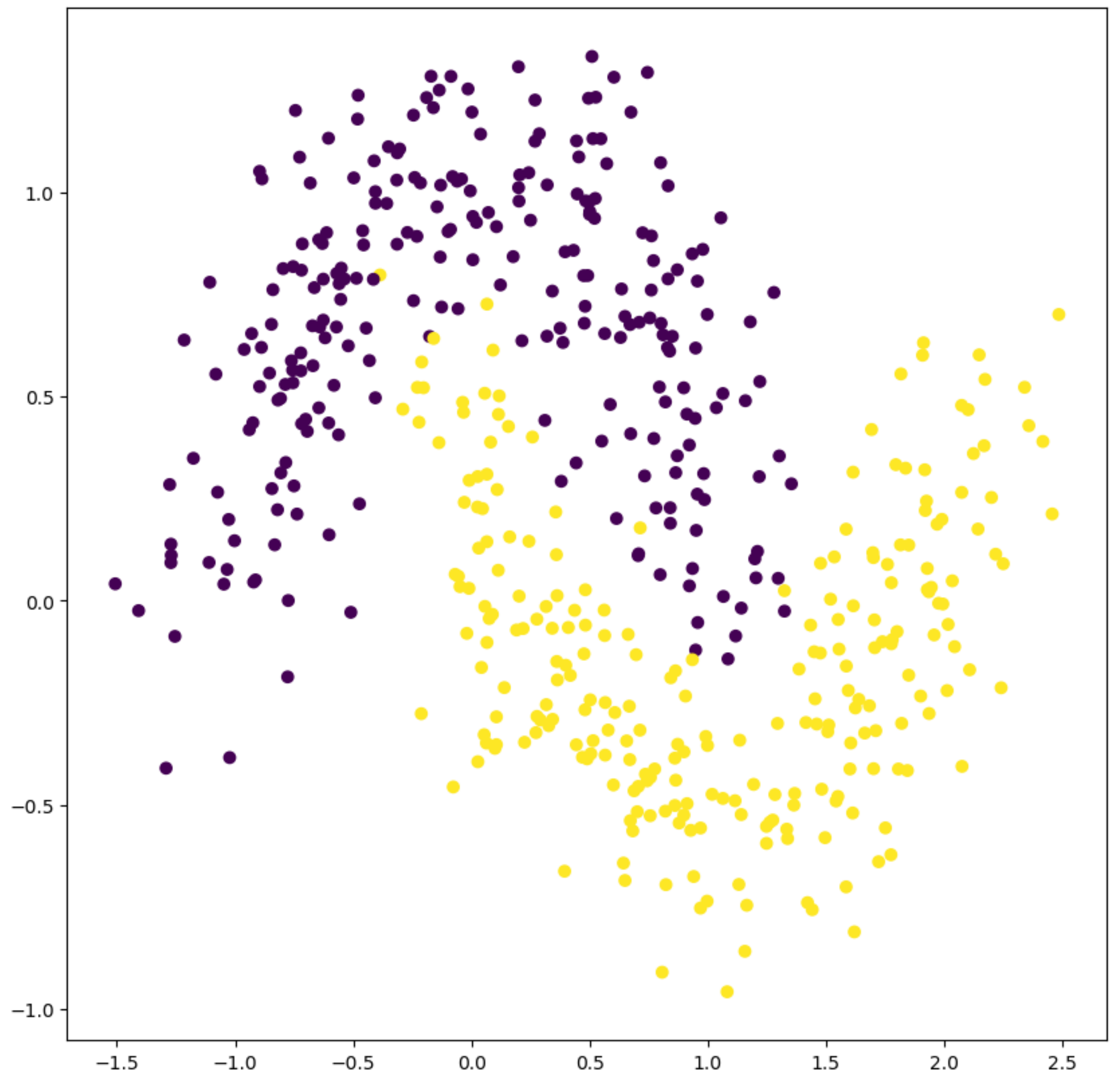
[13]:
```python
plot_decision(X_test, Y_test, lambda x: forward_one_layer(W, b, x))
```



