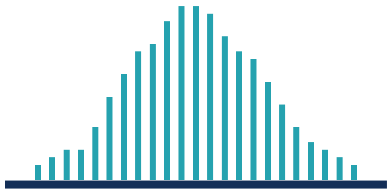
Modeling Risk with Monte Carlo Simulation

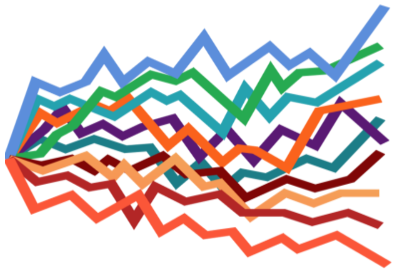
Monte Carlo Simulation Introduction

Course Introduction

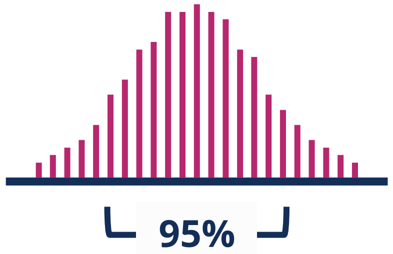
Past Observations



Probability Distribution



Many Future Scenarios

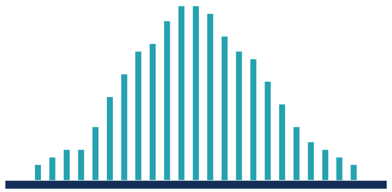


Quantify the Range of Scenarios

“It is better to be approximately right than precisely wrong.”

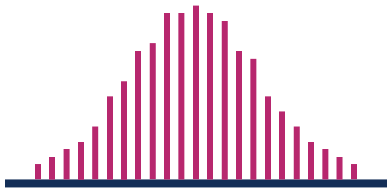
Warren Buffett

Course Introduction



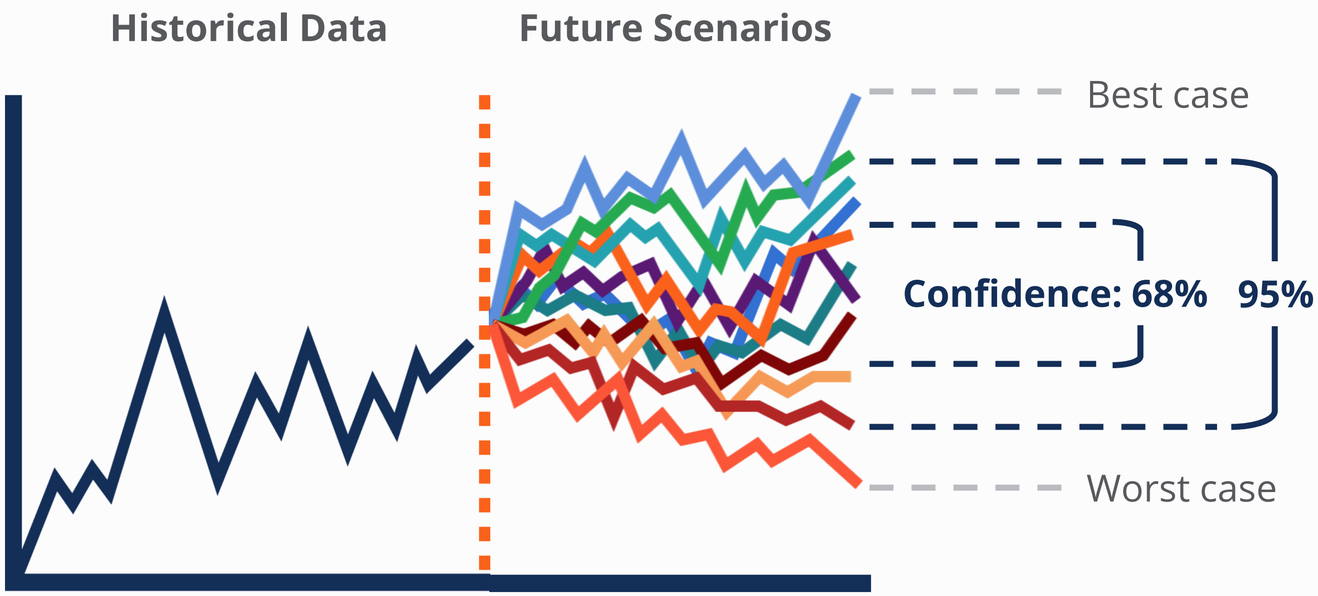
Past Events

Monte Carlo Simulation



Future Events

Stock Price Example



Mean of daily return: 0.13%

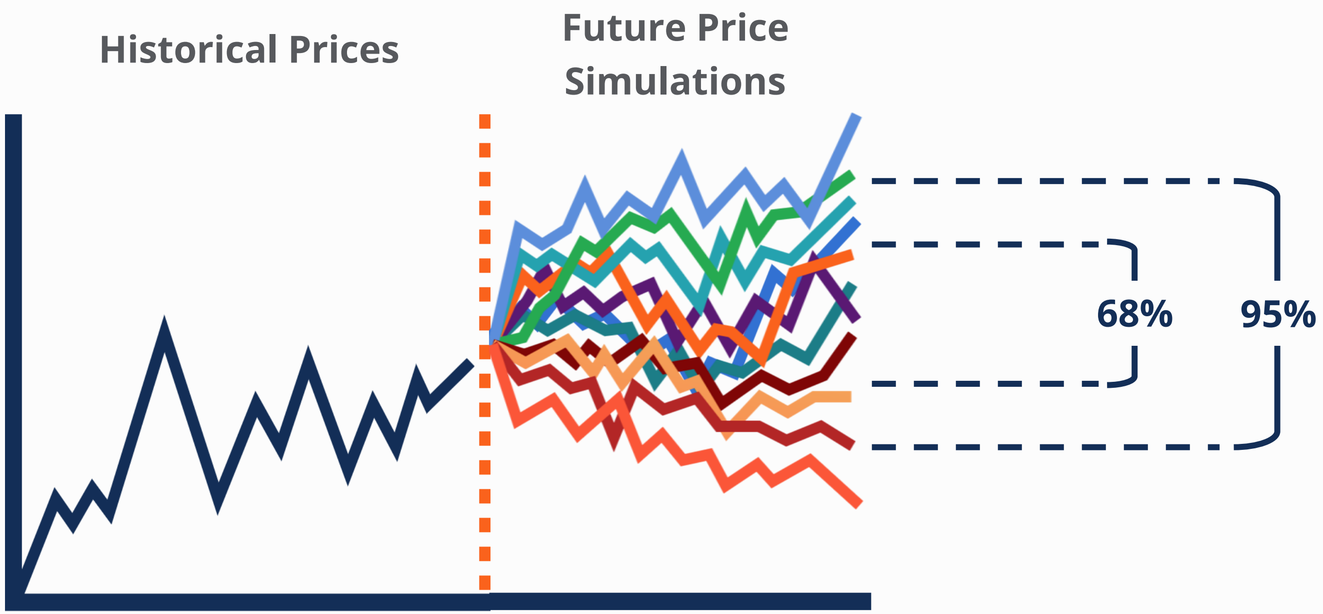
Std dev of daily return: 0.014%

Course Introduction

Case 1: Coin Flipping Model

Course Introduction

Case 2: Stock Price Model



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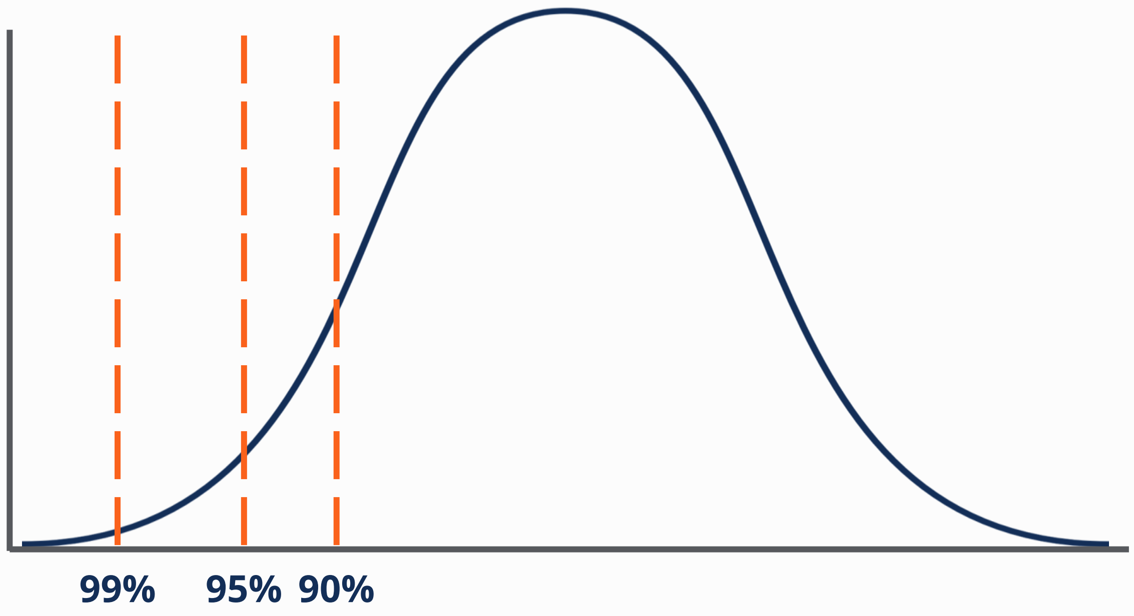
Case 3: Value at Risk Model



