Oligopoly (Ch28)

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Outline: Oligopoly

Market Structure

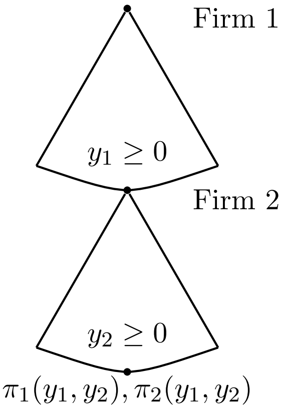
- Oligopoly: A few firms compete, each with noticeable impact on market price.
- We'll focus on the simplest case — **duopoly** (2 firms) with identical products.
- Complete information: each firm knows the market demand (inverse, $p(Q)$) and all individual cost functions $c_1(y), c_2(y)$.

Types of Strategic Interaction

1. **Stackelberg Model**: Sequential Quantity Setting
2. **Cournot Model**: Simultaneous Quantity Setting
3. **Collusion**: Forming a Cartel
4. **Bertrand Model**: Simultaneous Price Setting

1. Stackelberg Model: Sequential Quantity Setting

Backward Induction for Sub-game Perfect Equilibrium



- Firm 2 can observe y_1, then chooses y_2 to:

$$\max_{y_2} \quad \pi_2(y_1, y_2) = p(y_1 + y_2)y_2 - c(y_2)$$

$\Rightarrow$ Firm 2's **best response**: $BR^2(y_1)$

- Firm 1 anticipated $BR^2(y_1)$:

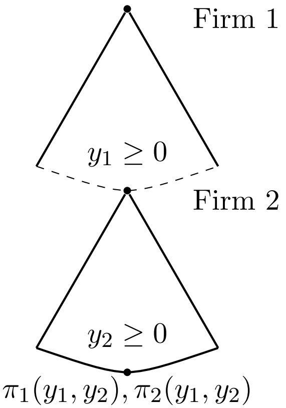
$$\max_{y_1} \quad \pi_1(y_1, y_2) = p(y_1 + BR^2(y_1))y_1 - c(y_1)$$

$\Rightarrow$ y_1^* from $(MR(y^*) = MC(y^*))$

- **Equilibrium**: $(y_1^*,\ y_2^* = BR^2(y_1*))$

2. Cournot Model: Simultaneous Quantity Setting

Bilateral Best Response for Nash Equilibrium



- If Firm 1 chooses y_1, Firm 2:

$$\max_{y_2} \quad \pi_2(y_1, y_2) = p(y_1 + y_2)y_2 - c(y_2)$$

$\Rightarrow$ Firm 2's **best response**: $BR^2(y_1)$

- If Firm 2 chooses y_2, Firm 1:

$$\max_{y_1} \quad \pi_2(y_1, y_2) = p(y_1 + y_2)y_1 - c(y_1)$$

$\Rightarrow$ Firm 1's **best response**: $BR^1(y_2)$

- **Equilibrium**: (y_1^*, y_2^*) that simultaneously solves $y_2^* = BR^2(y_1^*)$ and $y_1^* = BR^1(y_2^*)$.

3. Collusion: Forming a Cartel

Maximizing Total Profit

- Collusion: $\max_{(y_1,y_2)} \quad \pi(y_1, y_2)$
  $= p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2)$
- The **optimality condition:** $MR(y_1^* + y_2^*) = MC_1(y_1^*) = MC_2(y_2^*)$.

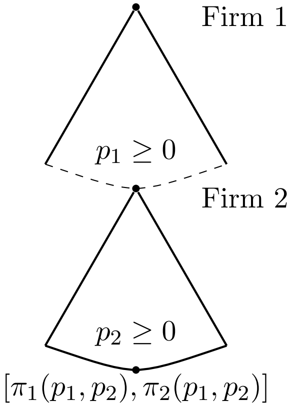
- Collusion result is not Nash Equilibrium.

$\Leftarrow$ Each firm has an incentive to deviate: $MR_i(y_i^*) > MC(y_i^*)$.

- Punishment Strategies are needed in the infinitely repeated games.

4. Bertrand Model: Simultaneous Price Setting

Suppose Firm with a Lower Price Captures the Whole Market



- If Nash Equilibrium exists, it will be a (roughly) **Competitive Equilibrium** ($p = MC$).

  Suppose $p_1 = p_2 > MC$, then each firm has an incentive to slightly undercut the other by ϵ.
- **Example 1**: $D(p) = 10 - p$, $c_1(y) = c_2(y) = \frac{1}{2}y^2$.

$\Rightarrow$ Equilibrium: $p_1^* = p_2^* = MC = \frac{10}{3}$.

- **Example 2:** $D(p) = 10 - p$, $MC_1 = 1$, $MC_2 = 2$, $p_i \in \{0, \epsilon, 2\epsilon, \ldots, MC_i - \epsilon, MC_i, \ldots\}$.

$\Rightarrow$ Equilibrium:$p_1^* = 2 - \epsilon$, $p_2^* = 2$.

- Example 3: $D(p) = 10 - p$, $MC_1 = 1$, $MC_2 = 2$, $p_i \geq 0$. $\Rightarrow$ No Nash Equilibrium (no best response of Firm 1 if $p_2 = 2$).

An Example for Comparison

Common Information

- Market Demand: $D(p) = 10 - p \Rightarrow$ Inverse Demand Function: $p(Q) = 10 - Q$
- Individual Cost Functions: $c_1(y) = \frac{1}{2}y^2$, $c_2(y) = y^2$

	Best Responses	Equlibrium Quantity	Price
Stackelberg Model	$BR^2 = \frac{10-y_1}{4}$, $BR^1 = 3$	$(y_1^* = 3, y_2^* = \frac{7}{4}y)$	$p^* = \frac{21}{4}$
Cournot Model	$BR^2 = \frac{10-y_1}{4}$, $BR^1 = \frac{10-y_2}{3}$	$(y_1^* = \frac{30}{11}, y_2^* = \frac{20}{11}y)$	$p^* = \frac{50}{11}$
Collusion	$MR = MC_1 = MC_2$	$y_1^* = \frac{5}{2}, y_2^* = \frac{5}{4}$	$p^* = \frac{25}{4}$
Competitive Market	$p = MC_1 = MC_2$	$y_1^* = 4, y_2^* = 2$	$p^* = 4$
Firm 1 Monopoly	$MR_1 = MC_1$	$y_1^* = \frac{10}{3}$	$p^* = \frac{20}{3}$
Firm 2 Monopoly	$MR_2 = MC_2$	$y_2^* = \frac{5}{2}$	$p^* = \frac{15}{2}$
Betrand Model	$(p_1^* = 4, p_2^* = 4)$ is not Nash Equilibrium		

Thank You!