# Surplus and Equilibrium (Ch14/15/16)

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# Consumption Theory

## What We Have Learned

- Consider non-extreme preferences. Given $p_1, p_2$ and $m$.

| | $p_1 \uparrow$ | $p_1 \downarrow$ |
|---|---|---|
| Consumption of Good 1 | Substitution: $-$<br>Income: $-/+$ | Substitution: $+$<br>Income: $+/-$ |
| Consumption of Good 2 | Substitution: $+$<br>Income: $-/+$ | Substitution: $-$<br>Income: $+/-$ |
| Consumer's Utility | ? | ? |

Table 1: Effect of rise/fall of $p_1$ in two good economy

## Measuring Gains and Losses from a Change

- Why is $u^{new} - u^{old}$ not a good measure?

# Outline

## Part I. Monetary Measures of Welfare Change

- The Relationship Triangle: Preferences, Choices, and Utilities
- Three Monetary Measures: Compensating Variation (CV), Equivalent Variation (EV), and Change in Consumer Surplus ($\Delta CS$) (Ch14)

## Part II. From Consumer Surplus to Consumers' Surplus

- Aggregate individual demand to derive market demand. (Ch15)
- Competitive equilibrium maximizes social surplus. (Ch16)
- Who bears the tax burden more? $\Rightarrow$ Depends on the price elasticity of demand and supply.

# Part I: Monetary Measures of Gain/Loss from a Price Change

## Why is $u^{new} - u^{old}$ not a good measure?

- The Relationship Triangle: Preferences, Choices, and Utilities

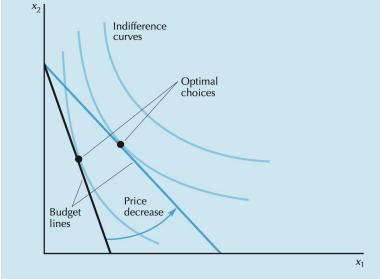
## Three Monetary Measures:

- Compensating Variation (CV)
- Equivalent Variation (EV)
- Change in Consumer Surplus ($\Delta CS$)

# Consumer will never Worse-off if Price Decreases

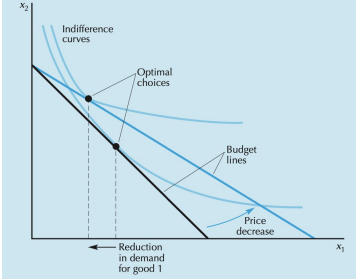
## A Price Decrease Expands the Budget Set

- *Ceteris paribus*, never get worse off with more options: $u^{new} \geq u^{old}$.
- Conversely, a price increase never benefits the consumer.




**An ordinary good**. Ordinarily, the demand for a good increases when its price decreases, as is the case here.

Good 1 is an Ordinary Good




**A Giffen good**. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

Good 1 is a Giffen Good

# Why is $u^{new} - u^{old}$ not a good measure?

## Choices Reveal Preferences: $x(\mathbf{p}, m)$

- Choices $\Leftarrow$ Preferences + Budget Constraints
- Choices are observable.

  Collecting choices under different prices $\Rightarrow$ Individual demand curve.

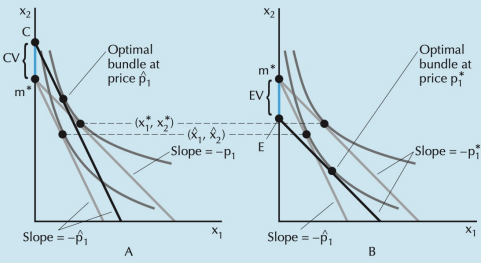
## Utility Functions Represent Preferences: $u(\mathbf{x})$

- Utility functions label indifference curves.
- Different utility functions can represent the same preferences.
- ***Lexicographical preferences*** cannot be represented by a utility function.

  For example, $(x_1^A, x_2^A) \succeq (x_1^B, x_2^B)$ if $x_1^A > x_1^B$ or $(x_1^A = x_1^B$ and $x_2^A \geq x_2^B)$.

# Monetary Measure of Welfare Loss under Rise of Price

## Compensating Variation (CV) Equivalent Variations (EV)




**The compensating and the equivalent variations.** Panel A shows the compensating variation (CV), and panel B shows the equivalent variation (EV).

