TÉCNICO
LISBOA

Circuit Theory and Electronics Fundamentals

T2 Laboratory Report

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Group 19

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1 Introduction

The goal of this laboratory assignment is to study a circuit with a sinusoidal voltage source and a capacitor. The equations below show how the value of the voltage source varies with time:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

with

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \tag{2}$$

In this circuit there are both a linearly dependent voltage and current source. The circuit also contains 7 resistors.

The nodes of the circuit were numbered arbitrarily (from V_0 to V_7), and it was considered that *node 0* was the ground node. The voltage-controlled current source I_b has a linear dependence on Voltage V_b, of constant K_b. The voltage V_b is the voltage drop at the ends of resistor R_3. The current-controlled voltage source V_d has a linear dependence on current I_d, of constant K_d. The control current I_d is the current that passes through the resistor R_6. The circuit can be seen in Figure 2.

These values for the capacitance, resistors and the constants for the dependent sources were obtained using the Python script provided by the Professor and using the number 95802 as the seed.The seed number can be altered in the top Makefile. By doing so, all figures and tables will be updated acording to the new values.

The goal of this laboratory assignment is to study a circuit with a sinusoidal voltage source and a capacitor. The equations below show how the value of the voltage source varies with time:

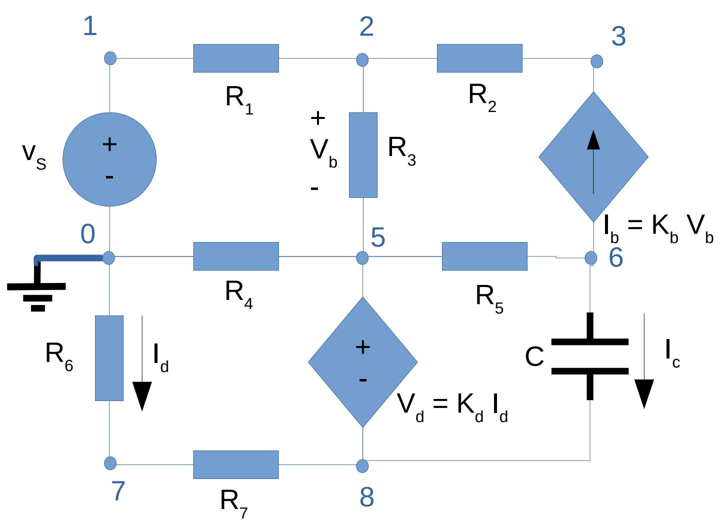

Figure 1: Circuit in study

2 Theoretical Analysis

In this section we are going to apply the methods as told in the exercise of the laboratory to be able to compare the results with the simulation results. As it is required in the ngspice we will add a voltage 0 source to the circuit, placing it betwin resistor 6 and node 7, creating a node 4 in betwin the sorce and the resistor.

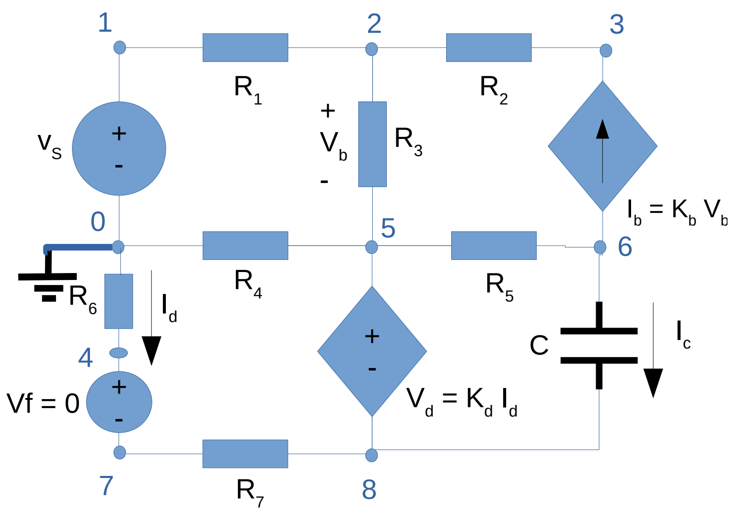

Figure 2: Circuit in study

2.1 Analysis for $t < 0$ - Nodal Method

For $t < 0$ the capacitor has no current thrue it, so it works as an open circuit. Aplying the Nodes Method we can find the following equations. From the ground placement on the circuit we get equation (3), from the 0 voltage source we get equation (4), from the information given about the dependent voltage source we get equation (5) and from the independent voltage source we get equation (6). Then, by aplying KCL to the nodes 2 (7), 3 (8), 6 (10), 7 (11) and 5, considering that the dependent voltage source doesn't change the current, (9).

