Non-Clairvoyant Dynamic Mechanism Design: Experimental Evidence

Shan Gui, Daniel Houser

George Mason University

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Main Takeaways

What do we do?

- ▶ Bring Non-Clairvoyant Environment (Mirrokni et al., 2020) into Lab.
- ▶ Test the optimal Non-Clairvoyant dynamic mechanism (NC).
- ► Compare the performance with Repeated Static optimal mechanism (RS).

What do we find?

- ▶ Dynamic NC works well as theory predicts More revenue in some scenarios.
- ▶ Participants overbid less in NC.
- ightharpoonup Risk aversion \rightarrow No full participation \rightarrow Revenue loss in NC.

Optimal Dynamic Mechanism Design

- ▶ How the seller establish the rules of allocation and price over **multi-period** for the best revenue, as the buyer receives private information over time.
 - ► Long-term principal-agent relationship
 - Repeated selling of perishable goods
- ▶ Dynamic mechanism improves revenues and efficiency (Baron & Besanko, 1984).

Non-Clairvoyance

Clairvoyance: Future demand is known at the beginning.

- ▶ Complicated, non-intuitive, lack of General Form.
- ▶ Buyers tend to have biased forecast on future demand (DellaVigna & Malmendier, 2006).

Non-Clairvoyance: Future demand is not accessible at the beginning.

- ▶ No need to share unbiased belief.
- ► General Form.



 F_2 is unknown in Day 1



 $v_1 \sim F_1$



 $v_2 \sim F_2$?

Mechanisms under Non-Clairvoyant Environment

 \Rightarrow Optimal clairvoyant revenue Rev^* is unachieveble.

RS: Repeated Static optimal mechanism (Myerson, 1981)

▶ Rules in two days are independent of each other

Maximize intra-period revenue for each period separately.

 $\Rightarrow \frac{Rev^{RS*}}{Rev^*}$ could be arbitrarily small (Papadimitriou et al., 2016)

NC: Non-Clairvoyant optimal dynamic mechanism (Mirrokni et al., 2020)

▶ Rules in Day 2 depends on bid in day 1

Best Revenue Guarantee:
$$\Rightarrow \frac{Rev^{NC*}}{Rev^*} \ge \frac{1}{\alpha}$$

Achieve at least $\frac{1}{2}$ revenue produced by optimal clairvoyant mechanism under all scenarios in **two-period single-buyer** case.

When can Non-Clairvoyant dynamic mechanism do better?

Theoretically, Non-Clairvoyant mechanism can not always outperform.

Optimal Intra-period Revenue Optimal Inter-period Revenue

| RS | 100% | zero |
|----|-------------|-------------|
| NC | $\geq 50\%$ | $\geq 50\%$ |

Relative size of optimal intra- and inter-period revenues is the key.

- ightharpoonup Scenario A: Optimal inter period revenue is larger \Rightarrow NC outperforms.
- ightharpoonup Scenario B: Optimal intra period revenue is larger \Rightarrow RS outperforms.

Experimental Design 2 * 2

Two Mechanisms

- ▶ Non-Clairvoyant Dynamic Mechanism (NC)
- ▶ Repeated Static Mechanism (RS)

Two Scenarios

$$F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\}, \mathbb{E}_A = 3.$$

$$F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\}, \mathbb{E}_B = 6.$$

▶ Scenario A (S_A) : NC generates more revenue than RS.

$$F_1 = F_A, \ F_2 = F_B \implies REV^{RS} = 4, \ REV^{NC} = 4.5 \uparrow 12.5\%$$

▶ Scenario B (S_B) : NC generates less revenue than RS.

$$F_1 = F_B, \ F_2 = F_A \implies REV^{RS} = 4, \ REV^{NC} = 3.5 \downarrow 12.5\%$$



Non-Clairvoyant Environment

- **Buyer**: Participant
- ▶ **Robot Seller**: Experimenter, c = 0
- **Two periods**: Buyer can buy one item in each period from seller, t = 1, 2.
- ▶ Non-clairvoyance: The distribution of buyer's value (F_t) is common knowledge only in that period
- ► Incomplete Information :
 - 1. Only buyer knows his value for the item in each period, v_t .
 - 2. Buyer's value is drawn **independently**.

