Non-Clairvoyant Dynamic Mechanism Design: Experimental Evidence

Shan Gui, Daniel Houser

George Mason University

July, 2023 AMES

Main Takeaways

What do we do?

- Bring Non-Clairvoyant Environment (Mirrokni et al., 2020) into Lab.
- Test the optimal Non-Clairvoyant dynamic mechanism (NC).
- Compare the performance with Repeated Static optimal mechanism (RS).

What do we find?

- Dynamic NC works well as theory predicts - more revenue.
- Participants overbid less in NC.
- Risk aversion $\rightarrow$ No full participation $\rightarrow$ Revenue loss in NC.

Optimal Dynamic Mechanism Design

- How the principal (seller) establish the rules of allocation and price over **multi-period** as the agent (buyer) receives private information over time.
  - Long-term principal-agent relationship
  - **Repeated selling of perishable goods**
- Dynamic mechanism improves revenues and efficiency (Baron & Besanko, 1984).

Non-Clairvoyance

Clairvoyance: Future demand distribution is known at the beginning.

- Form of the optimal dynamic mechanism depends on environment.
- Buyers tend to have biased forecast on future demand (DellaVigna & Malmendier, 2006).

Non-Clairvoyance: Future demand is not accessible at the beginning.

- No needs to share unbiased belief.
- General Form.

F_2 is unknown in Day 1

$v_1 \sim F_1$

$v_2 \sim F_2$?

Mechanisms under Non-Clairvoyant Environment

$\Rightarrow$ Optimal clairvoyant revenue Rev^* is unachieveble.

RS: Repeated Static optimal mechanism (Myerson, 1981)

- Rules in two days are independent of each other

  Maximize **intra-period revenue** for each period separately.

  $\Rightarrow \frac{Rev^{RS*}}{Rev^*}$ could be arbitrarily small (Papadimitriou et al., 2016)

NC: Non-Clairvoyant optimal dynamic mechanism (Mirrokni et al., 2020)

- Rules in Day 2 depends on bid in day 1

  Best Revenue Guarantee: $\Rightarrow \frac{Rev^{NC*}}{Rev^*} \geq \frac{1}{a}$

  Achieve at least $\frac{1}{2}$ revenue produced by optimal clairvoyant mechanism under all scenarios in **two-period single-buyer** case.

When can Non-Clairvoyant dynamic mechanism do better?

Theoretically, Non-Clairvoyant mechanism can not always outperform.

	Optimal Intra-period Revenue	**Optimal Inter-period Revenue**
RS	100%	zero
NC	$\geq 50\%$	$\geq 50\%$

Relative size of optimal intra- and inter-period revenues is the key.

- Scenario A: Optimal inter - period revenue is larger $\Rightarrow$ NC outperforms.
- Scenario B: Optimal intra - period revenue is larger $\Rightarrow$ RS outperforms.

Experimental Design $2 * 2$

Two Mechanisms

- Non-Clairvoyant Dynamic Mechanism (NC)
- Repeated Static Mechanism (RS)

Two Scenarios

$F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\}, \mathbb{E}_A = 3.$
$F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\}, \mathbb{E}_B = 6.$

- Scenario A (S_A): NC generates more revenue than RS.

$$F_1 = F_A,\ F_2 = F_B \quad \Rightarrow REV^{RS} = 4,\ REV^{NC} = 4.5\ \uparrow 12.5\%$$

- Scenario B (S_B): NC generates less revenue than RS.

$$F_1 = F_B,\ F_2 = F_A \quad \Rightarrow REV^{RS} = 4,\ REV^{NC} = 3.5\ \downarrow 12.5\%$$

Non-Clairvoyant Environment

- **Buyer**: Participant
- **Robot Seller**: Experimenter, $c = 0$
- **Two periods** : Buyer can buy one item in each period from seller, $t = 1, 2$.
- **Non-clairvoyance** : The distribution of buyer's value (F_t) is common knowledge **only in that period**
- **Incomplete Information** :
  1. **Only buyer** knows his value for the item in each period, v_t.
  2. Buyer's value is drawn **independently**.