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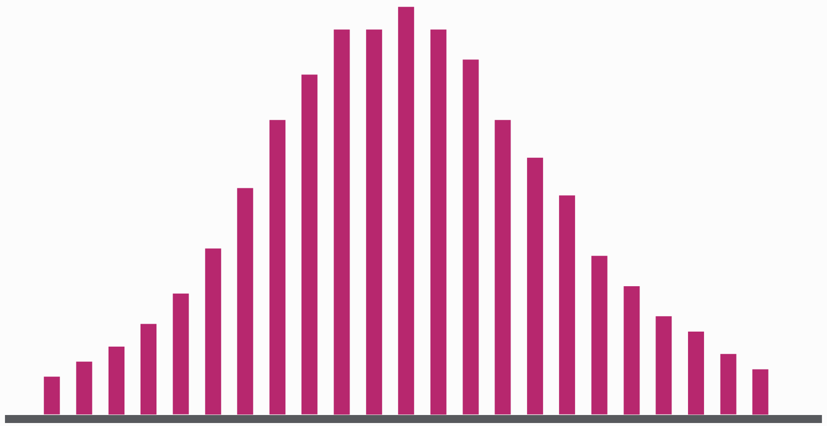
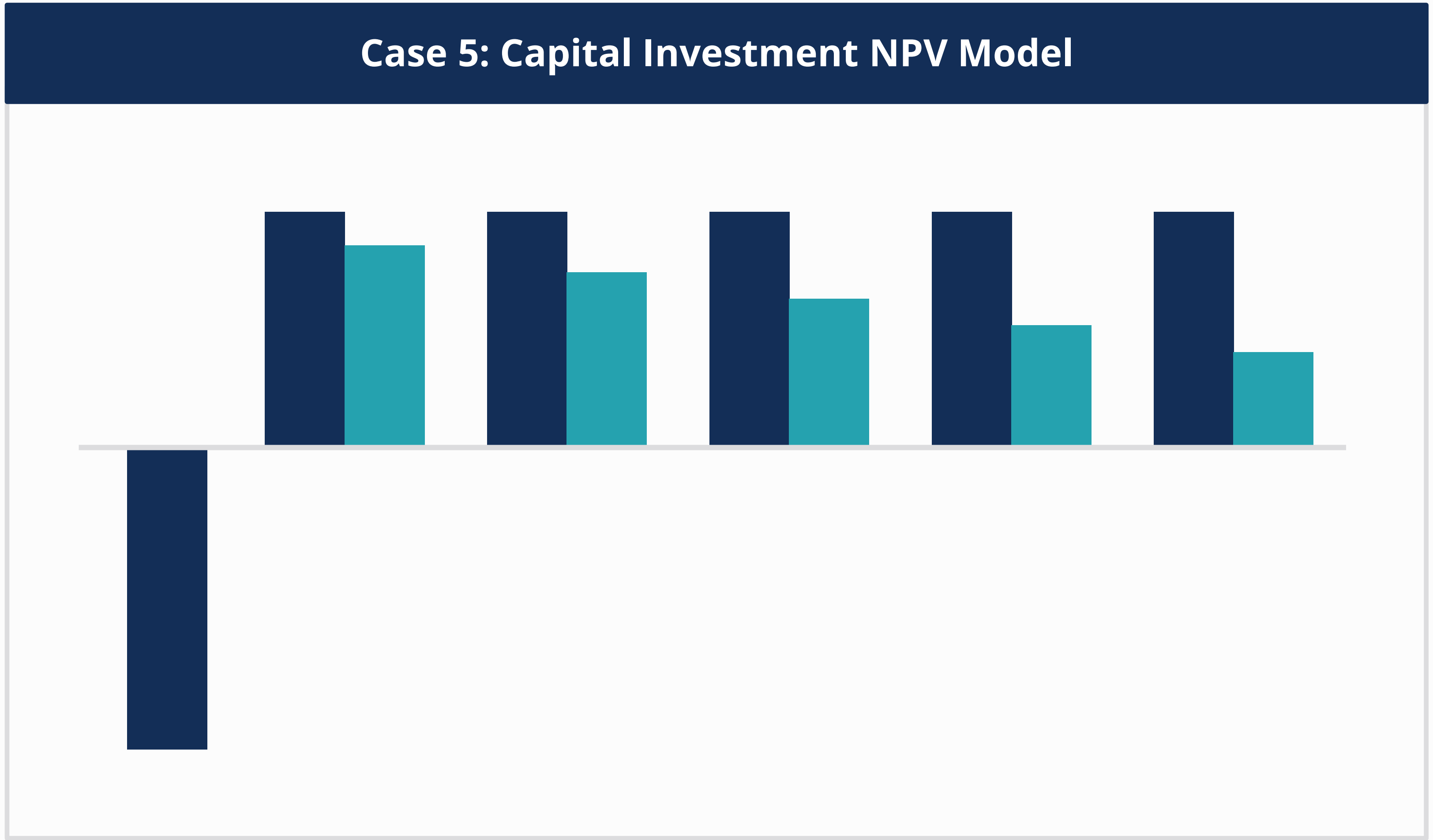
Case 4: Net Profit Model



Probability Distribution of Net Profit Simulations

Course Introduction



Course Introduction

Python Fundamentals

```python
# For replication from randomizer
import random
random.seed(3)
# Fun fact, different randomizer for numpy
np.random.seed(3)

for index in range(number_ports):
    #generate random weights
    numbers = np.array(np.random.random(4))
    weights = numbers/np.sum(numbers)

    #save weights
    all_weights[index, :] = weights

    #expected return
    returns_array[index] = np.sum(stock_return.mean() * 252 * weights)

    #expected volatility = square root(Weights-Transposed * Covariance Matrix * Weights)
    volatility_array[index] = np.sqrt(np.dot(weights.T, np.dot(stock_return.cov() * 252, weights)))

    #Sharpe ratio
    sharpe_array[index] = returns_array[index] / volatility_array[index]
```

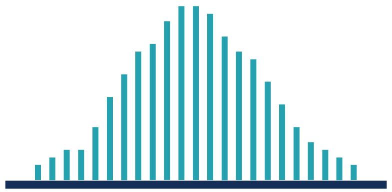
```python
#Print all weight combinations
print("All Weights:", all_weights)

#Print first weights
print("First combination:", all_weights[0])
```

Learning Objectives

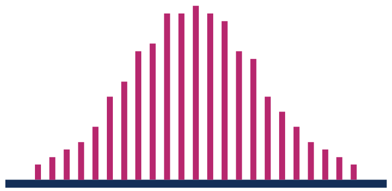
- Explain the main concepts of Monte Carlo simulation
- Use historical observations to estimate the probability distributions of data
- Simulate many possible outcomes for independent variables using Python
- Summarize the distribution of scenarios using confidence intervals
- Interpret the output of Monte Carlo simulation results and use it to guide business decisions

