Circuit Theory and Electronics Fundamentals

T5 Laboratory Report

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Group 61

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1 Introduction

In this laboratory assignment we analysed, theoreticaly and also in regards to simulation, a Pass-band Filter using an Operational Amplifier (OPAMP). This was composed by three stages: a High Pass stage, with the goal of passing high frequencie signals and cutting the ones with a low frequencie, and Amplification stage, where, using the OPAMP, the signals were amplified, and a Low Pass Stage that cuts off high frequency signals and allows the ones with low frequency to pass. For the OPAMP, we chose the model μA741 from Texas Instruments. In this report we will do first the theoretical analysis, followed by the simulation using NGSpice in 2. In the end of this section we will compare the simulation and the theoratical analysis. The conclusions are presented in 4. The stated circuit is presented in 1.

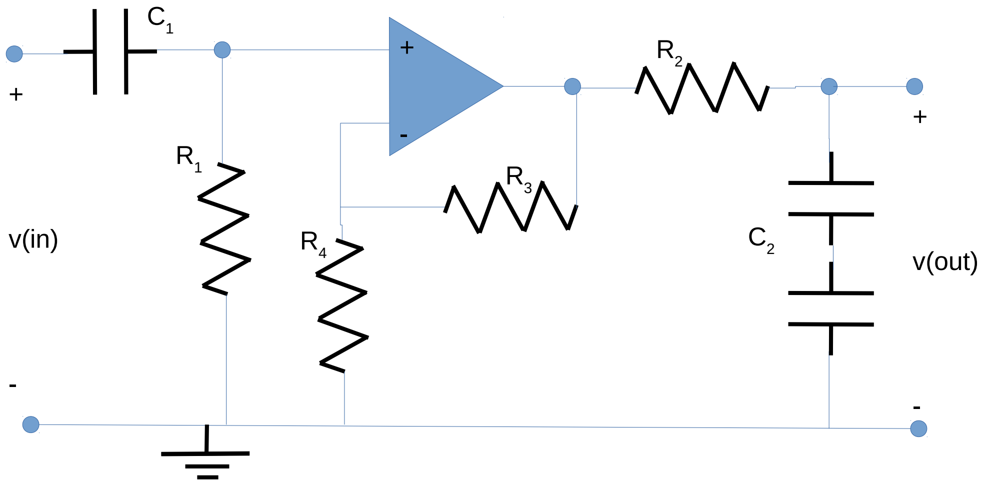


Figure 1: Active Passband Filter

2 Theoretical and Simulation Analysis

In this stage we are going to compare the theoretical and simulation analysis, but first we are going to present each part of the circuit by its self and discuss how it works. The constant values in the table below were the ones we used for this activity.

Name	Value	Units
R_1	1000	Ω [Ohms]
R_2	1000	Ω [Ohms]
R_3	100000	Ω [Ohms]
R_4	1000	Ω [Ohms]
C_1	220	ηF [ηFarads]
C_2	110	ηF [ηFarads]

Table 1: Constants Values

2.1 High Pass Stage

The first segment of the circuit woeks as a voltage divider, so we can write the equation:

$$v_{HP} = v_{R_1} = \frac{R_1}{R1 + \frac{1}{j\omega C_1}} v_i \tag{1}$$

or

$$A_{HP} = \frac{v_{R_1}}{v_{in}} = \frac{jR_1\omega C_1}{jR_1\omega C_1 + 1} \tag{2}$$

For high frequencies ($\omega >> 1$), the voltage at the resistor tends to v_i and for low frequencies ($\omega << 1$), it tends to 0. This is a simple RC Series circuit, behaving as a high pass filter.

2.2 Amplification Stage

In this stage the signal is going to be amplefied. It is where the gain is maximum. This configuration uses a Non-Inverting Amplifier, and the gain is given by:

$$AV_{AMP} = 1 + \frac{R_3}{R_4} \tag{3}$$

2.3 Low Pass Stage

In this last stage, we have a circuit similar to the the High Pass one, except we are interested in the voltage of the capacitor to cut high frquences.

With the voltage divider law we can write:

$$v_{LP} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + R_2} v_{amp} \tag{4}$$

or simplified:

$$A_{LP} = \frac{v_{LP}}{v_{amp}} = \frac{1}{1 + j\omega C_2 R_2} \tag{5}$$

For high frequencies ($\omega >> 1$), the voltage at the capacitor tends to 0 and for low frequencies ($\omega << 1$), it tends to v_{amp}.