Variables for Environment

- **Endowment**: $E = 50$

Mechanism - Repeated Static (RS)

Period 1

- Seller sets a reserve price r_1 based on the distributional knowledge F_1.
- Buyer learns his value (v_1), makes a bid : b_1
- Buyer can get the item only when $b_1 \geq r_1$ and pay $p_1 = r_1$.

Period 2

- $F_2 \Rightarrow r_2$, $v_2 \Rightarrow b_2$, pays $p_2 = r_2$ if $b_2 \geq r_2$

Myerson's Auction

monopoly price: $r_1 = r_2 = 2$

$r_A = 2 \in \{arg\max_r \ r \cdot P(v_A > r)\}, \quad r_B = 2 \in \{arg\max_r \ r \cdot P(v_B > r)\}$

Mechanism - Non-Clairvoyant Dynamic (NC)

How the dynamic mechanism work?

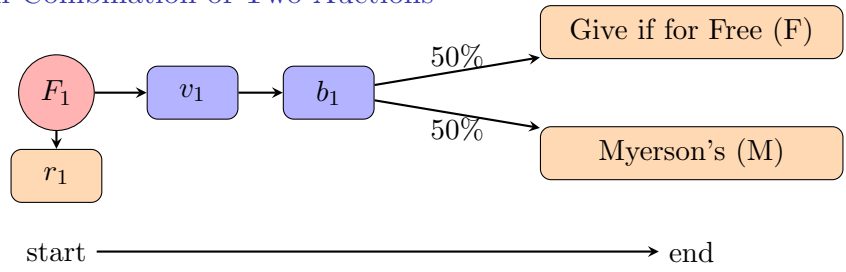
Half chance of free item in period 1

Half chance of upfront fee in period 2

Non-Clairvoyant Mechanism in Period 1

- Seller sets a fixed reserve price r_1 based on the distribution F_1.
- Buyer learns his value (v_1), makes a bid : b_1
- Buyer has 50% chance to get the item for free: $p_1 = 0$;
  Otherwise, buyer can get the item only when $b_1 \geq r_1$ and pay $p_1 = r_1$.

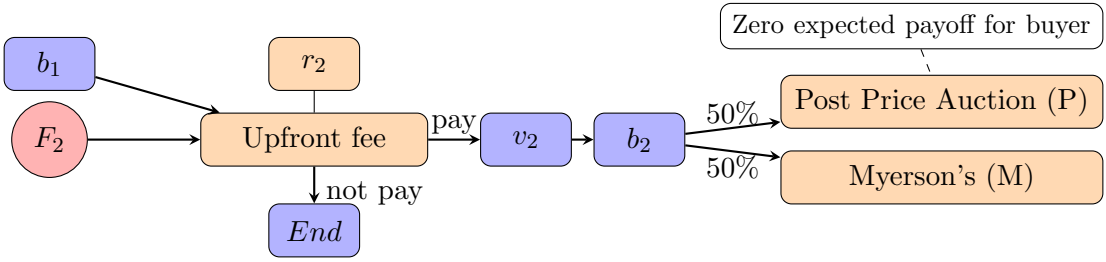
Uniform Combination of Two Auctions



Non-Clairvoyant Mechanism in Period 2

- Seller sets an upfront fee $s_2 = \min(b_1, E(v_2))$.
- Buyer decides pay or leave. If buyer leave ($enter = 0$), game over.
- If buyer pays, ($enter = 1$),
  - Buyer learns his value, v_2, and makes a bid: b_2
  - Buyer has 50% chance to get refund on the upfront fee ($luck = 1$).
  - Seller sets two reserve prices (r_2) based on the $F_2, luck$ for each given m_2, Buyer can get the item only when $b_2 \geq r_2$ and pay $p_2 = r_2$

Uniform Combination of Two Auctions



Hypotheses

Hypothesis 1 - On Revenue Comparison

- In Scenario A (S_A), the non-clairvoyant mechanism (NC) generates greater revenue than the repeated static mechanism (RS);
- In S_B, NC generates less revenue than RS.

Hypothesis 2 - On Individual Rationality

- Some buyers choose not to pay the upfront fee, such that the experimental revenue of the non-clairvoyant mechanism is less than its theoretical prediction.