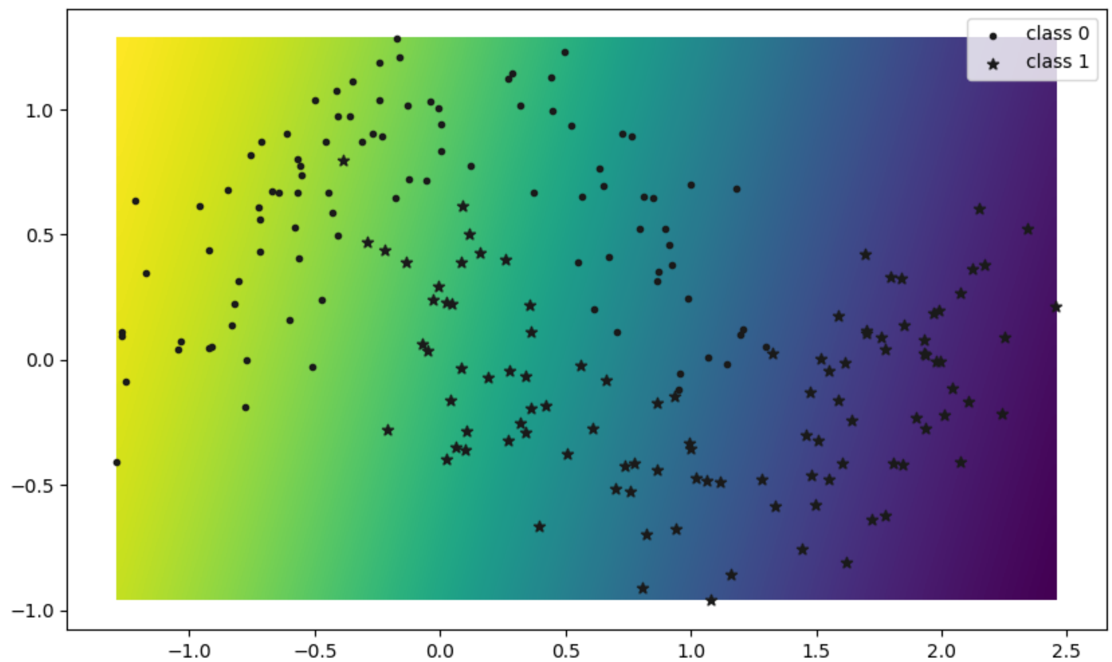
The data we used was linearly separable, but what if it is not ?

3.0.1 Task 5 (1 point): Using the following data train a new perceptron and visualize its decision function.

[14]:
```
X, Y = make_moons(n_samples=500, noise=.2)
plt.figure(figsize=(10,10))
plt.scatter(X[:,0], X[:,1], c=Y)
plt.show()
Y = OneHotEncoder(categories='auto').fit_transform(Y.reshape(-1,1)).toarray()

X_train, X_test, Y_train, Y_test = train_test_split(X, Y, random_state=42,␣
↪test_size=0.4)
```



[15]:
```
W, b, loss, acc, test_acc, grads, approx_grads = train_one_layer(X_train,␣
 ↪Y_train, X_test, Y_test, 0.001)
plot_decision(X_test, Y_test, lambda x: forward_one_layer(W, b, x))
```



4 Two Layer perceptron - Feed forward neural network

In neural networks, non linearity comes from having at least one hidden layer and a non linear activation function such as the $ReLU$ function.

$$h = ReLU({W^h}^\top x + b^h)$$

$$\hat{y} = {W^o}^\top h + b^o$$

$$\hat{P}(y_j = 1) = Softmax(\hat{y})_j$$

With $W^h \in M_{N,H}(\mathbb{R})$ and $W^o \in M_{C,H}(\mathbb{R})$ and b^h, b^o vectors of corresponding dimensions. H is the number of hidden units.

The $ReLU$ funciton is defined as:

$$ReLU(x) = \begin{cases} 0 \text{ if } x < 0 \\ x \text{ otherwise} \end{cases}$$

4.0.1 Question 3 (2 points): Derive the gradients with respect to W_h and b_h.

The gradient for the weights and the bias of the output layer is the same as before, however with the input of the hidden activations **h**:

$$\nabla_{W_o} loss = h\left(Softmax(\hat{y}) - y\right)^T$$
$$\nabla_{h_o} loss = \left(Softmax(\hat{y}) - y\right)^T$$

In the following step we have to consider the gradient of the loss function with respect to the hidden layers and the derivative of the ReLU function:

$$\nabla_h loss = W_o \left(Softmax(\hat{y}) - y\right)$$
$$\frac{d}{dx} ReLU(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

