Evaluating Non-Clairvoyant Mechanisms: Theory and Experiment

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Optimal Dynamic Mechanism Design

- To maximize the revenues (payoff), the seller (principle) sets rules of allocations and prices over multi-period as the buyer (agent) receives private information over time.
  - **Repeated selling of perishable goods**
  - Long-term principal-agent relationship
- Dynamic mechanism improves revenue and efficiency (Baron & Besanko, 1984).

A “Simple” Example

Scenario U (Mirrokni et al 2017)

- two-period, single-buyer (with a quasi-linear utility function)
- the seller sells one item in each period; zero production cost
- distribution of Buyer’s value: $F_1 = U[0,1] = F_2$, independent draws

What are the best rules of allocation and price?

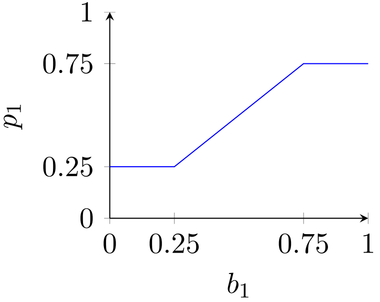
- <u>Dynamic IC</u>: the buyer reports the true value in each period
- <u>Ex-post IR</u>: the buyer gains a non-negative payoff after the realization of values

A complicated Answer

Buyer knows the clairvoyant bundle:

$$p_2 = 1 - \sqrt{2p_1 - 0.5}$$

- Buyer makes a bid in Period 1, pays p_1 if $b_1 \geq p_1$
- p_1 is a function of b_1



Clairvoyant mechanism is hard to solve, understand, and implement

Clairvoyant Mechanisms

- Full information design $\Rightarrow$ Future demand (F_2) is used to design the structure

Why not clairvoyant mechanism in real-life?

- Difficult to compute (Papadimitriou et al., 2022)
- Not intuitive (Mirrokni et el. 2020)
- Require to share a common unbiased belief
- Lack of a general form
- Real revenue is not as expected (Gui and Houser, 2024)

Non-Clairvoyance Environment: more practical

Future demand is not accessible at the beginning.

- No need to share the unbiased belief.
- General Form.

F_2 is unknown in Day 1

$v_1 \sim F_1$

$v_2 \sim F_2$?

Non-Clairvoyant Mechanisms: general form

The clairvoyant revenue Rev^* is not achievable.

RS: Optimal Repeated Static mechanism (Myerson, 1981)

- Rules in two days are independent of each other

  Maximize **intra-period revenue** for each period separately.

  $\Rightarrow \frac{Rev^{RS*}}{Rev^*}$ could be arbitrarily small (Papadimitriou et al., 2022)

NC: Optimal Non-clairvoyant dynamic mechanism (Mirrokni et al., 2020)

- Rules on Day 2 depend on bids on Day 1

  Best Revenue Guarantee: $\Rightarrow \frac{Rev^{NC*}}{Rev^*} \geq \frac{1}{a}$

  Achieve at least $\frac{1}{2}$ revenue produced by optimal clairvoyant mechanism under all scenarios in **two-period single-buyer** case.

Constructing Non-clairvoyant Dynamic Mechanisms

Three Basic Dynamic Mechanism satisfies IC and IR

- Myerson Auction (M): get the item if $b \geq r$, pay r
- Give-for-free (F): Free item
- Posted-price Auction (P): pay upfront fee $s = \min[u_{-1}, \mathbb{E}(v)]$,
  get the item if $b \geq r$, such that $\mathbb{E}[v - r | v \geq r] = s$

NC, RS in a two-period case

- RS: M in Period 1 and Period 2;
- NC: Uniform combination of F and M in Period 1;
  Uniform combination of P and M in Period 2

When can NC do better than RS?

Relative size of optimal intra- and inter-period revenues is the key.

