Choice and Demand (Ch5/6/8)

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2026 Spring

Consumption Theory

Components of Market Equilibrium

- Demand side: **Consumer Theory**
- Supply side: Producer Theory
- Equilibrium

Consumer Theory

- Consumer's (optimal) choices (Ch5): How to maximize utility (Ch3-4) subject to a budget constraint (Ch2).

  $\Rightarrow$ Demand functions (Ch6): The quantity demanded given prices and income
- The Law of Demand (Ch8)

Outline

Solving the Consumer's Problem

- Method 1 (Intuition): Sufficient for some extreme preferences.
  - Perfect substitutes: Consume only the cheaper good.
  - Perfect complements: Consume in a fixed proportion.
- Method 2 (Graphical): Identify the affordable bundle on the highest indifference curve.
- Method 3 (Mathematical): Analyze interior and corner solutions.

The Demand Function: $x_1^*(p_1, p_2, m)$, $x_2^*(p_1, p_2, m)$

- Comparative statics: How the quantity demanded changes with price or income.

Solving the Consumer's Problem

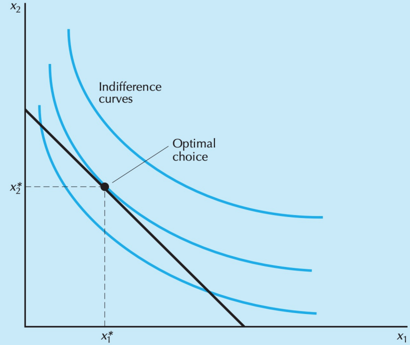
Basic Setting

- Consider two goods only: x_1 and x_2
- Given income (m), prices for two goods (p_1, p_2), and preferences, how does the consumer choose the optimal bundle?

A constrained optimization problem

$$\begin{aligned} \max_{x_1, x_2} \quad & U(x_1, x_2) \\ \text{subject to} \quad & p_1 x_1 + p_2 * x_2 \leq m \end{aligned}$$

Optimality Condition (Interior Solution)



Optimal choice. The optimal consumption position is where the indifference curve is tangent to the budget line.

- Tangency Condition: $-MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$
  Relative marginal utility of Good 1 =Relative cost
- Budget Line: $p_1 x_1 + p_2 x_2 = m$
  Spend all income to gain the highest utility

Cobb-Douglas Preferences

Basic Form: $U(x_1, x_2) = x_1^a x_2^b$.

- Optimality Conditions:
  1. $-MRS = \frac{ax_2}{bx_1} = \frac{p_1}{p_2}$
  2. $p_1 x_1 + p_2 x_2 = m$
- Optimal Choices (Demand Function):

  $x_1^* = \frac{a}{a+b}\frac{m}{p_1}$, $x_2^* = \frac{b}{a+b}\frac{m}{p_2}$
- Expenditure of Each Good:

  $p_1 x_1^* = \frac{a}{a+b}m$, $p_2 x_2^* = \frac{b}{a+b}m$

Monotonic Transformation to $V(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$

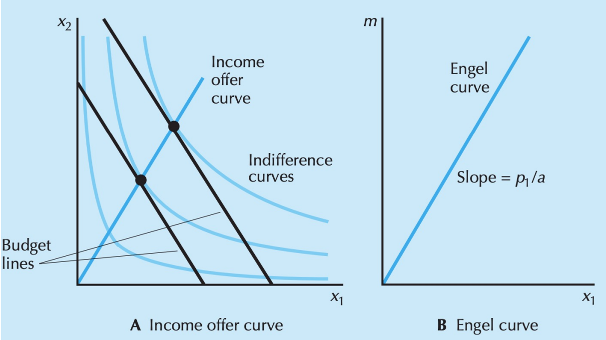
- $f(u) = u^{\frac{\alpha}{a}} = u^{\frac{1}{a+b}}$
- $\alpha = \frac{a}{a+b}$, the share of income the consumer spends on Good 1

Comparative Statics

1. How Demand Changes with Income?
   - The Demand Function: $x_1^*(p_1, p_2, m)$, $x_2^*(p_1, p_2, m)$
   - Income Offer Curve and Engel Curve
   - Income Elasticity of Demand

Cobb-Douglas Preferences: Income Offer Curve and Engel Curve

$U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$.