Variables for Environment

▶ Endowment: E = 50

Mechanism - Repeated Static (RS)

Period 1

- \triangleright Seller sets a reserve price r_1 based on the distributional knowledge F_1 .
- ▶ Buyer learns his value (v_1) , makes a bid : b_1
- ▶ Buyer can get the item only when $b_1 \ge r_1$ and pay $p_1 = r_1$.

Period 2

 $ightharpoonup F_2 \Rightarrow r_2, v_2 \Rightarrow b_2, \text{ pays } p_2 = r_2 \text{ if } b_2 \geq r_2$

Myerson's Auction

monopoly price:
$$r_1 = r_2 = 2$$
 $r_A = 2 \in \{arg \max_r r \cdot P(v_A > r)\}, \quad r_B = 2 \in \{arg \max_r r \cdot P(v_B > r)\}$



Mechanism - Non-Clairvoyant Dynamic (NC)

How the dynamic mechanism work?



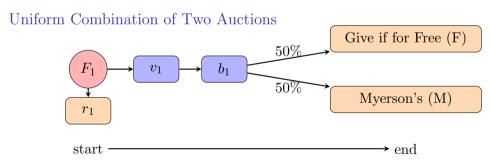
Half chance of free item in period 1



Half chance of upfront fee in period 2

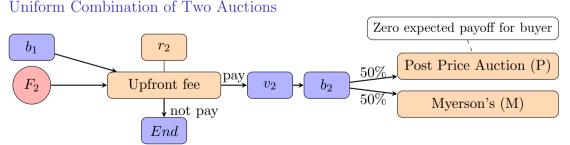
Non-Clairvoyant Mechanism in Period 1

- \triangleright Seller sets a fixed reserve price r_1 based on the distribution F_1 .
- ▶ Buyer learns his value (v_1) , makes a bid : b_1
- Buyer has 50% chance to get the item for free: $p_1 = 0$; Otherwise, buyer can get the item only when $b_1 \ge r_1$ and pay $p_1 = r_1$.



Non-Clairvoyant Mechanism in Period 2

- ▶ Seller sets an upfront fee $s_2 = \min(b_1, E(v_2))$.
- ▶ Buyer decides pay or leave. If buyer leave (enter = 0), game over.
- ightharpoonup If buyer pays, (enter = 1),
 - ▶ Buyer learns his value, v_2 , and makes a bid: b_2
 - ▶ Buyer has 50% chance to get refund on the upfront fee (luck = 1).
 - Seller sets two reserve prices (r_2) based on the F_2 , luck for each given m_2 , Buyer can get the item only when $b_2 \geq r_2$ and pay $p_2 = r_2$



Hypotheses

Hypothesis 1 - On Revenue Comparison

- ▶ In Scenario A (S_A) , the non-clairvoyant mechanism (NC) generates greater revenue than the repeated static mechanism (RS);
- ▶ In S_B , NC generates less revenue than RS.

Hypothesis 2 - On Individual Rationality

▶ Some buyers choose not to pay the upfront fee, such that the experimental revenue of the non-clairvoyant mechanism is less than its theoretical prediction.

Hypothesis 3 - On Incentive Compatibility

▶ Participants' bids are closer to true value under NC than RS.

Experiments

▶ 256 George Mason Students. September to November 2021.

| | ${\bf Scenario}{\bf A}$ | | | ario B |
|-----------------|-------------------------|-----------------|------|---------------|
| Treatment | Non-Clairvoyant | Repeated Static | NC | \mathbf{RS} |
| Age | 21.6 | 22.3 | 21.9 | 22.7 |
| Gender (Male=1) | 0.48 | 0.44 | 0.52 | 0.47 |
| Risk aversion | 4.46 | 4.90 | 4.55 | 4.63 |
| Observation | 64 | 64 | 64 | 64 |

Table 1: Summary Statistic

Results

Result 1.