Hypothesis 3 - On Incentive Compatibility

- Participants' bids are closer to true value under NC than RS.

Experiments

- 256 George Mason Students. September to November 2021.

Treatment	Scenario A		Scenario B	
	Non-Clairvoyant	Repeated Static	NC	RS
Age	21.6	22.3	21.9	22.7
Gender (Male=1)	0.48	0.44	0.52	0.47
Risk aversion	4.46	4.90	4.55	4.63
Observation	64	64	64	64

Table 1: Summary Statistic

Results

Result 1.

Experimental observations match theoretical predictions.

- In S_A, the non-clairvoyant dynamic mechanism gains more revenue than the repeated static mechanism.
- In S_B, the non-clairvoyant dynamic mechanism gains less revenue than the repeated static mechanism. mechanism.

Experimental Revenue Comparison - Period 1

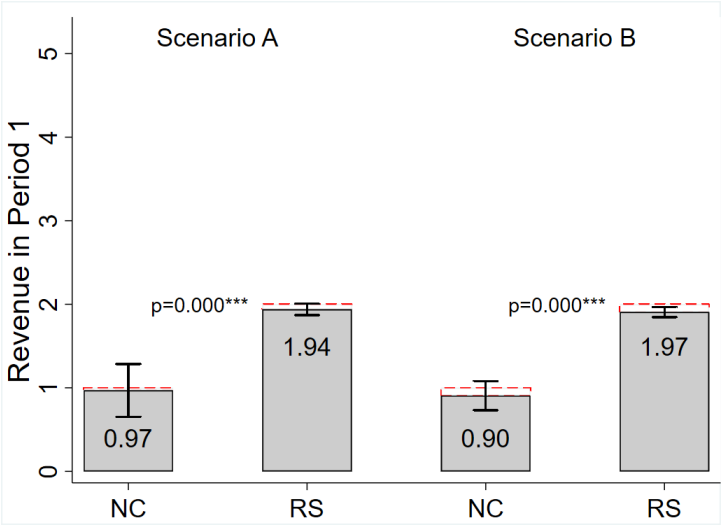


Figure 3: Revenues of Period 1 in each Treatment

Experimental Revenue Comparison - Period 1 & Period 2

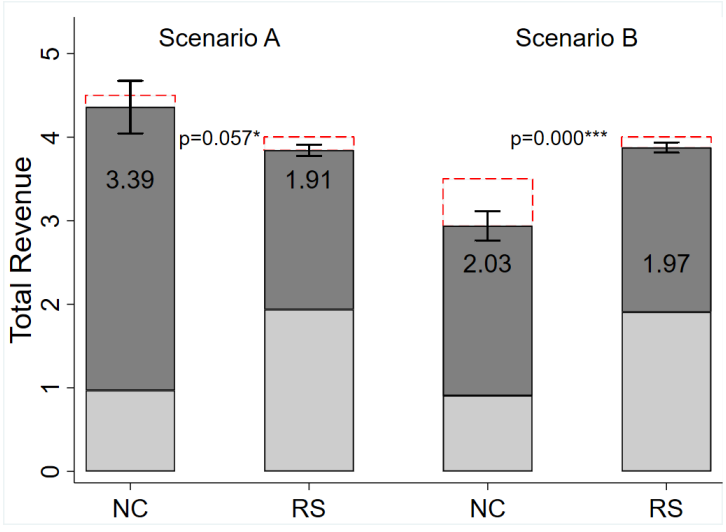


Figure 4: Revenues in each Treatment

Results

Result 2.

Risk aversion deters buyers from participating in the second period in NC.

- In S_A, 4 buyers quit the second period, and the number doubles in S_B.
- Revenue from NC being less than theoretically predicted.