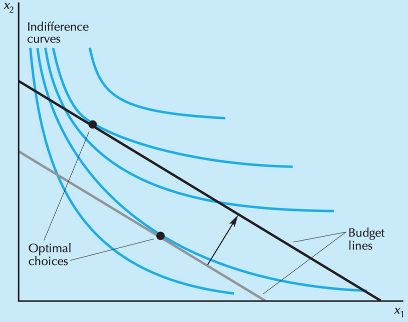


Cobb-Douglas. An income offer curve (A) and an Engel curve (B) for Cobb-Douglas utility.

- Demand Function:
  $x_1^* = \frac{\alpha m}{p_1}$, $x_2^* = \frac{(1-\alpha)m}{p_2}$
- Income Offer Curve: optimal **bundles** at the different levels of income
- Engel Curve: optimal quantity of **each good** at different levels of income
  $m = \frac{p_1}{\alpha} x_1$, $m = \frac{p_2}{1-\alpha} x_2$

How Demand Changes as Income Changes

Normal Goods ($\frac{\partial x}{\partial m} > 0$) and Inferior Goods ($\frac{\partial x}{\partial m} < 0$)



An inferior good. Good 1 is an inferior good, which means that the demand for it decreases when income increases.

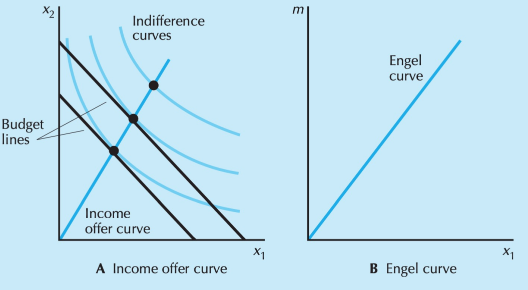
Figure 1: Good 1 is an Inferior Good

- Cobb-Douglas Preference: both are normal goods
- Inferiority is a "local" concept. A good **cannot** be inferior over the whole range of consumption or else it would never have been consumed in positive amounts in the first place! (Silberberg,1978)

  $\Rightarrow$ Engel curve has negative slope
- What about $\frac{\partial x}{\partial m} = 0$?

Income Elasticity of Demand (Ch15)

$$\epsilon^m = \frac{\text{\% change in quantity demanded}}{\text{\% change in income}} = \frac{\frac{\Delta x}{x}}{\frac{\Delta m}{m}}$$



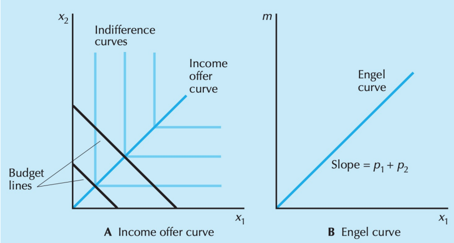
Homothetic preferences. An income offer curve (A) and an Engel curve (B) in the case of homothetic preferences.

- Normal Goods: $\epsilon^m > 0 (\frac{\partial x}{\partial m} > 0)$
  Luxury Goods: $\epsilon^m > 1$
  Necessary Goods: $\epsilon^m \in (0,1)$
  Homothetic Goods: $\epsilon^m = 1$

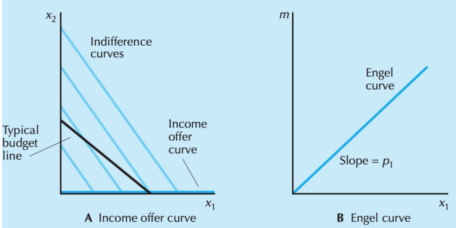
- Inferior Goods: $\epsilon^m < 0 (\frac{\partial x}{\partial m} < 0)$
- Sticky Goods:$\epsilon^m = 0$ $(\frac{\partial x}{\partial m} = 0)$

Homothetic Preferences ($\epsilon^m = 1$)

$$A(x_1, x_2) \succ B(y_1, y_2) \Rightarrow A'(tx_1, tx_2) \succ B(ty_1, ty_2), \forall t > 0$$



Perfect complements. The income offer curve (A) and an Engel curve (B) in the case of perfect complements.