$$V_0 = 0 \tag{3}$$

$$V_4 = V_7 \tag{4}$$

$$V_5 - V_8 = K_d \frac{V_0 - V_4}{R_6} \tag{5}$$

$$V_1 - V_0 = V_s \tag{6}$$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_5}{R_3} + \frac{V_2 - V_3}{R_2} = 0 \tag{7}$$

$$\frac{V_3 - V_2}{R_2} - K_b(V_2 - V_5) = 0 \tag{8}$$

$$\frac{V_5 - V_2}{R_3} + \frac{V_5 - V_0}{R_4} + \frac{V_5 - V_6}{R_5} + \frac{V_8 - V_7}{R_7} = 0 \tag{9}$$

$$\frac{V_6 - V_5}{R_5} + K_b(V_2 - V_5) = 0 \tag{10}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{11}$$

Name	Nodal method
@c	0
@Gb	-2.800095e-04
@r1	2.671901880857956e-04
@r2	2.800095398539263e-04
@r3	-1.281935176813124e-05
@r4	-1.201195269381554e-03
@r5	-2.800095398539267e-04
@r6	9.340050812957588e-04
@r7	9.340050812957588e-04
v(1)	5.21115954318
v(2)	4.94228369329
v(3)	4.36977885664
v(4)	-1.87084144744
v(5)	4.98155378993
v(6)	5.83532935279
v(7)	-1.87084144744
v(8)	-2.82758714243

Table 1: A variable that starts with "@" is of type *current* and expressed in Ampere (A); all the other variables that start with a "V" are of the type *voltage* and expressed in Volt (V).

2.2 Equivalent resistor as seen from the capacitor terminals

To obtain the equivalent resistence as viewd by C we need to remove the independt voltage source, by replacing it with a short circuit ($V_s = 0$). Because of the dependent source we also need to replace the capacitor with a voltage source $V_x = V_6 - V_8$. We use the V_6 and V_8 from the previous section beacause the voltage drop at the terminals of the capacitor needs to be a continuous function (there can not be an energy discontinuity in the capacitor).

Now we perform an other nodal analysis to figure out the current I_x supplied by the immaginary voltage source V_x, to then be able to find the equivalent resistance ($R_eq = V_x/I_x$).The equations considered for these calculations were 3, 4, 5, 6, 7, 8 and the following:

$$\frac{V_1 - V_2}{R_1} + \frac{V_0 - V_4}{R_6} + \frac{V_0 - V_5}{R_4} = 0 \tag{12}$$

$$K_b(V_2 - V_5) + \frac{V_6 - V_5}{R_5} + I_x = 0 \tag{13}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{14}$$

$$V_x = V_6 - V_8 \tag{15}$$

Name	Theoretical values
@Gb	0.00000000000
@r1	0
@r2	0
@r3	0
@r4	0
@r5	-2.841143934260936e-03
@r6	0
@r7	0
v(1)	0.00000000000
v(2)	0.00000000000
v(3)	0.00000000000
v(4)	0.00000000000
v(5)	0.00000000000
v(6)	8.66291649521
v(7)	0.00000000000
v(8)	0.00000000000
Ix	-0.00284114393
Vx	8.66291649521
Req	-3.049095e+03
τ	-3.081415e-03

Table 2: A variable that starts with a "V" is of type *voltage* and expressed in Volt (V). The variable R_{eq} is expressed in Ohm and the variable τ is expressed in seconds

2.3 Natural solution for V6

To calculate the natural solution for V6 we first need to simplify the circuit by removing all independent sources and aplying KVL. We then get the natural solution by the equation $V_{6n}(t) = Ae^{\frac{-t}{\tau}}$.