Experimental observations match theoretical predictions.

- ▶ In S_A , the non-clairvoyant dynamic mechanism gains more revenue than the repeated static mechanism.
- ▶ In S_B , the non-clairvoyant dynamic mechanism gains less revenue than the repeated static mechanism. mechanism.

Experimental Revenue Comparison - Period 1

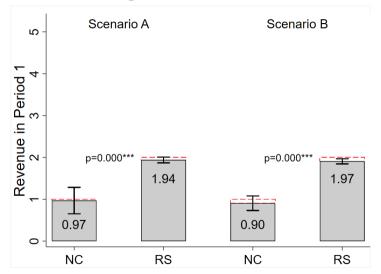


Figure 3: Revenues of Period 1 in each Treatment

Experimental Revenue Comparison - Period 1 & Period 2

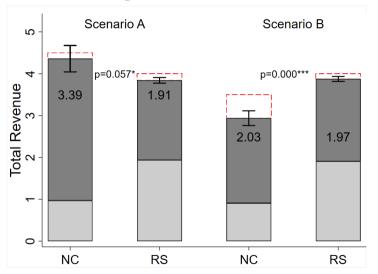


Figure 4: Revenues in each Treatment

Results

Result 2.

Risk aversion deters buyers from participating in the second period in NC.

- ▶ In S_A , 4 buyers quit the second period, and the number doubles in S_B .
- ▶ Revenue from NC being less than theoretically predicted.

Revenue Loss Decomposition

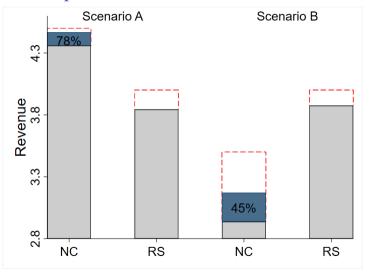


Figure 5: Revenues Increase if all Buyers enter in Period 2.

Why not Pay the Upfront Fee (membership fee)

- ▶ "Since I got a profit the first time I didn't want to go again with my luck"
- ▶ "Risk vs Reward..... I got lucky and did not have to pay."
- ▶ "Based on the membership fee."
- "didn't want to take any big risks so I just lowballed my offers and refused to take the membership"
- "i read the instructions carefully. i think the second period isn't worth losing the points i had to pay membership fee and could only get the item by bidding higher than the price set by the seller..... honestly, i haven't been feeling lucky so i'd rather not take my chances. so i tried not to lose money in the first period and just left it as is."

Risk Aversion Affects Second-period Participation Indirectly

| | DV: Enter in Period 2 (=1) | | | | |
|--------------------------|----------------------------|--------------|--|--|--|
| | (1) | (2) | | | |
| Scenario A (=1) | 0.17** | 0.25^{*} | | | |
| | (0.08) | (0.13) | | | |
| $notfree_1 (= 1)$ | 0.07 | 0.08 | | | |
| | (0.07) | (0.11) | | | |
| Scenario A * $notfree_1$ | -0.18^* | -0.14 | | | |
| | (0.10) | (0.17) | | | |
| risk aversion | -0.01 | -0.03 | | | |
| | (0.01) | (0.02) | | | |
| $pay of f_1$ | 0.00 | 0.00 | | | |
| | (0.01) | (0.01) | | | |
| $upfront_2$ | -0.01 | -0.03 | | | |
| | (0.03) | (0.05) | | | |
| Controls | | \checkmark | | | |

Table 2: Regression of Participation Choice on Risk attitude.

Results

Result 3.

- ► Generally overbid.
- ▶ Buyers overbid less under Non-Clairvoyant mechanism when the distribution of their valuation has low variance.