Monte Carlo Simulation Overview



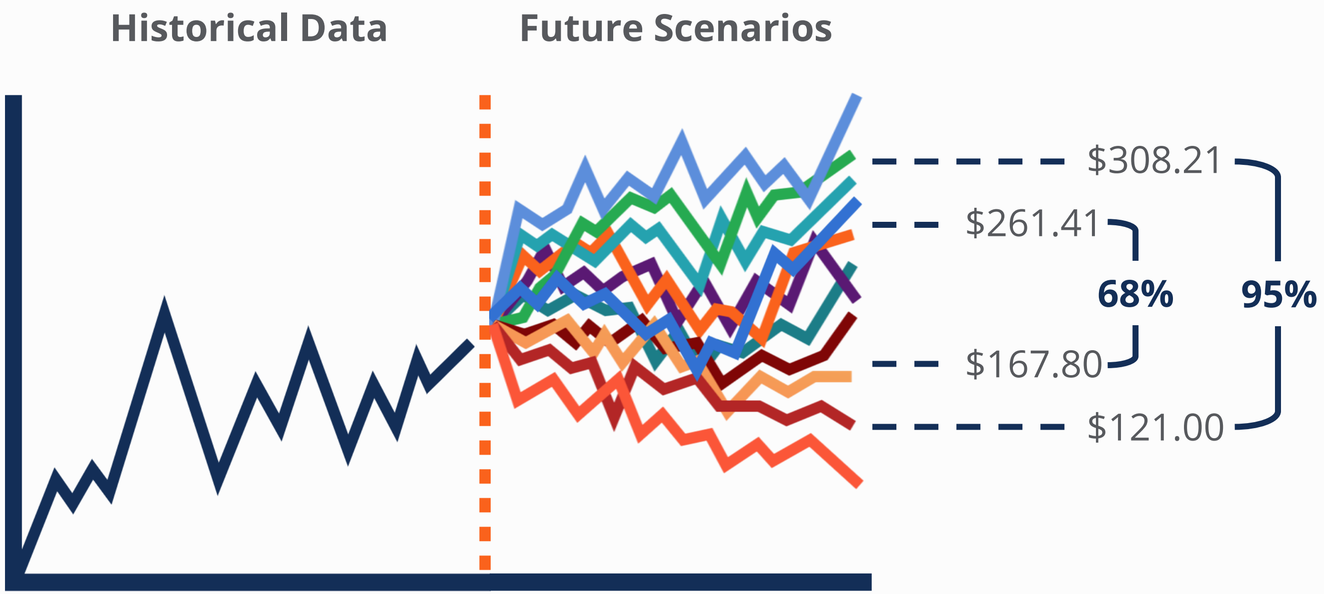
Past Events

Monte Carlo Simulation



Future Events

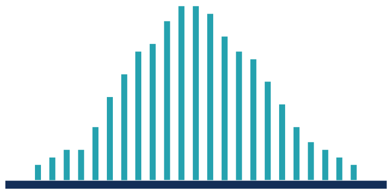
Stock Price Example



Mean of daily return: 0.13%

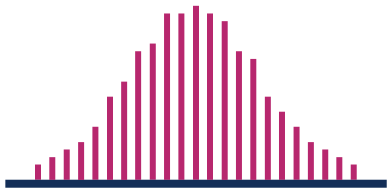
Std dev of daily return: 0.014%

Monte Carlo Simulation Overview



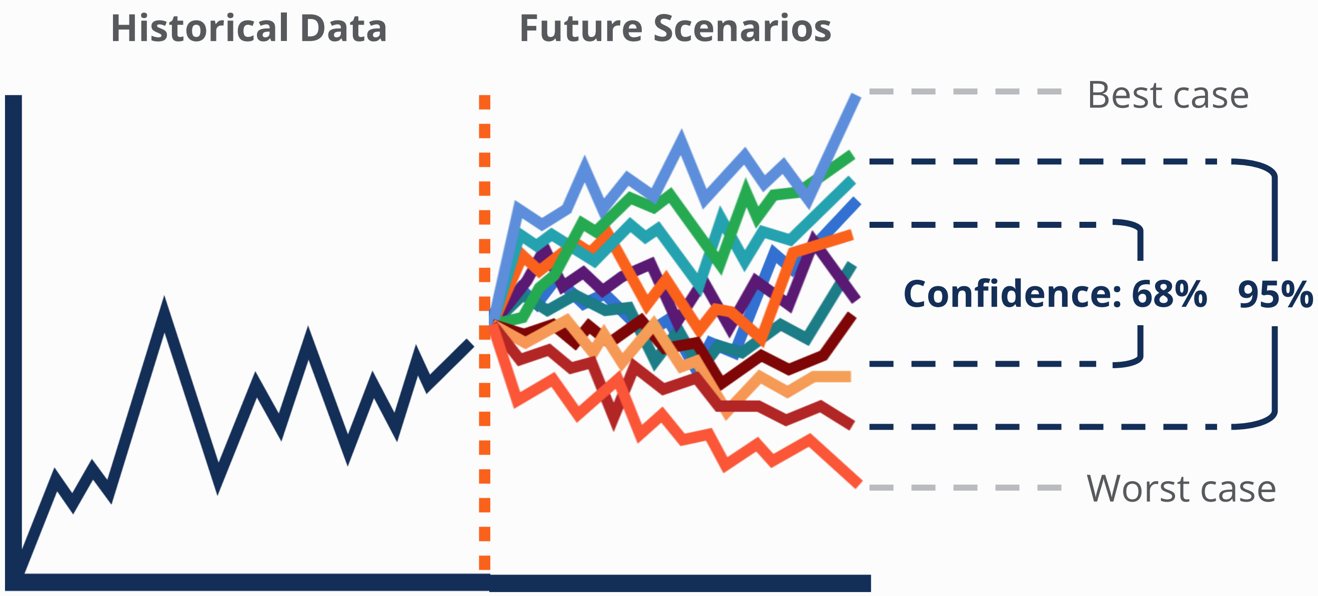
Past Events

Monte Carlo Simulation



Future Events

Stock Price Example



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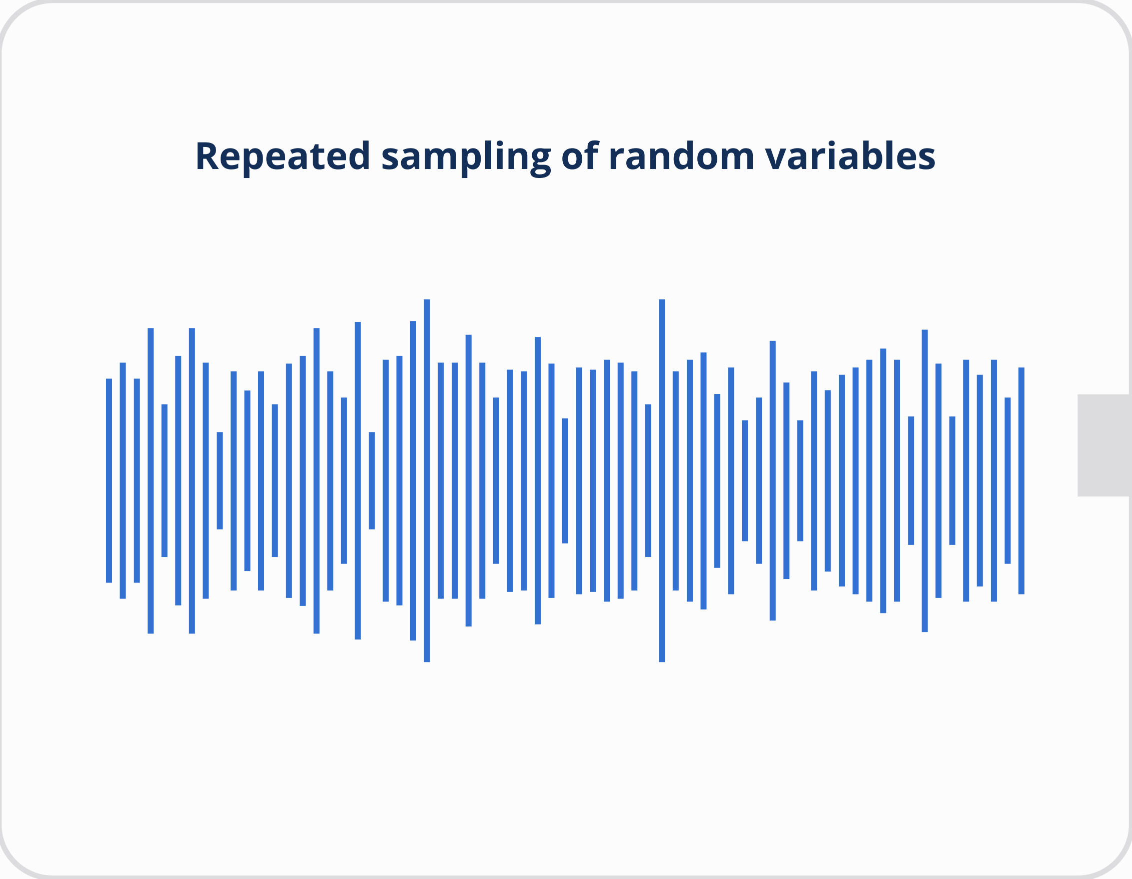
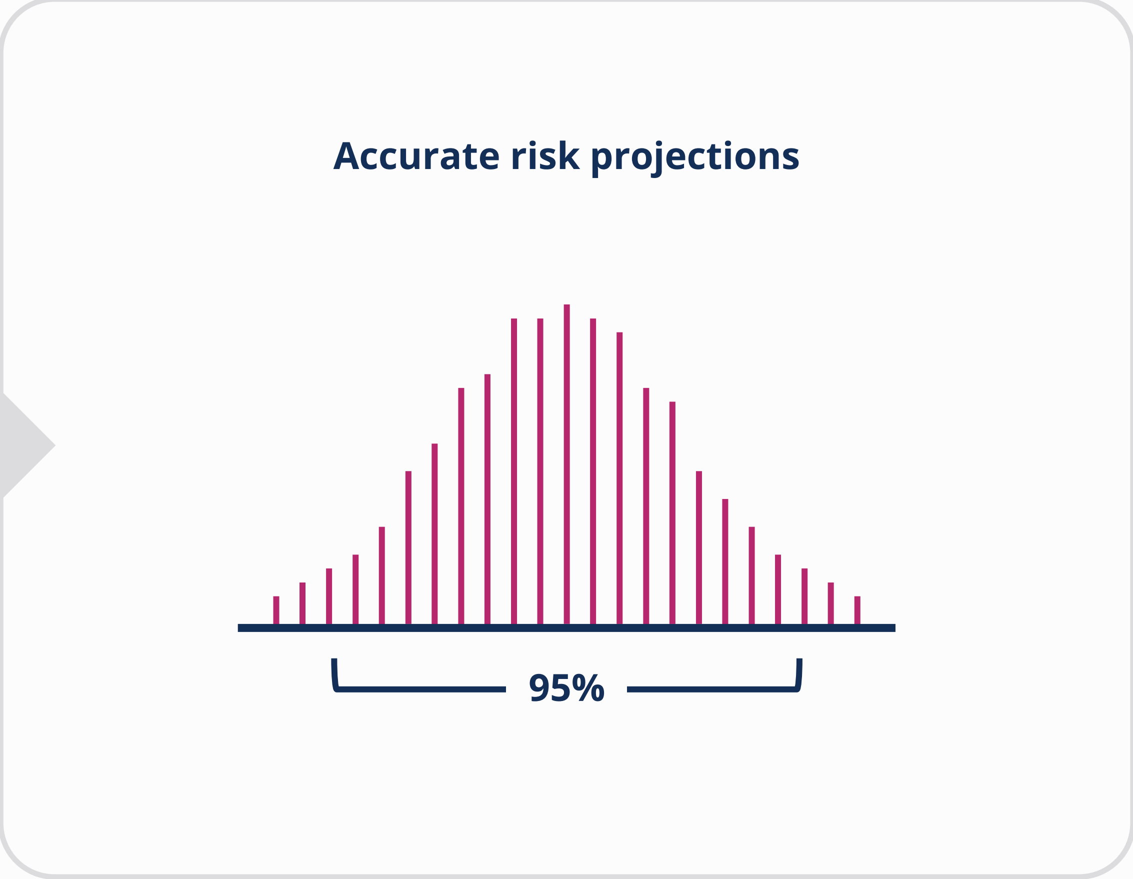
Std dev of daily return: 0.014%

Random Sampling and the Law of Large Numbers





Random Sampling and the Law of Large Numbers

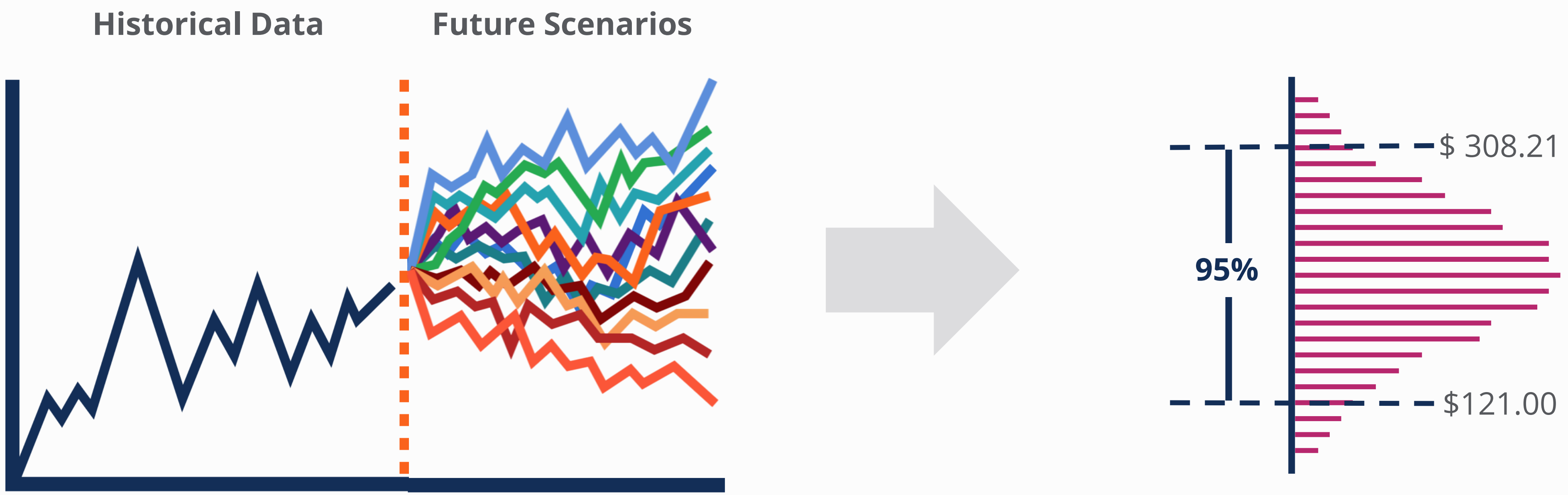
Probability is the chance of an event happening, which can be represented as a percentage.

Law of large numbers: As a sample size grows, the observed probability approaches the theoretical probability.

By performing a large number of random trials, we can start to accurately represent the theoretical probability distribution.