The process to get to this point is explained in the theoretical class nº3. τ is the time constant, calculated with the equivalent resistance from the previous section ($\tau = R_{eq} * C$) and A is taken from the boundry conditions also calculated in the last section ($t = 0, A = V_x$).

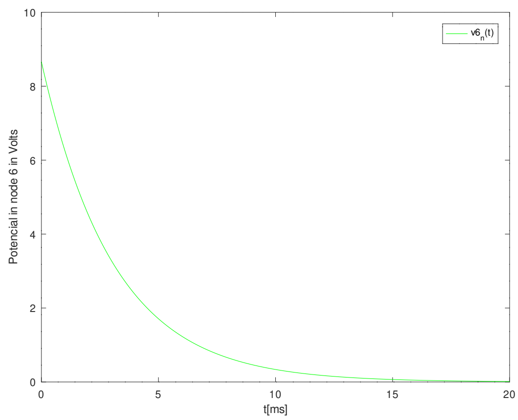

Figure 3: Natural response of V_6 as a function of time in the interval from [0,20] ms

2.4 Forced solution for V6 with f=1000Hz

Now we determine the forced solution for V_6, in the same time interval and with a frequency of 1kHz. For this, we will redo the nodal method but using impedances now, and considering that $V_s = 1$, taken from the initial condition of the source. We can determine the complex amplitudes in all nodes with the following equations:

$$Z = \frac{1}{jwC} \tag{16}$$

$$\tilde{V}_s = 1 \tag{17}$$

$$\tilde{V}_0 = 0 \tag{18}$$

$$\tilde{V}_4 = \tilde{V}_7 \tag{19}$$

$$\tilde{V}_5 - \tilde{V}_8 = K_d \frac{\tilde{V}_0 - \tilde{V}_4}{R_6} \tag{20}$$

$$\tilde{V}_1 - \tilde{V}_0 = \tilde{V}_s \tag{21}$$

$$\frac{\tilde{V}_2 - \tilde{V}_1}{R_1} + \frac{\tilde{V}_2 - \tilde{V}_5}{R_3} + \frac{\tilde{V}_2 - \tilde{V}_3}{R_2} = 0 \tag{22}$$

$$\frac{\tilde{V}_3 - \tilde{V}_2}{R_2} - K_b(\tilde{V}_2 - \tilde{V}_5) = 0 \tag{23}$$

$$\frac{\tilde{V}_1 - \tilde{V}_2}{R_1} + \frac{\tilde{V}_0 - \tilde{V}_4}{R_6} + \frac{\tilde{V}_0 - \tilde{V}_5}{R_4} = 0 \tag{24}$$

$$K_b(\tilde{V}_2 - \tilde{V}_5) + \frac{\tilde{V}_6 - \tilde{V}_5}{R_5} + \frac{\tilde{V}_6 - \tilde{V}_8}{Z} = 0 \tag{25}$$

$$\frac{\tilde{V}_4 - \tilde{V}_0}{R_6} + \frac{\tilde{V}_7 - \tilde{V}_8}{R_7} = 0 \tag{26}$$

$$\tilde{V}_x = \tilde{V}_6 - \tilde{V}_8 \tag{27}$$

The complex amplitudes of the phasors are presented in **Table 3**

Name	Complex amplitude voltage
V0	0
V1	1
V2	9.484038345673389e-01
V3	8.385425202267185e-01
V4	3.590067492545353e-01
V5	9.559396039702367e-01
V6	5.449495992733405e-01
V7	3.590067492545353e-01
V8	5.426022978184054e-01

Table 3: Complex amplitudes in all nodes in Volts

2.5 Final total solution V6(t)

In this section the final total solution V_6 for a frequency of 1KHz is determined using the natural and forced solutions determined in previous sections (V_6=V_{n6}+V_{6f}). In Figure: 4 the voltage of the independent source V_s and the voltage of V_6 were plotted for the time interval of [-5,20] ms.

