Preferences and Utility (Ch3/4)

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Consumption Theory

Components of Market Equilibrium

- Demand side: **Consumer Theory**
- Supply side: Producer Theory
- Equilibrium

Consumer Theory

- Simple intuition: assumes that consumers choose the best (Ch3, preference) bundle of goods they can afford (Ch2).
- Budget Constraint: describes what a consumer can afford
- Preferences: describe what a consumer thinks as “the best”

Outline

Describing Preferences

- Translating verbal statements into economic notation
- Establishing assumptions about rational preferences

Illustrating Preferences

- Using indifference curves

Utility Representation of Preferences

- Defining the utility function

Preference Refers to How Individuals Compare Options

Example: Ranking My Professors

- Each professor is a **bundle** of attributes (lecture quality, personality).
- Suppose their attributes are: **Prof. Zhao**: (9 stars, 5 stars); **Prof. Qian**: (8 stars, 4 stars); **Prof. Sun**: (7 stars, 9 stars)

Preference Refers to How Individuals Compare Options

Example: Ranking My Professors

- Each professor is a **bundle** of attributes (e.g., lecture quality, personality).
- Suppose their attributes are: **Prof. Zhao**: (9 stars, 5 stars); **Prof. Qian**: (8 stars, 4 stars); **Prof. Sun**: (7 stars, 9 stars)

Your ranking reflects your **preference**

- The ranking comes from binary comparisons: *"I prefer Prof. Zhao over Prof. Qian, Prof. Qian over Prof. Sun, and so on."*

How do you determine the ranking?

- Based on **satisfaction**: *"Prof. Zhao gives me the most satisfaction, followed by Prof. Qian, then Prof. Sun, etc."*
- **Utility** is a way to measure satisfaction. (Thus, rating professors is possible.)

Translating Verbal Statements into Economic Notation

Strictly Preferred ($\succ$), Weakly Preferred ($\succeq$), and Indifferent ($\sim$)

1.	"I like A more than B"	$A \succ B$	A is strictly preferred to B (by me)
2.	"I like B more than A"	$B \succ A$	I strictly prefers B to A
3.	"I like A and B the same"	$A \sim B$	A is indifferent to B
(4.)	"I think A is at least as good as B"	$A \succeq B$	A is weakly preferred to B

Q: Can One Symbol Represent All (Three) Possible Comparison Outcomes?

- Use $\succeq$

How Do You Describe Your Satisfaction Level (Utility)?

- The **utility function** assigns a numerical value to each bundle to represent your satisfaction level.
- $A \succeq B \Rightarrow U(A) \geq U(B)$

Three Assumptions on Preference $(\succeq)$ as a Binary Relationship

Completeness

- Every two options can be compared, leading to a complete ranking list.
- Either $A \succeq B$ or $B \succeq A$ (or both).

Reflexivity

- Any bundle is at least as good as itself.
- $A \succeq A$.

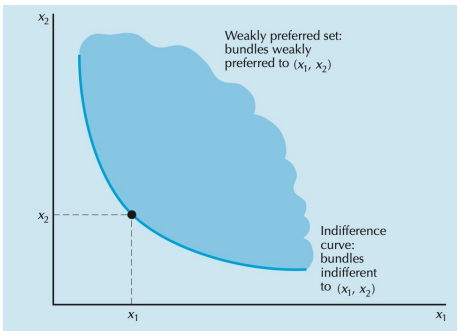
Transitivity

- Preferences are logically consistent, leading to a unique and stable ranking.
- If $A \succeq B$ and $B \succeq C$, then $A \succeq C$.

Are These Assumptions Obvious?

To Illustrate Preferences

The indifference curve represents all equally preferred bundles

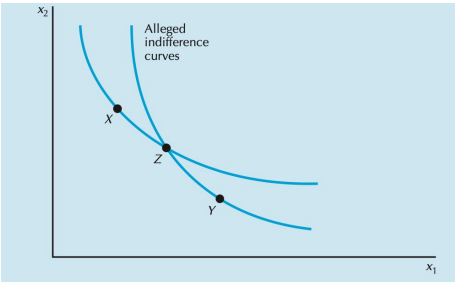


- Consider two goods only: x_1 and x_2
- x_1: what we focus on
- x_2: composite good that represents anything else other than x_1 income is m (RMB)
- Indifference Curve: $U(x_1, x_2) = u_0$

  Bundles at the same indifference curve are equally preferred

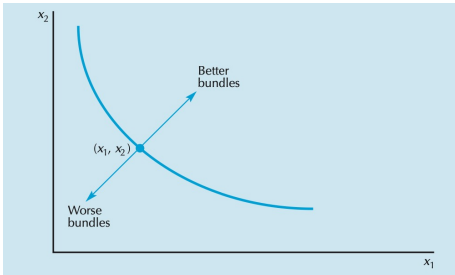
To Illustrate Preferences

Indifference curves cannot cross



- Different curves represent different sanctification levels (utilities).
- If indifference curves across X, Y, and Z would all have to be indifferent to each other
- and thus could not lie on distinct indifference curves.