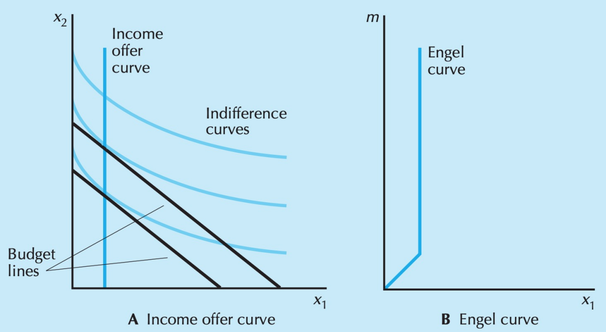
Perfect substitutes. The income offer curve (A) and an Engel curve (B) in the case of perfect substitutes.

Engel curve for Good 2?

- $\exists U$, such that $U(tx_1, tx_2) = tU(x_1, x_2)$, $\Rightarrow MRS_{(x_1,x_2)} = MRS_{(tx_1,tx_2)}$
- Engel curves are straight lines: quantity changes at the same rate as income.

Quasi-linear Preferences: Income Offer Curve and Engel Curve

Basic Form: $U(x_1, x_2) = v(x_1) + x_2$. $(v'(0) > \frac{p_1}{p_2})$



Quasilinear preferences. An income offer curve (A) and an Engel curve (B) with quasilinear preferences.

- Recall previous examples when we interpret Good 2 as the **money** spent on all goods except Good 1.
- $-MRS = v'(x_1)$
- Demand Function[a]:
  $x_1^* = \min[\frac{m}{p_1}, v'^{-1}(\frac{p_1}{p_2})]$,
  $x_2^* = ?$
- Zero income effect for Good 1 after a threshold.

[a] v'^{-1} is the inverse function of v'.

Comparative Statics

2. How Demand Changes with the Price of Other Goods?
   - The Demand Function: $x_1^*(p_1, p_2, m)$, $x_2^*(p_1, p_2, m)$
   - Cross (Price) Elasticity of Demand

How Demand Changes as Price of the Other Good Changes

Substitutes ($\frac{\partial x_1}{\partial p_2} > 0$) and Complements ($\frac{\partial x_1}{\partial p_2} < 0$)

- Two goods are substitutes: $\frac{\partial x_1}{\partial p_2} > 0$
- Two goods are complements: $\frac{\partial x_1}{\partial p_2} < 0$
- (Two goods are independent: $\frac{\partial x_1}{\partial p_2} = 0$)

Cross (Price) Elasticity of Demand

- $\eta_{21} = \frac{\text{\% change in quantity demanded of Good 1}}{\text{\% change in price of Good 2}} = \frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta p_2}{p_2}}$
- Good 2 is a (gross) substitute for Good 1: $\eta_{21} > 0$,
- Good 2 is a (gross) complement for Good 1: $\eta_{21} < 0$
- (Good 2 is independent for Good 1: $\eta_{21} = 0$)

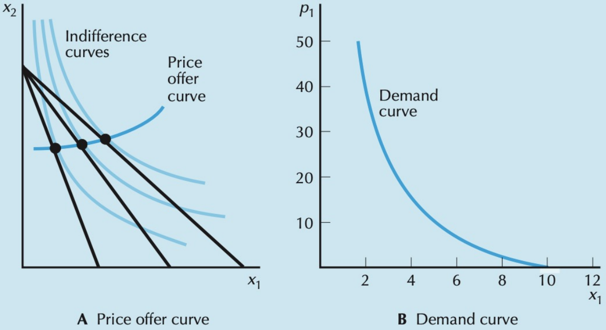
Comparative Statics

3. How Demand Changes with Its Own Price

- The Demand Function: $x_1^*(p_1, p_2, m)$, $x_2^*(p_1, p_2, m)$
- Price Offer Curve and Demand Curve
- Slope of the Demand Curve?