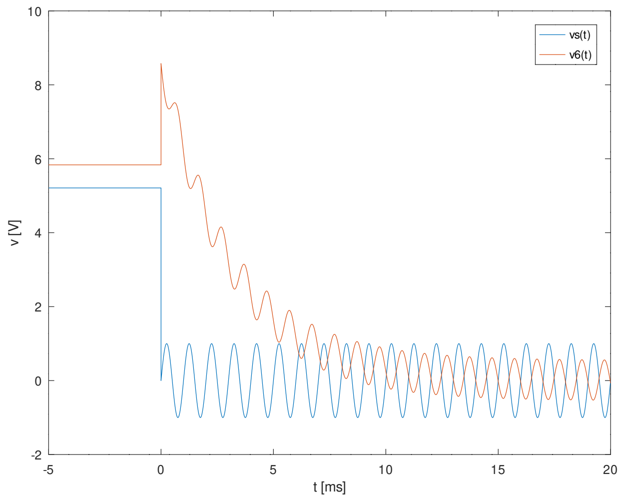

Figure 4: Voltage of $V_6(t)$ and $V_s(t)$ as functions of time from [-5,20] ms

2.6 Frequency responses $v_c(f)$, $v_s(f)$ and $v_6(f)$ for frequency range 0.1 Hz to 1 MHz

For this section, it was considered that $v_s(t) = sin(2\pi ft)$. This circuit will act aproximatly as a low pass-filter. This means when the frequency is low the capacitor has a long time to charge up, then acting as an open circuit, and achieving praticly the same voltage as the input, therefore, there is a considerable voltage drop between node 6 and 8 and the capacitor will remane almost in phase with the voltage source.

For high frequences the oposite is seen. The capacitor will have no time to charge before the input changes direction, so it will act as a short-circuit with almost no potential drop. Therefore, the capacitor will end up having a very diferent phase from the imput source as can be confirmed in the graphs bellow. This will be noted for frequences greater then the cutoff frequence ($f_c = \frac{1}{2.\pi.\tau}$). With our data f_c is close to 50Hz, this is also the section of the graphs where we can see a drop in potencial for the capacitor and a big difference in phase start to show up. Simplifying this circuit to something close to its Thevenan equivalent, using only V_{eq}, R_{eq} and a Capacitor leads to the following equations, allowing for a better understanding of the phase and magnitude decrease as the frequency increases:

$$V_c = \frac{V_s}{\sqrt{1 + (R_{eq}.C.2\pi.f)^2}} \tag{28}$$

$$\phi_{V_c} = -\frac{\pi}{2} + arctan(R_{eq}.C.2\pi.f) \tag{29}$$

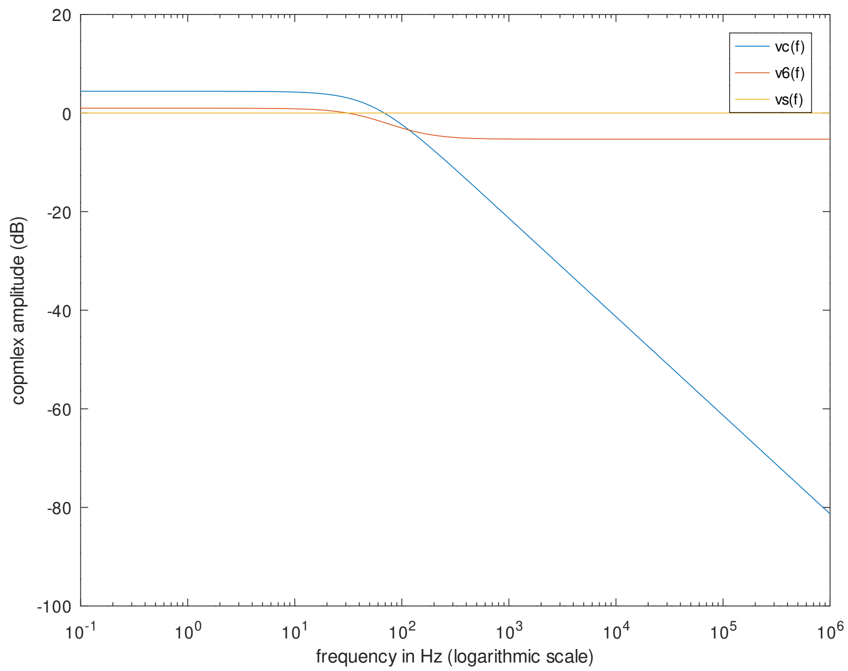

Figure 5: Graph for amplitude frequency response, in dB, of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz (logarithmic scale).

