Decision Making Under uncertainty

Hélène Fargier, fargier@irit.fr

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Exercice 1

The aim of this exercise is to experiment several criteria for decision making under (partial) ignorance on a small example.

In the following gamble, a ball is drawn from an urn containing containing 20 balls, numbered from 1 to 20. Balls are either White, Blue or Red. The player has to choose between 5 different possible gambles (or lotto tickets) about the possible color and numbering of a ball when drawn from the urn.

Alternative interpretation : gambles represent investments you could choose. Gambles 2 *and* 4 *represent more or less risky investments on the Iron market (that can either grow (even balls) or come down (odd balls)). Gambles* 1*,* 3 *and* 5 *represent more or less risky investments in steel industry (the steel stocks market shall either increase (red balls), be stable (white) or crash (blue balls))*

— Gamble 1 : Win 10 if the ball is Red, 0 otherwise.

— Gamble 2 : Win 10 if the ball is even, 0 if it is odd.

— Gamble 3 : Win 5 if the ball is Red, 2 if it is Blue, 0 otherwise.

— Gamble 4 : Win 5 if the ball is even, 1 otherwise.

— Gamble 5 : Win 3 if the ball is Red or White, 0 otherwise.

1. Rank the gambles according to the Mean value of utility, considering a neutral attitude with respect to risk (e.g. $u(x) = x$).

2. Rank the gambles according to the Mean value of utility, considering a optimistic attitude with respect to risk (e.g. $u(x) = exp(x)$).

3. Rank the gambles according to the Mean value of utility, considering a pessimistic attitude with respect to risk (e.g. $u(x) = ln(1 + x)$).

4. We now know that only 20% of the balls are Blue. How shall we rank the gambles ?

5. Same question but assuming only that the balls have been labeled without unicity of the numbers (still between 1 and 2) : Represent the knowledge by a mass function m and compute the pessimistic utility of each gamble

Exercice 2

A computer retailer (the decision maker) has a budget of 200 units (hundreds of euros). He can with his 200 units buy (decision A) or not (decision $barA$) from a wholesaler a batch of screens which can be, after purchase :

- either to be of good quality (event B), in which case the resale of the batch will bring him 600 units (hundreds of euros),
- or be of very bad quality (event M), in which case he will not resell anything

1. Which is the best decision for the criteria Wald, Maximax, Regret, Laplace, Hurwicz (for $u(0) = 0, u(200) = 200, u(600) = 300$) ?
2. Consider now the criterion of the average utility, and set $u(0) = 0$: at which condition on u will the decision-maker buy ?
3. The retailer knows that, depending on the arrivals, the probability that the lot is good varies between 0.5 and 0.8.
   — Compute the probabilistic pessimistic utility of each decision for the utility function $u : u(0) = 0, u(200) = 200, u(600) = 300$.
   — With $u(0) = 0$, under what condition on u does the pessimistic utility maximizing retailer buy ?
4. If the retailer esimates that the probability of event B is around 0.7 (between 0.6 and 0.8), still for the utility function $u : u(0) = 0, u(200) = 200, u(600) = 300$, should he buy ?
5. The retailer gives events B and M the probabilities : $P(B) = 0.6$, $P(M) = 0.4$.
   — Compute the expected utility of each decision for the utility function $u : u(0) = 0, u(200) = 200, u(600) = 300$.
   — With $u(0) = 0$, at what condition on u will the decision maker buy ?
   — With $u(0) = 0$, under what condition on u would you say the decision maker is optimistic ?
   .

Exercice 3

Consider a small investor who likes to save 3000 euros. He can either buy a portfolio of auctions, use his savings passbook , or buy an ethical porfolio . According to the financial press, the probability of growth of the economy will vary between 0.1 and 0.3, and the probability of stability is between 0.6 and 0.8 (a declining is thus non impossible). The following table gives an estimation of the value of each porfolio after 4 years (depending of the behaviour of the economy) :

	Growth	Stability	Crash
Auctions	6000	4000	1000
Ethical	4000	3000	2000
Passbook	3000	3000	3000

1. Compute the lower probability of each of the following events : $\{Growth, Stability, Crash\}$, $\{Growth\}$, $\{Stability\}$, $\{Growth, Stability\}$, $\{Crash\}$
2. Compute the Pessimistic utility (or equivalently the Choquet utility) of each decision, on the basis of :
   — a linear utility function, e.g. : $u(x) = \frac{x}{1000}$
   — a pessimistic utility function, e.g. : $u(x) = ln(x)$