Bid-Value Ratio Comparison

| Bid/value | Non-Clairvoyant Dynamic | Repeated Static | (p-value) 1 |
|-----------------------|-------------------------|------------------|----------------|
| F_A (Low variance) | $1.264 \ (0.04)$ | 1.379(0.04) | 0.060* |
| F_B (High variance) | $1.194\ (0.05)$ | $1.251 \ (0.04)$ | 0.392 |
| (p-value) | 0.116 | 0.008*** | |

Table 3: Bid-Value Ratio Comparison

Conclusions

- ▶ We find the experimental observations are consistent with theoretical predictions: the optimal Non-Clairvoyant dynamic mechanism outperforms the optimal Repeated Static mechanism when it is predicted to do so.
- ▶ Buyers' risk attitudes matter in the success of Non-Clairvoyant mechanism.
- ► Randomization in non-clairvoyant mechanism leads buyers to overbid less.

Discussion

How should sellers choose between mechanisms?

- ▶ NC generates more revenue when the scenario is "good".
- ▶ NC encourages more accurate valuation information.
- ▶ NC works better when buyers are not risk-averse.

Future work

- ► Can participants (human sellers) set up correct rules?
- Experiments on multi-buyer with more than 2 periods.

Thank you!

Requirements of Mechanisms under Non-clairvoyant Environment

Seller sets up:

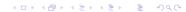
- ightharpoonup Allocation rule $x \in \{0,1\}$: whether buyer can get the item or not
- ▶ Price rule $p \in \mathcal{R}$: how much to pay if buyer gets the item

Buyer:
$$\max_{\{b_1,b_2\}} u_1 + u_2 = (x_1v_1 - p_1) + (x_2v_2 - p_2)$$

- Dynamic Incentive Compatibility (DIC)
 For a buyer, it is optimal to bid true value in each period
- Ex-post Individual Rationality (EPIR) $u_1 + u_2 > 0$, for all realization of v_1, v_2

Intra-period Revenue & Inter-period Revenue

- ▶ Intra-period revenue: independent revenue, using information within that period ⇒bounded my Myerson's revenue.
- ► Inter-period revenue: dependent revenue, linking past periods with current period ⇒ bounded by current-period expected value.



Revenue Comparison in Scenario A

$$F_1 = F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\} \mathbb{E}_1 = 3.$$

$$F_2 = F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\}, \mathbb{E} = 6.$$

▶ Non-Clairvoyant Mechanism increases revenue, ↑ 12.5%

| Revenue in S_A | Non-clairvoyant Dynamic | | Repeated Static | | |
|---|--|----------|-----------------------|-----|--|
| Period 1 | Give for Free (F) Myerson's Auction (M) | 0 2 | Myerson's Auction (M) | 2 | |
| Period 2 | Post Price Auction (P) Myerson's Auction (M) | 5 2 | Myerson's Auction (M) | 2 | |
| Total | | 4.5 | | 4 | |
| Intra-period Revenue Inter-period Revenue | | 2 2.5 | | 4 0 | |

Table 4: Theoretical Revenues in Scenario A.

Revenue Comparison in Scenario B

$$F_1 = F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\}, \ \mathbb{E}_1 = 6.$$

$$F_2 = F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\}, \ \mathbb{E}_2 = 3.$$

▶ Non-Clairvoyant Mechanism decreases revenue, ↓ 12.5%

| Revenue in S_B | Non-clairvoyant Dynamic | | Repeated Static | | |
|---|--|----------|-----------------------|-----|--|
| Period 1 | Give for Free (F) Myerson's Auction (M) | 0 2 | Myerson's Auction (M) | 2 | |
| Period 2 | Post Price Auction (P) Myerson's Auction (M) | 3 2 | Myerson's Auction (M) | 2 | |
| Total | | 3.5 | | 4 | |
| Intra-period Revenue Inter-period Revenue | | 2 1.5 | | 4 0 | |

Table 5: Theoretical Revenues in Scenario B.

