Circuit Theory and Electronics Fundamentals

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Laboratory 1 Report

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing several resistors R_n, two current sources, I_d and I_b, with one of them beeing dependant, and two tension sources, V_a and V_c, with the latter beeing dependant. The circuit can be seen if Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure **??** is analyzed theoretically with the Mesh Method and with the Nodal Method, in order to complement each other results.

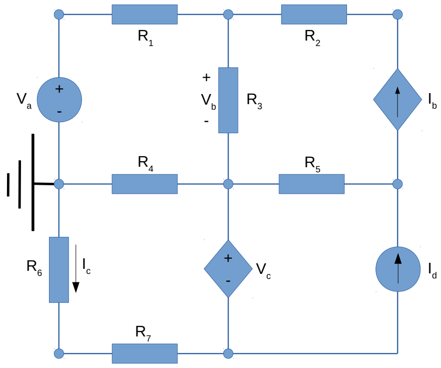

Figure 1: Voltage driven serial RC circuit.

2.1 Mesh Method

The Mesh Method consists in introducing currents that circulate in the meshes of the circuit, that is, in the loops that do not contain other loops, as shown in Figure **??**, and then evaluate the circuit based on the new currents.

After identifying the mesh currents, the next step in this method is to use the Kirchhoff Voltage Law (KVL) in the meshes that do not contain current sources (mesh α (1) and δ (2)) and to relate the mesh currents to the currents imposed by the sources (mesh β (3) and γ (4)):

$$R_1 I_\alpha + V_b + R_4(I_\alpha - I_\delta) - V_a = 0; \tag{1}$$

$$R_6 I_\delta + R_4(I_\delta - I_\alpha) + V_c + R_7 I_\delta = 0; \tag{2}$$

$$I_\beta = -I_b; \tag{3}$$

$$I_\gamma = -I_d. \tag{4}$$

Since there are 8 variables in the circuit, I_α, I_β, I_γ, I_δ, V_b, V_c, I_b, I_c, there must be more four independent equations: two of them are already given,

$$I_b = K_b V_b; \tag{5}$$

$$V_c = K_c I_c. \tag{6}$$

The other two are found by examining the circuit and with Ohm's Law:

$$I_c = -I_\delta; \tag{7}$$

$$V_b = R_3(I_\alpha - I_\beta) \tag{8}$$

The solution to this linear system of equations is determined by Octave:

2.2 Nodal Method

In the Nodal Method, the determination of the values of current and voltage is made by, firstly, finding all the knots in the circuit, as made in Figure **??**.

Then, it is used the Kirchhoff Current Law (KCL) in the nodes that are not connected to voltage sources (Equations 9 to 12) an relate the knots voltages with the voltage sources that are connected to them (Equations 13 and 14):

Name	Value [A or V]
$@I_\alpha$	2.671902e-04
$@I_\beta$	2.800095e-04
$@I_\gamma$	-1.010603e-03
$@I_\delta$	-9.340051e-04
V_b	-3.927010e-02
V_c	7.809141e+00
$@I_b$	-2.800095e-04
$@I_c$	9.340051e-04

Table 1: Variables in the Mesh Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

$$\frac{v_3 - v_2}{R_2} - \frac{V_b}{R_3} + \frac{v_1 - v_2}{R_1} = 0; \tag{9}$$

$$I_b + \frac{v_2 - v_3}{R_2} = 0; \tag{10}$$

$$\frac{v_5 - v_6}{R_5} - I_b + I_d = 0; \tag{11}$$

$$I_c + \frac{v_8 - v_7}{R_7} = 0; \tag{12}$$

$$v_1 - v_4 = V_a; \tag{13}$$

$$v_5 - v_8 = V_c. \tag{14}$$

Since there are 8+4 variables (v_1 to v_8, V_b, V_c, I_b, I_c), the system is defined by twelve independent equations. Two are provided in the circuit and are the same as in the Mesh Method, (5) and (6). By observing the circuit,

$$v_5 - v_8 = V_b. \tag{15}$$

Using Ohm's Law, we find the relation:

$$I_c = \frac{v_4 - v_7}{R_6}. \tag{16}$$

Because there needs to be a knot with a defined voltage, we chose v_4 to be connected to the ground:

$$v_4 = 0. \tag{17}$$

For the last equation, the continuity of current in the circuit can be used to create a "super-knot", bypassing the voltage sources V_a and V_c, from which the equations are (18) and (19), respectively:

$$\frac{v_5 - v_4}{R_4} + \frac{v_2 - v_1}{R_1} - I_c = 0; \tag{18}$$

$$\frac{v_7 - v_8}{R_7} - Id + \frac{v_6 - v_5}{R_5} + \frac{Vb}{R3} + \frac{v_4 - v_5}{R_4} = 0. \tag{19}$$

Given that there only one equation is needed, we chose to use the more simple (18). The solution to this linear system of equations is determined by Octave:

Name	Value [A or V]
v_1	5.198324e+00
v_2	4.956490e+00
v_3	4.450997e+00
v_4	0.000000e+00
v_5	4.991201e+00
v_6	9.002253e+00
v_7	-2.050829e+00
v_8	-3.043174e+00
V_b	-3.471054e-02
V_c	8.034375e+00
$@I_b$	-2.455590e-04
$@I_c$	9.770070e-04

Table 2: Variables in the Nodal Method. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
@gb[i]	9.516829e-04
@id[current]	1.010603e-03
@r1[i]	-9.08113e-04
@r2[i]	-9.51683e-04
@r3[i]	4.356980e-05
@r4[i]	-1.44473e-03
@r5[i]	9.516829e-04
@r6[i]	2.352843e-03
@r7[i]	1.342240e-03
v(1)	5.211160e+00
v(2)	6.125002e+00
v(3)	8.070804e+00
v(4)	0.000000e+00
v(5)	5.991533e+00
v(6)	3.089761e+00
v(7)	-4.71282e+00
v(8)	-6.08774e+00

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.