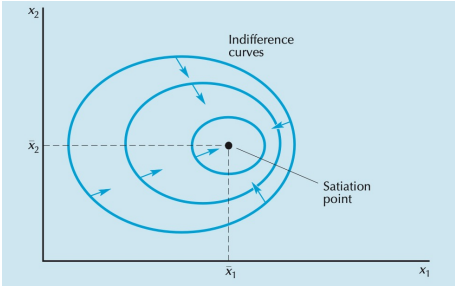
Well-behaved Preference: (Positive) Monotonicity



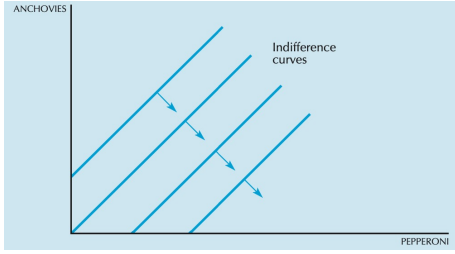
- **Monotonic Preference:** More of both goods is better,
  $A(x_1 + \Delta_{x_1}, x_2 + \Delta_{x_2}) \succ B(x_1, x_2)$
- Curve 1: $U(x_1, x_2) = u_1$; Curve 2: $U(x_1, x_2) = u_2$.
- If $u_2 > u_1$, then Curve 2 must lie above or to the right of Curve 1

- **Strict Monotonic Preference:** More of any good is better,
  $B(x_1 + \Delta_{x_1}, x_2) \succ A(x_1, x_2)$,
  $C(x_1, x_2 + \Delta_{x_2}) \succ A(x_1, x_2)$
- $\Rightarrow$ negative slope

Preference Violating Monotonicity



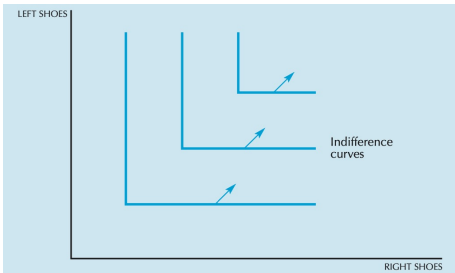
(a) Satiated Preference



(b) x_2 is a bad

Extreme Preferences: Two Goods Cannot Be Substituted

Perfect Complements

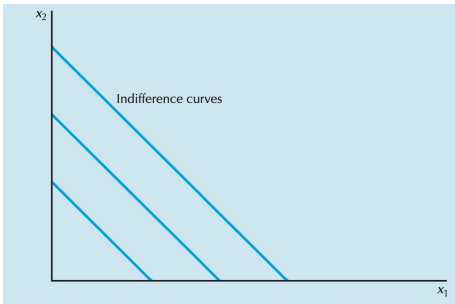


- **Perfect complements** are goods that are always consumed together in fixed proportions.
- Say one unit of x_1 must be consumed with one unit of x_2 together,
- Then the utility function: $U(x_1, x_2) = U(min\{x_1, x_2\})$

- Q: Is this a strict monotonic preference?

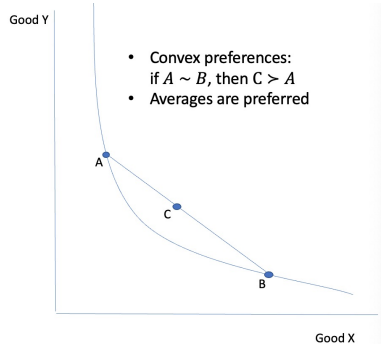
Extreme Preferences: Two Goods Can Be Fully Substituted

Perfect Substitutes



- Two goods are **perfect substitutes** if the consumer is willing to substitute one good for the other at a constant rate.
- Say one unit of x_1 leads to the same satisfactions level as one unit of x_2
- Then the utility function: $U(x_1, x_2) = U(x_1 + x_2)$
- Indifference curves are straight lines: $x_1 + x_2 = a_n$

