Exchange (Ch32)

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What We Have Learned

Demand Side: Consumer Theory

- Budget constraints, preferences, choices, individual demand for each good

$\Rightarrow$ Market demand for each good

Supply Side: Producer Theory

- Cost minimization for any producer
- Profit maximization for a competitive Seller/monopolist

$\Rightarrow$ Market supply for each good

Partial Equilibrium: How Demand and Supply Determine Price of One Good

- Competitive equilibrium and monopoly pricing
- Strategic interaction in duopoly markets

How Do Several Markets Interact?

General Equilibrium Analysis

- (Complex Problem): How do demand and supply conditions across multiple markets interact to determine the prices of goods?

Start from the Simplest Setting: Pure Exchange

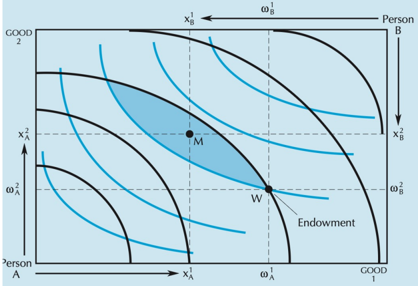
1. Focus on competitive markets
2. Two goods (1, 2), two persons (A, B) with well-behaved preferences
3. Initial **endowments** for each person: $w^A = (w_1^A, w_2^A)$, $w^B = (w_1^B, w_2^B)$
4. No production $\Rightarrow$ Fixed endowments

How might these two individuals trade goods among themselves?

- Final allocation? $x^A = (x_1^A, x_2^A)$, $x^B = (x_2^B, x_2^B)$
- Trading prices? p_1^*, p_2^*

The Edgeworth Box

Illustrating All Possible Allocations for Both People



An Edgeworth box. The width of the box measures the total amount of good 1 in the economy and the height measures the total amount of good 2. Person A's consumption choices are measured from the lower left-hand corner while person B's choices are measured from the upper right.

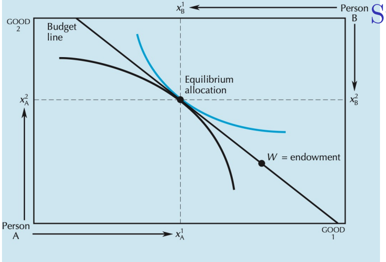
- Draw person A's indifference curves and initial endowment, $W = (w_1^A, w_2^A)$.
- The width of the box represents the total amount of good 1 in the economy: $w_1^A + w_1^B$.
- The height of the box represents the total amount of good 2: $w_2^A + w_2^B$.

$\Rightarrow$ W also represents B's initial endowment

- Draw person B's indifference curves starting from the upper-right corner.
- **Mutually advantageous allocations**: points inside the lens-shaped area, such as point M.

Competitive Equilibrium (Market Equilibrium, Walrasian Equilibrium)

Each person's indifference curve is tangent to their budget line.



Equilibrium in the Edgeworth box. In equilibrium, each person is choosing the most-preferred bundle in his budget set, and the choices exhaust the available supply.

Only $\frac{p_1}{p_2}$ matters: if (p_1, p_2) is an equilibrium, (kp_1, kp_2) is also an equilibrium.

Step1 Given prices (p_1, p_2) and $W(w_1^A, w_2^A)$:

- The "income" is determined by initial endowments.

$\Rightarrow$ (Budget Constraints)

$$p_1 x_1^A + p_2 x_2^A = p_1 w_1^A + p_2 w_2^A,$$
$$p_1 x_1^B + p_2 x_2^B = p_1 w_1^B + p_2 w_2^B.$$

$\Rightarrow$ (Tangency Conditions) $MRS^A = MRS^B = \frac{p_1}{p_2}$

- Find each person's optimal consumption bundle, A and B.

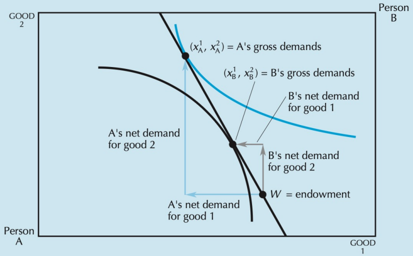
Step2 Find the equilibrium price ratio $\frac{p_1^*}{p_2^*}$:

- (Market Clearing): $x_1^A + x_1^B = w_1^A + w_1^B$.