When combining all the stages, we end up with the final Active Pass-band filter as seen in Figure 1.

2.4 Theoretical Analysis

On the Theoretical side, we will evaluate the central frequency (f_o), the two impedances, input and output, respectively (Z_I and Z_O), and the three gains associated with each stage (AV_{HP},AV_{amp} and AV_{LP}).

2.4.1 Central Frequency

The Central Frequency (f_o) is the frequency at the center of the passband of the filter. We want $f_o = \omega_o/2\pi$ to be as close to 1kHz as possible.

The Lower Cut-off Frequency (ω_L), the Higher Cut-off Frequency (ω_H) and the Central Frequency are given by:

$$\omega_L = \frac{1}{R_1C_1} \qquad \omega_H = \frac{1}{R_2C_2} \qquad \omega_o = \sqrt{w_L * w_H} = \sqrt{\frac{1}{R_1C_1R_2C_2}} \tag{6}$$

If both frequencies, f_L and f_H, are equal, f_o is going to be also equal and the passband width is going to be 0 - in the plot, we can see a peak instead of a highland.

The theoretical values of the lower cut-off frequency, the high cut-off frequency, and the central frequency are presented in the table below and will be better analysed and compared in Section 2.6.

Name	Value [Hz]
f_L	723.431560
f_H	1446.863119
f_O	1023.086723

Table 2: Frequencies (Lower, Higher and Central)

2.4.2 Impedances

We have the input and output impedances and we can write the following equations:

$$Z_I = R_1 + Z_{C_1} = R_1 + \frac{1}{j\omega C_1} \qquad Z_O = Z_{C_2}||R_2 = \frac{1}{j\omega C_2}||R_2 \tag{7}$$

Below, we present the values of these impedances, for 1kHz,

Name	Value [Hz]
Z_{in}	1000.000000-723.431560i
Z_{out}	676.732451-467.723894i

Table 3: Impedances

It is important to have a high input impedance to have a minimum degradation of the input signal. On the other hand, the output impedance needs to be as low as possible. Our results regarding the output impedance of the circuit are not close to being good, however, because of the limitations of the components we could't reduce them.

2.4.3 Gain

In order to calculate the total gain (A_V) we performed a simple multiplication, so that we have $A_{V_{HP}} A_{V_{amp}} A_{V_{LP}}$.

The Transfer Function is given by,

$$T(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \left(1 + \frac{R_3}{R_4}\right) \frac{1}{1 + R_2 C_2 s} \tag{8}$$

where $s = j\omega + \sigma$ and $\sigma = 0$ (for sinusoidal waves).

We can identify $T(s)$ to be the multiplication of $T_i(s) \equiv$ transfer function of the $i - th$ stage.

When substituting the frequency, $f = \frac{\omega}{2\pi} = 1000Hz$, we have the results below,

Name	**Value [dB]**
AV_{HP}	-1.828006
AV_{Amp}	40.086427
AV_{LP}	-1.695830
AV	36.562591

Table 4: Theoretical Gains

2.5 Simulation Analysis

In regards to the simulation analysis, using the software NGSpice we can get the values of Z_I, Z_O, f_O and A_v. The tables with the simulated data are presented below.

Name	Value [Ω]
Input Imp	999.991 + -723.534 j

Table 5: Experimental Input Impedance

Name	Value [Ω]
Output Imp	680.05 + -466.901 j

Table 6: Experimental Output Impedance

Name	Value	Units
AV	36.5323	dB
fO	1013.91	Hz

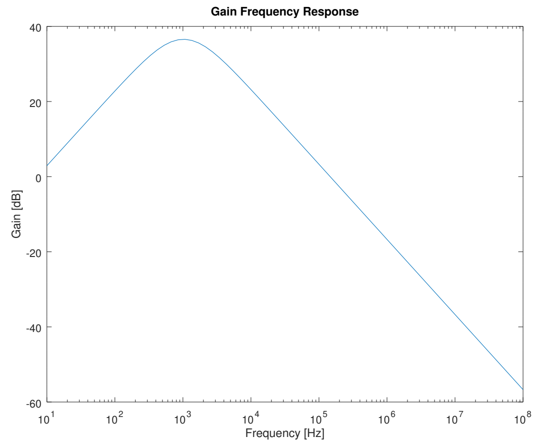
Table 7: Experimental Gain[1]and Central Frequency