Random Sampling and the Law of Large Numbers

Stock Price Example



Monte Carlo Simulation Process

Monte Carlo Simulation Process

- Observations
- Distributions
- Simulations
- Quantifications

Monte Carlo Simulation Process

Monte Carlo Simulation Process

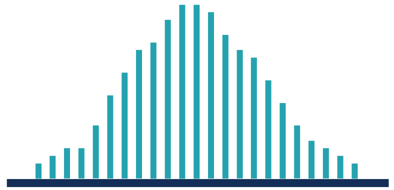
- **Observations**
- **Distributions**
- **Simulations**
- **Quantifications**

Past Observations

Monte Carlo Simulation Process

Monte Carlo Simulation Process

- Observations
- Distributions
- Simulations
- Quantifications



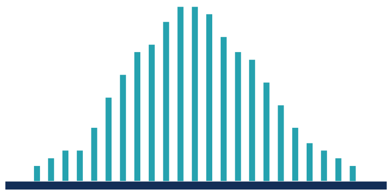
- Mean
- Standard deviation
- Other metrics

Monte Carlo Simulation Process

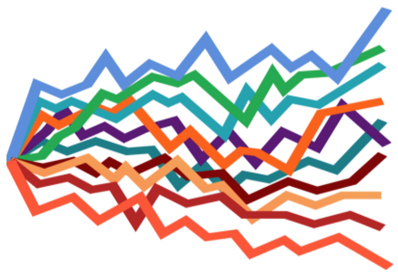
Monte Carlo Simulation Process

- Observations
- Distributions
- Simulations
- Quantifications

Past Observations



Probability Distribution



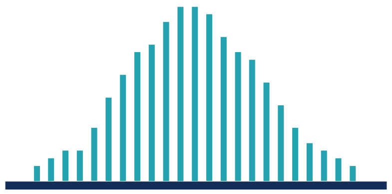
Many Future Scenarios

Monte Carlo Simulation Process

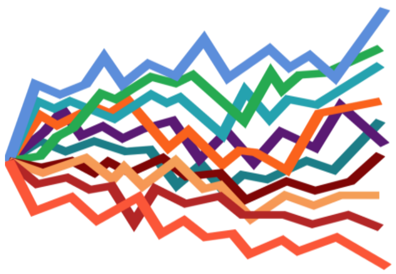
Monte Carlo Simulation Process

- Observations
- Distributions
- Simulations
- Quantifications

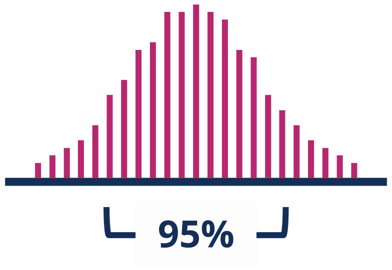
Past Observations



Probability Distribution

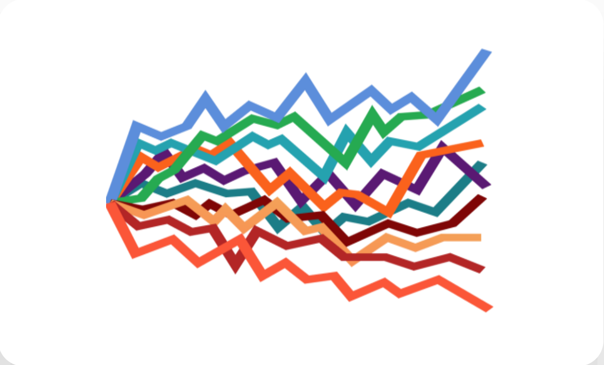


Many Future Scenarios



Quantify the Probability

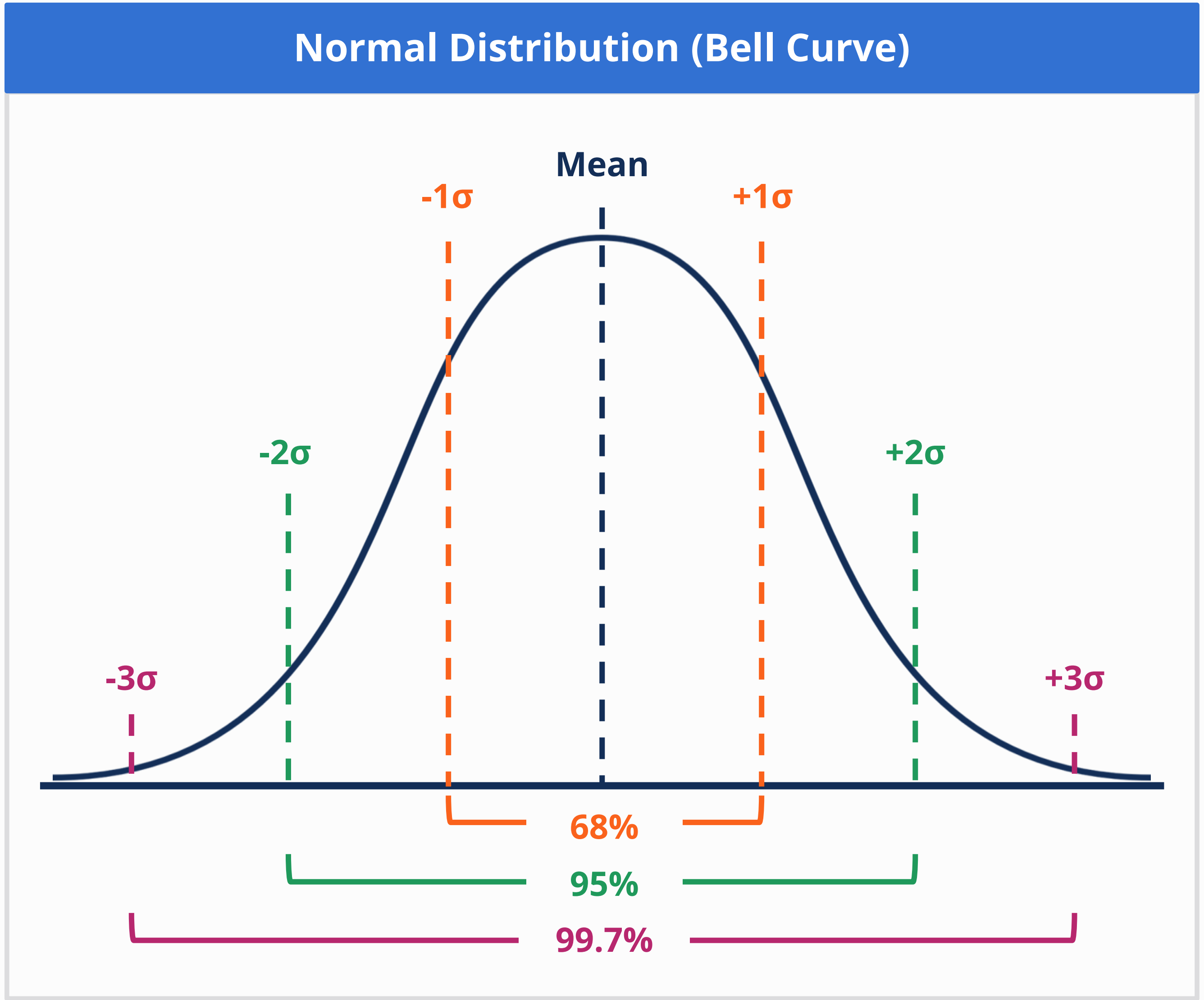
Normal Distribution



In our example, the stock prices have an equal probability of moving up or down, with large movements less common than small ones.

Normal distribution describes a dataset where values farther from its mean occur less frequently than values closer to its mean.

68-95-99.7 Empirical Rule

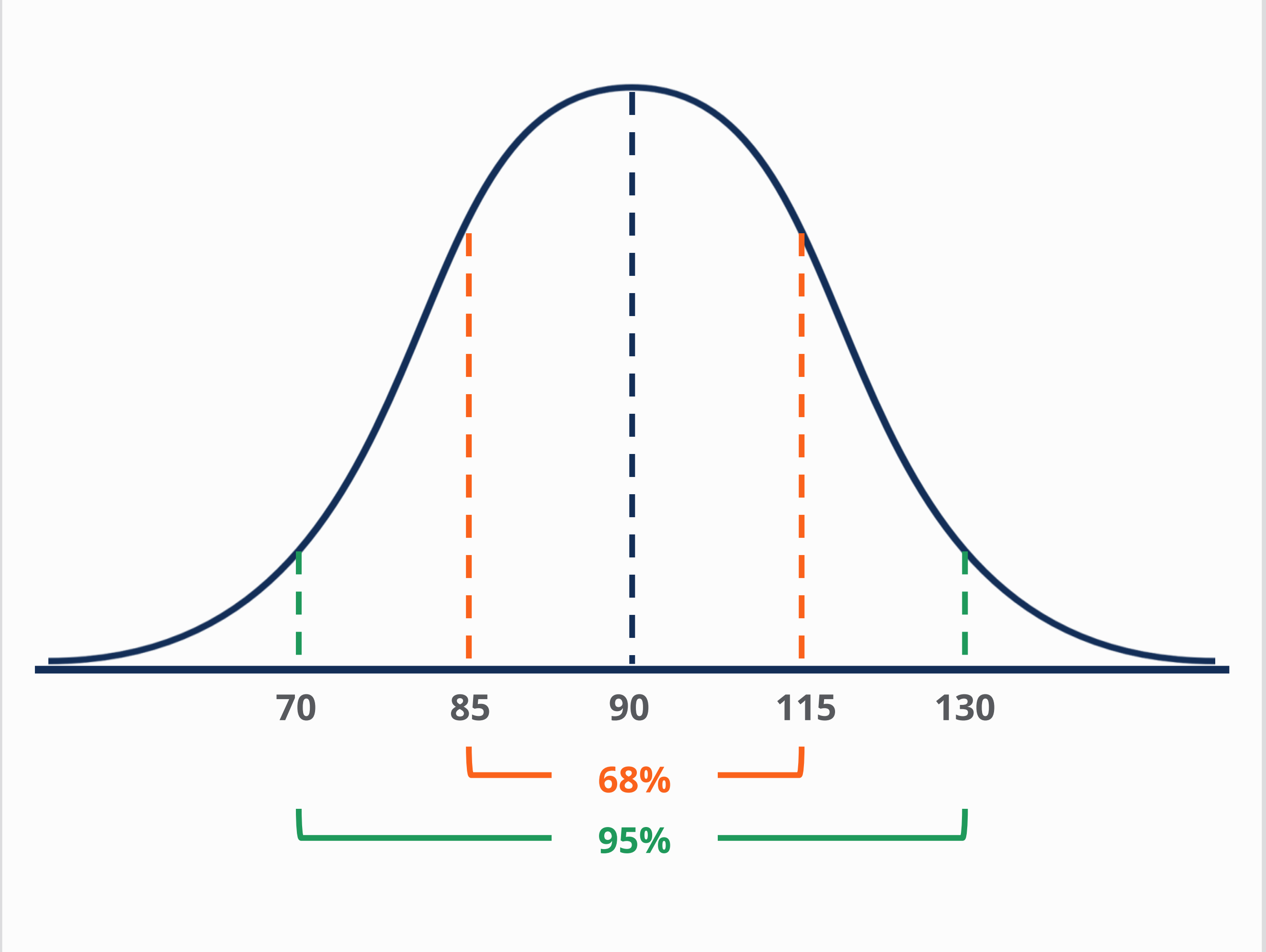


Normal Distribution

Normal Distribution Example: IQ Score

- 68% score between 85 and 115
- 5% score higher than 130 or lower than 70

IQ Score Distribution



CFI

Other Types of Distribution

Binomial Distribution

The probability of yes or no

Only two possible values when flipping a coin

Uniform Distribution

All events have an equal chance of occurring

The probability of landing on each side is the same

Poisson Distribution

Happens in discrete events for modeling how many times an event would happen in a time period

Other Types of Distribution

Beta Distribution

Best used when we have limited data to form a probability

Predict a student's GPA when having limited data

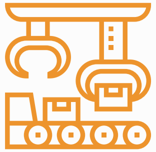
Gamma Distribution

Used for positive skewed continuous values

The probability of a bank teller gets more than 20 customers within an hour

Log Distribution

Commonly applied with a relatively small mean with a large variances



Milk production, expected life of machinery, etc.