Computing therefore the gradient of the loss with respect of the hidden layer before activation, wich we call out:

$$\nabla_{out} loss = \left(W_o \left(Softmax(\hat{y}) - y\right)\right) * \frac{dReLU(h)}{dh}$$

Where * denotes element-wise product. With the same process of the chain rule, we have the last two gradients:

$$\begin{aligned} \nabla_{W_h} loss &= x(\nabla_{out} loss)^T \\ \nabla_{h_h} loss &= \nabla_{out} loss \end{aligned}$$

4.0.2 Task 5 (5 points): Based on the previous implementation complete the following functions and train a 2 layers perceptron.

[16]:
```
def relu(x):
    """
    input:
        x (np.array)

    returns:
        x (np.array)
    """
    return (x>0)*x


def d_relu(x):
    """Computes the derivative of the relu
    input:
        x (np.array)

    returns:
        x (np.array)
    """
    return (x>0)*1

def forward_two_layers(Wo, bo, Wh, bh, x):
    """Forward pass of a teo layer perceptron with relu activation
    input:
```

```
        Wh (np.array): (INPUT_SHAPE, HIDDEN_SHAPE) The weight matrix of the␣
↪hidden layer
        Wo (np.array): (HIDDEN_SHAPE, N_CLASSES) The weight matrix of the␣
↪output layer
        bh (np.array): (HIDDEN_SHAPE, 1) The bias matrix of the hidden layer
        bo (np.array): (N_CLASSES, 1) The bias matrix of the output layer
        x (np.array): (INPUT_SHAPE, 1) The input of the perceptron

    returns:
        softmaxed (np.array): (N_CLASSES, 1) the output of the network after␣
↪final activation
        hidden (np.array): (HIDDENT_SHAPE, 1) the output of the hidden layer␣
↪after activation
        out (np.array): (N_CLASSES, 1) the output of the network before final␣
↪activation
    """
    hidden = relu( Wh.T @ x + bh )
    out = Wo.T @ hidden + bo
    softmaxed = softmax(out)


    return softmaxed, hidden, out

def compute_grads_two_layers(hidden, softmaxed, Wo, x, y):
    """Forward pass of a teo layer perceptron with relu activation
    input:
        hidden (np.array): (HIDDENT_SHAPE, 1) the output of the hidden layer␣
↪after activation
        softmaxed (np.array): (N_CLASSES, 1) the output of the network after␣
↪final activation
        Wo (np.array): (HIDDEN_SHAPE, N_CLASSES) The weight matrix of the␣
↪output layer
        x (np.array): (INPUT_SHAPE, 1) The input of the perceptron
        y (np.array): (N_CLASSES, 1) Ground truth class

    returns:
        d_Wo (np.array): (HIDDEN_SHAPE, N_CLASSES) Gradient with respect
                        to the weight matrix of the output layer
        d_bo (np.array): (N_CLASSES, 1) Gradient with respect to the bias␣
↪matrix of the output layer
        d_Wh (np.array): (INPUT_SHAPE, HIDDEN_SHAPE) Gradient with respect to␣
↪the
                        weight matrix of the hidden layer
        d_bh (np.array): (HIDDEN_SHAPE, 1) Gradient with respect to the bias␣
↪matrix of the hidden layer
    """
```

```
    d_Wo, d_bo = compute_grads_one_layer(softmaxed, hidden, y)

    d_bh = (Wo @ (softmaxed-y)) * d_relu(hidden)
    d_Wh = x @ d_bh.reshape(1,-1)

    return d_Wo, d_bo, d_Wh, d_bh
```