Exercice 3

Consider a decision problem with 3 states bearing on tomorrow's meteo : nice weather, variableand rain : normally it should not rain. Thisinformation is capture by the following mass function :

$$m(\{nice, variable\}) = 0.7, m(\{rain\}) = 0.3$$

Let us study two decision decisions : outdoors climbing and indoors climbing. The utility degrees are the following :
- Indoors climbing : 0.5 for any state
- Outdoors climbing : utility degree 1 it the weather is nice, 0.8 if variable, utilité 0 if it rains

Question 1 To what extend is it certain that it will not rain ? To what extend is it plausible that the weather will not be nice ?

Question 2 Which is the pessimisitc global utility of each decision ? which one shall we choose ?

Exercice 5

A judge knows this :
— Big Boss has decided that Mr. Jones must die ;
— 3 possible killers : Peter, Quentin, Mary ;
— Big Boss designates the sex of the killer (tossing a coin) ;
— No idea on the choice between Peter and Quentin, in the case where a man is chosen ;
— Mr. Jones is killed by a Big Boss killer
Question : Who killed Mr. Jones ?

Model the problem in a suitable framework ; to what extent can we think that Peter is guilty, Mary is guilty, the guilty is Quentin or Peter, the guilty is Quentin or Mary ?

If we estimate the utility of sentencing a guilty person to 100*, the utility of sentencing an innocent person to* 100*, and the utility of letting a guilty person go free to* 50*, what would be the best decision ?*

Exercice 6

A lotto ticket costs x euros. There are 3 possible oucomes : either a payoff of 10 euros (with a probability equal to 1/50) or a payoff of 1.000.000 euros a (with a probability equal to 1/2.000.000), or no payoff.

1. Which is the expected gain for such a ticket (when bought) ?
2. Consider a decision maker who maximizes the expected gain - write the condition on x that leads him to buy a ticket
3. Consider a decision maker who maximizes the expected utility - write the condition on x and u that leads him to buy a ticket
4. Suppose that $u(10) = 10 \times u(1)$ and that the price of a ticket is 1 euros ; for which minimal value of $u(1.000.000)$ a decision maker who maximizes the utility of the gain will buy a ticket ?
5. Sociological studies show that people with low incomes buy lots of lotto ticket while people with high incomes seldom buy such tickets. Does that mean that the first ones are "bad" decision makers ?

Exercice 7

In the state of Syldavia, like in many other countries, the situation of the national economy oscillate between *expansion* (growth of the production) and *recession*.

The government must decide, for the next period, between 3 different politics :
— *subvention* : increasing the funding aids allowed to the people and to firms is a way to combat unemployment and to support the demand ;
— *status quo* : the funding aids are not increased nor decreased ;
— *disengaging* : the funding aids are suppressed in order to reduce the public deficit.

The income of the country (gross domestic product - GDP) depend on (i) the politics chosen at the beginning of the period and (ii) the economical situation (expansion or recession) of the country (this situation may depend on external factors, on the e.g. the international financial market). This income for each situation is summarized in the following table :

GDP	$subvention$	$statusquo$	$disengaging$
$recession$	50	30	10
$expansion$	100	110	120

1. When nothing can be forecasted about the situation of the economy (total ignorance), which are the best decision according to the rules presented in the course (maximin, leximin, etc). Put your result on the moodle pool
2. Which is the decsion rule which best modelize the behaviour of the government ? justify your choice.

3. The government can used the probabilities provided by the National Institute of Statistics and Economics about the future situation : ($p(recession) = 1 - p$ and $p(expansion) = p$). As a utility function simply use a linear one (make the assumption that the GDP is really what the government is looking for).
   — For which values of p is "subvention" the best of the decisions ?
   — For which values of p is "status quo" the best of the decisions ?
   — For which values of p is "disengaging" the best of the decisions ?

4. If the government is maximizing the expected gain, will it choose the status quo ? (justify your answer).

5. Public decision makers and politicians are often more sensitive to the image that the citizen have about the GDP than to the GDP itself ; in general, their utility is cautious in the sense of Bernoulli. Which decision will then be chosen for $p = 2/3$?

6. Actually, the National Institute for Statistics and Economics does not provide a precise probability, but only an estimate of p : the probability interval $[2/3, 1]$. Which shall be the best decision for the cautious decision maker ? justify your answer.