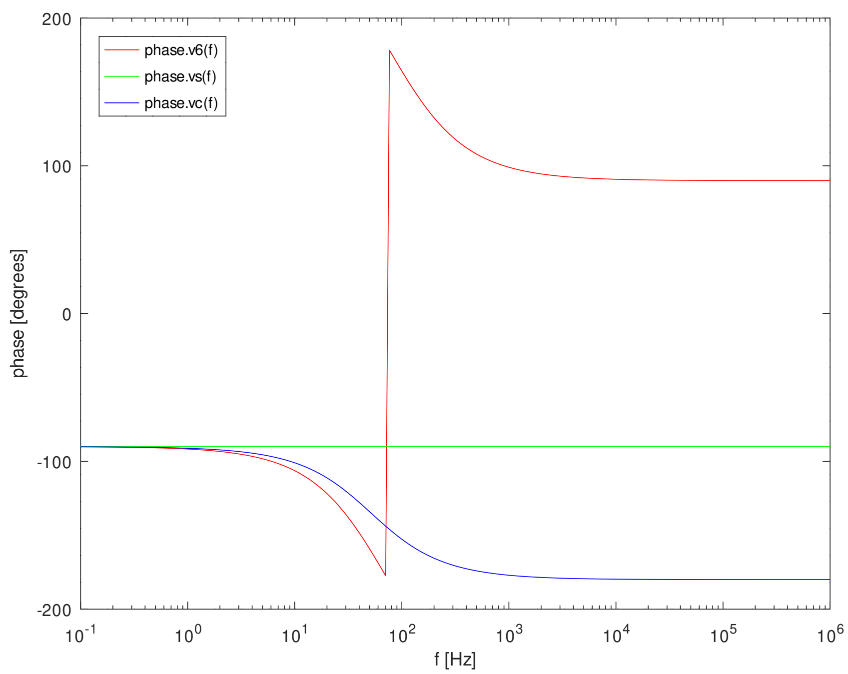

Figure 6: Graph for the phase response, in degrees of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz, displayed in a logarithmic scale. The phase of V_6 is actually continuous, and the reason for the apparent discontinuity in the graph is because of the domain of the arctan function - D_{arctan} =]-180, 180].

3 Simulation Analysis

In this section the steps needed to simulate this circuit using the software Ngspice are described. The analysis (operating point, frequency response and phase response) are the following:

- operating point for $t < 0$;

- operating point for $V_s(0) = 0$, replacing the capacitor with a a voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8 as obtained in the previous step (the reason for this is the fact that the equivalent resistance experienced by the capacitor must be calculated. Furthermore, because the energy discharge in the capacitor is continuous, it is required that the initial boundary conditions are computed);

- simulate the natural response of the circuit (using the boundary conditions V(6) and V(8) as obtained previously) using a transient analysis;

- repeating the third step, using V_s as given in **equation 2** and f = 1kHz in order to simulate for the total response on node 6

- simulate the frequency response in node 6 for a frequency range 0.1 Hz to 1MHz.

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{30}$$

with

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \tag{31}$$

3.1 Operating Point Analysis for $t < 0$

In this section, there was a need to create a Node 4, between the resistor R6 and Node 7, where an independent voltage source, V_x, providing 0V was inserted, to measure the voltage drop ($-V7$) felt by I_d. This was created in order to comply with NGSpice's requirements for defining the current controlled voltage source V_d. Table 4 shows the simulated operating point results for the circuit under analysis for $t < 0$.

Name	Value [mA or V]
@c[i]	0.000000e+00
@gb[i]	-2.80010e-04
@r1[i]	2.671903e-04
@r2[i]	2.800097e-04
@r3[i]	-1.28194e-05
@r4[i]	-1.20120e-03
@r5[i]	-2.80010e-04
@r6[i]	9.340050e-04
@r7[i]	9.340050e-04
v(1)	5.211160e+00
v(2)	4.942284e+00
v(3)	4.369779e+00
v(4)	-1.87084e+00
v(5)	4.981554e+00
v(6)	5.835330e+00
v(7)	-1.87084e+00
v(8)	-2.82759e+00

Table 4: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in milliAmpere; other variables are of type *voltage* and expressed in Volt.