- NC Better Scenario: Optimal inter - period revenue is larger $\Rightarrow$ NC outperforms.

  $$F_1 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\}, \qquad \mathbb{E}_A = 3.$$
  $$F_2 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\}, \qquad \mathbb{E}_B = 6.$$

  $$REV^{RS} = 4, \; REV^{NC} = 4.5 \; \uparrow 12.5\%$$

- RS Better Scenario: Optimal intra - period revenue is larger $\Rightarrow$ RS outperforms.

  $$F_1 = F_B, F_2 = F_A$$
  $$REV^{RS} = 4, \; REV^{NC} = 3.5 \; \downarrow 12.5\%$$

Experimental Design $2 * 2$

Two Mechanisms * Two Scenarios

- NC
- RS

- NC Better Scenario
- RS Better Scenario

Non-Clairvoyant Environment

- Participants as the **Buyer** trading with **Robot Seller**, $c = 0$,
- **Two periods**: The buyer can buy one item in each period from the seller
- **Non-clairvoyance**: The distribution of buyer's value (F_t) is common knowledge only in that period
- **Incomplete Information**: Only the buyer knows his value for the item in each period, v_t, independent draw.
- **Endowment** $= 50$

Mechanism - Optimal Repeated Static (RS)

Period 1

- Seller sets a reserve price r_1 based on the distributional knowledge F_1.
- Buyer learns his value (v_1), makes a bid: b_1
- Buyer can get the item only when $b_1 \geq r_1$ and pay $p_1 = r_1$.

Period 2

- $F_2 \Rightarrow r_2$, $v_2 \Rightarrow b_2$, pays $p_2 = r_2$ if $b_2 \geq r_2$

Myerson's Auction

monopoly price: $r_1 = r_2 = 2$

$r_A = 2 \in \{arg\max_r \; r \cdot P(v_A > r)\}, \quad r_B = 2 \in \{arg\max_r \; r \cdot P(v_B > r)\}$

Mechanism - Optimal Non-Clairvoyant Dynamic (NC)

How the dynamic mechanism work?

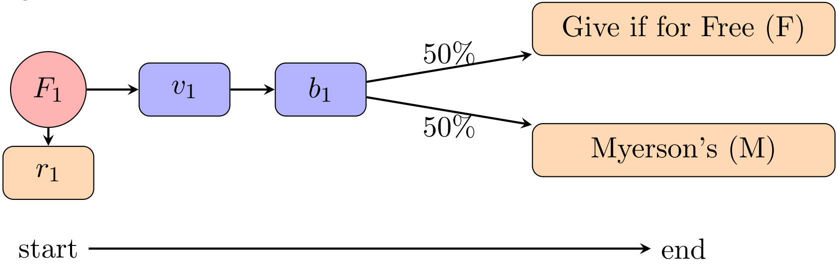
Half chance of free item in period 1

Half chance of upfront fee in period 2

NC in Period 1

- Seller sets a fixed reserve price r_1 based on the distribution F_1.
- Buyer learns his value (v_1), makes a bid : b_1
- Buyer has 50% chance to get the item for free: $p_1 = 0$;
  Otherwise, buyer can get the item only when $b_1 \geq r_1$ and pay $p_1 = r_1$.

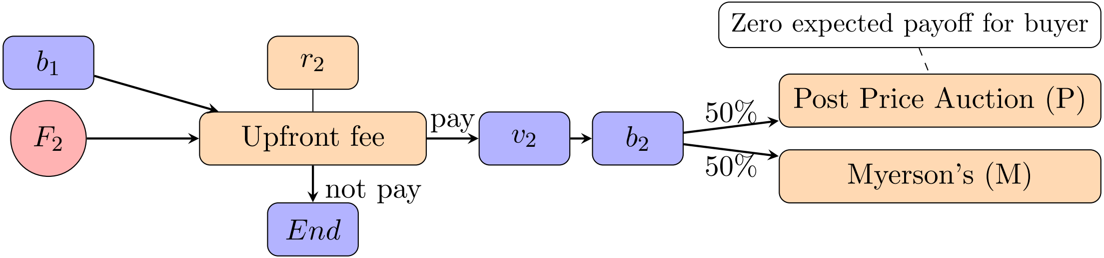
Uniform Combination of Two Auctions



NC in Period 2

- Seller sets an upfront fee $s_2 = \min(b_1, E(v_2))$.
- Buyer decides to pay or leave. Game ends if the buyer leaves ($enter = 0$).
- If the buyer pays ($enter = 1$),
  - Buyer learns their value (v_2) and makes a bid (b_2)
  - Buyer has 50% chance to get refund on the upfront fee ($luck = 1$).
  - Seller sets two reserve prices (r_2) based on the $F_2, luck$ for each given m_2, Buyer can get the item only when $b_2 \geq r_2$ and pay $p_2 = r_2$

Uniform Combination of Two Auctions



Hypotheses

Hypothesis 1 - On Revenue Comparison

- In the NC Better Scenario, NC gains more revenue than RS;
- In the RS Better Scenario, NC gains less revenue than RS.