How Demand Changes as its Own Price Changes

Price Offer Curve and Demand Curve



The price offer curve and demand curve. Panel A contains a price offer curve, which depicts the optimal choices as the price of good 1 changes. Panel B contains the associated demand curve, which depicts a plot of the optimal choice of good 1 as a function of its price.

- Demand Function For Cobb-Douglas:[a]

  $x_1^* = \frac{\alpha m}{p_1}, \; x_2^* = \frac{(1-\alpha)m}{p_2}$

- Price Offer Curve: optimal **bundles** at the different levels of price

- Demand Curve: optimal quantity of **each good** at different levels of price

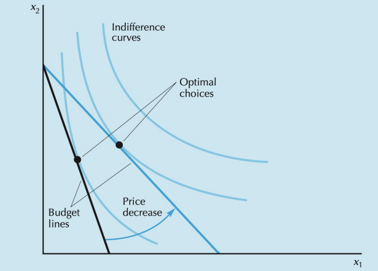
- Inverse Demand Function:

  $p_1 = \frac{am}{x_1}, \; p_2 = \frac{(1-a)m}{x_2}$

[a] The graph on the left does not represent a Cobb-Douglas preference.

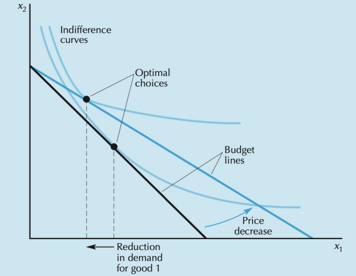
How Demand Changes as its Own Price Changes

Ordinal Goods ($\frac{\partial x_1}{\partial p_1} < 0$) and Giffen Goods ($\frac{\partial x_1}{\partial p_1} > 0$)



An ordinary good. Ordinarily, the demand for a good increases when its price decreases, as is the case here.

Good 1 is an Ordinary Good



A Giffen good. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

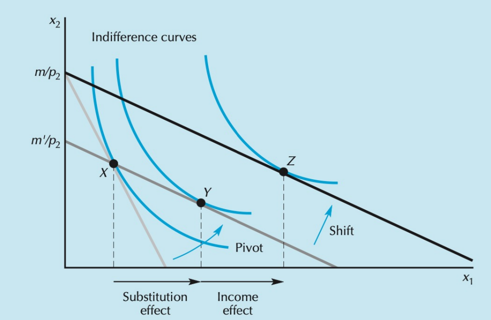
Good 1 is a Giffen Good

Slope of the Demand Curve?

- Price changes bring both <u>income effect</u> and <u>substitution effect</u>.

Decompose Demand Changes in its Own Price

Slutsky identity: $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$

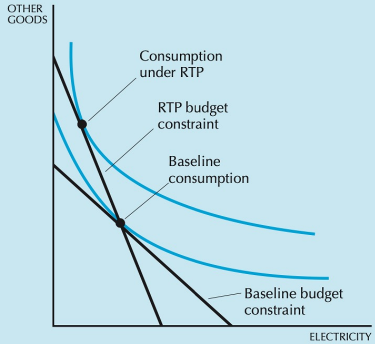


Substitution effect and income effect. The pivot gives the substitution effect, and the shift gives the income effect.