(For Good 2, $x_2^A + x_2^B = w_2^A + w_2^B$.)

Walras' Law

The Value of Aggregate Excess Demand is Identically Zero.



Gross demands and net demands. Gross demands are the amounts the person wants to consume; net demands are the amounts the person wants to purchase.

(Equilibrium) Market 1 is cleared $\Rightarrow$ Market 2 is also cleared; (Disequilibrium) Aggregate excess demand of Market 1 $\Rightarrow$ Aggregate excess supply of Market 1 .

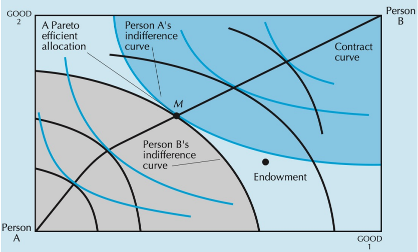
- Set $p_2 = 1$.
- Adding up (Budget Constraints), we have the **Walras' Law:** $p_1 z^1(p_1) + z^2(p_1) \equiv 0$.
- Net demand function for good 1 by agent A/B: $e_1^A = x_1^A(p_1) - w_1^A$, $e_1^B = x_1^B(p_2) - w_1^B$
- Aggregate excess demand for Good 1: $z^1(p_1) = e_1^A + e_1^B$

$\Rightarrow$ The value of the sum of the agents' excess demands must equal zero (for any price).

- No aggregate demand for each good in equilibrium, $z^1(p_1^*) = 0 \Rightarrow z^2(p_1^*) = 0$.

Walrasian Equilibrium is Pareto Efficient

Pareto Efficiency: there is no way to make all the people involved better off.



A Pareto efficient allocation. At a Pareto efficient allocation such as M, each person is on his highest possible indifference curve, given the indifference curve of the other person. The line connecting such points is known as the contract curve.

- Pareto efficient allocations: points that satisfy $MRS^A = MRS^B$
- **Contract curve/Pareto Set**: the line connecting all Parato efficient allocation points.
- **First Theorem of Welfare Economics**: given endowment, all market equilibria are Pareto efficient.
- **Second Theorem of Welfare Economics**: if all agents have convex preferences, each Pareto efficient allocation is a market equilibrium with an appropriate assignment of endowments.

An Example

Exchange Between Agents with Cobb-Douglas Preferences

- Amy: $u^A(x_1, x_2) = \frac{1}{2}\ln x_1 + \frac{1}{2}\ln x_2$; Bob: $u^B(x_1, x_2) = \frac{1}{4}\ln x_1 + \frac{3}{4}\ln x_2$.
- Initial Endowments: $w^A = (8, 2)$, $w^B = (2, 8)$.

- Set $p_1 = 1$ and focus on p_1.
- Given p_1, Amy's"income" $m^A = 8p_1 + 2$, Bob's"income" $m^B = 2p_1 + 8$.

$\Rightarrow$ Amy's optimal Bundle: $(x_1^A = \frac{1}{2}\frac{8p_1+2}{p_1}, x_2^A = \frac{1}{2}\frac{8p_1+2}{1})$

$\Rightarrow$ Bob's optimal Bundle: $(x_1^B = \frac{1}{4}\frac{2p_1+8}{p_1}, x_2^B = \frac{3}{4}\frac{2p_1+8}{1})$

- Market Clearing for Good 2: $x_2^A + x_2^B = 2 + 8 \Rightarrow p_1^* = \frac{6}{11}$.
- Final Allocation: $x^A(\frac{35}{6}, \frac{35}{11})$, $x^B(\frac{35}{6}, \frac{75}{11})$
- Pareto Set: $\{(x_1^A, x_2^A)\colon MRS^A = \frac{x_2^A}{x_1^A} = MRS^B = \frac{x_2^A}{3x_1^A} = \frac{10 - x_2^A}{3(10 - x_1^A)}\}$

$\Rightarrow$ Writing x_2^A as a function of x_1^A is the Contract Curve.

Thank You!