2.6 Comparison

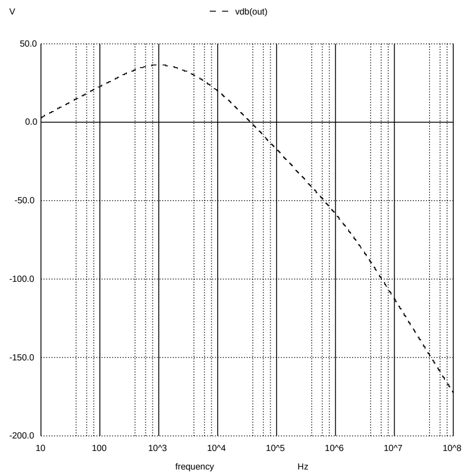
We will now compare the results obtained by the two approaches and, in order to do that, we present the graphs of the frequency response of the gain and the phase from both the theoretical and simulation analysis.

In general we achieved very satisfactory results. Looking to the gain response graphs, we can perceive similarities in the shapes, beeing easy to realise that both have similar behaviours. In terms of the phase response, both plots differ. This is due a fault in our theoretical model wich doesn't predict the existence of capacitors in the OPAMP, idealising its gain to be purely real, where there's no shift in phase. When it comes to the tables presented above, we have small relative errors for the impedances, total gain and the central frequency. This shows that from a fairly simple model, we can predict the OPAMP behaviour, even though some of the elements are non-linear.

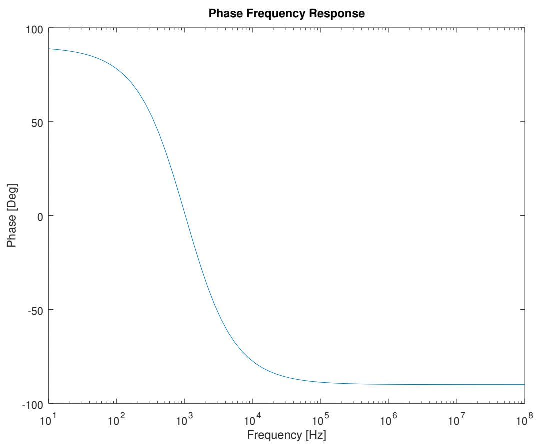
[1]Measured at 1000Hz



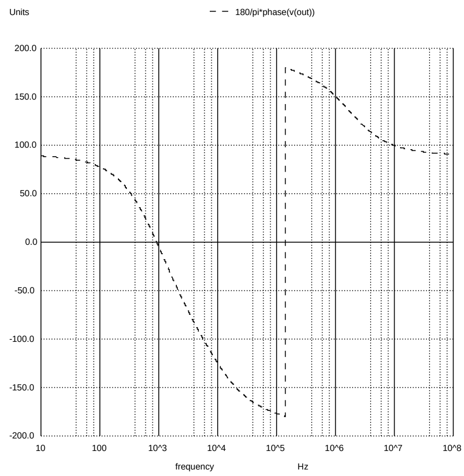
(a) Theoretical Gain Response



(b) Simulation Gain Response



(c) Theoretical Phase Response



(d) Simulation Phase Response

Figure 2: Frequency Response

3 Merit Results

From the results obtained through the *Ngspice* simulation (see Section 2.6) and considering we used the data shown in Section 1, we can compute the price and the merit using the *formulae* given in the lab assignment:

Name	**Value**	**Units**
AV dev	32.9164	
Freq dev	13.911	Hz
Cost	13426.8	MU
Merit	1.59048E-06	

Table 8: Merit Results

4 Conclusion

As a conclusion, we can state that, unlike the previous two lab assignments, there is a degree of similarity between both analysis, in terms of precision. This was rather surprising, for we are dealing with a high-complexity component - the OPAMP - at least in terms of architecture. But, when we analyse it in a macroscopical point of view, it's rather predictable and stable - at least in this circuit.

Overall, the laboratory gave us the opportunity to deepen our knowledge with OPAMPs and Active 2nd-order filters. We understood it as an extension of the second lab, where we analysed 1st-order RC circuits.

It is also worth noting that the concept of the merit figure proves itself to be extremely important, because it narrows the gap between an academic point of view and an industrial/engineering approach. It made us understand that there are many factors involved in the construction of a circuit, some of them being the cost, the space availability and the customers' needs. Another limiting factor was the fixed values and quantities for each of the components, which made us have to think more thoroughly and inventively about how and where we would use them. A complete circuit that takes into account all these factors is most of the times really hard to achieve, which leads us to sacrifice some characteristics in favour of some more important others, achieving a balance between them all.