- Consider a decrease of p_1 to p_1'.
  $\Rightarrow$ Optimal Bundle: X to Z
- Step 1 (Pivot): X to Y
  $\Rightarrow$ the same purchasing power
  <u>Substitution Effect</u>: $\Delta x_1^s = x_1^Y - x_1^X$
- Step 2 (Shift): Y to Z
  $\Rightarrow$ the new budget line
  <u>Income Effect</u>: $\Delta x_1^n = x_1^Z - x_1^Y$

Substitution Effect (in rates) is Non-Positive

$$\frac{\Delta x_1^s}{\Delta p_1} \leq 0$$



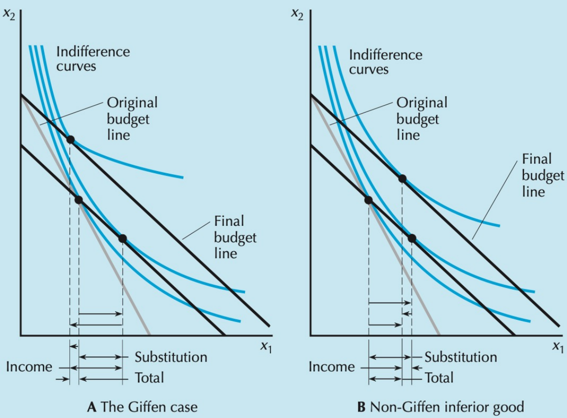
Voluntary real time pricing. Users pay higher rates for additional electricity when the real time price rises, but they also get rebates at the same price if they cut back their use. This results in a pivot around the baseline use and tends to make the customers better off.

- For perfect complements: $\frac{\Delta x_1^s}{\Delta p_1} = 0$
- Otherwise, pivoting the budget line (Step 1) makes the consumer better off.
- Why?

  Revealed Preference (Ch7)

Income Effect (in rates) can be Positive or Negative

Normal Goods ($\frac{\Delta x^m}{\Delta m} > 0$) and Inferior Goods ($\frac{\Delta x^m}{\Delta m} < 0$)



Inferior goods. Panel A shows a good that is inferior enough to cause the Giffen case. Panel B shows a good that is inferior, but the effect is not strong enough to create a Giffen good.

- Slutsky Equation in rates:
  $$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1$$
  increasing (decreasing) price makes consumers poorer (richer)
- Normal Goods ($\frac{\Delta x_1}{\Delta m} > 0$) are all ordinal Goods.
- Giffen Goods ($\frac{\Delta x_1}{\Delta p_1} > 0$) must be inferior Goods ($\frac{\Delta x_1}{\Delta m} < 0$).

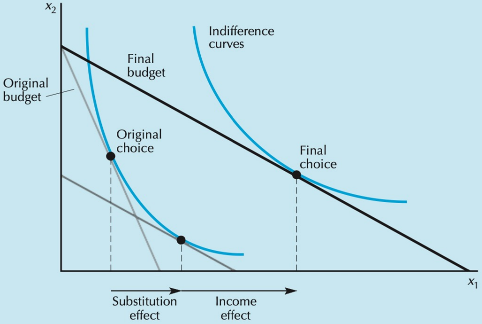
The Law of Demand

Consumer Theory only Restricts How Substitution Effect and Income Effect Interact

- The Law of Demand. *If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.*

Compensated Demand Curves: Non-Positive Slope

Another Substitution Effect



The Hicks substitution effect. Here we pivot the budget line around the indifference curve rather than around the original choice.

- Slutsky Substitution Effect: holding purchasing power the same
- <u>Hicks Substitution Effect</u>: holding the utility the same
- <u>Compensated demand curve:</u> Hicksian demand curve with utility held constant
- Hicksian Demand Function: $x_1(p_1, p_2, u)$

Summary

What We Have Learned

- Solving the Consumer's Problem
- How Demand Changes with Income
- How Demand Changes with the Price of Other Goods
- How Demand Changes with Its Own Price

  $\Rightarrow$ Substitution Effect and the Income Effect

What's Next?

- Consumer's Surplus (Ch14) and Market Demand (Ch15)

Thank You!