Revenue Loss Decomposition

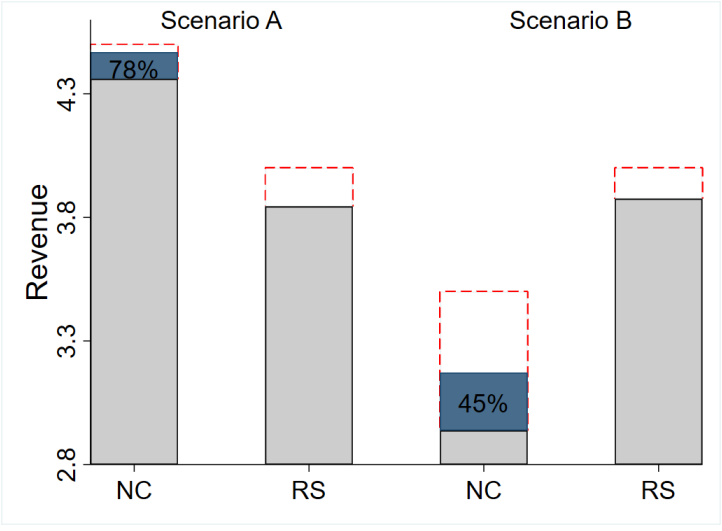


Figure 5: Revenues Increase if all Buyers enter in Period 2.

Why not Pay the Upfront Fee (membership fee)

- “Since I got a profit the first time I didn’t want to go again with my luck”
- “Risk vs Reward..... I got lucky and did not have to pay.”
- “Based on the membership fee. ”
- “didn’t want to take any big risks so I just lowballed my offers and refused to take the membership”
- “i read the instructions carefully. i think the second period isn’t worth losing the points - i had to pay membership fee and could only get the item by bidding higher than the price set by the seller..... honestly, i haven’t been feeling lucky so i’d rather not take my chances. so i tried not to lose money in the first period and just left it as is.”

Risk Aversion Affects Second-period Participation Indirectly

	DV: Enter in Period 2 (=1)	
	(1)	(2)
Scenario A (=1)	0.17**	0.25*
	(0.08)	(0.13)
$notfree_1(=1)$	0.07	0.08
	(0.07)	(0.11)
Scenario A * $notfree_1$	−0.18*	−0.14
	(0.10)	(0.17)
risk aversion	−0.01	−0.03
	(0.01)	(0.02)
$payoff_1$	0.00	0.00
	(0.01)	(0.01)
$upfront_2$	−0.01	−0.03
	(0.03)	(0.05)
Controls		✓

Standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 2: Regression of Participation Choice on Risk attitude.

Results

Result 3.

- Generally overbid.
- Buyers overbid less under Non-Clairvoyant mechanism when the distribution of their valuation has low variance.

Bid-Value Ratio Comparison

Bid/value	**Non-Clairvoyant Dynamic**	**Repeated Static**	(p-value) [1]
F_A (Low variance)	1.264 (0.04)	1.379 (0.04)	**0.060***
F_B (High variance)	1.194 (0.05)	1.251 (0.04)	0.392
(p-value)	0.116	**0.008*** **	

Table 3: Bid-Value Ratio Comparison

[1] We report two-sided p-value under t-test.

Conclusions

- We find the experimental observations are consistent with theoretical predictions: the optimal Non-Clairvoyant dynamic mechanism outperforms the optimal Repeated Static mechanism when it is predicted to do so.
- Buyers' risk attitudes matter in the success of Non-Clairvoyant mechanism.
- Randomization in non-clairvoyant mechanism leads buyers to overbid less.

Discussion

How should sellers choose between mechanisms?

- NC generates more revenue when the scenario is “good”.
- NC encourages more accurate valuation information.
- NC works better when buyers are not risk-averse.

Future work

- Can participants (human sellers) set up correct rules?
- Experiments on multi-buyer with more than 2 periods.

Thank you!

Requirements of Mechanisms under Non-clairvoyant Environment

Seller sets up:

- Allocation rule $x \in \{0, 1\}$: whether buyer can get the item or not
- Price rule $p \in \mathcal{R}$: how much to pay if buyer gets the item

Buyer: $\max_{\{b_1, b_2\}} u_1 + u_2 = (x_1 v_1 - p_1) + (x_2 v_2 - p_2)$

- Dynamic Incentive Compatibility (DIC)

  For a buyer, it is optimal to bid true value in each period

- Ex-post Individual Rationality (EPIR)

  $u_1 + u_2 \geq 0$, for all realization of v_1, v_2

Intra-period Revenue & Inter-period Revenue

- Intra-period revenue: independent revenue, using information within that period $\Rightarrow$bounded my Myerson's revenue.
- Inter-period revenue: dependent revenue, linking past periods with current period $\Rightarrow$ bounded by current-period expected value.