Distributions

Every event we try to simulate depends on 1 or more underlying variables.

Each variable behaves in a different way.

Which type of distribution best describes the past behavior of the variable?

- Normal Distribution
- Binomial Distribution
- Uniform Distribution
- Poisson Distribution
- Beta Distribution
- Gamma Distribution
- Log Distribution

Monte Carlo Simulation Applications

Monte Carlo simulations help us understand **the risk of uncertainty** in prediction and forecasting models.

Stock Price

Model the randomness of the stock price and help us assess the uncertainty of the investment

M&A Deals

Assess the probability of strike a deal or no deal

Option Pricing

View the possible future prices generated at different times

Cash Flow Analysis

Capture the variability of a company's cash flows to plan on unforeseen scenarios

Retirement Planning

Plan for the possibility of not having enough assets for retirement

Coin Flipping Example

Coin Flipping Simulation Overview

50% chance

50% chance

Binomial Distribution

Simulation: Flip 20 Coins

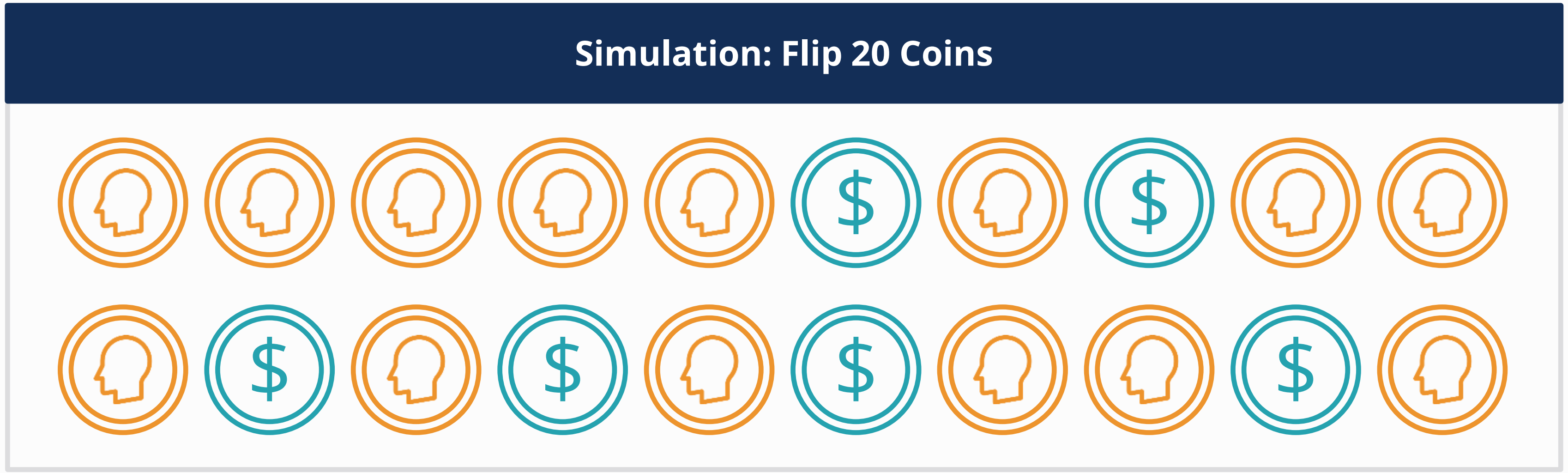
What's the probability of achieving a certain number of heads?

Coin Flipping Simulation Overview

Simulation: Flip 20 Coins

1st attempt: 6 heads

Coin Flipping Simulation Overview



1st attempt: 6 heads
2nd attempt: 14 heads
…

Only once we simulate this event many times do we start to understand the true probability of outcomes.

Assumptions:

- Number of assumptions: 10 and 10,000.
- Every time we flip the 20 coins, the simulation is totally independent from the previous simulation.
- The probability distribution of each 20-coin simulation is the same every time.

Coin Flipping Simulation in Practice

Define the Parameters

The number of binomial tests in each simulation: **20 coins**

The probability of success for each binomial trial: **50%**

The number of simulations:

- 1st model: **10**
- 2nd model: **10,000**

Create an Array of Simulation Results

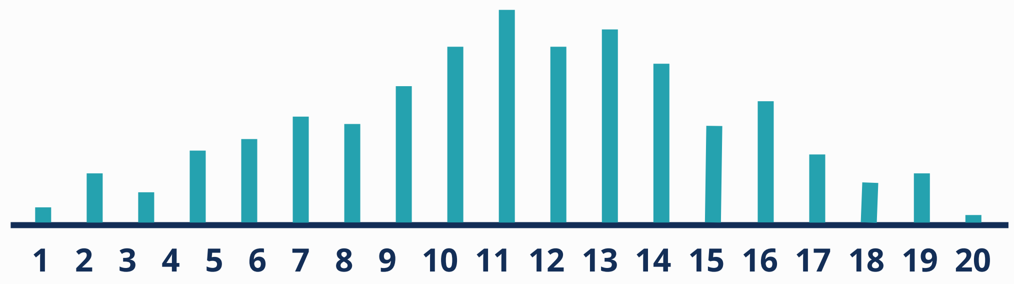
Array showing # of heads in 10 simulations

10	6	14	11	7	8	11	12	10	7

Calculate the Observed Probability

Probability of 10 heads:	1 out of 10 simulations
Probability of 6 heads:	1 out of 10 simulations
Probability of 11 heads:	2 out of 10 simulations

Plot the Distribution



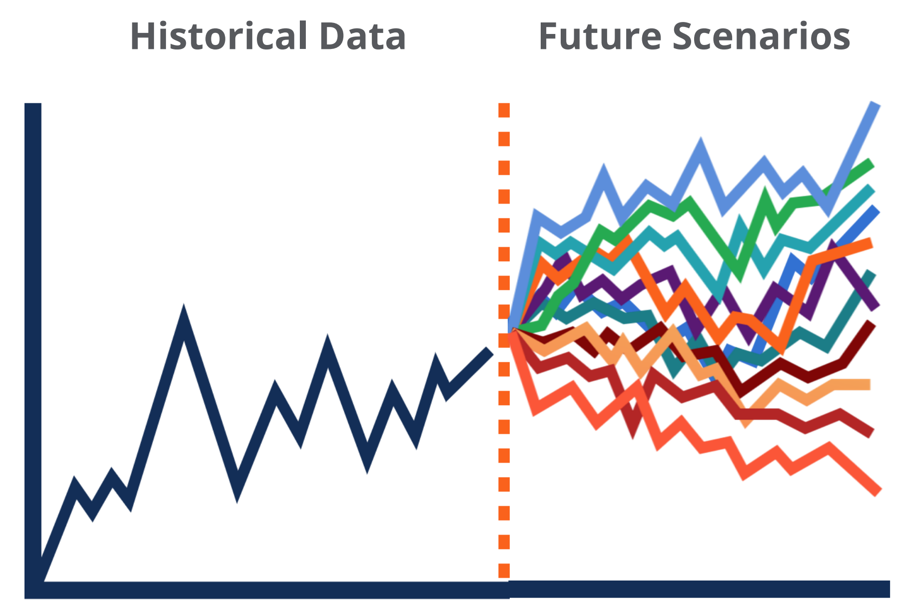
Coin Flipping Exam Recap

Observations	Distributions	Simulations	Quantifications
We did not analyze historical data in this model.	**Binomial distribution:** The probability of getting a head is 50%.	**Simulate flipping 20 coins:** The 1st model ran 10 simulations. The 2nd model ran 10,000 simulations.	**10 simulations:** Not able to generate representative probability distribution. **10,000 simulations:** Generate consistent normal distribution.

Stock Price Prediction

Case Overview

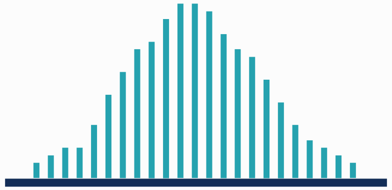
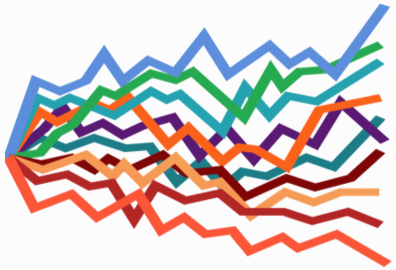
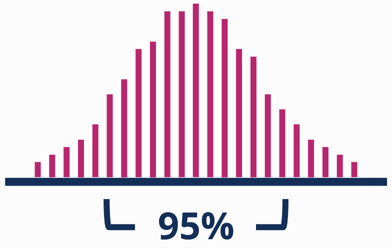
Monte Carlo simulations are commonly used to **forecast stock prices**.



- Stock returns have a random process.
- Any future changes in price are based on new, random occurring information.
- Monte Carlo simulations help us model the random process overtime.

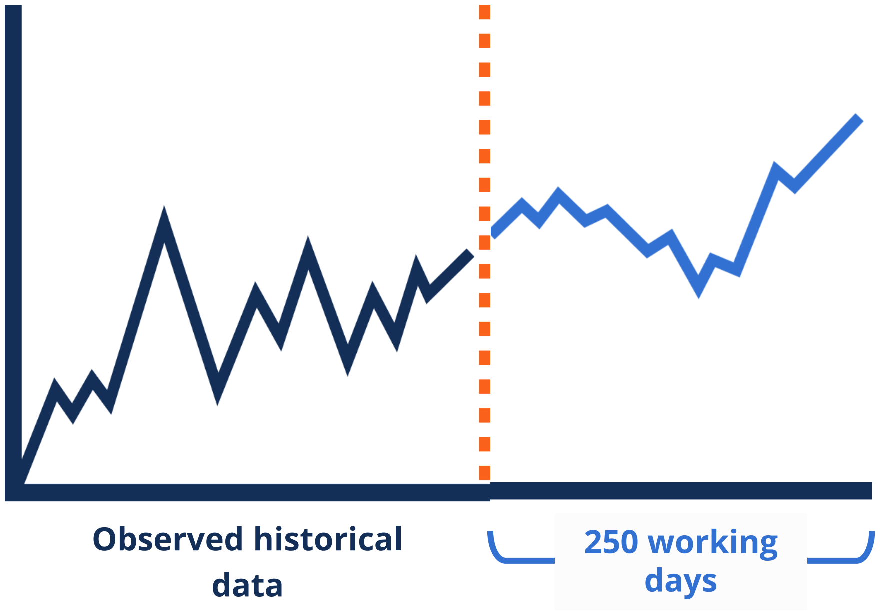
In this case, we will predict the plausible range of MSFT stock price based on the historical data.