Hypothesis 2 - On Individual Rationality

- Some buyers choose not to pay the upfront fee, such that the experimental revenue of NC is less than its theoretical prediction.

Hypothesis 3 - On Incentive Compatibility

- Participants' bids are closer to true value under NC than RS.

The Experiment

- 256 George Mason Students. September to November 2021.

Treatment	**NC Better Scenario**		**RS Better Scenario**	
	NC	**RS**	**NC**	**RS**
Age	21.6	22.3	21.9	22.7
Gender (Male=1)	0.48	0.44	0.52	0.47
Risk aversion	4.46	4.90	4.55	4.63
Observation	64	64	64	64

Table 1: Summary Statistic

Results

Result 1.

Experimental observations match theoretical predictions.

- In the NC Better Scenario, NC gains more revenue than RS.
- In the RS Better Scenario, NC gains less revenue than RS.

Experimental Revenue Comparison - Period 1

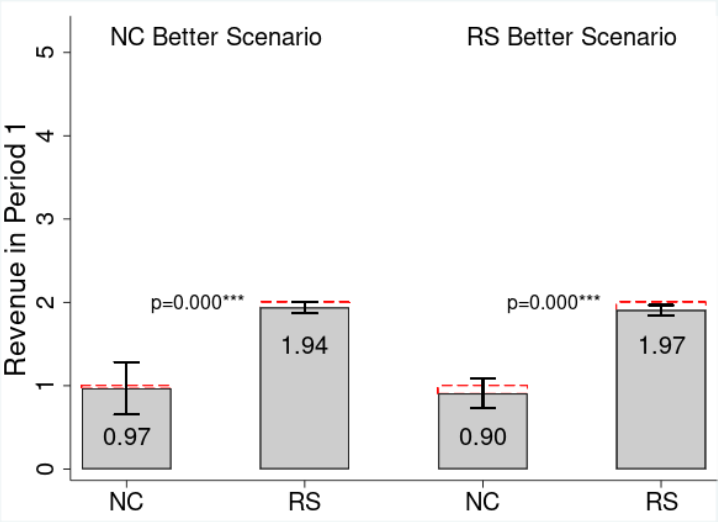


Figure 3: Revenues of Period 1 in each Treatment

Experimental Revenue Comparison - Period 1 & Period 2

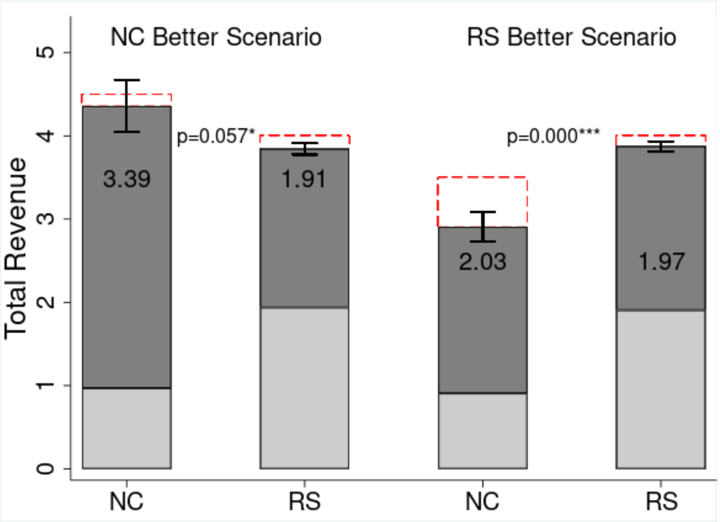


Figure 4: Revenues in each Treatment

Results

Result 2.

Risk aversion deters buyers from participating in the second period in NC.

- In the NC Better Scenario, 4 buyers quit the second period, and the number doubles in the RS Better Scenario.
- Revenue from NC being less than theoretically predicted.