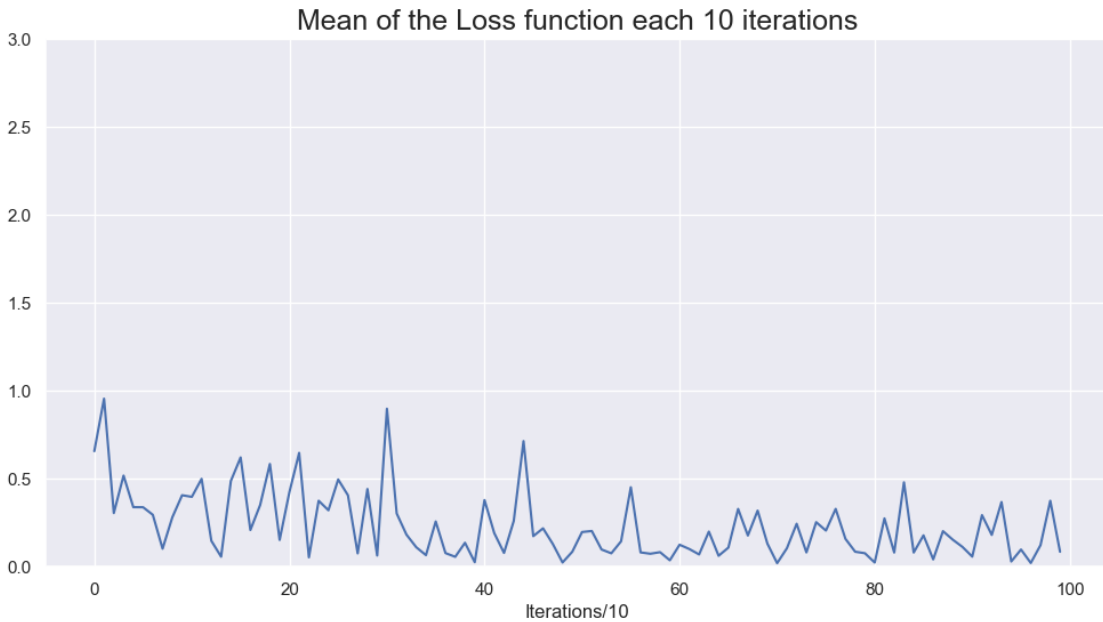
[17]:
```
def train_two_layer(X_train, Y_train, X_test, Y_test, lr, n_hidden,
                    n_it=1000, test_freq=10, random_seed=42):



    INPUT_SHAPE = X_train.shape[1]
    N_CLASSES = Y_train.shape[1]



    # Initialise metrics lists
    loss = []
    acc = []
    test_acc = []



    # Initialisation of the weigths
    np.random.seed(random_seed)
    bh = rand.normal(size=(n_hidden, 1))
    Wh = rand.normal(size=(INPUT_SHAPE, n_hidden))
    bo = rand.normal(size=(N_CLASSES, 1))
    Wo = rand.normal(size=(n_hidden, N_CLASSES))

    # Shuffling data
    indexes = rand.randint(X_train.shape[0], size=n_it)

    # training loop
    for it, i in enumerate(indexes): # 'i' represents iteration and 'it'␣
↪represents x vector
        x = X_train[i,:].reshape(-1,1)
        y = Y_train[i,:].reshape(-1,1)

        # Forward passs
        softmaxed, hidden, out = forward_two_layers(Wo, bo, Wh, bh, x)

        # Back probagaiton
        d_Wo, d_bo, d_Wh, d_bh = compute_grads_two_layers(hidden, softmaxed,␣
↪Wo, x, y)

        Wo -= lr * d_Wo
        bo -= lr * d_bo
        Wh -= lr * d_Wh
```

```
        bh -= lr * d_bh


        # Metrics recording
        loss.append(compute_loss(softmaxed, y))
        acc.append(np.argmax(softmaxed) == np.argmax(y))

        # Test loop
        if it % test_freq == 0:
            acc_temp = []
            for i in range(X_test.shape[0]):
                x = X_train[i,:].reshape(-1,1)
                y = Y_train[i,:].reshape(-1,1)
                softmaxed, _, _ = forward_two_layers(Wo, bo, Wh, bh, x)
                acc_temp.append(np.argmax(softmaxed) == np.argmax(y))

            test_acc.append(np.mean(acc_temp))




    return Wo, bo, Wh, bh, loss, acc, test_acc
```

[18]:
```
Wo, bo, Wh, bh, loss, acc, test_acc = train_two_layer(X_train, Y_train, X_test,␣
 ↪Y_test, 0.1, 16)
```