Case Overview

Observations	Distributions	Simulations	Quantifications
 20 years of historical data	 Daily Returns Probability Distribution **Mean, std dev, variance, drift**	 **Result of each simulation:** 250-day stock price movement **Number of simulations:** 10,000	 **Best, worst, and average** stock price scenarios **Distribution** of the outcomes

Calculate Daily Returns

How to calculate daily returns from our observed data?



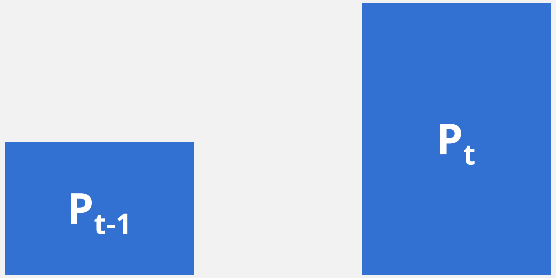
We need to generate plausible scenarios to watch the price evolve over a **250-day period**.

That means **250 consecutive, daily returns**.

Simple Returns

Daily Returns Definition

Previous day's price **Today's price**



Simple Return:
% change in price

$$\textbf{Simple Return} = (P_t - P_{t-1}) / P_{t-1}$$

Example Simple Returns

Day 0 Price: **$30.00**	Simple Return: N/A
Day 1 Price: **$30.90**	Simple Return: **3%**
Day 2 Price: **$31.52**	Simple Return: **2%**
Overall Return: **5.06% (from original price)**	

Simple Returns are **not additive**, which makes them difficult to work with.

Simple Returns

Example Simple Returns

Day 0 Price: **$30.00**	Simple Return: N/A
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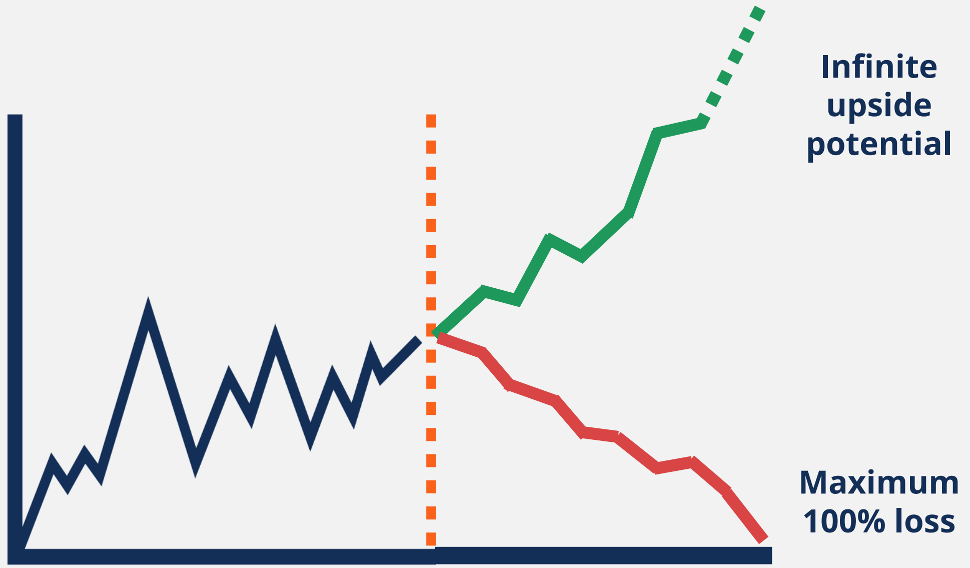


Simple returns are **not symmetric.**

Simple returns cannot be approximated by a normal distribution.

Log Returns to the Rescue

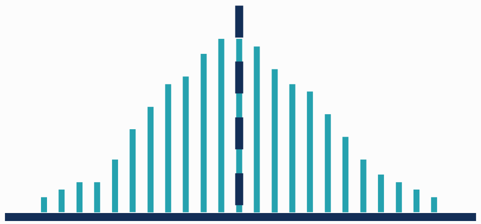
Log Return = log (1 + **Simple Return**)

The benefits of log returns:

- **Time additive**: The log returns over the whole period are equal to the sum of log returns over the period.
- **Symmetric**: The upside and downside movements are more balanced.
- The log returns are assumed to be more closely represented by a **normal distribution**.

Example Simple and Log Returns

Day 0 Price: **$30.00**	Simple Return: N/A	Log Return: N/A
Day 1 Price: **$30.90**	Simple Return: **3%**	Log Return: **2.96..%**
Day 2 Price: **$31.52**	Simple Return: **2%**	Log Return: **1.99..%**
	Overall Return = Sum of Daily Log Returns	Overall Return: **4.95..%**



Log returns can be approximated by a normal distribution

Convert the log returns back into simple returns by using the exponential function

Stock Price Monte Carlo in Python

1) Place the daily historical stock prices into a Pandas dataframe

2) Create a dataframe of **daily** log returns

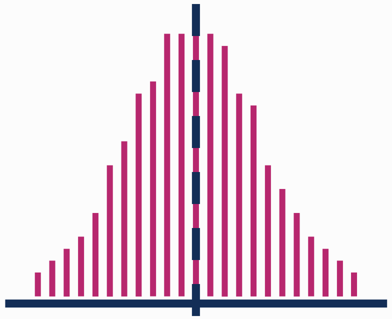
3) Plot a distribution of returns to confirm assumptions

4) Calculate the **historical** statistical measures of the daily log returns

Date	Price
2000-01-03	36.79
2000-01-04	35.55
2000-01-05	35.93
...	
2019-12-31	154.75

-0.034%
+0.011%
...
+1.460%

Log returns = log (1 + % Change of daily prices)



Plot of daily log returns

- **Mean**
- **Variance**
- **Std dev**

Stock Price Monte Carlo in Python

4) Calculate the **historical** statistical measures of the daily log returns

5) Simulate price movements over the next 250 days (random log returns)

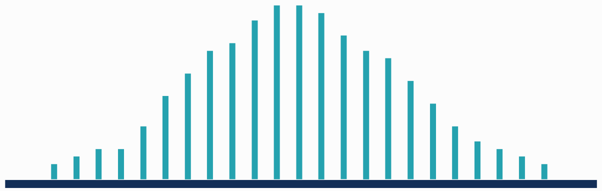
- **Mean**
- **Variance**
- **Std dev**

Day 1	Random Log $Return_1$
Day 2	Random Log $Return_2$
Day 3	Random Log $Return_3$

…

Day 250	Random Log $Return_{250}$

Simulate Random Daily Log Returns



Historical Std Dev of Log Returns x **Random number from normal distribution**

Stock Price Monte Carlo in Python

4) Calculate the **historical** statistical measures of the daily log returns

5) Simulate price movements over the next 250 days (random log returns)

6) Convert log returns back into **simple returns**

7) Calculate the **price progression** for each of our simulations

- **Mean**
- **Variance**
- **Std dev**

Day 1	Random Log $Return_1$
Day 2	Random Log $Return_2$
Day 3	Random Log $Return_3$

...

Day 250	Random Log $Return_{250}$

$\text{Simple Return}_1 = e^{(\text{LogReturn}_1)}$

$\text{Simple Return}_2 = e^{(\text{LogReturn}_2)}$

$\text{Simple Return}_3 = e^{(\text{LogReturn}_3)}$

...

$\text{Simple Return}_{250} = e^{(\text{LogReturn}_{250})}$

Known Start Price = 154.75

$P_{DAY1} = 154.75 \times \text{SimpleReturn}_{DAY1}$

$P_{DAY2} = P_{DAY1} \times \text{SimpleReturn}_{DAY2}$

$P_{DAY3} = P_{DAY2} \times \text{SimpleReturn}_{DAY3}$

...

$P_{DAY250} = P_{DAY249} \times \text{SimpleReturn}_{250}$

Stock Price Monte Carlo in Python

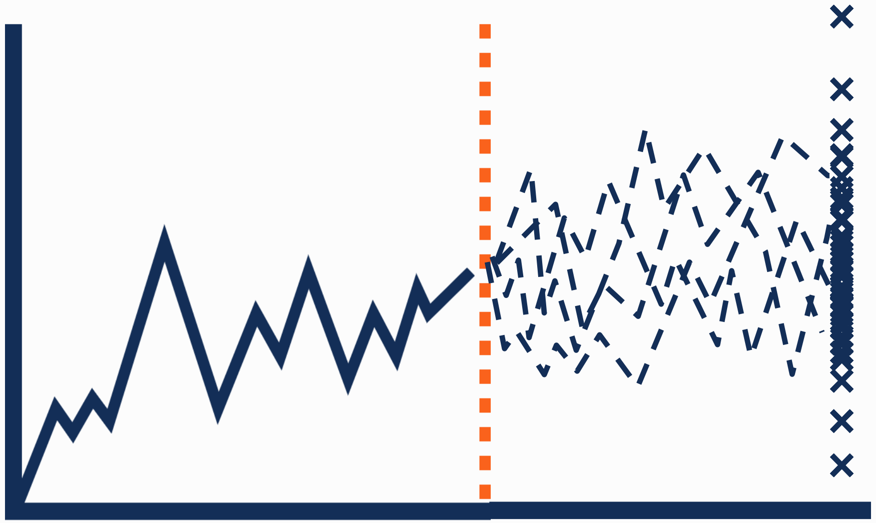
7) Calculate the **price progression** for each of our simulations

8) Repeat simulation steps 10,000 times

Known Start Price = 154.75
P_{DAY1} = 154.75 x $SimpleReturn_{DAY1}$
P_{DAY2} = P_{DAY1} x $SimpleReturn_{DAY2}$
P_{DAY3} = P_{DAY2} x $SimpleReturn_{DAY3}$

…

P_{DAY250} = P_{DAY249} x $SimpleReturn_{250}$



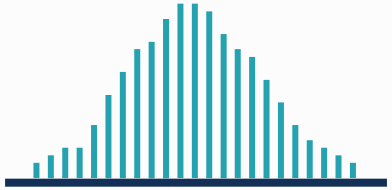
The density of simulations is far greater in the center. Extreme changes are less frequent.