3.2 Operating Point Analysis for $t = 0$

In this section the circuit is simulated using an operating point analysis with $V_s(0) = 0$ and with the capacitor replaced by a voltage source $V_x = V(6) - V(8)$, using the values obtained in the last step. This step was taken because we must compute the new initial conditions that guarantee continuity in the capacitor's discharge. Therefore, $V(6) - V(8)$ must be a continuous function, defined in branches (constant for $t < 0$ and varying in time for $t \geq 0$), as there can not be a energy discontinuity in the capacitor ($E_C = \frac{1}{2}CV^2$). However, that does not imply that that $V(6)$ and $V(8)$ are continuous functions in time. In **Table 5** the simulation results are presented.

Name	Value [mA or V and Ohm]
@gb[i]	-2.16631e-18
@r1[i]	2.067130e-18
@r2[i]	2.166308e-18
@r3[i]	-9.91776e-20
@r4[i]	4.283305e-19
@r5[i]	-2.84114e-03
@r6[i]	-4.33681e-19
@r7[i]	-8.86107e-19
v(1)	0.000000e+00
v(2)	-2.08017e-15
v(3)	-6.50938e-15
v(4)	8.686762e-16
v(5)	-1.77636e-15
v(6)	8.662916e+00
v(7)	8.686762e-16
v(8)	1.776357e-15
Ix	-2.84114e-03
Vx	8.662916e+00
Req	-3.04909e+03

Table 5: Operating point for $v_s(0) = 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; variables are of type *voltage* and expressed in Volt. The equivalent resistance is in Ohms

3.3 Natural solution for V_6 using transient analysis

In this section the natural response of the circuit in the interval [0,20] ms was studied using a transient analysis simulation. Using the previous simulations, the initial conditions for V(6) and V(8) were defined, using NGSpice's diretive *.ic*.

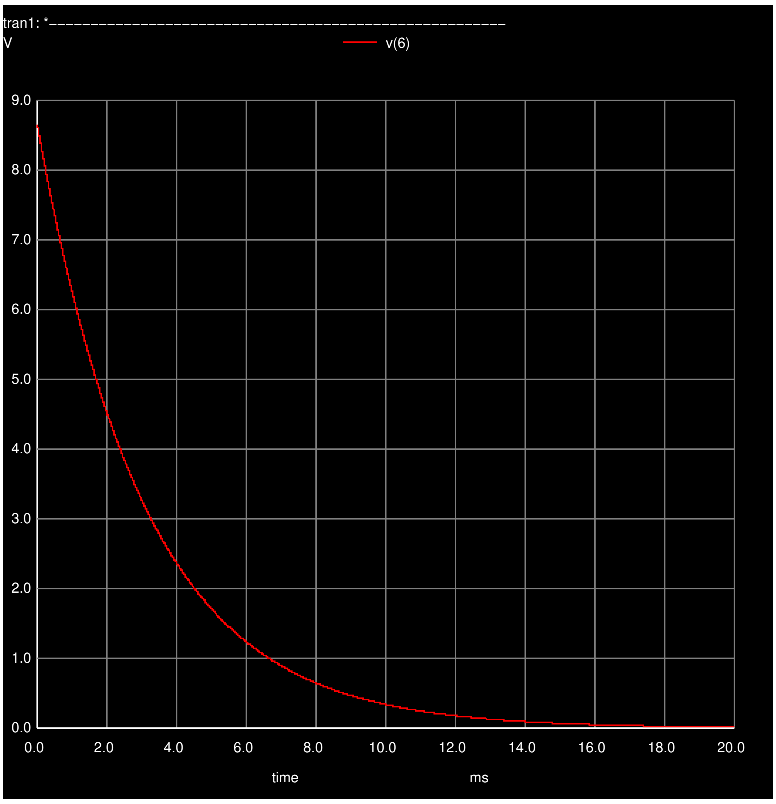

Figure 7: Simulated natural response of $V_6(t)$ in the interval [0,20] ms. The *x axis* represents the time in milliseconds and the *y axis* the Voltage in node 6 in Volts.