Well-behaved Preference: Convexity

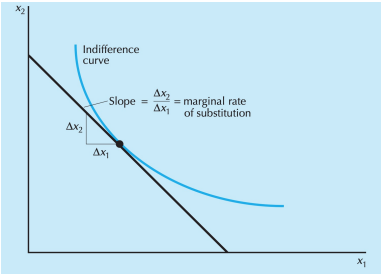


- **Convex preference:** Weighted averaged bundles are weakly preferred to extremes.
- Bundle $A(x_1, y_1)$, Bundle $B(x_2, y_2)$
- Weighted averaged bundles: $C(tx_1+(1-t)x_2, ty_1+(1-t)y_2), t \in (0,1)$
- Convexity: $C \succeq A, C \succeq B$

- **Strict Convex preference:** Weighted averaged bundles are strictly preferred to extremes.
- $\Rightarrow$ slope of the indifferent curve becomes flatter as you move right

Slope of Indifference Curve

Marginal Rate of Substitution (MRS)

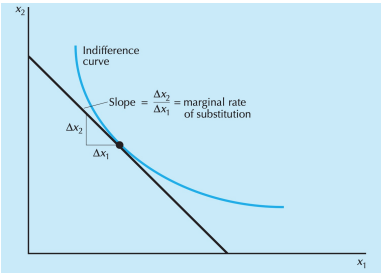


- Slope:= $\frac{\Delta x_2}{\Delta x_1}$
- Marginal Rate of Substitution (MRS) **of Good 1** (for Good 2):= $\frac{\Delta x_2}{\Delta x_1}$

- Strict Monotonicity: To maintain in the same indifference curve, taking a little of Good 1 ($\Delta x_1 < 0$) away requires giving him a little more of Good 2 ($\Delta x_2 \geq 0$)
- Strict Convexity: The more of Good 1 you have already consumed, the less of Good 2 is needed to substitute for additional units of Good 1.

Move Along A Given Indifference Curve

Marginal Rate of Substitution (MRS)

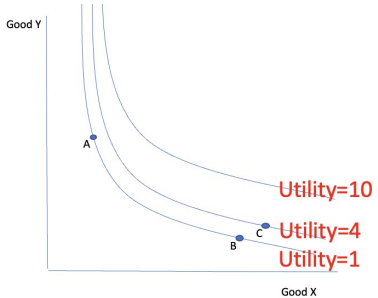


- MRS:= $\frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$
- Marginal Utility of Good 1 := MU_1
- $MU_1 := \frac{\partial U}{\partial x_1} = \frac{U(x_1+\Delta_{x_1},x_2)-U(x_1,x_2)}{\Delta x_1}$

- $MU_1 * \Delta x_1 + MU_2 * \Delta x_1 = \Delta U = 0$
- MRS represents the (relative) marginal willingness to pay for Good 1 in terms of Good 2.

Labeling Different Indifference Curves

Utility Function: Assigning a Number to Every Bundle



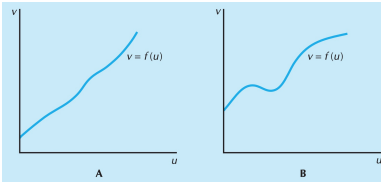
- The utility function should represent preferences: If $A \succeq B$, then $U(A) \geq U(B)$.
- The assigned number represents the utility level.

- The utility representation of a given preference is not unique:

  the slope of the indifference curve matters, but the specific labeling of different indifference curves does not.
- See Examples

Utility Representation of Preferences

(Positive) Monotonic Transformation



A positive monotonic transformation. Panel A illustrates a monotonic function—one that is always increasing. Panel B illustrates a function that is *not* monotonic, since it sometimes increases and sometimes decreases.

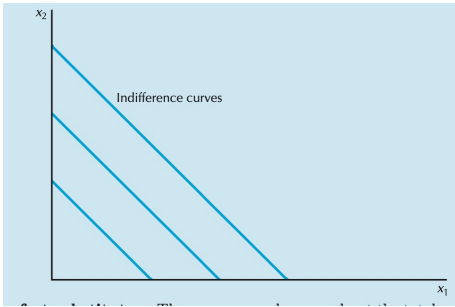
- Positive monotonic function $f(u)$: $u_1 > u_2 \Rightarrow f(u_1) > f(u_2)$.
- If $U(x_1, x_2)$ represents a preference, then $V = f(u)$ represents the same preference.