Revenue Loss Decomposition

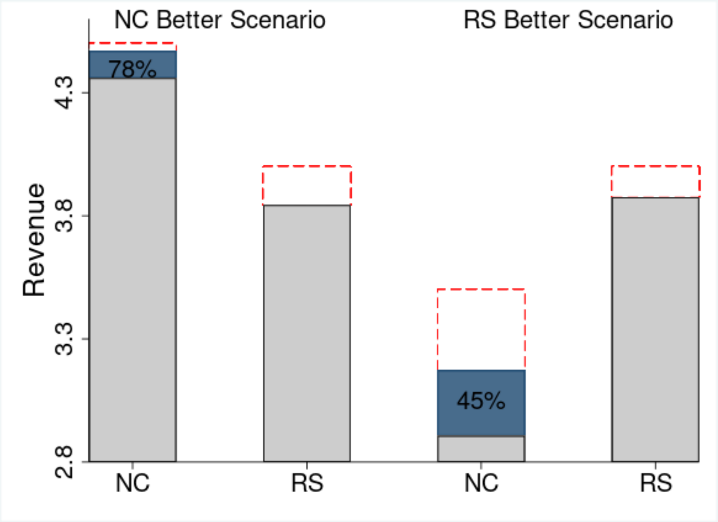


Figure 5: Revenues Increase if all Buyers enter in Period 2.

Why not Pay the Upfront Fee (membership fee)

- “Since I got a profit the first time I didn’t want to go again with my luck”
- “Risk vs Reward...... I got lucky and did not have to pay.”
- “Based on the membership fee. ”
- “didn’t want to take any big risks so I just lowballed my offers and refused to take the membership”
- “i read the instructions carefully. i think the second period isn’t worth losing the points - i had to pay membership fee and could only get the item by bidding higher than the price set by the seller...... honestly, i haven’t been feeling lucky so i’d rather not take my chances. so i tried not to lose money in the first period and just left it as is.”

Risk Aversion Affects Participation in Period 2 Indirectly

	DV: Enter in Period 2 (=1)	
	(1)	(2)
NC Better Scenario (=1)	0.17**	0.25*
	(0.08)	(0.13)
$notfree_1 (= 1)$	0.07	0.08
	(0.07)	(0.11)
NC Better Scenario * $notfree_1$	−0.18*	−0.14
	(0.10)	(0.17)
risk aversion	−0.01	−0.03
	(0.01)	(0.02)
$payoff_1$	0.00	0.00
	(0.01)	(0.01)
$upfront_2$	−0.01	−0.03
	(0.03)	(0.05)
Controls		✓

Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

Table 2: Regression of Participation Choice on Risk attitude.

Results

Result 3.

- Generally overbid.
- Buyers overbid less under NC when the distribution of their valuation has low variance.

Bid-Value Ratio Comparison

Bid/value	**Non-Clairvoyant Dynamic**	**Repeated Static**	(p-value) [1]
F_A (Low variance)	1.264 (0.04)	1.379 (0.04)	**0.060***
F_B (High variance)	1.194 (0.05)	1.251 (0.04)	0.392
(p-value)	0.116	**0.008*****	

Table 3: Bid-Value Ratio Comparison

[1] We report two-sided p-value under t-test.

What we learn (so far)

Practical value of non-clairvoyant mechanisms

- We find non-clairvoyant mechanisms work as intended: NC outperforms RS when it is predicted to do so.
- Buyers' risk attitudes matter in the success of NC.
- Randomization in NC leads buyers to overbid less.

Further Questions

- How does the optimal clairvoyant mechanism perform? $\Rightarrow$ New treatment OC
- How does the deterministic implementation of NC perform? $\Rightarrow$ New treatment NCD

The Optimal Clairvoyant Mechanism (OC) in the NC Better scenario

Clairvoyant menu:

$$\{(p_1, p_2)\} = \{(2, 8),\ (4, 2)\}$$

$\Rightarrow$ Cannot discriminate in Period 2: $REV_2^{OC} = 2$

$\Rightarrow$ Extract the whole expected value in Period 1: $REV_1^{OC} = 3$

Check IC and IR $u(b_1)$

- if $v_1 = 2$: $u(2) = 0 + 4 - \frac{1}{4} * 8 = 2, u(4) = -2 + 4 = 2$
- if $v_1 = 4$: $u(2) = 2 + 2 = 4, u(4) = 0 + 4 = 4$

Implementations of OC in the NC Better scenario

Free item in Period 1 (Give for Free)

- Buyer makes a bid in Period 1 (or quit), pays p_1 if $b_1 \geq p_1$
- $p_1 = 0$, get the item for free in Period 1