- **Compensating Variation**: How much income is needed to maintain $u^{old}$?
- **Method 1**: $v(p_1^{new}, p_2, m + CV) = u^{old}$

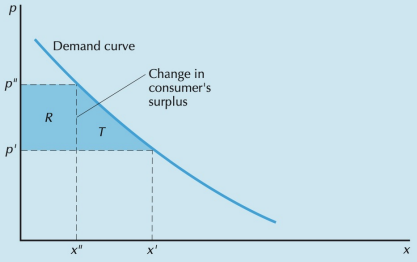
  $v$: maximum utility given $(p_1, p_2, m)$
- Method 2: $CV = e(p_1^{new}, p_2, u) - m$

  $e$: minimum expenditure given $(p_1, p_2, u)$

- **Equivalent Variation**: How much income loss is equivalent to $\uparrow p_1$?
- **Method 1**: $v(p_1, p_2, m - EV) = u^{new}$
- Method 2: $EV = m - e(p_1, p_2, u^{new})$

# Monetary Measure of Welfare Loss under Rise of Price

## Change in Consumer's Surplus ($\Delta CS$)




**Change in consumer's surplus.** The change in consumer's surplus is sum of the square area $R$ and the roughly triangular area $T$, and thus has a roughly trapezoidal shape.

- Optimal consumption: $x_1^*(p_1, p_2, m)$
- Inverse demand function: $p(x)$, write $p_1$ as a function of $x_1$, viewing $p_2, m$ as constant.
- **Gross Surplus** at $(x', p')$: area under demand curve, $\int_0^{x'} p(x)\,dx$
- **(Net) Surplus**: area under demand curve and above market price line $(p')$, $\int_0^{x'} [p(x) - p']\,dx$
- $p'$ to $p''$:
  $\Delta CS = \int_0^{x''} [p(x) - p'']\,dx - \int_0^{x'} [p(x) - p']\,dx$
  $\Delta CS = \int_{p''}^{p'} x(p)dp$

# Relationship Among $CV$, $EV$, and $\Delta CS$

## Marshallian Demand Curve and Hickisan Demand Curve

- Marshallian Demand: $x_1^*(p_1, p_2, m)$, given income $\Rightarrow$ normal demand curve

  $\Delta CS$: Trapezoidal area under Marshallian demand curve
- Hickisan Demand: $x_1^*(p_1, p_2, u)$, holding the utility the same

  $CV, EV$: Trapezoidal area under Hicksian demand curves

## For a normal good

- Hicksian demand curve is steeper than Marshallian demand curve. Why?
- For a price increase: $CV > -\Delta CS > EV > 0$
- For a price decrease: $0 > CV > -\Delta CS > EV$

## If $U(x_1, x_2) = v(x_1) + x_2$

- No income effect on Good 1 if income is enough.
- $CV = -\Delta CS = EV = -\Delta U$

# Part II. From Consumer Surplus to Consumers' Surplus

## Aggregate individual demand to derive market demand

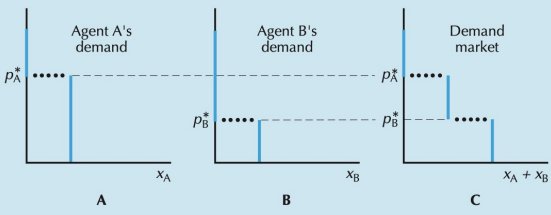
- Consumers' Surplus

## Market Demand, Market Supply, and Equilibrium

- Competitive equilibrium maximizes social surplus. (Ch16)
- Who bears the tax burden more? $\Rightarrow$ Depends on the price elasticity of demand and supply.

# From Individual to Market Demand

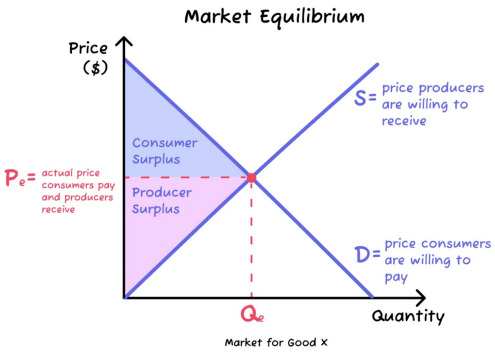
## Summing the Demand Curves Horizontally




**Market demand for a discrete good.** The market demand curve is the sum of the demand curves of all the consumers in the market, here represented by the two consumers A and B.

- Market Demand Function for Good 1:
  $X^1(p_1, p_2, m_1, m_2 ... m_n)$
  $= \sum_{i=1}^{n} x_i^1(p_1, p_2, m_i)$
- Market Demand Curve: $p_1(X^1)$, viewing $p_1, m_1, ... m_m$ as constant
- Market Supply: aggregation of individual supply

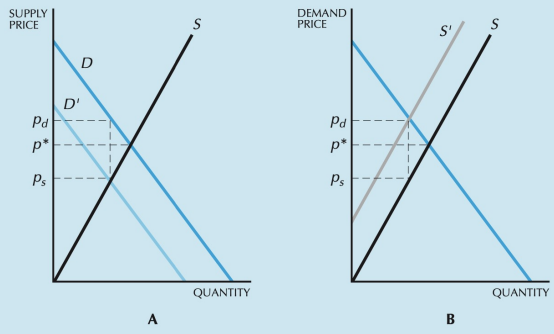
# From Consumer's Surplus to Consumers' Surplus