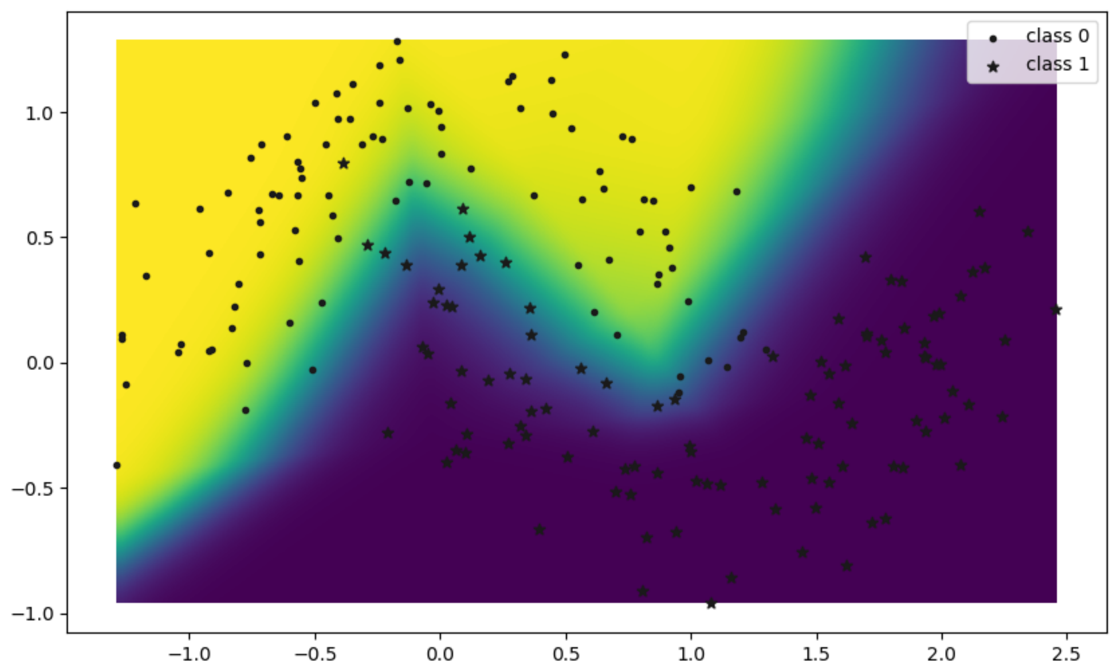
[19]:
```
sns.set()
n_iterations = len(acc)


#Function that returns the mean of 10 itens within a list
get_mean_list = lambda myList, i: ( np.sum([myList[i:i+10]]) / 10)


#LOSS
Loss = [get_mean_list(loss,i) for i in range(0, n_iterations, 10)]

plt.figure(figsize = (12,6))
plt.plot(Loss)
plt.ylim(0,3)
plt.title("Mean of the Loss function each 10 iterations", fontsize=18)
plt.xlabel("Iterations/10")
plt.show()


#ACCURACY
# Training data accuracy
```

```
Acc = [get_mean_list(acc,i) for i in range(0, n_iterations, 10)]

plt.figure(figsize = (12,6))
plt.plot(Acc, label="Train accuracy")
plt.plot(test_acc, label="Test accuracy")
plt.title("Accuracy", fontsize=18)
plt.ylim(0.2,1.05)
plt.xlabel("iteration")
plt.legend()
plt.show()
```