Upfront fee in Period 2 (Posted-Price Auction)

- Upfront fee equals to past bid: $s_2 = b_1$, buyer pays or quit
- Buyer makes a bid in Period 2 if enter pays p_2 if $b_2 \geq p_2$

The Optimal Clairvoyant Mechanism (OC) in the RS Better scenario

Clairvoyant menu:

$$\{(p_1, p_2)\} = \{(2,4),\ (4,1)\}$$

- Buyer makes a bid in Period 1 (or quit), pays p_1 if $b_1 \geq p_1$
- $p_1 = 2$ or $p_1 = 4$ with equal chance

$\Rightarrow$ Cannot discriminate in Period 2: $REV_2^{OC} = 2$

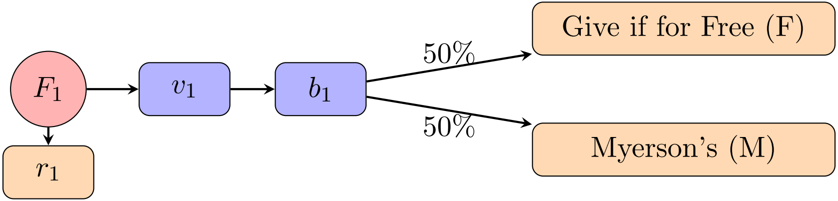
$\Rightarrow$ Cannot discriminate among $v_1 \geq 4$: $REV_1^{OC} = \frac{1}{2}(2+3) = 2.5$

Check IC and IR $u(b_1)$

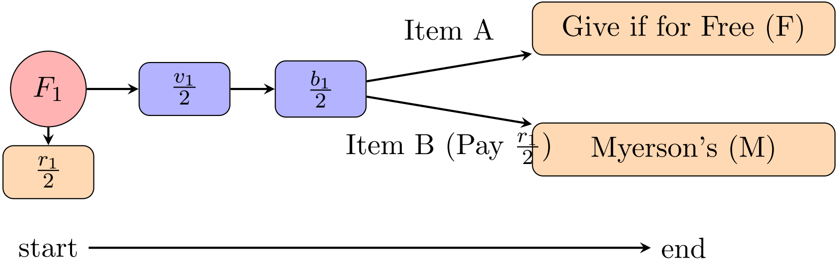
- if $v_1 = 2$: $u(2) = 0 + 0 = 0, u(4) = -2 + 2 = 0$
- if $v_1 = 4$: $u(2) = 2 + 0 = 2, u(4) = 0 + 2 = 2$

With and without Randomization in Period 1

With Randomization (NC)

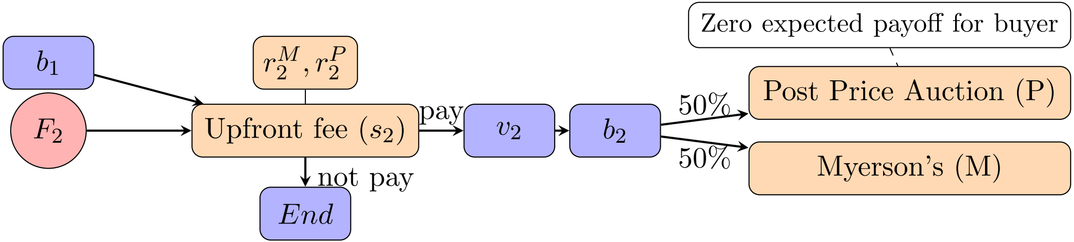


Without Randomization (NCD, two small items in Period 1)

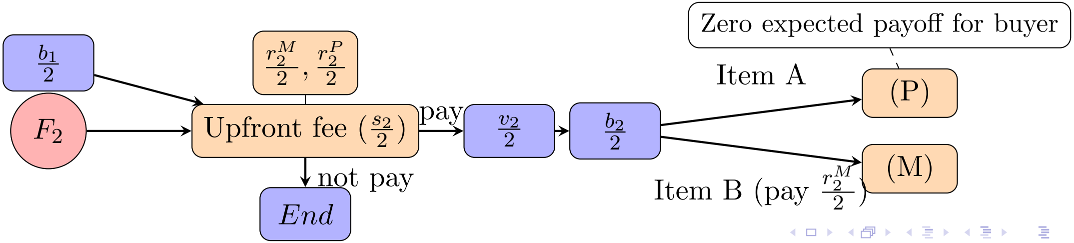


With or without Randomization in Period 2

With Randomization (NC)



Without Randomization (NCD, two small items in Period 2)



Design of Experiment 2

Two Mechanisms * Two Scenarios

- Optimal Clairvoyant Mechanism (OC)
- NC in Deterministic form (NCD)

- NC Better Scenario
- RS Better Scenario

- 128 Participants as the **Buyer** trading with **Robot Seller**, $c = 0$,
- **Between Subject**: Buyers are assigned to only one treatment
- **Within Subject**: All buyers participate two scenarios.