- **Consumers' Surplus**: area under market demand curve and above market price line ($p^e$), $\int_0^{x^e} [p(x) - p^e]\, dx$
- **Producers' Surplus**: area under market supply curve and below market price line ($p^e$), $\int_0^{x^e} [p(x) - p^e]\, dx$
- **Social Surplus**: Consumers' Surplus + Producers' Surplus + (Government's Revenue)

# Who Bears the Tax More?

## Tax on Producers or Consumers: It Doesn't Matter



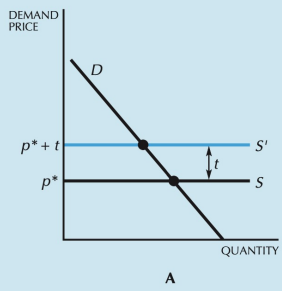

**The imposition of a tax.** In order to study the impact of a tax, we can either shift the demand curve down, as in panel A, or shift the supply curve up, as in panel B. The equilibrium prices paid by the demanders and received by the suppliers will be the same either way.

- A tax $t = p^d - p^s$
- Tax on consumer: shift the demand curve down
- Tax on producer: shift the supply curve up
- $\Rightarrow$ same equilibrium quantity, same price perceived by producer $p^d$, same price perceived by supplier $p^s$.

- Who Bears the Tax More? $p^d - p^* = \Delta p = ?$

# Extreme Cases

## Perfectly Elastic Supply and Perfectly Inelastic Supply

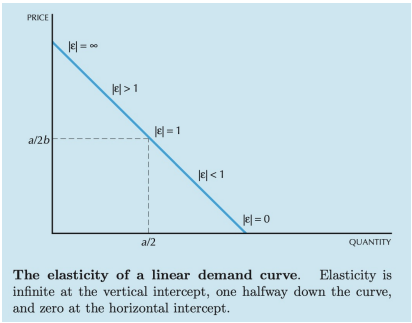



**Special cases of taxation.** (A) In the case of a perfectly elastic supply curve the tax gets completely passed along to the consumers. (B) In the case of a perfectly inelastic supply none of the tax gets passed along.

- Consumers bear all tax:
  $\Delta p = p^d - p^* = t$
  - Perfectly Elastic Supply
  - Perfectly Inelastic Demand
- Producers bear all tax:
  $\Delta p = p^d - p^* = 0$
  - Perfectly Inelastic Supply
  - Perfectly Elastic Demand

# Tax Incidence Depends on Price Elasticity of Demand and Supply

Price elasticity: $\epsilon = \frac{\Delta q/q}{\Delta p/p}$ :



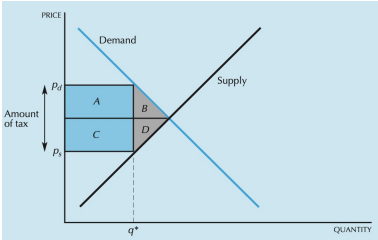

**The elasticity of a linear demand curve.** Elasticity is infinite at the vertical intercept, one halfway down the curve, and zero at the horizontal intercept.

- **Price Elasticity**: percentage change in quantity when price changes by one percent
- Price Elasticity of Demand: $\epsilon^D = \frac{\Delta D/D}{\Delta p/p}$
- Price Elasticity of Supply: $\epsilon^S = \frac{\Delta S/S}{\Delta p/p}$

- Before Tax: $S(p) = D(p)$, $\Delta t \Rightarrow \Delta p$
- After Tax: $S(p + \Delta p) = D(p + \Delta p + \Delta t)$
  $\Rightarrow \frac{\Delta p}{\Delta t} = \frac{D'(p)}{S'(p) - D'(p)} = \frac{\epsilon^D}{\epsilon^S - \epsilon^D}$
  The more elastic the market demand, the smaller the tax burden on consumers.

# Deadweight Loss of a Tax

## Less Than Market Equilibrium Quantity




**The deadweight loss of a tax.** The area $B + D$ measures the deadweight loss of the tax.

- Competitive Equilibrium: Maximum Social Surplus
- Output has been decreased by this tax.

  Change in Consumers' Surplus: -(A+B)

  Change in Producers' Surplus: -(C+D)

  Change in Government's Revenue: +(A+C)
- Deadweight Loss of a tax: area B+D

- subsidy = - tax
- The Deadweight Loss of a subsidy?

# End of Consumer Theory

## Midterm Exam

- Date: Tuesday, Oct 17th, during class time
- Location: Classroom 2-109
- Format: Closed book; answers may be written in English or Chinese.
- Chapters Covered: Ch. 1–6, Ch. 8, Ch. 14–16

- Good luck!

## Good luck!


Thank You!