3.4 Total solution for V_6 using transient analysis

In this section the total response of V_6 (natural + forced) is simulated using NGSpice's transient analysis capabilities. This is done by repeating the previous section, but using V_s as given in 2 and f = 1kHz.

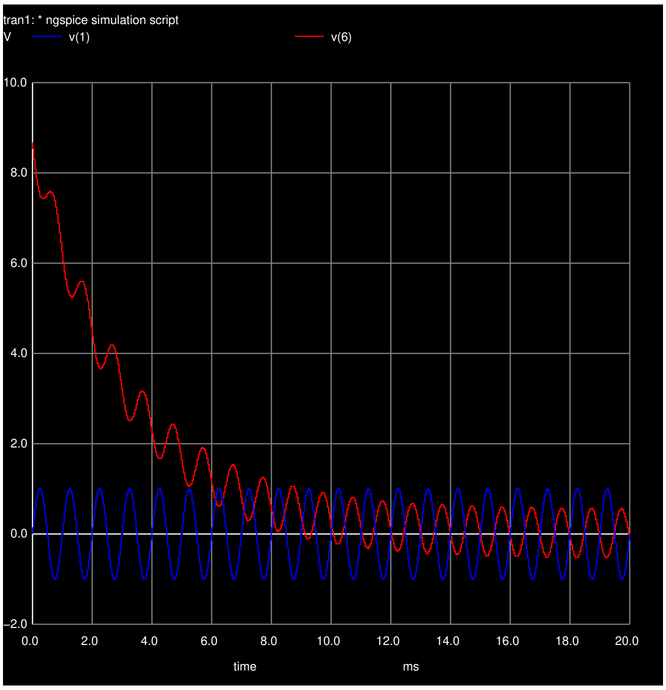

Figure 8: Simulated response of $V_6(t)$ and of the stimulus $V_s(t)$ as functions of time from [0,20] ms. The *x axis* represents the time in milliseconds and the *y axis* the Voltage in Volts.

3.5 Frequency response in node 6

In this section, the frequency response in node 6 is simulated for the frequency range from 0.1 Hz to 1 MHz, along with the phase response of the circuit. The reasons of how and why $V_6(t)$ and $V_s(t)$ differ have been covered in **subsection 2.6**.

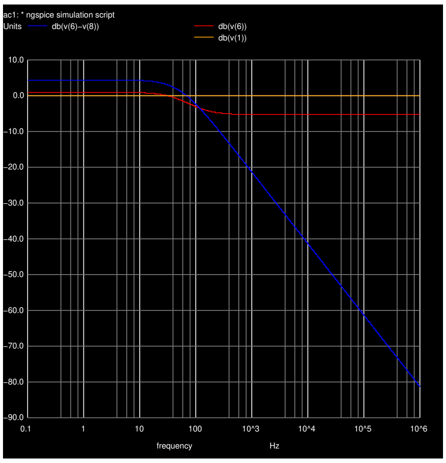

Figure 9: Magnitude of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in Hz, using a logarithmic scale and the *y axis* the magnitude in dB.

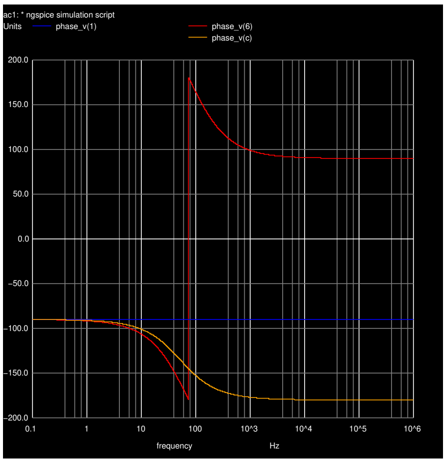

Figure 10: Phase of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in HZ, using a logarithmic scale and the *y axis* the phase in degrees.

4 Conclusion

In conclusion, after comparing the data obtained on the simulation and with the mathematical tools we can see a almost complete match with only some minor diferences in the last digit. The difference can be despised due to its insignificance because both tools use the same methods to solve the circuit so an equal output it's expected.