Results: OC does not do better than NC/NCD

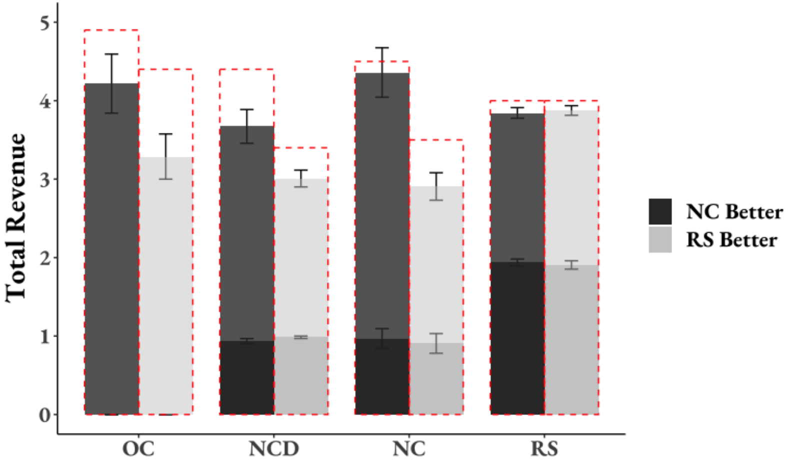


Figure 6: Revenue Comparison

Conclusion

- We highlight the practical importance of non-clairvoyant mechanisms.
  - consistent with theory prediction; robust performance
- We provide behavioral insights for future mechanism design theory.
  - risk attitude matters; randomization advantage

Discussion

- Middle ground of dynamic mechanism design: partially future information design
- Multi-period game with competition among buyers

From the seller side

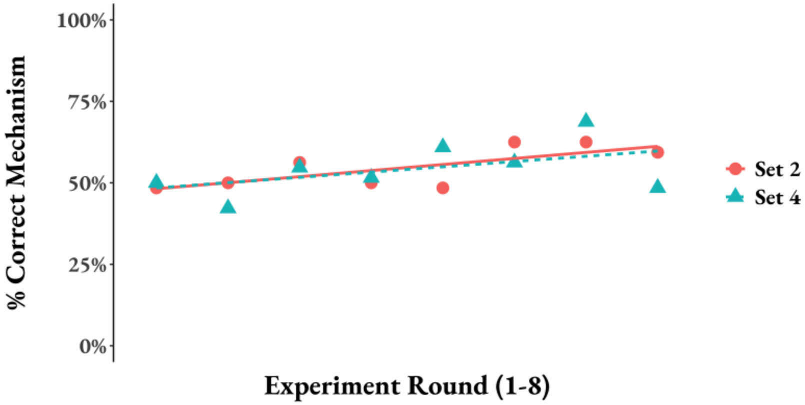
How should sellers choose between mechanisms?

- NC generates more revenue when the scenario is “good”.
- NC encourages more accurate valuation information.
- NC works better when buyers are not risk-averse.

What do human sellers select?

- Can they learn the intuition of the revenue comparison and choose correctly?
- Are they willing to select NC more if setting up NC is easier?

Sellers learn the intuition when they get experienced (Gui and Houser 2024b)



% of Choosing correct Mechanism

Thank you!

Requirements of Mechanisms under Non-clairvoyant Environment

Seller sets up:

- Allocation rule $x \in \{0, 1\}$: whether buyer can get the item or not
- Price rule $p \in \mathcal{R}$: how much to pay if buyer gets the item

Buyer: $\max_{\{b_1, b_2\}} u_1 + u_2 = (x_1 v_1 - p_1) + (x_2 v_2 - p_2)$

- Dynamic Incentive Compatibility (DIC)

  For a buyer, it is optimal to bid true value in each period

- Ex-post Individual Rationality (EPIR)

  $u_1 + u_2 \geq 0$, for all realization of v_1, v_2

Intra-period Revenue & Inter-period Revenue

- Intra-period revenue: independent revenue, using information within that period $\Rightarrow$bounded my Myerson's revenue.
- Inter-period revenue: dependent revenue, linking past periods with current period $\Rightarrow$ bounded by current-period expected value.