[ ]:
```
sns.reset_orig()
```

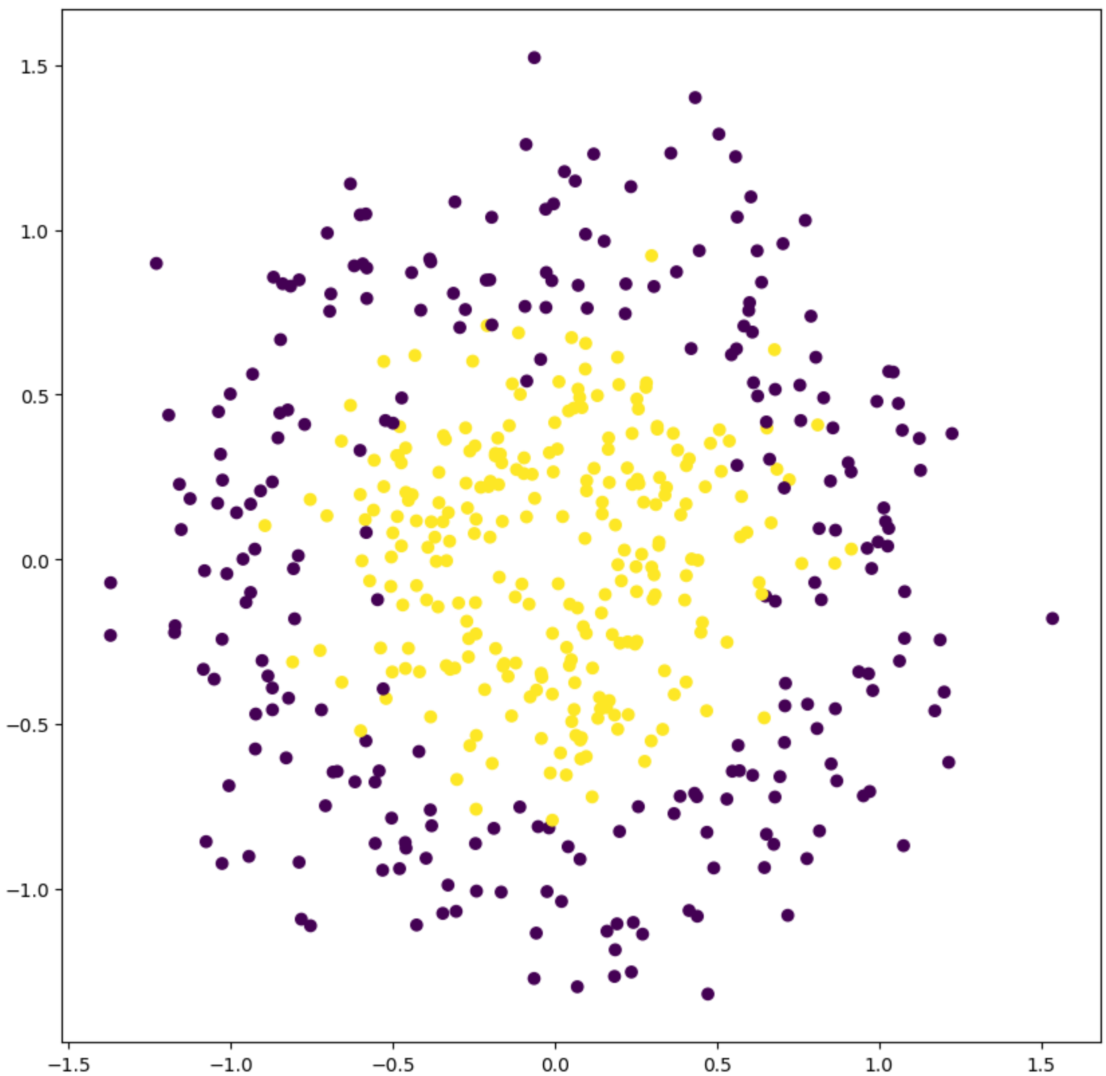
[21]:
```
plot_decision(X_test, Y_test, lambda x: forward_two_layers(Wo, bo, Wh, bh,␣
↪x)[0])
```



4.0.3 Task 7 (3 points): The non linearity of the decision function is conditionned by the number of units, visualize this effect. What can you comment on the smoothness of the boundary ? (Be carefull, to visualize this you need to properly train the networks)