Reserve price (r_1, r_2) in Scenario A

$$\begin{array}{l} F_1=F_A=\{v,p(v)\}=\{(2,\frac{1}{2}),(4,\frac{1}{2})\},\ \mathbb{E}_1=3.\\ F_2=F_B=\{v,p(v)\}=\{(2,\frac{1}{2}),(4,\frac{1}{4}),(8,\frac{1}{8}),(16,\frac{1}{16}),(32,\frac{1}{16})\},\ \mathbb{E}_2=6. \end{array}$$

Period 1

▶ Myserson's Auction: $r_1 = 2$

Period 2

- ▶ If luck = 1, Myserson's Auction: $r_2 = 2$
- ▶ If luck = 0, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfrong \ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \ge 6$, $r_2^P = 2$ if $b_1 = 4$, $r_2^P = 8$ if $b_1 = 2$, and $r_2^P = 32$ if $b_1 = 0$.

Reserve price (r_1, r_2) in Scenario B

$$F_1 = F_B = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{4}), (8, \frac{1}{8}), (16, \frac{1}{16}), (32, \frac{1}{16})\}, \ \mathbb{E}_1 = 6.$$

$$F_2 = F_A = \{v, p(v)\} = \{(2, \frac{1}{2}), (4, \frac{1}{2})\}, \ \mathbb{E}_2 = 3.$$

Period 1

▶ Myserson's Auction: $r_1 = 2$

Period 2

- ▶ If luck = 1, Myserson's Auction: $r_2 = 2$
- ▶ If luck = 0, Posted Price Auction: r_2 satisfies

$$E_{v_2}[(v_2 - r_2)^+] = min(b_1, E(v_2)) = upfront \ fee.$$

Piece-wise function: $r_2^P = 0$ if $b_1 \ge 3$, $r_2^P = 1$ if $b_1 = 2$ and $r_2^P = 4$ if $b_1 = 0$.

Experimental Revenue Decomposition in Scenario A

| | Non-clairvoyant Dynamic | | | Repeated Static | | | |
|------------------|-------------------------|-------------------|--------------------|-----------------|----------------------|------------|------------|
| Revenue in S_A | Theory | Theory Experiment | | Theory | | Experiment | |
| Deste I 1 | Give it for free | 0 | 0 | M | 0 | 1.04(0.04) | |
| Period 1 | Myerson's auction | 2 | 1.94(0.06) | Myerson's | 2 | 1.94(0.04) | |
| Period 2 | Post Price Auction | 5 | 4.84(0.47) | Mrzangan'a | 4.84(0.47) Myerson's | 9 | 1.91(0.05) |
| renod 2 | Myerson's auction | 2 | 1.94(0.06) | Myerson s | 2 | 1.91(0.05) | |
| Total | • | 5 | 4.35 (0.32) | | 4 | 3.84(0.07) | |

Table 6: Revenue decomposition in S_A

Experimental Revenue Decomposition in Scenario B

| | Non-clairvoyant Dynamic | | | Repeated Static | | |
|------------------|-------------------------|-----|--------------------|-----------------|---|--------------------|
| Revenue in S_B | Theory | | Experiment | Theory | | Experiment |
| D 1 1 | Give it for free | 0 | 0 | M | 0 | 1.01(0.05) |
| Period 1 | Myerson's auction | 2 | 1.93(0.06) | Myerson's | 2 | 1.91(0.05) |
| Period 2 | Post Price Auction | 3 | 2.25(0.21) | Myerson's | 2 | 1.97(0.03) |
| | Myerson's auction | 2 | 1.75(0.12) | Mycroon 5 | _ | 1.01 (0.00) |
| Total | | 3.5 | 2.91 (0.18) | | 4 | 3.88 (0.06) |

Table 7: Revenue decomposition in S_B