Stock Price Prediction Recap

Observations

20 years of historical data

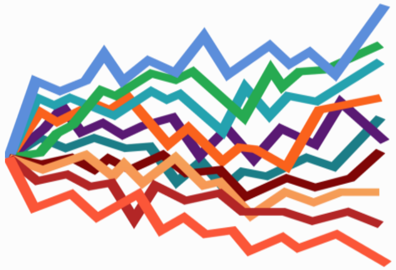
Distributions



Daily Returns Probability Distribution

Mean, std dev, variance, drift

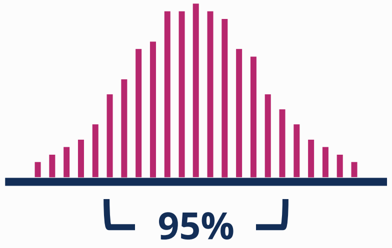
Simulations



Result of each simulation: 250-day stock price movement

Number of simulations: 10,000

Quantifications



Best, worst, and average stock price scenarios

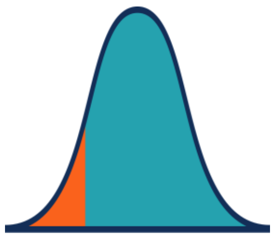
Distribution of the outcomes

Value at Risk Assessment

Value At Risk (VaR)

Risk Management

The range of possible future scenarios gives us good information to quantify and deal with financial risk.



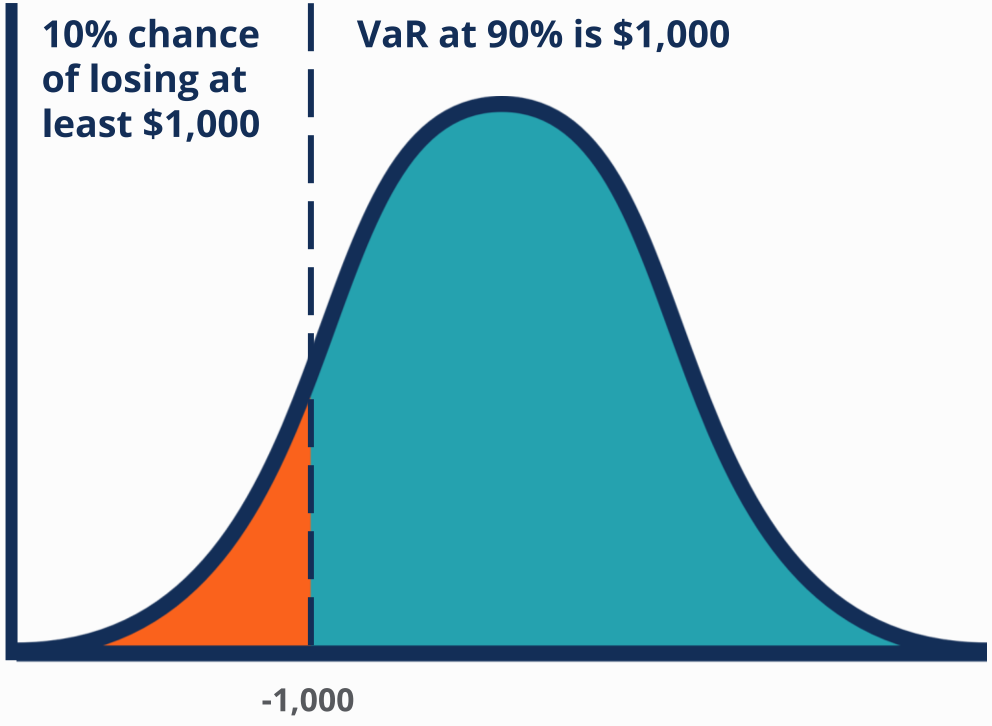
Value at Risk

Value at Risk (VaR) is a metric that estimates the worst-case risk exposure of an investment.

Value At Risk (VaR)

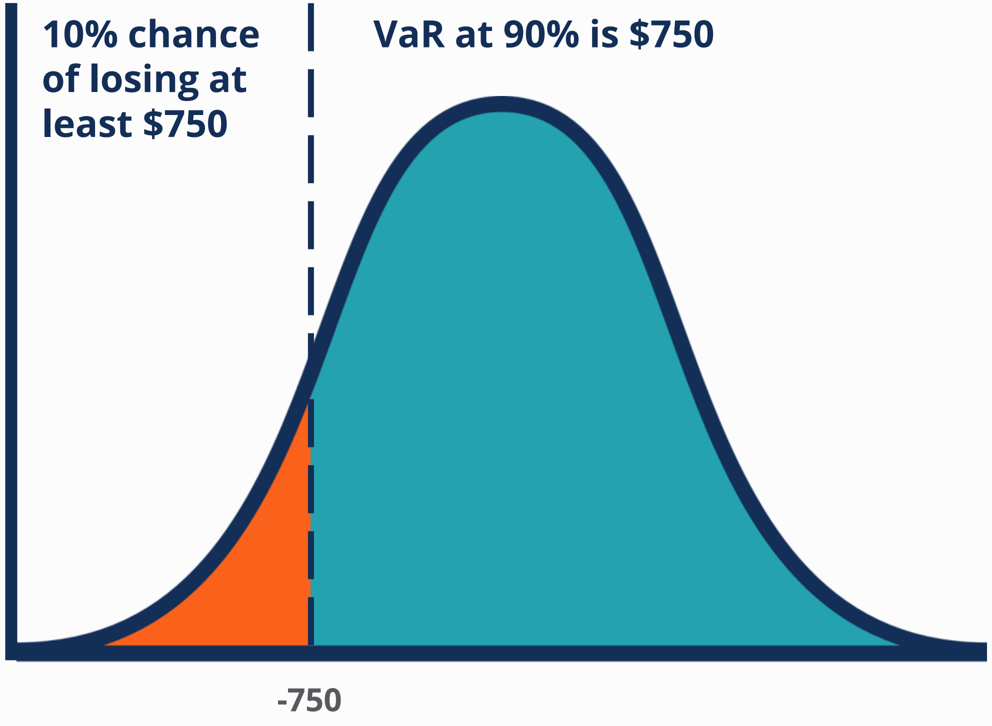
Project A

Expected Profit: $10,000



Project B

Expected Profit: $10,000



Case Overview

Assess the risk of purchasing 1,000 MSFT shares and holding them for one month

Observations	Distributions	Simulations	Quantifications
We will not directly analyze the 30-day historical data.	**Key information:** • Current value • Volatility	**Result of each simulation:** the return of holding the stocks for one month **Number of simulations:** 5,000	**One-month VaR at 90%, 95%, and 99% confidence**

Parametric Simulation

Coin Flipping Model	Stock Price Model	VaR Model
Each simulation represented the number of heads in a 20 coin flip.	Each simulation represents the price after 250 days.	Each simulation has an answer derived from a formula.
1 Sim Result = **Number of Heads**	1 Sim Result = **Price after 250 days**	1 Sim Result = **Formula Answer**

VAR Parametric Approach

The following formulas are used for parametric value at risk modeling:

Investment Return = End Value – Present Value (PV)

where **End Value = PV * e^((rfr – 0.5 * vol^2) * t + z * σ)**

Step 1: Calculate the **present value (PV)** of the investment

Step 2: For each simulation, use the formula to calculate a plausible **end value**

Step 3: Calculate an **investment return** for each simulation.

Step 4: Summarize the distribution of investment returns and calculate the value at risk

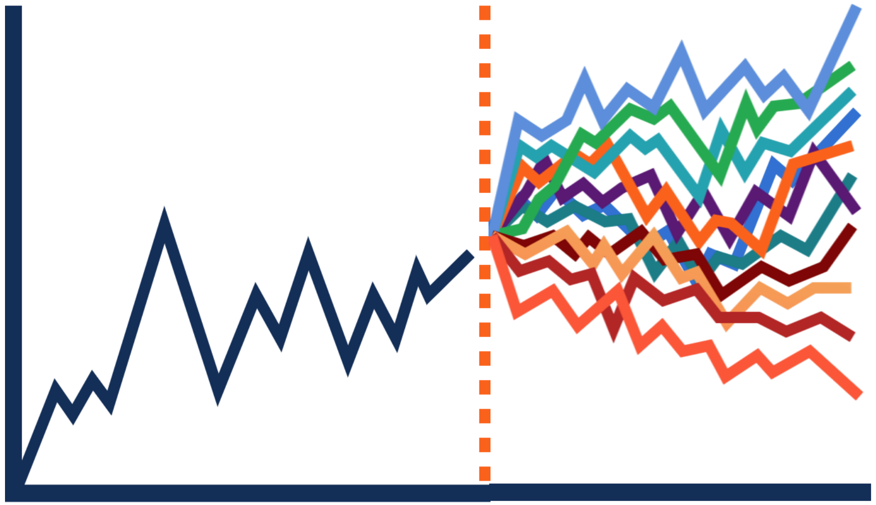
Again, we have a **single answer per simulation**.

- **z:** randomly generated variable from a standard normal distribution
- **σ:** historical standard deviation
- **t:** the time in years
- **rfr:** risk free rate
- **vol:** historical volatility

VaR Assessment Model Recap

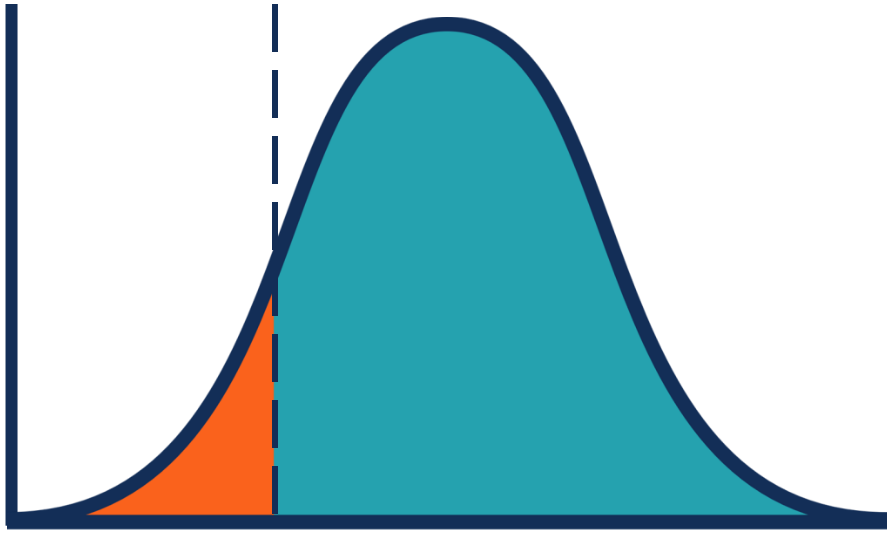
Stock Price Prediction Model

Output of simulation: stock price



Value at Risk Model

Output of simulation: profit or loss



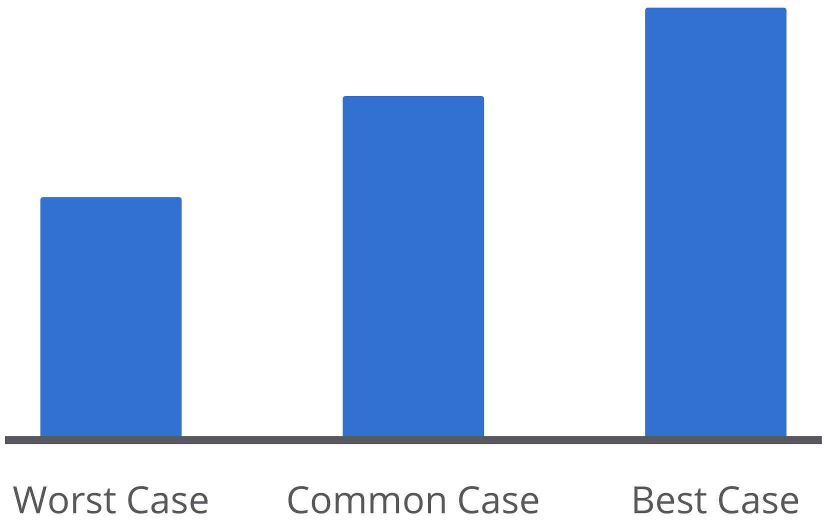
VaR Assessment Model Recap

Observations	Distributions	Simulations	Quantifications
We did not directly analyze the 30-day historical data.	**Parameters:** • Current investment value • Risk free rate • Volatility	**Use the VaR formula to calculate the return** **Number of simulations:** 5,000	**One-month VaR at 90%, 95%, and 99% confidence**