- Bundle $A(x_1, x_2)$ might located at difference indifference curve: $U(x_1, x_2)$ to $V(x_1, x_2) = f(U)$
- Slope of indifference curve is the same (Chain Rule):

$$-MRS = \frac{MU_1}{MU_2} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{f'(U)\frac{\partial U}{\partial x_1}}{f'(U)\frac{\partial U}{\partial x_2}} = \frac{\frac{\partial V}{\partial x_1}}{\frac{\partial V}{\partial x_2}}$$

Utility Representation of Perfect Substitutes

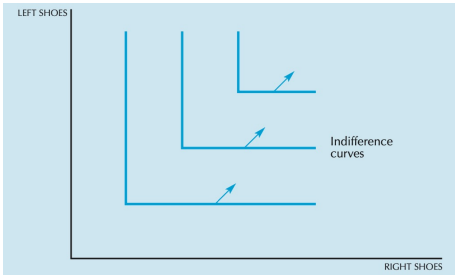
Basic Form: $U(x_1, x_2) = ax_1 + bx_2$.



- Two goods are **perfect substitutes** if the consumer is willing to substitute one good for the other at a constant rate.
- $MRS = -\frac{a}{b}$: one unit of x_1 leads to the same satisfaction level as $\frac{a}{b}$ unit of x_2
- Check the Marginal Utility and MRS of $V(x_1, x_2)$:
  - Case 1: $V := f(U) = kU + b$
  - Case 2: $V := f(U) = U^2$

Utility Representation of Perfect Complements

Basic Form: $U(x_1, x_2) = \min\{ax_1, bx_2\}$.

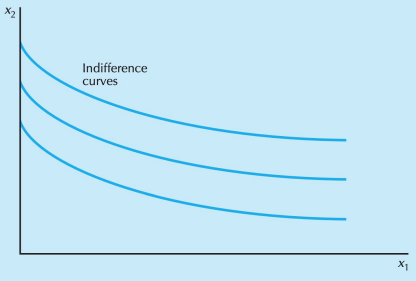


- **Perfect complements** are goods that are always consumed together in fixed proportions.
- one unit of x_1 must be consumed with $\frac{b}{a}$ unit of x_2 together
- MRS?

- Check the Marginal Utility and MRS of $V(x_1, x_2)$:
  - Case 1: $V := f(U) = kU + b$
  - Case 2: $V := f(U) = U^2$

Utility Representation of Quasi-linear Preferences

Basic Form: $U(x_1, x_2) = v(x_1) + x_2$.

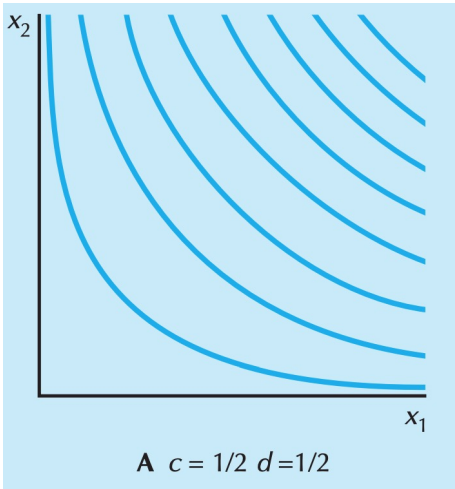


Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

- Recall previous examples when we interpret Good 2 as the **money** spent on all goods except Good 1.
- $MRS = v'(x_1)$
- Check the Marginal Utility and MRS of $V(x_1, x_2)$:
  - Case 1: $V := ln(x_1) + x_2$
  - Case 2: $V := \sqrt{x_1} + x_2$

Utility Representation of Cobb-Douglas Preferences

Basic Form: $U(x_1, x_2) = x_1^c x_2^d$.



- Named after Paul Douglas and Charles Cobb.
- Indifference curves look well-behaved.
- $MRS = \frac{cx_2}{dx_1}$
- Check the Marginal Utility and MRS of $V(x_1, x_2)$:
  - Case 1: $V := ln(U)$
  - Case 2: $V := U^2$

Summary

What We Have Learned

- Three fundamental assumptions of preference:
  Completeness, Reflexivity, and Transitivity.
- Two additional assumptions for well-behaved preferences:
  Monotonicity and Convexity.
- $\Rightarrow$ Diminishing Marginal Rate of Substitution.
- A monotonic transformation of a utility function represents the same preferences.

What's Next?

- Consumer Choices (Ch5): Finding the best affordable bundle (Ch2) that maximizes utility (Ch3-4).


Thank You!