Revenue Comparison in Scenario A

$$F_1 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\}\ \mathbb{E}_1 = 3.$$
$$F_2 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \ \mathbb{E} = 6.$$

- Non-Clairvoyant Mechanism increases revenue, $\uparrow 12.5\%$

Revenue in NC Better Scenario	NC		RS	
Period 1	**Give for Free (F)**	0	Myerson Auction (M)	2
	Myerson Auction (M)	2		
Period 2	**Posted-price Auction (P)**	5	Myerson Auction (M)	2
	Myerson's Auction (M)	2		
Total		**4.5**		**4**
Intra-period Revenue		2		4
Inter-period Revenue		2.5		0

Table 4: Theoretical Revenues in NC Better Scenario

Revenue Comparison in RS Better Scenario

$$F_1 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \mathbb{E}_1 = 6.$$
$$F_2 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\},\ \mathbb{E}_2 = 3.$$

- NC gains less revenue, $\downarrow 12.5\%$

Revenue in RS Better Scenario	NC		RS	
Period 1	**Give for Free (F)**	0	Myerson Auction (M)	2
	Myerson Auction (M)	2		
Period 2	**Posted-price Auction (P)**	3	Myerson Auction (M)	2
	Myerson Auction (M)	2		
Total		**3.5**		**4**
Intra-period Revenue		2		4
Inter-period Revenue		1.5		0

Table 5: Theoretical Revenues in RS Better Scenario

Reserve price (r_1, r_2) in NC Better Scenario

$F_1 = F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\},\ \mathbb{E}_1 = 3.$
$F_2 = F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\},\ \mathbb{E}_2 = 6.$

Period 1

- Myserson Auction: $r_1 = 2$

Period 2

- If $luck = 1$, Myserson's Auction: $r_2 = 2$
- If $luck = 0$, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfrong\ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \geq 6$, $r_2^P = 2$ if $b_1 = 4$, $r_2^P = 8$ if $b_1 = 2$, and $r_2^P = 32$ if $b_1 = 0$.

Reserve price (r_1, r_2) in RS Better Scenario

$$F_1 = F_B = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{4}), (8, \tfrac{1}{8}), (16, \tfrac{1}{16}), (32, \tfrac{1}{16})\},\ \mathbb{E}_1 = 6.$$
$$F_2 = F_A = \{v, p(v)\} = \{(2, \tfrac{1}{2}), (4, \tfrac{1}{2})\},\ \mathbb{E}_2 = 3.$$

Period 1

- Myserson's Auction: $r_1 = 2$

Period 2

- If $luck = 1$, Myserson's Auction: $r_2 = 2$
- If $luck = 0$, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfront\ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \geq 3$, $r_2^P = 1$ if $b_1 = 2$ and $r_2^P = 4$ if $b_1 = 0$.

Experimental Revenue Decomposition in NC Better Scenario

Revenue in NC Better Scenario	**NC**			**RS**		
	Theory		**Experiment**	**Theory**		**Experiment**
Period 1	Give for Free	0	0	Myerson	2	1.94(0.04)
	Myerson auction	2	1.94(0.06)			
Period 2	Post Price Auction	**5**	4.84(0.47)	Myerson	2	1.91(0.05)
	Myerson auction	2	1.94(0.06)			
Total		5	**4.35**(0.32)		4	**3.84**(0.07)

Table 6: Revenue decomposition in NC Better Scenario

Experimental Revenue Decomposition in RS Better Scenario

Revenue in RS Better Scenario	**NC**			**RS**		
	Theory		**Experiment**	**Theory**		**Experiment**
Period 1	Give for Free	0	0	Myerson	2	1.91(0.05)
	Myerson auction	2	1.93(0.06)			
Period 2	Post Price Auction	**3**	2.25(0.21)	Myerson	2	1.97(0.03)
	Myerson auction	2	1.75(0.12)			
Total		3.5	**2.91**(0.18)		4	**3.88**(0.06)

Table 7: Revenue decomposition in RS Better Scenario