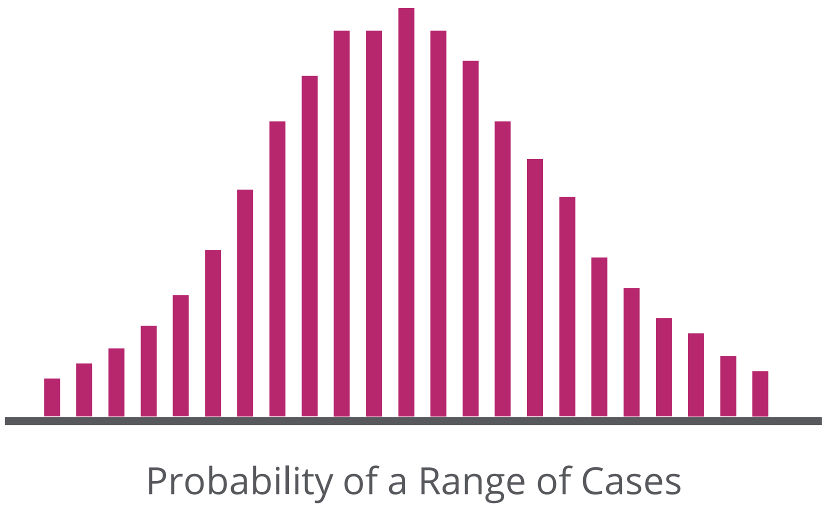
Net Income Forecast

Case Overview

Traditional Net Income Forecast



Monte Carlo Simulations



Case Overview

Forecast a simple company's net income based on the sales and the cost of goods sold.

Observations

We will not directly observe and analyze the historical data.

Distributions

Sales:

- mean = 50 (in millions)
- std dev = 5 (in millions)

Cost of Goods Sold (percentage of sales)

- mean = 15%
- std dev = 0.1

Simulations

Result of each simulation: net profit of the company

Number of simulations: 10,000

Quantifications

Best case, worst case, and average case

Probability distribution of the outcomes

Net Income Forecast Recap

Observations	Distributions	Simulations	Quantifications
We did not directly observe and analyze the historical data.	**Assumptions:** • Mean and std dev of sales • Mean and std dev of COGS (percentage of sales)	• 10,000 sales samples • 10,000 COGS samples **Net Income = Sales - COGS**	**Best case, worst case, and average case** **Probability range** with 68% and 95% confidence

Capital Investment (NPV) Forecasting

Net Present Value (NPV)

Forecast the profitability of investing in equipment that costs $750,000. It will generate revenue for 5 years.

Net Present Value (NPV) is the value of all future cash flows over the entire life of an investment discounted to the present.

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Present cash flow	Future cash flow discounted to the present	Future cash flow discounted to the present	Future cash flow discounted to the present	Future cash flow discounted to the present	Future cash flow discounted to the present

$\underbrace{\hspace{10em}}_{\text{Same Future Cash Flow Over the 5 Years}}$

Free Cash Flow

Free Cash Flow (FCF) = EBIT

x (1 – Tax Rate)

+ Depreciation & Amortization

– Net Increase in Working Capital

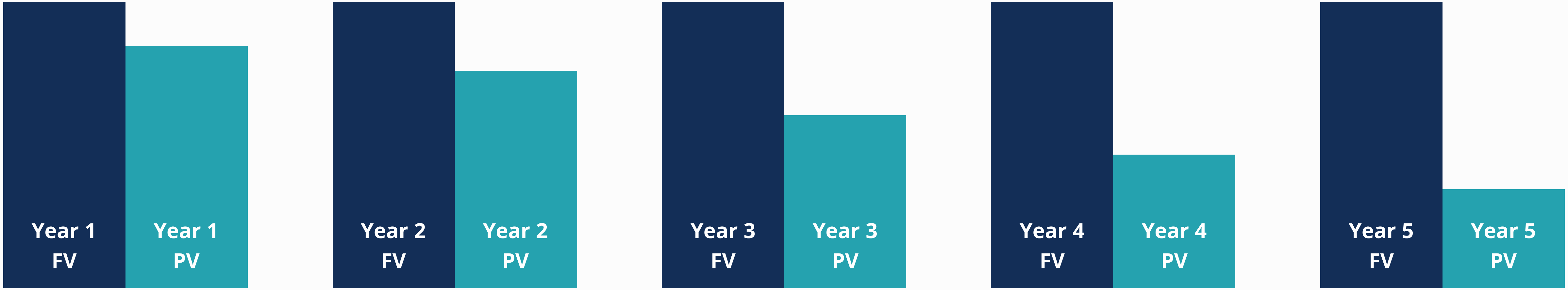
– Capital Expenditure (Equipment Cost)

NOPAT
(Net Operating Profit after Tax)

The cash flow available to all the investors, both debt and equity.

Future Value and Present Value

Discount the future cash flows (FV) to the present value (PV).



Why discount the future cash flows?

- To account for the time value of money
- To adjust for the risk of an investment opportunity

- **Discount rate**
- **Numpy's NPV function**

Case Overview

Net Income Forecast Model

Simulate the randomness of:

- Revenue
- Cost of goods sold

Net Income = Sales - COGS

NPV Model

- More uncertainties
- More fixed financial items
- Use assumptions to estimate future cash flows

Case Overview – Distributions

Three Independent Variables

Each variable is assumed to be normally distributed

	Mean	Standard Deviation
Price Per Unit	25	0.5
Number of Units	35,000	2,000
Discount rate	0.15	0.02

Case Overview – Distributions

Fixed Business Assumptions	
Cost of Goods Sold	37.5% of Revenue
Salaries & Benefits	160,427 (82,750 for Year 0)
Other Expenses	10,963
Net Increase in Working Capital	9,003
Tax Rate	25%

Case Overview – Simulations

Generate 10,000 simulations for the three uncertain metrics:

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Price Per Unit	0	Simulate	Simulate	Simulate	Simulate	Simulate
Number of Units	0	Simulate	Simulate	Simulate	Simulate	Simulate
Discount rate	n/a	Simulate	Simulate	Simulate	Simulate	Simulate

The Monte Carlo method tries to create a more realistic representation of what could happen in real life, where variables are truly independent.

The sample values of each variable are randomly generated from a distribution.

Case Overview – Simulations

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Sales	0	Price x Units	Price x Units	Price x Units	Price x Units	Price x Units
COGS	0	Sales x 0.375	Sales x 0.375	Sales x 0.375	Sales x 0.375	Sales x 0.375
Salaries	82,750	160,427	160,427	160,427	160,427	160,427
Other Expenses	0	10,963	10,963	10,963	10,963	10,963
Depreciation	0	Equipment / 5	Equipment / 5	Equipment / 5	Equipment / 5	Equipment / 5
Net increase in W.C	0	9,003	9,003	9,003	9,003	9,003
Equipment Cost	450,000	0	0	0	0	0
Free Cash Flows	Derived from above	Derived from above	Derived from above	Derived from above	Derived from above	Derived from above

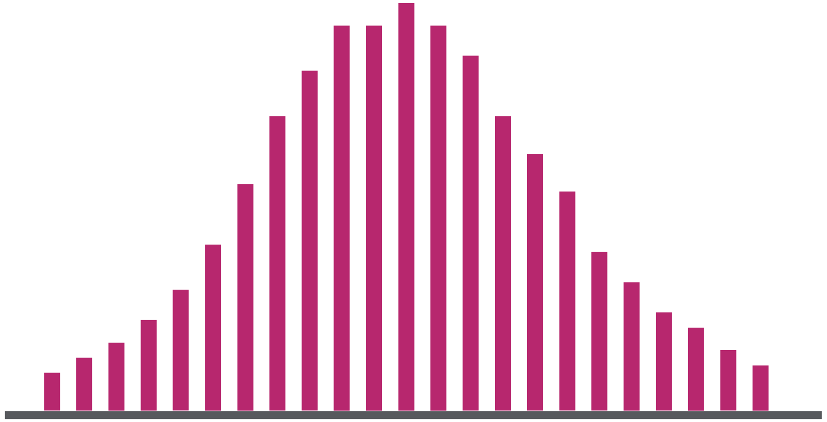
NPV: Sum of All Discounted Cash Flows

Final Simulated Value

Case Overview – Quantifications

Profitability Analysis

Worst Case Common Case Best Case



Probability Distribution of NPV Simulations

Capital Investment NPV Recap

Observations	Distributions	Simulations	Quantifications
We did not directly observe and analyze the historical data.	**Variables of uncertainty:** • Price per unit • Number of units • Discount rate Other business assumptions	**10,000 simulations** based on the mean and standard deviation Calculate **FCF** and **NPV** in each scenario	**Minimum, average, and maximum NPV** **Probability distribution**

Course Summary

Course Summary

Simulated the probability of getting heads when flipping 20 coins

Simulated 250-day stock price movement and predicted the range of the stock price

Simulated the investment return and identified the value at risk

Simulated the uncertainty of sales and COGS to forecast the net profit

Simulated multiple variables to estimate the NPV of investing in an equipment

Course Summary

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Be it known by all those present, that the board of directors of the Corporate Finance Institute® have conferred upon

STUDENT NAME

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with all the rights, privileges and honors everywhere pertaining to that degree. In testimony whereof we have hereunto subscribed our names on

CORPORATE FINANCE INSTITUTE VERIFIED CFI CERTIFICATE

Chair of the board

Director

Director