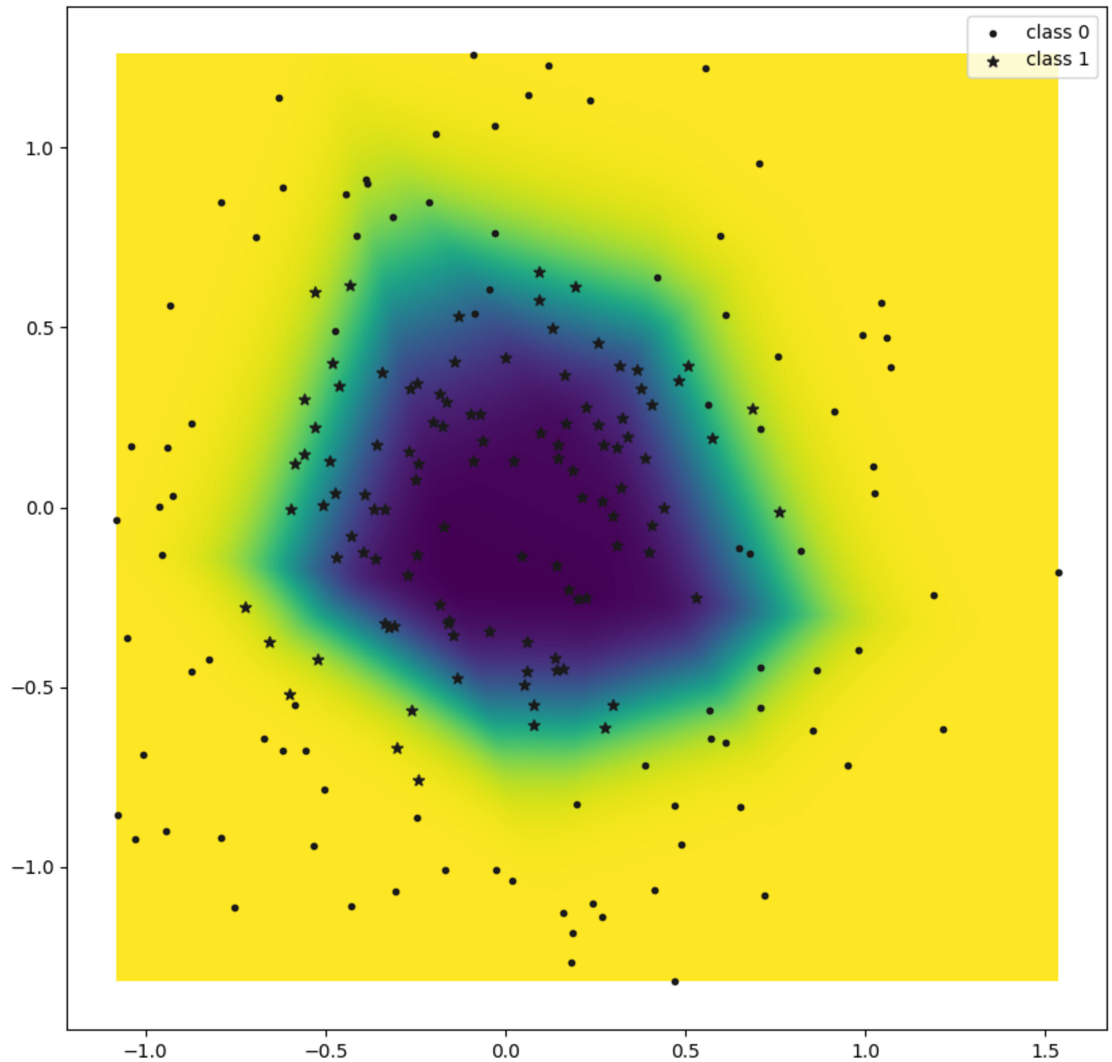
To better visualize this let's generate new data.

[22]:
```
X, Y = make_circles(n_samples=500, noise=.2, factor=0.4)
plt.figure(figsize=(10,10))
plt.scatter(X[:,0], X[:,1], c=Y)
plt.show()
Y = OneHotEncoder(categories='auto').fit_transform(Y.reshape(-1,1)).toarray()

X_train, X_test, Y_train, Y_test = train_test_split(X, Y, random_state=42,␣
↪test_size=0.4)
```



[23]:
```
Wo, bo, Wh, bh, loss, acc, test_acc = train_two_layer(X_train, Y_train, X_test,␣
↪Y_test, 0.1, 16)
```

[24]:
```
plot_decision(X_test, Y_test, lambda x: forward_two_layers(Wo, bo, Wh, bh,␣
↪x)[0])
```