Revenue Comparison in Scenario A

$F_1 = F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\}$ $\mathbb{E}_1 = 3.$
$F_2 = F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\},$ $\mathbb{E} = 6.$

- Non-Clairvoyant Mechanism increases revenue, $\uparrow 12.5\%$

Revenue in S_A	Non-clairvoyant Dynamic		Repeated Static	
Period 1	**Give for Free (F)** Myerson's Auction (M)	0 2	Myerson's Auction (M)	2
Period 2	**Post Price Auction (P)** Myerson's Auction (M)	5 2	Myerson's Auction (M)	2
Total		**4.5**		**4**
Intra-period Revenue		2		4
Inter-period Revenue		2.5		0

Table 4: Theoretical Revenues in Scenario A.

Revenue Comparison in Scenario B

$$F_1 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \mathbb{E}_1 = 6.$$
$$F_2 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\},\ \mathbb{E}_2 = 3.$$

- Non-Clairvoyant Mechanism decreases revenue, $\downarrow 12.5\%$

Revenue in S_B	Non-clairvoyant Dynamic		Repeated Static	
Period 1	**Give for Free (F)** Myerson's Auction (M)	0 2	Myerson's Auction (M)	2
Period 2	**Post Price Auction (P)** Myerson's Auction (M)	3 2	Myerson's Auction (M)	2
Total		**3.5**		**4**
Intra-period Revenue		2		4
Inter-period Revenue		1.5		0

Table 5: Theoretical Revenues in Scenario B.

Reserve price (r_1, r_2) in Scenario A

$$F_1 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\},\ \mathbb{E}_1 = 3.$$
$$F_2 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \mathbb{E}_2 = 6.$$

Period 1

- Myserson's Auction: $r_1 = 2$

Period 2

- If $luck = 1$, Myserson's Auction: $r_2 = 2$
- If $luck = 0$, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfrong\ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \geq 6$, $r_2^P = 2$ if $b_1 = 4$, $r_2^P = 8$ if $b_1 = 2$, and $r_2^P = 32$ if $b_1 = 0$.

Reserve price (r_1, r_2) in Scenario B

$$F_1 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \mathbb{E}_1 = 6.$$
$$F_2 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\},\ \mathbb{E}_2 = 3.$$

Period 1

- Myserson's Auction: $r_1 = 2$

Period 2

- If $luck = 1$, Myserson's Auction: $r_2 = 2$
- If $luck = 0$, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfront\ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \geq 3$, $r_2^P = 1$ if $b_1 = 2$ and $r_2^P = 4$ if $b_1 = 0$.

Experimental Revenue Decomposition in Scenario A

Revenue in S_A	**Non-clairvoyant Dynamic**			**Repeated Static**		
	Theory		**Experiment**	**Theory**		**Experiment**
Period 1	Give it for free	0	0	Myerson's	2	1.94(0.04)
	Myerson's auction	2	1.94(0.06)			
Period 2	Post Price Auction	**5**	4.84(0.47)	Myerson's	2	1.91(0.05)
	Myerson's auction	2	1.94(0.06)			
Total		5	**4.35**(0.32)		4	**3.84**(0.07)

Table 6: Revenue decomposition in S_A

Experimental Revenue Decomposition in Scenario B

Revenue in S_B	**Non-clairvoyant Dynamic**			**Repeated Static**		
	Theory		**Experiment**	**Theory**		**Experiment**
Period 1	Give it for free	0	0	Myerson's	2	1.91(0.05)
	Myerson's auction	2	1.93(0.06)			
Period 2	Post Price Auction	**3**	2.25(0.21)	Myerson's	2	1.97(0.03)
	Myerson's auction	2	1.75(0.12)			
Total		3.5	**2.91**(0.18)		4	**3.88**(0.06)

Table 7: Revenue decomposition in S_B