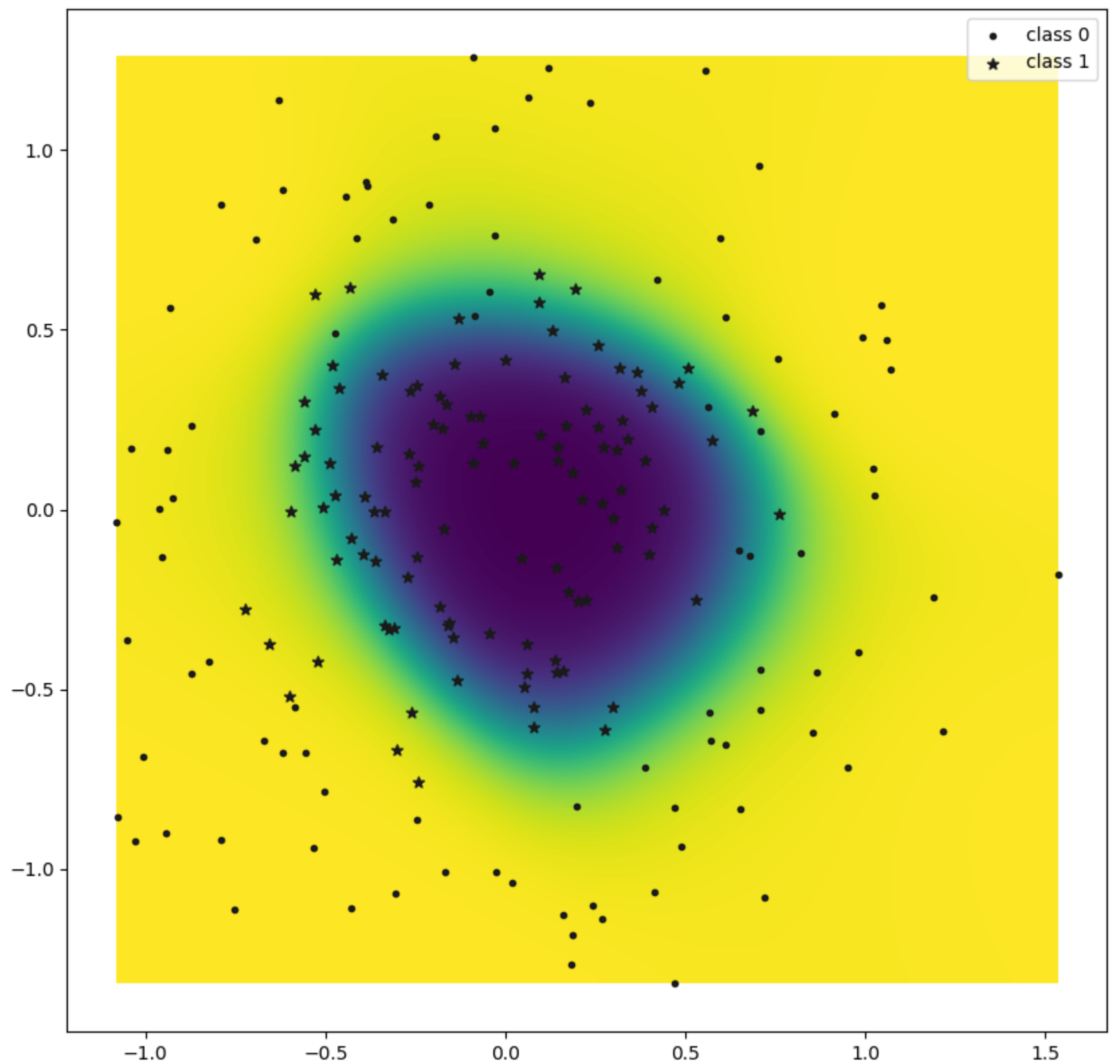
4.0.4 Task 8 (2 points): To have a smooth boundary we need to change the activation. Change the activation from relu to tanh, and vizualize the result.

[25]:
```
def forward_two_layers(Wo, bo, Wh, bh, x):
    """Change this funciton to uses tanh
    """
    hidden = np.tanh(Wh.T @ x + bh)
    out = Wo.T @ hidden + bo
```

```
    softmaxed = softmax(out)

    return softmaxed, hidden, out

def compute_grads_two_layers(hidden, softmaxed, Wo, x, y):
    """Change this funciton to uses tanh
    """
    d_bo = softmaxed - y
    d_Wo = hidden @ d_bo.T

    d_bh = (Wo @ d_bo) * (1 - (hidden ** 2))
    d_Wh = x @ d_bh.T

    return d_Wo, d_bo, d_Wh, d_bh
```

[26]:
```
Wo, bo, Wh, bh, loss, acc, test_acc = train_two_layer(X_train, Y_train, X_test,␣
 ↪Y_test, 0.1, 16)
```

[27]:
```
plot_decision(X_test, Y_test, lambda x: forward_two_layers(Wo, bo, Wh, bh,␣
 ↪x)[0])
```



class 0
class 1
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5